25 VERTICAL CURVES

Asterisks (*) indicate problems that have partial answers given in Appendix G.

25.1 What is the advantage of using a parabola in the vertical design of highways and railroads?

From Section 25.1, paragraph 1: "Because parabolas provide a constant rate of change of grade, ⌡"

25.2 What factors must be taken into account when designing a grade line on any highway or railroad?

From Section 25.1, paragraph 3: "There are several factors that must be taken into account when designing a grade line of tangents and curves on any highway or railroad project. They include (1) providing a good fit with the existing ground profile, thereby minimizing the depths of cuts and fills, (2) balancing the volume of cut material against fill, (3) maintaining adequate drainage, (4) not exceeding maximum specified grades, and (5) meeting fixed elevations such as intersections with other roads. In addition, the curves must be designed to (a) fit the grade lines they connect, (b) have lengths sufficient to meet specifications covering a maximum rate of change of grade (which affects the comfort of vehicle occupants), and (c) provide sufficient sight distance for safe vehicle operation (see Section 25.11)."

Tabulate station elevations for an equal-tangent parabolic curve for the data given in Problems 25.3 through 25.8. Check by second differences.

25.3* A +2.50% grade meets a −1.75% grade at station 44+25 and elevation 386.96 ft, 400-ft curve, stakeout at half stations.

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BVC Station = 42+25.00
BVC Elevation = 381.96
Station     x (Sta)        g1*x     r/2*x*x      Elevation
============================================================
46+25.00       4.00       10.00       -8.50         383.46
46+00.00       3.75        9.38       -7.47         383.86
45+50.00       3.25        8.13       -5.61         384.47
45+00.00       2.75        6.88       -4.02         384.82
44+50.00       2.25        5.63       -2.69         384.90
44+00.00       1.75        4.38       -1.63         384.71
43+50.00       1.25        3.13       -0.83         384.25
43+00.00       0.75        1.88       -0.30         383.54
42+50.00       0.25        0.63       -0.03         382.55
42+25.00          0           0           0         381.96
============================================================
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Maximum elevation = 384.90 @ station 44+60.29

25.4 A −3.00% grade meets a +2.50% grade at station 4 + 200 and elevation 105.568 m, 200-m curve, stakeout at 30-m increments.

BVC Station = 4+100.000
BVC Elevation = 108.568

Station	x (Sta)	g1*x	r/2*x*x	Elevation
4+300.000	2.000	-6.000	5.500	108.068
4+290.000	1.900	-5.700	4.964	107.832
4+260.000	1.600	-4.800	3.520	107.288
4+230.000	1.300	-3.900	2.324	106.992
4+200.000	1.000	-3.000	1.375	106.943
4+170.000	0.700	-2.100	0.674	107.142
4+140.000	0.400	-1.200	0.220	107.588
4+110.000	0.100	-0.300	0.014	108.282
4+100.000	0	0	0	108.568

Minimum elevation = 106.932 @ station 4+209.091

25.5 A 525-ft curve, grades of $g_1 = -2.00\%$ and $g_2 = +1.50\%$, VPI at station 78 + 60, and elevation 1255.35 ft, stakeout at full stations.

BVC Station = 75+97.500
BVC Elevation = 1260.600

Station	x (Sta)	g1*x	r/2*x*x	Elevation
81+22.500	5.250	-10.500	9.188	1,259.287
81+00.000	5.025	-10.050	8.417	1,258.967
80+00.000	4.025	-8.050	5.400	1,257.950
79+00.000	3.025	-6.050	3.050	1,257.600
78+00.000	2.025	-4.050	1.367	1,257.917
77+00.000	1.025	-2.050	0.350	1,258.900
76+00.000	0.025	-0.050	0.000	1,260.550
75+97.500	0	0	0	1,260.600

Minimum elevation = 1,257.600 @ station 78+97.500

25.6 A 550-ft curve, grades of $g_1 = -4.00\%$ and $g_2 = -2.25\%$, VPI at station 38 + 00, and elevation 5560.00 ft, stakeout at full stations.

BVC Station = 35+25.000
BVC Elevation = 5571.000

Station	x (Sta)	g1*x	r/2*x*x	Elevation
40+75.000	5.500	-22.000	4.813	5,553.813
40+00.000	4.750	-19.000	3.589	5,555.589
39+00.000	3.750	-15.000	2.237	5,558.237
38+00.000	2.750	-11.000	1.203	5,561.203
37+00.000	1.750	-7.000	0.487	5,564.487
36+00.000	0.750	-3.000	0.089	5,568.089

35+25.000	0	0	0	5,571.000

25.7 A 180-m curve, g_1 = +3.00%, g_1 = −2.00%, VPI station = 2 + 175, VPI elevation = 686.543 m, stakeout at 30-m increments.

BVC Station = 2+85.000
BVC Elevation = 683.843

Station	x (Sta)	g1*x	r/2*x*x	Elevation
2+265.000	1.800	5.400	-4.500	684.743
2+250.000	1.650	4.950	-3.781	685.012
2+220.000	1.350	4.050	-2.531	685.362
2+190.000	1.050	3.150	-1.531	685.462
2+160.000	0.750	2.250	-0.781	685.312
2+130.000	0.450	1.350	-0.281	684.912
2+100.000	0.150	0.450	-0.031	684.262
2+085.000	0	0	0	683.843

Maximum elevation = 685.463 @ station 2+193.000

25.8 A 200-ft curve, g_1 = −1.50%, g_2 = +2.50%, VPI station = 46 + 00, VPI elevation = 895.00 ft, stakeout at quarter stations.

BVC Station = 45+00.000
BVC Elevation = 896.500

Station	x (Sta)	g1*x	r/2*x*x	Elevation
47+00.000	2.000	-3.000	4.000	897.500
46+75.000	1.750	-2.625	3.063	896.938
46+50.000	1.500	-2.250	2.250	896.500
46+25.000	1.250	-1.875	1.563	896.188
46+00.000	1.000	-1.500	1.000	896.000
45+75.000	0.750	-1.125	0.563	895.938
45+50.000	0.500	-0.750	0.250	896.000
45+25.000	0.250	-0.375	0.063	896.188
45+00.000	0.000	-0.000	0.000	896.500

Minimum elevation = 895.938 @ station 45+75.000

25.9 A 90-m curve, g_1 = −1.50%, g_2 = −0.75%, VPI station = 6 + 280, VPI elevation = 235.600 m, stakeout at 10-m increments.

BVC Station = 6+235.000
BVC Elevation = 236.275

Station	x (Sta)	g1*x	r/2*x*x	Elevation
6+325.000	0.900	-1.350	0.337	235.262
6+320.000	0.850	-1.275	0.301	235.301
6+310.000	0.750	-1.125	0.234	235.384
6+300.000	0.650	-0.975	0.176	235.476
6+290.000	0.550	-0.825	0.126	235.576
6+280.000	0.450	-0.675	0.084	235.684

6+270.000	0.350	-0.525	0.051	235.801
6+260.000	0.250	-0.375	0.026	235.926
6+250.000	0.150	-0.225	0.009	236.059
6+240.000	0.050	-0.075	0.001	236.201
6+235.000	0	0	0	236.275

Field conditions require a highway curve to pass through a fixed point. Compute a suitable equal-tangent vertical curve and full-station elevations for Problems 25.10 through 25.12.

25.10* Grades of $g_1 = -2.50\%$ and $g_2 = +1.00\%$, VPI elevation 750.00 ft at station 30 + 00. Fixed elevation 753.00 ft at station 30 + 00.

L = 685.714 ft   reduced Equation (25.3): $0.4375L = 3$

BVC Station = 26+57.143
BVC Elevation = 758.571

Station	x (Sta)	g1*x	r/2*x*x	Elevation
33+42.857	6.857	-17.143	12.000	753.429
33+00.000	6.429	-16.071	10.547	753.047
32+00.000	5.429	-13.571	7.521	752.521
31+00.000	4.429	-11.071	5.005	752.505
30+00.000	**3.429**	**-8.571**	**3.000**	**753.000**
29+00.000	2.429	-6.071	1.505	754.005
28+00.000	1.429	-3.571	0.521	755.521
27+00.000	0.429	-1.071	0.047	757.547
26+57.143	0	0	0	758.571

Minimum elevation = 752.449 @ station 31+46.939

25.11 Grades of $g_1 = -2.50\%$ and $g_2 = +1.50\%$, VPI elevation 2430.00 ft at station 315 + 00 Fixed elevation 2436.50 ft at station 314 + 00.

***L* = 1165.68 ft**
BVC Station = 309+17.160
BVC Elevation = 2444.571

Station	x (Sta)	g1*x	r/2*x*x	Elevation
320+82.840	11.657	-29.142	23.314	2,438.743
320+00.000	10.828	-27.071	20.118	2,437.618
319+00.000	9.828	-24.571	16.574	2,436.574
318+00.000	8.828	-22.071	13.373	2,435.873
317+00.000	7.828	-19.571	10.515	2,435.515
316+00.000	6.828	-17.071	8.000	2,435.500
315+00.000	5.828	-14.571	5.828	2,435.828
314+00.000	4.828	-12.071	4.000	2,436.500
313+00.000	3.828	-9.571	2.515	2,437.515
312+00.000	2.828	-7.071	1.373	2,438.873
311+00.000	1.828	-4.571	0.574	2,440.574
310+00.000	0.828	-2.071	0.118	2,442.618
309+17.160	0	0	0	2,444.571

Minimum elevation = 2,435.464 @ station 316+45.710

25.12 Grades of g_1 = +5.00% and g_2 = +1.50% VPI station 6+300 and elevation205.920 m. Fixed elevation 205.610 m at station 6+400. (Use 100-m stationing)

L = <u>761.217 m</u>
BVC Station = 5+919.391
BVC Elevation = 186.900

Station	x (Sta)	g1*x	r/2*x*x	Elevation
6+680.608	7.612	38.061	-13.321	211.639
6+600.000	6.806	34.030	-10.649	210.281
6+500.000	5.806	29.030	-7.750	208.180
6+400.000	4.806	24.030	-5.310	205.620
6+300.000	3.806	19.030	-3.330	202.600
6+200.000	2.806	14.030	-1.810	199.120
6+100.000	1.806	9.030	-0.750	195.180
6+000.000	0.806	4.030	-0.149	190.781
5+919.391	0	0	0	186.900

25.13 A −1.10% grade meets a +0.60% grade at station 36 + 00 and elevation 800.00 ft. The +0.60% grade then joins a +2.40% grade at station 39 + 00. Compute and tabulate the notes for an equal-tangent vertical curve, at half-stations, that passes through the midpoint of the 0.60% grade.

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Midpoint = (3600 + 3900)/2 = 37+50
Elevation @ 37+50 = 800.00 + 1.5*0.6 = 800.90
Find station and elevation of P (PVI of g1 and g2)
Along g1: E1 = –1.10x; Y x = –E1/(1.10)      (a)
Along g3: E2 = 2.40(3 – x)                   (b)
Along g2: E1 + E2 = 0.60´3 = 1.80            (c)

Substitute (a) into (b):
E2 = 2.40[ 3 + E1/(1.10)]               (d)

Substitute (d) into (c) and solve for E1
2.40[3 + E1/(1.10)] = 1.80 – E1
E1 = –1.69714
From (a): x = 1.69714/1.10 = 1.54286 sta = 154.29
Sta of P = 3600 + 154.29 = 3754.29
Elev of P = 800 + E1 = 798.30

Find L:
800.90 = 798.30+1.10[L/2–(L/2–0.0429)]+(3.50/2L)(L/2–
0.0429)2
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L2 – 6.00653L + 0.007347 = 0

BVC Station = 34+54.025
BVC Elevation = 801.603

Station	x (Sta)	g1*x	r/2*x*x	Elevation
40+54.555	6.005	-6.606	10.509	805.506
40+50.000	5.960	-6.556	10.350	805.398
40+00.000	5.460	-6.006	8.687	804.284
39+50.000	4.960	-5.456	7.168	803.316
39+00.000	4.460	-4.906	5.796	802.493
38+50.000	3.960	-4.356	4.569	801.816
38+00.000	3.460	-3.806	3.488	801.285
37+50.000	2.960	-3.256	2.553	800.900
37+00.000	2.460	-2.706	1.763	800.660
36+50.000	1.960	-2.156	1.119	800.566
36+00.000	1.460	-1.606	0.621	800.618
35+50.000	0.960	-1.056	0.268	800.816
35+00.000	0.460	-0.506	0.062	801.159
34+54.025	0	0	0	801.603

Minimum elevation = 800.565 @ station 36+42.763

25.14 When is it advantageous to use an unequal-tangent vertical curve instead of an equal-tangent one?

From Section 25.8, paragraph 1: They are used to enable the vertical curve to closely fit the ground conditions, which is used to minimize excessive cut or fill quantities.

Compute and tabulate full-station elevations for an unequal-tangent vertical curve to fit the requirements in Problems 25.15 through 25.18.

25.15 A +3.50% grade meets a −2.25% grade at station 60+00 and elevation 1310.00 ft. Length of first curve 600 ft, second curve 400 ft.

BVC Station = 54+00.000
BVC Elevation = 1289.000

Station	x (Sta)	g1*x	r/2*x²	Elevation
64+00.000	4.000	4.800	-6.900	1,301.000
63+00.000	3.000	3.600	-3.881	1,302.819
62+00.000	2.000	2.400	-1.725	1,303.775
61+00.000	1.000	1.200	-0.431	1,303.869
60+00.000	0.000	0.000	-0.000	1,303.100
CVC				
60+00.000	6.000	21.000	-6.900	1,303.100
59+00.000	5.000	17.500	-4.792	1,301.708
58+00.000	4.000	14.000	-3.067	1,299.933
57+00.000	3.000	10.500	-1.725	1,297.775

56+00.000	2.000	7.000	-0.767	1,295.233
54+00.000	0	0	0	1,289.000

25.16 Grade g_1 = +2.25%, g_1 = +3.75%, VPI at station 62+00 and elevation 850.00 ft, L_1 = 700 ft and L_2 = 500 ft.

BVC Station = 54+00.000
BVC Elevation = 1289.000

Station	x (Sta)	g1*x	r/2*x²	Elevation
64+00.000	4.000	4.800	-6.900	1,301.000
63+00.000	3.000	3.600	-3.881	1,302.819
62+00.000	2.000	2.400	-1.725	1,303.775
61+00.000	1.000	1.200	-0.431	1,303.869
60+00.000	0.000	0.000	-0.000	1,303.100
CVC				
60+00.000	6.000	21.000	-6.900	1,303.100
59+00.000	5.000	17.500	-4.792	1,301.708
58+00.000	4.000	14.000	-3.067	1,299.933
57+00.000	3.000	10.500	-1.725	1,297.775
56+00.000	2.000	7.000	-0.767	1,295.233
54+00.000	0	0	0	1,289.000

25.17 Grades g_1 of +5.00% and g_2 of −2.00% meet at the VPI at station 4+300 and elevation 154.960 m. Lengths of curves are 200 m and 350 m. (Use 40-m stationing.)

BVC Station = 4+100.000
BVC Elevation = 144.960

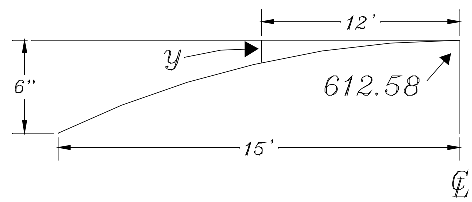
Station	x (Sta)	g1*x	r/2*x²	Elevation
4+650.000	2.750	1.909	-4.455	147.960
4+640.000	3.400	1.855	-4.204	148.156
4+600.000	3.000	1.636	-3.273	148.869
4+560.000	2.600	1.418	-2.458	149.465
4+520.000	2.200	1.200	-1.760	149.945
4+480.000	1.800	0.982	-1.178	150.309
4+440.000	1.400	0.764	-0.713	150.556
4+400.000	1.000	0.545	-0.364	150.687
4+360.000	0.600	0.327	-0.131	150.702
4+320.000	0.200	0.109	-0.015	150.600
CVC				
4+300.000	2.000	10.000	-4.455	150.505
4+240.000	1.400	7.000	-2.183	149.777
4+200.000	1.000	5.000	-1.114	148.846
4+160.000	0.600	3.000	-0.401	147.559
4+100.000	0	0	0	144.960

25.18 A −2.40% grade meets a +1.75% grade at station 95 + 00 and elevation 2320.64 ft. Length of first curve is 300 ft, of second curve, 500 ft.

BVC Station = 92+00.000
BVC Elevation = 2327.840

Station	x (Sta)	g1*x	r/2*x²	Elevation
100+00.000	5.000	0.969	3.891	2,329.390
99+00.000	4.000	0.775	2.490	2,327.796
98+00.000	3.000	0.581	1.401	2,326.512
97+00.000	2.000	0.388	0.622	2,325.541
96+00.000	1.000	0.194	0.156	2,324.880
95+00.000	0.000	0.000	0.000	2,324.531
CVC				
95+00.000	3.000	-7.200	3.891	2,324.531
94+00.000	2.000	-4.800	1.729	2,324.769
93+00.000	1.000	-2.400	0.432	2,325.872
92+00.000	0	0	0	2,327.840

25.19* A manhole is 12 ft from the centerline of a 30-ft wide street that has a 6-in. parabolic crown. The street center at the station of the manhole is at elevation 612.58 ft. What is the elevation of the manhole cover?



Elev = **612.26 ft**

$$y = \left(\frac{12}{15}\right)^2 6 = 3.84 \text{ in.} = 0.32 \text{ ft}$$

25.20 A 50-ft wide street has an average parabolic crown from the center to each edge of 1/4 in./ft. How much does the surface drop from the street center to a point 4 ft from the edge?

0.44 ft y = (25 − 4)(1/4 in.) = 5.25 in. = 0.44 ft

25.21 Determine the station and elevation at the high point of the curve in Problem 25.3.

384.90 ft @ station 44+60.29

25.22* Calculate the station and elevation at the low point of the curve in Problem 25.4.

106.932 m @ station 4+209.091

25.23 Compute the station and elevation at the low point of the curve of Problem 25.5.

1,257.60 ft @ station 78+97.50

25.24 What are the station and elevation of the high point of the curve of Problem 25.7?

685.463 @ station 2+193.000

25.25 What are the requirements for sight distances on a vertical curve?

From Section 25.12, paragraph 1: "The vertical alignments of highways should provide ample sight distance for safe vehicular operation. Two types of sight distances are involved: (1) stopping sight distance (the distance required, for a given "design speed,"1 to safely stop a vehicle thus avoiding a collision with an unexpected stationary object in the roadway ahead) and (2) passing sight distance (the distance required for a given design speed, on two-lane two-way highways to safely overtake a slower moving vehicle, pass it, and return to the proper lane of travel leaving suitable clearance for an oncoming vehicle in the opposing lane)."

25.26* Compute the sight distance available in Problem 25.3. (Assume h_1 = 3.50 ft and h_2 = =4.25 ft.)

563.85 ft By Eq. (251.10): $S = 0.5\left[4 + 2\frac{\left(\sqrt{3.5}+\sqrt{4.25}\right)^2}{2.5+1.75}\right] = 5.6385 \text{ sta}$

So $S > L$ is satisfied

25.27 Similar to Problem 25.26, except h_2 = 2.00 ft.

453.92 ft By Eq. (251.10): $S = 0.5\left[4 + 2\frac{\left(\sqrt{3.5}+\sqrt{2}\right)^2}{2.5+1.75}\right] = 4.53918 \text{ sta}$

So $S > L$ is satisfied

25.28 Similar to Problem 25.26, except for the data of Problem 25.7, where h_1 = 1.0 m and h_2 = 0.5 m.

144.852 m By Eq. (25.9): $S = \sqrt{\frac{180}{0.03+0.02}\left[2\left(\sqrt{1}+\sqrt{0.5}\right)^2\right]} = 144.852 \text{ m}$

So $S < L$ is satisfied

25.29 In determining sight distances on vertical curves, how does the designer determine whether the cars or objects are on the curve or tangent?

Try either formula in Section 25.11 and compare the derived sight distance with the length of the curve. If the derived sight distance does not fit on the curve, then use the other formula.

What is the minimum length of a vertical curve to provide a required sight distance for the conditions given in Problems 25.30 through 25.32?

25.30* Grades of +3.00% and −2.50%, sight distance 600 ft, h_1 = 3.50 ft and h_2 = 2.00 ft.

917.39 ft By Eq. (25.9): $L = \frac{6^2(3+2.5)}{2\left(\sqrt{3.5}+\sqrt{2}\right)^2} = 9.1739 \text{ sta}$

So $S < L$ is satisfied

25.31 A crest curve with grades of +4.50% and −3.00% sight distance 500 ft, h_1 = 3.50 ft and

h_2 = 4.25 ft.

606.26 ft By Eq. (25.9): $L = \frac{5^2(4.5+3.0)}{2\left(\sqrt{3.5}+\sqrt{4.25}\right)^2} = 6.0626$ sta ($S < L$)

25.32 Sight distance of 200 m, grades of +1.00% and −2.25%, h_1 = 1.0 m and h_2 = 0.5 m.

223.045 m By Eq. (25.9): $L = \frac{200^2(0.01+0.0225)}{2\left(\sqrt{1}+\sqrt{0.5}\right)^2} = 2.23045$ sta ($S < L$)

25.33* A backsight of 6.85 ft is taken on a benchmark whose elevation is 567.50 ft. What rod reading is needed at that HI to set a blue top at grade elevation of 572.55 ft?

1.80 ft = 567.50 + 6.85 − 572.55

25.34 A backsight of 4.52 ft is taken on a benchmark whose elevation is 658.28 ft. A foresight of 5.04 ft and a backsight of 7.04 ft are then taken in turn on TP_1 to establish a HI. What rod reading will be necessary to set a blue top at a grade elevation of 660.38 ft?

4.42 ft = 658.28 + 4.52 – 5.04 + 7.04 – 660.38