

11 COORDINATE GEOMETRY IN SURVEYING CALCULATIONS

Asterisks (*) indicate problems that have partial answers given in Appendix G.

- 11.1 The X and Y coordinates (in meters) of station Shore are 379.241 and 819.457, respectively, and those for station Rock are 437.854 and 973.482, respectively. What are the azimuth, bearing, and length of the line connecting station Shore to station Rock?

Az = 20°50'02"; Brg = N 20°50'02" E; Distance = 164.800 m

- 11.2 Same as Problem 11.1, except that the X and Y coordinates (in feet) of Shore are 2058.97 and 6980.06, respectively, and those for Rock are 1408.03 and 6980.06, respectively.

Az = 343°08'40"; Brg = N 16°51'20" W; Distance = 2244.92 ft

- 11.3* What are the slope, and y-intercept for the line in Problem 11.1?

$m = 2.62783$; $b = -177.124$ m

- 11.4 What are the slope, and the y-intercept for the line in Problem 11.2?

$m = -3.30057$; $b = 11,627.36$ ft

- 11.5* If the slope (XY plane) of a line is 0.800946, what is the azimuth of the line to the nearest second of arc? (XY plane)

51°18'26"

- 11.6 If the slope (XY plane) of a line is -0.689443 , what is the azimuth of the line to the nearest second of arc? (XY plane)

124°35'02" or 304°35'02"

- 11.7* What is the perpendicular distance of a point from the line in Problem 11.1, if the X and Y coordinates (in meters) of the point are 422.058 and 932.096, respectively?

$$Az_{SR} = 20^\circ 48' 47''; SR = 120.502 \text{ m}$$

0.044 m

$$\text{By (11.11): } \alpha = 0^\circ 01' 15''$$

$$\text{By (11.12):}$$

$$SP = 120.502 \sin 0^\circ 01' 15'' = 0.044$$

- 11.8** What is the perpendicular distance of a point from the line in Problem 11.2, if the X and Y coordinates (in feet) of the point are 1848.30 and 5528.73, respectively?

$$Az_{SR} = 343^{\circ}11'08.2''; SR = 728.276 \text{ ft}$$

0.52 ft

$$\text{By (11.11): } \alpha = 0^{\circ}02'28.6''$$

$$\text{By (11.12): } SP = 728.2761 \sin 0^{\circ}02'28.6'' = 0.525$$

- 11.9*** A line with an azimuth of $105^{\circ}46'33''$ from a station with X and Y coordinates of 5885.31 and 5164.15, respectively, is intersected with a line that has an azimuth of $200^{\circ}31'24''$ from a station with X and Y coordinates of 7337.08 and 5949.99, respectively. (All coordinates are in feet.) What are the coordinates of the intersection point?

(6932.18, 4868.39)

$$D_{12} = 1650.81 \text{ ft}; Az_{12} = 61^{\circ}34'24''; \angle 1 = 44^{\circ}12'09''; \angle P = 94^{\circ}44'51'' \quad D_{2P} = 1154.90 \text{ ft}$$

- 11.10** A line with an azimuth of $164^{\circ}28'17''$ from a station with X and Y coordinates of 2443.94 and 3563.84, respectively, is intersected with a line that has an azimuth of $81^{\circ}19'04''$ from a station with X and Y coordinates of 2126.86 and 3235.93, respectively. (All coordinates are in feet.) What are the coordinates of the intersection point?

(1725.06, 1454.41)

$$D_{12} = 368.688 \text{ ft}; Az_{12} = 98^{\circ}15'09''; \angle 1 = 23^{\circ}35'35''; \angle 2 = 23^{\circ}40'55'' \quad D_{1P} = 201.58 \text{ ft}$$

- 11.11** Same as Problem 11.9 except that the bearing of the first line is $S 22^{\circ}12'04'' E$ and the bearing of the second line is $S 38^{\circ}12'11'' W$.

(6169.87, 4466.89)

$$D_{12} = 1650.81 \text{ ft}; Az_{12} = 61^{\circ}34'24''; \angle 1 = 96^{\circ}13'09''; \angle P = 23^{\circ}21'51'' \quad D_{2P} = 752.81 \text{ ft}$$

- 11.12** In the accompanying figure, the X and Y coordinates (in meters) of station A are 2084.274 and 5579.124, respectively, and those of station B are 3012.870 and 3589.315, respectively. Angle BAP was measured as $310^{\circ}20'25''$ and angle ABP was measured as $44^{\circ}21'58''$. What are the coordinates of station P ?

(3568.763, 5172.375)

$$D_{12} = 2195.821 \text{ m}; Az_{12} = 154^{\circ}58'58''; \angle 1 = 49^{\circ}39'35''; \angle P = 85^{\circ}58'27'' \quad D_{1P} = 1539.205 \text{ m}; Az_{1P} = 105^{\circ}19'22.6''$$

- 11.13*** In the accompanying figure, the X and Y coordinates (in feet) of station A are 1248.16 and 3133.35, respectively, and those of station B are 1509.15 and 1101.89, respectively. The length of BP is 2657.45 ft, and the azimuth of line AP is $98^{\circ}25'00''$. What are the coordinates of station P ?

(3560.56, 2791.19)

$AZ_{AB} = 172^{\circ}40'45''$; $D_{AB} = 2048.157$; $D_{AP} = 2337.576$ or -1226.525 ; $\angle PAB = 74^{\circ}15'45''$;

$a = 1$; $b = 1111.051$; $c = -2,867,094.99$;

- 11.14** In the accompanying figure, the X and Y coordinates (in feet) of station A are 3539.51 and 5971.30, respectively, and those of station B are 3401.79 and 2708.06, respectively. The length of AP is 1987.54 ft, and angle ABP is $35^{\circ}22'43''$? What are the possible coordinates for station P ?

(5408.77, 5295.88) or (4658.84, 4328.92)

$AZ_{BA} = 2^{\circ}24'59.9''$; $AZ_{BP} = 37^{\circ}47'42.9''$; $D_{AB} = 3266.145$; $D_{BP} = 3274.873$ or 2051.190 ;
 $a = 1$; $b = 5326.063$; $c = 6,717,386.86$;

- 11.15*** A circle of radius 798.25 ft, centered at point A , intersects another circle of radius 1253.64 ft, centered at point B . The X and Y coordinates (in feet) of A are 3548.53 and 2836.49, respectively, and those of B are 4184.62 and 1753.52, respectively. What are the coordinates of station P in the figure?

(4330.13, 2998.69) or (3026.23, 2232.83)

$AB = 1255.96$; $AZ_{AB} = 149^{\circ}34'18.7''$; $\angle PAB = 71^{\circ}17'43.6''$; $AZ_{AP} = 78^{\circ}16'35''$ or $220^{\circ}52'02''$

- 11.16** The same as Problem 11.15, except the radii from A and B are 853.34 ft and 1389.54 ft, respectively, and the X and Y coordinates (in feet) of A are 2058.74 and 4311.32, respectively, and those of station B are 2581.52 and 2344.21, respectively.

(2683.45, 3730.01) or (1805.05, 3496.56)

$AB = 2035.392$; $AZ_{AB} = 165^{\circ}07'01.4''$; $\angle PAB = 32^{\circ}10'40.4''$; $AZ_{AP} = 132^{\circ}56'21.0''$ or $197^{\circ}17'41.9''$

- 11.17** For the subdivision in the accompanying figure, assume that lines AC , DF , GI , and JL are parallel, but that lines BK and CL are parallel to each other, but not parallel to AJ . If the X and Y coordinates (in feet) of station A are (5000.00, 5000.00), what are the coordinates of each lot corner shown?

Station	X	Y	Method
A	5000.00	5000.00	Given
B	5149.99	4997.99	Forward
C	5299.97	4995.99	Forward
D	5013.14	5078.91	Forward
E	5162.53	5076.92	Direction-Direction
F	5312.52	5074.91	Direction-Distance
G	5026.27	5157.83	Forward
H	5175.08	5155.84	Direction-Direction
I	5325.07	5153.83	Direction-Distance

<i>J</i>	5039.40	5236.74	Forward
<i>K</i>	5187.63	5234.76	Direction-Direction
<i>L</i>	5337.61	5232.75	Direction-Distance

- 11.18** If the *X* and *Y* coordinates (in feet) of station *A* are (1000.00, 1000.00), what are the coordinates of the remaining labeled corners in the accompanying figure?

Station	<i>X</i>	<i>Y</i>	Method
<i>A</i>	1000.00	1000.00	Given
<i>B</i>	1000.00	1400.01	Forward
<i>C</i>	1430.00	1400.01	Forward
<i>D</i>	1430.00	1000.00	Direction-Direction
<i>E</i>	1235.58	1193.82	Forward or Direction-Distance
<i>F</i>	1194.42	1193.82	Forward or Direction-Distance
<i>G</i>	1215.00	1171.99	Forward or Direction-Direction
<i>H</i>	1200.00	1146.01	Direction-Distance
<i>I</i>	1230.00	1146.01	Direction-Distance
<i>J</i>	1200.00	1000.00	Forward
<i>K</i>	1230.00	1000.00	Forward

- 11.19*** In Figure 11.8, the *X* and *Y* coordinates (in feet) of *A* are 1234.98 and 5415.48, respectively, those of *B* are 3883.94 and 5198.47, respectively, and those of *C* are 6002.77 and 5603.25, respectively. Also angle *x* is 36°59'21" and angle *y* is 44°58'06". What are the coordinates of station *P*?

(4538.67, 2940.13)

$$\begin{aligned}
 Az_{BA} &= 274^\circ 41' 00.1''; Az_{BC} = 79^\circ 11' 04.4''; BC(a) = 2157.148; AB(c) = 2657.834; \\
 \sphericalangle &= 195^\circ 29' 55.7''; A+C = 82^\circ 32' 37.3''; A = 32^\circ 09' 34.9''; C = 50^\circ 23' 02.4''; \\
 \sphericalangle_1 &= 110^\circ 51' 04.1''; AP = 4128.165; Az_{AP} = 126^\circ 50' 35.1''
 \end{aligned}$$

- 11.20** In Figure 11.8, the *X* and *Y* coordinates (in feet) of *A* are 4371.56 and 8987.63, those of *B* are 8531.05 and 8312.57, and those of *C* are 10,240.98 and 8645.07, respectively. Also angle *x* is 50°12'45" and angle *y* is 44°58'06". What are the coordinates of station *P*?

(6971.30, 6801.51)

$$\begin{aligned}
 Az_{BA} &= 347^\circ 31' 46.5''; Az_{BC} = 121^\circ 58' 49.1''; BC(a) = 2142.543; AB(c) = 1888.525; \\
 \sphericalangle &= 225^\circ 32' 57.4''; A+C = 44^\circ 22' 27.6''; A = 20^\circ 04' 39.3''; C = 24^\circ 17' 48.3''; \\
 \sphericalangle_1 &= 123^\circ 33' 52.7''; AP = 2654.469; Az_{AP} = 187^\circ 36' 25.8''
 \end{aligned}$$

- 11.21** In Figure 11.9, the following *EN* and *XY* coordinates for points *A* through *C* are given. In a 2-D conformal coordinate transformation, to convert the *XY* coordinates into the *EN* system, what are the

(a)* Scale factor? **0.3048277** $AB = 500.5768$ m; $ab = 1642.163$ ft

(b) Rotation angle? **23°12'53.0"**; $Az_{AB} = 45^\circ 24' 51.8''$; $Az_{ab} = 68^\circ 37' 44.8''$

- (c) Translations in X and Y? $T_x = 718,971.604 \text{ m}$ and $T_y = 109,056.362 \text{ m}$
 (d) Coordinates of points C in the EN coordinate system?

$E = 720,218.768 \text{ m}$ and $N = 111,593.722$

Point	State Plane Coordinates (m)		Arbitrary Coordinates (ft)	
	E	N	X	Y
A	719,542.829	111,493.468	4873.67	6609.04
B	719,899.341	11,844.860	6402.92	7207.45
C			7041.22	6037.23

11.22 Do Problem 11.21 with the following coordinates.

Point	State Plane Coordinates (m)		Arbitrary Coordinates (m)	
	E	N	X	Y
A	678,805.266	121,851.804	6182.848	6323.893
B	679,481.136	121,952.112	5430.607	3816.422
C			3957.467	5101.501

- (a) Scale factor? 0.2610028 $AB = 683.273$; $ab = 2617.876$
 (b) Rotation angle? $115^\circ 08' 27.8''$; $Az_{AB} = 81^\circ 33' 29.4''$; $Az_{ab} = 196^\circ 41' 57.3''$
 (c) Translations in X and Y? $T_x = 680,985.048$ and $T_y = 121,092.177$
 (d) Coordinates of points C in the EN coordinate system?

$E = 679,340.853$ and $N = 121,461.546$

11.23 In Figure 11.12, the elevations of stations A and B are 100.00 ft, and 98.45 ft, respectively. Instrument heights hi_A and hi_B are 5.20 ft, and 5.06 ft, respectively. What is the average elevation of point P if the other field observations are:

$AB = 128.46 \text{ ft}$
 $A = 62^\circ 06' 00''$ $B = 50^\circ 12' 07''$
 $v_1 = 36^\circ 33' 59''$ $v_2 = 33^\circ 22' 46''$

Elev = 184.34 ft

$AI = 106.676$; $BI = 122.707$; $IP_A = 79.128$; $IP_B = 80.848$

11.24 In Problem 11.23, assume station P is to the left of the line AB, as viewed from station A. If the X and Y coordinates (in feet) of station A are 159.19 and 101.20, respectively, and the azimuth of line AB is $69^\circ 22' 32''$ what are the X and Y coordinates of the inaccessible point?

(172.70, 207.02); $Az_{AI} = 7^\circ 16' 32''$

11.25 In Figure 11.12, the elevations of stations A and B are 1106.78 ft, and 1116.95 ft, respectively. Instrument heights hi_A and hi_B are 5.14 and 5.43 ft, respectively. What is the average elevation of point P if the other field observations are:

$$AB = 438.18 \text{ ft}$$

$$A = 49^\circ 31' 00'' \quad B = 52^\circ 35' 26''$$

$$v_1 = 27^\circ 40' 57'' \quad v_2 = 27^\circ 20' 51''$$

Elev = 1322.81 ft

$$AI = 355.971; BI = 340.859; IP_A = 186.750; IP_B = 224.580$$

- 11.26** In Problem 11.25, assume station P is to the left of line AB as viewed from station A . If the X and Y coordinates (in feet) of station A are 8975.18 and 7201.89, respectively, and the azimuth of line AB is $347^\circ 22' 38''$, what are the X and Y coordinates of the inaccessible point?

(8660.47, 7368.24); $Az_{AI} = 297^\circ 51' 38''$

- 11.27** In Figure 11.13, the X , Y , and Z coordinates (in feet) of station A are 5111.82, 4452.50, and 492.40, respectively, and those of B are 5627.41, 4440.12, and 501.65, respectively. Determine the three-dimensional position of the occupied station P with the following observations:

$$v_1 = 32^\circ 14' 00'' \quad PA = 513.06 \text{ ft} \quad hr_A = 6.53 \text{ ft} \quad \gamma = 79^\circ 06' 19''$$

$$v_2 = 37^\circ 06' 00'' \quad PB = 467.02 \text{ ft} \quad hr_B = 5.33 \text{ ft} \quad hi_p = 5.35 \text{ ft}$$

(5410.30, 4137.45, 219.92)

$$AB = 515.739; Az_{AB} = 91^\circ 22' 31.7''; PC = 433.989; PD = 372.488; \angle DCP = 45^\circ 10' 18.5''$$

$$AC = 273.650; BC = 281.710$$

- 11.28** Adapt Equations (11.43) and (11.47) so they are applicable for zenith angles.

(11.43): $PC = PA \sin(z_1)$; $PD = PB \sin(z_2)$

(11.47): $PA = PA \cos(z_1)$; $BD = PB \cos(z_2)$

- 11.29** In Figure 11.13, the X , Y , and Z coordinates (in meters) of station A are 1671.392, 1168.484, and 252.796, respectively, and those of B are 1569.635, 1395.155, and 245.809, respectively. Determine the three-dimensional position of occupied station P with the following observations:

$$z_1 = 110^\circ 33' 54'' \quad PA = 200.285 \text{ m} \quad hr_A = 1.676 \text{ m} \quad \gamma = 89^\circ 40' 58''$$

$$z_2 = 113^\circ 23' 37'' \quad PB = 177.196 \text{ m} \quad hr_B = 1.678 \text{ m} \quad hi_p = 1.676 \text{ m}$$

(1971.858, 1543.278, 428.459)

$$AB = 248.4637; Az_{AB} = 335^\circ 49' 25.6''; PC = 480.3649; PD = 428.6304; \angle DCP = 62^\circ 53' 41.5''; Az_{AP} = 38^\circ 43' 07.1''; AC = -180.222; BC = -185.428$$

- 11.30** Use WOLFPACK to do Problem 11.9. (See solution to 11.9)

- 11.31** Use WOLFPACK to do Problem 11.10. (See solution to 11.10)
- 11.32** Use WOLFPACK to do Problem 11.12. (See solution to 11.12)
- 11.33** Use WOLFPACK to do Problem 11.13. (See solution to 11.13)
- 11.34** Use WOLFPACK to do Problem 11.15. (See solution to 11.15)
- 11.35** Use WOLFPACK to do Problem 11.16. (See solution to 11.16)
- 11.36** Use WOLFPACK to do Problem 11.17. (See solution to 11.17)
- 11.37** Write a computational program that solves Example 11.6 using matrices. (Solution will vary.)
- 11.38** Write a computational program that solves Example 11.8. (Solutions will vary.)