5 LEVELING — FIELD PROCEDURES AND COMPUTATIONS

Asterisks (*) indicate problems that have answers given in Appendix G.

5.1 What errors are eliminated by keeping the lengths of the plus and minus sights equal?

From Section 5.4: Balancing plus and minus sights will eliminate errors due to instrument maladjustment and the combined effect of Earth curvature and refraction.

5.2 Why should sight lines be kept at least 0.5 m from any surface?

From Section 5.12.2: "Air boiling or heat waves near the ground surface or adjacent to heated objects make the rod appear to wave and prevent accurate sighting. Raising the line of sight by high tripod setups, taking shorter sights, avoiding any that pass close to heat sources (such as buildings and stacks), and using the lower magnification of a variable-power eyepiece reduce the effect."

5.3 Why is it advisable to set up a level with all three tripod legs on, or in, the same material (concrete, asphalt, soil), if possible?

From Section 5.12.2: " Therefore setups on spongy ground, blacktop, or ice should be avoided if possible, but if they are necessary, unusual care is required to reduce the resulting errors. This can include taking readings in quick order, using two rods and two observers to preclude walking around the instrument, and alternating the order of taking plus and minus sights. Additionally whenever possible, the instrument tripod’s legs can be set on long hubs that are driven to refusal in the soft material."

Simply stated, to avoid uneven settling of the tripod.

5.4 Discuss how the collimation correction factor can be used to remove instrumental errors in differential leveling.

From Section 5.12.1: The collimation correction factor can be used to correction for any collimation error in an instrument using the lengths of the plus sights and minus sights between benchamarks.

5.5 Explain how errors due to lack of instrument adjustment can be practically eliminated in running a line of differential levels?

From Section 5.4: “Balancing plus and minus sight distances will eliminate errors due to instrument maladjustment (most important) and the combined effects of the Earth’s curvature and refraction, as shown in Figure 5.6. Here e_1 and e_2 are the combined curvature and refraction errors for the plus and minus sights, respectively. If D_1 and D_2 are made equal, e_1 and e_2 are also equal. In calculations, e_1 is added and e_2 subtracted; thus they cancel each other.”

5.6 Why must the shoes of the tripod be snug?

From Section 5.12.1: Loose shoes cause unstable tripod setups."

5.7 List four considerations that govern a rodperson's selection of TPs and BMs.

1. From Chapter 4: BMs must be permanent.
2. From Section 5.4: "Turning points should be solid objects with a definite high point."
3. From Section 5.6: "...it is recommended that some turning points or benchmarks used in the first part of the circuit be included again on the return run. This creates a multi-loop circuit, and if a blunder or large error exists, its location can be isolated to one of the smaller loops."
4. From Section 5.12.2: "It (settlement) can be avoided by selecting firm, solid turning points or, if none are available, using a steel turning pin set firmly in the ground."
5. Find turning points that aid in the balancing of plus and minus sight distances.

5.8* What error is created by a rod leaning 10 min from plumb at a 12.51-ft reading on the leaning rod?

<u>Error = 0.000 ft</u>

Correct reading = 12.51cos(10') = 12.50995; So error is 0.00005 ft, or 0.000 ft

Problem is designed to show that even for a high reading and a mislevelment outside of a typical circular bubble, the resulting error is negligble.

5.9 Similar to Problem 5.6, except for a 3.5-m reading.

<u>Error is 0.000 m</u>

Correct reading = 3.5cos(10') = 3.499985, so error is 0.000015, again error is negligible.

Problem is designed to show that even for a high reading and a mislevelment outside of a typical circular bubble, the resulting error is negligble.

5.10 What error results on a 30-m sight with a level if the rod reading is 1.505 m but the top of the 4 m rod is 0.3 m out of plumb?

Correct reading = $\frac{0.3}{4} 1.505 = 1.50076$

<u>Error = 0.0042 m</u>

5.11 What error results on a 200-ft sight with a level if the rod reading is 6.307 ft but the top of the 7-ft rod is 0.2 ft out of plumb?

Correct reading = $\frac{0.2}{7} 6.307 = 6.3044$

<u>Error = 0.0026 ft</u>

5.12 Prepare a set of level notes for the data listed. Perform a check and adjust the misclosure. Elevation of BM 7 is 852.045 m. If the total loop length is 1500 m, what order of leveling is represented? (Assume all readings are in feet)

POINT	+S (BS)	−S (FS)
BM 7	9.432	
TP 1	6.780	8.363
BM 8	7.263	9.822
TP 2	3.915	9.400
TP 3	7.223	5.539
BM 7		1.477

STA	+	HI	−	ELEV
BM 7	9.432			852.045
		861.477	(−0.0024)	(853.112)
TP 1	6.780		8.363	853.114
		859.894	(−0.0048)	(850.067)
BM 8	7.263		9.822	850.072
		857.335	(−0.0072)	(847.928)
TP 2	3.915		9.400	847.935
		851.85	(−0.0096)	(846.301)
TP 3	7.223		5.539	846.311
		853.534	(−0.0120)	(852.045)
BM 7			1.477	852.057
	34.613		34.601	
Page check	852.045 + 34.613 − 34.601			852.057
Misclosure =	852.057 − 852.045		0.012	
Correction =	−0.012/5 =	−0.0024		

From Equation 5.3: $m = 12 \text{ mm}\big/\sqrt{0.5} = 9.8 \text{ mm}$, **third order**

5.13* Similar to Problem 5.12, except the elevation of BM 7 is 823.38 and the loop length 1500 ft. (Assume all readings are in meters)

STA	"+"	HI	"−"	ELEV
BM7	9.432			823.38
		832.812	(−0.0024)	(824.447)
TP1	6.780		8.363	824.449
		831.229	(−0.0048)	(821.402)
BM 8	7.263		9.822	821.407
		828.670	(−0.0072)	(819.263)
TP2	3.915		9.400	819.27
		823.185	(−0.0096)	(817.636)
TP3	7.223		5.539	817.646

		824.869	(−0.012)	(823.38)
BM7			1.477	823.392
	34.613		34.601	

Page check	823.38 + 34.613 − 34.601 =		823.392
Misclosure =	823.392 − 823.38 =	0.012	
Correction =	−0.012/5 =	−0.0024	

Misclosure = 0.012(12/39.37) = 3.66 mm; K = 1500(12/39.37) = 0.6096

From Equation 5.3: $\frac{3.66\ mm}{\sqrt{0.6096}} = 5.4\ mm$,Second order, class 1

5.14 A differential leveling loop began and closed on BM Tree (elevation 323.48 ft). The plus sight and minus sight distances were kept approximately equal. Readings (in feet) listed in the order taken are 3.18 (+S) on BM Tree, 4.76 (−S) and 2.44 (+S) on TP1, 3.05 (−S) and 6.63 (+S) on BM X, 3.64 (−S) and 2.35 (+S) on TP2, and 3.07 (−S) on BM Tree. Prepare, check, and adjust the notes.

Sta	+sight	HI	-sight	Elev	Adj. Elev	Corr.
BMTREE	3.18			323.48	323.48	
		326.66				
TP1	2.44		4.76	321.90	321.88	-0.020
		324.34				
BMX	6.63		3.05	321.29	321.25	-0.040
		327.92				
TP2	2.35		3.64	324.28	324.22	-0.060
		326.63				
BMTREE			3.07	323.56	323.48	-0.080
sum	14.60		14.52			

Misclosure: 323.56 - 323.48 = 0.08
Page Check: 323.48+14.60-14.52=323.56

5.15 A differential leveling circuit began on BM Hydrant (elevation 4823.65 ft) and closed on BM Rock (elevation 4834.47 ft). The plus sight and minus sight distances were kept approximately equal. Readings (in feet) given in the order taken are 2.65(+S) on BM Hydrant, 3.51 (−S) and 7.23 (+S) on TP1, 5.04 (−S) and 11.41 (+S) on BM 1, 8.58 (−S) and 7.65 (+S) on BM 2, 4.23 (−S) and 7.53 (+S), on TP2, and 4.34 (−S) on BM Rock. Prepare, check, and adjust the notes.

Sta	+sight	HI	-sight	Elev	Adj. Elev	Corr.
HYDRANT	2.65			4823.65	4823.65	
		4826.30				
TP1	7.23		3.51	4822.79	4822.80	0.010
		4830.02				
BM1	11.41		5.04	4824.98	4825.00	0.020
		4836.39				
BM2	7.65		8.58	4827.81	4827.84	0.030

Sta	+sight	HI	-sight	Elev	Adj. Elev	Corr.
		4835.46				
TP2	7.53		4.23	4831.23	4831.27	0.040
		4838.76				
ROCK			4.34	4834.42	4834.47	0.050
sum	36.47		25.70			

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Misclosure: 4834.42 - 4834.47 = -0.05
Page Check: 4823.65+36.47-25.70=4834.42
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5.16 A differential leveling loop began and closed on BM Bridge (elevation 814.687 m). The plus sight and minus sight distances were kept approximately equal. Readings (in meters) listed in the order taken are 0.548 (+S) on BM Bridge, 1.208 (−S) and 0.843 (+S) on TP1, 1.287 (−S) and 1.482 (+S) on BM *X*, 0.743 (−S) and 0.944 (+S) on TP2, and 0.571 (−S) on BM Bridge. Prepare, check, and adjust the notes.

Sta	+sight	HI	-sight	Elev	Adj. Elev	Corr.
BRIDGE	0.548			814.687	814.687	
		815.235				
TP1	0.843		1.208	814.027	814.025	-0.0020
		814.870				
BMX	1.482		1.287	813.583	813.579	-0.0040
		815.065				
TP2	0.944		0.743	814.322	814.316	-0.0060
		815.266				
BRIDGE			0.571	814.695	814.687	-0.0080
sum	3.817		3.809			

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Misclosure: 814.695 - 814.687 = 0.008
Page Check: 814.687+3.817-3.809=814.695
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5.17 A differential leveling circuit began on BM Rock (elevation 543.202 m) and closed on BM Manhole (elevation 542.546 m). The plus sight and minus sight distances were kept approximately equal. Readings (in meters) listed in the order taken are 1.559 (+S) on BM Rock, 0.987 (−S) and 1.105 (+S) on TP1, 0.842 (−S) and 0.679 (+S) on BM 1, 1.846 (−S) and 0.849 (+S) on BM 2, 1.895 (−S) and 1.436 (+S) on TP2, and 0.704 (−S) on BM Manhole. Prepare, check, and adjust the notes.

Sta	+sight	HI	-sight	Elev	Adj. Elev	Corr.
ROCK	1.559			543.202	543.202	
		544.761				
TP1	1.105		0.987	543.774	543.772	-0.0020
		544.879				
BM1	0.679		0.842	544.037	544.033	-0.0040
		544.716				
BM2	0.849		1.846	542.870	542.864	-0.0060
		543.719				
TP2	1.436		1.895	541.824	541.816	-0.0080
		543.260				
MANHOLE			0.704	542.556	542.546	-0.0100

sum	5.628		6.274

Misclosure: 542.556 - 542.546 = 0.010
Page Check: 543.202+5.628-6.274=542.556

5.18 A differential leveling loop started and closed on BM Juno, elevation 2485.19 ft. The plus sight and minus sight distances were kept approximately equal. Readings (in feet) listed in the order taken are 5.49 (+S) on BM Juno, 3.46 (−S) and 8.84 (+S) on TP1,5.34 (−S) and 6.51 (+S) on TP2, 8.27 (−S) and 4.03 (+S) on BM1, 9.46 (−S) and 7.89 (+S) on TP3, and 6.13 (−S) on BM Juno. Prepare, check, and adjust the notes.

Sta	+sight	HI	-sight	Elev	Adj. Elev	Corr.
JUNO	5.49			2485.19	2485.19	
		2490.68				
TP1	8.84		3.46	2487.22	2487.20	-0.020
		2496.06				
TP2	6.51		5.34	2490.72	2490.68	-0.040
		2497.23				
BM1	4.03		8.27	2488.96	2488.90	-0.060
		2492.99				
TP3	7.89		9.46	2483.53	2483.45	-0.080
		2491.42				
JUNO			6.13	2485.29	2485.19	-0.100
sum	32.76		32.66			

Misclosure: 2485.29 - 2485.19 = 0.10
Page Check: 2485.19+32.76-32.66=2485.29

5.19* A level setup midway between *X* and *Y* reads 6.29 ft on *X* and 7.91 ft on *Y*. When moved within a few feet of *X*, readings of 5.18 ft on *X* and 6.76 ft on *Y* are recorded. What is the true elevation difference, and the reading required on *Y* to adjust the instrument?

Correct ΔElev = 7.91 − 6.29 = 1.62 ft
Unbalanced ΔElev = 6.76 − 5.18 = 1.58 ft
Error at Y = 0.04 ft
Corrected reading at Y = 6.76 + 0.04 = 6.80 ft

5.20 To test its line of sight adjustment, a level is setup near *C* (elev 193.436 m) and then near *D*. Rod readings listed in the order taken are *C* = 1.315 m, *D* = 0.848 m, *D* = 1.296 m, and *C* = 1.767 m. Compute the elevation of *D*, and the reading required on *C* to adjust the instrument.

ΔElev from C = 1.315 − 0.848 = 0.467
ΔElev from D = 1.767 − 1.296 = 0.471
ΔElev = (0.467 + 0.471)/2 = 0.469 m
$Elev_D$ = 193.436 + 0.469 = **193.905 m**
Corrected reading at *C* = 1.767 − 0.004/2 = **1.765 m**; Check 1.767 − 1.296 = 0.469 m

5.21* The line of sight test shows that a level's line of sight is inclined downward 3 mm/50 m. What is the allowable difference between BS and FS distances at each setup (neglecting curvature and refraction) to keep elevations correct within 1 mm?

$$\frac{50}{3} = \frac{X}{1} \rightarrow X = \mathbf{16.7\ mm}$$

5.22 Reciprocal leveling gives the following readings in meters from a set up near *A:* on *A*, 1.365; on *B*, 4.928, 4.924, and 4.926. At the setup near *B:* on *B*, 4.251; on *A*, 0.687, 0.688, and 0.689. The elevation of *A* is 564.872 m. Determine the misclosure and elevation of *B*.

568.434 m

From A: $\Delta Elev_{AB} = \frac{4.928+4.924+4.926}{3} - 1.365 = 4.926 - 1.365 = 3.561$

From B: $\Delta Elev_{AB} = 4.251 - \frac{0.687+0.688+0.689}{3} = 4.251 - 0.688 = 3.563$

$$Elev_B = 564.872 - \frac{3.561 + 3.563}{2} = 564.872 + 3.562 = \mathbf{568.434\ m}$$

5.23* Reciprocal leveling across a canyon provides the data listed (in meters). The elevation of *Y* is 2265.879 ft. The elevation of *X* is required. Instrument at *X*: +3.182, −S = 9.365, 9.370, and 9.368. Instrument at *Y*: +S = 10.223; −S = 4.037, 4.041, and 4.038.

From *X*: $\Delta Elev = 3.182 - \frac{9.365 + 9.370 + 9.368}{3} = -6.1857$

From *Y*: $\Delta Elev = \frac{4.037 + 4.041 + 4.038}{3} - 10.223 = -6.1843$

$$Elev_B = 2265.879 - \frac{6.1857 + 6.1843}{2} = 2259.694 \text{ m}$$

5.24 Prepare a set of three-wire leveling notes for the data given and make the page check. The elevation of BM *X* is 733.387 m. Rod readings (in meters) are (*U* denotes upper cross-wire readings, *M* middle wire, and *L* lower wire): +S on BM *X:* *U* = 2.959, *M* = 2.707, *L* = 2.454; −S on TP 1: *U* = 1.683, *M* = 1.453, *L* = 1.224; +S on TP 1: *U* = 2.254, *M* = 2.054, *L* = 1.854; −S on BM *Y:* *U* = 1.013, *M* = 0.817, *L* = 0.620.

Station	+S	stadia	HI	−S	Stadia	Elev (m)
BM X	2.959					733.387
	2.707	0.252				
	2.454	0.253				
	2.706	0.505				
			736.0937			
TP1	2.254			1.683		734.6403
	2.054	0.200		1.453	0.230	
	1.854	0.200		1.224	0.229	
	2.054	0.400		1.341	0.2295	
			736.6943			
BMY				5.674		731.2417
				5.453	0.197	

				5.231	0.197	
				5.4527	0.2215	(cont.)
	4.7607	0.905		6.906	0.451	

Page check: 733.387 + 4.7607 − 6.906 = 731.2417✓

5.25 Similar to Problem 5.24, except the elevation of BM *X* is 1482.909 ft, and rod readings (in feet) are: +S on BM *X:* *U* = 6.573, *M* = 6.321, *L* = 6.070; −S on TP 1: *U* = 5.949, *M* = 5.653, *L* = 5.356; +S on TP 1: *U* = 5.470, *M* = 5.195, *L* = 4.921; −S on BM *Y:* *U* = 5.674, *M* = 5.653, *L* = 5.231.

Station	+S	stadia	HI	−S	Stadia	Elev (ft)
BM X	6.573					1482.909
	6.321	0.252				
	6.070	0.251				
	6.3213	0.503				
			1489.230			
TP1	5.470			5.949		1483.5777
	5.195	0.275		5.653	0.296	
	4.921	0.274		5.356	0.297	
	5.1953	0.549		5.6527	0.2965	
			1488.773			
BMY				6.074		1483.1203
				5.653	0.421	
				5.231	0.422	
				5.6527	0.4215	
	11.5167	1.052		11.3053	0.718	

Page check: 1482.909 + 11.5167 − 11.3053 = 1483.1203✓

5.26 Assuming a stadia constant of 99.987, what is the distance leveled in Problem 5.24?

D = 99.987(0.0.905 + 0.451) = **135.6 m**

5.27 Assuming a stadia constant of 101.5, what is the distance leveled in Problem 5.25?

D = 101.5(1.052 + 0.718) = **179.7 ft**

5.28 Prepare a set of profile leveling notes for the data listed and show the page check. All data is given in feet. The elevation of BM *A* is 659.08, and the elevation of BM *B* is 648.47. Rod readings are: +S on BM *A*, 5.68 intermediate foresight (IFS) on 11+00, 4.3; –S on TP1, 7.56; +S on TP 1, 8.02; intermediate foresight on 12+00, 6.6; on 12+50, 5.3; on 13+00, 5.8; on 14+00, 6.3; −S on TP 2, 10.15, +S on TP 2, 5.28; intermediate foresight on 14+73, 4.1; on 15+00, 4.9; on 16+00, 6.3; −S on TP3, 7.77; +S on TP3, 3.16; −S on BM *B*, 7.23.

Sta	+S	HI	−S	−IFS	Elev (ft)	Adj. Elev
BM A	5.68	(664.8) ~~664.76~~			659.08	659.08
11+00				4.3		660.5
TP1	8.02	(665.2) ~~665.22~~	7.56		657.20	657.19
12+00				6.6		658.6
12+50				5.3		659.9
13+00				5.8		659.4
14+00				6.3		658.9
TP2	5.28	(660.3) ~~660.35~~	10.15		655.07	655.05
14+73				4.1		656.2
15+00				4.9		655.4
16+00				6.3		654.0
TP3	3.16		7.77		652.58	652.55
		655.74				
BM *B*			7.23		648.51	648.47
	25.23		32.71			

Page check: 659.08 + 25.23 − 32.71 = 648.51✓
Misclosure = 648.51 − 648.47 = 0.04

5.29 Same as Problem 5.28, except the elevation of BM *A* is 356.98 ft, the elevation of BM *B* is 349.58 ft, and the +S on BM *A* is 8.77 ft.

Sta	+S	HI	−S	−IFS	Elev (ft)	Adj. Elev
BM A	5.68	(365.7) ~~365.75~~			356.98	356.98
11+00				4.3		361.4
TP1	8.02	(366.2) ~~366.21~~	7.56		358.19	358.17
12+00				6.6		359.6
12+50				5.3		360.9
13+00				5.8		360.4
14+00				6.3		359.9
TP2	5.28	(361.3) ~~361.34~~	10.15		356.06	355.78
14+73				4.1		357.2
15+00				4.9		356.4

16+00				6.3		355.0
TP3	3.16		7.77		353.57	353.25
		356.73				
BM *B*			7.23		349.50	349.16
	25.23		32.71			

Page check: 356.98 + 25.23 − 32.71 = 349.50✓
Misclosure = 3549.50 − 349.58 = −0.08

5.30 Plot the profile Problem 5.28 and design a grade line between stations 11 + 00 and 16 + 00 that balances cut and fill areas.

Plot of profile from 5.28. Response should vary, but cut and fill sections should visually balance.

5.31* What is the percent grade between stations 11+00 and 16 + 00 in Problem 5.28?

$$\frac{654.0 - 660.5}{1600 - 1100}100\% = -\mathbf{1.3}\%$$

5.32 Differential leveling between BMs *A, B, C, D*, and *A* gives elevation differences (in meters) of −15.632, +32.458, +38.214 and −55.025, and distances in km of 4.0, 6.0, 5.0, and 3.0, respectively. If the elevation of *A* is 634.597, compute the adjusted elevations of BMs *B, C,* and *D,* and the order of leveling.

Overall length of loop = 3.4 km.

Misclosure = −6.532 + 12.845 + 9.241 − 15.717 = 0.017 m

$$m = \frac{15 \text{ mm}}{\sqrt{18}} = 3.54 \text{ mm, First order, class I leveling}$$

Sta	ΔElev (m)	Length (km)	Correction	Adj. ΔElev.	Adj. Elev.
A					634.597
	−15.632	4.0	−0.015(4.0/18) = −0.0033	−15.6353	
B					**618.962**
	32.458	6.0	−0.015(6.0/18) = −0.0050	32.4530	
C					**651.415**
	38.214	5.0	−0.015(5.0/18) = −0.0042	38.2098	
D					**689.625**
	−55.025	3.0	−0.017(3.0/18) = −0.0025	−55.0275	
A					634.597
	0.015	18.0		0.0000	

5.33 Leveling from BM *X* to *W,* BM *Y* to *W,* and BM *Z* to *W* gives differences in elevation (in feet) of $-30.24, +26.20,$ and $+10.18,$ respectively. Distances between benchmarks are $XW = 2500, YW = 3000,$ and $ZW = 4000.$ True elevations of the benchmarks are $X = 571.93, Y = 515.47,$ and $Z = 531.58.$ What is the adjusted elevation of *W?* (Note: All data are given in feet.)

From BM X: 571.93 – 30.24 = 541.69 ft.
From BM Y: 515.47 + 26.20 = 541.67 ft.
From BM Z: 531.58 + 10.18 = 541.76 ft.

$$Elev_W = \frac{541.69\left(\frac{1}{2500}\right) + 541.67\left(\frac{1}{3000}\right) + 541.76\left(\frac{1}{4000}\right)}{\left(\frac{1}{2500}\right) + \left(\frac{1}{3000}\right) + \left(\frac{1}{4000}\right)} = 541.701\,ft.$$

5.34 A 3-m level rod was calibrated and its graduated scale was found to be uniformly expanded so that the distance between its 0 and 3.000 marks was actually 3.006 m. How will this affect elevations determined with this rod for (**a**) circuits run on relatively flat ground (**b**) circuits run downhill (**c**) circuits run uphill?

(**a**) On level ground the plus and minus sight readings will be approximately the same. Since the plus readings are added and minus readings subtracted, the net effect will tend to cancel the errors.

(**b**) In level downhill, the minus readings will tend to be higher on the rod than the plus readings. Thus the minus readings will tend to have more error than the plus readings. Thus the elevations will tend to be too high.

(**c**) In level uphill, the plus readings will be tend to be higher on the rod than the minus readings. Thus the plus readings will tend to have more error than the minus readings. Thus the elevations will tend to be too low.

5.35* A line of levels with 42 setups (84 rod readings) was run from BM Rock to BM Pond with readings taken to the nearest 3.0 mm; hence any observed value could have an error of ± 1.5 mm. For reading errors only, what total error would be expected in the elevation of BM Pond?

$$1.5\sqrt{84} = \pm 13.7 \text{ mm}$$

5.36 Same as Problem 5.35, except for 65 setups and readings to the nearest 0.01 ft with possible error of ± 0.005 ft each.

$$0.005\sqrt{65(2)} = \pm 0.057 \text{ ft}$$

5.37 Compute the permissible misclosure for the following lines of levels: (**a**) a 20-km loop of third-order levels (**b**) a 10-km section of second-order class I levels (**c**) a 30-km loop of first-order class I levels.

(**a**) $C = 12\sqrt{20} = 53.7$ mm

(**b**) $C = 6\sqrt{10} = 18.9$ mm

(**c**) $C = 4\sqrt{30} = 21.9$ mm