16 ADJUSTMENT BY LEAST SQUARES

Asterisks (*) indicate problems that have partial answers given in Appendix G.

16.1 What fundamental condition is enforced by the method of unweighted least squares?

From Section 16.2, paragraph 4: "For a group of equally weighted observations, the fundamental condition enforced by the least-squares method is that the sum of the squares of the residuals is a minimum. Suppose a group of m observations of equal weight were taken having residuals . Then, in equation form, the fundamental condition of least squares is

$\sum_{i=1}^{m} v_i^2 = v_1^2 + v_2^2 + v_3^3 + \cdots + v_m^2 \rightarrow$ minimum"

16.2 Why are adjustments performed using the least-squares method generally programmed?

From Section 16.1, paragraph 3: "Although the theory of least squares was developed in the late 1700s, because of the lengthy calculations involved, the method was not used commonly prior to the availability of computers. Instead, arbitrary, or "rule of thumb," methods such as the compass (Bowditch) rule were applied."

16.3 Why is the compass rule adjustment of a traverse considered an arbitrary adjustment??

From Section 16.1, paragraph 5: "A simple example can be used to illustrate the arbitrary nature of 'rule of thumb' adjustments, as compared to least squares. Consider the horizontal survey network shown in Figure 16.1. If the compass rule was used to adjust the observations in the network, several solutions would be possible. To illustrate one variation, suppose that traverse *ABCDEFGA* is adjusted first. Then holding the adjusted values of points *G* and *E*, traverse *GHKE* is adjusted, and finally, holding the adjusted values on *H* and *C*, traverse *HJC* is adjusted. This obviously would yield a solution, but there are other possible approaches. In another variation, traverse *ABCDEFGA* could be adjusted followed by *GHJC*, and then *HKE*. This sequence would result in another solution, but with different adjusted values for points *H, J*, and *K*. There are still other possible variations. This illustrates that the compass rule adjustment is properly referred to as an "arbitrary" method."

16.4* What is the most probable value for the following set of ten distance observations in meters? 532.688, 532.682, 532.682, 532.684, 532.689, 532.686, 532.690, 532.684, 532.686, 532.686

532.686 m

16.5 What is the standard deviation of the adjusted value in Problem 16.4?

±0.0028 m

16.6 Three horizontal angles were observed around the horizon of station A. Their values are 85°07′15″, 134°26′48″, and 140°26′15″. Assuming equal weighting, what are the most probable values for the three angles?

85°07'09", **134°26'42"**, and **140°26'09"**

Condition $x + y + z = 360°$ or $v_3 = 18$ » − $(v_1 + v_2)$

$$\sum \upsilon_1^2 + \upsilon_2^2 + \left(18'' - \upsilon_1^2 - \upsilon_2^2\right) \rightarrow \text{minimum}$$

Normal equations
$$2\upsilon_1 + \upsilon_2 = -18''$$
$$\upsilon_1 + 2\upsilon_2 = -18''$$

So, $v_1 = v_2 = v_3 = -6''$

16.7 What are the standard deviations of the adjusted values in Problem 16.6?

±2.8" $S_0 = \sqrt{\frac{6^2+6^2+6^2}{3-2}} = \pm 10.4\ ;\ \sigma = 10.4\sqrt{\frac{2}{3}} = \pm 8.5''$

16.8 In Problem 16.6, the standard deviations of the three angles are ±5.5″, ±6.0″, and ±4.9″ respectively. What are the most probable values for the three angles?

85°07'09.0", **134°26'40.8"**, and **140°26'10.2"**

Normal equations:
$$2w_1v_1 + w_3v_2 = -w_3 18''$$
$$w_3v_1 + 2w_2v_2 = -w_3 18''$$

Normal Equations		A^TWL	X (")
0.074707	0.041649	−0.74969	−6.03
0.041649	0.069427	−0.74969	−7.18

16.9* Determine the most probable values for the *x* and *y* distances of Figure 16.2, if the observed lengths of *AC*, *AB*, and *BC* (in meters) are 315.297, 155.046, and 160.258, respectively.

***x* = 135.469** and ***y* = 158.609**

Normal equations:
$2x + y = 470.343$
$x + 2y = 475.555$

Normal Equations		A^TL	X
2	1	470.343	155.0437
1	2	475.555	160.2557

16.10* What are the standard deviations of the adjusted values in Problem 16.9?

±0.0033 ft for both

$$S_0 = \pm 0.0040;\ \Sigma_{xx} = 0.0040^2 \begin{bmatrix} 2/3 & -1/3 \\ -1/3 & 2/3 \end{bmatrix}$$

16.11 A network of differential levels is run from existing benchmark Juniper through new stations A and B to existing benchmarks Red and Rock as shown in the accompanying figure. The elevations of Juniper, Red, and Rock are 101.968, 123.411, and 145.820 m, respectively. Develop the observation equations for adjusting this network by least squares, using the following elevation differences.

From	To	Elev. Diff. (m)	σ (m)
Juniper	*A*	3.295	0.022
A	*B*	31.833	0.016
B	Red	−13.638	0.029
B	Rock	8.752	0.020

$A = -101.968 + 3.295 = 105.263$

$-A + B = 31.833$

$-B = -123.411 + 13.638 = -137.049$

$-B = -145.820 + 8.752 = -137.068$

16.12 For Problem 16.11, following steps outlined in Example 16.6 perform a weighted least-squares adjustment of the network. Determine weights based upon the given standard deviations. What are the

(a)* Most probable values for the elevations of *A* and *B*? **105.247** and **137.071**

(b) Standard deviations of the adjusted elevations? **±0.0135** and **±0.0120**

(c) Standard deviation of unit weight? **±0.85**

(d) Adjusted elevation differences and their residuals?

From	To	ΔElev	V	S
Juniper	A	3.279	−0.0163	±0.0135
A	B	31.824	−0.0086	±0.0117
B	Red	−13.655	−0.0220	±0.0120
B	Rock	8.754	−0.0030	±0.0120

(e) Standard deviations of the adjusted elevation differences? (see d)

16.13 Repeat Problem 16.12 using distances for weighting. Assume the following course lengths for the problem.

From	To	Dist (ft)
Juniper	A	1500
A	B	300
B	Red	1200

B	Rock	2300

(a) Most probable values for the elevations of *A* and *B*? **105.240** and **137.068**

(b) Standard deviations of the adjusted elevations? **±0.0152** and **±0.0142**

(c) Standard deviation of unit weight? **±0.00061**

(d) Adjusted elevation differences and their residuals?

From	To	ΔElev	V	S
Juniper	A	3.272	−0.0235	±0.0152
A	B	31.828	−0.0047	±0.0099
B	Red	−13.657	−0.0188	±0.0142
B	Rock	8.752	0.0002	±0.0142

(e) Standard deviations of the adjusted elevation differences? (see d)

16.14 Use WOLFPACK to do Problem 16.12 and 16.13 and compare the solutions for *A* and *B*.

See solutions in Problem 16.12 and 16.13. The standard deviation of unit weight is different as well as small changes in the solution.

16.15 Repeat Problem 16.12 using the following data.

From	To	Elev. Diff. (m)	σ (m)
Juniper	*A*	24.402	0.027
A	*B*	1.515	0.024
B	Red	−4.492	0.031
B	Rock	17.862	0.026

(a) Most probable values for the elevations of *A* and *B*? **126.392** and **127.924**

(b) Standard deviations of the adjusted elevations? **±0.0264** and **±0.0225**

(c) Standard deviation of unit weight? **±1.3**

(d) Adjusted elevation differences and their residuals?

From	To	Elev. Diff.	V	S
Juniper	*A*	24.424	0.0215	±0.0264
A	*B*	1.532	0.0170	±0.0252
B	Red	−4.513	−0.0206	±0.0225
B	Rock	17.896	0.0344	±0.0225

(e) Standard deviations of the adjusted elevation differences? (see d)

16.16 A network of differential levels is shown in the accompanying figure. The elevations of benchmarks *A* and *G* are 835.24 ft and 865.64 ft, respectively. The observed elevation differences and the distances between stations are shown in the following table. Using WOLFPACK, determine the

From	To	Elev. Diff (ft)	S (ft)
A	*B*	30.55	0.022
B	*C*	−45.22	0.025
C	*D*	24.34	0.022
D	*E*	10.38	0.016
E	*F*	−15.16	0.013
F	*A*	−4.83	0.011
G	*F*	−25.59	0.008
G	*H*	−7.66	0.010
H	*D*	−13.10	0.009
G	*B*	0.14	0.010
G	*E*	−10.42	0.011

(a) Most probable values for the elevations of new benchmarks *B*, *C*, *D*, *E*, *F*, and *H*?

Adjusted Elevations

Station	Elevation	S
B	865.78	0.010
C	820.54	0.021
D	844.87	0.012
E	855.23	0.009
F	840.06	0.007
H	857.97	0.010

(b) Standard deviations of the adjusted elevations? See (a)

(c) Standard deviation of unit weight? 1.2

(d) Adjusted elevation differences and their residuals?

Adjusted Elevation Differences

From	To	Elevation Difference	V	S
A	B	30.54	-0.011	0.010
B	C	-45.24	-0.017	0.021
C	D	24.33	-0.013	0.020
D	E	10.36	-0.023	0.013
E	F	-15.17	-0.007	0.010
F	A	-4.82	0.011	0.007
G	F	-25.58	0.009	0.007
G	H	-7.67	-0.006	0.010
H	D	-13.11	-0.005	0.009
G	B	0.14	-0.001	0.010
G	E	-10.41	0.006	0.009

(e) Standard deviations of the adjusted elevation differences? See (d).

16.17 Develop the observation equations for line *AB* and *BC* in Problem 16.16.

$B = 865.79 + v_1$
$-B + C = -45.22 + v_2$

16.18 A network of GNSS observations shown in the accompanying figure was made with two receivers using the static method. Known coordinates of the two control stations are in the geocentric system. Develop the observation equations for the following baseline vector components.

Station	X (m)	Y (m)	Z (m)
Jim	1,161,510.5022	−4,667,575.5684	4,174,209.5623
Al	1,171,820.5926	−4,640,316.7293	4,202,588.1131

Jim to Troy					**Al to Troy**		
−13,024.396	2.82E−4	9.93E−6	1.24E−4	−23,334.463	2.63E−4	1.68E−5	−5.35E-7
14,982.023		2,92E−4	2.87E−6	−12,276.800		2.63E−4	7.11E−6
20,654.719			2.82E−4	−7723.869			2.70E−4

$$X_{Troy} = 1{,}148{,}486.106 + \upsilon_1$$
$$Y_{Troy} = -4{,}652{,}593.545 + \upsilon_2$$
$$Z_{Troy} = 4{,}194{,}864.281 + \upsilon_3$$
$$X_{Troy} = 1{,}148{,}486.130 + \upsilon_4$$
$$Y_{Troy} = -4{,}652{,}593.529 + \upsilon_5$$
$$Z_{Troy} = 4{,}194{,}864.244 + \upsilon_6$$

16.19 For Problem 16.18, construct the *A* and *L* matrices.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad L = \begin{bmatrix} 1{,}148{,}486.106 \\ -4{,}652{,}593.545 \\ 4{,}194{,}864.281 \\ 1{,}148{,}486.130 \\ -4{,}652{,}593.529 \\ 4{,}194{,}864.244 \end{bmatrix}$$

16.20 For Problem 16.18, construct the covariance matrix.

$$\Sigma = \begin{bmatrix} 2.82\text{E}-4 & 9.93\text{E}-4 & 1.24\text{E}-3 & 0 & 0 & 0 \\ 9.93\text{E}-6 & 2.92\text{E}-3 & 2.87\text{E}-5 & 0 & 0 & 0 \\ 1.24\text{E}-4 & 2.87\text{E}-6 & 2.82\text{E}-4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2.63\text{E}-4 & 1.68\text{E}-5 & -5.35\text{E}-7 \\ 0 & 0 & 0 & 1.68\text{E}-5 & 2.63\text{E}-4 & 7.11\text{E}-6 \\ 0 & 0 & 0 & -5.35\text{E}-7 & 7.11\text{E}-6 & 2.70\text{E}-4 \end{bmatrix}$$

16.21 Use WOLFPACK to adjust the baselines of Problem 16.18.

```
Degrees of Freedom = 3
Reference Variance = 1.624
Standard Deviation of Unit Weight = ±1.3

************************
Adjusted Distance Vectors
************************
From       To        dX           dY           dZ       Vx       Vy       Vz
==============================================================================
 Jim      Troy   -13024.389   14982.031    20654.704   0.0071   0.0084  -0.0154
  Al      Troy   -23334.479  -12276.808    -7723.847  -0.0163  -0.0077   0.0218

**************************************************
Advanced Statistical Values
**************************************************
      From        To      ±S   Vector Length    Prec
=====================================================
       Jim      Troy  0.0254      28,648.085 1,126,000
        Al      Troy  0.0254      27,475.002 1,080,000

*******************
Adjusted Coordinates
*******************
Station            X                 Y                 Z       Sx      Sy      Sz
===================================================================================
=
    Jim    1,161,510.5022   -4,667,575.5684    4,174,209.5623
     Al    1,171,820.5926   -4,640,316.7293    4,202,588.1131
   Troy    1,148,486.1133   -4,652,593.5370    4,194,864.2659  0.0145  0.0150  0.0146
```

16.22 Convert the geocentric coordinates obtained for station Troy in Problem 16.21 to geodetic coordinates using the WGS84 ellipsoidal parameters.

(41°23'16.39651" N, 76°08'01.76564" W, 38.806 m)

16.23 A network of GNSS observations shown in the accompanying figure was made with two receivers using the static method. Use WOLFPACK to adjust the network, given the following data.

Station	X (m)	Y (m)	Z (m)
Bonnie	1,161,121.599	−4,655,872.977	4,188,330.232
Tom	1,176,398.558	−4,653,039.613	4,187,198.360

Bonnie to Ray

3,377.788	3.40E−05	2.82E−06	1.57E−05
−4,727.902		3.23E−05	8.04E−07
-6,172.019			3.37E−05

Bonnie to Herb

7,826.248	4.33E−05	6.62E−07	6.09E−07
5,106.722		4.31E−05	9.51E−07
3,521.039			4.29E−05

Tom to Ray

−11,899.158	8.34E−05	5.37E−06	1.30E−08
−7,561.271		8.54E−05	4.01E−07
−5,040.146			8.36E−05

Tom to Herb

−7,450.717	3.73E−05	1.75E−06	8.70E−07
2,273.364		3.69E−05	−9.74E−07
4,652.903			3.60E−05

Bonnie to Tom (Fixed line—Don't use in adjustment.)

$$\begin{matrix} 15{,}276.953 & 8.99\text{E}-05 & 1.77\text{E}-06 & 1.88\text{E}-06 \\ 2{,}833.370 & & 8.96\text{E}-05 & 6.11\text{E}-06 \\ -1{,}131.871 & & & 9.01\text{E}-05 \end{matrix}$$

Degrees of Freedom = 6
Reference Variance = 0.3520
Standard Devaiation of Unit Weight = ±0.59

****
Adjusted Distance Vectors
****

From	To	dX	dY	dZ	Vx	Vy	Vz
Bonnie	Ray	3377.780	-4727.918	-6172.016	0.0006	0.0018	-0.0015
Bonnie	Herb	7826.242	5106.717	3521.030	0.0044	0.0016	0.0045
Tom	Ray	-11899.179	-7561.282	-5040.144	-0.0014	-0.0042	0.0035
Tom	Herb	-7450.717	2273.353	4652.902	-0.0036	-0.0014	-0.0035

****
Advanced Statistical Values
****

From	To	±S	Vector Length	Prec
Bonnie	Ray	0.0051	8,476.815	1,678,000
Bonnie	Herb	0.0046	9,986.304	2,150,000
Tom	Ray	0.0051	14,972.191	2,964,000
Tom	Herb	0.0046	9,073.632	1,953,000

****
Adjusted Coordinates
****

Station	X	Y	Z	Sx	Sy	Sz
Bonnie	1,161,121.599	-4,655,872.977	4,188,330.232			
Tom	1,176,398.558	-4,653,039.613	4,187,198.360			
Ray	1,164,499.379	-4,660,600.895	4,182,158.216	0.0029	0.0029	0.0029
Herb	1,168,947.841	-4,650,766.260	4,191,851.262	0.0027	0.0027	0.0027

16.24 For Problem 16.23, write the observation equations for the baselines "Bonnie to Ray" and "Tom to Herb."

$$X_{Ray} = 1{,}148{,}486.106 + \upsilon_x \qquad X_{Herb} = 1{,}148{,}486.130 + \upsilon_x$$

$$Y_{Ray} = -4{,}652{,}593.545 + \upsilon_y \qquad Y_{Herb} = -4{,}652{,}593.529 + \upsilon_y$$

$$Z_{Ray} = 4{,}194{,}864.281 + \upsilon_z \qquad Z_{Herb} = 4{,}194{,}864.244 + \upsilon_z$$

16.25 For Problem 16.23, construct the *A*, *X*, and *L* matrices for the observations.

$$A=\begin{bmatrix}1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\\1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\end{bmatrix}\quad X=\begin{bmatrix}X_{Ray}\\Y_{Ray}\\Z_{Ray}\\X_{Herb}\\Y_{Herb}\\Z_{Herb}\end{bmatrix}\quad L=\begin{bmatrix}1,164,499.387\\-4,660,600.879\\4,182,158.213\\1,164,499.400\\-4,660,600.884\\4,182,158.214\\1,168,947.847\\-4,650,766.255\\4,191,851.271\\-1,168,947.841\\-4,650,766.349\\4,191,851.263\end{bmatrix}$$

16.26 For Problem 16.23, construct the covariance matrix.

$$\Sigma=\begin{bmatrix}3.40E-5&2.82E-6&1.57E-5&0&0&0&0&0&0&0&0&0\\2.82E-6&3.23E-5&8.04E-7&0&0&0&0&0&0&0&0&0\\1.57E-5&8.04E-7&3.37E-5&0&0&0&0&0&0&0&0&0\\0&0&0&8.34E-5&5.37E-6&1.30E-8&0&0&0&0&0&0\\0&0&0&5.37E-6&8.54E-5&4.01E-7&0&0&0&0&0&0\\0&0&0&1.30E-8&4.01E-7&8.36E-5&0&0&0&0&0&0\\0&0&0&0&0&0&4.33E-5&6.62E-7&6.09E-7&0&0&0\\0&0&0&0&0&0&6.62E-7&4.31E-5&9.51E-7&0&0&0\\0&0&0&0&0&0&6.09E-7&9.51E-7&4.29E-5&0&0&0\\0&0&0&0&0&0&0&0&0&3.73E-5&1.75E-6&8.70E-7\\0&0&0&0&0&0&0&0&0&1.75E-6&3.69E-5&-9.74E-7\\0&0&0&0&0&0&0&0&0&8.70E-7&-9.74E-7&3.60E-5\end{bmatrix}$$

16.27* After completing Problem 16.23, convert the geocentric coordinates for station Ray and Herb to geodetic coordinates using the WGS84 ellipsoidal parameters. (Hint: See Section 13.4.3)

Station	Latitude	Longitude	h (m)
Ray	41°13'58.16047" N	75°58'16.63454" W	387.630
Herb	41°20'55.13809" N	75°53'28.45776" W	428.569

16.28 Following the procedures discussed in Section 14.5.2, analyze the fixed baseline from station Bonnie to Tom.

Dist: 15,578.6		**ppm**
dX	0.006	0.38
dY	0.006	0.38
dZ	0.001	0.06

16.29 For the horizontal survey of the accompanying figure, determine initial approximations for the unknown stations. The observations for the survey are

Station	X (ft)	Y (ft)
Dave	1000.00	1000.00

From	To	Azimuth	S
Dave	Wes	7°38'26"	0.001"

From	To	Distance (ft)	σ (ft)
Dave	Steve	2222.32	0.014
Steve	Frank	1488.20	0.013
Frank	Wes	2038.42	0.014
Wes	Dave	2360.91	0.014
Dave	Frank	3540.62	0.016
Steve	Wes	1852.34	0.013

Backsight Station	Instrument Station	Foresight Station	Angle	σ"
Wes	Dave	Frank	33°24'50"	±3.0
Frank	Dave	Steve	14°08'29"	±3.1
Dave	Steve	Wes	70°08'56"	±3.1
Wes	Steve	Frank	74°18'46"	±3.2
Steve	Frank	Dave	21°23'56"	±3.1
Dave	Frank	Wes	39°37'50"	±3.1
Frank	Wes	Steve	44°39'30"	±3.1
Steve	Wes	Dave	62°17'42"	±3.1

Initial approximations can vary slightly:

Steve: **(2824.81, 2268.38)**

Frank: **(3325.45, 3669.85)**

Wes: **(1313.90, 3339.98)**

16.30* Using the data in Problem 16.29, write the linearized observation equation for the distance from Steve to Frank.

$$-0.3364dx_{\text{Steve}} - 0.9417dy_{\text{Steve}} + 0.3364dx_{\text{Frank}} + 0.9417dy_{\text{Steve}} = -0.0065$$

16.31 Using the data in Problem 16.29, write the linearized observation equation for the angle Wes-Dave-Frank.

$$43.9298dx_{\text{Frank}} - 38.2630dy_{\text{Frank}} - 86.5899dx_{\text{Wes}} + 11.6157dy_{\text{Wes}} = 6.0955''$$

where coefficients are multiplied by 206264.8"/rad.

16.32 Assuming a standard deviation of ±0.001" for the azimuth line Dave-Wes, use WOLFPACK to adjust the data in Problem 16.29.

Adjusted stations

Station	Northing	Easting	Sn	Se	Su	Sv	t
Steve	2,268.39	2,824.81	0.015	0.010	0.015	0.010	176.47°
Frank	3,669.86	3,325.45	0.018	0.013	0.019	0.011	151.52°
Wes	3,339.96	1,313.90	0.012	0.002	0.012	0.000	7.64°

Adjusted Distance Observations

Station Occupied	Station Sighted	Distance	V	S
Dave	Steve	2,222.33	-0.009	0.012
Steve	Frank	1,488.21	-0.008	0.011
Frank	Wes	2,038.42	0.001	0.012
Wes	Dave	2,360.93	-0.015	0.012
Dave	Frank	3,540.60	0.018	0.012
Steve	Wes	1,852.33	0.009	0.012

Adjusted Angle Observations

Station Backsighted	Station Occupied	Station Foresighted	Angle	V	S
Wes	Dave	Frank	33°24'55"	-5.3"	1.1"
Frank	Dave	Steve	14°08'31"	-1.5"	1.1"
Dave	Steve	Wes	70°08'51"	4.7"	1.7"
Wes	Steve	Frank	74°18'45"	0.8"	2.0"
Steve	Frank	Dave	21°23'53"	3.0"	1.6"
Dave	Frank	Wes	39°37'50"	0.3"	1.3"
Frank	Wes	Steve	44°39'32"	-2.1"	1.4"
Steve	Wes	Dave	62°17'43"	-0.9"	1.5"

16.33* Given the following inverse matrix and a standard deviation of unit weight of 1.13, determine the parameters of the error ellipse.

$$(A^TWA)^{-1} = \begin{bmatrix} q_{xx} & q_{xy} \\ q_{xy} & q_{yy} \end{bmatrix} = \begin{bmatrix} 0.00016159 & -0.00001827 \\ -0.00001827 & 0.00028020 \end{bmatrix}$$

t = 171°26'19.7"; $S_u = 1.13\sqrt{0.00028295} = 0.019$; $S_v = 1.13\sqrt{0.000015884} = 0.014$

16.34 Compute S_x and S_y in Problem 16.33.

$$S_x = 1.13\sqrt{0.00016159} = 0.014;\ S_y = 1.13\sqrt{0.00028020} = 0.019$$

16.35 Given the following inverse matrix and a standard deviation of unit weight of 1.45, determine the parameters of the error ellipse.

$$(A^TWA)^{-1} = \begin{bmatrix} q_{xx} & q_{xy} \\ q_{xy} & q_{yy} \end{bmatrix} = \begin{bmatrix} 0.0004894 & 0.0000890 \\ 0.0000890 & 0.0002457 \end{bmatrix}$$

$t = 71°55'40''$; $S_u = \pm 0.033$ ft; $S_v = \pm 0.021$ ft

16.36 Compute S_x and S_y in Problem 16.35.

$$S_x = 1.45\sqrt{0.0004894} = \mathbf{0.032};\ S_y = 1.45\sqrt{0.0002457} = \mathbf{0.023}$$

16.37 The well-known observation equation for a line is $mx + b = y + v_y$. What is the slope and y-intercept of the best fit line for a set of points with coordinates (1446.81, 2950.79), (2329.79, 2432.66), (3345.74, 1837.13), (478.72, 3517.64), (4382.98, 1229.16)?

m = −0.5860; b = 3797.95

16.38 Use WOLFPACK and the following standard deviations for each observation to do a least squares adjustment of Example 10.4, and describe any differences in the solution. What advantages are there to using the least squares method in adjusting this traverse?

Stations	Angle ± S	Stations	Distance ± S
E-A-B	100°45′37″ ± 16.7″	*AB*	647.25 ± 0.027
A-B-C	231°23′43″ ± 22.1″	*BC*	203.03 ± 0.026
B-C-D	17°12′59″ ± 21.8″	*CD*	720.35 ± 0.027
C-D-E	89°03′28″ ± 10.2″	*DE*	610.24 ± 0.027
D-E-A	101°34′24″ ± 16.9″	*EA*	285.13 ± 0.026
AZIMUTH *AB*	126°55′17″ ± 0.001″		

Adjusted stations

Sta	Northing	Easting	Sn	Se	Su	Sv	t
B	4,611.179	10,517.459	0.0099	0.0132	0.0165	0.0000	126.92°
C	4,408.224	10,523.432	0.0172	0.0178	0.0193	0.0154	130.51°
D	5,102.267	10,716.279	0.0232	0.0192	0.0256	0.0160	147.58°
E	5,255.934	10,125.709	0.0150	0.0149	0.0175	0.0119	44.56°

Adjusted Distance Observations

Station Occupied	Station Sighted	Distance	V	S
A	B	647.26	-0.010	0.016
B	C	203.04	-0.013	0.016
C	D	720.34	0.013	0.017
D	E	610.23	0.005	0.017
E	A	285.14	-0.011	0.017

Adjusted Angle Observations

Station Backsighted	Station Occupied	Station Foresighted	Angle	V	S
E	A	B	100°45′44″	-6.9″	9.1″
A	B	C	231°23′34″	8.8″	12.3″
B	C	D	17°12′51″	7.6″	10.2″
C	D	E	89°03′24″	4.4″	6.1″
D	E	A	101°34′27″	-2.9″	8.4″

Adjusted Azimuth Observations

Station Occupied	Station Sighted	Azimuth	V	S
A	B	126°55′17″	0.0″	0.0″

-----Standard Deviation of Unit Weight = 0.700781