19 CONTROL SURVEYS AND GEODETIC REDUCTIONS

Asterisks (*) indicate problems that have partial answers given in Appendix G.

19.1 Define the *geoid* and *ellipsoid*.

From Section 19.2, paragraph 1: "The geoid is an equipotential gravitational surface located approximately at mean sea level, which is everywhere perpendicular to the direction of gravity."

From Section 19.2, paragraph 2: "The ellipsoid is a mathematical surface obtained by revolving an ellipse about the Earth's polar axis. The dimensions of the ellipse are selected to give a good fit of the ellipsoid to the geoid over a large area, and are based upon surveys made in the area."

19.2 What are the possible monumentation types for a control station with a quality code of *A*?

From Section 19.11, paragraph 3: "Qualty code A monuments are the most reliable and are expected to hold a precise elevation. These monuments are typically rock outcrops, bedrock, and similar features plus massive structures with deep foundations; large structures with foundations on bedrock; or sleeved deep settings (10 ft or more) with galvanized steel pipe or galvanized steel, stainless steel, or aluminum rods."

19.3 What is *precession*?

From Section 19.3, paragraph 1: "Precession is the greater of the two and is the wander of the polar axis over a long period of time. The pole makes a complete revolution about once every 26,000 years. Additionally, the pole wanders in much smaller radial arcs that are superimposed upon precession."

19.4* What is the difference between the equatorial circumference of the Clarke 1866 ellipsoid and that of the WGS84 ellipsoid?

$\underline{\mathbf{\Delta C = 436.0\ m}}$ $= 2\pi(6378206.4 - 6378137.0)$

19.5 Determine the first and second eccentricities for the GRS80 ellipsoid.

By Equation (19.2): $\underline{\mathbf{e = 0.081819191;}}$ $\underline{\mathbf{e' = 0.082094438}}$

19.6 Discuss the motions of the Earth's instantaneous pole with respect to the conventional terrestrial pole.

From Section 19.3, paragraph 1: "Precession is the greater of the two and is the wander of the polar axis over a long period of time. The pole makes a complete revolution about once every 26,000 years. Additionally, the pole wanders in much smaller radial arcs that are superimposed upon precession. These smaller circles are known as a nutation, and are completed about once every 18.6 years."

19.7 What are the radii in the meridian and prime vertical for a station with latitude 43°06'58.29740" using the GRS80 ellipsoid?

By Equation 19.5: **R_M = 6,365,274.646 m**

By Equation 19.4: **R_N = 6,388,133.456 m**

19.8 For the station listed in Problem 19.7, what is the radius of the great circle at the station that is at an azimuth of 66°49'21" using the GRS80 ellipsoid?

By Equation 19.6: **$R_\forall$ = 6,384,581.750 m**

19.9* What are the radii in the meridian and prime vertical for a station with latitude 42°37'26.34584" using the GRS80 ellipsoid?

By Equation 19.5: **R_M = 6,364,725.399 m**

By Equation 19.4: **R_N = 6,387,949.711 m**

19.10 For the station listed in Problem 19.9, what is the radius of the great circle at the station that is at an azimuth of 203°29'32" using the GRS80 ellipsoid?

By Equation 19.6: **$R_\forall$ = 6,368,404.487 m**

19.11* The orthometric height at Station Y 927 is 304.517 m, and the geoidal height at that station −31.893 m. What is its geodetic height?

By Equation (19.7): h = **272.624 m**

19.12 The geodetic height at Station Z104 is 452.054 m. Its geoidal undulation is −25.089 m. What is its orthometric height?

By Equation (19.7): H = **477.143 m**

19.13 The orthometric height of a particular benchmark is 887.95 ft. The geoidal height at the station is –30.66 m. What is the geodetic height of the benchmark? Draw a sketch depicting the geoid, ellipsoid, and benchmark.

By Equation (19.7): $h = 887.95(12/39.37) - 30.66 =$ **787.36 ft**.

19.14 The instantaneous position of the pole at the time of an azimuth observation is x −1.06" and y = 1.23". The position of the station is (29°37'23.0823" N, 108°56'01.0089" W) and the observed azimuth of a line is 88°52'37". What is the astronomic azimuth of the line corrected for polar motion?

By Equation (19.3): **88°52'36.7"** $Az_A = Az_{obs} - (x \sin\lambda + y \cos\lambda) \sec\phi$

19.15* The deflection of the vertical components ξ and η are −2.85" and −5.94", respectively. The observed zenith angle is 42°36'58.8". What is the geodetic zenith angle and for the observations in Problem 19.14?

z = 42°36'52.8"; ∀ = 88°52'36.7" by Equations (19.34) and (19.32), respectively.

19.16 To within what tolerance should the elevations of two benchmarks 15 km apart be established if second-order class II standards were used to set them? What should it be if first-order class I standards were used?

By equations in Table 19.5

Second order Class II: ±5.0 mm

First order, Class I: ±1.9 mm

19.17 Name the orders and classes of accuracy of both horizontal and vertical control surveys, and give their relative accuracy requirements.

See Table 19.4 and 19.5

19.18* Given the following information for stations *JG00050* and *KG0089*, what should be the leveled height difference between them?

Station	Height (m)	Gravity (mgal)
JG0050	474.442	979,911.9
KG0089	440.552	979,936.2

−33.880 m; By Equations (19.42) and (19.43).

19.19 Similar to Problem 19.18 except that the station data for EY5664 and EY1587 is

Station	Height (m)	Gravity (mgal)
EY5664	453.278	980,678.9
EY1587	336.908	980,579.4

−116.414 m by Equations (19.42) and (19.43)

19.20 Similar to Problem 19.18 except that the station data for CV0178 and DQ0080 is

Station	Height (m)	Gravity (mgal)
CV0178	97.841	979,523.1
DQ0080	47.072	979,614.6

−50.762 m by Equations (19.42) and (19.43)

19.21* A slope distance of 2458.663 m is observed between two points *Gregg* and *Brian* whose orthometric heights are 458.966 m and 566.302 m, respectively. The geoidal undulations are −25.66 m and −25.06 m at *Gregg* and *Brian*, respectively. The height of the instrument at station *Gregg* at the time of the observation was 1.525 m and the height of the reflector at station *Brian* was 1.603 m. What are the geodetic and mark-to-mark distances for this observation? (Use an average radius for the Earth of 6,371,000 m for R_α)

D_2 (geodetic) = **2456.310 m**; D_3 (mark-to-mark) = **2458.868 m** by Equations (19.21) through (19.23)

19.22 If the latitude of station *Gregg* in Problem 19.21 was 56°16'22.4450" and the azimuth of the line was 135°48'26.8" what are the geodetic, and mark-to-mark distances for this observation? (Use the GRS80 ellipsoid).

D_2 (geodetic) = **2456.310 m**; D_1 (mark-to-mark) = **2458.868 m** by Equations (19.21) through (19.23)

where R_M = 6,379,700.520 m; R_N = 6,392,955.708; and $R_\forall$ = 6,386,134.467 m

19.23 A slope distance of 6365.780 m is observed between two stations *A* and *B* whose geodetic heights are 24.483 m and 115.097 m, respectively. The height of the instrument at the time of the observation was 1.544 m, and the height of the reflector was 2.000 m. The latitude of Station *A* is 43°08'36.2947" and the azimuth of *AB* is 32°28'21.9". What are the geodetic, and mark-to-mark distances for this observation?

D_2 (geodetic) = **6365.174 m**; D_1 (mark-to-mark) = **6365.889 m** by Equations (19.21) through (19.23)

where R_M = 6,365,305.044 m; R_N = 6,388,143.625; and $R_\forall$ = 6,371,871.722 m

19.24 Describe the differences between a geodetic distance and observed distance.

Observed distances are the slope lengths between instrument and reflector whereas geodetic distances are the lengths on the ellipsoid between these same points as projected onto the ellipsoid.

19.25* Compute the back azimuth of a line 5863 m long at a mean latitude of 45°01'32.0654" whose forward azimuth is 88°16'33.2" from north. (Use an average radius for the Earth of 6,371,000 m.)

268°19'43.1" = 88°16'33.2 – 189.9" + 180° and θ = 189.9" from Equation (19.15).
Length of E-W line is 5863*sin(88°16'33.2") = 5860.3457 m

19.26 Compute the back azimuth of a line 8720.245 m long at a mean latitude of 48°52'02" whose forward azimuth is 104°24'37.5" from north. (Use an average radius for the Earth of 6,371,000 m.)

284°29'50.6" = 104°24'37.5" – 313.089" + 180° and θ = 303.1" from Equation (19.15).
Length of E-W line is 8720.245*sin(104°24'37.5") = 8445.888 m

19.27 In Figure 19.14 azimuth of *AB* is 102°36'20" and the angles to the right observed at *B, C, D, E*, and *F* are 132°01'05", 241°45'12", 141°15'01", 162°09'24", and 202°33'19", respectively. An astronomic observation yielded an azimuth of 82°24'03" for line *FG*. The mean latitude of the traverse is 42°16'00", and the total departure between points *A* and *F* was 24,986.26 ft. Compute the angular misclosure and the adjusted angles. (Assume the angles and distances have already been corrected to the ellipsoid.)

Misclosure = −15.3" Correction +3"/angle.

Adjusted Angles: 132°01'08", 241°45'15", 141°15'04", 162°09'27", and 202°33'22"

θ = 206.7" by Equation (19.15)

19.28 In Figure 19.20 slope distance *S* and vertical angles ∀ and β were observed as 18,320.96 ft, +5°26'37", and −5°34'14", respectively. Ellipsoid height of point *A* is 11402.11 ft. What is length *A'B'* on the ellipsoid? (Use an average radius for the Earth of 6,371,000 m.)

18,233.880 ft ∗ = 5°30'25.5"; AB_1 = 18,236.3967; BB_1 = 1758.2407; P = 0°09'50.4";

B_1B_2 = 2.5162; AB_2 = 18,233.8805

19.29 In Figure 19.19 slope distance *S* was observed as 5438.015 m. The orthometric elevations of points *A* and *B* were 343.460 m and 632.180m, respectively, and the geoid height at both stations was −28.620 m. The instrument and reflector heights were both set at 1.200 m. Calculate geodetic distance *A'B'* (Use an average radius for the Earth of 6,371,000 m.)

5429.953 m D_2 = 5429.953 m; D_3 = 5429.953 m

19.30 In Figure 19.20, slope distance *S* and zenith angle ∀ at station *A* were observed as 2072.33 m and 82°17'18", respectively to station *B*. If the elevation of station *A* is 435.967 m and the geoid height at stations *A* and *B* are both −28.04m, what is ellipsoid length *A'B'*? (Use an average radius for the Earth of 6,371,000 m.)

2053.556 m ∗ = 7°43'05.7"; AB_1 = 2053.556; BB_1 = 278.318; P = 0°01'06.5";

B_1B_2 = 0.0448; AB_2 = 2053.511 m

19.31* Components of deflection of the vertical at an observing station of latitude 43°15'47.5864" are ξ = −6.87" and η = −3.24" If the observed zenith angle on a course with an astronomic azimuth of 204°32'44" is 85°56'07", what are the azimuth and zenith angles corrected for deviation of the vertical?

z = 85°56'00.1"; ∀ = 204°32'47.3" by Equations (19.34) and (19.32), respectively.

19.32 At the same observation station as for Problem 19.31, the observed zenith angle on a course with an azimuth of 154°00'59" is 84°22'21", what are the azimuth and zenith angles corrected for deviation of the vertical?

z = 84°22'14.1"; ∀ = 154°01'02.4" by Equations (19.34) and (19.32), respectively.

19.33 Using the reduced azimuths of Problems 19.31 and 19.32, what is the reduced geodetic angle that is less than 180°?

Angle: **50°31'44.9"**

19.34 What is a orthometric height of a point? ...geodetic height of a point?

Requires students to go back to Sections 4.2 and 14.1.3 definitions:

Section 14.1.3: “..., geodetic height of a point is the vertical distance between the ellipsoid and the point.”

Section 4.2: Elevation (orthometric height) is the distance measured along a vertical line from a vertical datum (geoid) to a point.

19.35 Compute the collimation correction factor *C* for the following field data, taken in accordance with the example and sketch in the field notes of Figure 19.18. With the instrument at station 1, high, middle, and low cross-hair readings were 5.512, 5.401, and 5.290 ft on station *A* and 4.978, 3.728, and 2.476 ft on station *B*. With the instrument at station 2, high, middle, and low readings were 7.211, 6.053, and 4.894 ft on *A* and 4.561, 4.358, and 4.155 ft on *B*.

0.0022 ft/ft

	r1	i1	R1	I1		
1	5.512		4.978			
	5.401	0.111	3.728	1.250	C	0.0022
	5.290	0.111	2.476	1.252		
	5.401	0.222	3.727	2.502		

	R2	I2	r2	i2
2	7.211		4.651	
	6.053	1.158	4.358	0.293
	4.894	1.159	4.155	0.203
	6.053	2.317	4.388	0.496

19.36 A leveling instrument having a collimation factor of 0.0005 m/m of interval was used to run a section of three-wire differential levels from BM *A* to BM *B*. Sums of backsights and foresights for the section were 1320.892 m and 933.695 m, respectively. Backsight stadia intervals totaled 1557.48, while the sum of foresight intervals was 805.67. What is the corrected elevation difference from BM *A* to BM *B*?

−387.573 m by Equation (19.17): 1320.892 – 933.695 + 0.0005(1557.48 – 805.67)

19.37 The relative error of the difference in elevation between two benchmarks directly connected in a level circuit and located 70 km apart is ±0.006 m. What order and class of leveling does this represent?

Second order, Class I; c = 0.72 mm

19.38 Similar to Problem 19.37, except the relative error is 0.015 ft for benchmarks located 25 km apart.

First Order, Class I c = 0.09 mm

19.39 The baseline components of a GPS baseline vector observed at a station *A* in meters are (1204.869, 798.046, −666.157). The geodetic coordinates of the first base station are 24°27'36.0894" N latitude and 104°44'09.4895" W longitude. What are the changes in the local geodetic coordinate system of (Δn, Δe, Δu)?

(−159.902, 962.244, −1257.330)

R			neu
0.105322	0.400439	0.91025	-159.902
0.967108	-0.25437	0	962.2437
-0.23154	-0.88031	0.414058	-1257.33

19.40 In Problem 19.39, what are the slant distance, zenith angle, and azimuth for the baseline vector?

S = 1591.337, Az = 99°26'05.8", z = 142°11'44.0" by Equation (19.50)

19.41 If the slant distance between two stations is 1243.273 m, the zenith angle between them is 98°58'44" and the azimuth of the line is 32°23'59", what are the changes in the local geodetic coordinates?

(−103.970, −658.010, −194.038) by Equation (19.51)