24 HORIZONTAL CURVES

Asterisks (*) indicate problems that have partial answers given in Appendix *G*.

24.1 What features make a spiral curve a particularly useful easement curve?

From Section 24.1, paragraph 3: "A spiral makes an excellent easement curve because its radius decreases uniformly from infinity at the tangent to that of the curve it meets."

24.2 For the following circular curves having a radius *R*, what is their degree of curvature by (1) arc definition and (2) chord definition?

(a)* 500.00 ft (1) **11°27′33″** (2) **11°28′42″**

(b) 900.00 ft (1) **6°21′58″** (2) **6°22′10″**

(c) 2500.00 ft (1) **2°17′31″** (2) **2°17′31″**

Compute *L, T, E, M, LC, R*, and stations of the PC and PT for the circular curves in Problems 24.3 through 24.6. Use the chord definition for the railroad curve and the arc definition for the highway curves.

24.3* Railroad curve with $D_c = 4°00′$, $I = 24°00′$, and PI station = 36 + 45.00 ft.

24.4 Highway curve with $D_a = 4°20′$, $I = 24°30′$, and PI station = 32 + 55.00 ft.

24.5 Highway curve with $R = 600.000$ m, $I = 12°30′$, and PI station = 6+517.500 m.

24.6 Highway curve with $R = 900.000$ m, $I = 15°30′$, and PI station = 1+984.000 m.

	24.3	**24.4**	**24.5**	**24.6**
PI	36+45.00	24+65.00	6+517.500	1+984.000
D_c	4			
D_a		4°20’		
R	1432.68	2148.59	600	900
I	24	24°30’	12°30’	15°30’
L	600	565.38	130.900	243.473
T	304.53	287.08	65.711	122.485
E	32.01	30.81	3.588	8.296
M	31.31	30.11	3.566	8.221
LC	595.74	561.09	130.640	242.732
PC	33+40.47	21+77.92	6+451.789	1+861.515
PT_{Back}	42+45.00	30+30.38	6+648.400	2+227.473

$PT_{Forward}$	39+49.53	27+52.08	6+583.211	2+106.485

Tabulate *R* or *D, T, L, E, M,* PC, PT, deflection angles, and incremental chords to lay out the circular curves at full stations (100 ft or 30 m) in Problems 24.7 through 24.14.

24.7 Highway curve with $D_a = 3°30'$, $I = 15°30'$, and PI station = 36 +44.50 ft.

```
                Intersection Angle = 15°30'00"
               Degree of Curvature = 3°30'00"
                           Radius = 1,637.02
          Circular Curve Length = 442.86
               Tangent Distance = 222.79
     Circular Curve Long Chord = 441.51
                Middle Ordinate = 14.95
                       External = 15.09

                  PI Stationing = 30+44.50
                  32+64.57 Back = 32+67.29 Ahead
   ============================================================

        Station |      Chord |   Defl. Increment |       Defl. Angle |
   ============================================================================
       32+64.57 |      64.56 |          1°07'48" |          7°45'00" |
       32+00.00 |      99.98 |          1°45'00" |          6°37'12" |
       31+00.00 |      99.98 |          1°45'00" |          4°52'12" |
       30+00.00 |      99.98 |          1°45'00" |          3°07'12" |
       29+00.00 |      78.28 |          1°22'12" |          1°22'12" |
       28+21.71
   ============================================================================
```

24.8 Railroad curve with $D_c = 3°00'$, $I = 15°0'$, and PI station = 24 + 50.50 ft.

```
                Intersection Angle = 15°00'00"
               Degree of Curvature = 3°00'00"
                           Radius = 1,910.08
          Circular Curve Length = 500.00
               Tangent Distance = 251.47
     Circular Curve Long Chord = 498.63
                Middle Ordinate = 16.34
                       External = 16.48

                  PI Stationing = 24+50.50
                  26+99.03 Back = 27+01.97 Ahead
   ============================================================

        Station |      Chord |   Defl. Increment |       Defl. Angle |
   ============================================================================
       26+99.03 |      99.03 |          1°29'08" |          7°30'00" |
       26+00.00 |     100.00 |          1°30'00" |          6°00'52" |
       25+00.00 |     100.00 |          1°30'00" |          4°30'52" |
       24+00.00 |     100.00 |          1°30'00" |          3°00'52" |
       23+00.00 |     100.00 |          1°30'00" |          1°30'52" |
       22+00.00 |       0.97 |          0°00'52" |          0°00'52" |
       21+99.03
```

24.9 Highway curve with $R = 650$ m, $I = 10°00'$, and PI station = 3 + 290.600 m.

```
                Intersection Angle = 10°00'00"
              Degree of Curvature = 8°48'53"
                           Radius = 650.000
          Circular Curve Length = 113.446
               Tangent Distance = 56.868
     Circular Curve Long Chord = 113.302
                Middle Ordinate = 2.473
                       External = 2.483

                  PI Stationing = 32+90.600
                 33+47.179 Back = 33+47.468 Ahead
```

Station	Chord	Defl. Increment	Defl. Angle
33+47.179	17.178	0°45'26"	5°00'00"
33+30.000	29.997	1°19'20"	4°14'34"
33+00.000	29.997	1°19'20"	2°55'14"
32+70.000	29.997	1°19'20"	1°35'54"
32+40.000	6.268	0°16'34"	0°16'34"
32+33.732			

24.10 Highway curve with $R = 600$ m, $I = 12°30'$, and PI station = 4+200.600 m.

```
                Intersection Angle = 12°30'00"
              Degree of Curvature = 2°54'38"
                           Radius = 600.000
          Circular Curve Length = 130.900
               Tangent Distance = 65.711
     Circular Curve Long Chord = 130.640
                Middle Ordinate = 3.566
                       External = 3.588

                  PI Stationing = 42+00.600
                 42+65.789 Back = 42+66.311 Ahead
```

Station	Chord	Defl. Increment	Defl. Angle
42+65.789	5.789	0°16'35"	6°15'00"
42+60.000	29.997	1°25'57"	5°58'25"
42+30.000	29.997	1°25'57"	4°32'28"
42+00.000	29.997	1°25'57"	3°06'32"
41+70.000	29.997	1°25'57"	1°40'35"
41+40.000	5.111	0°14'38"	0°14'38"
41+34.889			

24.11 Highway curve with $R = 850$ ft, $I = 40°00'$, and PI station = 45 + 50.00 ft.

```
                Intersection Angle = 40°00'00"
               Degree of Curvature = 4°42'21"
                            Radius = 1,217.54
          Circular Curve Length = 850.00
               Tangent Distance = 443.15
     Circular Curve Long Chord = 832.84
                Middle Ordinate = 73.43
                       External = 78.14

                  PI Stationing = 45+50.00
                  49+56.85 Back = 49+93.15 Ahead
    ============================================================
```

Station	Chord	Defl. Increment	Defl. Angle
49+56.85	56.85	1°20'16"	20°00'00"
49+00.00	99.97	2°21'11"	18°39'44"
48+00.00	99.97	2°21'11"	16°18'34"
47+00.00	99.97	2°21'11"	13°57'23"
46+00.00	99.97	2°21'11"	11°36'12"
45+00.00	99.97	2°21'11"	9°15'02"
44+00.00	99.97	2°21'11"	6°53'51"
43+00.00	99.97	2°21'11"	4°32'41"
42+00.00	93.12	2°11'30"	2°11'30"
41+06.85			

24.12 Highway curve with $L = 270$ m, $R = 600$ m, and PI station = 4 + 350.000 m.

```
                Intersection Angle = 25°46'59"
               Degree of Curvature = 2°54'38"
                            Radius = 600.000
          Circular Curve Length = 270.000
               Tangent Distance = 137.325
     Circular Curve Long Chord = 267.728
                Middle Ordinate = 15.124
                       External = 15.515
                  PI Stationing = 4+350.000
                  4+482.675 Back = 4+487.325 Ahead
    ============================================================
```

Station	Chord	Defl. Increment	Defl. Angle
4+482.675	12.675	0°36'19"	12°53'30"
4+470.000	29.997	1°25'57"	12°17'11"
4+440.000	29.997	1°25'57"	10°51'14"
4+410.000	29.997	1°25'57"	9°25'18"
4+380.000	29.997	1°25'57"	7°59'21"
4+350.000	29.997	1°25'57"	6°33'24"
4+320.000	29.997	1°25'57"	5°07'28"
4+290.000	29.997	1°25'57"	3°41'31"
4+260.000	29.997	1°25'57"	2°15'35"
4+230.000	17.325	0°49'38"	0°49'38"
4+212.675			

24.13 Highway curve with $T = 229.23$ ft, $R = 1300$ ft, and PI station = 87 + 50.00 ft.

$$I = 2\,\text{atan}\left(\frac{229.23}{1300}\right) = 20°00'02"$$

```
Intersection Angle = 20°00'02"
Degree of Curvature = 4°24'27"
Radius = 1,300.00
Circular Curve Length = 453.80
Tangent Distance = 229.23
Circular Curve Long Chord = 451.50
Middle Ordinate = 19.75
External = 20.06

PI Stationing = 87+50.00
89+74.57 Back = 89+79.23 Ahead
```

Station	Chord	Defl. Increment	Defl. Angle
89+74.57	74.56	1°38'36"	10°00'01"
89+00.00	99.98	2°12'13"	8°21'25"
88+00.00	99.98	2°12'13"	6°09'12"
87+00.00	99.98	2°12'13"	3°56'59"
86+00.00	79.22	1°44'46"	1°44'46"
85+20.77			

24.14 Railroad curve with $T = 150.00$ ft, $D_C = 2°30'$, and PI station = 48 + 00.00 ft.

$$R = 50/\sin\left(\frac{2°30'}{2}\right) = 2292.01 \text{ ft}$$

$$I = 2\,\text{atan}\left(\frac{150}{2292.01}\right) = 7°29'20"$$

```
Intersection Angle = 7°29'20"
Degree of Curvature = 2°30'00"
Radius = 2,292.00
Circular Curve Length = 299.55
Tangent Distance = 150.00
Circular Curve Long Chord = 299.36
Middle Ordinate = 4.89
External = 4.90

PI Stationing = 48+00.00
49+49.55 Back = 49+50.00 Ahead
```

Station	Chord	Defl. Increment	Defl. Angle
49+49.55	49.55	0°37'010"	3°44'40"
49+00.00	100.00	1°15'00"	3°07'30"
48+00.00	100.00	1°15'00"	1°52'30"
47+00.00	50.01	0°37'30"	0°37'30"
46+50.00			

In Problems 24.15 through 24.18 tabulate the curve data, deflection angles, and total chords needed to lay out the following circular curves at full-station increments using a total station instrument set up at the PC.

24.15 The curve of Problem 24.7.

```
                    Intersection Angle = 15°30'00"
                  Degree of Curvature = 3°30'00"
                               Radius = 1,637.02
              Circular Curve Length = 442.86
                   Tangent Distance = 222.79
          Circular Curve Long Chord = 441.51
                    Middle Ordinate = 14.95
                           External = 15.09

                      PI Stationing = 30+44.50
                      32+64.57 Back = 32+67.29 Ahead
     ============================================================

        Station |      Chord |   Defl. Increment |       Defl. Angle |
    ============================================================================
       32+64.57 |     441.51 |          1°07'48" |          7°45'00" |
       32+00.00 |     377.45 |          1°45'00" |          6°37'12" |
       31+00.00 |     277.95 |          1°45'00" |          4°52'12" |
       30+00.00 |     178.20 |          1°45'00" |          3°07'12" |
       29+00.00 |      78.28 |          1°22'12" |          1°22'12" |
       28+21.71
    ============================================================================
```

24.16 The curve of Problem 24.8

```
                    Intersection Angle = 15°00'00"
                  Degree of Curvature = 3°00'00"
                               Radius = 1,910.08
              Circular Curve Length = 500.00
                   Tangent Distance = 251.47
          Circular Curve Long Chord = 498.63
                    Middle Ordinate = 16.34
                           External = 16.48

                      PI Stationing = 24+50.50
                      26+99.03 Back = 27+01.97 Ahead
     ============================================================

        Station |      Chord |   Defl. Increment |       Defl. Angle |
    ============================================================================
       26+99.03 |     498.63 |          1°29'08" |          7°30'00" |
       26+00.00 |     400.28 |          1°30'00" |          6°00'52" |
       25+00.00 |     300.69 |          1°30'00" |          4°30'52" |
       24+00.00 |     200.90 |          1°30'00" |          3°00'52" |
       23+00.00 |     100.97 |          1°30'00" |          1°30'52" |
       22+00.00 |       0.97 |          0°00'52" |          0°00'52" |
       21+99.03
    ============================================================================
```

24.17 The curve of Problem 24.9

```
                    Intersection Angle = 10°00'00"
                  Degree of Curvature = 2°41'12"
                              Radius = 650.000
                 Circular Curve Length = 113.446
                      Tangent Distance = 56.868
            Circular Curve Long Chord = 113.302
                       Middle Ordinate = 2.473
                              External = 2.483

                        PI Stationing = 3+290.600
                       3+347.179 Back = 3+347.468 Ahead
      ============================================================

        Station |      Chord |   Defl. Increment |       Defl. Angle |
=================================================================================
      3+347.179 |    113.302 |          0°45'26" |          5°00'00" |
      3+330.000 |     96.180 |          1°19'20" |          4°14'34" |
      3+300.000 |     66.239 |          1°19'20" |          2°55'14" |
      3+270.000 |     36.263 |          1°19'20" |          1°35'54" |
      3+240.000 |      6.268 |          0°16'34" |          0°16'34" |
      3+233.732
=================================================================================
```

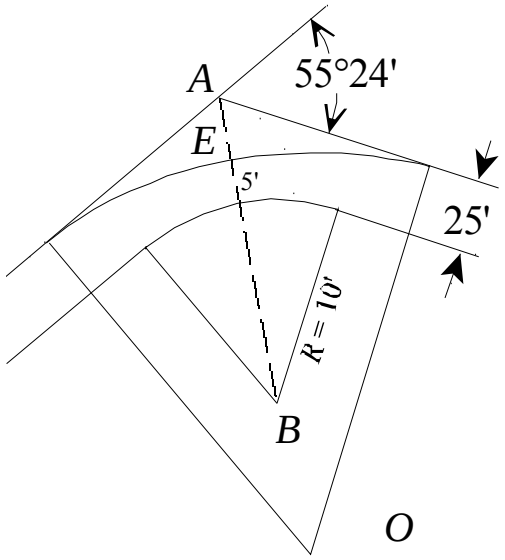
24.18 The curve of Problem 24.10

```
                    Intersection Angle = 12°30'00"
                  Degree of Curvature = 2°54'38"
                              Radius = 600.000
                 Circular Curve Length = 130.900
                      Tangent Distance = 65.711
            Circular Curve Long Chord = 130.640
                       Middle Ordinate = 3.566
                              External = 3.588

                        PI Stationing = 4+200.600
                       4+265.789 Back = 4+266.311 Ahead
      ============================================================

        Station |      Chord |   Defl. Increment |       Defl. Angle |
=================================================================================
      4+265.789 |    130.640 |          0°16'35" |          6°15'00" |
      4+260.000 |    124.884 |          1°25'57" |          5°58'25" |
      4+230.000 |     95.011 |          1°25'57" |          4°32'28" |
      4+200.000 |     65.079 |          1°25'57" |          3°06'32" |
      4+170.000 |     35.106 |          1°25'57" |          1°40'35" |
      4+140.000 |      5.111 |          0°14'38" |          0°14'38" |
      4+134.889
=================================================================================
```

24.19 A rail line on the center of a 80-ft street makes a 55°24′ turn into another street of equal width. The corner curb line has R = 10 ft. What is the largest R that can be given a circular curve for the track centerline if the law requires it to be at least 5 ft from the curb?



$$AB = (25 + 10)/\cos\left(\frac{55°24'}{2}\right) = 39.530 \text{ ft}$$

$AE = 39.53 - (10+5) = 24.53$ ft

$$R = \frac{24.53}{1/\cos\left(\frac{55°24'}{2}\right) - 1} = 189.51 \text{ ft}$$

Tabulate all data required to lay out by deflection angles and incremental chords, at the indicated stationing, for the circular curves of Problems 24.20 and 24.21.

24.20 The R for a highway curve (arc definition) will be rounded off to the nearest larger multiple of 100 ft. Field conditions require M to be approximately 24 ft to avoid an embankment. The PI = 94 + 18.70 and I = 25°00′ with stationing at 100 ft.

$$R = \frac{24}{1-\cos\left(\frac{25}{2}\right)} = 1012.49 \text{ ft so round } R \text{ to } 1100 \text{ ft.}$$

```
                Intersection Angle = 25°00'00"
               Degree of Curvature = 5°12'31"
                            Radius = 1,100.00
          Circular Curve Length = 479.97
               Tangent Distance = 243.86
        Circular Curve Long Chord = 476.17
                 Middle Ordinate = 26.07
                        External = 26.71

                     PI Stationing = 94+18.70
                     96+54.80 Back = 96+62.56 Ahead
```

Station	Chord	Defl. Increment	Defl. Angle
96+54.80	54.80	1°25'38"	12°30'00"
96+00.00	99.97	2°36'16"	11°04'22"
95+00.00	99.97	2°36'16"	8°28'06"
94+00.00	99.97	2°36'16"	5°51'51"
93+00.00	99.97	2°36'16"	3°15'35"
92+00.00	25.16	0°39'19"	0°39'19"
91+74.84			

24.21 For a highway curve R will be rounded off to the nearest larger multiple of 10 m. Field measurements show that T should be approximately 85 m to avoid an overpass. The PI = 6 + 356.400 m and $I = 13°20'$ with stationing at 30 m.

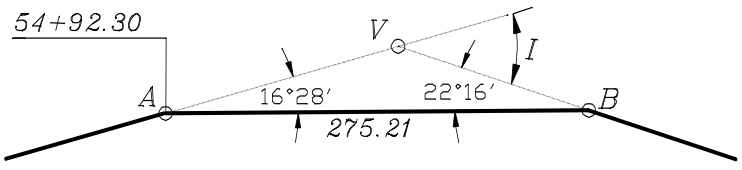
$$R = \frac{85}{\tan\frac{13°20'}{2}} = 727.221 \text{ m, so round to 730 m.}$$

```
                Intersection Angle = 13°20'00"
              Degree of Curvature = 2°23'32"
                          Radius = 730.000
         Circular Curve Length = 169.879
               Tangent Distance = 85.325
     Circular Curve Long Chord = 169.496
                Middle Ordinate = 4.936
                        External = 4.970

                  PI Stationing = 6+356.400
                 6+440.954 Back = 6+441.725 Ahead
=============================================================
```

Station	Chord	Defl. Increment	Defl. Angle
6+440.954	20.953	0°49'20"	6°40'00"
6+420.000	29.998	1°10'38"	5°50'40"
6+390.000	29.998	1°10'38"	4°40'01"
6+360.000	29.998	1°10'38"	3°29'23"
6+330.000	29.998	1°10'38"	2°18'45"
6+300.000	28.923	1°08'06"	1°08'06"
6+271.075			

24.22 A highway survey PI falls in a pond, so a cut off line AB = 275.12 ft is run between the tangents. In the triangle formed by points A, B, and PI, the angle at $A = 16°28'$ and at $B = 22°16'$. The station of A is 54+92.30 ft. Calculate and tabulate curve notes to run, by deflection angles and incremental chords, a 4°30′ (arc definition) circular curve at full-station increments to connect the tangents.



$I = 16°28' + 22°16' = 38°44'$

$$AV = \frac{275.21\sin(22°16')}{\sin(180° - 38°44')} = 166.665 \text{ ft}$$

PI = 54 + 92.30 + 166.66 = 56 + 58.96

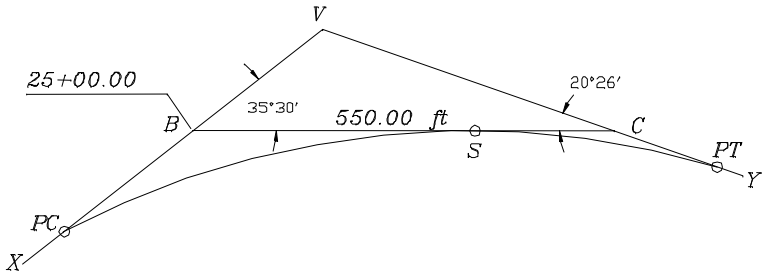
```
                Intersection Angle = 38°44'00"
              Degree of Curvature = 4°30'00"
                          Radius = 1,273.24
         Circular Curve Length = 860.74
               Tangent Distance = 447.55
```

```
          Circular Curve Long Chord = 844.44
                   Middle Ordinate = 72.05
                          External = 76.37

                     PI Stationing = 56+58.96
                     60+72.15 Back = 61+06.51 Ahead
=============================================================

        Station |      Chord |   Defl. Increment |       Defl. Angle |
=============================================================================
       60+72.15 |      72.15 |          1°37'25" |         19°22'00" |
       60+00.00 |      99.97 |          2°15'00" |         17°44'35" |
       59+00.00 |      99.97 |          2°15'00" |         15°29'35" |
       58+00.00 |      99.97 |          2°15'00" |         13°14'35" |
       57+00.00 |      99.97 |          2°15'00" |         10°59'35" |
       56+00.00 |      99.97 |          2°15'00" |          8°44'35" |
       55+00.00 |      99.97 |          2°15'00" |          6°29'35" |
       54+00.00 |      99.97 |          2°15'00" |          4°14'35" |
       53+00.00 |      88.57 |          1°59'35" |          1°59'35" |
       52+11.41
=============================================================================
```

24.23 In the figure, a single circular highway curve (arc definition) will join tangents *XV* and *VY* and also be tangent to *BC*. Calculate *R*, *L*, and the stations of the PC and PT.



$$I = 35°30' + 20°26' = 55°56'$$

$$R = \frac{550}{\tan(35°30'/2) + \tan(20°20'/2)} = 1099.27 \text{ ft}$$

$$L = 1099.27I = 1073.13 \text{ ft}$$

$$T = 1099.27 \tan(55°56'/2) = 583.67 \text{ ft}$$

$$BV = \frac{550.00 \sin(20°26')}{\sin(55°56')} = 231.79 \text{ ft}$$

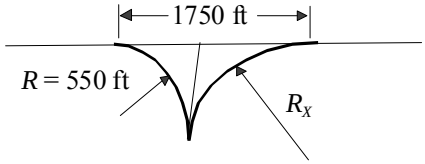
PI = 2500 + 231.79 = 27 + 31.79

PC = 2731.79 – 583.67 = 21 + 48.12

PT_{Back} = 2148.12 + 1073.13 = 32 + 21.25

$PT_{Forward}$ = 2731.79 + 583.67 = 33 + 15.46

24.24* Compute R_x to fit requirements of the figure and make the tangent distances of the two curves equal.



<u>1392.04 ft</u>

$T_x = 1750/2 = 875$

$I = 2\tan^{-1}(875/550) = 115°41'43.5''$

$I_x = 180° - I = 64°18'16.5''$

$$R_x = \frac{875}{\tan(64°18'16.5'')} = 1392.04 \text{ ft}$$

24.25 After a backsight on the PC with $0°00'$ set on the instrument, what is the deflection angle to the following circular curve points?

(a)* Setup at curve midpoint, deflection to the PT. ***I*/2**

(b) Instrument at curve midpoint, deflection to the 3/4 point. **3/8*I***

(c) Setup at 1/4 point of curve, deflection to 3/4 point. **3/8*I***

24.26 In surveying a construction alignment, why should the *I* angle be measured using both faces of the instrument?

To account for possible instrumental errors, to increase the precision of the observation, and to check for possible mistakes.

24.27 A highway curve (arc definition) to the right, having R = 550 m and $I = 18°30'$, will be laid out by coordinates with a total station instrument setup at the PI. The PI station is 3 + 855.200 m, and its coordinates are X = 75,428.863 m and Y = 36,007.434 m. The azimuth (from north) of the back tangent proceeding toward the PI is $48°17'12''$. To orient the total station, a backsight will be made on a POT on the back tangent. Compute lengths and azimuths necessary to stake the curve at 30-m stations.

Station	δ_a	Total δ_a	Chord	Chord Azimuth
3+943.214	0°17'55"	9°15'00"	160.743	57°32'12"
3+930.000	1°43'08"	8°57'05"	155.595	57°14'17"
3+900.000	1°43'08"	7°13'57"	125.895	55°31'09"
3+870.000	1°43'08"	5°30'49"	96.082	53°48'01"
3+840.000	1°43'08"	3°47'41"	66.182	52°04'53"
3+810.000	1°43'08"	2°04'33"	36.222	50°21'45"
3+780.000	0°21'25"	0°21'25"	6.230	48°38'37"
3+765.627				48°17'12"

24.28 In Problem 24.27, compute the *XY* coordinates at 30-m stations.

Station	Azimuth	Chord	*X*	*Y*
3+943.214	57°32'12"	160.74	75,497.623	36,034.112
3+930.000	57°14'17"	155.6	75,492.842	36,032.032
3+900.000	55°31'09"	125.9	75,465.775	36,019.105
3+870.000	53°48'01"	96.082	75,439.533	36,004.578

3+840.000	52°04'53"	66.182	75,414.208	35,988.503
3+810.000	50°21'45"	36.222	75,389.893	35,970.939
3+780.000	48°38'37"	6.230	75,366.674	35,951.948
3+765.627	48°17'12"		75,361.998	35,947.832

24.29 A exercise track must consist of two semicircles and two tangents, and be exactly 1000 m along its centerline. The two tangent sections are 300 m each. Calculate the radius for the curves.

Curves = 1000 − 600 = 400 m

R = 400/(2π) = **31.831 m**

What sight distance is available if there is an obstruction on a radial line through the PI inside the curves in Problems 24.31 and 24.32?

24.30* For Problem 24.7, obstacle 15 ft from curve.

By Equation 24.24: $C = \sqrt{8(15)1637.02} = \mathbf{443\ ft}$

24.31 For Problem 24.12, obstacle 10 m from curve.

By Equation 24.24: $C = \sqrt{8(10)600} = \mathbf{219\ m}$

24.32 If the misclosure for the curve of Problem 24.7, computed as described in Section 24.8, is 0.12 ft, what is the field layout precision?

Precision = 0.12/[2(222.79) + 442.86], or 1:7400

24.33 Assume that a 100-ft entry spiral will be used with the curve of Problem 24.7. Compute and tabulate curve notes to stake out the alignment from the TS to ST at full stations using a total station and the deflection-angle, total chord method.

```
                          Spiral Angle: 1°45'00"
                          Spiral Throw: 0.25
                  Spiral Long Tangent: 66.67
                 Spiral Short Tangent: 33.34
                        Spiral Length: 100.00
               Spiral Long Chord Length: 100.00

             Exit spiral notes for layout from ST to CS
                    with tangent as backsight.

             Station |      Chord |       Defl. Angle |
            ============================================
ST          33+14.54 |                                 |
            33+00.00 |      14.54 |          0°00'44" |
CS          32+14.54 |     100.00 |          0°35'00" |

               Horizontal Curve Notes -- Arc Definition
               ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
```

```
                        Defining Curve Parameters
          ============================================================
          Curve to the right of the back tangent as viewed from the PC.
                Intersection Angle = 15°30'00" (Back to Forward
Tangent)
      Circular Curve Intersection Angle = 12°00'00"
                   Degree of Curvature = 3°30'00"
                               Radius = 1,637.02
                  Circular Curve Length = 342.86
                Tangent Distance (TS-PI) = 272.82
               Circular Curve Long Chord = 342.23
                    Long Chord (TS - ST) = 540.66
                               External = 16.12
         Circular Curve Tangent Distance = 172.06

                           PI Stationing = 30+44.50
                           33+14.54 Back = 33+17.32 Ahead
          ============================================================

         Station |      Chord |   Defl. Increment |       Defl. Angle |
=================================================================================
       32+14.54 |     342.23 |          0°15'16" |          6°00'00" |
       32+00.00 |     327.77 |          1°45'00" |          5°44'44" |
       31+00.00 |     228.14 |          1°45'00" |          3°59'44" |
       30+00.00 |     128.29 |          1°45'00" |          2°14'44" |
       29+00.00 |      28.32 |          0°29'44" |          0°29'44" |
       28+71.68
=================================================================================

                        ************************************
                        *******  Spiral Staking Notes  *******
                        ************************************

                    Station |      Chord |       Defl. Angle |
                   ============================================
       SC            28+71.68 |     100.00 |          0°35'00" |
                     28+00.00 |      28.32 |          0°02'48" |
       TS            27+71.68 |                               |
```

24.34 Same as Problem 24.33, except use a 200-ft spiral for the curve of Problem 24.8.

```
                                  Spiral Angle: 2°59'59"
                                  Spiral Throw: 0.87
                            Spiral Long Tangent: 133.35
                           Spiral Short Tangent: 66.68
                                  Spiral Length: 200.00
                        Spiral Long Chord Length: 199.98

                       Exit spiral notes for layout from ST to CS
                              with tangent as backsight.

                    Station |      Chord |       Defl. Angle |
                   ============================================
       ST            27+98.98 |                               |
                     27+00.00 |      98.98 |          0°14'42" |
                     26+00.00 |     198.96 |          0°59'23" |
```

```
CS        25+98.98 |     199.98 |          1°00'00" |


                  Horizontal Curve Notes -- Chord Definition
                  ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~

                              Defining Curve Parameters
            ============================================================
              Curve to the right of the back tangent as viewed from the PC.
                   Intersection Angle = 15°00'00" (Back to Forward Tangent)
          Circular Curve Intersection Angle = 9°00'02"
                        Degree of Curvature = 3°00'00"
                                     Radius = 1,910.08
                        Circular Curve Length = 300.06
                     Tangent Distance (TS-PI) = 351.57
                    Circular Curve Long Chord = 299.75
                         Long Chord (TS - ST) = 697.13
                                    External = 20.00
            Circular Curve Tangent Distance = 150.34

                              PI Stationing = 24+50.50
                              27+98.98 Back = 28+02.07 Ahead
            ============================================================

          Station |      Chord |   Defl. Increment |       Defl. Angle |
        ==========================================================================
          25+98.98 |     299.75 |          1°29'05" |          4°30'01" |
          25+00.00 |     200.97 |          1°30'00" |          3°00'56" |
          24+00.00 |     101.04 |          1°30'00" |          1°30'56" |
          23+00.00 |       1.07 |          0°00'58" |          0°00'56" |
          22+98.93
        ==========================================================================

                            ************************************
                            *******  Spiral Staking Notes  *******
                            ************************************

                     Station |      Chord |       Defl. Angle |
                    ============================================
          SC          22+98.93 |     199.98 |          1°00'00" |
                      22+00.00 |     101.07 |          0°15'19" |
                      21+00.00 |       1.07 |          0°00'00" |
          TS          20+98.93 |                               |
```

24.35 Same as Problem 24.33, except for the curve of Problem 24.9, with a 50-m entry spiral using stationing of 30 m and a total station instrument.

```
                                  Spiral Angle: 2°12'13"
                                  Spiral Throw: 0.160
                             Spiral Long Tangent: 33.336
                            Spiral Short Tangent: 16.669
                                  Spiral Length: 50.000
                         Spiral Long Chord Length: 49.997

                        Exit spiral notes for layout from ST to CS
```

with tangent as backsight.

	Station	Chord	Defl. Angle
ST	3+372.166		
	3+360.000	12.166	0°02'37"
	3+330.000	42.165	0°31'21"
CS	3+322.166	49.997	0°44'04"

Horizontal Curve Notes -- Arc Definition

Defining Curve Parameters

Curve to the right of the back tangent as viewed from the PC.
Intersection Angle = 10°00'00" (Back to Forward Tangent)
Circular Curve Intersection Angle = 5°35'33"
Degree of Curvature = 2°41'12"
Radius = 650.000
Circular Curve Length = 63.446
Tangent Distance (TS-PI) = 81.880
Circular Curve Long Chord = 63.421
Long Chord (TS - ST) = 163.138
External = 3.126
Circular Curve Tangent Distance = 31.748

PI Stationing = 3+290.600
3+372.166 Back = 3+372.480 Ahead

Station	Chord	Defl. Increment	Defl. Angle
3+322.166	63.421	0°58'37"	2°47'47"
3+300.000	41.273	1°19'20"	1°49'010"
3+270.000	11.280	0°29'50"	0°29'50"
3+258.720			

******* Spiral Staking Notes *******

	Station	Chord	Defl. Angle
SC	3+258.720	49.997	0°44'04"
	3+240.000	31.280	0°17'15"
	3+210.000	1.280	0°00'02"
TS	3+208.720		

24.36 Compute the area bounded by the two arcs and tangent in Problem 24.24.

306,460 ft^2

Area of parallelogram; = 1,699,285

Area of Sector R = 305,414

Area of Sector R_x = 1,087,408

Area = 1699285 − 305,414 - 1,087,408 = 306,460

24.37 In an as-built survey, the XY coordinates in meters of three points on the centerline of a highway curve are determined to be A: (3770.52, 4913.84); B: (3580.80, 4876.37); C: (3399.27, 4809.35). What are the radius, and coordinates for the center of the curve in meters?

```
Center of Circle at: X = 3,911.375
                     Y = 3,701.621
With A Radius of   : 1,220.375
```

Matrix setup:

$$\begin{bmatrix} 7541.04 & 9827.68 & -1 \\ 7161.60 & 9752.74 & -1 \\ 6798.54 & 9618.70 & -1 \end{bmatrix} \begin{bmatrix} X_O \\ Y_O \\ f \end{bmatrix} = -\begin{bmatrix} 3770.52^2 + 4913.84^2 \\ 3580.80^2 + 4876.37^2 \\ 3399.27^2 + 4809.35^2 \end{bmatrix} = -\begin{bmatrix} 38{,}362{,}644.616 \\ 36{,}601{,}113.0169 \\ 34{,}684{,}883.9554 \end{bmatrix}$$

Solution: $X = A^{-1}L = \begin{bmatrix} -3911.375 \\ -3701.621 \\ 27{,}511{,}534.994 \end{bmatrix}$

By Eq. (24.35): $R = \sqrt{(-3911.375)^2 + (-3701.621)^2 - 27{,}511{,}534.994} = 1220.375$

24.38 In Problem 24.37, if the (x, y) coordinates in meters of two points on the centerline of the tangents are (3042.28, 4616.77) and (4435.66, 4911.19), what are the coordinates of the PC, PT, and the curve parameters L, T, and I?

PC: (3324.32, 4771.52)
PT: (3937.16, 4921.72)
***I* = 29°57′52″**
***L* = 638.23**
***T* = 326.59**

$$O1 = \sqrt{(3042.28 - 3911.38)^2 + (4616.77 - 3701.62)^2} = 1262.07$$

$$Az_{O1} = \tan^{-1}\left(\frac{-869.10}{915.15}\right) + 360° = 316°28'43"$$

$$O2 = \sqrt{(4435.66 - 3911.38)^2 + (4911.19 - 3701.62)^2} = 1318.31$$

$$Az_{O2} = \tan^{-1}\left(\frac{524.28}{1209.57}\right) + 0° = 23°26'03.2"$$

Solve triangle O-1-PC: $1 - PC = \sqrt{1262.07^2 - 1220.38^2} = 321.70$

$\angle$1-O-PC = acos(1220.38/122.07) = 14°46′04″

Solve triangle O-2-PT: $2 - PT = \sqrt{1318.31^2 - 1220.38^2} = 498.61$

$\angle$PT-O-2 = acos(1220.38/1318.31) = 22°13′25″

Az_{O1} = 316°28'43" + 14°46'04" = 331°14'47"
Az_{O2} = 23°26′03″ – 22°13′25″ = 1°12′38″

X_{PC} = 3911.38 + 1220.38 sin(331°14′47″) = 3324.32
Y_{PC} = 3701.62 + 1220.38 cos(331°14′47″) = 4771.52
X_{PT} = 3911.38 + 1220.38 sin(1°12′38″) = 3937.16
Y_{PT} = 3701.62 + 1220.38 cos(1°12′38″) = 4921.72

I = 360° + 1°12′38″ – 331°14′47″ = 29°57′52″
L = (1220.38)(29°57′52″)(π/180°) = 638.23
T = 1220.38 tan(29°57′52″/2) = 326.59