3 THEORY OF ERRORS IN OBSERVATIONS

3.1 Discuss the difference between an error and a residual.

From Section 3.3, an error is the difference between the observation and its true value, or $E = X - \bar{X}$ whereas a residual, which is defined in Section 3.11 is the difference between the mean of a set of observations and the observation or $v = \bar{M} - M$

3.2 Give two examples of **(a)** direct and **(b)** indirect measurements.

From Section 3.2: A direct observation is made by applying a measurement instrument directly to a quantity to be measured and an indirect observation is made by computing a quantity from direct observations.

Examples should vary by student response.

3.3 Define the term *systematic error*, and give two surveying examples of a systematic error.

See Section 3.6

3.4 Define the term *random error,* and give two surveying examples of a random error.

See Section 3.6

3.5 Discuss the difference between accuracy and precision.

From Section 3.7, accuracy is the nearness of the observed quantities to the true value, which is never known. Precision is the degree of refinement or consistency of a group of observations and is evaluated on the basis of discrepancy size.

A distance *AB* is observed repeatedly using the same equipment and procedures, and the results, in meters, are listed in Problems 3.6 through 3.10. Calculate **(a)** the line's most probable length, **(b)** the standard deviation and **(c)** the standard deviation of the mean for each set of results.

3.6* 65.401, 65.400, 65.402, 65.396, 65.406, 65.401, 65.396, 65.401, 65.405, and 65.404

(a) **<u>65.401</u>** $\sum 654.012$

(b) **<u>±0.003</u>** $\sum v^2 = 0.000104$

(c) **<u>±0.001</u>**

3.7 Same as Problem 3.6, but discard one observation, 65.406.

(a) **<u>65.401</u>** $\sum 588.606$

(b) **±0.003** $\sum v^2 = 0.00008$

(c) **±0.001**

3.8 Same as Problem 3.6, but discard two observations, 65.405 and 65.406.

(a) **65.402** ∑523.201

(b) **±0.003** $\sum v^2 = 0.000048$

(c) **±0.001**

3.9 Same as Problem 3.6, but include two additional observations, 65.408 and 65.409.

(a) **65.402** ∑784.829

(b) **±0.004** $\sum v^2 = 0.000191$

(c) **±0.001**

3.10 Same as Problem 3.6, but include three additional observations, 65.408, 65.409, and 65.410.

(a) **65.403** ∑850.239

(b) **±0.004** $\sum v^2 = 0.000244$

(c) **±0.001**

In Problems 3.10 through 3.14, determine the range within which observations should fall **(a)** 90% of the time and **(b)** 95% of the time. List the percentage of values that actually fall within these ranges.

(a) $E_{90} = 1.6449\sigma$ (3.7) (b) $E_{95} = 1.9599\sigma$ (3.8)

3.11 For the data of Problem 3.6.

***(a)** **65.4012±0.0052 (65.3960, 65.4064), 100%**

(b) **65.4012±0.0062 (65.3950, 65.4074), 100%**

3.12 For the data of Problem 3.7.

(a) **65.4007±0.0048 (65.3959, 65.4054), 100%**

(b) **65.4007±0.00057 (65.3950, 65.4064), 100%**

3.13 For the data of Problem 3.8.

(a) **65.4001±0.0043 (65.3958, 65.4044), 100%**

(b) **65.4001±0.0051 (65.3950, 65.4053), 100%**

3.14 For the data of Problem 3.9.

(a) **65.4024±0.0068 (65.3956, 65.4093), 91.6%**

(b) **<u>65.4024±0.0068 (65.3943, 65.4106), 100%</u>**

In Problems 3.15 through 3.17, an angle is observed repeatedly using the same equipment and procedures. Calculate **(a)** the angle's most probable value, **(b)** the standard deviation, and **(c)** the standard deviation of the mean.

3.15* $23°30'00''$, $23°29'40''$, $23°30'15''$, and $23°29'50''$.

(a) **<u>23°29′56″</u>**

(b) **<u>±14.9″</u>**

(c) **<u>±7.5″</u>**

3.16 Same as Problem 3.15, but with three additional observations, $23°29'40''$, $23°29'45''$, and $23°20'50''$.

(a) **<u>23°29′51″</u>**

(b) **<u>±12.5″</u>**

(c) **<u>±4.7″</u>**

3.17 Same as Problem 3.15, but with two additional observations, $23°30'05''$ and $23°29'55''$.

(a) **<u>23°29′58″</u>**

(b) **<u>±12.1″</u>**

(c) **<u>±5.0″</u>**

3.18* A field party is capable of making taping observations with a standard deviation of $\pm 0.010\text{ ft}$ per 100-ft tape length. What standard deviation would be expected in a distance of 200 ft taped by this party?

By Equation (3.12): **<u>±0.014 ft</u>** $= 0.010\sqrt{2}$

3.19 Repeat Problem 3.18, except that the standard deviation per 30-m tape length is $\pm 0.005\text{ m}$ and a distance of 90 m is taped. What is the expected 95% error in 90 m?

By Equation (3.12): **<u>±0.009 m</u>** $= 0.005\sqrt{3}$;

By Equation (3.8): **<u>±0.017 m</u>** $= 1.9559(0.00866)$

3.20 A distance of 200 ft must be taped in a manner to ensure a standard deviation smaller than $\pm 0.05\text{ ft}$. What must be the standard deviation per 100 ft tape length to achieve the desired precision?

<u>±0.035 ft</u> $= \pm 0.05/\sqrt{2}$ by Equation (3.12) rearranged.

3.21 Lines of levels were run requiring n instrument setups. If the rod reading for each backsight and foresight has a standard deviation σ, what is the standard deviation in each of the following level lines?

(a) $n = 15, \sigma = \pm 0.015 \text{ ft}$ By Equation (3.12): **<u>±0.058 ft</u>** = $0.015\sqrt{15}$

(b) $n = 28, \sigma = \pm 5 \text{ mm}$ By Equation (3.12): **<u>±26.4 mm</u>** = $5\sqrt{28}$

3.22 A line *AC* was observed in 2 sections *AB* and *BC*, with lengths and standard deviations listed below. What is the total length *AC*, and its standard deviation?

(a)* $AB = 60.00 \pm 0.015 \text{ ft}; BC = 86.13 \pm 0.018 \text{ ft}$; **<u>146.13±0.023 ft</u>** by Equation (3.11)

(b) $AB = 30.000 \pm 0.005 \text{ m}; 15.413 \pm 0.005 \text{ m}$; **<u>45.413±0.007 m</u>** by Equation (3.11)

3.23 Line *AD* is observed in three sections, *AB*, *BC*, and *CD*, with lengths and standard deviations as listed below. What is the total length *AD* and its standard deviation?

(a) $AB = 236.57 \pm 0.01 \text{ ft}; BC = 608.99 \pm 0.01 \text{ ft}; CD = 426.87 \pm 0.01 \text{ ft}$

<u>1272.43 ± 0.017 ft</u> by Equation (3.11)

(b) $AB = 688.980 \pm 0.003 \text{ m}; BC = 1274.865 \text{ m} \pm 0.003 \text{ m}; CD = 2542.373 \text{ m} \pm 0.005 \text{ m}$

<u>4506.218 ± 0.006 m</u> by Equation (3.11)

3.24 The difference in elevation between *A* and *B* was observed four times as 29.85, 29.83, 29.88 and 29.79 ft. The observations were given weights of 2, 1, 3 and 2, respectively, by the observer. ***(a)** Calculate the weighted mean for distance *AB*. (**b**) What difference results if later judgment revises the weights to 2, 3, 1, and 1, respectively?

By Equation (3.17):

(a)* **<u>29.831 ft</u>**

(b) **29.837 ft**

3.25 Determine the weighted mean for the following angles:

By Equation (3.17):

(a) $222°12'36''$, wt 2; $222°12'42''$, wt 1; $222°12'34''$, wt 3 **<u>222°12′36″</u>**

(b) $106°28'54'' \pm 1''; 106°28'46'' \pm 3''; 106°28'56'' \pm 1''$ **<u>106°28′54.5″</u>**

$$\frac{\frac{54}{1^2} + \frac{46}{3^2} + \frac{56}{1^2}}{\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{1^2}} = 54.5''$$

3.26 Specifications for observing angles of an n-sided polygon limit the total angular misclosure to E. How accurately must each angle be observed for the following values of n and E?

By rearranged Equation (3.12):

(a) $n = 8, E = 8''$ **<u>±2.8″</u>**

(b) $n = 16, E = 12''$ **<u>±3.0″</u>**

S1 **3.27** What is the area of a rectangular field and its estimated error for the following recorded values:

By Equation (3.13):

(a)* 243.89 ± 0.05 ft, by 208.65 ± 0.04 ft **<u>50,888 ± 14 ft²</u>** or **<u>1.1682 ± 0.0003 ac</u>**

(b) 660.23 ± 0.012 ft by 1425.67 ± 0.013 ft **<u>941,270 ± 20 ft²</u>** or **<u>21.608 ± 0.0004 ac</u>**

(c) 304.206 ± 0.005 m, by 758.234 ± 0.006 m **<u>230,659 ± 4.4 m²</u>** or **<u>23.0659 ± 0.00044 ha</u>**

3.28 Adjust the angles of triangle *ABC* for the following angular values and weights:

By Equation (3.17):

(a)* $A = 49°24'22''$, wt 2; $B = 39°02'16''$, wt 1; $C = 91°33'00''$, wt 3

Misclosure = −22″

	Obs. Ang.	**Wt**	**Corr.**	**Num. Cor.**	**Rnd. Cor.**	**Adj. Ang.**
A	49°24′22″	2	3x	6″	6″	**49°24′28″**
B	39°02′16″	1	6x	12″	12″	**39°02′28″**
C	91°33′00″	<u>3</u>	<u>2x</u>	4″	4″	**91°33′04″**
	179°59′38″	6	11x			
			11x = 22″	x = 2″		

(b) $A = 79°23'55''$, wt 3; $B = 56°41'05''$, wt 2; $C = 43°55'33''$, wt 1

Misclosure = 33″

	Obs. Ang.	**Wt**	**Corr.**	**Num. Cor.**	**Adj. Ang.**
A	79°23′55″	3	2x	−6″	79°23′49″
B	56°41′05″	2	3x	−9″	56°40′56″
C	43°55′33″	<u>1</u>	<u>6x</u>	−18″	43°00′15″
	180°00′33″	6	11x		
			11x = −33″	x = −3″	

3.29 Determine relative weights and perform a weighted adjustment (to the nearest second) for angles *A*, *B*, and *C* of a plane triangle, given the following four observations for each angle:

Angle *A*	**Angle *B***	**Angle *C***
44°28′16″	65°56′13″	69°35′20″
44°28′12″	65°56′10″	69°35′24″
44°28′17″	65°56′06″	69°35′18″

44°28′11″ 65°56′08″ 69°35′24″

A = 44°28′14.0″ ± 2.9″; **B** = 65°56′09.3″ ± 3.0″; **C** = 69°35′21.5″ ± 3″

Misclosure = −15.3″

	Obs. Ang.	Wt	Corr. Multiplier	Num. Cor.	Rnd. Cor.	Adj. Ang.
A	44°28′14.0″	0.115385	0.338645/wt = 2.93	4.97″	5.0″	44°28′19″
B	65°56′09.3″	0.11215	0.338645/wt = 3.02	5.12″	5.1″	65°56′14″
C	69°35′21.5″	0.111111	0.338645/wt= 3.05	5.16″	5.2″	69°35′27″
	179°59′44.7″	0.338645	9.00x			
			9.0 = 15.3″	x = 1.69″		

3.30 A line of levels was run from benchmarks *A* to *B*, *B* to *C*, and *C* to *D*. The elevation differences obtained between benchmarks, with their standard deviations, are listed below. What is the difference in elevation from benchmark *A* to *D* and the standard deviation of that elevation difference?

(a) BM *A* to BM $B = +12.68 \pm 0.10$ ft; BM *B* to BM $C = -8.23 \pm 0.18$ ft; and BM *C* to BM $D = -14.66 \pm 0.06$ ft

By Equation (3.11): **−10.21 ± 0.21 ft**

(b) BM *A* to BM $B = -15.324 \pm 0.022$ m; BM *B* to BM $C = -10.250 \pm 0.015$ m; and BM *C* to BM $D = -16.892 \pm 0.008$ m

By Equation (3.11): **−42.466 ± 0.028 m**