4 Leveling Theory, Methods, and Equipment

4.1 Define the following leveling terms: (**a**) vertical control, (**b**) elevation, and (**c**) vertical datum.

From Section 4.2:

(a) A series of benchmarks or other point of known elevation established throughout an area.

(b) The distance along a vertical line from a vertical datum to a point or object.

(c) Any level surface to which elevations are referenced.

4.2* How far will a horizontal line depart from the Earth's surface in 1 km? 5 km? 10 km? (Apply both curvature and refraction)

1 km? $C_m = 0.0675(1)^2 = 0.068$ m

5 km? $C_m = 0.0675(5)^2 = 1.688$ m

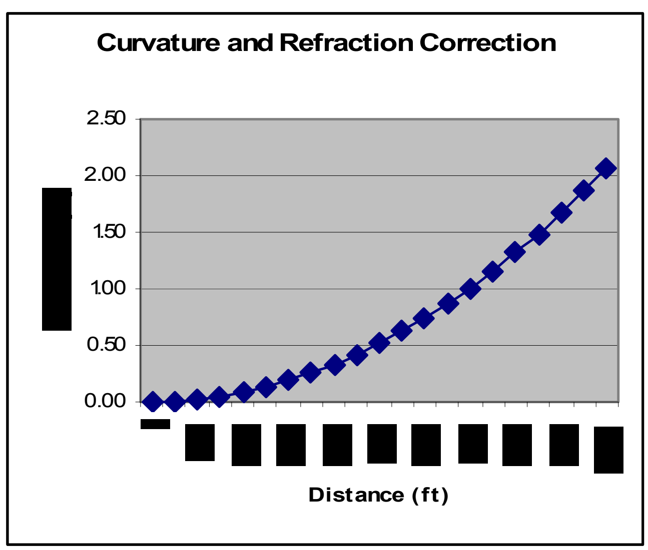
10 km? $C_m = 0.0675(10)^2 = 6.750$ m

4.3 Visit the website of the National Geodetic Survey, and obtain a data sheet description of a benchmark in your local area.

Solutions should vary.

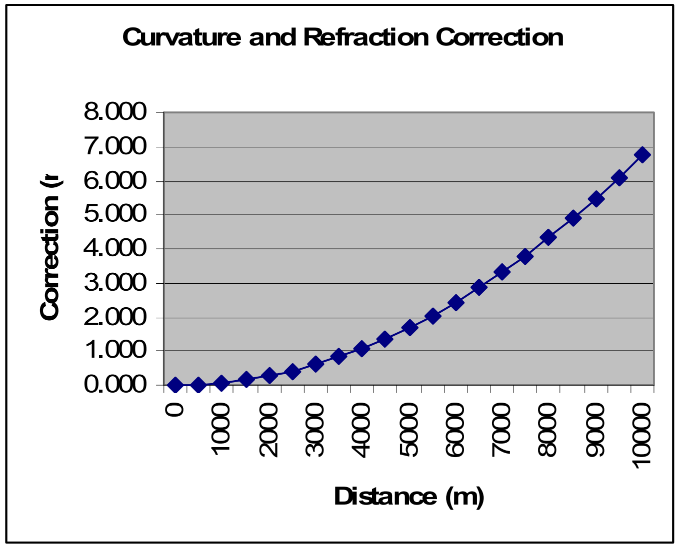
4.4 Create plot of the curvature and refraction correction for sight lines going from 0 ft to 10,000 ft in 500 ft increments.

Sight (ft)	CR (ft)
0	0.00
500	0.01
1000	0.02
1500	0.05
2000	0.08
2500	0.13
3000	0.19
3500	0.25
4000	0.33
4500	0.42
5000	0.52



4.5 Create a plot of curvature and refraction corrections for sight lines going from 0 m to 10,000 m in 500 m increments.

Sight (m)	CR (m)
0	0.000
500	0.017
1000	0.068
1500	0.152
2000	0.270
2500	0.422
3000	0.608
3500	0.827
4000	1.080
4500	1.367
5000	1.688



4.6 Why is it important for a benchmark to be a stable, relatively permanent object?

It is important for a benchmark to be a stable, relatively permanent object so that is elevation will remain constant and useable over a long period of time.

4.7* On a large lake without waves, how far from shore is a sailboat when the top of its 30-ft mast disappears from the view of a person lying at the water's edge?

$$F = 1000\sqrt{\frac{30}{0.0206}} = 38{,}161 \text{ ft} = 7.228 \text{ mi}$$

4.8 Similar to Problem 4.7, except for a 10-m mast and a person whose eye height is 1.5 m above the water's edge.

$$K = \sqrt{\frac{1.5}{0.0675}} + \sqrt{\frac{10}{0.0675}} = 16.886 \text{ km}$$

4.9 Readings on a line of differential levels are taken to the nearest 2 mm. For what maximum distance can the Earth's curvature and refraction be neglected?

$$F = 1000\sqrt{\frac{0.001}{0.0675}} = 121.7 \text{ m}$$

4.10 Similar to Problem 4.9 except readings are to the 0.02 ft.

$$m = 1000\sqrt{\frac{0.01}{0.0206}} = 696.7 \text{ ft}$$

4.11 Describe how readings are determined in a digital level when using a bar coded rod.

From Section 4.11:

"At the press of a button, the image of bar codes in the telescope's field of view is captured and processed. This processing consists of an on-board computer comparing the captured image to the rod's entire pattern, which is stored in memory. When a match is found, which takes about 4 sec, the rod reading is displayed digitally."

Successive plus and minus sights taken on a downhill line of levels are listed in Problems 4.12 and 4.13. The values represent the horizontal distances between the instrument and either the plus or minus sights. What error results from curvature and refraction?

4.12* 20, 225; 50, 195; 40, 135; 30, 250 ft.

Plus	CR	Minus	CR
20	0.00000824	225	0.001043
50	0.0000515	195	0.000783
40	0.00003296	135	0.000375
30	0.00001854	250	0.001288
Sum	0.00011124		0.003489

Combined **−0.003 ft**

4.13 30, 55; 30, 50; 25, 45; 55, 60 m.

Plus	CR (mm)	Minus	CR (mm)
30	0.06075	55	0.204188
30	0.06075	50	0.16875
25	0.042188	45	0.136688
55	0.204188	60	0.243
	0.367875		0.75625

Combined −0.38 mm

4.14 What error results if the curvature and refraction correction is neglected in trigonometric leveling for sights: (**a**) 2000 ft long (**b**) 1000 m long (**c**) 3000 ft long?

(**a**) $h_f = 0.0206\left(\frac{2000}{1000}\right)^2 = 0.082$ ft

(**b**) $h_m = 0.0675\left(\frac{1000}{1000}\right)^2 = 0.068 \text{ m} = 68 \text{ mm}$

(**c**) $h_f = 0.0206\left(\frac{3000}{1000}\right)^2 = 0.18$ ft

4.15* The slope distance and zenith angle observed from point *P* to point *Q* were 2013.875 m and $95°13'04''$, respectively. The instrument and rod target heights were equal. If the elevation of point *P* is 188.988 m, above datum, what is the elevation of point *Q*?

$$Elev_Q = 188.988 + 2013.875\cos(95°13'04'') + 0.0675\left(\frac{2013.875\sin(95°13'04'')}{1000}\right)^2$$

$$\underline{Elev_Q = 188.988 - 183.145 + 0.271 = \mathbf{6.114\ m}}$$

4.16 The slope distance and zenith angle observed from point *X* to point *Y* were 1501.85 ft and $86°27'15''$. The instrument and rod target heights were equal. If the elevation of point *X* is 102.09 ft above datum, what is the elevation of point *Y*?

$$Elev_Y = 102.09 + 1501.85\cos\left(86°27'15''\right) + 0.0206\left(\frac{1501.85\sin\left(86°27'15''\right)}{1000}\right)^2$$

$$= 102.09 + 92.885 + 0.046$$

$$= 195.02 \text{ ft}$$

4.17 Similar to Problem 4.15, except the slope distance was 606.430 m, the zenith angle was $95°14'44''$, and the elevation of point *P* was 908.884 m above datum.

$$Elev_Q = 908.884 + 606.430\cos\left(95°14'44''\right) + 0.0675\left(\frac{606.430\sin\left(95°14'44''\right)}{1000}\right)^2$$

$$= 908.884 - 55.4425 + 0.0246$$

$$= 853.466 \text{ m}$$

4.18 In trigonometric leveling from point *A* to point *B*, the slope distance and zenith angle measured at *A* were 7929.464 m and $88°42'50''$ At *B* these measurements were 7929.473 m and $91°17'16''$ respectively. If the instrument and rod target heights were equal, calculate the difference in elevation from *A* to *B*.

$$Z_{avg} = \frac{88°42'50'' + 180° - 91°17'16''}{2} = 88°08'22''$$

$$S_{avg} = \frac{7929.464 + 7929.473}{2} = 7929.4685$$

$$\Delta Elev = 7929.7685\cos\left(88°42'47''\right) = 178.092$$

4.19 Describe how parallax in the viewing system of a level can be detected and removed.

From Section 4.7:

"After focusing, if the cross hairs appear to travel over the object sighted when the eye is shifted slightly in any direction, *parallax* exists. The objective lens, the eyepiece, or both must be refocused to eliminate this effect if accurate work is to be done."

4.20 What is the sensitivity of a level vial with 2-mm divisions for: (**a**) a radius of 40.4 m (**b**) a radius of 20.6 m?

(**a**) $\theta = \left[\frac{2}{40.4\left(1000\right)}\right]206264.8 = 10.03$

(b) $\theta = \left[\dfrac{2}{20.6(1000)}\right]206264.8 = 20.0''$

4.21* An observer fails to check the bubble, and it is off two divisions on a 500-ft sight. What error in elevation difference results with a 10-sec bubble?

angular error = 2(10) = 20 sec

Error = 250 tan(20) = **0.048 ft**

4.22 An observer fails to check the bubble, and it is off two divisions on a 200-m sight. What error results for a 10-sec bubble?

angular error = 2(10) = 20 sec

Error = 200 tan(20) = **0.019 m**

4.23 Similar to Problem 4.22, except a 20-sec bubble is off three divisions on a 300-ft sight.

angular error = 3(20) = 60 sec

Error = 300 tan(60) = **0.087 ft**

4.24 With the bubble centered, a 100-m-length sight gives a reading of 1.352 m. After moving the bubble four divisions off center, the reading is 1.410 m. For 2-mm vial divisions, what is: (**a**) the vial radius of curvature in meters (**b**) the angle in seconds subtended by one division?

Δrdg = 1.410 − 1.352 = 0.058 m

$$4\theta = \text{atan}\left(\frac{0.058}{100}\right) = 120''$$

(a) $R = 0.002/\tan(120'') = \mathbf{3.438\ m}$

(b) $120''/4 = \mathbf{30''}$

4.25 Similar to Problem 4.24, except the sight length was 300 ft, the initial reading was 5.132 ft, and the final reading was 5.190 ft.

Δrdg = 5.190 – 5.132 = 0.0.058 ft

$$4\theta = \text{atan}\left(\frac{0.058}{300}\right) = 40''$$

(a) $R = 0.002/\tan(40'') = \mathbf{10.31\ ft}$

(b) $40''/4 = \mathbf{10''}$

4.26 Sunshine on the forward end of a $20''/2\text{ mm}$ level vial bubble draws it off 1-1/2 divisions, giving a plus sight reading of 4.63 ft on a 200-ft sight. Compute the correct reading.

Correction = 200 tan(1.5·20″) = 0.0.029 ft

Correct reading = 4.63 − 0.029 = **4.60 ft**

Note: the correction is subtracted since the bubble was drawn off on the forward end of the level, thus raising the line of sight.

4.27 List in tabular form, for comparison, the advantages and disadvantages of an automatic level versus a digital level.

See Section 4.10 and 4.11.

4.28* If a plus sight of 3.54 ft is taken on BM *A*, elevation 850.48 ft, and a minus sight of 7.84 ft is read on point *X*, calculate the HI and the elevation of point *X*.

HI = 850.48 + 3.54 = **854.02 ft**

Elev = 854.02 − 7.84 = **846.18 ft**

4.29 If a plus sight of 1.097 m is taken on BM *A*, elevation 305.348 m, and a minus sight of 0.832 m is read on point *X*, calculate the HI and the elevation of point *X*.

HI = 305.348 + 1.097 = **306.445 m**

Elev = 607.834 − 0.468 = **305.613 m**

4.30 Similar to Problem 4.28, except a plus sight of 3.36 ft is taken on BM *A*, elevation 1265.58 ft, and a minus sight of 6.32 ft read on point *X*.

HI = 1265.58 + 3.36 = **1268.94 ft**

Elev = 1268.94 – 6.32 = **1262.62 ft**

4.31 Describe the procedure used to test if the level vial is perpendicular to the vertical axis of the instrument.

See Section 4.15.5

4.32 A horizontal collimation test is performed on an automatic level following the procedures described in Section 4.15.5. With the instrument setup at point 1, the rod reading at *A* was 3.886 ft, and to *B* it was 3.907 ft. After moving and leveling the instrument at point 2, the rod reading to *A* was 4.094 ft and to *B* was 4.107 ft. What is the collimation error of the instrument and the corrected reading to *A* from point 2?

$$\varepsilon = \frac{3.907 - 3.886 - 4.107 + 4.094}{2} = 0.004 \text{ ft}$$

Correct reading at *A* = 4.094 – 2(0.004) = **4.086 ft**

4.33 The instrument tested in Problem 4.32 was used in a survey immediately before the test where the observed elevation difference between two benchmarks was +23.78 ft. The sum of the plus sight distances between the benchmarks was 560 ft and the sum of the minus sight distances was 1210 ft. What is the corrected elevation difference between the two benchmarks?

+23.81 ft; = 23.78 + 0.004/100(560 − 1210) = 23.806

4.34 Similar to Problem 4.32 except that the rod readings are 1.894 m and 1.923 m to *A* and *B*, respectively, from point 1, and 1.083 m and 1.100 m to *A* and *B*, respectively, from point 2. The distance between the points in the test was 100 m.

$$\varepsilon = \frac{1.923 - 1.894 - 1.100 + 1.083}{2} = 0.006 \text{ m}$$

Correct reading at A = 1.083 – 2(0.006) = **1.071 m**

4.35 The instrument tested in Problem 4.34 was used in a survey immediately before the test where the observed elevation difference between two benchmarks was −13.068 m. The sum of the plus sight distances between the benchmarks was 1540 m and the sum of the minus sight distances was 545 m. What is the corrected elevation difference between the two benchmarks?

−13.128 m; = −13.068 – 0.006/100(1540 − 545)