12 AREA

Asterisks $(*)$ indicate problems that have answers given in Appendix G.

12.1* Compute the area enclosed within polygon *ABDFGA* of Figure 12.1 using triangles.

418,320 ft^2 0r 9.6040 ac;

$AGD = 137,481$ ft^2; $GFD = 61,900$ ft^2; $GDB = 81,462$ ft^2; $GBA = 137,481$ ft^2

12.2 Similar to Problem 12.1, except for polygon *BGFDB* of Figure 12.1.

143,360 ft^2 or 3.2911 ac; $GFD = 61,900$ ft^2; $GDB = 81,462$ ft^2

12.3 Compute the area enclosed between line *AGBA* and the shoreline of Figure 12.1 using the offset method.

183,390 ft^2 or 4.2101 ac; $ABG = 137,481$ ft^2; shoreline $= 45,911$ ft^2

Shoreline

0	69	257	535	610.5	
0	102	71	92	0	Sum
3,519.0	16,262.0	22,657.0	3,473.0		45,911.0

12.4 By rule of thumb, what is the estimated uncertainty in an 870,684 ft^2 if the estimated error in the coordinates was ±0.2 ft?

$E =$ **121 ft^2**; By Equation (12.9)

12.5* Compute the area between a lake and a straight line *AG*, from which offsets are taken at irregular intervals as follows (all distances in feet):

Offset Point	A	B	C	D	E	F	G
Stationing	0.00	0 + 54.80	1 + 32.54	2 + 13.02	2 + 98.74	3 + 45.68	4 + 50.17
Offset	12.3	34.2	56.5	85.4	69.1	68.9	23.9

25,220 ft^2 or 0.5789 ac.

Computations

0	54.8	132.54	213.02	298.74	345.68	450.17
12.3	34.2	56.5	85.4	69.1	68.9	23.9
1274.1	3525.5	5710.06	6621.87	3238.9	4848.3	

12.6   Repeat Problem 12.5 with the following offset in meters.

Offset Point	A	B	C	D	E	F	G
Stationing	0.000	20.000	78.940	148.963	163.654	203.691	250.454
Offset	2.15	3.51	4.04	6.57	5.87	4.64	1.65

1099 m^2 or 0.1099 ha;

Computations

0	20	78.94	148.963	163.65	203.69	250.454
2.15	3.51	4.04	6.57	5.87	4.64	1.65
56.6	222.5	371.472	91.378	210.39	147.07	

12.7   Use the coordinate method to compute the area enclosed by the traverse of Problem 10.8.

3,570,400 ft^2 or 81.965 ac

Area = 0.5(7,140,808.1) = 3,570,400

X	Y	XY (+)	YX (-)
0.00	5000.00		9093650
1818.73	3839.67	0	10790317.43
2810.22	5248.32	9545277.034	7992194.179
1522.81	5531.30	15544169.89	2939720.011
531.47	6704.64	10209892.84	0
0.00	5000.00	2657350	
		37,956,689.8	30,815,881.6

12.8   Calculate by coordinates the area within the traverse of Problem 10.11.

66,810 m^2 or 6.681 ha

Area = 0.5(133,626) = 66,813

X	Y	XY (+)	YX (-)
630.890	1311.410		714652.9
544.950	1105.620	697524.6	747786.1
676.350	999.640	544753.8	822723.7
823.020	1108.000	749395.8	927717.3
837.290	1336.600	1100049	843247.6
630.890	1311.410	1098030	
		4189753	4056128

12.9   Compute the area enclosed in the traverse of Problem 10.8 using DMDs.

3,570,400 ft^2 or 81.965 ac

Dep	Lat	DMD	D-Area
1818.731	−1160.328	1818.731	−2110324.504
991.487	1408.640	4628.949	6520522.719
−1287.409	282.984	4333.027	1226177.313
−991.338	1173.340	2054.280	2410368.895
−531.471	−1704.640	531.471	−905966.7254
			7140777.698

12.10*   Determine the area within the traverse of Problem 10.11 using DMDs.

66,810 m^2 or 6.681 ha

Dep	Lat	DMD	D-Area
−85.9474	−205.7924	−85.947	17687.32
131.4089	−105.9817	−40.486	4290.765
146.6660	108.3591	237.589	25744.93
14.2742	228.6006	398.529	91104.01
−206.4017	−25.1855	206.402	−5198.33
			133628.7

12.11   By the DMD method, find the area enclosed by the traverse of Problem 10.20.

4,938,000 m^2 or 4.937 ha

Departure	Latitude	DMD	D-Area
−2014.132	662.305	−2014.132	−1333969.694
−1656.629	−4358.189	−5684.893	24775838.14
3670.761	3695.884	−3670.761	−13566706.85
			9,875,161.597
			4,937,580.798

12.12   Compute the area within the traverse of Problem 10.17 using the coordinate method. Check by DMDs.

21,081,500 ft^2 or 483.965 ac

X	Y	XY (+)	YX(−)
6,521.95	7037.072		45950201.26
6,529.73	4174.528	27226067.06	44737068.2
10,716.68	4042.141	26394101.48	51839920.72
12,824.87	7221.93	77395091.13	68873141.51
9,536.67	8795.595	112802336.1	57364439.61
6521.951	7037.072	67110212.32	
		310927808	268764771.3

21,081,518   483.965

Check by DMD

Dep	Lat	DMD	D-Area
7.7822	−2862.544	7.782	−22276.88992
4186.9438	−132.3871	4202.508	−556357.8733
2108.19	3179.7891	10497.642	33380287.61
−3288.1997	1573.6647	9317.632	14662829.04
−3014.7163	−1758.5227	3014.716	−5301447.048
			42163034.83
			21,081,517
			483.965

12.13 Calculate the area inside the traverse of Problem 10.18 by coordinates and check by DMDs.

302,010 ft^2 or 6.9332 ac

X	Y	XY (+)	YX(-)
10,000.00	5,000.00		50290850
10,058.17	5,399.50	53995000	57666498.02
10,679.97	5,431.69	54632861.41	57104932.16
10,513.29	4,800.13	51265244.4	48001300
10000.00	5000.00	52566450	
		212459555.8	213063580.2
			302012

Check by DMD

Dep	Lat	DMD	D-Area
58.17	399.502	58.17	23239.03134
621.798	32.185	738.138	23756.97153
−166.68	−631.559	1193.256	−753611.5661
−513.289	199.873	513.287	102592.2126
			−604023.3507
			302012

12.14 Compute the area enclosed by the traverse of Problem 10.19 using the DMD method. Check by coordinates.

<u>419,700 ft^2 or 9.6351 ac</u>

Dep	Lat	DMD	D-Area
359.571	−140.204	359.571	−50413.29248
430.594	921.177	1149.736	1059110.359
−296.458	273.72	1283.872	351421.4438
−493.707	−1054.69	493.707	−520709.317
			839409.1937
			419705

Check by coordinates

X	Y	XY (+)	YX(-)
10,000.00	5,000.00		51797850
10,359.57	4,859.80	48598000	52438068.17
10,790.17	5,780.97	59888363.38	60663822.7
10,493.71	6,054.69	65331134.4	60546900
10,000.00	5,000.00	52468550	
		226286047.8	225446640.9
			419703

12.15 Find the area of the lot in Problem 10.25.

<u>115,640 ft^2 or 2.6547 ac</u>

X	Y	XY (+)	YX (-)
5000.00	5000.00		26448350
5289.67	5436.12	27180600	26555229
4884.96	5354.54	28323750	25469726
4756.66	5068.37	24758785	25341850
5000.00	5000.00	23783300	
		104046434	103815155
		115640	2.6547

12.16* Determine the area of the lot in Problem 10.26.

8,868,600 m^2 or 886.86 ha

X	Y	XY (+)	YX (-)
8000.000	5000.000		13250000
2650.000	4702.906	37623248	8239623
1752.028	2015.453	5340950	3854157
1912.303	1511.635	2648427	12093080
8000.000	5000.000	9561515	
		55174140	37436860
		8868640	886.8640242

12.17 Calculate the area of Lot 16 in Figure 21.2.

11,370 ft^2 or 0.2610 ac; Lot rectangular = 80(142.09) = 11,367.2

12.18 Plot the lot of Problem 10.25 to a scale of 1 in. = 100 ft. Determine its surrounded area using a planimeter.

About **115,640 ft^2 or 2.6547 ac**

12.19 Similar to Problem 12.18, except for the traverse of Problem 10.26.

About **8,868,600 m^2 or 886.86 ha**

12.20 Plot the traverse of Problem 10.19 to a scale of 1 in. = 200 ft, and find its enclosed area using a planimeter.

About **419,700 ft^2 or 9.6351 ac**

12.21 The (X,Y) coordinates (in feet) for a closed-polygon traverse *ABCDEFA* follow. *A* (1000.00, 1000.00), *B* (1645.49, 1114.85), *C* (1675.95, 1696.05), *D* (1178.99, 1664.04), *E* (1162.62, 1337.78) and *F* (996.53, 1305.30). Calculate the area of the traverse by the method of coordinates.

349,610 ft^2 or 8.0260 ac

X	Y	XY (+)	YX (-)
1000.00	1000.00		1645490
1645.49	1114.85	1114850	1868433
1675.95	1696.05	2790833.315	1999626
1178.99	1664.04	2788847.838	1934646
1162.62	1337.78	1577229.242	1333138
996.53	1305.30	1517567.886	1305300
1000.00	1000.00	996530	
		10785858.28	10086633
		349613	8.026002588

12.22 Compute by DMDs the area in hectares within a closed-polygon traverse *ABCDEFA* by placing the *X* and *Y* axes through the most southerly and most westerly stations, respectively. Departures and latitudes (in meters) follow. *AB*: E dep. 30, N lat. = 40; *BC*: E dep. = 70, N lat. = 10; *CD* = E dep. = 30, S lat = 50; *DE*: W dep = 60, S lat. = 40; *EF*: W dep = 90, S lat = 30; *FA*: E dep. = 20, N lat = 70.

1 ha or 10,000 m^2

Dep	Lat	DMD	D-Area
30	40	30	1200
70	10	130	1300
30	−50	230	−11500
−60	−40	200	−8000
−90	−30	50	−1500
20	70	-20	−1400
			−19900
			9950

12.23 Calculate the area of a piece of property bounded by a traverse and circular arc with the following coordinates at angle points: *A* (1275.11,1356.11), *B* (1000.27, 1365.70), *C* (1000.00, 1000.00), *D* (1450.00, 1000.00) with a circular arc of radius *CD* starting at *D* and ending at *A* with the curve outside the course *AD*.

142,700 ft^2 or 3.2759 ac

Area of ABCD is 130,381 ft^2

Angle *ACD* = 90° − 37°41'15" = 52°18'45"

Area of segment is 12,319 ft^2

X	Y	XY (+)	YX (-)
1275.11	1356.11		1356476
1000.27	1365.70	1741417.7	1365700
1000.00	1000.00	1000270	1450000
1450.00	1000.00	1000000	1275110
1275.11	1356.11	1966359.5	
		5708047.2	5447286
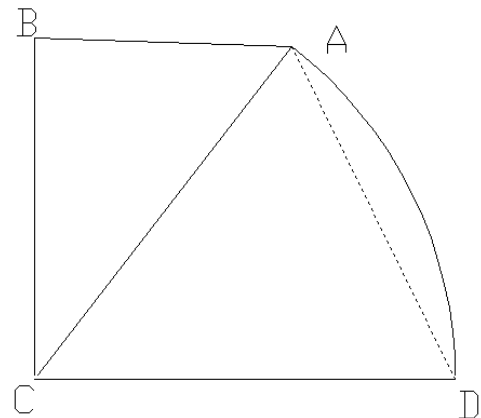
12.24 Calculate the area of a piece of property bounded by a traverse and circular arc with the following coordinates in feet at angle points: *A* (526.68, 823.98), *B* (535.17, 745.61), *C* (745.17, 745.61), *D* (745.17, 845.61), *E* (546.62, 846.14) with a circular arc of radius 25 ft starting at *E*, tangent to *DE*, and ending at *A*.

21,570 ft² or 0.49526 ac

Az_{AB} = 173°49'01.7"
Azimuth to center of circle: 83°49'01.7"
Az_{ED} = 90°09'10.6"
Azimuth to center of circle: 180°09'10.6"
θ = 96°20'08.9"
Area of segment = 214.8 ft²
Area of remainder = 21,358.6 ft²

X	Y	XY (+)	YX (-)
526.68	823.98		440969
535.17	745.61	392697.87	555606
745.17	745.61	399028.1	555606
745.17	845.61	630123.2	462227
546.62	846.14	630518.14	445645
526.68	823.98	450403.95	
		2502771.3	2460054

12.25 Divide the area of the lot in Problem 12.23 into two equal parts by a line through point *B*. List in order the lengths and azimuths of all sides for each parcel.

Parcel A

Course	Length	Azimuth
AB	275.007	271°59'54"
BE	534.593	133°09'46"
ED	597.796	90°00'00"
DA	396.738	153°50'38"

Parcel B

Course	Length	Azimuth
BC	365.700	180°02'32"
CE	390.210	270°00'00"
EB	534.593	313°09'46"
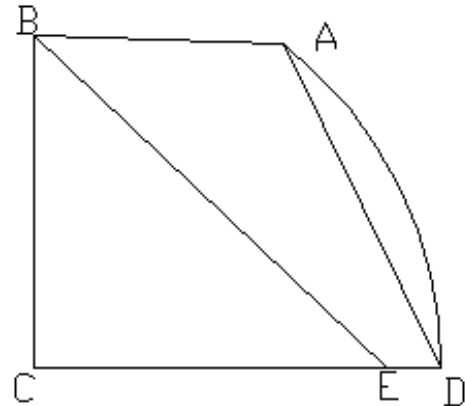
1/2 Area = 71,350 ft²; Area$_{BCD}$ = 82,282.5 ft²; Area$_{BCE}$ = 10,932.5 ft²
Az_{CB} = 309°06'59" Az_{DB} = 270°
BD = 579.65 ft; $\angle CDB$ = 39°06'59"
$1/2(BD)(DE)\sin(CDB)$ = 10932.5 ft²; DE = 59.79 ft

12.26 Partition the lot of Problem 12.24 into two equal areas by means of a line parallel to *BC*. Tabulate in clockwise consecutive order the lengths and azimuths of all sides of each parcel.

Parcel A

Course	Length	Azimuth
AG	28.02	173°49'02"
GF	215.47	90°00'00"
FD	49.48	0°00'00"
DE	198.55	270°09'11"
EA	29.81	221°58'53"

Parcel B

Course	Length	Azimuth
GB	50.82	173°49'02"
BC	210.00	90°00'00"
CF	50.52	0°00'00"
FG	215.47	180°00'00"

Area = 10,747.5 ft^2; $\angle ABC$ = 96°10'58"; *BC* = 210.00 ft
10,747.5 = h/2[210.00 + (210.00 + $h$tan 6°10'58")]
quadratic equation: 0 = 0.108331h^2 + 420h – 21495; h = 50.52 ft;

12.27 Lot *ABCD* between two parallel street lines is 350.00 ft deep and has a 220.00 ft frontage (*AB*) on one street and a 260.00 ft frontage (*CD*) on the other. Interior angles at *A* and *B* are equal, as are those at *C* and *D*. What distances *AE* and *BF* should be laid off by a surveyor to divide the lot into two equal areas by means of a line *EF* parallel to *AB*?
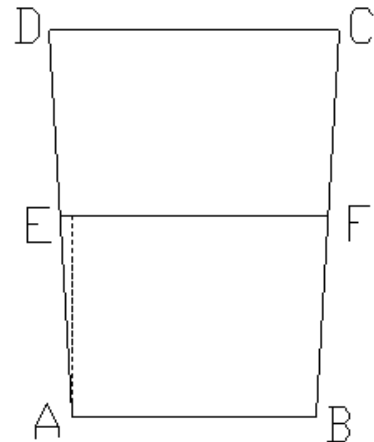
AE = BF = 182.58 ft

Area = 84,000 ft^2; 1/2 area = 42,000 ft^2
$\angle @A$ = tan^{-1} (20/350) = 93°16'14"
42000 = h/2[220.00 + (220.00 + 2h tan 3°16'14")]
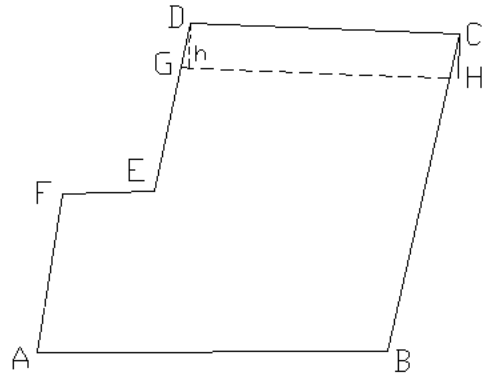quadratic equation: 0.057142857h^2 + 220h – 42000 = 0
h = 182.28 ft
EF = 220 + 2(182.28)tan 3°16'14" = 240.83 ft

12.28 Partition 1-acre parcel from the northern part of lot *ABCDEFA* in Problem 12.21 such that its southern line is parallel to the northern line.

Lay off 88.04 ft along *DE* to *G*, and 88.18 ft along *CB* to *H*

Required area: 43,560 sq. ft.
$x = 498.00 - h \tan a_2 + h \tan a_1$

$$CD = \sqrt{(1178.99 - 1675.95)^2 + (1664.04 - 1696.05)^2} = 498.00$$

$$Az_{CD} = \tan^{-1}\left(\frac{1178.99 - 1675.95}{1664.04 - 1696.05}\right) + 180° = 266°18'52.5''$$

$$Az_{DE} = \tan^{-1}\left(\frac{1166.62 - 1178.99}{1337.78 - 1664.04}\right) + 180° = 182°10'16.7''$$

$$Az_{CB} = \tan^{-1}\left(\frac{1645.49 - 1675.95}{1114.85 - 1696.05}\right) + 180° = 183°00'00.2''$$

$a_1 = 182°10'16.7'' - (266°18'52.5'' - 90°) = 5°51'24.2''$
$a_2 = 183°00'00.2'' - (266°18'52.5'' - 90°) = 6°41'07.7''$
$x = 498.00 - h\,(0.014639447)$
$43,560 = ½\,(498.00 + x)h$
So, $0.014639447\,h^2 - 996h + 87,120 = 0$

$$h = \frac{996.00 \pm \sqrt{(-996.00)^2 - 4(0.014639447)(87,120)}}{2(0.014639447)}$$

$$= \frac{2.564322415}{2(0.014639447)} = 87.58 \, ft$$

Check Area = ½ (498.00 + 496.73) 87.58 = 43,560 sq ft. (check)

$$DG = \sqrt{87.58^2 + (87.58 \tan 5°51'24.2'')^2} = 88.04 \, ft$$

$$CH = \sqrt{87.58^2 + (87.58 \tan 6°41'07.7'')^2} = 88.18 \, ft$$

12.29 Write a computational spreadsheet for calculating areas within closed polygon traverses by the coordinate method.

Solutions will vary.