7 ANGLES, AZIMUTHS, AND BEARINGS

Asterisks (*) indicate problems that have partial answers given in Appendix G.

7.1 Define the different reference meridians that can be used for the direction of a line.

See Section 7.4.

Geodetic or true: Reference to geodetic north, which is referenced to the average position of the poles between 1900.0 and 1905.0

Astronomic: Reference meridians taken from instantaneous position of Earth's pole, which wanders over time.

Magnetic: Reference meridian taken from current position of magnetic poles.

Grid: Reference meridian chosen from some map projection system such as the state plane coordinate system where all meridians are parallel to the central meridian of the projection.

Record or deed: Reference meridian selected from a recorded deed by using the value given for one line in the deed.

Assumed: A value of 0° arbitrarily assigned to a line on the ground.

7.2 List the three basic requirements in determining an angle.

From Section 7.1, paragraph 2:

"…they are (1) *reference* or *starting line*, (2) *direction of turning*, and (3) *angular distance* (value of the angle)."

7.3 Why is it important to adopt a standard angle measuring procedure, such as always measuring angles to the right?

From Section 7.3, paragraph 3: "*To avoid this confusion, it is recommended that a uniform procedure of always observing angles to the right be adopted*, and the direction of turning noted in the field book with a sketch."

7.4 What is the relationship of a forward and back azimuth?

See Section 7.5, paragraph 2: "A line's forward direction can be given by its *forward* azimuth, and its reverse direction by its *back* azimuth. In plane surveying, forward azimuths are converted to back azimuths, and vice versa, by adding or subtracting 180°."

7.5 Convert: ***(a)** 203°26′48″ to grads **(b)** 2.341539 radians to degrees, minutes, and seconds **(c)** 43°38′05″ to radians.

(a) 226.0518 grad

(b) 134°09'37"

(c) 0.761570 rad

In Problems 7.6 through 7.7, convert the azimuths from north to bearings, and compute the angles, smaller than 180° between successive azimuths.

7.6 43°00′36″, 141°25′34″, 230°12′20″, and 330°35′48″

Bearings	Angles
N43°00'36"E	98°24'58"
S38°34'26"E	88°46'46"
S50°12'20"W	100°23'28"
N29°24'12"W	72°24'48"

7.7 98°12′55″,153°26′40″,192°56′22″, and 288°12′50″

Bearings	Angles
S81°47'05"E	55°13'45"
S26°33'20"E	39°29'42"
S12°56'22"W	95°16'28"
N71°47'10"W	170°00'05"

Convert the bearings in Problems 7.8 through 7.9 to azimuths from north and compute the angle, smaller than 180°, between successive bearings.

7.8 N44°50′38″E, S38°42′54″E, S45°06′02″W, and N13°24″30″W

Azimuths	Angles
44°50'38"	96°26'28"
141°17'06"	83°48'56"
225°06'02"	121°29'28"
346°35'30"	58°15'08"

7.9 N32°42′38″E, S54°02′02″E, S22°42′56″W, and N44°35′26″W

Azimuths	Angles
32°42'38"	93°15'20"
125°57'58"	76°44'58"
202°42'56"	112°41'38"
315°24'34"	77°18'04"

Compute the azimuth from north of line CD in Problems 7.10 through 7.12. (Azimuths of AB are also from north.)

***7.10** Azimuth $AB = 101°26'32"$; angles to the right $ABC = 50°54'26", BCD = 38°36'38"$.

Az_{CD} = **<u>190°57'36"</u>**; Az_{BC} = 332°20'58"

7.11 Bearing $AB = S74°26'12"E$; angles to the right $ABC = 98°20'06", BCD = 104°21'08"$.

Brg_{CD} = **<u>N51°44'58"W</u>**; Brg_{BC} = N23°53'54"E

7.12 Azimuth $AB = 275°32'20"$; angles to the right $ABC = 66°36'10", BCD = 82°16"24"$.

Az_{CD} = **<u>64°24'54"</u>**; Az_{BC} = 162°08'30"

***7.13** For a bearing $DE = N08°53'56"W$ and angles to the right, compute the bearing of *FG* if angle $DEF = 88°12"29"$ and $EFG = 40°20'30"$.

Brg_{FG} = **<u>S60°20'57"E</u>**; Brg_{EF} = S79°18'33"W

7.14 Similar to Problem 7.13, except the azimuth of *DE* is $12°02'18"$ and angles to the right *DEF* and *EFG* are $21°44'52"$ and $86°10'14"$, respectively.

Az_{FG} = **<u>119°57'24"</u>**; Az_{EF} = 213°47'10"

Course AB of a five-sided traverse runs due north. From the given balanced interior angles to the right, compute and tabulate the bearings and azimuths from north for each side of the traverses in Problems 7.15 through 7.17.

7.15 $A = 82°13'15", B = 106°35'18", C = 28°45'06", D = 205°14'56", E = 117°11'25"$

Course	Bearing	Azimuth
AB	Due North	0°00'00"
BC	N73°24'42"W	286°35'18"
CD	S44°39'36"E	135°20'24"
DE	S19°24'40"E	160°35'20"
EA	S82°13'15"E	97°46'45"

***7.16** $A = 90°29'18", B = 107°54'36", C = 104°06'37", D = 129°02'57", E = 108°26'32"$

Course	Bearing	Azimuth
AB	Due North	0°00'00"
BC	N72°05'24"W	287°54'36"
CD	S32°01'13"W	212°01'13"
DE	S18°55'50"E	161°04'10"
EA	N89°30'42"	89°30'42"

7.17 $A = 156°23'48", B = 41°37'02", C = 94°30'15", D = 154°11'50", E = 93°17'05"$

Course	Bearing	Azimuth
AB	Due North	0°00'00"
BC	S41°37'02"W	221°37'02"
CD	S43°52'43"E	136°07'17"
DE	S69°40'53"E	110°19'07"

EA	N23°36'12"E	23°36'12"

In Problems 7.18 and 7.19, compute and tabulate the azimuths of the sides of a regular pentagon (polygon with five equal angles), given the starting direction of side AB.

7.18 Bearing of $AB = \text{N}37°26'05''\text{E}$ (Station C is westerly from B.)

Course	Azimuths
AB	37°26'05"
BC	337°26'05"
CD	277°26'05"
DE	217°26'05"
EF	157°26'05"
FA	37°26'05"

7.19 Azimuth of $AB = 207°53'14''$ (Station C is westerly from B.)

Course	Azimuths
AB	207°53'14"
BC	147°53'14"
CD	87°53'14"
DE	27°53'14"
EF	327°53'14"
FA	267°53'14"

7.20 Azimuth of $AB = 202°02'00''$ (Station *C* is westerly from *B*.)

Course	Azimuths
AB	202°02'00"
BC	142°02'00"
CD	82°02'00"
DE	22°02'00"
EF	322°02'00"
FA	262°02'00"

Compute azimuths of all lines for a closed traverse ABCDEFA that has the following balanced angles to the right, using the directions listed in Problems 7.21 and 7.22.
$FAB = 118°26'59'', ABC = 123°20'28'', BCD = 104°10'32'', CDE = 133°52'50'',$
$DEF = 108°21'58'', \ EFA = 131°47'13''.$

7.21 Bearing $AB = \text{N}88°18'42''\text{W}.$

Course	Azimuths
AB	271°41'18"
BC	215°01'46"
CD	139°12'18"

DE	93°05'08"
EF	21°27'06"
FA	333°14'19"

7.22 Azimuth $DE = 36°10'20''$.

Course	Azimuths
AB	214°46'30"
BC	158°06'58"
CD	82°17'30"
DE	36°10'20"
EF	324°32'18"
FA	276°19'31"

7.23 Similar to Problem 7.21, except that bearings are required, and fixed bearing $AB = \text{S}44°46'25''\text{E}$.

Course	Bearings
AB	S44°46'25"E
BC	N11°53'07"W
CD	N87°42'35"W
DE	S46°10'15"W
EF	S25°27'47"E
FA	S73°40'34"E

7.24 Similar to Problem 7.22, except that bearings are required, and fixed azimuth $DE = 206°22'40''$ (from north).

Course	Bearings
AB	S26°22'40"E
BC	N45°15'22"W
CD	S86°31'51"W
DE	S24°58'50"W
EF	S31°40'42"E
FA	N72°29'50"E

7.25 Geometrically show how the sum of the interior angles of a pentagon (five sides) can be computed using the formula $(n-2)180°$?

A sketch showing that a pentagon can be divided into three triangles each of which as a sum of angles of 180°.

7.26 Determine the predicted declinations on January 1, 2013 using the WMM-10 model at the following locations.

(a)* latitude = 42°58′28″N, longitude = 77°12′36″W, elevation = 310.0 m; **11.8° W**

(b) latitude = 37°56′44″N, longitude = 110°50′40″W, elevation = 1500 m; **11.1°E**

(c) latitude = 41°18′15″N, longitude = 76°00′26″W, elevation = 240 m **12.1°W**

7.27 Using Table 7.4, what was the total difference in magnetic declination between Boston, MA and San Francisco, CA on January 1, 2013?

28°58'; 14°01'E − 14°57'W

7.28 The magnetic declination at a certain place is 18°06′W. What is the magnetic bearing there: **(a)** of true north **(b)** of true south **(c)** of true east?

(a) N18°06'E

(b) S18°06'W

(c) N61°54'W

7.29 Same as Problem 7.28, except the magnetic declination at the place is 9°30′E.

(a) N9°30'W

(b) S9°30'E

(c) S80°30'W

For Problems 7.30 through 7.32 the observed magnetic bearing of line AB and its true magnetic bearing are given. Compute the amount and direction of local attraction at point A.

	Observed Magnetic Bearing	True Magnetic Bearing	**Local Attraction**
7.30*	N32°30'E	N32°15'E	**0°15'E**
7.31	S15°25'W	S10°15′W	**5°10'E**
7.32	N9°56'W	N8°20′E	**1°36'E**

What magnetic bearing is needed to retrace a line for the conditions stated in Problems 7.33 through 7.36?

	1875 Magnetic Bearing	1875 Declination	Present Declination	**Present Magnetic Bearing**
7.33*	N32°45′E	8°12′W	2°30′E	**N22°03'E**
7.34	S63°40′W	3°40′E	2°20′W	**S57°40'W**
7.35	S69°20′W	14°20′W	12°30′W	**S67°30'W**
7.36	N24°30′W	2°30′E	2°30′W	**N19°30'W**

In Problems 7.37 through 7.38 calculate the magnetic declination in 1870 based on the following data from an old survey record.

	1870 Magnetic Bearing	Present Magnetic Bearing	Present Magnetic Declination	1870 Magnetic Declination
7.37	N14°20'E	N16°30'E	10°15'W	**0°15'W**

7.38 S40°40'W S54°35'W 8°30′E **22°25'E**

7.39 An angle *APB* is measured at different times using various instruments and procedures. The results, which are assigned certain weights, are as follows: 89°43'38", wt 2; 89°43'42", wt 1; and 89°43'30", wt 3. What is the most probable value of the angle?

46°13'35"; $sec = \frac{38(2)+42(1)+30(3)}{2+1+3} = 34.7''$

7.40 Similar to Problem 7.39, but with an additional measurement of 43°13′32″, wt 4.

46°13'34"; $sec = \frac{38(2)+42(1)+30(3)+32(4)}{2+1+3+4} = 33.6''$