# PART I:
# SOLUTIONS TO PROBLEMS

# 1 INTRODUCTION

*NOTE:* Answers for some of these problems, and some in later chapters, can be obtained by consulting the bibliographies, later chapters, websites, or professional surveyors.

**1.1** List 10 uses for surveying in areas other than boundary surveying.

Answers may vary many are included in Section 1.6, which lists control, topographic hydrographic, alignment, construction, as-built, mine, solar, optical tooling, ground, aerial, and satellite surveys. This list is not complete and could also include other types of surveys such as hydrographic surveys, for example.

**1.2** Explain the difference between geodetic and plane surveys.

From Section 1.4:
In geodetic surveys the curved surface of the earth is considered by performing the computations on an ellipsoid (curve surface approximating the size and shape of the earth). In plane surveys, except for leveling, the reference base for fieldwork and computations is assumed to be a flat horizontal surface. The direction of a plumb line (and thus gravity) is considered parallel throughout the survey region, and all measured angles are presumed to be plane angles.

**1.3** Describe some surveying applications in:

**(a)** Archeology
There are many different uses of surveying in archeology. Some include using sonar to identify possible underground or underwater archeology sites, LiDAR to help identify possible ancient human settlements in unexplored forest and jungles, and traditional surveying and laser scanning to help locate artifacts in site excavations.

**(b)** Gas exploration
There are several stages of surveying in gas exploration, which include but are not limited to determining anomalies in the gravity field, which identify possible gas deposits, boundary surveys identifying properties that have mineral rights to the gas deposits, alignment surveys for placement of pipelines to transport extracted gas.

**(c)** Agriculture
In agriculture, surveying is used to determine the acreage of fields, to locate lines of constant elevation for strip farming, to track harvesting machinery to enable the size of the harvest, and to track the position of the planting equipment to allow for precise applications of seeds and fertilizers. The field is known as high-precision agriculture.

**1.4** List some application of surveying in geology, forestry, and archeology.

Applications in each are multiple. For some in geology and archeology see the answer to Problem 1.3 (a) and (b). Some uses of surveying in forestry identifying forest boundaries, locating spread of diseases and insects through remote sensing, using GIS to help inventory and keep records on resources in forested regions.

**1.5** Why is it important to make accurate surveys of underground utilities?

To provide an accurate record of the locations of these utilities so they can be found if repairs or servicing is needed, and to prevent their accidental destruction during excavation for other projects.

**1.6** Discuss the uses for topographic surveys.

Topographic surveys are used whenever elevation data is required in the end product. Some examples include (1) creating maps for highway design; (2) creating maps for construction surveys; (3) creating maps for flood plain delineation; (4) creating maps for site location of buildings; and so on.

**1.7** What are hydrographic surveys, and why are they important?

From Section 1.6, hydrographic surveys define shorelines and depths of lakes, streams, oceans, reservoirs, and other bodies of water. Sea surveying is associated with port and offshore industries and the marine environment, including measurements and marine investigations made by ship borne personnel.

**1.8** Print a view of your location using Google Earth.®

Answers will vary but should be an image in your region.

**1.9** Briefly explain the procedure used by Eratosthenes in determining the Earth's circumference.

From Section 1.3, paragraph 8 of text: His procedure, which occurred about 200 B.C., is illustrated in Figure 1.3. Eratosthenes had concluded that the Egyptian cities of Alexandria and Syene were located approximately on the same meridian, and he had also observed that at noon on the summer solstice, the sun was directly overhead at Syene. (This was apparent because at that time of that day, the image of the sun could be seen reflecting from the bottom of a deep vertical well there.) He reasoned that at that moment, the sun, Syene, and Alexandria were in a common meridian plane, and if he could measure the arc length between the two cities, and the angle it subtended at the earth's center, he could compute the earth's circumference. He determined the angle by measuring the length of the shadow cast at Alexandria from a tall vertical staff of known length. The arc length was found from multiplying the number of caravan days between Syene and Alexandria by the average daily distance traveled. From these measurements

Eratosthenes calculated the earth's circumference to be about 25,000 mi. Subsequent precise geodetic measurements using better instruments, but techniques similar geometrically to Eratosthenes', have shown his value, though slightly too large, to be amazingly close to the currently accepted one.

**1.10** Describe the steps a land surveyor would need to do when performing a boundary survey.

Briefly, the steps should include (1) preliminary walking of property with owner; (2) courthouse research to locate deed of property and adjoiners to determine ownership, possible easements, right-of-ways, conflicts of interest, and so on; (3) location survey of property noting any encroachments; conflicting elements; and so on; (4) resolution of conflicting elements between deed and survey; (5) delivery of surveying report to owner.

**1.11** What is the name of the state-level professional surveying organization in your state or region?

Answer will vary by location.

**1.12** What organizations in your state furnish maps and reference data to surveyors and engineers?

Responses will vary but some common organizations are the (1) county surveyor, (2) register of deeds, (3) county engineer or county highway department (4) Department of Transportation, (5) Department of Natural Resources of its equivalent, and so on.

**1.13** List the legal requirements for registration as a land surveyor in your state.

Responses will vary. Contact with your licensing board can be found on the NCEES website at http://www.ncees.org/licensure/licensing_boards/.

**1.14** Briefly describe an Earth-Centered, Earth-Fixed coordinate system.

From Section 1.4 and 13.4.3, a ECEF coordinate system is an Earth-based three-dimensional coordinate system with its origin a the mass-center of the Earth, it $Z$ axis aligned with the semi-minor (spin) axis of the Earth defined at some epoch, it $X$ axis in the plane of the equator passing through mean Greenwich meridian, and it $Y$ axis in the plane of the equator and creating a right-handed coordinate system. At this stage of their introduction to surveying it should be sufficient for students to simply know that it is an Earth-based three-dimensional coordinate system.

**1.15** List the professional societies representing the geospatial industry in the

**(a)** United States.

There are several including AAGS, ASCE, ASPRS, GLIS, NSPS, and SaGES.

**(b)** Canada.

Canadian Institute of Geomatics (CIG)

**(c)** International.

International Federation of Surveyors (FIG)

**1.15** Explain how aerial photographs and satellite images can be valuable in surveying.

Photogrammetry presently has many applications in surveying. It is used, for example, in land surveying to compute coordinates of section corners, boundary corners, or point of evidence that help locate these corners. Large–scale maps are made by photogrammetric procedures for many uses, one being subdivision design. Photogrammetry is used to map shorelines, in hydrographic surveying, to determine precise ground coordinates of points in control surveying, and to develop maps and cross sections for route and engineering surveys. Photogrammetry is playing an important role in developing the necessary data for modern Land and Geographic Information Systems.

**1.16** Search the Internet and define a Very Long Baseline Interferometry (VLBI) station. Discuss why these stations are important to the geospatial industry.

VLBI stands for Very Long Baseline Interferometry. Responses will vary. These stations provide extremely accurate locations on the surface of the Earth. The stations are used to develop world-wide reference frameworks such as ITRF08 and thus provide a worldwide coordinate system that links continents. They also may provide tracking information for satellites.

**1.17** Describe how a GIS can be used in flood emergency planning.

Responses will vary but may mention the capabilities of a GIS to overlay soil type and their permeability with slopes, soil saturation, and watershed regions. A GIS can also be used to provide a list of business and residences that will be affected by possible flooding for evacuation purposes. It can provide "best" routes out of a flooded region.

**1.19** Visit one of the surveying websites listed in Table 1.1, and write a brief summary of its contents. Briefly explain the value of the available information to surveyors.

Responses will vary with time, but below are brief responses to the question

- NGS – control data sheets, CORS data, surveying software
- USGS – maps, software
- BLM – cadastral maps, software, ephemerides
- U.S. Coast Guard Navigation Center - GPS information
- U.S. Naval Observatory –Notice Advisory for NAVSTAR Users (NANU) and other GPS related links
- National Society of Professional Surveyors – professional organization for boundary and construction
- American Association for Geodetic Surveying – professional organization for control surveying
- Geographic and Land Information Society – professional organization for developers and users of geographic and land information systems

- American Society for Photogrammetry and Remote Sensing – professional organization for photogrammetry and remote sensing
- The Pearson Prentice Hall publishers access to software and support materials that accompany this book.
- SaGES – An organization to advance surveying/geomatics education

**1.20** Read one of the articles cited in the bibliography for this chapter, or another of your choosing, that describes an application where satellite surveying methods were used. Write a brief summary of the article.

Answer will vary.

**1.21** Same as Problem 1.20, except the article should be on safety as related to surveying.

Answers will vary but should be related to safety issues in surveying.

**1.20** Read one of the articles cited in the bibliography for this chapter, or another of your choosing, that describes an application where satellite surveying methods were used. Write a brief summary of the article.

Answers will vary. Students should be told to look in trade journals for articles.

**1.21** Same as Problem 1.20, except the article should be on safety as related to surveying.

Answers will vary. Students should be told to look in trade journals for articles.

# 2 UNITS, SIGNIFICANT FIGURES, AND FIELD NOTES

**2.1** List the five types of measurements that form the basis of traditional plane surveying.

From Section 2.1, they are (1) horizontal angles, (2) horizontal distances, (3) vertical (altitude or zenith) angles, (4) vertical distances, and (5) slope (or slant) distances.

**2.2** Give the basic units that are used in surveying for length, area, volume, and angles in

**(a)** The English system of units.

From Section 2.2:

length (U.S. survey ft or in some states international foot), area (sq. ft. or acres), volume (cu. ft. or cu. yd.), angle (sexagesimal)

**(b)** The SI system of units.

From Section 2.3:

length (m), area (sq. m. or hectare), volume (cu. m.), angle (sexagesimal, grad, or radian)

**2.3** The easting coordinate for a point is 725,316.911 m. What is the coordinate using the

**(a)** Survey foot definition?

**(b)** International foot definition?

**(c)** Why was the survey foot definition maintained in the United States?

**(a)** **2,379,643.90 sft**; $725{,}316.911\left(\frac{39.37}{12}\right) = 2{,}379{,}643.899$ sft

**(b)** **2,379,648,66 ft**; $725{,}316.911/0.3048 = 2{,}379{,}648.658$ ft

**(c)** From Section 2.2: "Because of the vast number of surveys performed prior to 1959, it would have been extremely difficult and confusing to change all related documents and maps that already existed. Thus the old standard, now called the *U.S. survey foot*, is still used."

**2.4** Convert the following distances given in meters to U.S. survey feet:

***(a)** 4129.574 m **13,548.44 sft**

**(b)** 588.234 m **1929.90 sft**

**(c)** 102,302.103 m **335,636.15 sft**

**2.5** Convert the following distances given in survey feet to meters:

***(a)** 537.52 sft **163.836 m**

**(b)** 2,405,687.82 sft **733,255.114 m**

**(c)** 5783.12 sft **1762.699 m or 1762.70 m**

**2.6** Compute the lengths in survey feet corresponding to the following distances measured with a Gunter's chain:

***(a)** 10 ch 13 lk **668.6 sft**

**(b)** 56 ch 83 lk **3750.8 sft**

**(c)** 124 ch 35 lk **8207.1 sft**

**2.7** Express 5,377,700 $sft^2$ in:

***(a)** acres **123.46 ac**

**(b)** hectares **49.961 ha**

**(c)** square Gunter's chains **1234.6 sq. ch.**

**2.8** Convert 23.4587 ha to:

**(a)** square survey feet **2,525,070 $sft^2$**

**(b)** acres **57.9676 ac**

**(b)** square Gunter's chains **579.676 sq. ch**

**2.9** What are the lengths in feet and decimals for the following distances shown on a building blueprint:

**(a)** 12 ft 6-1/4 in. **12.5 ft** 601/4/12

**(b)** 10 ft 6-1/2 in. **10.5 ft** 253/2/12

**2.10** What is the area in acres of a rectangular parcel of land measured with a Gunter's chain if the recorded sides are as follows:

***(a)** 9.17 ch and 10.64 ch **9.76 ac**

**(b)** 16 ch 78 lk and 52 ch 49 lk **88.08 ac**

**2.11** Compute the area in acres of triangular lots shown on a plat having the following recorded right-angle sides:

**(a)** 335.36 ft and 804.02 ft **3.0945 ac**

**(b)** 93.064 m and 30.346 m **0.69785 ac**

**2.12** A distance is expressed as 9756.12 sft. What is the length in

***(a)** international feet? **9756.14 ft**

**(b)** meters? **2973.67 m**

**2.13** What are the radian and degree-minute-second equivalents for the following angles given in grads:

**(a)*** 136.000 grads **<u>122°24′00"; 2.13628 rad</u>**

**(b)** 115.089 grads **<u>103°34′50″; 1.80781 rad</u>**

**(c)** 363.809 grads **<u>327°25′40″; 5.71469 rad</u>**

**2.14** Give answers to the following problems in the correct number of significant figures:

***(a)** sum of 23.15, 0.984, 124, and 12.5 **<u>161</u>**

**(b)** sum of 2.115, 23.04, 13.8, and 199.66 **<u>238.6</u>**

**(c)** product of 127.08 and 13.1 **<u>1660</u>**

**(d)** quotient of 4466.83 divided by 35.61 **<u>125.4</u>**

**2.15** Express the value or answer in powers of 10 to the correct number of significant figures:

**(a)** 4586.49 **<u>$4.58679 \times 10^3$</u>**

**(b)** 2450 **<u>$2.45 \times 10^3$</u>**

**(c)** square of 199.99 **<u>$3.9996 \times 10^4$</u>**

**(d)** sum of (32.087 + 1.56 + 206.44) divided by 2.3 **<u>$1.95 \times 10^1$</u>**

**2.16** Convert the angles of a triangle to radians and show a computational check:

***(a)** 39°41′54, 91°30'16", 48°47′50" **<u>0.692867, 1.59705,</u>** and **<u>0.851672</u>**

**0.6928666 + 1.597054 + 0.8516721 = <u>3.14159 check</u>**

**(b)** 96°23'18, 44°56′53", 38°39′49" **<u>1.68229, 0.784492</u>**, and **<u>0.674807</u>**

**1.682294 + 0.784492 + 0.674807 = <u>3.14159 check</u>**

**2.17** Why should a ball point pen not be used in field notekeeping?

From Section 2.7: "Books so prepared (with 3h or higher pencil) will withstand damp weather in the field (or even a soaking) and still be legible, whereas graphite from a soft pencil, or ink from a pen or ballpoint, leaves an undecipherable smudge under such circumstances."

**2.18** Explain why one number should not be superimposed over another or the lines of sketches.

From Section 2.7: This can be explained with the need for integrity since it would raise the issue of what are you hiding, legibility since the numbers are often hard to interpret when so written, or by clarity since the notes are being crowded.

**2.19** Explain why data should always be entered directly into the field book at the time measurements are made, rather than on scrap paper for neat transfer to the field book later.

From Section 2.7: Data should always be entered into the field book directly at the time of the measurements to avoid loss of data.

**2.20** Why should the field notes show the precision of the measurements?

Field notes should show the precision of the measurements made to indicate the accuracy of the measurements.

**2.21** Explain the reason for item 7 in Section 2.11 when recording field notes.

In general a sketch will show more than a table of numbers. As the saying goes, "A picture is worth a thousand words."

**2.22** Explain the reason for item 13 in Section 2.11 when recording field notes.

A standard set of symbols and signs improve the clarity of drawings.

**2.23** Explain the reason for item 18 in Section 2.11 when recording field notes.

A zero should be placed before a decimal point for the sake of clarity.

**2.24** When should sketches be made instead of just recording data?

Sketches should be made instead of recording data anytime observations need to be clarified so that the personnel interpreting the notes can have a clear understanding of the field conditions. This also serves as a reminder of the work performed and any unusual conditions in later references to the project.

**2.25** Justify the requirement to list in a field book the makes and serial numbers of all instruments used on a survey.

Listing the makes and serial numbers of the instruments used in the survey may help isolate instrumental errors later when reviewing the project.

**2.26** Discuss the advantages of survey controllers that can communicate with several different types of instruments.

The ability of survey controllers to communicate with several different types of instruments allows the surveyor to match the specific conditions of the project with the instrument that this is ideally suited for the job. Thus total station, digital levels, and GNSS receivers can all be used in a single project.

**2.27** Discuss why data should always be backed up at regular intervals.

From Section 2.13, paragraph 1: "At regular intervals, usually at lunchtime and at the end of a day's work, or when a survey has been completed, the information stored in files within a data collector is transferred to another device. This is a safety precaution to avoid accidentally losing substantial amounts of data."

**2.28** Search the Internet and find at least two sites related to

**(a)** Manufacturers of survey controllers.

**(b)** Manufacturers of total stations.

**(c)** Manufacturers of global navigation satellite system (GNSS) receivers.

**Answers should vary with students.**

**2.29** Why do many survey controllers contain digital cameras?

From Section 2.15: "Many modern survey controllers also contain digital cameras that allow field personnel to capture a digital image of the survey."

**2.30** What are the dangers involved in using a survey controller?

From Section 2.15: "Although survey controllers have many advantages, they also present some dangers and problems. There is the slight chance, for example, the files could be accidentally erased through carelessness or lost because of malfunction or damage to the unit."

**2.31** Describe what is meant by the phrase "field-to-finish."

From Section 2.15, "The field codes can instruct the drafting software to draw a map of the data complete with lines, curves and mapping symbols. The process of collecting field data with field codes that can be interpreted later by software is known as a *field-to-finish* survey. This greatly reduces the time needed to complete a project."

**2.32** Why are sketches in field books not usually drawn to scale?

This is true since this would require an overwhelming amount of time. The sketches are simply to provide readers of the notes an approximate visual reference to the measurements.

# 3 THEORY OF ERRORS IN OBSERVATIONS

**3.1** Discuss the difference between an error and a residual.

From Section 3.3, an error is the difference between the observation and its true value, or $E = X - \bar{X}$ whereas a residual, which is defined in Section 3.11 is the difference between the mean of a set of observations and the observation or $v = \bar{M} - M$

**3.2** Give two examples of **(a)** direct and **(b)** indirect measurements.

From Section 3.2: A direct observation is made by applying a measurement instrument directly to a quantity to be measured and an indirect observation is made by computing a quantity from direct observations.

Examples should vary by student response.

**3.3** Define the term *systematic error*, and give two surveying examples of a systematic error.

See Section 3.6

**3.4** Define the term *random error*, and give two surveying examples of a random error.

See Section 3.6

**3.5** Discuss the difference between accuracy and precision.

From Section 3.7, accuracy is the nearness of the observed quantities to the true value, which is never known. Precision is the degree of refinement or consistency of a group of observations and is evaluated on the basis of discrepancy size.

**3.6** The observations of 124.53, 124.55, 142.51, and 124.52 are obtained when taping the length of a line. What should the observer consider doing before a mean length is determined from the set of observations?

It appears that the observation 142.51 is an outlier and a possible mistake in the data set. The observer should collect another tape observation of the line and discard the offending observation(s).

A distance *AB* is observed repeatedly using the same equipment and procedures, and the results, in meters, are listed in Problems 3.7 through 3.10. Calculate **(a)** the line's most probable length, **(b)** the standard deviation and **(c)** the standard deviation of the mean for each set of results.

***3.7** 65.401, 65.400, 65.402, 65.396, 65.406, 65.401, 65.396, 65.401, 65.405, and 65.404

**(a)** **<u>65.401</u>** $\sum 654.012$

**(b)** **<u>±0.003</u>** $\sum v^2 = 0.000091$

**(c)** **<u>±0.001</u>**

**3.8** Same as Problem 3.7 but discard only one 65.396 observation.

**(a)** **<u>65.402</u>** ∑588.616

**(b)** **<u>±0.003</u>** $\sum v^2 = 0.000064$

**(c)** **<u>±0.0009</u>**

**3.9** Same as Problem 3.7, but discard both 65.396 observations.

**(a)** **<u>65.402</u>** ∑523.220

**(b)** **<u>±0.002</u>** $\sum v^2 = 0.000030$

**(c)** **<u>±0.0007</u>**

**3.10** Same as Problem 3.7, but include two additional observations, 65.402 and 65.405.

**(a)** **<u>65.402</u>** ∑784.819

**(b)** **<u>±0.003</u>** $\sum v^2 = 0.000115$

**(c)** **<u>±0.0009</u>**

In Problems 3.11 through 3.14, determine the range within which observations should fall (a) 90% of the time and (b) 95% of the time. List the percentage of values that actually fall within these ranges.

**3.11** For the data of Problem 3.7.

***(a)** **<u>65.4012±0.0052 (65.3960, 65.4064), 100%</u>**

**(b)** **<u>65.4012±0.0062 (65.3950, 65.4074), 100%</u>**

**3.12** For the data of Problem 3.8.

**(a)** **<u>65.4018±0.0046 (65.3971, 65.4064), 90%, 65.396 outside of range</u>**

**(b)** **<u>65.4018±0.0055 (65.3963, 65.4073), 90%, 65.396 outside of range</u>**

**3.13** For the data of Problem 3.9.

**(a)** **<u>65.4025±0.0034 (65.3991, 65.4059), 90%, 65.406 outside of range</u>**

**(b)** **<u>65.4025±0.0040 (65.3985, 65.4065), 100%</u>**

**3.14** For the data of Problem 3.10.

**(a)** **<u>65.4016±0.0053 (65.3963, 65.4069), 83.3%, both 65.396 outside of range</u>**

**(b)** **<u>65.4016±0.0063 (65.3952, 65.4079), 100%</u>**

In Problems 3.15 through 3.17, an angle is observed repeatedly using the same equipment and procedures. Calculate **(a)** the angle's most probable value, **(b)** the standard deviation, and **(c)** the standard deviation of the mean.

***3.15** $23°30'00''$, $23°30'10''$, $23°30'10''$, and $23°29'55''$.

**(a)** **<u>23°30′04″</u>**

**(b)** **<u>±7.5″</u>**

**(c)** **<u>±3.8″</u>**

**3.16** Same as Problem 3.15, but with three additional observations, $23°29'55''$, $23°29'50''$ and $23°30'05''$.

**(a)** **<u>23°30′01″</u>**

**(b)** **<u>±7.9″</u>**

**(c)** **<u>±3.0″</u>**

**3.17** Same as Problem 3.15, but with two additional observations, $23°30'05''$ and $23°29'55''$.

**(a)** **<u>23°30′02″</u>**

**(b)** **<u>±6.9″</u>**

**(c)** **<u>±2.8″</u>**

**3.18*** A field party is capable of making taping observations with a standard deviation of ±0.02 ft per 100 ft tape length. What standard deviation would be expected in a distance of 400 ft taped by this party?

By Equation (3.12): **<u>±0.04 ft</u>** $= 0.020\sqrt{400/100}$

**3.19** Repeat Problem 3.18, except that the standard deviation per 30-m tape length is ±3mm and a distance of 60 m is taped. What is the expected 95% error in 60 m?

$S$ = by Equation (3.12): **<u>±0.004 m</u>** $= 0.003\sqrt{60/30}$

$S_{95}$ = by Equation (3.8): **<u>±0.008 m</u>** = 0.0042(1.9599)

**3.20** A distance of 200 ft must be taped in a manner to ensure a standard deviation smaller than ±0.04 ft. What must be the standard deviation per 100 ft tape length to achieve the desired precision?

**<u>±0.028 ft</u>** $= \pm 0.04 / \sqrt{200/100}$ by Equation (3.12) rearranged.

**3.21** Lines of levels were run requiring $n$ instrument setups. If the rod reading for each backsight and foresight has a standard deviation σ, what is the standard deviation in each of the following level lines?

**(a)** $n = 12$, $\sigma = \pm 0.005$ ft; By Equation (3.12): **<u>±0.017 ft</u>** $= 0.005\sqrt{12}$.

**(b)** $n$ = 32, σ = ±3 mm; By Equation (3.12): **±17.0 mm** = $3\sqrt{32}$.

**3.22** A line *AC* was observed in 2 sections *AB* and *BC*, with lengths and standard deviations listed below. What is the total length *AC*, and its standard deviation?

*__(a)__ $AB = 60.00 \pm 0.015$ ft; $BC = 86.13 \pm 0.018$ ft; **146.13±0.023 ft** by Equation (3.11)

**(b)** $AB = 30.000 \pm 0.004$ m; $BC = 23.150 \pm 0.003$ m; **53.150±0.005 m** by Equation (3.11)

**3.23** Line *AD* is observed in three sections, *AB, BC*, and *CD*, with lengths and standard deviations as listed below. What is the total length *AD* and its standard deviation?

**(a)** $AB = 456.78 \pm 0.03$ ft; $BC = 524.56 \pm 0.04$ ft; $CD = 692.35 \pm 0.05$ ft
**1673.69 ± 0.071 ft** by Equation (3.11)

**(b)** $AB = 229.090 \pm 0.005\ m$; $BC = 336.447 \pm 0.006$ m; $CD = 465.837 \pm 0.008$ m

**1031.374 ± 0.011 m** by Equation (3.11)

**3.24** The difference in elevation between *A* and *B* was observed four times as 32.05, 32.03, 32.08, and 32.01 ft. The observations were given weights of 2, 1, 3 and 2, respectively, by the observer. ***(a)** Calculate the weighted mean for distance *AB*. (**b**) What difference results if later judgment revises the weights to 2, 3, 1, and 1, respectively?

By Equation (3.17):

*__(a)__ **32.036 ft**; $m_w = \frac{32.05(2)+32.03(1)+32.08(3)+32.01(2)}{2+1+3+2}$

**(b)** **32.040 ft**; $m_w = \frac{32.05(2)+32.03(3)+32.08(1)+32.01(1)}{2+3+1+1}$

**3.25** Determine the weighted mean for the following angles:

By Equation (3.17):

**(a)** $222°12'36''$, wt 2; $222°12'42''$, wt 1; $222°12'34''$, wt 3; **222°12′40.2″**; $m_w = \frac{36(2)+42(1)+34(3)}{2+1+3}$

**(b)** $96°14'20'' \pm 3''$; $96°14'24'' \pm 2''$; $96°14'18'' \pm 1''$; **96°14′19.3″**; $m_w = \frac{20\left(\frac{1}{3}\right)^2+42\left(\frac{1}{2}\right)^2+34\left(\frac{1}{1}\right)\hat{}2}{\frac{1}{3^2}+\frac{1}{2^2}+\frac{1}{1^2}}$

**3.26** Specifications for observing angles of an *n*-sided polygon limit the total angular misclosure to *E*. How accurately must each angle be observed for the following values of *n* and *E*?

By rearranged Equation (3.12):

**(a)** $n$ = 6, $E$ = ±10″; **±4.1″**; $10/\sqrt{6}$

**(b)** $n$ = 10, $E$ = ±10″; **±3.2″**; $10/\sqrt{10}$

**3.27** What is the area of a rectangular field and its estimated error for the following recorded values:

By Equation (3.13):

***(a)** $243.89 \pm 0.05$ ft by $208.65 \pm 0.04$ ft; **50,888 ± 14 ft²** or **1.1682 ± 0.0003 ac**;

$\sqrt{[243.89(0.04)]^2 + [208.65(0.05)]^2}$

**(b)** $1203.45 \pm 0.08$ ft by $906.78 \pm 0.06$ ft; **1,091,300 ± 100 ft²** or **25.052 ± 0.002 ac**; $\sqrt{[1203.45(0.06)]^2 + [906.78(0.08)]^2}$

**(c)** $344.092 \pm 0.006$ m by $180.403 \pm 0.005$ m; **62,075.0 ± 2.0 m²** or **6.2075 ± 0.00020 ha**; $\sqrt{[344.092(0.005)]^2 + [180.403(0.006)]^2}$

**3.28** Adjust the angles of triangle *ABC* for the following angular values and weights:

By Equation (3.17):

***(a)** $A = 49°24'22''$, wt 2; $B = 39°02'16''$, wt 1; $C = 91°33'00''$, wt 3

Misclosure = −22″

| | Obs. Ang. | Wt | Corr. | Num. Cor. | Rnd. Cor. | Adj. Ang. |
|---|---|---|---|---|---|---|
| *A* | 49°24′22″ | 2 | 3x | 6″ | 6″ | **49°24′28″** |
| *B* | 39°02′16″ | 1 | 6x | 12″ | 12″ | **39°02′28″** |
| *C* | 91°33′00″ | 3 | 2x | 4″ | 4″ | **91°33′04″** |
| | 179°59′38″ | 6 | 11x | | | |
| | | | 11x = 22″ | x = 2″ | | |

**(b)** $A = 81°06'44''$, wt 2; $B = 53°33'56''$, wt 2; $C = 45°19'20''$, wt 3

Misclosure = –10″

| | Obs. Ang. | Wt | Corr. | Num. Cor. | Rnd Cor. | Adj. Ang. |
|---|---|---|---|---|---|---|
| *A* | 81°06′44″ | 2 | 21x | 3.8″ | 4″ | **81°06′48″** |
| *B* | 53°33′56″ | 2 | 21x | 3.8″ | 4″ | **53°34′00″** |
| *C* | 45°19′10″ | 3 | 14x | 2.5″ | 2″ | **45°19′12″** |
| | 179°59′53″ | 7 | 56x | | 10″ | |
| | | | 56x = 10″ | x = 0.178″ | | |

**3.29** Determine relative weights and perform a weighted adjustment (to the nearest second) for angles *A, B*, and *C* of a plane triangle, given the following four observations for each angle:

| Angle *A* | Angle *B* | Angle *C* |
|---|---|---|
| 44°28′16″ | 65°56′13″ | 69°35′20″ |
| 44°28′12″ | 65°56′10″ | 69°35′24″ |
| 44°28′17″ | 65°56′06″ | 69°35′18″ |
| 44°28′11″ | 65°56′08″ | 69°35′24″ |

***A*** = 44°28′14.0″ ± 2.9″; ***B*** = 65°56′09.3″ ± 3.0″; ***C*** = 69°35′21.5″ ± 3″

Misclosure = −15.3″

| | Obs. Ang. | Wt | Corr. Multiplier | Num. Cor. | Rnd. Cor. | Adj. Ang. |
|---|---|---|---|---|---|---|
| *A* | 44°28′14.0″ | 0.115385 | 0.338645/wt = 2.93 | 4.97″ | 5.0″ | 44°28′19″ |
| *B* | 65°56′09.3″ | 0.11215 | 0.338645/wt = 3.02 | 5.12″ | 5.1″ | 65°56′14″ |
| *C* | 69°35′21.5″ | 0.111111 | 0.338645/wt= 3.05 | 5.16″ | 5.2″ | 69°35′27″ |
| | 179°59′44.7″ | 0.338645 | 9.00x | | | |
| | | | 9.0 = 15.3″ | x = 1.69″ | | |

**3.30** A line of levels was run from benchmarks *A* to *B, B* to *C*, and *C* to *D*. The elevation differences obtained between benchmarks, with their standard deviations, are listed below. What is the difference in elevation from benchmark *A* to *D* and the standard deviation of that elevation difference?

**(a)** BM *A* to BM *B* = +37.78 ± 0.12 ft; BM *B* to BM *C* = –73.50 ± 0.16 ft; and BM *C* to BM *D* = –84.09 ± 0.08 ft

By Equation (3.11): **−119.81 ± 0.22 ft**

**(b)** BM *A* to BM *B* = –60.821 ± 0.015 m; BM *B* to BM *C* = +94.378 241 ± 0.020 m; and BM *C* to BM *D* +56.805 ± 0.015 m

By Equation (3.11): **90.362 ± 0.029 m**

# 4 LEVELING THEORY, METHODS, AND EQUIPMENT

**4.1** Define the following leveling terms: (**a**) vertical line, (**b**) level surface, and (**c**) benchmark.

From Section 4.2:

**(a)** Vertical line: "A line that follows the local direction of gravity as indicated by a plumb line"

**(b)** Level surface: ". A curved surface that at every point is perpendicular to the local plumb line (the direction in which gravity acts)."

**(c)** Benchmark: "A relatively permanent object, natural or artificial, having a marked point whose elevation above or below a reference datum is known or assumed."

***4.2** How far will a horizontal line depart from the Earth's surface in 1 km? 5 km? 10 km? (Apply both curvature and refraction)

1 km? $C_m = 0.0675(1)^2 = 0.068$ m

5 km? $C_m = 0.0675(5)^2 = 1.688$ m

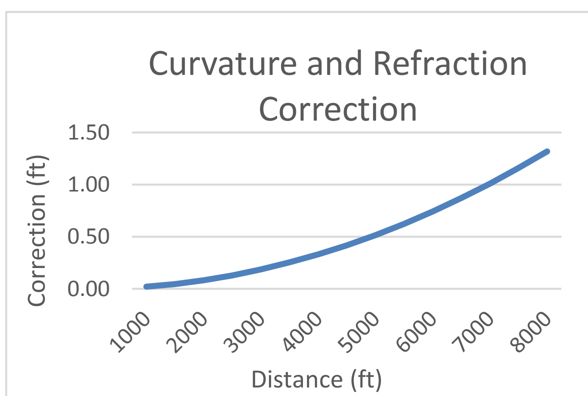
10 km? $C_m = 0.0675(10)^2 = 6.750$ m

**4.3** Visit the website of the National Geodetic Survey, and obtain a data sheet description of a benchmark in your local area.

Solutions should vary.

**4.4** Create plot of the curvature and refraction correction for sight lines going from 0 ft to 10,000 ft in 500 ft increments.

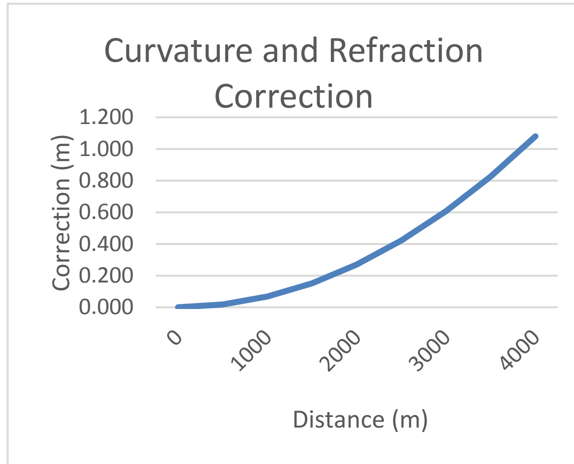
| Dist. (ft) | CR (ft) |
|---|---|
| 1000 | 0.02 |
| 1500 | 0.05 |
| 2000 | 0.08 |
| 2500 | 0.13 |
| 3000 | 0.19 |
| 3500 | 0.25 |
| 4000 | 0.33 |
| 4500 | 0.42 |
| 5000 | 0.52 |
| 5500 | 0.62 |



| | |
|---|---|
| 6000 | 0.74 |
| 6500 | 0.87 |
| 7000 | 1.01 |
| 7500 | 1.16 |
| 8000 | 1.32 |

**4.5** Create a plot of curvature and refraction corrections for sight lines going from 0 m to 10,000 m in 500 m increments.

| Dist. (m) | CR (m) |
|---|---|
| 0 | 0.000 |
| 500 | 0.017 |
| 1000 | 0.068 |
| 1500 | 0.152 |
| 2000 | 0.270 |
| 2500 | 0.422 |
| 3000 | 0.608 |
| 3500 | 0.827 |
| 4000 | 1.080 |




**4.6** Why are elevations today not referred to as mean sea-level heights?

From Section 4.3: "This adjustment (NAVD88) shifted the position of the reference surface from the mean of the 26 tidal gage stations to a single tidal gage benchmark called *Father Point*, which is in Rimouski, Quebec, Canada, near the mouth of the St. Lawrence Seaway. Thus, elevations in NAVD88 are no longer referenced to mean sea level. Benchmark elevations that were defined by the NGVD29 datum have changed by relatively small, but nevertheless significant amounts in the eastern half of the continental United States (see Figure 19.7). However, the changes are much greater in the western part of the country and reach 1.5 m in the Rocky Mountain region. It is therefore imperative that surveyors positively identify the datum to which their elevations are referred. After the adjustment, adjusted elevations are properly known as orthometric heights."

***4.7** On a large lake without waves, how far from shore is a sailboat when the top of its 30-ft mast disappears from the view of a person lying at the water's edge?

**38,160 ft or 7.3 mi**; $F = 1000\sqrt{\frac{30}{0.0206}} = 38{,}161\ ft = \mathbf{7.228\ mi}$

**4.8** Similar to Problem 4.7, except for a 5-m mast and a person whose eye height is 1.5 m above the water's edge.

**13.3** km; $K = \sqrt{1.5/0.0675} + \sqrt{5/0.0675} = \mathbf{13.32\ km}$

**4.9** Readings on a line of differential levels are taken to the nearest 0.2 mm. For what maximum distance can the Earth's curvature and refraction be neglected?

$M = 1000\sqrt{0.0002/0.0675} = \mathbf{54.4\ m}$

**4.10** Similar to Problem 4.9 except readings are to the 0.001 ft.

$F = 1000\sqrt{0.001/0.0206} = \mathbf{220.3\ ft}$

**4.11** Describe how readings are determined in a digital level when using a bar coded rod.

From Section 4.11: "At the press of a button, the image of bar codes in the telescope's field of view is captured and processed. This processing consists of an on-board computer comparing the captured image to the rod's entire pattern, which is stored in memory. When a match is found, which takes about 4 sec, the rod reading is displayed digitally."

Successive plus and minus sights taken on a downhill line of levels are listed in Problems 4.12 and 4.13. The values represent the horizontal distances between the instrument and either the plus or minus sights. What error results from curvature and refraction?

***4.12** 20, 225; 50, 195; 40, 135; 30, 250 ft.

| Plus | CR (ft) | Minus | CR (ft) |
|---|---|---|---|
| 20 | 0.00000824 | 225 | 0.001043 |
| 50 | 0.0000515 | 195 | 0.000783 |
| 40 | 0.00003296 | 135 | 0.000375 |
| 30 | 0.00001854 | 250 | 0.001288 |
| Sum | 0.00011124 | | 0.003489 |

Combined **−0.003 ft**

**4.13** 20, 70; 25, 60; 20, 55; 15, 60 m.

| Plus | CR (mm) | Minus | CR (mm) |
|---|---|---|---|
| 20 | 0.027 | 70 | 0.33075 |
| 25 | 0.042188 | 60 | 0.243 |
| 20 | 0.027 | 55 | 0.204188 |
| 15 | 0.015188 | 60 | 0.243 |
| | 0.111375 | | 1.020938 |

Combined: –0.91 mm

**4.14** What error results if the curvature and refraction correction is neglected in trigonometric leveling for sights: (**a**) 3000 ft long (**b**) 1200 m long (**c**) 4500 ft long?

(**a**) $h_f = 0.0206\left(\frac{3000}{1000}\right)^2 = \mathbf{0.18\ ft}$

(**b**) $h_m = 0.0675\left(\frac{1200}{1000}\right)^2 = \mathbf{0.097}\ \boldsymbol{m}$

(**c**) $h_f = 0.0206\left(\frac{4500}{1000}\right)^2 = \mathbf{0.42\ ft}$

***4.15** The slope distance and zenith angle observed from point *P* to point *Q* were 2406.787 m and 84°13′07″ respectively. The instrument and rod target heights were equal. If the elevation of point *P* is 30.245 m, above datum, what is the elevation of point *Q*?

$$Elev_Q = 30.245 + 2406.787\cos(84°13'07'') + 0.0675\left(\frac{2406.787\sin(84°13'07'')}{1000}\right)^2$$

$$Elev_Q = 30.245 + 242.443 + 0.387 = \mathbf{273.075\ m}$$

**4.16** The slope distance and zenith angle observed from point X to point Y were 2907.45 ft and 97°25'36″. The instrument and rod target heights were equal. If the elevation of point X is 6547.89 ft above datum, what is the elevation of point Y?

$$Elev_y = 6547.89 + 2907.45\cos(97°25'36'') + 0.0206\left(\frac{2907.45\sin(97°25'36'')}{1000}\right)^2$$

$$= 6547.89 - 375.809 + 0.171 = \mathbf{6172.25\ ft}$$

**4.17** Similar to Problem 4.15, except the slope distance was 1543.853 m, the zenith angle was 83°44'08″ and the elevation of point P was 1850.567 m above datum.

$$Elev_y = 1850.567 + 1543.853\cos(83°44'08'') + 0.0675\left(\frac{1543.853\sin(83°44'08'')}{1000}\right)^2$$

$$= 1850.567 + 168.462 + 0.159 = \mathbf{2016.187\ m}$$

**4.18** In trigonometric leveling from point A to point B, the slope distance and zenith angle measured at A were 5462.46 ft and 94°08'36″. At B these measurements were 5462.58 ft and 85°51'47″, respectively. If the instrument and rod target heights were equal, calculate the difference in elevation from A to B.

$$z_{avg} = \frac{94°08'36'' + 180° - 85°51'47''}{2} = 94°08'24''$$

$$S_{avg} = \frac{5462.58 + 5462.46}{2} = 5462.52$$

$$\Delta Elev = 5462.52\cos(94°08'24'') = \mathbf{-394.36\ ft}$$

**4.19** Describe how parallax in the viewing system of a level can be detected and removed.

From Section 4.7:

"After focusing, if the cross hairs appear to travel over the object sighted when the eye is shifted slightly in any direction, *parallax* exists. The objective lens, the eyepiece, or both must be refocused to eliminate this effect if accurate work is to be done."

**4.20** What is the sensitivity of a level vial with 2-mm divisions for: (a) a radius of 13.75 m (b) a radius of 10.31 m?

**(a)** $\theta = \left[\frac{2}{13.75(1000)}\right]206264.8 = \mathbf{30''}$

**(b)** $\theta = \left[\frac{2}{10.31(1000)}\right]206264.8 = \mathbf{40''}$

***4.21** An observer fails to check the bubble, and it is off two divisions on a 500-ft sight. What error in elevation difference results with a 10-sec bubble?

angular error = 2(10) = 20 sec

Error = 250 tan(20) = **0.048 ft**

**4.22** An observer fails to check the bubble, and it is off two divisions on a 300-m sight. What error results for a 20-sec bubble?

angular error = 2(20) = 40 sec

Error = 200 tan(40) = **0.058 m**

**4.23** Similar to Problem 4.22, except a 30-sec bubble is off three divisions on a 300-ft sight.

angular error = 3(30) = 90 sec

Error = 300 tan(90) = **0.13 ft**

**4.24** With the bubble centered, a 100-m sight gives a reading of 1.352 m. After moving the bubble three divisions off center, the reading is 1.396 m. For 2-mm vial divisions, what is: (**a**) the vial radius of curvature in meters (**b**) the angle in seconds subtended by one division?

Δrdg = 1.410 − 1.352 = 0.058 m

$$4\theta = \text{atan}\left(\frac{0.058}{100}\right) = 120''$$

(**a**) $R = 0.002/\tan(120'') = \mathbf{3.438\ m}$

(**b**) 120″/4 = **30″**

**4.25** Similar to Problem 4.24, except the sight length was 300 ft, the initial reading was 5.132 ft, and the final reading was 5.176 ft.

Δrdg = 5.176 – 5.132 = 0.0.044 ft

$$3\theta = \text{atan}\left(\frac{0.044}{300}\right) = 3\mathbf{0}''$$

(**a**) $R = 0.002/\tan(30'') = \mathbf{13.75\ ft}$

(**b**) 30″/3 = **10″**

**4.26** Sunshine on the forward end of a 20″/2 mm level vial bubble draws it off 2 divisions, giving a plus sight reading of 4.63 ft on a 250-ft sight. Compute the correct reading.

Correction = 200 tan(2·20″) = 0.048 ft

Correct reading = 4.63 − 0.048 = **4.58 ft**

Note: the correction is subtracted since the bubble was drawn off on the forward end of the level, thus raising the line of sight.

**4.27** List in tabular form, for comparison, the advantages and disadvantages of an automatic level versus a digital level.

See Section 4.10 and 4.11.

***4.28** If a plus sight of 3.54 ft is taken on BM *A*, elevation 850.48 ft, and a minus sight of 7.84

ft is read on point *X*, calculate the HI and the elevation of point *X*.

HI = 850.48 + 3.54 = **854.02 ft**

Elev = 854.02 − 7.84 = **846.18 ft**

**4.29** If a plus sight of 0687 m is taken on BM *A*, elevation 85.476 m, and a minus sight of 1.564 m is read on point *X*, calculate the HI and the elevation of point *X*.

HI = 85.476 + 0.687 = **86.163 m**

Elev = 86.163 – 1.564 = **84.599 m**

**4.30** Similar to Problem 4.28, except a plus sight of 8.98 ft is taken on BM *A*, elevation 606.33 ft, and a minus sight of 4.32 ft read on point *X*.

HI = 606.33 + 8.98 = **615.31 ft**

Elev = 615.31 – 4.32 = **610.99 ft**

**4.31** Describe the procedure used to test if the level vial is perpendicular to the vertical axis of the instrument.

See Section 4.15.5

**4.32** A horizontal collimation test is performed on an automatic level following the procedures described in Section 4.15.5. With the instrument setup at point 1, the rod reading at *A* was 5.548 ft, and to *B* it was 5.126 ft. After moving and leveling the instrument at point 2, the rod reading to *A* was 5.540 ft and to *B* was 5.126 ft. What is the collimation error of the instrument and the corrected reading to *A* from point 2?

$$\varepsilon = \frac{5.126 - 5.548 - 5.126 + 5.540}{2} = -0.004$$

Correct reading at *A* = 5.540 – 2(–0.004) = **5.548 ft**

**4.33** The instrument tested in Problem 4.32 was used in a survey immediately before the test where the observed elevation difference between two benchmarks was +44.65 ft. The sum of the plus sight distances between the benchmarks was 250 ft and the sum of the minus sight distances was 490 ft. What is the corrected elevation difference between the two benchmarks?

**+44.64 ft**; = 44.65– 0.004/100(250 − 490) = 44.64 ft

**4.34** Similar to Problem 4.32 except that the rod readings are 1.894 m and 1.923 m to *A* and *B*, respectively, from point 1, and 1.083 m and 1.100 m to *A* and *B*, respectively, from point 2. The distance between the points in the test was 100 m.

$$\varepsilon = \frac{1.923 - 1.894 - 1.100 + 1.083}{2} = 0.006 \text{ m}$$

Correct reading at *A* = 1.083 – 2(0.006) = **1.071 m**

**4.35** The instrument tested in Problem 4.34 was used in a survey immediately before the test where the observed elevation difference between two benchmarks was −13.068 m. The sum of the plus sight distances between the benchmarks was 1540 m and the sum of the minus sight distances was 545 m. What is the corrected elevation difference between the two benchmarks?

**−13.128 m;** $= -13.068 - 0.006/100(1540 - 545)$

# 5 LEVELING — FIELD PROCEDURES AND COMPUTATIONS

Asterisks (*) indicate problems that have answers given in Appendix G.

**5.1** Explain the left-thumb rule when centering a level.

From Section 5.2, paragraph 3: A simple but useful rule in centering a bubble, illustrated in Figure 5.1, is: *A bubble follows the left thumb when turning the screws*.

**5.2** What is the difference between a benchmark and a turning point in differential leveling?

A benchmark is a relatively permanent object, natural or artificial, having a marked point whose elevation above or below a reference datum is known, assumed, or will be established during the leveling process whereas a turning point is an intermediate, temporary point between benchmarks which are created to perform the differential leveling process. They are usually temporary points whose elevations are lost after the differential leveling process is complete.

**5.3** Discuss how stadia can be used to determine the plus and minus sight distances in differential leveling.

From Section 5.4: "The stadia method determines the horizontal distance to points through the use of readings on the upper and lower (stadia) wires on the reticle. The method is based on the principle that in similar triangles, corresponding sides are proportional. … Thus the equation for a distance on a horizontal stadia sight reduces to $D = KI$ (5.2) … It should be realized by the reader that in differential leveling, the actual sight distances to the rod are not important. All one needs to balance is the rod intervals on the plus and minus sights between benchmarks to ensure that the sight distances are balanced."

**5.4** What is the collimation error, and how can it be removed from the differential leveling process.

From Section 5.12.1: It is caused by the line of sight not being parallel with the axis of the level vial. When this condition exists, the line of sight will not be horizontal and thus result in incorrect readings. This is a systematic error and can be removed by balancing the backsight and foresight distances between benchmarks.

**5.5** Discuss how errors due to Earth curvature and refraction can be eliminated from the differential leveling process.

From Section 5.4: "Balancing plus and minus sight distances will eliminate errors due to instrument maladjustment (most important) and the combined effects of the Earth's curvature and refraction, as shown in Figure 5.6. Here $e_1$ and $e_2$ are the combined curvature and refraction errors for the plus and minus sights, respectively. If $D_1$ and $D_2$ are made equal, $e_1$ and $e_2$ are also equal. In calculations, $e_1$ is added and $e_2$ subtracted; thus they cancel each other."

**5.6** When is it appropriate to use the reciprocal leveling procedure?

From Section 5.7: "Sometimes in leveling across topographic features such as rivers, lakes, and canyons, it is difficult or impossible to keep plus and minus sights short and equal. Reciprocal leveling may be utilized at such locations."

**5.7** List four considerations that govern a rodperson's selection of TPs and BMs.

1. From Chapter 4: BMs must be permanent.
2. From Section 5.4: "Turning points should be solid objects with a definite high point."
3. From Section 5.6: "...it is recommended that some turning points or benchmarks used in the first part of the circuit be included again on the return run. This creates a multi-loop circuit, and if a blunder or large error exists, its location can be isolated to one of the smaller loops."
4. From Section 5.12.2: "It (settlement) can be avoided by selecting firm, solid turning points or, if none are available, using a steel turning pin set firmly in the ground."
5. Find turning points that aid in the balancing of plus and minus sight distances.

***5.8** What error is created by a rod leaning 10 min from plumb at a 12.51-ft reading on the leaning rod?

**Error = 0.000 ft**

Correct reading = 12.51 cos(10') = 12.50995; So error is 0.00005 ft, or 0.000 ft

Problem is designed to show that even for a high reading and a mislevelment outside of a typical circular bubble, the resulting error is negligble.

**5.9** Similar to Problem 5.6, except for a 5-m reading.

**Error is 0.000 m**

Correct reading = 5 cos(10') = 4.9999785, so error is 0.000021. The error is negligible.

Problem is designed to show that even for a high reading and a mislevelment outside of a typical circular bubble, the resulting error is negligble.

**5.10** What error results on a 30-m sight with a level if the rod reading is 2.865 m but the top of the 3 m rod is 0.3 m out of plumb?

Correct reading = $\frac{0.3}{3} 2.865 = 2.8506$ m

**Error = 0.014 m**

**5.11** What error results on a 200-ft sight with a level if the rod reading is 6.307 ft but the top of the 7-ft rod is 0.2 ft out of plumb?

Correct reading = $\frac{0.2}{7} 6.307 = 6.3044$

**Error = 0.0026 ft**

**5.12** Prepare a set of level notes for the data listed. Perform a check and adjust the misclosure. Elevation of BM 7 is 2303.45 ft. If the total loop length is 2400 ft, what order of leveling is represented? (Assume all readings are in feet)

| POINT | +S (BS) | –S (FS) |
|---|---|---|
| BM 7 | 5.68 | |
| TP 1 | 9.42 | 7.58 |
| TP 2 | 9.26 | 5.81 |
| BM 8 | 6.45 | 4.59 |
| TP 3 | 9.59 | 8.50 |
| BM 7 | | 13.95 |

| STA | Plus | HI | Minus | ELEV |
|---|---|---|---|---|
| BM 7 | 5.68 | | | 2303.45 |
| | | 2309.13 | (0.006) | (2301.556) |
| TP 1 | 9.42 | | 7.58 | 2301.55 |
| | | 2310.97 | (0.012) | (2305.172) |
| TP 2 | 9.26 | | 5.81 | 2305.16 |
| | | 2314.42 | (0.018) | (2309.848) |
| BM 8 | 6.45 | | 4.590 | 2309.83 |
| | | 2316.28 | (0.024) | (2307.804) |
| TP 3 | 9.59 | | 8.500 | 2307.78 |
| | | 2317.37 | (0.03) | (2303.45) |
| BM 7 | | | 13.950 | 2303.42 |
| | 40.40 | | 40.43 | |

Page check 2303.45+40.4-40.43 = 2303.42
Misclosure = 2303.42 – 2303.45 = –0.03
Correction = – –0.03/5 = 0.006

2400 ft ≈ 0.7315 km and 0.03 ft ≈ 9.1 mm; From Equation 5.3: $m = 12/\sqrt{0.7315} = 10.7$ mm, <u>**Third Order**</u>

***5.13** Similar to Problem 5.12, except the elevation of BM 7 is 132.05 ft and the loop length 2400 ft. (Assume all readings are in feet)

| STA | Plus | HI | Minus | ELEV |
|---|---|---|---|---|
| BM7 | 5.68 | | | 132.05 |
| | | 137.73 | (0.006) | (130.156) |
| TP1 | 9.420 | | 7.58 | 130.15 |
| | | 139.57 | (0.012) | (133.772) |
| BM 8 | 9.26 | | 5.81 | 133.76 |
| | | 143.02 | (0.018) | (138.448) |
| TP2 | 6.45 | | 4.590 | 138.43 |

| | | | | |
|---|---|---|---|---|
| | | 144.88 | (0.024) | (136.404) |
| TP3 | 9.59 | | 8.500 | 136.38 |
| | | 145.97 | (0.03) | (132.05) |
| BM7 | | | 13.950 | 132.02 |
| | 40.4 | | 40.43 | |

Page check: 132.05 + 40.4 – 40.43 = 132.02
Misclosure = 132.02 – 132.05 = –0.03
Correction = – –0.03/5 = 0.006

Misclosure = 0.03(12/39.37) = 9.14 mm; *K* = 2400(12/39.37) = 0.7315

From Equation 5.3: $\frac{9.14\text{ mm}}{\sqrt{0.7315}} = 10.69\text{ mm}$, **Third Order**

**5.14** A differential leveling loop began and closed on BM Tree (elevation 323.48 ft). The plus sight and minus sight distances were kept approximately equal. Readings (in feet) listed in the order taken are 3.18 (+S) on BM Tree, 4.76 (−S) and 2.44 (+S) on TP1, 3.05 (−S) and 6.63 (+S) on BM *X*, 3.64 (−S) and 2.35 (+S) on TP2, and 3.07 (−S) on BM Tree. Prepare, check, and adjust the notes.

| STA | Plus | HI | Minus | ELEV |
|---|---|---|---|---|
| BM TREE | 3.18 | | | 1314.08 |
| | | 1317.26 | (-0.02) | (1312.48) |
| TP 1 | 2.44 | | 4.76 | 1312.50 |
| | | 1314.94 | (-0.04) | (1311.85) |
| BM X | 6.63 | | 3.05 | 1311.89 |
| | | 1318.52 | (-0.06) | (1314.82) |
| TP2 | 2.35 | | 3.640 | 1314.88 |
| | | 1317.23 | (-0.08) | (1314.08) |
| BM TREE | 0.00 | | 3.070 | 1314.16 |
| | 14.60 | | 14.52 | |

Page check: 1314.08 + 14.6 – 14.52 =1314.16
Misclosure = 1314.16 – 1314.08 = 0.08
Correction = –0.08/4 = –0.02

**5.15** A differential leveling circuit began on BM Hydrant (elevation 6012.03 ft) and closed on BM Rock (elevation 6022.90 ft). The plus sight and minus sight distances were kept approximately equal. Readings (in feet) given in the order taken are 1.85(+S) on BM Hydrant, 3.56 (−S) and 8.80 (+S) on TP1, 5.63 (−S) and 9.78 (+S) on BM 1, 6.88 (−S) and 5.54 (+S) on BM 2, 3.11 (−S) and 6.98 (+S), on TP2, and 3.00 (−S) on BM Rock. Prepare, check, and adjust the notes.

| STA | Plus | HI | Minus | ELEV |
|---|---|---|---|---|

| | | | | |
|---|---|---|---|---|
| BM Hydrant | 1.85 | | | 6012.03 |
| | | 6013.88 | (0.02) | (6010.34) |
| TP 1 | 8.80 | | 3.56 | 6010.32 |
| | | 6019.12 | (0.04) | (6013.53) |
| BM 1 | 9.78 | | 5.63 | 6013.49 |
| | | 6023.27 | (0.06) | (6016.45) |
| BM 2 | 5.54 | | 6.880 | 6016.39 |
| | | 6021.93 | (0.08) | (6018.9) |
| TP 2 | 6.98 | | 3.110 | 6018.82 |
| | | 6025.80 | (0.1) | (6022.9) |
| BM Rock | | | 3.000 | 6022.80 |
| | 32.95 | | 22.18 | |

Page check: 6012.03 + 32.95 – 22.18 = 6022.80
Misclosure = 6022.8 – 6022.9 = –0.10
Correction = – –0.1/5 = 0.02

**5.16** A differential leveling loop began and closed on BM Bridge (elevation 12.063 m). The plus sight and minus sight distances were kept approximately equal. Readings (in meters) listed in the order taken are 0.687 (+S) on BM Bridge, 1.109 (−S) and 2.843 (+S) on TP1, 0.866 (−S) and 2.708 (+S) on BM *X*, 2.752 (−S) and 1.064 (+S) on TP2, and 2.563 (−S) on BM Bridge. Prepare, check, and adjust the notes.

| STA | Plus | HI | Minus | ELEV |
|---|---|---|---|---|
| BM Bridge | 0.687 | | | 12.063 |
| | | 12.750 | (-0.003) | (11.638) |
| TP 1 | 2.843 | | 1.109 | 11.641 |
| | | 14.484 | (-0.3) | (13.612) |
| BM X | 2.708 | | 0.866 | 13.618 |
| | | 16.326 | (-0.32) | (13.565) |
| TP2 | 1.064 | | 2.752 | 13.574 |
| | | 14.638 | (-0.34) | (12.063) |
| BM Bridge | 0.000 | | 2.563 | 12.075 |
| | 7.302 | | 7.290 | |

Page check: 12.063 + 7.302 – 7.29 = 12.075
Misclosure = 12.075 – 12.063 = 0.012
Correction = –0.012/4 = –0.003

**5.17** A differential leveling circuit began on BM Rock (elevation 645.879 m) and closed on BM Manhole (elevation 649.159 m). The plus sight and minus sight distances were kept approximately equal. Readings (in meters) listed in the order taken are 1.849 (+S) on BM

Rock, 0.553 (−S) and 2.777 (+S) on TP1, 0.634 (−S) and 1.056 (+S) on BM 1, 1.648 (−S) and 0.458 (+S) on BM 2, 1.409 (−S) and 1.897 (+S) on TP2, and 0.503 (−S) on BM Manhole. Prepare, check, and adjust the notes.

| STA | Plus | HI | Minus | ELEV |
|---|---|---|---|---|
| BM Hydrant | 1.849 | | | 645.879 |
| | | 647.728 | (-0.002) | (647.173) |
| TP 1 | 2.777 | | 0.553 | 647.175 |
| | | 649.952 | (-0.03) | (649.314) |
| BM 1 | 1.056 | | 0.634 | 649.318 |
| | | 650.374 | (-0.006) | (648.72) |
| BM 2 | 0.458 | | 1.648 | 648.726 |
| | | 649.184 | (-0.008) | (647.767) |
| TP 2 | 1.897 | | 1.409 | 647.775 |
| | | 649.67 | (-0.01) | (649.159) |
| BM Rock | | | 0.503 | 649.169 |
| | 8.037 | | 4.747 | |

Page check: 645.879 + 8.037 – 4.747 = 649.169
Misclosure = 649.169 – 649.159 = 0.010
Correction = –0.01/5 = –0.002

**5.18** A differential leveling loop started and closed on BM Juno, elevation 2485.19 ft. The plus sight and minus sight distances were kept approximately equal. Readings (in feet) listed in the order taken are 5.46 (+S) on BM Juno, 3.42 (−S) and 8.88 (+S) on TP1, 5.34 (−S) and 6.46 (+S) on TP2, 8.37 (−S) and 4.26 (+S) on BM1, 9.66 (−S) and 7.89 (+S) on TP3, and 6.13 (−S) on BM Juno. Prepare, check, and adjust the notes.

| STA | Plus | HI | Minus | ELEV |
|---|---|---|---|---|
| BM 7 | 5.46 | | | 2485.19 |
| | | 2490.65 | (-0.006) | (2487.224) |
| TP 1 | 8.88 | | 3.42 | 2487.23 |
| | | 2496.11 | (0.216) | (2490.758) |
| TP 2 | 6.46 | | 5.34 | 2490.77 |
| | | 2497.23 | (0.222) | (2488.842) |
| BM 8 | 4.26 | | 8.37 | 2488.86 |
| | | 2493.12 | (0.228) | (2483.436) |
| TP 3 | 7.89 | | 9.66 | 2483.46 |
| | | 2491.35 | (0.234) | (2485.19) |
| BM 7 | | | 6.13 | 2485.22 |
| | 32.95 | | 32.92 | |

Page check: 2485.19 + 32.95 – 32.92 = 2485.22

Misclosure = 2485.22 – 2485.19 = 0.03
Correction = –0.03/5 = –0.006

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***5.19** A level setup midway between *X* and *Y* reads 6.29 ft on *X* and 7.91 ft on *Y*. When moved within a few feet of *X*, readings of 5.18 ft on *X* and 6.76 ft on *Y* are recorded. What is the true elevation difference, and the reading required on *Y* to adjust the instrument?

Correct ΔElev = 7.91 − 6.29 = **1.62** ft
Unbalanced ΔElev = 6.76 − 5.18 = 1.58 ft
Error at Y = 0.04 ft
Corrected reading at Y = 6.76 + 0.04 = **6.80** ft

**5.20** To test its line of sight adjustment, a level is setup near *C* (elev 193.436 m) and then near *D*. Rod readings listed in the order taken are *C* = 1.815 m, *D* = 1.789 m, *D* = 1.691 m, and *C* = 1.719 m. Compute the elevation of *D,* and the reading required on *C* to adjust the instrument.

ΔElev from C = 1.789 − 1.815 = –0.026
ΔElev from D = 1.691 − 1.719 = –0.028
ΔElev = (–0.026 – 0.028)/2 = –0.027 m
$Elev_D$ = 193.436 – 0.027 = **193.409 m**
Corrected reading at *C* = 1.719 + (–0.002/2) = **1.718 m**;
Check 1.691 – 1.718 = –0.027 m

***5.21** The line of sight test shows that a level's line of sight is inclined downward 3 mm/50 m. What is the allowable difference between BS and FS distances at each setup (neglecting curvature and refraction) to keep elevations correct within 1 mm?

$$\frac{50}{3} = \frac{X}{1} \rightarrow X = \mathbf{16.7\ mm}$$

**5.22** Reciprocal leveling gives the following readings in meters from a set up near *A:* on *A*, 1.110; on *B,* 0.987, 0.995, and 0.984. At the setup near *B:* on *B,* 1.549; on *A,* 1.673, 1.670, and 1.666. The elevation of *A* is 324.583 m. Determine the misclosure and elevation of *B*.

**324.462 m**

From A: $\Delta Elev_{AB} = \frac{0.987+0.995+0.984}{3} - 1.110 = 0.9887 - 1.110 = -0.1213$

From B: $\Delta Elev_{AB} = 1.549 - \frac{1.673+1.670+1.666}{3} = 1.549 - 1.6697 = -0.1207$

$$Elev_B = 324.583 + \frac{-0.1213 - 0.1207}{2} = 324.583 - 0.121 = \mathbf{324.462\ m}$$

***5.23** Reciprocal leveling across a canyon provides the data listed (in meters). The elevation of *Y* is 2265.879 ft. The elevation of *X* is required. Instrument at *X*: +3.182, +S = 3.182; –S = 9.365, 9.370, and 9.368. Instrument at *Y*: : +S = 10.223; –S = 4.037, 4.041, and 4.038.

From *X*: $\Delta Elev = 3.182 - \frac{9.365 + 9.370 + 9.368}{3} = -6.1857$

From $Y$: $\Delta Elev = \dfrac{4.037 + 4.041 + 4.038}{3} - 10.223 = -6.1843$

$$Elev_B = 2265.879 - \frac{6.1857 + 6.1843}{2} = \mathbf{2259.694\ m}$$

**5.24** Prepare a set of three-wire leveling notes for the data given and make the page check. The elevation of BM $X$ is 83.097 m. Rod readings (in meters) are ($U$ denotes upper cross-wire readings, $M$ middle wire, and $L$ lower wire): +S on BM $X$: $U = 1.076$, $M = 0.829$, $L = 0.573$; −S on TP 1: $U = 1.131$, $M = 0.858$, $L = 0.586$; +S on TP 1: $U = 0.989$, $M = 0.767$, $L = 0.555$; −S on BM $Y$: $U = 1.089$, $M = 0.851$, $L = 0.611$.

| Sta | Plus | stadia | HI | Minus | stadia | Elev (m) |
|---|---|---|---|---|---|---|
| BM X | 1.076 | | | | | 83.097 |
| | 0.829 | 0.247 | | | | |
| | 0.573 | 0.256 | | | | |
| | 2.478 | 0.503 | | | | |
| | 0.8260 | | | | | |
| | | | 83.923 | | | |
| TP1 | 0.989 | | | 1.131 | | 83.0647 |
| | 0.767 | 0.222 | | 0.858 | 0.273 | |
| | 0.555 | 0.212 | | 0.586 | 0.272 | |
| | 2.311 | 0.434 | | 2.575 | 0.545 | |
| | 0.7703 | | | 0.8583 | | |
| | | | 83.835 | | | |
| BM Y | | | | 1.089 | | 82.9847 |
| | | | | 0.851 | 0.238 | |
| | | | | 0.611 | 0.24 | |
| | | | | 2.551 | 0.478 | |
| | | | | 0.8503 | | |
| | 1.5963 | 0.937 | | 1.7087 | 1.023 | |
| Page check: | | 83.097 + 1.5963 – 1.7087 = | | | 82.9847 | |

**5.25** Similar to Problem 5.24, except the elevation of BM $X$ is 543.56 ft, and rod readings (in feet) are: +S on BM $X$: $U = 4.898$, $M = 4.146$, $L = 3.396$; −S on TP 1: $U = 5.875$, $M = 4.948$, $L = 4.023$; +S on TP 1: $U = 6.504$, $M = 5.487$, $L = 4.472$; −S on BM $Y$: $U = 5.874$, $M = 5.026$, $L = 4.176$.

| Sta | Plus | stadia | HI | Minus | stadia | Elev (ft) |
|---|---|---|---|---|---|---|
| BM X | 4.898 | | | | | 543.56 |
| | 4.146 | 0.752 | | | | |
| | 3.396 | 0.750 | | | | |
| | 12.44 | 1.502 | | | | |
| | 4.1467 | | | | | |
| | | | 547.7067 | | | |
| TP1 | 6.504 | | | 5.875 | | 542.7580 |
| | 5.487 | 1.017 | | 4.948 | 0.927 | |
| | 4.472 | 1.015 | | 4.023 | 0.925 | |

| | | | | | | |
|---|---|---|---|---|---|---|
| | 16.463 | 2.032 | | 14.846 | 0.926 | |
| | 5.4877 | | | 4.9487 | | |
| | | | 548.2457 | | | |
| BM Y | | | | 5.874 | | 543.2203 |
| | | | | 5.026 | 0.848 | |
| | | | | 4.176 | 0.850 | |
| | | | | 15.076 | 0.849 | |
| | | | | 5.0253 | | |
| | 9.6343 | 3.534 | | 9.9740 | 1.775 | |

Page check: 543.56 + 9.6343 – 9.974 = 543.2203

**5.26** Assuming a stadia constant of 99.987, what is the distance leveled in Problem 5.24?

D = 99.987(0.9370 + 1.7087) = **195.97 m**

**5.27** Assuming a stadia constant of 101.5, what is the distance leveled in Problem 5.25?

D = 101.5(3.534 + 1.775) = **538.9 ft**

**5.28** Prepare a set of profile leveling notes for the data listed and show the page check. All data is given in feet. The elevation of BM *A* is 1213.65, and the elevation of BM *B* is 1234.15. Rod readings are: +S on BM *A,* 6.02 intermediate foresight (IFS) on 11+00, 5.6; –S on TP1, 3.28; +S on TP 1, 8.18; intermediate foresight on 12+00, 6.6; on 12+50, 5.3; on 13+00, 5.8; on 14+00, 6.3; −S on TP 2, 2.98, +S on TP 2, 8.76; intermediate foresight on 14+73, 4.1; on 15+00, 4.9; on 16+00, 6.3; −S on TP3, 3.11; +S on TP3, 9.88; −S on BM *B*, 3.05.

| STA | Plus | HI | Minus | IFS | Elev (ft) | Adj. Elev |
|---|---|---|---|---|---|---|
| BM A | 6.02 | (1219.7) | | | 1213.65 | 1213.65 |
| | | 1219.67 | | | | |
| 11+00 | | | | 5.6 | | 1214.1 |
| TP1 | 8.18 | (1224.6) | 3.28 | | 1216.39 | 1216.41 |
| | | 1224.57 | | | | |
| 12+00 | | | | 6.6 | | 1218.0 |
| 12+50 | | | | 5.3 | | 1219.3 |
| 13+00 | | | | 5.8 | | 1218.8 |
| 14+00 | | | | 6.3 | | 1218.3 |
| TP2 | 8.76 | (1230.4) | 2.98 | | 1221.59 | 1221.63 |
| | | 1230.35 | | | | |
| 14+73 | | | | 4.1 | | 1226.3 |
| 15+00 | | | | 4.9 | | 1225.5 |
| 16+00 | | | | 6.3 | | 1224.1 |
| TP3 | 9.88 | | 3.11 | | 1227.24 | 1227.30 |
| | | 1237.12 | | | | |
| BM B | | | 3.05 | | 1234.07 | 1234.15 |
| | 32.84 | | 12.42 | | | |

Page check: 1213.65 + 32.84 – 12.42 = 1234.07
Misclosure = 1234.07 – 1234.15= –0.08
Correction per setup = 0.02

**5.29** Same as Problem 5.28, except the elevation of BM *A* is 868.99 ft. and the elevation of BM *B* is 883.77 ft.

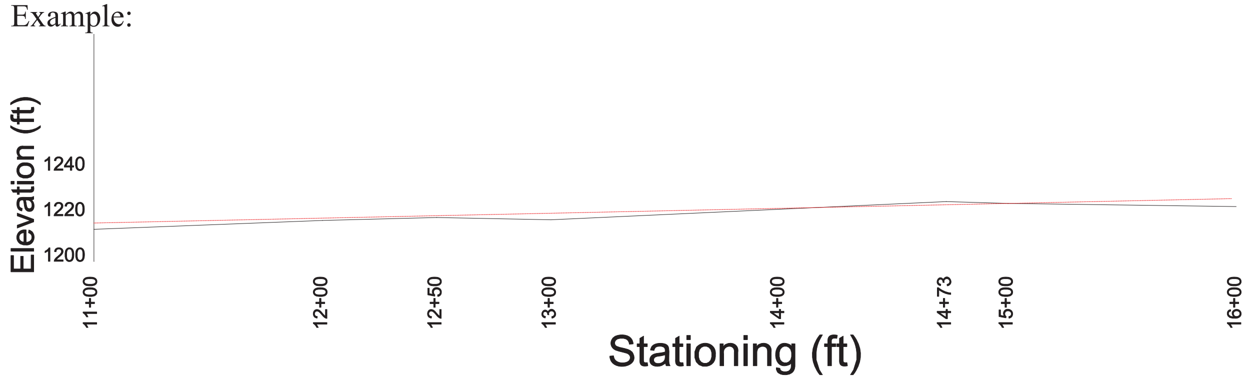
| STA | Plus | HI | Minus | -IFS | Elev (ft) | Adj. Elev |
|---|---|---|---|---|---|---|
| BM A | 6.02 | (875) | | | 868.99 | 868.99 |
| | | 875.01 | | | | |
| 11+00 | | | | 5.6 | | 869.4 |
| TP1 | 8.18 | (880) | 3.28 | | 871.73 | 871.75 |
| | | 879.91 | | | | |
| 12+00 | | | | 6.6 | | 1218.0 |
| 12+50 | | | | 5.3 | | 1219.3 |
| 13+00 | | | | 5.8 | | 1218.8 |
| 14+00 | | | | 6.3 | | 1218.3 |
| TP2 | 8.76 | (885.8) | 2.98 | | 876.93 | 876.97 |
| | | 885.69 | | | | |
| 14+73 | | | | 4.1 | | 1226.3 |
| 15+00 | | | | 4.9 | | 1225.5 |
| 16+00 | | | | 6.3 | | 1224.1 |
| TP3 | 9.88 | | 3.11 | | 882.58 | 882.64 |
| | | 892.46 | | | | |
| BM B | | | 8.77 | | 883.69 | 883.77 |
| | 32.84 | | 18.14 | | | |

Page check: 868.99 + 32.84 – 18.14 = 883.69
Misclosure = 883.69 – 883.77 = –0.08
Correction per setup: 0.02

**5.30** Plot the profile Problem 5.28 and design a grade line between stations 11 + 00 and 16 + 00 that balances cut and fill areas.

Plot of profile from 5.28. Response can vary, but cut and fill sections should visually balanced.

Example:



***5.31** What is the percent grade between stations 11+00 and 16 + 00 in Problem 5.28?

$$\frac{1224.1 - 1214.1}{1600 - 1100} 100\% = +\mathbf{2.0}\%$$

**5.32** Differential leveling between BMs *A, B, C, D*, and *A* gives elevation differences (in meters) of −15.632, +32.458, +38.214 and −55.025, and distances in km of 2.0, 3.0, 1.0, and 4.0, respectively. If the elevation of *A* is 634.597, compute the adjusted elevations of BMs *B, C,* and *D,* and the order of leveling.

Overall length of loop = 10 km.

Misclosure = 0.015 m

$m = \frac{15}{\sqrt{10}} = 4.74$ mm; First order, class II leveling

| Sta | ΔElev | Length (km) | Correction | Adj ΔElev | Adj Elev |
|---|---|---|---|---|---|
| A | | | | | 634.597 |
| | –15.632 | 2 | –0.0030 | –15.6350 | |
| B | | | | | 618.962 |
| | 32.458 | 3 | –0.0045 | 32.4535 | |
| C | | | | | 651.416 |
| | 38.214 | 1 | –0.0015 | 38.2125 | |
| D | | | | | 689.628 |
| | –55.025 | 4 | –0.0060 | –55.0310 | |
| A | | | | | 634.597 |
| | ∑ 0.015 | 10 | | 0.0000 | |

**5.33** Leveling from BM *X* to *W,* BM *Y* to *W,* and BM *Z* to *W* gives differences in elevation (in feet) of –30.26, +26.17, and +10.14, respectively. Distances between benchmarks are *XW* = 2000, *YW* = 2900, and *ZW* = 3550. True elevations of the benchmarks are *X* = 898.83, *Y* 842.39, and *Z* = 858.41. What is the adjusted elevation of *W?* (Note: All data are given in feet.)

From BM X: 898.83 – 30.26 = 868.57 ft.
From BM Y: 842.39 + 26.17 = 868.56 ft.
From BM Z: 858.41 + 10.14 = 868.55 ft.

$$Elev_W = \frac{868.57\left(\frac{1}{2000}\right) + 868.56\left(\frac{1}{2900}\right) + 868.55\left(\frac{1}{3550}\right)}{\frac{1}{2000} + \frac{1}{2900} + \frac{1}{3550}} = \mathbf{868.56\ ft}$$

**5.34** A 3-m level rod was calibrated and its graduated scale was found to be uniformly expanded so that the distance between its 0 and 3.000 marks was actually 3.006 m. How will this affect elevations determined with this rod for (**a**) circuits run on relatively flat ground (**b**) circuits run downhill (**c**) circuits run uphill?

(**a**) On level ground the plus and minus sight readings will be approximately the same. Since the plus readings are added and minus readings subtracted, the net effect will tend to cancel the errors.

(**b**) In level downhill, the minus readings will tend to be higher on the rod than the plus readings. Thus the minus readings will tend to have more error than the plus readings. Thus the elevations will tend to be too high.

(**c**) In level uphill, the plus readings will be tend to be higher on the rod than the minus readings. Thus the plus readings will tend to have more error than the minus readings. Thus the elevations will tend to be too low.

***5.35** A line of levels with 42 setups (84 rod readings) was run from BM Rock to BM Pond with readings taken to the nearest 3.0 mm; hence any observed value could have an error of $\pm 1.5$ mm. For reading errors only, what total error would be expected in the elevation of BM Pond?

$$1.5\sqrt{84} = \pm\mathbf{13.7\ mm}$$

**5.36** Same as Problem 5.35, except for 28 setups and readings to the nearest 0.01 ft with possible error of $\pm 0.005$ ft each.

$$0.005\sqrt{56} = \pm\mathbf{37\ mm}$$

**5.37** Compute the permissible misclosure for the following lines of levels: (**a**) a 10-km loop of third-order levels (**b**) a 20-km section of second-order class I levels (**c**) a 50-km loop of first-order class I levels.

(**a**) $C = 12\sqrt{10} = \mathbf{37.9\ mm}$

(**b**) $C = 6\sqrt{20} = \mathbf{26.8\ mm}$

(**c**) $C = 4\sqrt{50} = \mathbf{28.2\ mm}$

# 6 DISTANCE MEASUREMENT

Asterisks (*) indicate problems that have partial answers given in Appendix G.

**6.1** What distance in travel corresponds to 1 $\mu$sec of time for electromagnetic energy?

**0.299792 m** = 299,792,458(0.000001)

**6.2*** A student counted 92, 90, 92, 91, 93, and 91 paces in six trials of walking along a course of 200 ft known length on level ground. Then 85, 86, 86, and 84 paces were counted in walking four repetitions of an unknown distance AB. What is (a) the pace length and (b) the length of AB?

**(a)** pace length = 200(6)/(92+90+92+91+93+91) = **2.18 ft/pace**

**(b)** *AB* = (85+86+86+84)2.18/4 = **186 ft**

**6.3** What difference in temperature from standard, if neglected in use of a steel tape, will cause an error of 1 part in 5,000?

**31.0° F** or **17.2° C**

$1 = 0.00000645(\Delta T)5000$ or $1 = 0.0000116(\Delta T)5000$

$$\Delta T = \frac{1}{0.00000645(5000)} = 31.0° \text{ F} \qquad \text{or} \qquad \Delta T = \frac{1}{0.0000116(5000)} = 17.2° \text{ C}$$

**6.4** An add tape of 101 ft is incorrectly recorded as 100 ft resulting in a recorded 200-ft distance. What is the correct distance?

**202 ft**;

**6.5*** List five types of common errors in taping. (See Section 6.14 and Table 6.1)

**6.6** List the proper procedures taping a horizontal distance of about 84 ft down a 4% slope.

See Section 6.12 and 6.13.

**6.7** For the following data, compute the horizontal distance for a recorded slope distance *AB*,

**(a)** *AB* = 210.19 ft, slope angle = 3°14′28″; *H* = 210.19cos 3°14′28″ = **209.85 ft**

**(b)** *AB* = 125.657 m, difference in elevation A to B = 3.566 m;

$$H = \sqrt{125.657^2 - 3.566^2} = \mathbf{125.606\ m}$$

**6.8*** When measuring a distance AB, the first taping pin was placed 0.05 ft to the right of line *AB* and the second pin was set 0.5 ft left of line AB. The recorded distance was 268.87 ft. Calculate the corrected distance. (Assume three taped segments, the first two 100 ft each.)

**268.86 ft**

A-Pin1: $\sqrt{100^2 - 0.5^2} = 99.998$; Pin1-Pin2: $\sqrt{100^2 - 1^2} = 99.995$; Pin1-B: $\sqrt{68.87^2 - 0.5^2} = 68.868$; *AB* = 99.998+99.995+68.868 = 268.862 ft

**6.9** List the possible errors that can occur when measuring a distance with an EDM.

See Section 6.24, Table 6.1: (alignment, and length, temp, tension, and support differ from standard, tape not level, plumbing, marking, reading)

**6.10** Briefly describe how a distance can be measured by the method of phase comparison.

See Section 6.18

**6.11** Describe why the sight line for electronic distance measurement should be at least 0.5 m away from a parked vehicle near the line of sight.

Due to the fact that index of refraction will be significantly different near surface of the vehicle due a microclimate that lies immediately around it.

**6.12*** Assume the speed of electromagnetic energy through the atmosphere is 299,836,182 m/sec for measurements with an EDM instrument. What time lag in the equipment will produce an error of 800 m in a measured distance?

**0.0027 msec or 2.7 μsec**; $t = \frac{800}{299{,}836{,}182} = 0.00000267\text{sec}$

**6.13** What is the length of the partial wavelength for electromagnetic energy with a frequency of 29.988 MHz and a phase shift of 156°?

**4.332 m**; $\lambda = \frac{299{,}792{,}458}{29{,}988{,}000} = 9.99708\text{ m}$; $\Delta\lambda = 9.99708\left(\frac{156}{360}\right) = 4.332068\text{ m}$

**6.14** What "actual" wavelength results from transmitting electromagnetic energy through an atmosphere having an index of refraction of 1.0043, if the frequency is:

***(a)** 29.988 MHz; **9.9543 m**; $\lambda = \frac{299{,}792{,}458/1.0043}{29{,}988{,}000} = 9.9543\text{ m}$

**(b)** 14.998 MHz; **19.903 m**; $\lambda = \frac{299{,}792{,}458/1.0043}{14{,}998{,}000} = 19.9032\text{ m}$

**6.15** What index of refraction in the atmosphere will produce the speed of light given in Problem 6.12?

**1.0001458**; $n = \frac{299{,}836{,}182}{299{,}792{,}458} = 1.000145848$

**6.16** To calibrate an EDM instrument, distances *AC*, *AB*, and *BC* along a straight line were observed as 91.694 m, 60.025 m, and 31.698 m, respectively. What is the system measurement constant for this equipment? Compute the length of each segment corrected for the constant.

**–0.029 m**; From Equation (6.16): *K* = 91.694 – 60.025 – 31.698 = –0.029 m

**6.17** Which causes a greater error in a line measured with an EDM instrument: **(a)** A disregarded 5° C temperature variation from standard or **(b)** a neglected atmospheric pressure difference from standard of 25 mm of mercury?

**(a) Pressure**; From Figure 6.16, a 5° C difference in temperature will cause about a 4.7 ppm error in the distance whereas a 25 mm of Hg difference in pressure will cause about a 9.4 ppm error in the distance. So the pressure difference will cause the largest error.

**6.18*** In Figure 6.14, $h_e$, $h_r$, elev$_A$, elev$_B$ and the measured slope length $L$ were 5.56, 6.00, 603.45, 589.06, and 408.65 ft, respectively. Calculate the horizontal length between A and B.

**408.41 ft**; From Equation (6.13) $d = (603.45 + 5.56) - (589.06 + 6.00) = 13.95$ ft

By Equation (6.2): $H = \sqrt{408.65^2 - 13.95^2} = 408.41$ ft

**6.19** Similar to Problem 6.18, except that the values were 1.389, 1.500, 236.489, 254.876, and 312.049 m, respectively.

**311.500 m**; From Equation (6.13) $d = (1.389+236.489) - (1.500+254.876) = -18.498$ m; By Equation (6.2): $H = \sqrt{312.049^2 - 18.498^2} = 311.5002$ m

**6.20** In Figure 6.14, $h_e$, $h_r$, $z$, and the measured slope length $L$ were 5.15 ft, 6.00 ft, 92°32′44″ and 698.75 ft, respectively. Calculate the horizontal length between $A$ and $B$ if a total station measures the distance.

**698.06 ft**; $H$ = 698.75 sin(92°32'44") = 698.060 ft

**6.21*** Similar to Problem 6.20, except that the values were 1.45 m, 1.55 m, 96°05′33″ and 1663.254 m, respectively.

**1653.860 m**; $H$ = 1663.254 sin(96°05'33") = 1653.8597 m

**6.22** What is the actual wavelength and velocity of a near-infrared beam ($\lambda = 0.901\ \mu m$) of light modulated at a frequency of 320 MHz through an atmosphere with a dry bulb temperature, T, of 20° C, a relative humidity, h, of 88%, and an atmospheric pressure of 1034.56 hPa?

**0.936 m**

$$N_g = 287.6155 + \frac{4.88660}{0.901^2} + \frac{0.06800}{0.901^4} = 293.738139$$

$$a = \frac{7.5(20)}{237.3 + 20} + 0.7858 = 1.36877707$$

$$E = 10^a = 23.3763698$$

$$e = E\frac{75}{100} = 20.57120545$$

$$n_a = 1 + \left(\frac{273.15}{1013.25}\frac{293.7381(1034.56)}{20 + 273.15} - \frac{11.27e}{20 + 273.15}\right)10^{-6} = 1.000278663$$

$$V = \frac{299,792,458}{1.000278663} = 299,708,940 \text{ m/s}$$

$$\lambda = \frac{299,708,940}{320 \cdot 10^6} = 0.93659 \text{ m}$$

**6.23** What is the actual wavelength and velocity of a near-infrared beam ($\lambda = 0.901$ μm) of light modulated at a frequency of 340 MHz through an atmosphere with a dry bulb temperature, T, of 25° C, a relative humidity, h, of 75%, and an atmospheric pressure of 893 hPa?

**0.882 m**

$$N_g = 287.6155 + \frac{4.88660}{0.901^2} + \frac{0.06800}{0.901^4} = 293.738139$$

$$a = \frac{7.5(25)}{237.3+25} + 0.7858 = 1.500630347$$

$$E = 10^a = 31.66870809$$

$$e = E\frac{75}{100} = 23.75153107$$

$$n_a = 1 + \left(\frac{273.15}{1013.25}\frac{293.738139(893)}{25+273.15} - \frac{11.27e}{25+273.15}\right)10^{-6} = 1.000236273$$

$$V = \frac{299,792,458}{1.000236273} = 299,721,642 \text{ m/s}$$

$$\lambda = \frac{299,721,642}{340 \cdot 10^6} = 0.881534 \text{ m}$$

**6.24** If the temperature and pressure at measurement time are 18°C and 760 mm Hg, what will be the error in electronic measurement of a line 3 km long if the temperature at the time of observing is recorded 10°C too high? Will the observed distance be too long or too short?

From Figure 6.16: $ppm_T$ = 10 ppm; error = $10/10^6(3000)$ = 0.030 m, which is too short

**6.25*** The standard deviation of taping a 30 m distance is ±5 mm. What should it be for a 90-m distance?

**8.7 mm**; $5\sqrt{90/30} = 8.66$ mm

**6.26** Determine the most probable length of a line *AB*, the standard deviation, and the 95% error of the measurement for the following series of taped observations made under the same conditions: 215.380, 215.381, 215.382, 215.378, 215.388, 215.382, 215.374, 215.382, 215.389, and 215.387 m.

**215.382 ± 0.005 m; ±0.009 m**; Sum = 2153.823;

**6.27** If an EDM instrument has a purported accuracy capability of ±(3.0 mm + 0.5 ppm), what error can be expected in a measured distance of **(a)** 10 m **(b)** 586.08 ft **(c)** 405.957 m? (Assume that the instrument and target miscentering errors are equal to zero.)

**(a)** **±3.0 mm**; $\sqrt{3.0^2 + \left[\frac{0.5(10,000)}{10^6}\right]^2} = 3.000$ mm

**(b)** **±0.010 ft**; 586.08 ft = 178.6375 m; $\sqrt{3.0^2 + \left[\frac{0.5(178,637.5)}{10^6}\right]^2} = 3.001$ mm

**(c)** **<u>±3.0 mm</u>**; $\sqrt{3.0^2 + \left[\frac{0.5(405{,}957)}{10^6}\right]^2} = 3.007 \text{ mm}$

**6.28** The estimated error for both instrument and target miscentering errors is ±1.5 mm. For the EDM in Problem 6.27, what is the estimated error in the observed distances?

**(a)** **<u>±3.7 mm</u>**; $\sqrt{2(1.5^2) + 3.0^2 + \left[\frac{0.5(10{,}000)}{10^6}\right]^2} = 3.67 \text{ mm}$

**(b)** **<u>±0.012 ft</u>**; $\sqrt{2(1.5^2) + 3.0^2 + \left[\frac{0.5(178{,}637.5)}{10^6}\right]^2} = 3.68 \text{ mm}$

**(c)** **<u>±3.7 mm</u>**; $\sqrt{2(1.5^2) + 3.0^2 + \left[\frac{0.5(405{,}957)}{10^6}\right]^2} = 3.68 \text{ mm}$

**6.29** If a certain EDM instrument has an accuracy capability of ±(2 mm + 2 ppm), what is the precision of measurements, in terms of parts-per-million, for line lengths of: **(a)** 30.000 m **(b)** 300.000 m **(c)** 3000.000 m? (Assume that the instrument and target miscentering errors are equal to zero.)

**(a)** **<u>67 ppm</u>**; *E* = ±2.0009; ppm = (2.0009/30,000)1,000,000 = 66.7 ppm

**(b)** **<u>7 ppm</u>**; *E* = ±2.088; ppm = (2.088/300,000)1,000,000 = 7.0 ppm

**(c)** **<u>2.1 ppm</u>**; *E* = ±6.324; ppm = (6.324/3,000,000)1,000,000 = 2.1 ppm

**6.30** The estimated error for both instrument and target miscentering errors is ±1.5 mm. For the EDM and distances listed in Problem 6.29, what is the estimated error in each distance? What is the precision of the measurements in terms of part-per-million?

**(a)** **<u>97 ppm</u>**; *E* = ±2.92 mm; ppm = (2.92/30,000)1,000,000 = 97.2 ppm

**(b)** **<u>9.9 ppm</u>**; *E* = ±2.98 mm; ppm = (2.98/300,000)1,000,000 = 9.9 ppm

**(c)** **<u>2.2 ppm</u>**; *E* = ±6.67 mm; ppm = (6.67/3,000,000)1,000,000 = 2.2 ppm

**6.31** Create a computational program that solves Problem 6.22. Response should vary.

# 7 ANGLES, AZIMUTHS, AND BEARINGS

Asterisks (*) indicate problems that have partial answers given in Appendix G.

**7.1** Define the different reference meridians that can be used for the direction of a line.

See Section 7.4.

**Geodetic or true**: Reference to geodetic north, which is referenced to the average position of the poles between 1900.0 and 1905.0

**Astronomic**: Reference meridians taken from instantaneous position of Earth's pole, which wanders over time.

**Magnetic**: Reference meridian taken from current position of magnetic poles.

**Grid:** Reference meridian chosen from some map projection system such as the state plane coordinate system where all meridians are parallel to the central meridian of the projection.

**Record or deed:** Reference meridian selected from a recorded deed by using the value given for one line in the deed.

**Assumed:** A value of 0° arbitrarily assigned to a line on the ground.

**7.2** List the three basic requirements in determining an angle.

From Section 7.1, paragraph 2:

"…they are (1) *reference* or *starting line*, (2) *direction of turning*, and (3) *angular distance* (value of the angle)."

**7.3** Why is it important to adopt a standard angle measuring procedure, such as always measuring angles to the right?

From Section 7.3, paragraph 3: "*To avoid this confusion, it is recommended that a uniform procedure of always observing angles to the right be adopted*, and the direction of turning noted in the field book with a sketch."

**7.4** What is the relationship of a forward and back azimuth?

See Section 7.5, paragraph 2: "A line's forward direction can be given by its *forward* azimuth, and its reverse direction by its *back* azimuth. In plane surveying, forward azimuths are converted to back azimuths, and vice versa, by adding or subtracting 180°."

**7.5** Convert: ***(a)** 203°26′48″ to grads **(b)** 1.789546 radians to degrees, minutes, and seconds **(c)** 156°07′34″ to radians.

**(a) <u>226.0518 grad</u>**

**(b) <u>102°32'00"</u>**

**(c) <u>2.724915 rad</u>**

In Problems 7.6 through 7.7, convert the azimuths from north to bearings, and compute the angles, smaller than 180° between successive azimuths.

**7.6** $65°26'37'', 127°25'46'', 254°23'07''$, and $295°14'08''$

| Bearings | Angles |
|---|---|
| N65°26'37"E | 61°59'09" |
| S52°34'14"E | 126°57'21" |
| S74°23'07"W | 40°51'01" |
| N64°45'52"W | 130°12'29" |

**7.7** $87°08'04'', 165°44'58'', 203°16'38''$, and $313°59'02''$

| Bearings | Angles |
|---|---|
| S87°08'04"E | 78°36'54" |
| S14°15'02"E | 37°31'40" |
| S23°16'38"W | 110°42'24" |
| N46°00'58"W | 133°09'02" |

Convert the bearings in Problems 7.8 through 7.9 to azimuths from north and compute the angle, smaller than 180°, between successive bearings.

**7.8** $N36°17'54''E, S78°13'15''E, S52°34'09''W$, and $N67°23'14''W$

| Azimuths | Angles |
|---|---|
| 36°17'54" | 65°28'51" |
| 101°46'45" | 130°47'24" |
| 232°34'09" | 60°02'37" |
| 292°36'46" | 103°41'08" |

**7.9** $N88°05'23''E, S23°56'23''E, S44°00'48''W$, and $N14°24'44''W$

| Azimuths | Angles |
|---|---|
| 88°05'23" | 67°58'14" |
| 156°03'37" | 67°57'11" |
| 224°00'48" | 121°34'28" |
| 345°35'16" | 102°30'07" |

Compute the azimuth from north of line CD in Problems 7.10 through 7.12. (Azimuths of AB are also from north.)

***7.10** Azimuth $AB = 101°26'32''$; angles to the right $ABC = 50°54'26'', BCD = 38°36'38''$.

$Az_{CD}$ = **<u>190°57'36"</u>**; $Az_{BC}$ = 332°20'58"

**7.11** Bearing $AB = S23°08'24''W$; angles to the right $ABC = 98°20'06'', BCD = 104°21'08''$.

$Brg_{CD}$ = **<u>N45°49'38"W</u>**; $Brg_{BC}$ = S58°31'30"E

**7.12** $Azimuth_{AB}$ $343°26'14''$; angles to the right $ABC = 66°36'10'', BCD = 82°16'24''$.

$Az_{CD}$ = **<u>132°18'48"</u>**; $Az_{BC}$ = 230°02'24"

***7.13** For a bearing $DE = N08°53'56''W$ and angles to the right, compute the bearing of *FG* if angle $DEF = 88°12''29''$ and $EFG = 40°20'30''$.

$Brg_{FG}$ = **<u>S60°20'57"E</u>**; $Brg_{EF}$ = S79°18'33"W

**7.14** Similar to Problem 7.13, except the azimuth of *DE* is $85°58'06''$ and angles to the right *DEF* and *EFG* are $21°44'52''$ and $86°10'14''$, respectively.

$Brg_{FG}$ = **<u>S13°53'12"W</u>**; $Az_{EF}$ = 287°42'58"

Course *AB* of a five-sided traverse runs due north. From the given balanced interior angles to the right, compute and tabulate the bearings and azimuths from north for each side of the traverses in Problems 7.15 through 7.17.

**7.15** $A = 82°13'15'', B = 116°35'18'', C = 28°45'22'', D = 195°16'28'', E = 117°09'37''$

| Course | Bearing | Azimuth |
|---|---|---|
| *AB* | Due North | 0°00'00" |
| *BC* | N63°24'42"W | 296°35'18" |
| *CD* | S34°39'20"E | 145°20'40" |
| *DE* | S19°22'52"E | 160°37'08" |
| *EA* | S82°13'15"E | 97°46’45" |

***7.16** $A = 90°29'18'', B = 107°54'36'', C = 104°06'37'', D = 129°02'57'', E = 108°26'32''$

| Course | Bearing | Azimuth |
|---|---|---|
| *AB* | Due North | 0°00'00" |
| *BC* | N72°05'24"W | 287°54'36" |
| *CD* | S32°01'13"W | 212°01'13" |
| *DE* | S18°55'50"E | 161°04'10" |
| *EA* | N89°30'42" | 89°30'42" |

**7.17** $A = 156°23'53'', B = 53°36'08'', C = 83°15'58'', D = 153°52'00'', E = 92°52'01''$

| Course | Bearing | Azimuth |
|---|---|---|
| *AB* | Due North | 0°00'00" |
| *BC* | S53°36'08"W | 233°36'08" |
| *CD* | S43°07'54"E | 136°52'06" |
| *DE* | S69°15'54"E | 110°44'06" |
| *EA* | N23°36'07"E | 23°36'07" |

In Problems 7.18 and 7.19, compute and tabulate the azimuths of the sides of a regular hexagon (polygon with six equal angles), given the starting direction of side *AB*.

**7.18** Bearing of $AB = 45°04'20''$ (Station *C* is westerly from *B*.)

| Course | Azimuths |
|---|---|
| *AB* | 45°04'20" |
| *BC* | 345°04'20" |
| *CD* | 285°04'20" |
| *DE* | 225°04'20" |
| *EF* | 165°04'20" |
| *FA* | 105°04'20" |

**7.19** Azimuth of $AB = 303°16'22''$ (Station *C* is westerly from *B*.)

| Course | Azimuths |
|---|---|
| *AB* | 303°16'22" |
| *BC* | 243°16'22" |
| *CD* | 183°16’22" |
| *DE* | 123°16'22" |
| *EF* | 63°16'22" |
| *FA* | 3°16'22" |

**7.20** Azimuth of $AB = 256°23'50''$ (Station *C* is westerly from *B*.)

| Course | Azimuths |
|---|---|
| *AB* | 256°23'50" |
| *BC* | 196°23'50" |
| *CD* | 136°23'50" |
| *DE* | 76°23'50" |
| *EF* | 16°23'50" |
| *FA* | 316°23'50" |

Compute azimuths of all lines for a closed traverse *ABCDA* that has the following balanced angles to the right, using the directions listed in Problems 7.21 and 7.22. $FAB = 108°21'58'', ABC = 78°20'28'', BCD = 92°10'32'', CDA = 81°07'02''$.

**7.21** Bearing $AB = N1°43'22''W$.

| Course | Azimuths |
|---|---|
| *AB* | 358°16'38" |
| *BC* | 256°37'06" |
| *CD* | 168°47'38" |
| *DA* | 69°54'40" |

**7.22** Azimuth $AB = 96°10'20''$.

| Course | Azimuths |
|---|---|
| *AB* | 96°10'20" |
| *BC* | 354°30'48" |
| *CD* | 266°41'20" |
| *DA* | 167°48'22" |

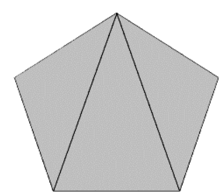
**7.23** Similar to Problem 7.21, except that bearings are required, and fixed bearing $B = \text{S}44°46'25''\text{W}$.

| Course | Bearings |
|---|---|
| *AB* | S44°46’25”W |
| *BC* | S56°53'07"E |
| *CD* | N35°17'25"E |
| *DA* | N63°35'33"W |

**7.24** Similar to Problem 7.22, except that bearings are required, and fixed azimuth $B = 106°48'29''$ (from north).

| Course | Bearings |
|---|---|
| *AB* | S73°11'21"E |
| *BC* | N5°09'07"E |
| *CD* | N82°40'21"W |
| *DA* | S1°33'19"E |

**7.25** Geometrically show how the sum of the interior angles of a pentagon (five sides) can be computed using the formula $(n-2)180°$?



A sketch showing that a pentagon can be divided into three triangles each of which as a sum of angles of 180°.

**7.26** Determine the predicted declinations on January 1, 2019 using the WMM 2015 model at the following locations.

**(a)*** latitude = 42°58′28″N, longitude = 77°12′36″W, elevation = 310.0 m; **11.7° W**

**(b)** latitude = 37°56′44″N, longitude = 110°50′40″W, elevation = 1500 m; **10.5°E**

**(c)** latitude = 41°18′15″N, longitude = 76°00′26″W, elevation = 240 m **12.0°W**

**7.27** Using Table 7.4, what is the predicted total difference in magnetic declination between Boston, MA and Seattle, WA on January 1, 2019?

**30°09'**; 15°34'E – 14°35'W

**7.28** The magnetic declination at a certain place is 8°15′E. What is the magnetic bearing there: **(a)** of true north **(b)** of true south **(c)** of true east?

**(a) N8°15'W**

**(b) S8°15'W**

**(c) N81°45'E**

**7.29** Same as Problem 7.28, except the magnetic declination at the place is 14°50′W.

**(a) N14°50'E**

**(b) S14°50'W**

**(c) S75°10'E**

For Problems 7.30 through 7.32 the observed magnetic bearing of line AB and its true magnetic bearing are given. Compute the amount and direction of local attraction at point A.

| | Observed Magnetic Bearing | True Magnetic Bearing | **Local Attraction** |
|---|---|---|---|
| **7.30*** | N32°30’E | N32°15’E | **0°15'E** |
| **7.31** | S14°30’W | S14°15′W | **0°15'E** |
| **7.32** | N9°30’W | N8°20′E | **1°10'E** |

What magnetic bearing is needed to retrace a line for the conditions stated in Problems 7.33 through 7.36?

| | 1875 Magnetic Bearing | 1875 Declination | Present Declination | **Present Magnetic Bearing** |
|---|---|---|---|---|
| **7.33*** | N32°45′E | 8°12′W | 2°30′E | **N22°03'E** |
| **7.34** | S63°40′W | 3°40′E | 2°20′W | **S62°20'W** |
| **7.35** | S69°20′E | 14°20′W | 12°30′W | **S67°30'W** |
| **7.36** | N24°30′W | 2°30′E | 1°30′E | **N23°30'W** |

In Problems 7.37 through 7.38 calculate the magnetic declination in 1870 based on the following data from an old survey record.

| | 1870 Magnetic Bearing | Present Magnetic Bearing | Present Magnetic Declination | 1870 Magnetic Declination |
|---|---|---|---|---|
| **7.37** | N14°20’E | N15°50’E | 8°15’W | **6°45'W** |
| **7.38** | S40°30’E | S44°35’E | 6°00′E | **1°55'E** |

# 8 TOTAL STATION INSTRUMENTS; ANGLE OBSERVATIONS

Asterisks (*) indicate problems that have partial answers given in Appendix G.

**8.1** Describe the proper procedure for transporting a total station instrument in the field.

From Section 8.5, paragraph 5: "When moving between setups in the field, proper care should be taken. Before the total station is removed from the tripod, the foot screws should be returned to the midpoints of the posts. Many instruments have a line on the screw post that indicates the halfway position. The instrument should NEVER be transported on the tripod since this causes stress to tripod head, tribrach, and instrument base. Figure 8.6(a) depicts the proper procedure for carrying equipment in the field. With adjustable-leg tripods, retracting them to their shortest positions and lightly clamping them in position can avoid stress on the legs. Since the screws on the instrument are made of brass typically, over-tightening screws on tripods and the instrument can cause serious harm to the instruments. Screws and locks should only be "finger" tight. Inexperienced users sometimes over-tighten screws to the detriment of the equipment."

**8.2** Why should a total station never be transported on a tripod?

From Section 8.5, paragraph 5: "The instrument should NEVER be transported on the tripod since this causes stress to tripod head, tribrach, and instrument base."

**8.3** What are the primary sources of random instrumental error in a total station instrument.

Random instrumental errors are:

1. From Section 8.21: Ability to resolve the reading for the horizontal and vertical circles, which is stated as the IOS 17123-3 accuracy of the instrument.
2. From Section 8.20.1: Ability of the operator and/or electronic sensor to level the instrument.
3. Ability to center instrument and target over a point.
4. From Section 6.22: Ability to resolve the slope distance, which is given as a constant error and a scalar error (ppm).

**8.4** Describe under what conditions parallax can exist and how it can be detected.

From Section 81.4, subsection 1: If the focusing of the two lenses is not coincident, a condition known as parallax will exist. The existence of parallax can be observed by quickly shifting one's eye position slightly and watching for movement of the object in relation to the cross-hairs. Careful adjustment of the eyepiece and objective lens will result in a sharp image of both the object and the reticle with no visible parallax.

**8.5** Name and briefly describe the three main components of a total station.

The three main components are (1) a digital theodolite, (2) and electronic distance measuring device, and (3) a microprocessor.

**8.6** Why are the bases of total station instruments designed to be interchanged with other equipment?

From Section 8.4.6: “The *bases* of total stations are often designed to permit interchange of the instrument with sighting targets and prisms in tribrachs without disturbing previously established centering over survey points. This can save a considerable amount of time.”

**8.7** Why is it important not to sight the EDM reflector when turning an angle?

From Section 8.14, paragraph 3: “Errors can result if the prism itself is sighted, especially on short lines since any misalignment of the face of the prism with the line of sight will cause and offset pointing on the prism.”

**8.8** How are the relative precisions of total station instruments differentiated?

From Section 8.21, paragraph 2: “The original German standards, called DIN 18723, provide values for estimated errors in the mean of two direction observations, one each in the direct and reverse modes. It has since been replaced by the International Standards Organization (ISO) 17123-3 standard.”

**8.9** What is meant by an angular position?

From Section 8.10, paragraph 2: "A set of readings around the horizon in both the direct and reverse modes constitutes a so-called *position.*"

**8.10** What is the purpose of the jog/shuttle mechanism on a robotic total station?

From 8.4, subsection 4: In servo-driven total stations (see Figure 8.7), the lock and tangent screw are replaced with a jog/shuttle mechanism. This device actuates an internal servo-drive motor that rotates the telescope about its horizontal and vertical axes.

**8.11** How can the maladjustment of a level vial be detected on a total station?

From Section 8.20.1, paragraph 2: “The plate bubble is out of adjustment if after centering it runs when the instrument is rotated 180° in azimuth.”

**8.12** Determine the angles subtended for the following conditions:

**(a)*** a 1-cm diameter pipe sighted by total station from 100 m.

<u>**21"**</u>; $\frac{0.010}{100}206,264.8 \approx 20.6''$

**(b)** a 1/4-in. stake sighted by total station from 300 ft.

<u>**14"**</u>; $\frac{\frac{1/4}{12}}{400} * 206{,}264.8''/rad = 14.3''$

**(c)** a 1/4-in. diameter chaining pin observed by total station from 600 ft.

**<u>7"</u>**; $\frac{1/4}{12*600}206{,}264.8 = 7.2''$

**8.13** What is the error in an observed direction for the situations noted?

**(a)** setting a total station 3 mm to the side of a tack on a 50-m sight.

**<u>12″</u>;** $\frac{0.003}{50}206{,}264.8 = 12.4''$

**(b)** lining in the edge (instead of center) of an 8-mm diameter chaining pin at 200 ft.

**<u>14"</u>**; $\frac{\frac{8/25.4}{12(2)}}{200}206264.8''/rad = 13.5''$

**(c)** sighting the edge (instead of center) of a 3-cm diameter range pole 200 m.

**<u>16"</u>**; $\frac{0.03/2}{200}206{,}264.8 = 15.5''$

**(d)** sighting the top of a 6-ft range pole that is 5′ off-level on a 200-ft sight.

**<u>9"</u>**; $6\left(\frac{5(60)}{206264.8}\right) = 0.0087;\ \frac{0.0087}{200}206264.8''/\mathrm{rad} = 9.0''$

***8.14** Intervening terrain obstructs the line of sight so only the top of a 6-ft long pole can be seen on a 250-ft sight. If the range pole is out of plumb and leaning sideways 0.025-ft per vertical foot what maximum angular error results?

**<u>124"</u>**; $S = 0.025(6) = 0.15$ ft; $\frac{0.015}{250}206264.8''/\mathrm{rad} = 123.8''$

**8.15** Same as Problem 8.14, except that it is a 1.5-m pole that is out of plumb and leaning sideways 1 cm per meter on a 200 m sight.

**<u>31"</u>**; $S = 0.01(1.5) = 0.015$ m; $\frac{0.015}{100}206264.8''/\mathrm{rad} = 30.9''$

**8.16** Discuss the advantages of a robotic total station instrument.

From Section 8.6: "The computer retrieves the direction to the point from storage or computes it and activates a servomotor to turn the telescope to that direction within a few seconds. This feature is particularly useful for construction stakeout, but it is also convenient in control surveying when multiple observations are made in observing angles. In this instance, final precise pointing is done manually."

In essence, it speeds the field operations.

**8.17** What instrumental errors are compensated by averaging an equal number of observations with the telescope direct and reversed?

From Section 8.20.1, paragraphs 2 and 3: Averaging an equal number of direct and reversed observations compensates for the instrumental errors of (1) vertical axis not perpendicular to the horizontal axis and the (2) axis of the line of sight not perpendicular to the horizontal axis.

**8.18** Describe the process of placing a point on line between two existing stations when the two stations are not intervisible.

The process known as balancing-in or wiggling-in is described in Section 8.16.

**8.19** An interior angle *x* and its explement *y* were turned to close the horizon. Each angle was observed once direct and once reversed using the repetition method. Starting with an initial backsight setting of 0°00'00" for each angle, the readings after the first and second turnings of angle *x* were 62°38'24" and 62°38'22", and the readings after the first and second turnings of angle *y* were 297°21'38" and 297°21'40" Calculate each angle and the horizon misclosure.

**62°38'23", 297°21'39", 2";**

$x = 62°38' + \frac{24+24}{2} = 62°38'23''$ ; $y = 297°21' + \frac{38+40}{2} = 297°21'39''$;

$misclosure = 360° - (62°38'23'' + 297°21'39'') = 2''$

***8.20** A zenith angle is measured as $84°13'56''$ in the reversed position. What is the equivalent zenith angle in the direct position?

**275°46'04"** = 360° − 84°13'56"

**8.21** What is the average zenith angle given the following direct and reversed readings.

Direct: 93°45′24″, 93°45′26″, 93°45′22″
Reversed: 266°14′40″, 266°14′36″, 266°14′38″

**93°45'23"**

$\sum z_D = 281°16'12''$; $\sum z_R = 798°43'54''$;

By Equation (8.3): $\frac{281°16'12''}{3} + \frac{3(360)-(281°16'12''+798°43'54'')}{2(3)} = 93°45'23''$

In Figure 8.9(c), direct and reversed directions observed with a total station instrument from A to points B, C, and D are listed in Problems 8.23 and 8.24. Determine the values of the three angles, and the horizon misclosure.

**8.22** Direct: 0°00′00″, 84°02′36″, 202°22′26″, 285°14′15″, 0°00′02″

Reverse: 0°00′00″, 86°02′40″, 266°22′26″, 285°14′18″, 359°59′58″

**84°02'38"; 118°19'48"; 82°51'50.5"; 74°45'43.5"; misclosure = 0"**

**8.23** Direct: 0°00′00″, 98°22′58″, 189°19′33″, 267°42′44″, 360°00′02″
Reverse: 0°00′00″, 98°23′00″, 189°19′33″, 267°42′40″, 360°00′02″

**98°22'59"; 90°56'34"; 78°23'09"; 92°17'20"; misclosure = 2"**

***8.24** The angles at point *X* were observed with a total station instrument. Based on 4 readings, the standard deviation of the angle was ±5.6″. If the same procedure is used in observing each angle within a six-sided polygon, what is the estimated standard deviation of closure at a 95% level of probability?

**±27"**; $1.9599(5.6)\sqrt{6} = 26.9''$

**8.25** The line of sight of a total station is out of adjustment by 5″.

**(a)** In prolonging a line by plunging the telescope between backsight and foresight, but not double centering, what angular error is introduced?

**10"**; 2(5")

**(b)** What off-line linear error results on a foresight of 200 m?

**9.7 mm**; 200 tan(10") = 0.009696 m

**8.26** A line PQ is prolonged to point R by double centering. Two foresight points $R'$ and $R''$ are set. What angular error would be introduced in a single plunging based on the following lengths of QR and $R'R''$ respectively?

**(a)*** 650.50 ft and 0.35 ft.

**56"**; $\tan^{-1}\left(\dfrac{0.35/2}{650.05}\right) = 55.5''$

**(b)** 298.406 m and 22 mm.

**7.6"**; $\tan^{-1}\dfrac{0.022/2}{312.6298.406} = 7.6''$

**8.27** Explain why the "principal of reversion" is important in angle measurement.

From Section 8.15 and 8.20.1: The principle of reversion is applied when angles are measured in both the direct and reversed positions. The procedure negates the effect of the horizontal axis not being perpendicular to the vertical axis. It is also used in detecting a maladjusted level and in prolonging a line of sight.

***8.28** A total station with a 20″/div. level bubble is one divisions out of level on a point with an altitude angle of 38°50′44″. What is the error in the horizontal pointing?

**16"**; By Equation (8.4): $E_H = (20'')\tan(38°50'44'') = 16.1''$

**8.29** What is the equivalent altitude angle for a zenith angle of 86°14′38″?

**3°45'22"**

**8.30** What is the equivalent altitude angle for a zenith angle of 265°13′56″?

**<u>−6°46'04"</u>**

**8.31** What error in horizontal angles is consistent with the following linear precisions?

**(a)** 1/5000, 1/10,000, 1/20,000, and 1/50,000

**<u>41.3", 20.6", 10.3", 4.1"</u>**

**(b)** 1/3000, 1/15,000, 1/30,000, and 1/100,000

**<u>68.8", 13.8", 6.9", 2.1"</u>**

**8.32** Why is it important to check for sloppiness in a tribrach?

From Section 8.19.3: "The clamping mechanism consists of three slides that secure three posts that protrude from the base of the instrument or tribrach adapter. As the tribrach wears, the clamping mechanism may not sufficiently secure the instrument during observation procedures."

**8.33** Describe the procedure to adjust a level bubble on a total station.

From Section 8.19.1: "To make the test, the instrument should first be leveled following the procedures outlined in Section 8.5. Then after carefully centering the bubble, the telescope should be rotated 180° from its first position. If the level vial is in adjustment, the bubble will remain centered. If the bubble deviates from center, the axis of the plate-level vial is not perpendicular to the vertical axis. The amount of bubble run indicates twice the error that exists. Level vials usually have a capstan adjusting screw for raising or lowering one end of the tube. If the level vial is out of adjustment, it can be adjusted by bringing the bubble *halfway back* to the centered position by turning the screw. Repeat the test until the bubble remains centered during a complete revolution of the telescope. If the instrument is equipped with an electronic level, follow the procedures outlined in the operator's manual to adjust the leveling mechanism."

**8.34** List the procedures for prolonging a line of sight.

See Section 8.15

**8.35** A zenith angle was read twice direct giving values of 83°15'34″ and 83°15'30″, and twice reverse yielding readings of 276°44'16″ and 276°44′20″. What is the mean direct-face zenith angle? What is the indexing error?

By Equation 8.3: **<u>83°15'37"; indexing error = 5″</u>**

**8.36** A zenith angle was read twice direct giving values of 94°05'24″ and 94°05'28″ and twice reverse yielding readings of 265°54′20″ and 265°54′22″. What is the mean direct-face zenith angle? What is the indexing error?

By Equation 8.3: **<u>94°05'32"; indexing error = 6″</u>**

**8.37** A total station has an ISO 17123-3 specified accuracy of ±2″. What is the estimated precision of an angle observed with 4 repetitions?

By Equation (8.5): **<u>±4.2”</u>**; $E = \frac{2(2")}{\sqrt{4}} = \pm 2.0"$

**8.38** A total station has an ISO 17123-3 specified accuracy of ±5″. What is the estimated precision of an angle observed with 2 repetitions?

By Equation (8.5): **<u>±7.1”</u>**; $E = \frac{2(5")}{\sqrt{2}} = \pm 7.1"$

# 9 TRAVERSING

Asterisks (*) indicate problems that have partial answers given in Appendix G.

**9.1** Explain the difference between closed and open traverses.

From Section 9.1: Open traverses are mathematically open and offer no means of checking for observational errors or mistakes whereas closed traverses are mathematically and sometimes geometrically closed and allow for mathematical checks on closure.

**9.2** List the disadvantages of an open traverse.

From Section 9.1, paragraph 4: "*Open traverse should be avoided because they offer no means of checking for observational errors or mistakes*."

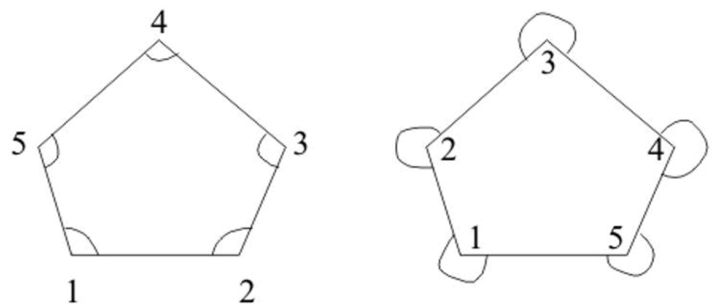
**9.3** Discuss the guidelines to consider when selecting positions for stations of a control traverse for topographic mapping.

From Section 9.4, paragraph 1: "Of course, intervisibility between adjacent stations, forward and back, must be maintained for angle and distance measurements. The stations should also ideally be set in convenient locations that allow for easy access. Ordinarily, stations are placed to create lines that are as long as possible. This not only increases efficiency by reducing the number of instrument setups, but it also increases accuracy in angle measurements. Utility may override using very long lines, however, because intermediate hubs, or stations at strategic locations, may be needed to complete the survey's objectives."

**9.4** In your own words define an angle to the right.

From Section 91.2.2: "Angles measured clockwise from a backsight on the "rearward" traverse station to a foresight on the "forward" traverse station are called *angles to the right*."

**9.5** Draw two five-sided closed polygon traverses with station labels 1 to 5. The first traverse should show angles to the right that are interior angles, and the second should show angles to the right that are exterior angles.




**9.6** Discuss the criteria to consider when making reference ties to traverse stations.

From Section 9.5, paragraph 2: "As illustrated, these ties consist of distance observations made to nearby fixed objects. Short lengths (less than 100 ft) are convenient if a steel tape is being used, but, of course, the distance to definite and unique points is a controlling factor. Two ties, preferably at about right angles to each other, are sufficient, but three should be used to allow for the possibility that one reference mark may be destroyed. Ties to trees can be observed in hundredths of a foot if nails are driven into them. However, *permission must be obtained from the landowner before driving nails into trees*."

**9.7** Explain why it is advisable to use two instrument stations, as O and O′ in Figure 9.7(b), when performing radial traverses.

From Section 9.9, paragraph 2: To provide checks in computed positions for observed stations.

**9.8** Explain the importance of always turning interior angles in a clockwise direction.

From Section 9.2.1: "… *to reduce mistakes in reading, recording, and computing, they should always be turned clockwise* from the backsight station to the foresight station."

**9.9** What should be the sum of the interior angles for a closed-polygon traverse that has: **(a)*** 6 sides **(b)** 12 sides **(c)** 20 sides.

From Equation (9.1):

**(a)*** 720°

**(b)** 1800°

**(c)** 3240°

**9.10** What should the sum of the exterior angles for a closed-polygon traverse that are listed in Problem 9.9.

From Equation (9.2):

**(a)** 1440°

**(b)** 2520°

(c) 3960°

**9.11** Four interior angles of a five-sided polygon traverse were observed as *A*=100°32'46″, *B*=112°36' 48″, *C*=108°13' 08″, and *D*=115°42' 58″. The angle at *E* was not observed. If all observed angles are assumed to be correct, what is the value of angle *E*?

**<u>102°54'20"</u>**; Sum of A – D = 437°05'40"

**9.12** Similar to Problem 9.11, except the traverse had six sides with observed angles of $A = 122°13'14''$, $B = 118°46'33''$, $C = 125°56'48''$, $D = 111°26'53''$, and $E = 133°38'27''$. Compute the angle at *F*, which was not observed.

**<u>107°58'05"</u>**; Sum of *A* – *E* = 612°01'55"

**9.13** What is the angular misclosure of a six-sided polygon traverse with observed angles of $126°10'10''$, $110°45'58''$, $122°23'10''$, $119°50'54''$, $E = 127°32'02''$, and $F = 113°17'36''$.

**<u>10"</u>**

**9.14** What FGCS standard would the angular misclosure in Problem 9.13 meet?

**<u>2nd order, class II</u>**; $c = 10''/\sqrt{6} = 4.1''$; allowable by 2nd order, class II is 4.5"

***9.15** According to FGSC standards, what is the maximum acceptable angular misclosure for a second order, class I traverse having 20 angles?

**<u>13"</u>**; by Equation (9.3) using *K* = 3"

***9.16** What is the angular misclosure for a five-sided polygon traverse with observed exterior angles of 252°26′37″, 255°55′13″, 277°15′53″, 266°35′02″, and 207°47′05″?

**<u>10"</u>**

**9.17** What is the angular misclosure for a five-sided polygon traverse with observed interior angles of $110°26'48''$, $105°55'04''$, $113°15'34''$, $100°35'24''$, and $109°47'20''$?

**<u>10"</u>**; Σangles = 540°00'10"

**9.18** Discuss how a data collector can be used to check the setup of a total station in traversing.

From Section 9.8, paragraph 4: "Mistakes in orientation can be minimized when a data collector is used in combination with a total station. In this process, the coordinates of each backsight station are checked before proceeding with the angle and distance observations to the next foresight station. For example in Figure 9.1(a), after the total station is leveled and oriented at station *B,* an observation is taken "back" on *A.* If the newly computed coordinates of *A* do not closely match their previously stored values, the instrument setup, leveling, and orientation should be rechecked, and the problem resolved before proceeding with any further measurements. This procedure often takes a minimal amount of time and typically identifies most field mistakes that occur during the observational process."

***9.19** If the standard error for each measurement of a traverse angle is ±5.4″, what is the expected standard error of the misclosure in the sum of the angles for an eight-sided traverse?

<u>**±15.3"**</u>; by Equation (3.12)

**9.20** If the angles of a traverse are turned so that the 95% error of any angle is ±2.2″ what is the 95% error in a ten-sided traverse?

<u>**±13.6"**</u>; by Equation (3.12) using an $E_{95}$ multiplier of 1.96(6.96″)

**9.21** What common mistake can be caught by observing the backsight station with a data collector?

From Section 9.6, paragraph 2: "A common field mistake uncovered in this procedure is failure to set the horizontal circle to zero on the backsight. Although it rarely happens, it is also possible to uncover distance observation errors."

***9.22** The azimuth from station *A* of a link traverse to an azimuth mark is 212°12′36″. The azimuth from the last station of the traverse to an azimuth mark is 192°12′16″. Angles to the right are observed at each station: $A = 136°15'40''$, $B = 119°15'36''$, $C = 93°48'54''$, $D = 136°04'16''$, $E = 108°30'10''$, $F = 42°48'03''$, and $G = 63°17'16''$. What is the angular misclosure of this link traverse?

<u>**14"**</u>

| | | | |
|---|---|---|---|
| $Az_{A\text{-}Mk}$ | 212°12'36" | *DC* | 21°32'46" |
| +*A* | +136°15'40" | +*D* | +136°04'16" |
| *AB* | 348°28'16" | *DE* | 157°37'02" |
| | −180° | | +180° |
| *BA* | 168°28'16" | *ED* | 337°37'02" |
| +*B* | +119°15'36" | +*E* | +108°30'10" |
| *BC* | 287°43'52" | *EF* | 446°07'12" |
| | −180° | | −180° |
| *CB* | 107°43'52" | *FE* | 266°07'12" |
| +*C* | +93°48'54" | +*F* | +42°48'02" |
| *CD* | 201°32'46" | *FG* | 308°55'14" |
| | −180° | | −180° |
| *DC* | 21°32'46" | *GF* | 128°55'14" |
| | | +*G* | +63°17'16" |
| | | $Az_{G\text{-}Mk}$ | 192°12'30" |

Misclosure = 192°12'30" − 192°12'16" = 14"

**9.23** What FGCS order and class does the traverse in Problem 9.22 meet?

<u>**Third order, Class I**</u>; By Equation (9.3): $K = \frac{14''}{\sqrt{7}} = 5.3''$; allowable 10″

***9.24** The interior angles in a five-sided closed-polygon traverse were observed as $A = 108°28'36''$, $B = 110°26'54''$, $C = 106°25'58''$, $D = 102°27'02''$, and $E = 116°11'15''$.

Compute the angular misclosure. For what FGCS order and class is this survey adequate?

**<u>−15"; Third order, Class I</u>**; By Equation (9.3): $K = 15''/\sqrt{5} = 6.7''$, allowable 10”

**9.25** Similar to Problem 9.24, except for a six-sided traverse with observed exterior angles of $A = 244°28'38''$, $B = 238°26'50''$, $C = 246°25'56''$, $D = 234°27'02''$, $E = 235°08'55''$, and $F = 241 \pm 02'45''$.

**<u>6"; Second order, Class I</u>**, By Equation (9.3): $K = 6''/\sqrt{6} = 2.4''$; allowable 3″

**9.26** In Figure 9.6, what is the average interior angle with the instrument at station 102.

**<u>95°32'06"</u>**; $\frac{95°32'10+(275°32'08-180°00'02'')}{2} = 95°32'06''$

**9.27** Same as Problem 9.26 except at instrument station 103.

**<u>49°33'46"</u>**; $\frac{49°33'46''+(229°33'47''-180°00'00'')}{2} = 49°33'46.5''$

# 10 TRAVERSE COMPUTATIONS

Asterisks (*) indicate problems that have partial answers given in Appendix G.

**10.1** In adjusting measured traverse angles, why aren't adjustments made in proportion to the angle sizes?

From Section 10.2, paragraph 5: "Unlike corrections for linear observations (described in Section 10.7), *adjustments applied to angles are independent of the size of the angle*."

***10.2** The sum of seven interior angles of a closed-polygon traverse each read to the nearest $3''$ is $899°59'39''$. What is the misclosure, and what correction would be applied to each angle in balancing them by method 1 of Section 10.2?

**Misclosure = −21"; Apply +3" correction per angle**

**10.3** Similar to Problem 10.2, except the angles were read to the nearest $2''$ and their sum was $719°59'48''$ for a six-sided polygon traverse.

**Misclosure = −12"; Apply +2" correction per angle.**

**10.4** Similar to Problem 10.2, except the angles were read to the nearest $1''$ and their sum for a nine-sided polygon traverse was $1260°00'12''$.

**Misclosure = +12"; Apply −1" correction per angle.**

***10.5** Balance the angles in Problem 9.22. Compute the preliminary azimuths for each course.

Preliminary computations are in the solution of Problem 9.22. The misclosure was 14".

Balanced angles (Correction −2" per angle):
$A = \mathbf{136°15'38''}$, $B = \mathbf{119°15'34''}$, $C = \mathbf{93°48'52''}$, $D = \mathbf{136°04'14''}$, $E = \mathbf{108°30'08''}$, $F = \mathbf{42°48'00''}$, **and** $G = \mathbf{63°17'14''}$.

Preliminary azimuths: ***AB* = 348°28'14"; *BC* = 287°43'48"; *CD* = 201°32'40"; *DE* = 157°36'54"; *EF* = 86°07'02"; *FG* = 308°55'02"**

**10.6** Balance the following interior angles (angles-to-the-right) of a five-sided closed polygon traverse using method 1 of Section 10.2. If the azimuth of side $AB$ is fixed at $218°59'30''$ calculate the azimuths of the remaining sides. $A = 132°47'06''$; $B = 108°46'18''$; $C = 107°19'37''$; $D = 81°50'36''$; $E = 109°16'18''$. (Note: line *BC* bears NE.)

<u>**Misclosure = −5"; Correction = +2" per angle**</u>

Balanced angles: . $\boldsymbol{A = 132°47'07''}$; $\boldsymbol{B = 108°46'19''}$; $\boldsymbol{C = 107°19'38''}$; $\boldsymbol{D = 81°50'37''}$; $\boldsymbol{E = 109°16'19''}$

Azimuths: ***AB* = 218°59'30"; *BC* = 147°45'49"; *CD* = 75°05'27"; *DE* = 336°56'04"; *EA* = 266°12'23"**

***10.7** Compute departures and latitudes, linear misclosure, and relative precision for the traverse of Problem 10.6 if the lengths of the sides (in feet) are as follows: *AB* = 202.74; *BC* = 283.87; *CD* = 498.37; *DE* = 320.33; and *EA* = 380.78. (Note: Assume units of feet for all distances.)

| Course | Length | Dep | Lat |
|---|---|---|---|
| *AB* | 202.74 | −127.565 | −157.577 |
| *BC* | 283.87 | 151.42 | −240.113 |
| *CD* | 498.37 | 481.592 | 128.224 |
| *DE* | 320.33 | −125.5 | 294.722 |
| *EA* | 380.78 | −379.946 | −25.193 |
| | ∑ = 1686.09 | 0.001 | 0.063 |

<u>**LEC = 0.063 ft; Relative precision = 1:26,800**</u>

**10.8** Using the compass (Bowditch) rule, adjust the departures and latitudes of the traverse in Problem 10.7. If the coordinates of station *A* are *X* = 20,000 ft and *Y* = 15,000 ft, calculate (a) coordinates for the other stations, (b) adjusted lengths and azimuths of lines *BC* and *CD*, and (c) the final adjusted angles at stations *B* and *C*.

**(a)** Balanced departures and latitudes and coordinates.

| Course | Dep | Lat | Point | *X* | *Y* |
|---|---|---|---|---|---|
| *AB* | −127.566 | −157.585 | *A* | 20,000.00 | 15,000.00 |
| *BC* | 151.42 | −240.123 | *B* | 19,872.43 | 14,842.42 |
| *CD* | 481.592 | 128.206 | *C* | 20,023.85 | 14,602.29 |
| *DE* | −125.5 | 294.71 | *D* | 20,505.45 | 14,730.50 |
| *EA* | −379.946 | −25.208 | *E* | 20,379.95 | 15,025.21 |

**(b)** <u>***BC* = 283.88 ft, $Az_{BC}$ = 147°45'53"; *CD* = 498.36 ft, $Az_{CD}$ = 75°05'34"**</u>

**(c)** Adjusted angles at *B* and *C*.

| Point | Angle |
|---|---|
| *B* | 108°46'28" |
| *C* | 107°19'41" |

**10.9** Balance the following interior angles-to-the-right for a polygon traverse to the nearest 1″ using method 1 of Section 10.2. Compute the azimuths assuming a fixed azimuth of $35°09'32''$ for line $AB$. $A = 57°00'34''$; $B = 88°24'40''$; $C = 126°37'20''$; $D = 46°03'46''$; $E = 221°53'30''$. (Note: Line *BC* bears NW.)

**Angular misclosure:** –10″ for correction of +2″/angle
**Adjusted Angles:** *A* = 57°00′36″; *B* = 88°24′42″; *C* = 126°37′22″; *D* = 46°03′48″; *E* = 221°53′32″.
**Preliminary Azimuths:** *AB* = 35°09′32″; *BC* = 303°34′19″; *CD* = 250°11′19″; *DE* = 116°14′46″; *EA* =158°08′40″.

**10.10** Determine departures and latitudes, linear misclosure, and relative precision for the traverse of Problem 10.9 if lengths of the sides (in meters) are as follows: *AB* = 383.808; *BC* = 360.209; *CD* = 342.204; *DE* = 336.210; and *EA* = 267.527.

| Course | Length | Azimuth | Dep | Lat |
|---|---|---|---|---|
| AB | 383.846 | 35°09'32.0" | 221.0361 | 313.8165 |
| *BC* | 360.256 | 303°34'14.0" | −300.1673 | 199.2084 |
| *CD* | 342.244 | 250°11'36.0" | −321.9973 | −115.9685 |
| *DE* | 336.228 | 116°15'24.0" | 301.5364 | −148.7449 |
| *EA* | 267.550 | 158°08'56.0" | 99.5810 | −248.3276 |
| | ∑ 1690.124 | | 0.0110 | −0.0162 |

**Linear misclosure = 0.0814; Relative precision = 1:20,800**

**10.11** Using the compass (Bowditch) rule adjust the departures and latitudes of the traverse in Problem 10.10. If the coordinates of station *A* are *X* = 310,630.892 m and *Y* = 121,311.411 m, calculate (a) coordinates for the other stations and, from them, (b) the lengths and bearings of lines *BC* and *EA*, and (c) the final adjusted angles at *B* and *D*.

(a)

| Course | Dep | Lat | Point | X | Y |
|---|---|---|---|---|---|
| *AB* | 221.0386 | 313.8202 | *A* | 310,630.892 | 121,311.411 |
| *BC* | −300.1649 | 199.2118 | *B* | 310,851.931 | 121,625.231 |
| *CD* | −321.9951 | −115.9652 | *C* | 310,551.766 | 121,824.443 |
| *DE* | 301.5386 | −148.7417 | *D* | 310,229.771 | 121,708.478 |
| *EA* | 99.5828 | −248.3251 | *E* | 310,531.309 | 121,559.736 |

(b)

| **Course** | **Distance** | **Bearing** |
|---|---|---|
| *BC* | 360.256 | N56°25'44"W |
| *EA* | 267.548 | S41°53'32"E |

(c) *B* = **88°24'44"**; *D* = **46°03'44"**

**10.12** Same as Problem 10.9, except assume line *AB* has a fixed azimuth of 125°09′32″ and line *BC* bears *NE*.

**Angular misclosure:** –10″ for correction of +2″/angle
**Adjusted Angles:** *A* = 57°00′36″; *B* = 88°24′42″; *C* = 126°37′22″; *D* = 46°03′48″; *E* = 221°53′32″.
**Preliminary Azimuths:** *AB* = 125°09′32″; *BC* = 33°34′14″; *CD* = 340°11′36″; *DE* = 206°15′24″; *EA* =248°08′56″.

**10.13** Using the lengths from Problem 10.10 and azimuths from Problem 10.12, calculate departures and latitudes, linear misclosure, and relative precision of the traverse.

| Course | Length | Dep | Lat |
|---|---|---|---|
| *AB* | 383.846 | 313.8165 | −221.0361 |
| *BC* | 360.256 | 199.2084 | 300.1673 |
| *CD* | 342.244 | −115.9685 | 321.9973 |
| *DE* | 336.228 | −148.7449 | −301.5364 |
| *EA* | 267.550 | −248.3276 | −99.5810 |
| | ∑ = 1690.124 | −0.0162 | 0.0110 |

**Linear misclosure = 0.0196; Relative precision = 1:86,300**

**10.14** Adjust the departures and latitudes of Problem 10.13 using the compass (Bowditch) rule, and compute coordinates of all stations if the coordinates of station *A* are $X = 243{,}605.596$ m and $Y = 25{,}393.201$ m. Compute the length and azimuth of line *CD*.

| Course | Dep | Lat | Point | X | Y |
|---|---|---|---|---|---|
| *AB* | 313.8202 | −220.0386 | *A* | 243,605.596 | 25,393.201 |
| *BC* | 199.2118 | 300.1649 | *B* | 243,919.416 | 25,172.162 |
| *CD* | −115.9652 | 321.9951 | *C* | 244,118.628 | 25,472.327 |
| *DE* | −148.7417 | −301.5386 | *D* | 244,002.663 | 25,794.322 |
| *EA* | −248.3251 | −99.5828 | *E* | 243,853.921 | 25,492.784 |

| **Course** | **Length** | **Azimuth** |
|---|---|---|
| *CD* | 342.241 | 340°11'37" |

**10.15** Compute and tabulate for the following closed-polygon traverse: (a) preliminary bearings (b) unadjusted departures and latitudes (c) linear misclosure and (d) relative precision. (Note: line *BC* bears *NE*.)

| Course | Azimuth | Length (m) | Interior Angles |
|---|---|---|---|
| *AB* | 75°14′47″ | 409.838 | A = 12°25’31” |
| *BC* | | 360.225 | *B* = 48°19’25” |
| *CD* | | 342.213 | *C* = 126°37’24” |
| *DE* | | 337.191 | *D* = 46°03’36” |

| | | |
|---|---|---|
| *EA* | 85.125 | *E* = 306°34’19” |

(a) Preliminary azimuths are listed below.

(b) Unadjusted latitudes and departures are listed below.

(c) **1.0024 m**

(d) **1:1500**

From WolfPack:

| Station | Unadj. Ang. | Adj. Ang. |
|---|---|---|
| *A* | 12°25'31.0" | 12°25'28.0" |
| *B* | 48°19'25.0" | 48°19'22.0" |
| *C* | 126°37'24.0" | 126°37'21.0" |
| *D* | 46°03'36.0" | 46°03'33.0" |
| *E* | 306°34'19.0" | 306°34'16.0" |
| Angular | misclosure: | 15" |

| Course | Length | Azimuth | Dep | Lat |
|---|---|---|---|---|
| *AB* | 409.838 | 75°14'47" | 396.326 | 104.371 |
| *BC* | 360.225 | 303°34'09" | −300.146 | 199.184 |
| *CD* | 342.213 | 250°11'30" | −321.965 | −115.97 |
| *DE* | 337.191 | 116°15'03" | 302.415 | −149.14 |
| *EA* | 85.125 | 242°49'19" | −75.7265 | −38.882 |
| | ∑=1534.592 | | 0.9033 | −0.4345 |

***10.16** In Problem 10.15, if one side and/or angle is responsible for most of the error of closure, which is it likely to be?

The azimuth of the misclosure line is 115°41’18”. The line most closely matching this bearing is *DE*. Thus *DE* is the line most likely course with a distance blunder.

**Instructor’s note:** The course *DE* had a +1 meter error placed in it. The correct observed length is 336.191 m.

***10.17** Adjust the traverse of Problem 10.15 using the Compass Rule. If the coordinates in meters of point *A* are 10,356.548 E and 98,761.391 N, determine the coordinates of all other points. Find the length and bearing of line *EA*.

| Course | Dep | Lat | Point | X | Y |
|---|---|---|---|---|---|
| *AB* | 396.0843 | 104.4866 | *A* | 10,356.548 | 98,761.391 |
| *BC* | −300.3583 | 199.2860 | *B* | 10,752.632 | 98,865.878 |
| *CD* | −322.1662 | −115.8705 | *C* | 10,452.274 | 99,065.164 |
| *DE* | 302.2168 | −149.0447 | *D* | 10,130.108 | 98,949.293 |
| *EA* | −75.7766 | −38.8574 | *E* | 10,432.325 | 98,800.248 |

***EA* = 85.159 m; Az$_{EA}$ = 242°51’06”**

For the closed-polygon traverses given in Problem 10.18 through 10.19 (lengths in feet), compute and tabulate: (a) unbalanced departures and latitudes (b) linear misclosure (c) relative precision and (d) preliminary coordinates if $X_A = 10{,}000.00$ and $Y_A = 5000.00$. Balance the traverses by coordinates using the compass rule.

| | Course | *AB* | *BC* | *CD* | *DA* |
|---|---|---|---|---|---|
| **10.18** | Bearing | N 22°36′40″ W | S60°39′24″W | S 38°46′10″E | N 88°22′58″ E |
| | Length | 314.02 | 264.49 | 213.50 | 217.69 |
| **10.19** | Azimuth | 252°36′35″ | 124°51′20″ | 50°41′28″ | 320°27′42″ |
| | Length | 449.60 | 427.28 | 301.49 | 243.23 |

**10.18** Solution from WolfPack

```
                                                Unbalanced
Course          Length       Bearing          Dep          Lat
------------------------------------------------------------
       1-2        314.02   N22°36'40.0"W  -120.733    289.883
       2-3        264.49   S60°39'24.0"W  -230.556   -129.611
       3-4        213.50   S38°46'10.0"E   133.691   -166.460
       4-1        217.69   N88°22'58.0"E   217.603      6.144
              ----------                 ---------  ---------
        Sum =   1,009.70                     0.006     -0.044

           Balanced                         Coordinates
        Dep          Lat      Point          X              Y
------------------------------------------------------------
     -120.735    289.897      1      10,000.00       5,000.00
     -230.557   -129.600      2       9,879.27       5,289.90
      133.690   -166.451      3       9,648.71       5,160.30
      217.602      6.153      4       9,782.40       4,993.85

     Linear misclosure  = 0.045
     Relative Precision = 1 in 22,500

     Area: 59,800 sq. ft.
            1.372 acres {if distance units are feet}

Adjusted Observations
---------------------
      Course     Distance    Bearing
--------------------------------------
         1-2       314.03  N22°36'38"W
         2-3       264.49  S60°39'33"W
         3-4       213.49  S38°46'15"E
         4-5       217.69  N88°22'49"E
```

**10.19** Solutions from WolfPack

```
                                                Unbalanced
Course          Length       Azimuth          Dep          Lat
------------------------------------------------------------
       1-2        449.60    252°36'35.0"  -429.049   -134.376
       2-3        427.28    124°51'20.0"   350.624   -244.195
       3-4        301.49     50°41'28.0"   233.275    190.994
       4-1        243.23    320°27'42.0"  -154.839    187.579
              ----------                 ---------  ---------
```

```
    Sum =   1,421.60                         0.011      0.002

        Balanced                           Coordinates
    Dep          Lat      Point          X              Y
------------------------------------------------------------
     -429.053   -134.377     1      10,000.00       5,000.00
      350.621   -244.195     2       9,570.95       4,865.62
      233.273    190.994     3       9,921.57       4,621.43
     -154.841    187.578     4      10,154.84       4,812.42

     Linear misclosure  = 0.012
     Relative Precision = 1 in 122,300

     Area: 112,600 sq. ft.
            2.585 acres {if distance units are feet}

Adjusted Observations
---------------------
      Course     Distance    Azimuth
--------------------------------------
        1-2       449.60   252°36'35"
        2-3       427.28   124°51'21"
        3-4       301.49    50°41'27"
        4-5       243.23   320°27'41"
```

**10.20** Compute the linear misclosure, relative precision, and adjusted lengths and azimuths for the sides after the departures and latitudes are balanced by the compass rule in the following closed-polygon traverse.

| Course | Length (m) | Departure (m) | Latitude (m) |
|---|---|---|---|
| *AB* | 412.516 | −216.2394 | −351.2975 |
| *BC* | 513.185 | +512.9654 | +15.0112 |
| *CA* | 448.495 | −296.7364 | +336.2964 |

Linear Misclosure = 0.0145
Relative Precision: 1: 94,600

| Course | Length | Dep | Lat | Adj Dep | Adj Lat | Length | Azimuth |
|---|---|---|---|---|---|---|---|
| ***AB*** | 412.516 | −216.2394 | −351.2975 | −216.236 | −351.301 | 412.517 | 211°36'49" |
| ***BC*** | 513.185 | +512.9654 | +15.0112 | +512.969 | +15.008 | 513.189 | 88°19'27" |
| ***CA*** | 448.495 | −296.7364 | +336.2964 | −296.733 | +336.293 | 448.490 | 318°34'34" |
| | 1374.196 | −0.0145 | 0.0101 | 0.000 | 0.000 | | |

**10.21** The following data apply to a closed link traverse [like that of Figure 9.1(b)]. Compute preliminary azimuths, adjust them, and calculate departures and latitudes, misclosures in departure and latitude, and traverse relative precision. Balance the departures and latitudes using the compass rule, and calculate coordinates of points *B*, *C*, and *D*. Compute the final lengths and azimuths of lines *AB*, *BC*, *CD*, and *DE*.

| Station | Measured Angle (to the right) | Adjusted Azimuth | Measured Length (ft) | X (ft) | Y (ft) |
|---|---|---|---|---|---|

| | | | | | |
|---|---|---|---|---|---|
| *AzMk*$_1$ | | | | | |
| | | 322°41'31" | | | |
| A | 267°19'13" | | | 2,517,347.31 | 395,025.36 |
| | | | 243.07 | | |
| B | 248°59'20" | | | | |
| | | | 258.93 | | |
| C | 110°29'24" | | | | |
| | | | 197.41 | | |
| D | 92°03'00" | | | 2,517,910.07 | 395,184.30 |
| | | 321°32'20" | | | |
| *AzMk*$_2$ | | | | | |

From WolfPack:

Angle Summary

| Station | Unadj. Angle | Adj. Angle |
|---|---|---|
| A | 267°19'13.0" | 267°19'11.0" |
| B | 248°59'20.0" | 248°59'18.0" |
| C | 110°29'24.0" | 110°29'22.0" |
| D | 92° 3' 0.0" | 92°02'58.0" |

Angular misclosure (sec): 8"

| Course | Length | Azimuth | Unbalanced Dep | Lat |
|---|---|---|---|---|
| AB | 243.07 | 50°00'42.0" | 186.234 | 156.204 |
| BC | 258.93 | 119°00'00.0" | 226.465 | -125.532 |
| CD | 197.41 | 49°29'22.0" | 150.088 | 128.235 |
| Sum = | 699.41 | | 562.788 | 158.908 |

Misclosure in Departure = 562.788 - 562.760 = 0.028
Misclosure in Latitude = 158.908 - 158.940 = -0.032

| Balanced Dep | Lat | Point | Coordinates X | Y |
|---|---|---|---|---|
| 186.225 | 156.216 | 1 | 2,517,347.31 | 395,025.36 |
| 226.455 | -125.520 | 2 | 2,517,533.53 | 395,181.58 |
| 150.080 | 128.244 | 3 | 2,517,759.99 | 395,056.06 |
| | | 4 | 2,517,910.07 | 395,184.30 |

Linear misclosure = 0.042
Relative Precision = 1 in 16,500

Adjusted Observations

| Course | Distance | Azimuth | Point | Angle |
|---|---|---|---|---|
| AB | 243.07 | 50°00'30" | A | 267°18'59" |
| BC | 258.92 | 118°59'56" | B | 248°59'26" |
| CD | 197.41 | 49°29'010" | C | 110°29'14" |
| | | | D | 92°02'58" |

**10.22** Similar to Problem 10.21, except use the following data:

| Station | Measured Angle (to right) | Adjusted Azimuth | Measured Length (m) | X (m) | Y (m) |
|---|---|---|---|---|---|
| *AzMk*$_1$ | | | | | |
| | | 314°09′23″ | | | |
| *A* | 263°48′52″ | | | 194,325.090 | 25,353.988 |
| | | | 957.957 | | |
| *B* | 263°45′24″ | | | | |
| | | | 945.617 | | |
| *C* | 98°55′04″ | | | | |
| | | | 857.724 | | |
| *D* | 110°21′40″ | | | 196,277.341 | 26,262.583 |
| | | 331°00′27″ | | | |
| *AzMk*2 | | | | | |

From WolfPack:

```
Angle Summary
Station   Unadj. Angle     Adj. Angle
---------------------------------------
    1     263°48'52.0"     263°48'53.0"
    2     263°45'24.0"     263°45'25.0"
    3      98°55'04.0"      98°55'05.0"
    4     110°21'40.0"     110°21'41.0"

Angular misclosure (sec): -4"
```

```
                                             Unbalanced
Course          Length       Azimuth          Dep          Lat
------------------------------------------------------------
       1-2       957.957      37°58'16.0"  589.3965   755.1777
       2-3       945.617     121°43'41.0"  804.2980  -497.2888
       3-4       857.724      40°38'46.0"  558.7086   650.7958
             ----------                 ---------- ----------
        Sum =  2,761.298                 1952.4032   908.6847

Misclosure in Departure = 1,952.4032 - 1,952.2510 = 0.1522
Misclosure in Latitude  = 908.6847 - 908.5950 = 0.0897
```

```
           Balanced                          Coordinates
        Dep          Lat      Point         X              Y
------------------------------------------------------------
     589.3437   755.1466       1    194,325.090      25,353.988
     804.2459  -497.3195       2    194,914.434      26,109.135
     558.6613   650.7679       3    195,718.680      25,611.815
                               4    196,277.341      26,262.583

     Linear misclosure  = 0.1766
     Relative Precision = 1 in 15,600
```

```
Adjusted Observations
---------------------
      Course     Distance    Azimuth         Point        Angle
---------------------------------------------------------------
         1-2      957.900    37°58'11"           1    263°48'48"
         2-3      945.589   121°43'53"           2    263°45'42"
         3-4      857.672    40°38'42"           3     98°54'49"
                                                 4    110°21'41"
```

The azimuths (from north of a polygon traverse are $AB = 38°17'02''$, $BC = 121°26'30''$, $CD = 224°56'59''$, and $DA = 308°26'56''$. If one observed distance contains a mistake, which course is most likely responsible for the closure conditions given in Problems 10.23 and 10.24? Is the course too long or too short?

***10.23** Algebraic sum of departures = 5.12 ft latitudes = −3.13 ft.

$Az_{LEC}$ = 121°26'19", which matches course ***BC*** closely

**10.24** Algebraic sum of departures = 3.723 m latitudes = +4.713 m.

$Az_{LEC}$ = 38°18'24", which closely matches course ***AB*** closely

**10.25** Determine the lengths and bearings of the sides of a lot whose corners have the following *X* and *Y* coordinates (in feet): *A* (5000.00, 5000.00); *B* (5289.67, 5436.12); *C* (4884.96, 5354.54); *D* (4756.66, 5068.37).

| Course | Length | Azimuth |
|---|---|---|
| *AB* | 523.55 | 33°35'31" |
| *BC* | 412.85 | 258°36'12" |
| *CD* | 313.61 | 204°08'54" |
| *DA* | 252.76 | 223°10'58" |

**10.26** Compute the lengths and azimuths of the sides of a closed-polygon traverse whose corners have the following X and Y coordinates (in meters): A (8000.000, 5000.000); B (2650.000, 4702.906); C (1752.028, 2015.453); D (1912.303, 1511.635).

| Course | Length | Azimuth |
|---|---|---|
| *AB* | 5358.24 | 266°49'18" |
| *BC* | 2833.51 | 198°28'35" |
| *CD* | 528.70 | 162°21'11" |
| *DA* | 7016.32 | 231°40'28" |

**10.27** In searching for a record of the length and true bearing of a certain boundary line which is straight between *A* and *B*, the following notes of an old random traverse were found (survey by compass and Gunter's chain, declination 4°45′W). Compute the true bearing and length (in feet) of *BA*.

| **Course** | ***A*-1** | **1-2** | **2-3** | **3-*B*** |
|---|---|---|---|---|
| **Magnetic bearing** | Due North | N 20°00′ E | Due East | S 46°30′ E |

| | | | | |
|---|---|---|---|---|
| **Distance (ch)** | 11.90 | 35.80 | 24.14 | 12.72 |

| | |
|---|---|
| **Course** | **BA** |
| **Distance (ch)** | 58.60 |
| **Bearing** | S55°51'50"W |

Convert direction to true north and then compute departures and latitudes shown below.

| Course | *A*-1 | 1-2 | 1-3 | 3-*B* | Total |
|---|---|---|---|---|---|
| **Departure** | 0.985 | 14.988 | 24.057 | 8.470 | 48.501 |
| **Latitude** | 11.859 | 32.512 | −1.999 | −9.490 | 32.882 |

**10.28** Describe how a blunder may be located in a traverse.

From Section 10.16: If a single blunder in a distance exists, the azimuth of the misclosure line will closely approximate the azimuth of the course with the blunder. If the blunder is in an angle, the perpendicular bisector of the misclosure line will come close to bisecting the angle with the blunder.

**Instructor's Note:** The Mathcad worksheet C10.xmcd and its equivalent html file demonstrate a traverse with a single angle blunder. A graphic of the traverse contained in the files shows how the perpendicular bisector of the misclosure line points at the station with the angular blunder.

## 11 COORDINATE GEOMETRY IN SURVEYING CALCULATIONS

Asterisks (*) indicate problems that have partial answers given in Appendix G.

**11.1** The $X$ and $Y$ coordinates (in meters) of station Shore are 246.873 and 659.457, respectively, and those for station Rock are 437.854 and 973.482, respectively. What are the azimuth, bearing, and length of the line connecting station Shore to station Rock?

**Az = 31°18'24"; Brg = N 31°18'24" E; Distance = 367.540 m**

**11.2** Same as Problem 11.1, except that the $X$ and $Y$ coordinates (in feet) of Shore are 5048.64 and 3278.59, respectively, and those for Rock are 3303.33 and 5876.93, respectively.

**Az = 326°06'38"; Brg = N 33°53'22" W; Distance = 3130.09 ft**

***11.3** What are the slope, and y-intercept for the line in Problem 11.1?

$m$ = **1.644274; *b* = 253.530 m**

**11.4** What are the slope, and the y-intercept for the line in Problem 11.2?

$m$ = **−1.48876;** $b$ = **10794.78 ft**

***11.5** If the slope ($XY$ plane) of a line is 0.800946, what is the azimuth of the line to the nearest second of arc? ($XY$ plane)

**51°18'26"**

**11.6** If the slope ($XY$ plane) of a line is −0.3250683, what is the azimuth of the line to the nearest second of arc? ($XY$ plane)

**288°00'28"**

***11.7** What is the perpendicular distance of a point from the line in Problem 11.1, if the $X$ and $Y$ coordinates (in meters) of the point are 422.058 and 947.653, respectively?

$$Az_{SR} = 31°17'38.6''; \; SR = 337.2636\,\text{m}$$

**0.075 m** By (11.11): $\alpha = 0°00'45.7''$

By (11.12): $SP = 120.502 \sin 0°00'45.7'' = 0.0748$

**11.8** What is the perpendicular distance of a point from the line in Problem 11.2, if the *X* and *Y* coordinates (in feet) of the point are 4405.68 and 4236.01, respectively?

$Az_{SR} = 326°06'59.3''$; $SR = 1153.278$ ft

**0.12 ft**

By (11.11): $\alpha = 0°00'59.3''$

By (11.12): $SP = 1153.278 \sin 0°00'59.3'' = 0.117$

***11.9** A line with an azimuth of 105°46′33″ from a station with *X* and *Y* coordinates of 5885.31 and 5164.15, respectively, is intersected with a line that has an azimuth of 200°31′24″ from a station with *X* and *Y* coordinates of 7337.08 and 5949.99, respectively. (All coordinates are in feet.) What are the coordinates of the intersection point?

**(6932.18, 4868.39)**

$D_{12}$ = 1650.812 ft; $Az_{12}$ = 61°34'24.3"; ∠1 = 44°12'08.7"; ∠$P$ = 94°44'51" $D_{2P}$ = 1154.90 ft

**11.10** A line with an azimuth of 37°30′48″ from a station with *X* and *Y* coordinates of 1234.87 and 898.56, respectively, is intersected with a line that has an azimuth of 314°51′43″ from a station with *X* and *Y* coordinates of 2034.79 and 962.48, respectively. (All coordinates are in feet.) What are the coordinates of the intersection point?

**(1609.14, 1386.08)**

$D_{12}$ = 802.470 ft; $Az_{12}$ = 85°25'52.7"; ∠1 =41°54'19.7"; ∠2 = 49°25'50.2" $D_{1P}$ = 614.620 ft

**11.11** Same as Problem 11.9 except that the azimuth of the first line is 43°31′06″ and the azimuth of the second line is 331°06′00″.

**(1763.15, 1454.75)**

$D_{12}$ = 802.470 ft; $Az_{12}$ = 85°25'52.7"; ∠1 = 41°54'19.7"; ∠2 = 65°40'40.2"; $D_{2P}$ = 767.085 ft

**11.12** In the accompanying figure, the *X* and *Y* coordinates (in meters) of station *A* are 1005.594 and 1868.720, respectively, and those of station *B* are 1564.865 and 644.491, respectively. Angle *BAP* was measured as 308°56′39″ and angle *ABP* was measured as 58°53′30″. What are the coordinates of station *P*?

**(2193.045, 1564.021)**

$D_{12}$ = 1345.927 m; $Az_{12}$ = 155°26'50.7"; ∠1 = 51°03'21"; ∠$P$ = 70°03'09" $D_{1P}$ = 12525.921 m; $Az_{1P}$ = 104°23'29.7"

***11.13** In the accompanying figure, the *X* and *Y* coordinates (in feet) of station *A* are 1248.16 and 3133.35, respectively, and those of station *B* are 1509.15 and 1101.89, respectively. The length of *BP* is 2657.45 ft, and the azimuth of line *AP* is 98°25′00″. What are the coordinates of station P?

**(3560.56, 2791.19)**

$Az_{AB}$ = 172°40'45"; $D_{AB}$ = 2048.157; $D_{AP}$ = 2337.576 or −1226.525; ∠*PAB* = 74°15'45";

*a* = 1; *b* = 1111.051; *c* = −2,867,094.99;

**11.14** In the accompanying figure, the *X* and *Y* coordinates (in feet) of station *A* are 1912.76 and 2238.63, respectively, and those of station *B* are 2342.39 and 1454.77, respectively. The length of *AP* is 694.50 ft, and angle *ABP* is 43°10′44″? What are the possible coordinates for station *P*?

**(2503.49, 1873.43) or (2590.93, 2388.35)**

$Az_{BA}$ = 326°27'32.9"; $Az_{BP}$ = 9°38'16.9"; $D_{AB}$ = 940.453; $D_{BP}$ = 424.652 or 946.944; *a* = 1; *b* = 1371.596; *c* = 402,121.586;

***11.15** A circle of radius 975.80 ft, centered at point *A*, intersects another circle of radius 963.09 ft, centered at point *B*. The *X* and *Y* coordinates (in feet) of *A* are 533.70 and 1157.86, respectively, and those of *B* are 1142.93 and 269.83, respectively. What are the coordinates of station *P* in the figure?

**(1509.50, 1160.43) or (180.08, 248.39)**

*AB* = 1076.921; $Az_{AB}$ = 145°32'53.0"; ∠*PAB* = 55°41'56.6"; $Az_{AP}$ = 89°50'56.4" or 201°14′49.6″

**11.16** The same as Problem 11.15, except the radii from *A* and *B* are 837.45 ft and 1062.16 ft, respectively.

**(1357.19, 1310.16) or (95.21, 444.38)**

*AB* = 1076.92; $Az_{AB}$ = 145°32'53.0"; ∠*PAB* = 66°01'33.6"; $Az_{AP}$ = 79°31'19.4" or 211°34'26.6"

**11.17** For the subdivision in the accompanying figure, assume that lines *AC*, *DF*, *GI*, and *JL* are parallel, but that lines *BK* and *CL* are parallel to each other, but not parallel to *AJ*. If the *X* and *Y* coordinates (in feet) of station *A* are (5000.00, 5000.00), what are the coordinates of each lot corner shown?

| Station | *X* | *Y* | Method |
|---|---|---|---|
| *A* | 5000.00 | 5000.00 | Given |
| *B* | 5149.99 | 4997.99 | Forward |
| *C* | 5299.97 | 4995.99 | Forward |
| *D* | 5013.14 | 5078.91 | Forward |
| *E* | 5162.53 | 5076.92 | Direction-Direction |
| *F* | 5312.52 | 5074.91 | Direction-Distance |
| *G* | 5026.27 | 5157.83 | Forward |
| *H* | 5175.08 | 5155.84 | Direction-Direction |
| *I* | 5325.07 | 5153.83 | Direction-Distance |
| *J* | 5039.40 | 5236.74 | Forward |
| *K* | 5187.63 | 5234.76 | Direction-Direction |
| *L* | 5337.61 | 5232.75 | Direction-Distance |

**11.18** If the *X* and *Y* coordinates (in feet) of station *A* are (1000.00, 1000.00), what are the coordinates of the remaining labeled corners in the accompanying figure?

| Station | *X* | *Y* | Method |
|---|---|---|---|
| *A* | 1000.00 | 1000.00 | Given |
| *B* | 1000.00 | 1400.01 | Forward |
| *C* | 1430.00 | 1400.01 | Forward |
| *D* | 1430.00 | 1000.00 | Direction-Direction |
| *E* | 1235.58 | 1193.82 | Forward or Direction-Distance |
| *F* | 1194.42 | 1193.82 | Forward or Direction-Distance |
| *G* | 1215.00 | 1171.99 | Forward or Direction-Direction |
| *H* | 1200.00 | 1146.01 | Direction-Distance |
| *I* | 1230.00 | 1146.01 | Direction-Distance |
| *J* | 1200.00 | 1000.00 | Forward |
| *K* | 1230.00 | 1000.00 | Forward |

***11.19** In Figure 11.8, the *X* and *Y* coordinates (in feet) of *A* are 616.31 and 1348.88, respectively, those of *B* are 1261.68 and 1137.20, respectively, and those of *C* are 1852.83 and 1385.02, respectively. Also angle *x* is 27°08′55″ and angle *y* is 25°23′38″. What are the coordinates of station *P*?

**(1284.92, 114.94)**

$Az_{BA}$ = 288°09'33.8"; $Az_{BC}$ = 67°15'20.7"; BC(a) = 640.994; AB(c) = 679.199;
$\forall$ = 220°54'13.1"; A+C = 86°33'13.9"; A = 43°23'22.5"; C = 43°09'51.5";
$\forall_1$ = 109°27'42.5"; *AP* = 1403.445; $Az_{AP}$ = 151°32'56.2"

**11.20** In Figure 11.8, the X and Y coordinates (in feet) of *A* are 2265.86 and 3008.76, those of *B* are 2983.51 and 2802.25, and those of *C* are 3742.46 and 3026.83, respectively. Also angle *x* is 28°00′18″ and angle *y* is 26°31′34″. What are the coordinates of station *P*?

**(2903.49, 1591.53)**

$Az_{BA}$ = 286°03'13.1"; $Az_{BC}$ = 73°30'57.8"; BC(a) = 791.480; AB(c) = 746.772;
$\forall$ = 212°32'15.3"; A+C = 92°55'52.7"; A = 49°43'21.7"; C = 43°12'31.0";
$\forall_1$ = 102°16'20.3"; *AP* = 1554.06; $Az_{AP}$ = 155°46'34.8"

**11.21** In Figure 11.9, the following *EN* and *XY* coordinates for points *A* through *C* are given. In a 2-D conformal coordinate transformation, to convert the *XY* coordinates into the *EN* system, what are the

**(a)*** Scale factor? **0.304826** *AB* = 418.98532 m; *ab* = 1374.5065 ft

**(b)** Rotation angle? **10°50'15.8"**; $Az_{AB}$ = 229°33'45.3"; $Az_{ab}$ = 240°24'01.1"

**(c)** Translations in X and Y? **Tx = 598,423.040 m** and **Ty = 385,610.572 m**

**(d)** Coordinates of points *C* in the *EN* coordinate system?

***E* = 599,044.806 m** and ***N* = 386,613.744 m**

| State Plane Coordinates (m) | Arbitrary Coordinates (ft) |
|---|---|

| Point | E | N | X | Y |
|---|---|---|---|---|
| *A* | 599,368.087 | 386,573.866 | 3639.18 | 2520.84 |
| *B* | 599,049.191 | 386,302.105 | 2444.05 | 1841.92 |
| *C* | | | 2622.15 | 2848.74 |

**11.22** Do Problem 11.21 with the following coordinates.

| | State Plane Coordinates (m) | | Arbitrary Coordinates (ft) | |
|---|---|---|---|---|
| **Point** | **E** | **N** | **X** | **Y** |
| *A* | 651,779.322 | 290,831.220 | 5504.32 | 3623.76 |
| *B* | 651,169.151 | 290,542.891 | 3409.59 | 2906.42 |
| *C* | | | 3849.59 | 3857.64 |

**(a)** Scale factor? **<u>0.3047959</u>** $AB = 674.866$; $ab = 2214.15$

**(b)** Rotation angle? **<u>6°23'19.5"</u>**; $Az_{AB}$ = 244°42'26.9"; $Az_{ab}$ = 251°05'46.5"

**(c)** Translations in X and Y? **<u>Tx = 650,234.949</u>** and **<u>Ty = 289,546.888</u>**

**(d)** Coordinates of points *C* in the *EN* coordinate system?

**<u>*E* = 651,270.167</u>** and **<u>*N* = 290,845.941</u>**

**11.23** In Figure 11.12, the elevations of stations *A* and *B* are 210.05 ft, and 208.53 ft, respectively. Instrument heights $hi_A$ and $hi_B$ are both 5.50 ft. What is the average elevation of point P if the other field observations are:

*AB* = 136.52 ft; *A* = 50°11′22″; *B* = 51°06′08″; $v_1$ = 36°33′59″; $v_2$ =37°26′46″

**<u>Elev = 295.92 ft</u>**

*AI* = 108.346; *BI* = 106.940; $IP_A$ = 80.367; $IP_B$ = 81.898

**11.24** In Problem 11.23, assume station *P* is to the left of the line *AB*, as viewed from station *A*. If the *X* and *Y* coordinates (in feet) of station *A* are 2041.19 and 2938.76, respectively, and the azimuth of line *AB* is 57°59′56″ what are the *X* and *Y* coordinates of the inaccessible point?

**<u>(2055.91, 3046.10)</u>**; $Az_{AI}$ = 7°48'34"

**11.25** In Figure 11.12, the elevations of stations *A* and *B* are 26.776 and 26.949 m, respectively. Instrument heights $hi_A$ and $hi_B$ are both 1.600 m. What is the average elevation of point P if the other field observations are:

*AB* = 112.531 m; *A* =57°52′55″; *B* = 62°13′08″; $v_1$ = 32°18′39″’ $v_2$ = 33°22′46″

**<u>Elev = 101.144 m</u>**

*AI* = 115.079; *BI* = 110.165; $IP_A$ = 72.780; $IP_B$ = 72.584

**11.26** In Problem 11.25, assume station *P* is to the left of line *AB* as viewed from station *A*. If the *X* and *Y* coordinates (in meters) of station *A* are 2985.465 and 3077.035, respectively, and the azimuth of line *AB* is 125°32′15″ what are the *X* and *Y* coordinates of the inaccessible point?

**(3091.903, 3408.576)**; $Az_{AI}$ = 67°39'20.4"

**11.27** In Figure 11.13, the *X*, *Y*, and *Z* coordinates (in feet) of station *A* are 2897.37, 3406.73, and 234.56, respectively, and those of *B* are 3126.27, 3394.46, and 241.69, respectively. Determine the three-dimensional position of the occupied station *P* with the following observations:

$v_1 = 32°14'00''$ $PA = 243.67$ ft $hr_A = 6.53$ ft $\gamma = 64°39'51''$

$v_2 = 31°29'58''$ $PB = 260.10$ ft $hr_B = 5.33$ ft $hi_P = 5.50$ ft

**(2987.58, 3221.40, 105.62)**

*AB* = 229.229; $Az_{AB}$ = 93°04'06.1"; *PC* = 206.116; *PD* = 221.773;

∠*DCP* = 60°58'34.9"; *AC* = 129.9659; *BC* = 135.8997

**11.28** Adapt Equations (11.43) and (11.47) so they are applicable for zenith angles.

**(11.43): *PC* = *PA* sin($z_1$); *PD* = *PB* sin($z_2$)**

**(11.47): *PA* = *PA* cos($z_1$); *BD* = *PB* cos($z_2$)**

**11.29** In Figure 11.13, the *X*, *Y*, and *Z* coordinates (in meters) of station *A* are 2634.100, 3119.530, and 252.796, respectively, and those of *B* are 2540.210, 3277.250, and 245.809, respectively. Determine the three-dimensional position of occupied station *P* with the following observations:

$z_1 = 69°26'06''$ $PA = 179.439$ m $hr_A = 2.000$ m $\gamma = 57°51'53''$

$z_2 = 74°44'01''$ $PB = 212.851$ m $hr_B = 2.000$ m $hi_P = 1.600$ m

**(2743.335, 3247.174, 190.164)**

*AB* = 183.551; $Az_{AB}$ = 329°14'05.5"; *PC* = 168.004; *PD* = 205.340;

∠*DCP* = 71°19'17.1"; *AC* = 63.032; *BC* = 56.045

**11.30** Use WOLFPACK to do Problem 11.9. (See solution to 11.9)

**11.31** Use WOLFPACK to do Problem 11.10. (See solution to 11.10)

**11.32** Use WOLFPACK to do Problem 11.12. (See solution to 11.12)

**11.33** Use WOLFPACK to do Problem 11.13. (See solution to 11.13)

**11.34** Use WOLFPACK to do Problem 11.15. (See solution to 11.15)

**11.35** Use WOLFPACK to do Problem 11.16. (See solution to 11.16)

**11.36** Use WOLFPACK to do Problem 11.17. (See solution to 11.17)

# 12 AREA

Asterisks (*) indicate problems that have answers given in Appendix G.

***12.1** Compute the area enclosed within polygon *ABDFGA* of Figure 12.1 using triangles.

**418,320 ft$^2$ 0r 9.6040 ac**;

*AGD* = 137,481 ft$^2$; *GFD* = 61,900 ft$^2$; *GDB* = 81,462 ft$^2$; *GBA*= 40,320 ft$^2$

**12.2** Similar to Problem 12.1, except for polygon *BGDCB* of Figure 12.1.

**121,780 ft$^2$ or 2.7957 ac**; *BGD* = 81,460 ft$^2$; *DCB* = 81,462 ft$^2$

**12.3** Compute the area enclosed between line *AGBA* and the shoreline of Figure 12.1 using the offset method.

**183,390 ft$^2$ or 4.2101 ac**; *ABG* = 137,481 ft$^2$; shoreline = 45,911 ft$^2$

**Shoreline**

| | | | | | |
|---|---|---|---|---|---|
| 0 | 69 | 257 | 535 | 610.5 | |
| 0 | 102 | 71 | 92 | 0 | Sum |
| 3,519.0 | 16,262.0 | 22,657.0 | 3,473.0 | | 45,911.0 |

**12.4** By rule of thumb, what is the estimated uncertainty in an 683,905 ft$^2$ if the estimated error in the coordinates was ±0.1 ft?

*E* = **61 or 100 ft$^2$**; By Equation (12.9)

***12.5** Compute the area between a lake and a straight line *AG*, from which offsets are taken at irregular intervals as follows (all distances in feet):

| Offset Point | *A* | *B* | *C* | *D* | *E* | *F* | *G* |
|---|---|---|---|---|---|---|---|
| Stationing | 0.00 | 0 + 54.80 | 1 + 32.54 | 2 + 13.02 | 2 + 98.74 | 3 + 45.68 | 4 + 50.17 |
| Offset | 12.3 | 34.2 | 56.5 | 85.4 | 69.1 | 68.9 | 23.9 |

**25,220 ft$^2$ or 0.5789 ac.**

Computations

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 54.8 | 132.54 | 213.02 | 298.74 | 345.68 | 450.17 |
| 12.3 | 34.2 | 56.5 | 85.4 | 69.1 | 68.9 | 23.9 |
| 1274.1 | 3525.5 | 5710.06 | 6621.87 | 3238.9 | 4848.3 | |

**12.6** Repeat Problem 12.5 with the following offset in meters.

| Offset Point | *A* | *B* | *C* | *D* | *E* | *F* | *G* |
|---|---|---|---|---|---|---|---|
| Stationing | 0.000 | 20.000 | 78.940 | 148.963 | 163.654 | 203.691 | 250.454 |
| Offset | 2.15 | 3.51 | 4.04 | 6.57 | 5.87 | 4.64 | 1.65 |

**<u>1099 m² or 0.1099 ha</u>**;

Computations

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 20 | 78.94 | 148.963 | 163.65 | 203.69 | 250.454 |
| 2.15 | 3.51 | 4.04 | 6.57 | 5.87 | 4.64 | 1.65 |
| 56.6 | 222.5 | 371.472 | 91.378 | 210.39 | 147.07 | |

**12.7** Use the coordinate method to compute the area enclosed by the traverse of Problem 10.8.

**<u>3,570,400 ft² or 81.965 ac</u>**

Area = 0.5(7,140,808.1) = 3,570,400

| X | Y | XY (+) | YX (-) |
|---|---|---|---|
| 0.00 | 5000.00 | | 9093650 |
| 1818.73 | 3839.67 | 0 | 10790317.43 |
| 2810.22 | 5248.32 | 9545277.034 | 7992194.179 |
| 1522.81 | 5531.30 | 15544169.89 | 2939720.011 |
| 531.47 | 6704.64 | 10209892.84 | 0 |
| 0.00 | 5000.00 | 2657350 | |
| | | 37,956,689.8 | 30,815,881.6 |

**12.8** Calculate by coordinates the area within the traverse of Problem 10.11.

**<u>66,810 m² or 6.681 ha</u>**

Area = 0.5(133,626) = 66,813

| X | Y | XY (+) | YX (-) |
|---|---|---|---|
| 630.890 | 1311.410 | | 714652.9 |
| 544.950 | 1105.620 | 697524.6 | 747786.1 |
| 676.350 | 999.640 | 544753.8 | 822723.7 |
| 823.020 | 1108.000 | 749395.8 | 927717.3 |
| 837.290 | 1336.600 | 1100049 | 843247.6 |
| 630.890 | 1311.410 | 1098030 | |
| | | 4189753 | 4056128 |

**12.9** Compute the area enclosed in the traverse of Problem 10.8 using DMDs.

**3,570,400 ft$^2$ or 81.965 ac**

| Dep | Lat | DMD | D-Area |
|---|---|---|---|
| 1818.731 | −1160.328 | 1818.731 | −2110324.504 |
| 991.487 | 1408.640 | 4628.949 | 6520522.719 |
| −1287.409 | 282.984 | 4333.027 | 1226177.313 |
| −991.338 | 1173.340 | 2054.280 | 2410368.895 |
| −531.471 | −1704.640 | 531.471 | −905966.7254 |
| | | | 7140777.698 |

***12.10** Determine the area within the traverse of Problem 10.11 using DMDs.

**66,810 m$^2$ or 6.681 ha**

| **Dep** | **Lat** | **DMD** | **D-Area** |
|---|---|---|---|
| −85.9474 | −205.7924 | −85.947 | 17687.32 |
| 131.4089 | −105.9817 | −40.486 | 4290.765 |
| 146.6660 | 108.3591 | 237.589 | 25744.93 |
| 14.2742 | 228.6006 | 398.529 | 91104.01 |
| −206.4017 | −25.1855 | 206.402 | −5198.33 |
| | | | 133628.7 |

**12.11** By the DMD method, find the area enclosed by the traverse of Problem 10.20.

**4,938,000 m$^2$ or 4.937 ha**

| **Departure** | **Latitude** | **DMD** | **D-Area** |
|---|---|---|---|
| −2014.132 | 662.305 | −2014.132 | −1333969.694 |
| −1656.629 | −4358.189 | −5684.893 | 24775838.14 |
| 3670.761 | 3695.884 | −3670.761 | −13566706.85 |
| | | | 9,875,161.597 |
| | | | 4,937,580.798 |

**12.12** Compute the area within the traverse of Problem 10.17 using the coordinate method. Check by DMDs.

**21,081,500 ft$^2$ or 483.965 ac**

| **X** | **Y** | **XY (+)** | **YX(-)** |
|---|---|---|---|
| 6,521.95 | 7037.072 | | 45950201.26 |
| 6,529.73 | 4174.528 | 27226067.06 | 44737068.2 |
| 10,716.68 | 4042.141 | 26394101.48 | 51839920.72 |
| 12,824.87 | 7221.93 | 77395091.13 | 68873141.51 |
| 9,536.67 | 8795.595 | 112802336.1 | 57364439.61 |
| 6521.951 | 7037.072 | 67110212.32 | |
| | | 310927808 | 268764771.3 |

| | |
|---|---|
| 21,081,518 | 483.965 |

Check by DMD

| Dep | Lat | DMD | D-Area |
|---|---|---|---|
| 7.7822 | −2862.544 | 7.782 | −22276.88992 |
| 4186.9438 | −132.3871 | 4202.508 | −556357.8733 |
| 2108.19 | 3179.7891 | 10497.642 | 33380287.61 |
| −3288.1997 | 1573.6647 | 9317.632 | 14662829.04 |
| −3014.7163 | −1758.5227 | 3014.716 | −5301447.048 |
| | | | 42163034.83 |
| | | | 21,081,517 |
| | | | 483.965 |

**12.13** Calculate the area inside the traverse of Problem 10.18 by coordinates and check by DMDs.

**<u>302,010 ft$^2$ or 6.9332 ac</u>**

| X | Y | XY (+) | YX(-) |
|---|---|---|---|
| 10,000.00 | 5,000.00 | | 50290850 |
| 10,058.17 | 5,399.50 | 53995000 | 57666498.02 |
| 10,679.97 | 5,431.69 | 54632861.41 | 57104932.16 |
| 10,513.29 | 4,800.13 | 51265244.4 | 48001300 |
| 10000.00 | 5000.00 | 52566450 | |
| | | 212459555.8 | 213063580.2 |
| | | | 302012 |

**Check by DMD**

| Dep | Lat | DMD | D-Area |
|---|---|---|---|
| 58.17 | 399.502 | 58.17 | 23239.03134 |
| 621.798 | 32.185 | 738.138 | 23756.97153 |
| −166.68 | −631.559 | 1193.256 | −753611.5661 |
| −513.289 | 199.873 | 513.287 | 102592.2126 |
| | | | −604023.3507 |
| | | | 302012 |

**12.14** Compute the area enclosed by the traverse of Problem 10.19 using the DMD method. Check by coordinates.

**<u>419,700 ft$^2$ or 9.6351 ac</u>**

| Dep | Lat | DMD | D-Area |
|---|---|---|---|
| 359.571 | −140.204 | 359.571 | −50413.29248 |
| 430.594 | 921.177 | 1149.736 | 1059110.359 |
| −296.458 | 273.72 | 1283.872 | 351421.4438 |
| −493.707 | −1054.69 | 493.707 | −520709.317 |
| | | | 839409.1937 |
| | | | 419705 |

**Check by coordinates**

| X | Y | XY (+) | YX(-) |
|---|---|---|---|
| 10,000.00 | 5,000.00 | | 51797850 |
| 10,359.57 | 4,859.80 | 48598000 | 52438068.17 |
| 10,790.17 | 5,780.97 | 59888363.38 | 60663822.7 |
| 10,493.71 | 6,054.69 | 65331134.4 | 60546900 |
| 10,000.00 | 5,000.00 | 52468550 | |
| | | 226286047.8 | 225446640.9 |
| | | | 419703 |

**12.15** Find the area of the lot in Problem 10.25.

**<u>115,640 ft$^2$ or 2.6547 ac</u>**

| X | Y | XY (+) | YX (-) |
|---|---|---|---|
| 5000.00 | 5000.00 | | 26448350 |
| 5289.67 | 5436.12 | 27180600 | 26555229 |
| 4884.96 | 5354.54 | 28323750 | 25469726 |
| 4756.66 | 5068.37 | 24758785 | 25341850 |
| 5000.00 | 5000.00 | 23783300 | |
| | | 104046434 | 103815155 |
| | | 115640 | 2.6547 |

***12.16** Determine the area of the lot in Problem 10.26.

**8,868,600 $m^2$ or 886.86 ha**

| X | Y | XY (+) | YX (-) |
|---|---|---|---|
| 8000.000 | 5000.000 | | 13250000 |
| 2650.000 | 4702.906 | 37623248 | 8239623 |
| 1752.028 | 2015.453 | 5340950 | 3854157 |
| 1912.303 | 1511.635 | 2648427 | 12093080 |
| 8000.000 | 5000.000 | 9561515 | |
| | | 55174140 | 37436860 |
| | | 8868640 | 886.8640242 |

**12.17** Calculate the area of Lot 16 in Figure 21.2.

**11,370 $ft^2$ or 0.2610 ac**; Lot rectangular = 80(142.09) = 11,367.2

**12.18** Plot the lot of Problem 10.25 to a scale of 1 in. = 100 ft. Determine its surrounded area using a planimeter.

**About 115,640 $ft^2$ or 2.6547 ac**

**12.19** Similar to Problem 12.18, except for the traverse of Problem 10.26.

**About 8,868,600 $m^2$ or 886.86 ha**

**12.20** Plot the traverse of Problem 10.19 to a scale of 1 in. = 200 ft, and find its enclosed area using a planimeter.

**About 419,700 $ft^2$ or 9.6351 ac**

**12.21** The $(X,Y)$ coordinates (in feet) for a closed-polygon traverse *ABCDEFA* follow. *A* (1000.00, 1000.00), *B* (1645.49, 1114.85), *C* (1675.95, 1696.05), *D* (1178.99, 1664.04), *E* (1162.62, 1337.78) and *F* (996.53, 1305.30). Calculate the area of the traverse by the method of coordinates.

**349,610 $ft^2$ or 8.0260 ac**

| X | Y | XY (+) | YX (-) |
|---|---|---|---|
| 1000.00 | 1000.00 | | 1645490 |
| 1645.49 | 1114.85 | 1114850 | 1868433 |
| 1675.95 | 1696.05 | 2790833.315 | 1999626 |
| 1178.99 | 1664.04 | 2788847.838 | 1934646 |
| 1162.62 | 1337.78 | 1577229.242 | 1333138 |
| 996.53 | 1305.30 | 1517567.886 | 1305300 |
| 1000.00 | 1000.00 | 996530 | |
| | | 10785858.28 | 10086633 |
| | | 349613 | 8.026002588 |

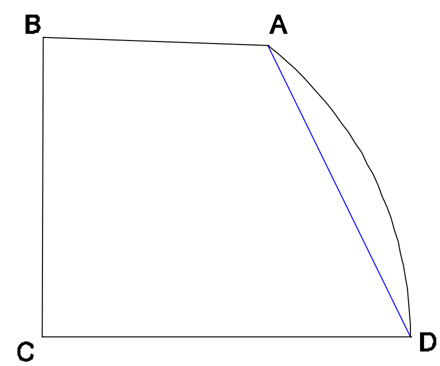
**12.22** Compute by DMDs the area in hectares within a closed-polygon traverse *ABCDEFA* by placing the *X* and *Y* axes through the most southerly and most westerly stations, respectively. Departures and latitudes (in meters) follow. *AB*: E dep. 30, N lat. = 40; *BC*: E dep. = 70, N lat. = 10; *CD* = E dep. = 30, S lat = 50; *DE*: W dep = 60, S lat. = 40; *EF*: W dep = 90, S lat = 30; *FA*: E dep. = 20, N lat = 70.

**<u>1 ha</u> or 10,000 m²**

| Dep | Lat | DMD | D-Area |
|---|---|---|---|
| 30 | 40 | 30 | 1200 |
| 70 | 10 | 130 | 1300 |
| 30 | −50 | 230 | −11500 |
| −60 | −40 | 200 | −8000 |
| −90 | −30 | 50 | −1500 |
| 20 | 70 | -20 | −1400 |
| | | | −19900 |
| | | | 9950 |

**12.23** Calculate the area of a piece of property bounded by a traverse and circular arc with the following coordinates in meters at the angle points: *A* (1479.838, 1803.583), *B* (1204.998, 1813.173), *C* (1204.728, 1447.473), *D* (1654.728, 1447.473) with a circular arc of radius *CD* starting at *D* and ending at *A* with the curve outside the course *AD*.

**<u>142,700 m² or 14.27 ha</u>**




Area of ABCD is 130,381 m²

Angle *ACD* = 90° − 37°41'15.4" = 52°18'44.6" = 0.913023617 radians

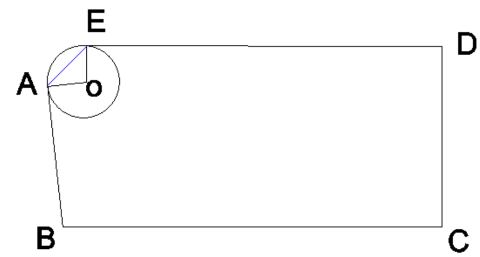
$R = CD = CA$ = 450.000 m

Area of segment is $0.5(450^2)(0.913023617 - \sin 52°18'6'')$ = 12,319 m²

| X | Y | XY (+) | YX (-) |
|---|---|---|---|
| 1479.838 | 1803.583 | | 2173313.9 |
| 1204.998 | 1813.173 | 2683202.3 | 2184380.3 |
| 1204.728 | 1447.473 | 1744202.1 | 2395174.1 |
| 1654.728 | 1447.473 | 1743811.3 | 2142025.5 |
| 1479.838 | 1803.583 | 2984439.3 | |
| | | 9155654.9 | 8894893.8 |

**12.24** Calculate the area of a piece of property bounded by a traverse and circular arc with the following coordinates in feet at angle points: *A* (445.18, 913.88), *B* (453.67, 835.51), *C* (663.67, 835.51), *D* (663.67, 935.51), *E* (465.12, 936.04) with a circular arc of radius 19.99 ft starting at *E*, tangent to *DE*, and ending at *A*.

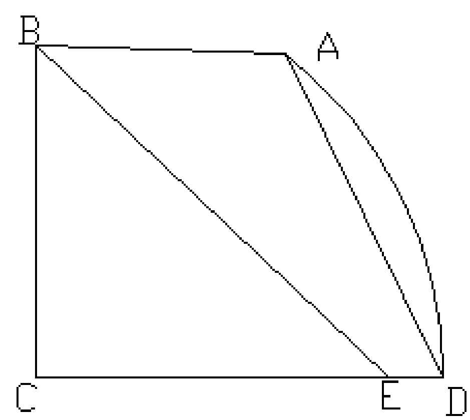
**21,500 ft² or 0.4935 ac**




$Az_{AB}$ = 173°49'01.7"
Azimuth to center of circle: 83°49'01.7"
$Az_{ED}$ = 90°09'10.6"
Azimuth to center of circle: 180°09'10.6"
θ = 96°20'08.9"
Area of segment = 137 ft² =
$0.5(20^2)(1.6813771 - \sin 96°20'08.9'')$
Area of remainder = 21,358.6 ft²

| X | Y | XY (+) | YX (-) |
|---|---|---|---|
| 445.18 | 913.88 | | 414,599.9 |
| 453.67 | 835.51 | 371,952.3 | 554,502.9 |
| 663.67 | 835.51 | 379,045.8 | 554,502.9 |
| 663.67 | 935.51 | 620,869.9 | 435,124.4 |
| 465.12 | 936.04 | 621,221.7 | 416,706.3 |
| 445.18 | 913.88 | 425,063.9 | |
| | | 2,418,153.6 | 2,375,436.5 |

**12.25** Divide the area of the lot in Problem 12.23 into two equal parts by a line through point *B*. List in order the lengths and azimuths of all sides for each parcel.




**Parcel A**

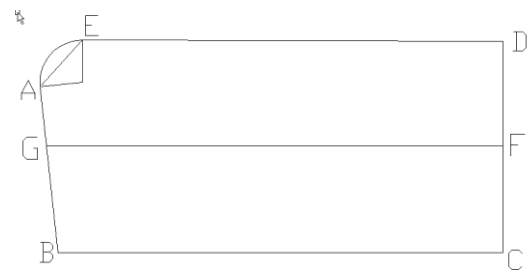
| Course | Length | Azimuth |
|---|---|---|
| *AB* | 275.007 | 271°59'54" |
| *BE* | 534.593 | 133°09'46" |
| *ED* | 597.796 | 90°00'00" |
| *DA* | 396.738 | 153°50'38" |

**Parcel B**

| Course | Length | Azimuth |
|---|---|---|
| *BC* | 365.700 | 180°02'32" |
| *CE* | 390.210 | 270°00'00" |
| *EB* | 534.593 | 313°09'46" |

1/2 Area = 71,350 ft²; $\text{Area}_{BCD}$ = 82,282.5 ft²; $\text{Area}_{BEDA}$ = 10,932.8 ft²
$Az_{CB}$ = 309°06'59" $Az_{DC}$ = 270°
*BD* = 579.65 ft; ∠*CDB* =∠*EDB* = 39°06'59"
1/2(*BD*)(*DE*)sin(*EDB*) = 10932.5 ft²;  *DE* = 59.79 ft
Coordinates of *E*: (1594.937, 1447.473)

**12.26** Partition the lot of Problem 12.24 into two equal areas by means of a line parallel to *BC*. Tabulate in clockwise consecutive order the lengths and azimuths of all sides of each parcel.




**Parcel A**

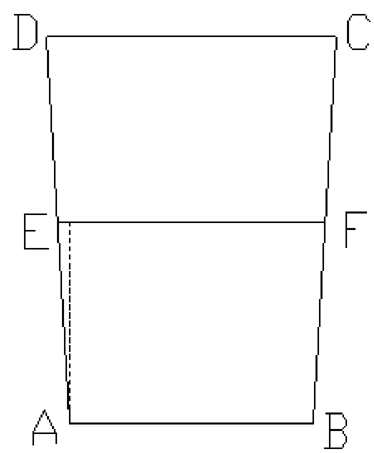
| Course | Length | Azimuth |
|---|---|---|
| *AG* | 28.02 | 173°49'02" |
| *GF* | 215.47 | 90°00'00" |
| *FD* | 49.48 | 0°00'00" |
| *DE* | 198.55 | 270°09'11" |
| *EA* | 29.81 | 221°58'53" |

**Parcel B**

| Course | Length | Azimuth |
|---|---|---|
| *GB* | 50.82 | 173°49'02" |
| *BC* | 210.00 | 90°00'00" |
| *CF* | 50.52 | 0°00'00" |
| *FG* | 215.47 | 180°00'00" |

Area = 10,747.5 $ft^2$; $\angle ABC$ = 96°10'58"; *BC* = 210.00 ft
10,747.5 = $h/2$[210.00 + (210.00 + $h$tan 6°10'58")]
quadratic equation: $0 = 0.108331h^2 + 420h - 21495$; $h$ = 50.52 ft;

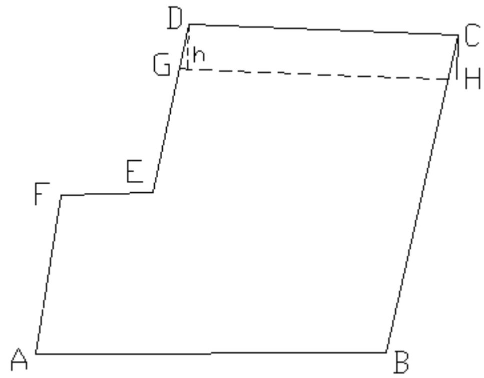
**12.27** Lot *ABCD* between two parallel street lines is 350.00 ft deep and has a 220.00 ft frontage (*AB*) on one street and a 260.00 ft frontage (*CD*) on the other. Interior angles at *A* and *B* are equal, as are those at *C* and *D*. What distances *AE* and *BF* should be laid off by a surveyor to divide the lot into two equal areas by means of a line *EF* parallel to *AB*?




***AE* = *BF* = 182.58 ft**

Area = 84,000 $ft^2$; 1/2 area = 42,000 $ft^2$
$\angle$@*A* = $\tan^{-1}$ (20/350) = 93°16'14"
42000 = $h/2$[220.00 + (220.00 + 2$h$ tan 3°16'14")]
quadratic equation: $0.057142857h^2 + 220h - 42000 =$ 0
$h$ = 182.28 ft
*EF* = 220 + 2(182.28)tan 3°16'14" = 240.83 ft

**12.28** Partition 1-acre parcel from the northern part of lot *ABCDEFA* in Problem 12.21 such that its southern line is parallel to the northern line.




**<u>Lay off 88.04 ft along *DE* to *G*, and 88.18 ft along *CB* to *H*</u>**

Required area: 43,560 sq. ft.
$x = 498.00 - h \tan a_2 + h \tan a_1$

$$CD = \sqrt{(117899 - 167595)^2 + (166404 - 169605)^2} = 49800$$

$$Az_{CD} = \tan^{-1}\left(\frac{117899 - 167595}{166404 - 169605}\right) + 180° = 266°18'52.5''$$

$$Az_{DE} = \tan^{-1}\left(\frac{116662 - 117899}{133778 - 166404}\right) + 180° = 182°10'16.7''$$

$$Az_{CB} = \tan^{-1}\left(\frac{164549 - 167595}{111485 - 169605}\right) + 180° = 183°00'00.2''$$

$a_1$ = 182°10’16.7” – (266°18’52.5” – 90°) = 5°51’24.2”
a2 = 183°00’00.2” – (266°18’52.5” – 90°) = 6°41’07.7”
$x = 498.00 - h\ (0.014639447)$
$43{,}560 = ½\ (498.00 + x)h$
So, $0.014639447\ h^2 - 996h + 87{,}120 = 0$

$$h = \frac{99600 \pm \sqrt{(-99600)^2 - 4(0.014639447)(87{,}120)}}{2(0.014639447)}$$

$$= \frac{2.564322415}{2(0.014639447)} = 87.58\ ft$$

Check Area = ½ (498.00 + 496.73) 87.58 = 43,560 sq ft. (check)

$$DG = \sqrt{87.58^2 + (87.58 \tan 5°51'24.2'')^2} = 88.04\ ft$$

$$CH = \sqrt{87.58^2 + (87.58 \tan 6°41'07.7'')^2} = 88.18\ ft$$

# 13 GLOBAL NAVIGATION SATELLITE SYSTEMS—INTRODUCTION AND PRINCIPLES OF OPERATION

Asterisks (*) indicate problems that have partial answers given in Appendix G.

**13.1** What does the GNSS receiver provide the user?

From Section 13.1, paragraph 1: "These systems provide precise timing and positioning information anywhere on the Earth with high reliability and low cost."

**13.2** What are the three segments in GPS?

See Section 13.2n paragraph 2: "The global positioning system can be arbitrarily broken into three parts: (a) the space segment, (b) the control segment, and (c) the user segment.

**13.3** What is the frequency of the $L1$ signal and its relationship to the fundamental frequency of the satellite clock?

From Section 13.3: The L1 is broadcast at a frequency of 1575.42 MHz and is $154f_0$.

***13.4** Discuss the purpose of the pseudorandom noise codes.

See Section 13.2, paragraphs 3 thru 8. "The individual satellites are normally identified by their *PseudoRandom Noise* (PRN) number. The receiver simultaneously generates a duplicate PRN code. Matching the incoming satellite signal with the identical receiver-generated signal derives the time it takes for the signal to travel from satellite to receiver. This yields the signal delay that is converted to travel time. From the travel time, and the known signal velocity, the distance to the satellite can be computed."

**13.5** What type of information is broadcast in the GPS signal?

From Section 13.3, paragraph 2: "Several different types of information (messages) are modulated upon these carrier waves using a phase modulation technique. Some of the information included in the broadcast message is the almanac, broadcast ephemeris, satellite clock correction coefficients, ionospheric correction coefficients, and satellite condition (also termed *satellite health*)."

**13.6** How is the line of apsides defined?

From Section 13.4.1, paragraph 1: "*Perigee* and *apogee* are points where the satellite is closest to, and farthest away from $G$, respectively, in its orbit. The *line of apsides* joins these two points, passes through the two foci, and is the reference axis $X_S$"

**13.7** What new signals were added to the Block IIF satellites?

From Section 13.3, paragraph 4: "*Modernized satellites* are being equipped with new codes. The modernized satellites include a second civilian code on the L2 signal called the L2C. This code has both a civilian moderate (CM) and civilian long (CL) version. Additionally, the P code is being replaced by two new military codes, known as *M* codes. In 1999, the Interagency GPS Executive Board (IGEB) decided to add a third civilian signal known as the L5 to provide safety of life applications to GPS. L5 will be broadcast at a frequency of 1176.45 MHz, which is $115 \times f_0$. The L5 signal will carry both civilian codes along with a codeless component."

**13.8** What is the purpose of anti-spoofing?

From Section 13.3, paragraph 6: "This encryption process is known as *anti-spoofing* (A-S). Its purpose is to deny access to the signal by potential enemies who could deliberately modify and retransmit it with the intention of "spoofing" unwary friendly users."

**13.9** Describe the geocentric coordinate system.

From Section 13.4.1, paragraph 2: "This three-dimensional rectangular coordinate system has its origin at the mass center of the Earth. Its $X_e$ axis passes through the Greenwich meridian in the plane of the equator, and its $Z_e$ axis coincides with the *Conventional Terrestrial Pole* (CTP). Its $Y_e$ axis lies in the plane of the equator and creates a right-handed coordinate system."

**13.10** Define the terms "geodetic height," "geoid height," and "orthometric height." Include their relationship to each other.

See Section 13.4.3: $h = H + N$ where $h$ is the geodetic height, $H$ is the elevation/orthometric height, or $N$ is the geoid height is the vertical distance between the ellipsoid and geoid.

**13.11** Define PDOP, HDOP, and VDOP.

From Section 13.6.4:

- Positional dilution of precision: $\sqrt{\sigma_X^2 + \sigma_Y^2 + \sigma_Z^2}$
- Horizontal dilution of precision: $\sqrt{\sigma_n^2 + \sigma_e^2}$
- Vertical dilution of precision: $\sqrt{\sigma_Z^2}$

**13.12** What reference ellipsoid is used in the broadcast message of GPS?

From Section 13.4.1, footnote 3: WGS84

**13.13** Why was the P code encrypted with the W code?

From Section 13.3, paragraph 6: "Its purpose is to deny access to the signal by potential enemies who could deliberately modify and retransmit it with the intention of "spoofing" unwary friendly users."

**13.14** Why is it important to check the reference frame of previously determined coordinates for a station?

From Section 13.4.4, paragraph 2: “Thus each change in WGS84 indicates a different set of coordinate values for any station.”

**13.15** Approximately how far from a GPS satellite will a signal travel in 0.07 sec?

By Equation (13.11); $r = 299{,}792{,}458(0.07) = \mathbf{20,985,472\ m}$

**13.16** What are the unknowns in Equation (13.13)?

From Section 13.5.1: The position of the receiver in geocentric coordinates,$(X_A, Y_A, Z_A)$, and the receiver and satellite clock biases, $\delta_A(t)$ and $\delta^{\#}(t)$, respectively.

**13.17** What is integer ambiguity?

From Section 13.5.2, paragraph 1: "However, it does not account for the number of full wavelengths or *cycles* that occurred as the signal traveled between the satellite and receiver. This number is called the *integer ambiguity* or simply *ambiguity*."

**13.18** What are the major sources of error in a GPS pseudorange?

From Section 13.6, paragraph 1: “Some of the larger errors include (1) satellite and receiver clock biases and (2) ionospheric and tropospheric refraction. Other errors in satellite surveying work stem from (a) satellite ephemeris errors, (b) multipathing, (c) instrument miscentering, (d) antenna height measurements, and (e) satellite geometry.”

**13.19** What errors can single differencing of phase-shift observations remove?

From Section 13.9.1, paragraph 1: “This difference eliminates the satellite clock bias and much of the ionospheric and tropospheric refraction from the solution” for short baselines.

***13.20** What errors can double differencing of phase-shift observations remove?

From Section 13.9.2, paragraph 1: “The procedure eliminates the receiver clock bias.”

For Problems 13.21 through 13.26 use the WGS84 ellipsoidal parameters.

***13.21** What are the geocentric coordinates in meters of a station in meters that has a latitude of 49°27’32.20144” N longitude of 122°46’53.56027” W and height of 303.436 m.

From WolfPack: (**−2,249,118.734, −3,492,419.151, 4,824,120.725**)

$R_N$ = 6,372,357.535 m

**13.22** Same as Problem 13.21 except with geodetic coordinates of 44°53’52.1918” N, longitude of 68°40’07.3487” W and height of 38.405 m?

From WolfPack: **(1,646,246.705, −4,215,600.147, 4,479,339.792)**

$R_N$ = 6,367,2678.421 m

**13.23** Same as Problem 13.21 except with geodetic coordinates of 27°42'45.18186" N longitude of 97°19'27.98983" W and height of 4.267 m?

From WolfPack: (**−720,383.135, −5,604,499.64293, 2,948,344.16328**)

$R_N$ = 6,349,222.145 m

***13.24** What are the geodetic coordinates in meters of a station with geocentric coordinates of (136,153.995, −4,859,278.535,4,115,642.695)?

From WolfPack: (**40°26'29.65168" N, 88°23'42.09876" W, 182.974 m**)

**13.25** Same as Problem 13.24, except with geocentric coordinates in meters are (738,640.328, −5,498,206.005, 3,136,724.170)?

From WolfPack: (**29°38'59.50029 " N, 82°20'54.88020" W, 45.110 m**)

**13.26** Same as Problem 13.24, except with geocentric coordinates in meters are (−1,556,234.893, −5,169,323.859, 3,387,440.832)?

From WolfPack: (**32°16'51.13326" N, 106°45'16.28109 " W, 1193.043 m**)

**13.27** The GNSS determined height of a station is 288.038 m. The geoid height at the point is −31.068 m.

**319.106 m** by Equation (13.8): $H = h - N$

***13.28** The GNSS determined height of a station is 84.097 m. The geoid height at the point is −30.025 m. What is the elevation of the point?

**114.122 m** by Equation (13.8) : $H = h - N$

**13.29** Same as Problem 13.28, except the height is 414.805 m and the geoid height is −28.968 m.

**443.773 m** by Equation (13.8): $H = h - N$

***13.30** The orthometric height of a point is 124.886 m. The geoid height of the point is −28.998 m. What is the geodetic height of the point?

**95.888 m** by Equation (13.8): $H = h - N$

**13.31** Same as Problem 13.30, except the orthometric height is 1350.984 m, and the geoid height is −22.232 m.

**1350.984 m** by Equation (13.8): $H = h - N$

**13.32** The geodetic heights of two stations are 324.685 m and 309.879 m, and their orthometric heights are 356.496 m and 341.707 m, respectively. These stations have model-derived geoid heights of −31.827 and −31.835 respectively. What is the orthometric height of a station with a GNSS measured height of 305.645 m and a model-derived geoid height of −31.802 m?

**134.376 m**

| $N_{GPS}$ | $\Delta N$ |
|---|---|
| 324.685 − 356.496 = −31.811 | −31.811 + 31.827 = 0.016 |
| 309.879 − 341.707 = −31.828 | −31.828 + 31.835 = 0.007 |

Avg $\Delta N$ = 0.0115 m H = 305.645 −(−31.802 + 0.0115) = **337.434 m**

**13.33** Why are satellites at an elevation below 10° from the horizon eliminated from the positioning solution?

From Section 13.6.2, last paragraph: To minimize the errors caused by refraction.

**13.34** Why should NOAA's space weather center be consulted at the beginning of any day where a GNSS survey is planned?

From Section 13.6.3: "During periods of high solar activity, the errors due to ionospheric refraction can be large. Since the ionosphere will remain charged for extensive periods of time, there will be some days when a satellite survey simply should not be attempted. In periods of very high solar activity, radio signals from the satellites may be interrupted. Additionally during these periods, radio communication between the base and roving receivers in an RTK survey (see Chapter 15) may be compromised. The National Oceanic and Atmospheric Administration (NOAA) Space Weather Prediction Center11 provides forecasts for solar activity and its effect on radio communications. In particular, users should monitor geomagnetic storms, solar radiation storms, and radio blackouts. Geomagnetic storms may cause satellite orientation problems, increasing broadcast ephemeris errors, satellite communication problems and can lead to problems in initialization. Solar radiation storms may also create problems with satellite operations, orientation, and communications, which can cause increased noise at the receiver resulting in degraded positioning. Radio blackouts can cause intermittent loss of satellite and radio communications, which can increase noise at the receiver degrading positional accuracy. These are identified on the NOAA website in five categories from mild to extreme. In general, a satellite survey should not be attempted when any of the three is rated in the range from strong to extreme. There is a global positioning dashboard on the space weather site that provides GPS user with a quick check on solar activity of the day and the predicted future."

# 14 GLOBAL NAVIGATION SATELLITE SYSTEMS—STATIC SURVEYS

Asterisks (*) indicate problems that have partial answers given in Appendix G.

**14.1** For a 5-km baseline using a dual-frequency receiver, (a) what static surveying method should be used, (b) for what time period should the baseline be observed, and (c) what epoch rate should be used?

(a) Rapid static survey (b) 15 min (c) 5 sec

From Section 14.2.2: Baselines over 20 km should be surveyed using the static survey method. From Table 14.1 the length of the session with a dual-frequency receiver should be 10 + 1(5) = 15 min. From Section 14.2.1, the epoch rate using the static survey method is typically set at 5 sec.

**14.2*** When using the static surveying method, what is the minimum recommended length of the session required to observe a baseline that is 30-km long with a dual-frequency receiver?

From Table 14.1: **80 min** = 20 min + 2(30) min

**14.3** What would be the recommended epoch rate for the survey given in Problem 14.2?

From Section 14.2.2, the typical epoch rate is **15 sec**.

**14.4** Crew B starts their static session 5 minutes after crew A does. Both collect data for 20 min. What is the actual session time?

**15 min**

From Section 14.1: "An observation session denotes the period of time during which all receivers being employed on a project have been setup on designated stations, and are simultaneously engaged in receiving signals from the same satellites." Thus since crew B started 5 min after A, they only performed 15 min of simultaneous data collection.

**14.5** What variables affect the accuracy of a static survey?

From Section 14.1: Besides what is mentioned in Section 13.6, other important variables that bear on the accuracy of a static survey include the (1) accuracy of the reference station(s) to which the survey will be tied, (2) number of satellites visible during the survey, (3) geometry of the satellites during the observation sessions, (4) atmospheric conditions during the observations, (5) lengths of observation sessions, (6) number and nature of obstructions at the proposed receiver stations, (7) number of redundant observations taken in the survey, and (8) method of reduction used by the software.

**14.6** Why are dual-frequency and GNSS receivers preferred for high-accuracy control stations?

From Section 14.1, paragraph 3: "In precise surveys, GNSS and dual-frequency receivers are preferred for several reasons: they can (a) collect the needed data faster; (b) observe longer baselines with greater accuracy; and (c) almost eliminate certain errors, such as ionospheric refraction, and therefore yield higher positional accuracies."

**14.7** What site conditions are required for a good GNSS session?

From Section 14.3.4: "Once the existing nearby control points and new stations have been located on paper, a reconnaissance trip to the field should be undertaken to check the selected observation sites for (1) overhead obstructions that rise above 10° to 20° from the horizon, (2) reflecting surfaces that can cause multipathing, (3) nearby electrical installations that can interfere with the satellite's signal, and (4) other potential problems."

**14.8** Why is it recommended to use a precise ephemeris when processing a static survey?

From 14.5, paragraph 3 and Section 13.6.3: The broadcast ephemeris is a near-future prediction of the location of the satellites whereas a precise ephemeris is their tracked position. Thus orbital errors are removed by processing with a precise ephemeris.

**14.9** What are the recommended rates of data collection in a **(a)** static survey, and a ***(b)** rapid static survey?

**(a)** <u>**15 sec**</u>

***(b)** <u>**5 sec**</u>

**14.10** What restrictions apply to performing a rapid static survey?

From Section 14.2.2: "The rapid static method is suitable for observing baselines up to 20 km in length under good observation conditions. … However to achieve these accuracies, optimal satellite configurations (good PDOP), lack of multipathing, and favorable ionospheric conditions must exist."

**14.11*** How many nontrivial baselines will be observed in one session with four receivers?

<u>3</u> by Equation (14.1a): (4 – 1) = 3

**14.12** How many trivial baselines will be created in one session with four receivers?

<u>3</u> by Equation (14.1c): $t = \frac{(4-1)(4-2)}{2} = 3$

**14.13** A site has some overhead obstructions that are over 10° in altitude. What steps should occur in presurvey planning?

From Section 14.3.1: An obstruction diagram should be overlaid a satellite sky plot to find the best times of day to occupy the station.

**14.14** A baseline of approximately 10 mi is to be observed with a GNSS receiver. **(a)** What method of GNSS survey should be used? **(b)** How long should the observing session be? **(c)** What should the epoch rate be set to?

**(a)** **<u>rapid static survey method</u>**

**(b)** **<u>26 min</u>**; 10 mi ≈ 16 km; from Table 14.1: $10 + 1(16) = 26$ min

**(c)** **<u>5 sec epoch rate</u>**

**14.15** Describe what a trivial baseline is in a static survey.

From Section 14.3.4 "If more than two receivers are used, both nontrivial and *trivial* (*mathematically dependent*) baselines will result."

**14.16** What problems can overhead obstructions cause in a GNSS survey?

From Section 14.3.1, paragraph 3: "Canopy restrictions may possibly block satellite signals, thus reducing observations and possibly adversely affecting satellite geometry."

**14.17** A planned survey site has several potential obstructions near its border. What tools can a surveyor use to find the optimal time for GNSS sessions?

From Section 14.3.1, paragraph 9: "When overhead obstructions are a concern, the elevations and azimuths of the obstructions can be overlaid with the sky plot to form *obstruction diagrams*. The diagram will then show whether crucial satellites are removed by the obstructions and also indicate the best times to occupy the station to avoid the obstructions."

**14.18*** When using three receivers, how many sessions will it take to independently observe all the baselines of a hexagon?

**<u>5 sessions</u>**; Using Equations (14.1): 2 nontrivial lines per session with a total of 10 lines.

**14.19** What determines the appropriate method to use in a GNSS survey?

From Section 14.3.2, paragraph 1: "Because of the variability in requirements, the selection of the appropriate survey method is dependent on the (1) desired level of accuracy in the final coordinates, (2) intended use of the survey, (3) type of equipment available for the survey, (4) size of the survey, (5) canopy and other local conditions for the survey, and (6) available software for reducing the data, there is seldom only one method for accomplishing the work."

**14.20** What items should be checked during a field reconnaissance to ensure the site is suitable for a GNSS survey?

From Section 14.3.3, paragraph 1: "Once the existing nearby control points and new stations have been located on paper, a reconnaissance trip to the field should be undertaken to check the selected observation sites for (1) overhead obstructions that rise above 10° to 15° from the horizon, (2) reflecting surfaces that can cause multipathing, (3) nearby electrical installations that can interfere with the satellite's signal, and (4) other potential problems."

**14.21** In preplanning it is noticed that for 20 min there is only one satellite in the NW quadrant of the sky plot. An obstruction will block this satellite for five min during this time.

**(a)** What concerns should this raise about the survey at this site?

The obstruction will cause a loss of lock on the satellite in the NW quadrant and will likely cause high PDOP during this time.

**(b)** What can be done to ensure a successful survey at this site?

The easiest solution is to avoid occupation of this station during this time period. The other possibility is to use the static survey method, which can often survive poor observation conditions due to the length of the session and the changes in geometry over the session.

**14.22** Why should the height of the antenna be listed on the site log sheet?

From Section 14.5: "As the observation files are downloaded, special attention should be given to checking station information that is read directly from the file with the site log sheets. Catching incorrectly entered items such as station identification, antenna heights, and antenna offsets at this point can greatly reduce later problems during processing."

**14.23** What is a satellite availability chart and how is it used?

From Section 14.3.1, paragraph 5: "To aid in selecting suitable observation windows, a satellite availability plot, as shown in Figure 14.7, can be applied." It also shows the PDOP, HDOP, and VDOP, which allows the user to pick the optimal time to collect observations.

**14.24*** What order of accuracy does a survey with a standard deviation in the geodetic height difference of 15 mm between two control stations that are 5 km apart meet?

**Fourth order, Class II**; $\frac{15}{\sqrt{5}} = 6.71$, which is under 4th order, class II of 15.0.

**14.25** Do Problem 14.24 when the standard deviation in the geodetic height difference is 5 mm for two control points 15 km apart?

**Second order, Class II**; $\frac{5}{\sqrt{15}} = 1.29$, which is under 2rd order, class II of 1.3.

**14.26** Use the NGS web site to download the station coordinates for the nearest CORS station.

Answers will vary.

**14.27** What steps may be taken to identify an obstruction problem during the post-processing of a GNSS static survey?

From Section 14.5, paragraph 3: "Timelines that show discontinuity indicate loss of lock problems. If images are available for the setup, they may show the problem at the station. However when no images are available, the station's geodetic coordinates can be entered into software, such as GOOGLE EARTH® to view the setup conditions. Of course in the latter case it should be realized that the site conditions may have changed since the images were taken, and thus a site visit may be warranted."

**14.28** Why should repeat baselines be performed in a static survey?

From Section 14.5.3: "Another procedure employed in evaluating the consistency of the observed data and in weeding out blunders is to make repeat observations of certain baselines. These repeat measurements are taken in different observing sessions and the results compared. Significant differences in repeat baselines indicate problems with field procedures or hardware."

**14.29** What order of survey should be performed for a mapping control survey?

From Section 14.5.1, paragraph 1: "The FGCS document also makes recommendations concerning categories of surveys for which the different orders of accuracy are appropriate. Some of these recommendations include: order AA for global and regional geodynamics and deformation measurements; order A for "primary" networks of the National Spatial Reference System (NSRS), and regional and local geodynamics; order B for "secondary" NSRS networks and high-precision engineering surveys; and the various classes of order C for mapping control surveys, property surveys, and engineering surveys."

**14.30** Using loop *AEDCA* from Figure 14.6, and the data from Table 14.6, what is the

**(a)** Misclosure in the X component? **29.0 mm**
**(b)** Misclosure in the Y component? **6.9 mm**
**(c)** Misclosure in the Z component? **21.6 mm**
**(d)** Length of the loop misclosure? **24,631.1 m**
**(e)*** Derived ppm for the loop? **1.49 ppm**

**14.31** Do Problem 14.30 with loop *AFEA*.

Using observation *AF*

**(a)** Misclosure in the X component? **37.7 mm**
**(b)** Misclosure in the Y component? **10.5 mm**
**(c)** Misclosure in the Z component? **18.4 mm**
**(d)** Length of the loop misclosure? **11,692.5 m**
**(e)** Derived ppm for the loop? **3.7 ppm**

Using observation *FA*

**(a)** Misclosure in the X component? **32.3 mm**
**(b)** Misclosure in the Y component? **16.2 mm**
**(c)** Misclosure in the Z component? **0.5 mm**
**(d)** Length of the loop misclosure? **11,692.5 m**
**(e)** Derived ppm for the loop? **3.1 ppm**

**14.32** Do Problem 14.30 with loop *BFDB*.

Using observation *BF*

**(a)** Misclosure in the X component? **−2.2 mm**
**(b)** Misclosure in the Y component? **−4.4 mm**
**(c)** Misclosure in the Z component? **9.7 mm**
**(d)** Length of the loop misclosure? **17,877.7 m**
**(e)** Derived ppm for the loop? **0.38 ppm**

Using observation *FB*
**(a)** Misclosure in the X component? **−2.1 mm**
**(b)** Misclosure in the Y component? **6.3 mm**
**(c)** Misclosure in the Z component? **−1.3 mm**
**(d)** Length of the loop misclosure? **17,877.7 m**
**(e)** Derived ppm for the loop? **0.61 ppm**

**14.33** A survey list the standard deviation in geodetic height of ±2.5 mm, which was derived from a baseline that is 5 km long. What NGS geodetic height order and class does this meet?

**2nd Order, Class II**, since *b* is less than 3.0; $b = \frac{2.5}{\sqrt{5}} = 1.12$

**14.34** The observed baseline vector components in meters between two control stations is (2405.654, −5618.606, 1243.666). The geocentric coordinates of the control stations are (1,162,247.650, −4,882,012.315, 4,182,563.098) and (1,164,653.289, −4,887,630.930, 4,183,806.756). What are:

***(a)** ΔX ppm? **2.40 ppm**

**(b)** ΔY ppm? **1.44 ppm**

**(c)** ΔZ ppm? **1.28 ppm**

**14.35** Same as Problem 14.34 except the two control station have coordinates in meters of (1,130,295.165, −5,498,572.893, 3,018,271.182) and (1,130,753.822, −5,497,667.568, 3,019,353.825), and the baseline vector between them was (458.657, 905.322, 1082.640).

**(a)** ΔX ppm? **6.06 ppm**

**(b)** ΔY ppm? **2.02 ppm**

**(c)** ΔZ ppm? **2.02 ppm**

**14.36** Baseline *EA* for Figure 14.6 is resurveyed at a later time as (5321.7135, −3634.0712, −3173,6583). What is

***(a)** ΔX ppm? 0.40 ppm

**(b)** ΔY ppm? 0.58 ppm

**(c)** ΔZ ppm? 0.96 ppm

**14.37** Repeat Problem 14.36 for baseline *CF* of Figure 14.6, which had vector components of (−10,527.7683, 994.9506, 956.6045).

**(a)** $\Delta X$ ppm? **<u>1.59 ppm</u>**

**(b)** $\Delta Y$ ppm? **<u>1.21 ppm</u>**

**(c)** $\Delta Z$ ppm? **<u>1.89 ppm</u>**

**14.38** During the first three minutes of a session the observer notices that the number of locked satellites varies between 7 and 8. What is the possible reason for this and how could it be checked in the field?

The variation in the number of satellites locked in the survey could be caused by an obstruction such as a tree where signals from the satellite are being repeatedly interrupted by the obstruction. This possibility can be verified by checking the skyplot of the satellites, properly orienting it, and checking for a satellite that is just above the mask angle and in the direction of a potential obstruction.

# 15 GLOBAL NAVIGATION SATELLITE SYSTEMS—KINEMATIC SURVEYS

Asterisks (*) indicate problems that have partial answers given in Appendix G.

**15.1*** What are the two types of kinematic survey?

**Post-processed kinematic (PPK) and Real-time kinematic (RTK)**

From Section 15.1, paragraph 2.

**15.2** What are the commonalities between a static and kinematic survey?

From Section 15.1, paragraph 3: "For example, a kinematic survey requires two receivers collecting observations simultaneously from a pair of stations with one receiver, the base, occupying a station of known position and another, the rover, collecting data on points of interest. It also uses relative positioning computational procedures similar to those used in static surveys. Thus, it requires that the integer ambiguities (see Section 13.5.2) be resolved before the survey is started."

**15.3** What is the main difference between a static survey and a kinematic survey?

From Section 15.1, paragraph 3: "The main difference between static and kinematic surveying techniques is the length of time per session. In a kinematic survey, observations from a single epoch may be all that are used to determine position of the roving receiver." Also, it is a lower accuracy than static surveys but is more productive.

**15.4** Why are kinematic surveys vulnerable to poor observation conditions?

From Section 15.2, paragraph 1: "However, kinematic surveys are particularly vulnerable to poor observation conditions due to the relatively low number of observations typically taken at any location and the lack of significant changes in satellite geometry."

**15.5** What items should be included in planning for a kinematic survey?

From Section 15.2: There are several including high solar activity, multipathing, canopy restrictions, and PDOP.

**15.6*** How much error in horizontal position occurs if the antenna is mounted on a 2.000 m pole that is 10 min out of level?

**±3.8 mm**; $\sigma_\theta = 10'\sqrt{0.5(4-\pi)} = \pm 6.55'$ $E_R = \frac{6.55(60)}{206264.8} 2{,}000 = \pm 3.8 \text{ mm}$

**15.7** Do Problem 15.6, but this time assume the level is 20 min out of level.

**±7.6 mm**; $\sigma_\theta = 20'\sqrt{0.5(4-\pi)} = \pm 13.1'$ $E_R = \frac{13.1(60)}{206264.8} 2{,}000 = \pm 7.6 \text{ mm}$

**15.8** How much error in vertical position occurs with the situation described in Problem 15.6?

**<u>±0.008 mm</u>** $2 - 2\cos 10'$

**15.9** How much error in the vertical position occurs if the GNSS antenna is mounted on a 2.000 m pole that is 20 min out of level?

**<u>±0.034 mm</u>**; $2 - 2\cos 20'$

**15.10*** Why should the radio antenna at the base station be mounted as high as possible?

From Section 15.4, paragraph 6: "Mounting the radio antenna high can increase the range of the base radio."

**15.11** A storm front passes over the base station, which is 4 km away from the rover. Why should the GNSS survey be stopped until the same front passes over the rover?

From Section 15.2, paragraph 4: "Except when using real-time networks (see Section 15.8), the software used in kinematic surveys assumes that both base and roving receivers are in the same atmospheric conditions. Thus, baselines should be less than 20 km and surveys should be suspended when the base and roving receiver are not in similar conditions such as when a storm front passes thru the project area." These condition cause differing tropospheric refraction conditions at the base and rover.

**15.12** Why should a control station be established at the start of the kinematic survey?

From Section 15.3, last paragraph: "Prudent surveyors will check the OTF solution for the ambiguities. To do this, a control baseline is established immediately after the ambiguities have been resolved. This baseline is established using the initial ambiguity solution. The receiver is then "dumped," which is a process of losing lock on all the satellites by inverting the receiver. The OTF method is then allowed to resolve the ambiguities for each satellite again. Once this is accomplished, the baseline established previously is checked. The two solutions should match within the accuracy of the survey, which should be less than 3 cm typically. If the baseline does not check, a second dump of the receiver can be performed to check either of the previous solutions. If none of these attempts produce a suitable check on the baseline, other conditions such as canopy restrictions, multipathing, high solar activity, and so on should be investigated before the survey is attempted. Once satisfied with the solution, the surveyor can use this baseline to re-establish the ambiguities as necessary. This process ensures the same solution for the ambiguities during the entire project." As stated later in the chapter, this baseline can also be used to re-establish the same ambiguities or check the survey later to ensure that everything is working properly.

**15.13** Why is a 2-m rod recommended for the roving receiver?

From Section 15.4, paragraph 1: "The operator's body can be an obstruction when performing a kinematic survey. Thus as shown in Figure 15.1, the antenna is often mounted on a fixed-height rod that is 2 m in length to avoid operator obstructions."

**15.14** What are autonomous coordinates?

From Section 15.6, paragraph 2: "When no reference station is available for the base, it can be started using *autonomous* coordinates. These coordinates are simply the code-based solution for the position of the receiver, which is only good to several feet in accuracy."

**15.15** The manufacturer-specified accuracy for a PPK survey is ±(10 mm + 1 ppm) at 68% probability. What is the 95% error for a control baseline that is 3 km long, which is observed using the stop-and-go method?

**±2.0 cm**, By Equation (3.8) & (3.11): $1.9599\sqrt{10^2 + \left(\frac{1}{10^6}3{,}000{,}000\right)^2} = \pm 20.5\text{ mm}$

**15.16** What items on the survey controller indicate that the satellites may be obstructed?

From Section 15.6, paragraph 5: "Thus, it is important to watch the number of visible satellites, PDOP, and RMS of the solution while surveying. If canopy restrictions obscure satellites that are crucial to an accurate solution, the displayed PDOP and the RMS of the coordinates will increase."

**15.17** What is the difference between the true kinematic mode and the stop-and-go mode of data collection?

From Section 15.5: "In *true kinematic* mode, data is collected at a specific rate. … An alternative to the true kinematic mode is to stop for several epochs of data at each point of interest, which are averaged."

**15.18*** What frequencies found in RTK radios require licensure?

**450 - 470 MHz.** From Section 15.3: "In North America and in other areas of the world, frequencies in the range of 150–174 MHz in the VHF radio spectrum, and from 450–470 MHz in the UHF radio spectrum can be used for RTK transmissions." and from Section 15.6, paragraph 2: "The Federal Communications Commission (FCC) does not require a license for radios that broadcast in the range from 157 – 174 MHz. However, all other frequencies given in Section 15.4 do require a FCC license"

**15.19** What is localization of a survey?

From Section 15.9: It is the process of transforming satellite-derived coordinates into some local reference frame.

**15.20** How can an RTK survey become a PPK survey?

From Section 15.9, paragraph 2: "In fact since observational data can be saved during an RTK survey, an RTK survey can become a PPK survey if this data is later post-processed. This may be helpful when problems are experienced in the field. However, this would serve no purpose on a stakeout survey. Thus whether the survey is an RTK or PPK type of survey is often confused in conversation. Simply stated an RTK survey is required when positions are needed immediately in the field such as in a stakeout survey. A PPK survey is performed when locations are determined after post-processing of the data, which would be appropriate for a mapping survey."

**15.21** An RTN survey is being performed where the closest physical base station is 40 km away from the project area. It estimated horizontal precision is ±(8 mm + 0.5 ppm). What is the 95% horizontal precision using the stop-and-go methods?

**±4.2 cm**; By Equations (13.8) and (13.11): $1.9599\sqrt{0.8^2+\left(\frac{0.5}{10^6}400{,}000\right)^2} = 4.2$ cm

**15.22** What three surveying elements are needed in machine guidance and control?

From Section 15.10:

1. Digital terrain model
2. Sufficient horizontal and vertical control to support machine control in all areas of project and during all phases of the project.
3. Site calibration parameters for localization of project.

**15.23** What errors are modeled in a VRS?

From Section 15.8, paragraph 1: “Using the known positions of the base receivers and their observational data, the central processor models errors in the satellite ephemerides, range errors caused by ionospheric and tropospheric refraction, and the geometric integrity of the network stations.”

**15.24** What is the current broadcast ephemeris coordinate reference frame?

**WGS84 (G1674)**, from Section 15.8, paragraph 3.

**15.25** Why should a geoid model be included in the localization process?

From Section 15.8, paragraph 5: “It is important to include the geoid model when occupying control stations since the geoid model represents a systematic error in the vertical, which must be removed from the data.”

**15.26** What factors may determine the best location for a base station in a RTK survey?

From Section 15.3, paragraph 7: "Several factors may determine the “best” location for the base station in a RTK survey. Since the range of the radio can be increased with increasing height of the radio antenna, it is advantageous to locate the base station on a local high point. Additionally, since the base station in an RTK survey requires the most equipment, it is also preferable to place the base station in an easily accessible location."

**15.27** Why is the use of an RTN discouraged in machine guidance and control applications?

**Data latency**: From Section 15.8, paragraph 2: “The application software typically stops survey operations if the data latency becomes greater than a specified time interval. This value may be as great as four seconds! For this reason, real-time networks are not recommended in machine guidance and control operations (see Section 15.9).”

**15.28*** How many total pseudorange observations will be observed using a 1-sec epoch rate for a total of 3 min with 12 usable satellites?

**2160**; 60 obs/min(3 min)12 sats

**15.29** How many pseudorange observations will be observed using a 1-sec epoch rate for a total of 5 min with 10 usable satellites?

**3000**; 60 obs/min(5 min)10 sats

**15.30** Why are laser levels used in machine guidance and control?

From Section 15.10, paragraph 3: "The accuracy in using RTK is about 1 cm in horizontal and 1.5 cm in vertical at a 68% probability. This accuracy is sufficient for excavation purposes. However, finished surfaces need to have accuracies under 0.02 ft (5 mm). This accuracy is achieved by augmenting the machine guidance system with laser levels as shown in Figure 23.2 or robotic total stations."

**15.31** Why should periods of PDOP spikes be avoided in a kinematic survey?

Section 15.1, last paragraph: "Other factors that limit the positioning accuracy of kinematic surveys are its susceptibility to errors such as DOP spikes, atmospheric and ionospheric refraction, multipathing, and obstructions to satellite signals. For example if a kinematic survey collected data during the time of the PDOP spikes shown in Figure 14.3, the resulting positions during this time would be of considerably lower quality than those collected at other times of the day."

**15.32** Why should fixed height tripods or rods be used in a kinematic survey?

From Section 15.4, first paragraph: "In any case, the advantages of fixed-height rods and tripods in all GNSS surveys are that they minimize measurement errors in the height of the receiver and help avoid operator caused obstructions."

# 16 ADJUSTMENT BY LEAST SQUARES

Asterisks (*) indicate problems that have partial answers given in Appendix G.

**16.1** What fundamental condition is enforced by the method of weighted least squares?

From Section 16.2, paragraph 5: "If observed values are to be weighted in least-squares adjustment, then the fundamental condition to be enforced is that the sum of the weights times their corresponding squared residuals is minimized or, in equation form

$$\sum_{i}^{m} w_i v_i^2 = w_1 v_1^2 + w_2 v_2^2 + w_3 v_3^2 + \ldots + w_m v_m^2 \rightarrow \text{minimum"}$$

**16.2** What is true about the set of residuals produced by a least squares adjustment?

From Section 16.2, paragraph 1: "It produces that unique set of residuals for a group of observations that have the highest probability of occurrence."

**16.3** How are observations weighted in a least squares adjustment?

From Section 16.2, paragraphs 3&4: "$w_i = \frac{1}{\sigma_i^2}$ … This equations states that the weights are inversely proportional to variances."

**16.4*** What is the most probable value for the following set of ten distance observations in meters? 532.688, 532.682, 532.682, 532.684, 532.689, 532.686, 532.690, 532.684, 532.686, 532.686

<u>**532.686 m**</u>

**16.5** What is the standard deviation of the adjusted value in Problem 16.4?

<u>**±0.0028 m**</u>

**16.6** Three horizontal angles were observed around the horizon of station A. Their values are 42°12′13″, 59°56′15″, and 257°51′38″. Assuming equal weighting, what are the most probable values for the three angles?

<u>**42°12′11″**</u>, <u>**59°56′12″**</u>, and <u>**257°51′36″**</u>

Condition $x + y + z = 360°$ or $v_3 = -6'' - (v_1 + v_2)$

$$\sum v_1^2 + v_2^2 + \left(-6'' - v_1^2 - v_2^2\right) \rightarrow \text{ minimum}$$

Normal equations $\begin{matrix} 2v_1 + v_2 = -6'' \\ v_1 + 2v_2 = -6'' \end{matrix}$ yields $\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} -6 \\ -6 \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$

So, $v_1 = v_2 = v_3 = -2''$

**16.7** What are the standard deviations of the adjusted values in Problem 16.6?

**±2.8"** $S_0 = \sqrt{\frac{(-2)^2+(-2)^2+(-2)^2}{3-2}} = \pm 3.5\ ;\ \sigma = 3.5\sqrt{\frac{2}{3}} = \pm 2.8''$

**16.8** In Problem 16.6, the standard deviations of the three angles are ±2.5″, ±1.0″, and ±1.9″ respectively. What are the most probable values for the three angles?

**42°12′09.6″, 59°56′14.4″**, and **257°51′36″**

Normal equations:
$$2w_1v_1 + w_3v_2 = -w_3(-6")$$
$$w_3v_1 + 2w_2v_2 = -w_3(-6)"$$

| Normal Equations | | $A^TWL$ | X (″) |
|---|---|---|---|
| 0.437008 | 0.277008 | −1.66205 | −3.45 |
| 0.277008 | 1.277008 | −1.66205 | −0.55 |

$v_3 = -6" - (-3.45" - 0.55") = -2"$

**16.9*** Determine the most probable values for the *x* and *y* distances of Figure 16.2, if the observed lengths of *AC*, *AB*, and *BC* (in meters) are 315.297, 155.046, and 160.258, respectively.

***x* = 135.469** and ***y* = 158.609**

Normal equations:
$$2x + y = 470.343$$
$$x + 2y = 475.555$$

| Normal Equations | | $A^TL$ | X |
|---|---|---|---|
| 2 | 1 | 470.343 | 155.0437 |
| 1 | 2 | 475.555 | 160.2557 |

**16.10*** What are the standard deviations of the adjusted values in Problem 16.9?

**±0.0033 ft** for both

$$S_0 = \pm 0.0040;\ \Sigma_{xx} = 0.0040^2\begin{bmatrix} 2/3 & -1/3 \\ -1/3 & 2/3 \end{bmatrix}$$

**16.11** A network of differential levels is run from existing benchmark Juniper through new stations A and B to existing benchmarks Red and Rock as shown in the accompanying figure. The elevations of Juniper, Red, and Rock are 685.673, 696.745, and 705.253 m, respectively. Develop the observation equations for adjusting this network by least squares, using the following elevation differences.

| From | To | Elev. Diff. (m) | σ (m) |
|---|---|---|---|
| Juniper | *A* | 29.783 | 0.0019 |
| *A* | *B* | −17.000 | 0.0023 |
| *B* | Red | −1.713 | 0.0025 |
| *B* | Rock | 6.789 | 0.0028 |

$A = -685.673 + 29.783 = 715.456$

$-A + B = -17.000$

$-B = -696.745 - 1.713 = -698.458$

$-B = -705.253 + 6.789 = -698.464$

**16.12** For Problem 16.11, following steps outlined in Example 16.6 perform a weighted least-squares adjustment of the network. Determine weights based upon the given standard deviations. What are the

***(a)** Most probable values for the elevations of *A* and *B*? **715.457** and **698.459**

**(b)** Standard deviations of the adjusted elevations? **±0.0023** and **±0.0023**

**(c)** Standard deviation of unit weight? **±1.5**

**(d)** Adjusted elevation differences and their residuals?

| From | To | ΔElev (m) | V (m) | S |
|---|---|---|---|---|
| Juniper | A | 29.784 | 0.0013 | ±0.0023 |
| A | B | −16.998 | 0.0020 | ±0.0025 |
| B | Red | −1.714 | −0.0013 | ±0.0023 |
| B | Rock | 6.794 | 0.0047 | ±0.0023 |

**(e)** Standard deviations of the adjusted elevation differences? (See part d)

**16.13** Repeat Problem 16.12 using distances for weighting. Assume the following course lengths for the problem.

| From | To | Dist (m) |
|---|---|---|
| Juniper | A | 500 |
| A | B | 800 |
| B | Red | 1000 |
| B | Rock | 1300 |

**(a)** Most probable values for the elevations of *A* and *B*? **715.457** and **698.459**

**(b)** Standard deviations of the adjusted elevations? **±0.0022** and **±0.0023**

**(c)** Standard deviation of unit weight? **±0.00012**

**(d)** Adjusted elevation differences and their residuals?

| From | To | ΔElev (m) | V (m) | S |
|---|---|---|---|---|
| Juniper | A | 29.784 | 0.0012 | ±0.0022 |
| A | B | −16.998 | 0.0020 | ±0.0025 |
| B | Red | −1.714 | −0.0012 | ±0.0023 |
| B | Rock | 6.794 | 0.0048 | ±0.0023 |

**(e)** Standard deviations of the adjusted elevation differences? (See d)

**16.14** Use WOLFPACK to do Problem 16.12 and 16.13 and compare the solutions for *A* and *B*.

See solutions in Problem 16.12 and 16.13.

**16.15** Repeat Problem 16.12 using the following data.

| From | To | Elev. Diff. (m) | σ (m) |
|---|---|---|---|
| Juniper | *A* | 15.779 | 0.0023 |
| *A* | *B* | −2.448 | 0.0017 |
| *B* | Red | −2.261 | 0.0027 |
| *B* | Rock | 6.243 | 0.0023 |

**(a)** Most probable values for the elevations of *A* and *B*? **701.454** and **699.007**

**(b)** Standard deviations of the adjusted elevations? **±0.0020** and **±0.0018**

**(c)** Standard deviation of unit weight? **±1.2**

**(d)** Adjusted elevation differences and their residuals?

| From | To | Elev. Diff. | V | S |
|---|---|---|---|---|
| Juniper | *A* | 15.781 | 0.0020 | ±0.0020 |
| *A* | *B* | −2.447 | 0.0011 | ±0.0018 |
| *B* | Red | −2.262 | −0.0011 | ±0.0018 |
| *B* | Rock | 6.246 | 0.0029 | ±0.0018 |

**(e)** Standard deviations of the adjusted elevation differences? (See d)

**16.16** A network of differential levels is shown in the accompanying figure. The elevations of benchmarks *A* and *G* are 835.24 ft and 865.64 ft, respectively. The observed elevation differences and the distances between stations are shown in the following table. Using WOLFPACK, determine the

| From | To | Elev. Diff (ft) | S (ft) |
|---|---|---|---|
| *A* | *B* | 30.55 | 0.022 |
| *B* | *C* | −45.22 | 0.025 |
| *C* | *D* | 24.34 | 0.022 |
| *D* | *E* | 10.38 | 0.016 |
| *E* | *F* | −15.16 | 0.013 |
| *F* | *A* | −4.83 | 0.011 |
| *G* | *F* | −25.59 | 0.008 |
| *G* | *H* | −7.66 | 0.010 |
| *H* | *D* | −13.10 | 0.009 |
| *G* | *B* | 0.14 | 0.010 |
| *G* | *E* | −10.42 | 0.011 |

**(a)** Most probable values for the elevations of new benchmarks *B*, *C*, *D*, *E*, *F*, and *H*?

Adjusted Elevations

| Station | Elevation | S |
|---|---|---|
| B | 865.78 | 0.010 |
| C | 820.54 | 0.021 |
| D | 844.87 | 0.012 |
| E | 855.23 | 0.009 |
| F | 840.06 | 0.007 |
| H | 857.97 | 0.010 |

**(b)** Standard deviations of the adjusted elevations? See (a)

**(c)** Standard deviation of unit weight? 1.2

**(d)** Adjusted elevation differences and their residuals?

Adjusted Elevation Differences

| From | To | Elevation Difference | V | S |
|---|---|---|---|---|
| A | B | 30.54 | -0.011 | 0.010 |
| B | C | -45.24 | -0.017 | 0.021 |
| C | D | 24.33 | -0.013 | 0.020 |
| D | E | 10.36 | -0.023 | 0.013 |
| E | F | -15.17 | -0.007 | 0.010 |
| F | A | -4.82 | 0.011 | 0.007 |
| G | F | -25.58 | 0.009 | 0.007 |
| G | H | -7.67 | -0.006 | 0.010 |
| H | D | -13.11 | -0.005 | 0.009 |
| G | B | 0.14 | -0.001 | 0.010 |
| G | E | -10.41 | 0.006 | 0.009 |

**(e)** Standard deviations of the adjusted elevation differences? See (d).

**16.17** Develop the observation equations for line *AB* and *BC* in Problem 16.16.

$B = 865.79 + v_1$
$-B + C = -45.22 + v_2$

**16.18** A network of GNSS observations shown in the accompanying figure was made with two receivers using the static method. Known coordinates of the two control stations are in the geocentric system. Develop the observation equations for the following baseline vector components.

| Station | X (m) | Y (m) | Z (m) |
|---|---|---|---|
| Jim | 1,644,485.933 | −4,214,787.021 | 4,480,735.160 |
| Al | 1,642,889.045 | −4,212,512.282 | 4,483,439.750 |

| **Jim to Troy** | | | | | **Al to Troy** | | |
|---|---|---|---|---|---|---|---|
| 2411.495 | 7.15E−6 | 2.52E−7 | 3.14E−6 | 4008.387 | 7.53E−6 | 4.79E−7 | −1.53E-8 |
| 2507.803 | | 7.41E−6 | 7.28E−8 | 233.067 | | 7.51E−6 | 2.03E−7 |
| 1464.623 | | | 7.15E−6 | −1239.971 | | | 7.73E−6 |

$$X_{Troy} = 1,646,897.428 + v_1$$
$$Y_{Troy} = -4,212,279.218 + v_2$$
$$Z_{Troy} = 4,482,199.783 + v_3$$
$$X_{Troy} = 1,646,897.432 + v_4$$
$$Y_{Troy} = -4,212,279.215 + v_5$$
$$Z_{Troy} = 4,452,199.779 + v_6$$

**16.19** For Problem 16.18, construct the *A* and *L* matrices.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad L = \begin{bmatrix} 1,646,897.428 \\ -4,212,279.218 \\ 4,482,199.783 \\ 1,646,897.432 \\ -4,212,279.215 \\ 4,452,199.779 \end{bmatrix}$$

**16.20** For Problem 16.18, construct the covariance matrix.

$$\Sigma = \begin{bmatrix} 7.15\text{E}-6 & 2.52\text{E}-7 & 3.14\text{E}-6 & 0 & 0 & 0 \\ 2.52\text{E}-7 & 7.41\text{E}-6 & 7.28\text{E}-8 & 0 & 0 & 0 \\ 3.14\text{E}-6 & 7.28\text{E}-8 & 7.15\text{E}-6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 7.53\text{E}-6 & 4.79\text{E}-7 & -1.53\text{E}-8 \\ 0 & 0 & 0 & 4.79\text{E}-7 & 7.51\text{E}-6 & 2.03\text{E}-7 \\ 0 & 0 & 0 & -1.53\text{E}-8 & 2.03\text{E}-7 & 7.73\text{E}-6 \end{bmatrix}$$

**16.21** Use WOLFPACK to adjust the baselines of Problem 16.18.

```
Degrees of Freedom = 3
Reference Variance = 1.096
Standard Deviation of Unit Weight = ±1.0

************************
Adjusted Distance Vectors
************************
From        To          dX           dY           dZ        Vx       Vy       Vz
===============================================================================
Jim      Troy     2411.496     2507.804     1464.622   0.0014   0.0015  -0.0014
 Al      Troy     4008.384      233.065    -1239.968  -0.0026  -0.0015   0.0026

*****************************************************
Advanced Statistical Values
*****************************************************
      From        To      ±S      Slope Dist    Prec       ppm
==============================================================
       Jim      Troy  0.0034       3,774.853 1,101,000    0.91
        Al      Troy  0.0034       4,202.260 1,225,000    0.82

*******************
```

```
Adjusted Coordinates
********************
Station           X                Y                 Z       Sx      Sy      Sz
===============================================================================
 Jim    1,644,485.933   -4,214,787.021    4,480,735.160
  Al    1,642,889.045   -4,212,512.282    4,483,439.750
Troy    1,646,897.429   -4,212,279.217    4,482,199.782  0.0020  0.0020  0.0020
```

**16.22** Convert the geocentric coordinates obtained for station Troy in Problem 16.21 to geodetic coordinates using the WGS84 ellipsoidal parameters.

**(44°56′03.13148″ N, 68°38′44.61231″ W, 35.023 m)**

**16.23** A network of GNSS observations shown in the accompanying figure was made with two receivers using the static method. Use WOLFPACK to adjust the network, given the following data.

| Station | X (m) | Y (m) | Z (m) |
|---|---|---|---|
| Bonnie | −2,660,581.015 | −1,513,935.768 | 5,576,785.765 |
| Tom | −2,648,294.114 | −1,526,048.226 | 5,579,322.060 |

**Bonnie to Ray**

| | | | |
|---|---|---|---|
| −3,886.055 | 3.06E−5 | −1.04E−7 | 6.28E−7 |
| −15,643.129 | | 3.14E−5 | 6.86E−7 |
| −6,079.276 | | | 3.06E−5 |

**Bonnie to Herb**

| | | | |
|---|---|---|---|
| 10,207.052 | 1.68E−5 | 7.85E−7 | 3.15E−7 |
| −2,006.464 | | 1.93E−5 | 4.63E−7 |
| 4,295.068 | | | 1.70E−5 |

**Tom to Ray**

| | | | |
|---|---|---|---|
| −16,172.951 | 3.43E−5 | 1.64E−6 | 4.06E−7 |
| −3,530.664 | | 3.49E−5 | 7.71E−7 |
| −8,615.580 | | | 3.68E−5 |

**Tom to Herb**

| | | | |
|---|---|---|---|
| −2,079.844 | 1.51E−5 | 3.78E−7 | −1.90E−6 |
| 10,105.996 | | 1.51E−5 | 1.45E−6 |
| 1,758.774 | | | 1.54E−5 |

**Bonnie to Tom (Fixed line—Don't use in adjustment.)**

| | | | |
|---|---|---|---|
| 12,286.899 | 3.21E−5 | −6.99E−7 | 1.20E−6 |
| −12,112.451 | | 3.11E−5 | 6.65E−7 |
| 2,536.295 | | | 3.21E−5 |

```
Degrees of Freedom = 9
Reference Variance = 0.5519
Standard Deviation of Unit Weight = ±0.74

*************************
Adjusted Distance Vectors
*************************
      From       To        dX          dY          dZ       Vx       Vy       Vz
=================================================================================
    Bonnie      Ray   -3886.053  -15643.126   -6079.280   0.0022   0.0033  -0.0041
    Bonnie     Herb   10207.055   -2006.463    4295.069   0.0027   0.0011   0.0007
       Tom      Ray  -16172.954   -3530.668   -8615.575  -0.0028  -0.0037   0.0049
       Tom     Herb   -2079.846   10105.995    1758.774  -0.0023  -0.0009  -0.0003
    Bonnie      Tom   12286.901  -12112.458    2536.295   0.0020  -0.0070  -0.0000

*****************************************************
Advanced Statistical Values
*****************************************************
      From       To      ±S      Slope Dist    Prec       ppm
```

```
==============================================================
    Bonnie       Ray  0.0052      17,226.910 3,300,000    0.30
    Bonnie      Herb  0.0037      11,254.220 3,067,000    0.33
       Tom       Ray  0.0052      18,661.677 3,575,000    0.28
       Tom      Herb  0.0037      10,466.622 2,853,000    0.35
```

```
********************
Adjusted Coordinates
********************
   Station            X                Y                Z       Sx      Sy      Sz
=====================================================================================
    Bonnie   -2,660,581.015   -1,513,935.768    5,576,785.765
       Tom   -2,648,294.114   -1,526,048.226    5,579,322.060
       Ray   -2,664,467.068   -1,529,578.894    5,570,706.485  0.0030  0.0030  0.0030
      Herb   -2,650,373.960   -1,515,942.231    5,581,080.834  0.0021  0.0022  0.0021
```

**16.24** For Problem 16.23, write the observation equations for the baselines "Bonnie to Ray" and "Tom to Herb."

$$X_{Ray} = -2,664,467.070 + v_x \qquad X_{Herb} = -2,650,373.958 + v_x$$

$$Y_{Ray} = -1,529,578.897 + v_y \qquad Y_{Herb} = -1,515,942.230 + v_y$$

$$Z_{Ray} = 5,570,706.489 + v_z \qquad Z_{Herb} = 5,581,080.834 + v_z$$

**16.25** For Problem 16.23, construct the $A$, $X$, and $L$ matrices for the observations.

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad X = \begin{bmatrix} X_{Ray} \\ Y_{Ray} \\ Z_{Ray} \\ X_{Herb} \\ Y_{Herb} \\ Z_{Herb} \end{bmatrix} \quad L = \begin{bmatrix} -2,664,467.070 \\ -1,529,578.897 \\ 5,570,706.489 \\ -2,650,373.963 \\ -1,515,942.232 \\ 5,581,080.833 \\ -2,664,467.065 \\ -1,529,578.890 \\ 5,570,706.480 \\ -2,650,373.958 \\ -1,515,942.230 \\ 5,581,080.834 \end{bmatrix}$$

**16.26** For Problem 16.23, construct the covariance matrix.

$$\Sigma = \begin{bmatrix}
3.06E-5 & -1.04E-7 & 6.28E-7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1.04E-7 & 3.14E-5 & 6.86E-7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6.28E-7 & 6.86E-7 & 3.06E-5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1.68E-5 & 7.85E-7 & 3.15E-7 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 7.85E-7 & 1.93E-5 & 4.63E-7 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 3.15E-7 & 4.63E-7 & 1.70E-5 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 3.43E-5 & 1.64E-6 & 4.06E-7 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1.64E-6 & 3.49E-5 & 7.71E-7 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 4.06E-7 & 7.71E-7 & 3.68E-5 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.51E-5 & 3.78E-7 & -1.90E-6 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3.78E-7 & 1.51E-5 & -1.45E-6 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1.90E-6 & -1.45E-6 & 1.54E-5
\end{bmatrix}$$

**16.27*** After completing Problem 16.23, convert the geocentric coordinates for station Ray and Herb to geodetic coordinates using the WGS84 ellipsoidal parameters. (Hint: See Section 13.4.3)

| Station | Latitude | Longitude | h (m) |
|---|---|---|---|
| Ray | 61°17'07.15657" N | 150°08'28.85600" W | 21.952 |
| Herb | 61°28'46.72051" N | 150°13'53.79964" W | 24.991 |

**16.28** Following the procedures discussed in Section 14.5.2, analyze the fixed baseline from station Bonnie to Tom.

| **Dist:** 17,438.818 | | **ppm** |
|---|---|---|
| dX | 0.002 | 0.11 |
| dY | 0.007 | 0.40 |
| dZ | 0.000 | 0.00 |

**16.29** For the horizontal survey of the accompanying figure, determine initial approximations for the unknown stations. The observations for the survey are

| Station | X (ft) | Y (ft) |
|---|---|---|
| Dave | 2340.12 | 3363.45 |

| From | To | Azimuth | S |
|---|---|---|---|
| Dave | Wes | 19°37′57″ | 0.001” |

| From | To | Distance (ft) | σ (ft) |
|---|---|---|---|
| Dave | Steve | 330.23 | 0.01 |
| Steve | Frank | 435.36 | 0.01 |
| Frank | Wes | 351.07 | 0.01 |
| Wes | Dave | 442.82 | 0.01 |
| Dave | Frank | 543.29 | 0.01 |
| Steve | Wes | 567.75 | 0.01 |

| Backsight Station | Instrument Station | Foresight Station | Angle | σ″ |
|---|---|---|---|---|
| Wes | Dave | Frank | 40°06′22″ | ±6.7 |

| | | | | |
|---|---|---|---|---|
| Frank | Dave | Steve | 53°15′24″ | ±7.9 |
| Dave | Steve | Wes | 51°08′22″ | ±7.9 |
| Wes | Steve | Frank | 38°10′30″ | ±6.7 |
| Steve | Frank | Dave | 37°26′01″ | ±6.8 |
| Dave | Frank | Wes | 54°21′20″ | ±7.6 |
| Frank | Wes | Steve | 50°02′10″ | ±7.6 |
| Steve | Wes | Dave | 35°29′47″ | ±6.6 |

Initial approximations can vary slightly:

Steve: **(2644.10, 3234.42)**

Frank: **(2809.38, 3637.20)**

Wes: **(2488.91, 3780.55)**

**16.30*** Using the data in Problem 16.29, write the linearized observation equation for the distance from Steve to Frank.

$$-0.3796dx_{\text{Steve}} - 0.9251dy_{\text{Steve}} + 0.3796dx_{\text{Frank}} + 0.9251dy_{\text{Steve}} = -0.012$$

**16.31** Using the data in Problem 16.29, write the linearized observation equation for the angle Wes-Dave-Frank.

$$191.3134dx_{\text{Frank}} - 327.9478dy_{\text{Frank}} - 438.6960dx_{\text{Wes}} + 156.4938dy_{\text{Wes}} = 12.5886''$$

where coefficients are multiplied by 206264.8”/rad.

**16.32** Assuming a standard deviation of ±0.001” for the azimuth line Dave-Wes, use WOLFPACK to adjust the data in Problem 16.29.

```
*****************
Adjusted stations
*****************
                                                        Standard error ellipse computed
   Station            X             Y          Sx        Sy         Su         Sv      t
==========================================================================================
     Steve      2,644.11      3,234.43      0.009      0.009      0.011      0.007  51.23°
     Frank      2,809.38      3,637.20      0.007      0.011      0.011      0.007 176.06°
       Wes      2,488.91      3,780.54      0.003      0.007      0.008      0.000  19.63°

*******************************
Adjusted Distance Observations
*******************************
   Station   Station
  Occupied   Sighted      Distance         V          S
======================================================
      Dave     Steve        330.24     0.009      0.008
     Steve     Frank        435.37     0.007      0.008
     Frank       Wes        351.07    -0.002      0.008
       Wes      Dave        442.84     0.017      0.008
      Dave     Frank        543.28    -0.012      0.008
     Steve       Wes        567.74    -0.010      0.008

***************************
Adjusted Angle Observations
***************************
    Station   Station     Station
Backsighted  Occupied Foresighted          Angle       V        S"
=================================================================
        Wes      Dave       Frank      40°06'32"     10.2"      3.8
      Frank      Dave       Steve      53°15'20"     -3.5"      4.9
```

| | | | | | |
|---|---|---|---|---|---|
| Dave | Steve | Wes | 51°08'16" | -6.4" | 4.8 |
| Wes | Steve | Frank | 38°10'29" | -1.4" | 3.8 |
| Steve | Frank | Dave | 37°25'55" | -5.6" | 3.8 |
| Dave | Frank | Wes | 54°21'20" | -0.4" | 4.7 |
| Frank | Wes | Steve | 50°02'16" | 6.4" | 4.6 |
| Steve | Wes | Dave | 35°29'52" | 4.8" | 3.7 |

```
****************************
Adjusted Azimuth Observations
****************************
```

| Station Occupied | Station Sighted | Azimuth | V | S" |
|---|---|---|---|---|
| Dave | Wes | 19°37'59" | 0.0" | 0.0" |

```
***************************************
        Adjustment Statistics
***************************************
                Iterations = 2
              Redundancies = 9

        Reference Variance = 1.328
              Reference So = ±1.2
```

**16.33*** Given the following inverse matrix and a standard deviation of unit weight of 1.13, determine the parameters of the error ellipse.

$$(A^TWA)^{-1} = \begin{bmatrix} q_{xx} & q_{xy} \\ q_{xy} & q_{yy} \end{bmatrix} = \begin{bmatrix} 0.00016159 & -0.00001827 \\ -0.00001827 & 0.00028020 \end{bmatrix}$$

$t$ = 171°26'19.7"; $S_u = 1.13\sqrt{0.00028295} = 0.019$; $S_v = 1.13\sqrt{0.000015884} = 0.014$

**16.34** Compute $S_x$ and $S_y$ in Problem 16.33.

$$S_x = 1.13\sqrt{0.00016159} = 0.014;\ S_y = 1.13\sqrt{0.00028020} = 0.019$$

**16.35** Given the following inverse matrix and a standard deviation of unit weight of 1.15, determine the parameters of the error ellipse.

$$(A^TWA)^{-1} = \begin{bmatrix} q_{xx} & q_{xy} \\ q_{xy} & q_{yy} \end{bmatrix} = \begin{bmatrix} 0.0000532 & 0.0000149 \\ 0.0000149 & 0.0000418 \end{bmatrix}$$

$t$ angle: 92°20′06.1″; $S_u = \pm 0.008$ ft; $S_v = \pm 0.024$ ft;

**16.36** Compute $S_x$ and $S_y$ in Problem 16.35.

$$S_x = 1.15\sqrt{0.0000532} = \pm\mathbf{0.008};\ S_y = 1.15\sqrt{0.0000418} = \pm\mathbf{0.024}$$

**16.37** The well-known observation equation for a line is $mx + b = y + v_y$. What is the slope and y-intercept of the best fit line for a set of points with coordinates (1446.81, 2950.79), (2329.79, 2432.66), (3345.74, 1837.13), (478.72, 3517.64), (4382.98, 1229.16)?

**m = −0.5860; b = 3797.95**

**16.38** Use WOLFPACK and the following standard deviations for each observation to do a least squares adjustment of Example 10.4, and describe any differences in the solution. What advantages are there to using the least squares method in adjusting this traverse?

| Stations | Angle ± S | Stations | Distance ± S |
|---|---|---|---|
| *E-A-B* | 100°45′37″ ± 16.7″ | *AB* | 647.25 ± 0.027 |
| *A-B-C* | 231°23′43″ ± 22.1″ | *BC* | 203.03 ± 0.026 |
| *B-C-D* | 17°12′59″ ± 21.8″ | *CD* | 720.35 ± 0.027 |
| *C-D-E* | 89°03′28″ ± 10.2″ | *DE* | 610.24 ± 0.027 |
| *D-E-A* | 101°34′24″ ± 16.9″ | *EA* | 285.13 ± 0.026 |
| AZIMUTH *AB* | 126°55′17″ ± 0.001″ | | |

Adjusted stations

| Sta | Northing | Easting | Sn | Se | Su | Sv | t |
|---|---|---|---|---|---|---|---|
| B | 4,611.179 | 10,517.459 | 0.0099 | 0.0132 | 0.0165 | 0.0000 | 126.92° |
| C | 4,408.224 | 10,523.432 | 0.0172 | 0.0178 | 0.0193 | 0.0154 | 130.51° |
| D | 5,102.267 | 10,716.279 | 0.0232 | 0.0192 | 0.0256 | 0.0160 | 147.58° |
| E | 5,255.934 | 10,125.709 | 0.0150 | 0.0149 | 0.0175 | 0.0119 | 44.56° |

Adjusted Distance Observations

| Station Occupied | Station Sighted | Distance | V | S |
|---|---|---|---|---|
| A | B | 647.26 | -0.010 | 0.016 |
| B | C | 203.04 | -0.013 | 0.016 |
| C | D | 720.34 | 0.013 | 0.017 |
| D | E | 610.23 | 0.005 | 0.017 |
| E | A | 285.14 | -0.011 | 0.017 |

Adjusted Angle Observations

| Station Backsighted | Station Occupied | Station Foresighted | Angle | V | S |
|---|---|---|---|---|---|
| E | A | B | 100°45′44″ | -6.9″ | 9.1″ |
| A | B | C | 231°23′34″ | 8.8″ | 12.3″ |
| B | C | D | 17°12′51″ | 7.6″ | 10.2″ |
| C | D | E | 89°03′24″ | 4.4″ | 6.1″ |
| D | E | A | 101°34′27″ | -2.9″ | 8.4″ |

Adjusted Azimuth Observations

| Station Occupied | Station Sighted | Azimuth | V | S |
|---|---|---|---|---|
| A | B | 126°55′17″ | 0.0″ | 0.0″ |

-----Standard Deviation of Unit Weight = 0.700781

# 17 MAPPING SURVEYS

Asterisks (*) indicate problems that have answers given in Appendix G.

**17.1** What methods can be used to display relief on a map?

From Section 17.1, paragraph 3: "Relief is displayed on maps by using various conventions and procedures. For topographic maps, *contours* are commonly used and are preferred by surveyors and engineers. *Digital elevation models* (DEMs) and *three-dimensional perspective models* are methods for depicting relief, made possible by computers. *Color, hachures, shading*, and *tinting* can also be used to show relief, but these methods are not quantitative enough and thus are generally unsuitable for surveying and engineering work."

**17.2** What is the difference between a planimetric and topographic map?

From Section 17.1, paragraph 2: "Two different types of maps, *planimetric* and *topographic*, are prepared as a result of mapping surveys. The former depicts natural and cultural features in the plan (*X-Y*) views only. Objects displayed are called *planimetric features*. Topographic maps also include planimetric features, but in addition they show the configuration of the Earth's surface."

**17.3** Define map scale.

From Section 17.3, paragraph 1: "Map scale is the ratio of the length of a feature on a map to the true length of the feature."

**17.4** What different methods can be used to show a map's scale?

From Section 17.3, paragraph 1: "Map scales are given in three ways: (1) by ratio or representative fraction, such as 1:2000 or 1/2000; (2) by an equivalence, for example, and (3) by graphically using either a bar scale or labeled grid lines spaced throughout the map at uniform distances apart."

**17.5** Define the term large-scale map.

From Section 17.3, paragraph 3: A map with a scale of 1 in. = 200 ft or 1:2400 or larger.

**17.6*** On a map sheet having a scale of 1 in. = 360 ft, what is the smallest distance (in feet) that can be plotted with an engineer's scale? (Minimum scale graduations are 1/60th in.)

$$\underline{\textbf{6 ft}} = \frac{1}{60} 360 \text{ ft}$$

**17.7** What equivalent scales are suitable to replace the following ratio scales: 1:480, 1:720, 1:3600, and 1:6000?

**1 in. = 40 ft, 1 in. = 60 ft, 1 in. = 300 ft, 1 in. = 500 ft, respectively.**

**17.8** A topographic map has a contour interval of 1 ft and a scale of 1:600. If two adjacent contours are 0.25 in. apart, what is the average slope of the ground between the contours?

**8%**; scale =50 ft/in. H = 50(0.25) = 12.5 ft; slope = 1/12.5*100% = 8%

**17.9** What are the ratio scales for the equivalent scales of 1 in. = 20 ft, 1 cm = 5 m, and 1 in. = 30 ft?

**1:240, 1:500, 1:360, respectively.**

**17.10*** On a map whose scale is 1 in. = 40 ft, how far apart (in inches) would 1-ft contours be on a uniform slope (grade) of 2%?

**1.25 in.** x = 1/0.02 = 50 ft, map dist = 50 ft/40 ft/in. = 1.25 in.

**17.11** Why should caution be used in enlarging a map?

From Section 17.4, paragraph 6: "*However, enlargements must be produced with caution since any errors in the original maps or digital data are also magnified, and the enlarged product may not meet required accuracy standards*."

**17.12** On a map drawn to a scale of 1:1000, contour lines are 2 mm apart at a center location. The contour interval is 0.1 m. What is the ground slope in percent, between adjacent contours?

**5%**, H = 0.002(1000) = 2 m; Slope = (0.1/2)100% = 5%

**17.13** The line width on a map of scale 1:2000 is 0.2 mm. What should the accuracy of the survey be to draw this map?

**0.2 m**; 0.2(2000) = 400 mm; 0.4 m/2 = 0.2 m

**17.14** When should points be located for contours connected by straight lines? When by smooth curves?

Straight line: When slope of ground is uniform as on a highway

Smooth curves: When slope of ground is gradually changing on gently rolling land.

**17.15*** What conditions in the field need to exist when using kinematic satellite survey?

**No overhead obstructions/Canopy restrictions or multipathing conditions**

**17.16** What is a digital terrain model?

From Section 17.8, paragraph 1: "Data for use in automated contouring systems is collected in arrays of points whose horizontal positions are given by their *X* and *Y* coordinates and whose elevations are given as *Z* coordinates. Such three-dimensional arrays provide a *digital* representation of the continuous variation of relief over an area and are known as *digital elevation models* (DEMs). Alternatively, the term *digital terrain model* (DTM) is sometimes used."

**17.17** What is a contour? …a contour interval?

From Section 17.5, paragraph 2: "A contour is a line connecting points of equal elevation."

And from paragraph 3: "The vertical distance between consecutive level surfaces forming the contours on a map (the elevation difference represented between adjacent contours) is called the *contour interval*."

**17.18** Why are spot elevations placed on maps?

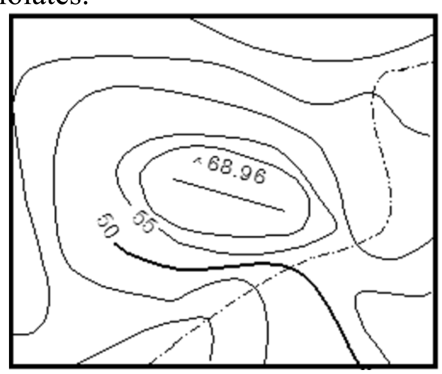
From Section 17.5, paragraph 5: "Spot elevations are used on maps to mark unique or critical points such as peaks, potholes, valleys, streams, and highway crossings. They may also be used in lieu of contours for defining elevations on relatively flat terrain that extends over a large area."

**17.19** Between the elevations of 649 and 659 at what elevations will contours be drawn for a 2-ft contour interval and which contours will be labeled?

**650**, 652, 654 656, and 658 with 650 labeled.

**17.20** Using the rules of contours, list the contouring mistakes that are shown in the accompanying figure and list the contouring rule it violates.

1. 65-ft contour violates item 10, which states that contours go in pairs along sides of ridges and item 11.
2. 55-ft contour should not be labeled.
3. 50-ft contour is not continuous which violates item 1.
4. 45-ft contour violates item 9 since it points both up and down the stream




**17.21** Describe the purpose of a breakline in defining contours by automated processes.

From Section 17.8, paragraph 6: "Breaklines are linear topographic features that delineate the intersection of two surfaces that have uniform slopes, and thus define changes in grade. *Automated mapping algorithms use these lines to define sides of the triangles that form the TIN model, and thus elevations are interpolated along them*. Streams, lake shores, roads, railroads, ditches, ridgelines, etc. are examples of controlling features or breaklines. Curved breaklines such as streams must have enough data points so that when adjacent ones are connected with straight lines, they adequately define the feature's alignment."

**17.22** Assuming *xy* coordinates for the instrument station of (5000, 5000), a backsight azimuth of 32°15′28″ and height of instrument of 53.76 ft, determine the coordinates and elevations for points 3, 4, and 5 in Table 17.1.

| Point | Hor. Angle | Zen. Angle | Distance | Notes |
|---|---|---|---|---|
| 3 | 16°37'44" | 90°25'50" | 165.85 | DTREE 24-in. Maple |
| 4 | 70°35'24" | 91°15'48" | 236.47 | MH Sanitary manhole |
| 5 | 225°14'22" | 88°30'36" | 665.93 | .BDG SE Corner of Building |

| Point | Azimuth | H-Dist | V-Dist | *x* | *y* | H |
|---|---|---|---|---|---|---|
| 3 | 48°53'12" | 165.85 | -4.25 | **5124.95** | **5109.06** | **49.51** |
| 4 | 102°50'52" | 436.36 | -9.62 | **5550.39** | **5012.02** | **39.88** |
| 5 | 257°29'50" | 265.84 | 6.91 | **5290.86** | **4954.47** | **46.80** |

**17.23** How is a tree with significant width located?

From Section 17.9.1, paragraph 4: "As shown in Figure 17.6, details that have width such as trees are located with two separate observations. The first observation locates the azimuth to the object by observing an angle from a reference line to the front center of the object. The second shot measures the distance to the side center of the object. Using the azimuth of the first shot and the distance from the second shot, coordinates near the center of the object can be determined."

**17.24** Discuss how a survey controller with a total station instrument can be combined with GPS methods to collect data for a topographic map.

A survey controller can often interface with both a GNSS receiver and a total station. Thus GNSS receivers can be used to establish a coordinate system for the survey, control points, and perform as much of the survey as physically possible. In areas where signal loss or multipathing are a problem, a total station can be connected to the same controller project and fill in the remaining data in the same project using GNSS established points for occupation and azimuth.

**17.25** What does the term "point cloud" describe in laser-scanning?

From Section 17.9.5, paragraph 2: "The resulting grid of scanned, three-dimensional points can be so dense that a visual image of the scene is formed. This so-called "point-cloud" image differs from a photographic image in that every point has a three-dimensional coordinate assigned to it. These coordinates can be used to obtain dimensions between any two observed points in the scene."

**17.26** What factors must be considered when planning a laser scanning survey?

From Section 17.10: Part of the planning for a laser scanning survey is to determine the ideal locations to setup the scanner. Establishing a method of creating a consistent set of coordinates from multiple scan through the use of a traverse or the placement of multiple targets on the surface of the object being scanned that will extend the coordinates from the first scan to subsequent scans. Additionally, the resolution of the scan must be set to a sufficient density to satisfy project requirements.

For Problems 17.27 through 17.30, calculate the *X, Y* and *Z* coordinates of point *B* for radial readings taken to *B* from occupied station *A*, if the backsight azimuth at *A* is 63°03'18", the elevation of *A* = 1210.06 ft, and *hi* = 5.63 ft. Assume the *XY* coordinates of *A* are (10,000.000, 5,000.000).

**17.27*** Clockwise horizontal angle = 55°37'42", zenith angle 92°34'18", slope distance = 435.09 ft, *hr* = 6.00 ft.

**(10,381.31, 4791.38, 1190.17)**

| Az | H-Dist | V-Dist | x | y | H |
|---|---|---|---|---|---|
| 118°41'00" | 434.65 | -19.52 | 10,381.31 | 4,791.38 | 1190.17 |

**17.28** Clockwise horizontal angle = 162°38′08″, zenith angle = 88°28'16", slope distance = 158.90 ft, *hr* = 5.80 ft.

**(9886.33, 4889.04, 1214.13)**

| Az | H-Dist | V-Dist | x | y | H |
|---|---|---|---|---|---|
| 225°41'26" | 158.8519 | 4.239607 | 9886.3291 | 4889.037 | 1214.1296 |

**17.29** Clockwise horizontal angle = 55°15'06", zenith angle = 88°52′36″, slope distance = 168.59 ft, *hr* = 5.50 ft.

**(10,148.40, 4920.07, 1213.50)**

| Az | H-Dist | V-Dist | x | y | H |
|---|---|---|---|---|---|
| 118°18'24" | 168.5576 | 3.305141 | 10148.402 | 4920.072 | 1213.4951 |

**17.30** Clockwise horizontal angle = 203°15′44″, zenith angle = 93°43′24″, slope distance = 253.48 ft, *hr* = 5.50 ft.

**(9747.58, 4983.75, 1193.73)**

| Az | H-Dist | V-Dist | x | y | H |
|---|---|---|---|---|---|
| 266°19'2" | 252.945 | -16.4607 | 9747.5774 | 4983.753 | 1193.7293 |

**17.31** What determines the field method using in a mapping survey?

From Section 17.12, paragraph 1: "Selection of the field method to be used on any topographic survey depends on many factors, including (1) purpose of the survey, (2) map use (accuracy required), (3) map scale, (4) contour interval, (5) size and type of terrain involved, (6) costs, (7) equipment and time available, and (8) experience of the survey personnel."

**17.32** What is the advantage of coding data points when combined with field-to-finish software?

From Section 17.13, paragraph 2: "No matter the method employed, correctly entering the field data at the time of collection will greatly reduce the time in generating the final map product."

**17.33*** On a map having a scale of 200 ft/in. the distance between plotted fixes 49 and 50 of Figure 17.14 is 3.15 in. From measurements on the profile of Figure 17.13, determine how far from fix 50 the 20-ft contour (existing between fixes 49 and 50) should be plotted on the map.

**0.47 in.** Ratio between fixes 49 and 50 = 0.15. Solution = 0.15(3.15) = 0.47 in.

**17.34** Similar to Problem 17.33, except locate the 16-ft contour between fixes 50 and 51 if the corresponding map distance is 2.98 in.

**2.12 in.** Ratio = 0.71 in. Solution = 0.71(2.98) = 2.12 in.

**17.35** What is a hydrographic survey?

From Section 17.14, paragraph 1: "Hydrographic surveys determine depths and terrain configurations of the bottoms of water bodies. Usually the survey data are used to prepare hydrographic maps."

# 18 MAPPING

Asterisks (*) indicate problems that have partial answers given in Appendix G.

**18.1** What is the scale of the USGS 7-1/2 minute quadrangle map?
**1:24,000**

**18.2*** On a map drawn to a scale of 1:4800, a point has a plotting error of 1/30-in. What is the equivalent ground error in units of feet?

**160 in.** = 4800(1/30)

**18.3** Define the term cartography.

From Section 18.1, paragraph 4: "Cartography, the term applied to the overall process of map production, includes map design, preparing or compiling manuscripts, final drafting, and reproduction."

**18.4** What federal agency in the United States is responsible for coordinating all mapping activities at the federal level?

From Section 18.2, paragraph 1: **U.S. Geological Survey**

**18.5** What is contained in a digital line graph?

From Section 18.3, paragraph 2: "The digital line graphs contain only linear features or planimetry in an area. Included are political boundaries, hydrography, transportation networks, and the subdivision lines of the U.S. Public Land Survey System (see Chapter 22)."

**18.6** What is contained in a digital elevation model?

From Section 18.3, paragraph 2: "The digital elevation models are arrays of elevation values, produced in grids of varying dimensions, depending on the source of the information."

**18.7** According to U.S. National Map Accuracy Standards, how much horizontal positional error is allowed in a map of scale 1:10,000?

From Section 18.4, paragraph 2: "…for maps produced at scales larger than 1:20,000, **not more than 10% of well-defined points tested shall be in error by more than 1/30 in. (0.8 mm).**"

**18.8** What is the 95% horizontal positional accuracy for the following set of data?

**0.117 m**

| Point | Map Coordinates x (m) | y (m) | Check Coordinates X (m) | Y (m) | Discrepancies Δx (m) | Δy (m) |
|---|---|---|---|---|---|---|
| 1 | 556,987.128 | 196,408.156 | 556,987.063 | 196,408.136 | 0.065 | 0.020 |
| 2 | 555,783.419 | 198,183.208 | 555,783.444 | 198,183.119 | −0.025 | 0.089 |
| 3 | 554,986.083 | 197,205.500 | 554,986.105 | 197,205.557 | −0.022 | −0.057 |
| 4 | 557,063.183 | 194,318.153 | 557,063.150 | 194,318.174 | 0.033 | −0.021 |
| | | | | RMSE | 0.040 | 0.055 |
| | | | | 95% RMSE | 0.117 | |

**18.9** What is the 95% positional vertical accuracy for the following data?

**0.095 m**

| Point | Map Coordinates h (m) | Check Coordinates E (m) | Discrepancies Δz |
|---|---|---|---|
| 1 | 348.986 | 349.001 | −0.015 |
| 2 | 339.642 | 339.560 | 0.082 |
| 3 | 355.483 | 355.491 | −0.008 |
| 4 | 368.913 | 368.864 | 0.049 |
| | | RMSE | 0.049 |
| | | 95% RMSE | 0.095 |

**18.10*** For a 20-ft contour interval, what is the greatest error in elevation expected of any definite point read from a map if it complies with National Map Accuracy Standards?

**±20 ft**;

From Section 18.4, paragraph 3: "The NMAS vertical accuracy requirements specify that not more than 10 percent of elevations tested shall be in error by more than one-half the contour interval, and **none can exceed the interval**."

**18.11** An area that varies in elevation from 556 to 623 ft is being mapped. What contour intervals will be drawn if a 10-ft interval is used? Which lines are emphasized?

**560, 570, 580, 590, 600, 610, and 620 with 600 being emphasized**

**18.12** Similar to Problem 18.11, except elevations vary from 103 to 142 m and a 5-m interval is used.

**105, 110, 115, 120, 125, 130, 135, and 145 with 75 and 125 being emphasized**

**18.13** If a map is to have a 10-ft contour interval, which contours are labeled between the elevations of 1130 and 1210 ft?

**1150 and 1200**

**18.14** How is maximum effectiveness achieved in map design?

From Section 18.6, paragraph 2: “To achieve maximum effectiveness in map design, the following elements or factors should be considered: (1) clarity, (2) order, (3) balance, (4) contrast, (5) unity, and (6) harmony.”

**18.15*** What is the largest acceptable error in position for 90% of the well-defined points on a map with a 1:24,000 scale that meets national map accuracy standards.

From Section 18.4, paragraph 2: "To meet the NMAS horizontal position specification, for maps produced at scales larger than 1:20,000, not more than 10 percent of well-defined points tested shall be in error by more than 1/30 in. (0.8 mm)."

**18.16** What questions must be answered before designing a map?

From Section 18.6, paragraph 1: “Before beginning the design of a map, the following two basic questions should be answered: (1) What is the purpose of the map? and (2) Who is the map intended to serve? All maps have a purpose, which in turn dictates the information that the map must convey.”

**18.17** Discuss why insets are sometimes used on maps.

From Section 18.6, paragraph 3: **<u>To enhance clarity</u>**: “If considerable detail must be included on a map, the information could be placed in a table. Other alternatives consist of preparing larger-scale *inset maps* of areas that contain dense detail, or creating an overlay to display some of the detail.”

**18.18*** If a map is to have a 1-in. border, what is the largest nominal scale that may be used for a subject area with dimensions of 604 ft and 980 ft on a paper of dimensions 24 by 36 in?

**<u>1 in./30 ft, or 1:360</u>**

Usable height = 22 in. Usable width = 34 in.

Scale for width = 604/22 = 27.5 ft/in. Scale for length = 980/34 = 28.8 ft/in.

**18.19** Similar to Problem 18.18, except the dimensions of the subject area are 1210 ft and 1575 ft.

**<u>60 ft/in. or 1:720</u>**

Scale for height= 1210/22 = 55 ft/in. Scale for length = 1475/34 = 46.3 ft/in.

**18.20** If a map is to have 1-1/2 in. borders on the top and left sides and 1/2 in. borders on the bottom and right sides what is the largest nominal scale that may be used for a subject area with dimensions of 506 and 995 ft on a paper of dimensions 24 by 36 in?

**<u>30 ft/in. or 1:360</u>**

Scale for height= 506/22 = 23 ft/in. Scale for length = 995/34 = 29.3 ft/in.

**18.21** If 90 percent of all elevations on a map must be interpolated to the nearest ±1 ft, what contour interval is necessary according to the National Map Accuracy Standards? Explain.

**2 ft,** From Section 18.4, paragraph 3: "The NMAS vertical accuracy requirements specify that not more than 10 percent of elevations tested shall be in error by more than one-half the contour interval, and **none can exceed the interval**."

**18.22** If an area having an average slope of 8% is mapped using a scale of 1:2000 and contour interval of 0.5 m, how far apart will contours be on the map?

**9.1 mm**; run = 0.5m/0.08 = 6.25 m; x = 6250 mm/2000 = 3.125 mm

**18.23** Similar to Problem 18.22, except average slope is 3.3%, map scale is 400 ft/in., and contour interval is 5 ft.

**0.31 in.**; run = 5 ft/0.033 = 151.52 ft; x = 151.52/400 = 0.379 in.

**18.24*** Similar to Problem 18.22, except average slope is 4%, map scale is 1:1000, and contour interval is 0.5 m.

**12.5 mm**; run = 0.5 m/0.04 = 12.5 m; x = 12,500 mm/1000 = 12.5 mm = 1.25 cm

**18.25*** The three-dimensional (*X*, *Y*, *Z*) coordinates in meters of vertexes *A*, *B*, and *C* in Figure 18.14 are (5412.456, 4480.621, 248.147), (5463.427, 4459.660, 253.121) and (5456.081, 4514.382, 236.193), respectively. What are the coordinates of the intersection of the 250-m contour with side AB? With side BC?

***AB*: (5431.445, 4472.812); *BC*: (5462.073, 4469.745)**

$AB$ = 55.113 m; $Az_{AB}$ = 112°21'14.8"; $BC$ = 55.213 m; $Az_{BC}$ = 352°21'15.1"

Interpolations:

$$AB: x = 55.113\frac{1.853}{4.974} = 20.532 \text{ m};\ \ BC: x = 55.213\frac{-3.121}{-16.928} = 10.180 \text{ m}$$

**18.26** The three-dimensional (*X*, *Y*, *Z*) coordinates in meters for vertices *A*, *B*, and *C* in Figure 18.14 are (5412.456, 4480.621, 124.381), (5463.427, 4459.660, 132.457) and (5456.081, 4514.382, 119.604), respectively. What are the coordinates of the intersections of the 125-m contour as it passes through the sides of the triangle?

***AB*: (5416.363, 4479.014); *AC*: (5459.165, 4491.408)**

$AB$ = 55.113 ft; $Az_{AB}$ = 112°21'14.8"; $AC$ = 55.213 ft; $Az_{AC}$ = 352°21'15.1"

Interpolations:

$$AB: x = 55.113\frac{0.619}{8.076} = 4.224 \text{ m};\ \ AC: x = 55.213\frac{-7.457}{-12.853} = 32.033 \text{ m}$$

**18.27** Similar to Problem 18.26, except compute the coordinates of the intersection of the 128-m contour.

***AB*: (5435.297, 4471.228); *BC* = (5460.880, 4478.636)**

$AB$ = 55.113 ft; $Az_{AB}$ = 112°21'14.8"; $AC$ = 55.213 ft; $Az_{AC}$ = 352°21'15.1"

Interpolations:

$$AB\!: x = 55.113\,\frac{3.619}{8.076} = 24.697\text{ m};\;\; AC\!: x = 55.213\,\frac{-4.457}{-12.853} = 19.146\text{ m}$$

**18.28** Discuss how balance is achieved on a map.

From Section 18.6, paragraph 5: "All elements on a map have weight, and they should be distributed uniformly around the "visual center" of the map to create good overall *balance.* The visual center is slightly above the geometrical center of the map sheet. In general, the weight of an element is affected by factors such as size, color, font, position, and line width. Map elements that appear at the center have less weight than those on the edges. Elements in the top or right half of the map will appear to have more weight than those in the bottom or left half of the map. Also map elements identified with thicker line widths will appear to have heavier weights than their slimmer counterparts. Colors such as red appear heavier than blue or yellow. ... The use of thumbnail sketches can often help to achieve a balanced layout for a map. It is important to place highest weights on those elements that enhance the purpose of the map."

The following table gives elevations at the corners of 50-ft coordinate squares, and they apply to Problems 18.29 and 18.31.

| | | | | | |
|---|---|---|---|---|---|
| 74 | 70 | 66 | 67 | 66 | 69 |
| 76 | 73 | 71 | 68 | 63 | 70 |
| 77 | 69 | 68 | 66 | 69 | 68 |

**18.29** At a horizontal scale of 1 in. = 50 ft draw 2-ft contours for the area.

Plot showing 2-ft contour interval

**18.30** Similar to Problem 18.29, except at the bottom of the table add a fourth line of elevations: 79, 78, 70, 66, 61, and 65 (from left to right).

Plot showing 2-ft contour interval

**18.31** If a map is drawn with 1-ft contour intervals, what contours between 243 and 273 ft are drawn with heavier line weight?

**245, 250, 255, 260, 265, and 270**

# 19 CONTROL SURVEYS AND GEODETIC REDUCTIONS

Asterisks (*) indicate problems that have partial answers given in Appendix G.

**19.1** What are the defining parameters to describe an ellipsoid such as GRS80?

From Section 19.2, paragraph 4: "For GRS80 and WGS84, the defining parameters are the semimajor axis *a* and flattening factor *f*."

**19.2** Define the geoid?

From Section 19.2, paragraph 1: "The geoid is an equipotential gravitational surface located approximately at mean sea level, which is everywhere perpendicular to the direction of gravity."

**19.3** Using the defining parameters for the PZ-90 ellipsoid from Table 19.1, what are the first and second eccentricities of the ellipsoid?

Using Equation (19.2): $\underline{\boldsymbol{e = 0.081819107;\ e' = 0.082094353}}$

**19.4*** What is the difference between the equatorial circumference of the Clarke 1866 ellipsoid and that of the WGS84 ellipsoid?

$\underline{\boldsymbol{\Delta C = 436.0\text{ m}}}$ $= 2\pi(6378206.4 - 6378137.0)$

**19.5** Determine the first and second eccentricities for the WGS84 ellipsoid.

By Equation (19.2): $\underline{\boldsymbol{e = 0.081819191;}}$ $\underline{\boldsymbol{e' = 0.082094438}}$

**19.6** Define the conventional terrestrial pole.

From Section 19.3, paragraph 1: "By international convention, the mean rotational axis of the Earth was defined as the "mean" position of the pole between the years of 1900.0 and 1905.0. This position is known as the *Conventional Terrestrial Pole* (CTP)."

**19.7** What are the radii in the meridian and prime vertical for a station with latitude 39°06′33.29742″ using the GRS80 ellipsoid?

By Equation 19.5: $\underline{\boldsymbol{R_M = 6{,}360{,}838.020\text{ m}}}$

By Equation 19.4: $\underline{\boldsymbol{R_N = 6{,}386{,}648.925\text{ m}}}$

**19.8** For the station listed in Problem 19.7, what is the radius of the great circle at the station that is at an azimuth of 123°14′08″ using the GRS80 ellipsoid?

By Equation 19.6: $\underline{\boldsymbol{R_\alpha = 6{,}378{,}873.492\text{ m}}}$

**19.9*** What are the radii in the meridian and prime vertical for a station with latitude

42°37'26.34584" using the GRS80 ellipsoid?

By Equation 19.5: $\underline{\boldsymbol{R_M} = \mathbf{6,364,725.399\ m}}$

By Equation 19.4: $\underline{\boldsymbol{R_N} = \mathbf{6,387,949.711\ m}}$

**19.10** For the station listed in Problem 19.9, what is the radius of the great circle at the station that is at an azimuth of 322°19′48″ using the GRS80 ellipsoid?

By Equation 19.6: $\underline{\boldsymbol{R_\alpha} = \mathbf{6,373,378.937\ m}}$

**19.11*** The orthometric height at Station Y 927 is 304.517 m, and the modeled geoid height at that station −31.893 m. What is its geodetic height?

By Equation (19.7): $h$ = **272.624 m**

**19.12** The geodetic height at Station Apple is 243.983 m, and the modeled geoid height is −31.70 m. What is its orthometric height?

By Equation (19.7): $H$ = **275.689 m**

**19.13** The orthometric height of a particular benchmark is 2513.68 ft, and the modeled geoid height at the station is –25.03 m. What is the geodetic height of the benchmark? Draw a sketch depicting the geoid, ellipsoid, and benchmark.

By Equation (19.7): $h = [2513.68(12/39.37) - 30.66]39.37/12 =$ **2431.56 ft**.

**19.14** What organization monitors the instantaneous position of the pole?

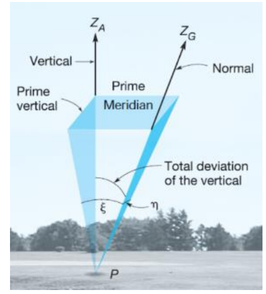
From Section 19.3, paragraph 3: "Since 1988, the *International Earth Rotation and Reference Systems Service* (IERS)[3] has monitored the instantaneous position of the pole with respect to the CTP using observations made by participating organizations employing advanced space methods including *Very Long Baseline Interferometry* (VLBI), and lunar and satellite laser ranging."

**19.15*** The deflection of the vertical components ξ and η are −2.85″ and −5.94″, respectively, at a station with geodetic coordinates of (29°37′23.0823″ N, 108°56′01.0089″ W). The observed zenith angle is 42°36′58.8″ along a line with azimuth of 88°52′36.7″. What is the geodetic zenith angle?

$\underline{\boldsymbol{z} = \mathbf{42°36'52.8''}}$ by Equation (19.34).

**19.16** Repeat Problem 19.15 with ξ and η of −1.06″ and 3.24″, respectively, and an observed zenith angle of 35°05′26.6″ along a line with an azimuth of 122°11′03.5″.

$z$ **= 35°05′29.9″** by Equation (19.34)




**19.17** Define deflection of the vertical.

From Section 19.5, paragraph 5: "As illustrated in the figure, the *deflection of the vertical* (also called *deviation of the vertical*) at any ground point *P* is the angle between the vertical (direction of gravity) and the normal to the ellipsoid."

**19.18** Discuss the evolution of NAD83.

See Sections 19.6.2 and 19.6.3.

**19.19*** Given the following information for stations *JG00050* and *KG0089*, what should be the leveled height difference between them?

| Station | Height (m) | Gravity (mgal) |
|---|---|---|
| JG0050 | 474.442 | 979,911.9 |
| KG0089 | 440.552 | 979,936.2 |

**−33.880 m**; By Equations (19.44) and (19.45).

**19.20** Similar to Problem 19.19 except for stations R 164 and C 65 near Fairbanks, AK.

| Station | Height (m) | Gravity (mgal) |
|---|---|---|
| R 164 | 485.699 | 982,192.3 |
| C 65 | 185.018 | 982,197.2 |

**−300.688 m** by Equations (19.44) and (19.45)

**19.21** Similar to Problem 19.19 except for two stations W 368 and C 593 near Gainesville, FL.

| Station | Height (m) | Gravity (mgal) |
|---|---|---|
| W 368 | 47.853 | 979,329.6 |
| C 593 | 32.110 | 979,245.5 |

**−15.746 m** by Equations (19.44) and (19.45)

**19.22*** A slope distance of 2458.663 m is observed between two points *Gregg* and *Brian* whose orthometric heights are 458.966 m and 566.302 m, respectively. The geoidal undulations are −25.66 m and −25.06 m at *Gregg* and *Brian*, respectively. The height of the instrument at station *Gregg* at the time of the observation was 1.525 m and the height of the reflector at station *Brian* was 1.603 m. What are the geodetic and mark-to-mark distances for this observation? (Use an average radius for the Earth of 6,371,000 m for $R_\alpha$)

$D_2$ (geodetic) = **2456.310 m**; $D_3$ (mark-to-mark) = **2458.868 m** by Equations (19.23) and (19.24)

**19.23** If the latitude of station *Gregg* in Problem 19.22 was 56°16′22.4450″ and the azimuth of the line was 135°48′26.8″ what are the geodetic, and mark-to-mark distances for this observation? (Use the GRS80 ellipsoid).

$D_2$ (geodetic) = **2456.310 m**; $D_1$ (mark-to-mark) = **2458.868 m** by Equations (19.23) through (19.25)

where $R_M$ = 6,379,700.520 m; $R_N$ = 6,392,955.708; and $R_\forall$ = 6,386,134.467 m

**19.24** A slope distance of 6365.780 m is observed between two stations *A* and *B* whose geodetic heights are 24.483 m and 115.097 m, respectively. The height of the instrument at the time of the observation was 1.544 m, and the height of the reflector was 2.000 m. The latitude of Station *A* is 43°08'36.2947" and the azimuth of *AB* is 32°28′21.9″. What are the geodetic, and mark-to-mark distances for this observation?

$D_2$ (geodetic) = **6365.174 m**; $D_1$ (mark-to-mark) = **6365.889 m** by Equations (19.23) through (19.25)

where $R_M$ = 6,365,305.044 m; $R_N$ = 6,388,143.625; and $R_\forall$ = 6,371,871.722 m

**19.25** What does the NGS horizontal time-dependent positioning software provide to users?

From Section 19.7.1, paragraph 2: "This software allows users to transform coordinates across epochs in time and between reference frames."

**19.26*** Compute the back azimuth of a line 5863 m long at a mean latitude of 45°01'32.0654" whose forward azimuth is 88°16'33.2" from north. (Use an average radius for the Earth of 6,371,000 m.)

**268°19'43.1"** = 88°16'33.2 – 189.9" + 180° and θ = 189.9" from Equation (19.17). Length of E-W line is 5863*sin(88°16'33.2") = 5860.3457 m

**19.27** Compute the back azimuth of a line 6832.519 m long at a mean latitude of 47°33′31.29897″ whose forward azimuth is 35°50′26.7″ from north. (Use an average radius for the Earth of 6,371,000 m.)

**215°52′48.3″** = 35°50′26.7″ + 141.6″ + 180° and θ = 141.6″ Equation (19.17). Length of E-W line is 6832.519 sin(35°50′26.7″) = 4000.675 m

**19.28** In Figure 19.14 azimuth of *AB* is 102°36'20" and the angles to the right observed at *B, C, D, E*, and *F* are 132°01'05", 241°45'12", 141°15'01", 162°09'24", and 202°33'19", respectively. An astronomic observation yielded an azimuth of 82°24'03" for line *FG*. The mean latitude of the traverse is 42°16'00", and the total departure between points *A* and *F* was 24,986.26 ft. Compute the angular misclosure and the adjusted angles. (Assume the angles and distances have already been corrected to the ellipsoid.)

**Misclosure = −15.3"** Correction +3"/angle.

**Adjusted Angles: 132°01'08", 241°45'15", 141°15'04", 162°09'27", and 202°33'22"**

θ = 206.7" by Equation (19.17)

**19.29** In Figure 19.19 slope distance *S* is observed as 4845.641 m. The orthometric elevations of points *A* and *B* are 343.460 m and 432.183 m, respectively, and the geoid heights at both stations is −22.86 m. The instrument and reflector heights were both set at 1.250 m. Calculate geodetic distance *A′B′* (Use an average radius for the Earth of 6,371,000 m.)

**4844.550 m** $D_1$ = 4845.640 m; $D_3$ = 4844.550 m

**19.30** In Figure 19.20, slope distance *S* and zenith angle ∀ at station *A* were observed as 2072.33 m and 82°17'18", respectively to station *B*. If the elevation of station *A* is 435.967 m and the geoid height at stations *A* and *B* are both −28.04m, what is ellipsoid length *A'B'*? (Use an average radius for the Earth of 6,371,000 m.)

**2053.556 m** ∗ = 7°43'05.7"; $AB_1$ = 2053.556; $BB_1$ = 278.318; P = 0°01'06.5";

$B_1B_2$ = 0.0448; $AB_2$ = 2053.511 m

**19.31*** Components of deflection of the vertical at an observing station of latitude 43°15'47.5864" are ξ = −6.87″ and η = −3.24″ If the observed zenith angle on a course with an astronomic azimuth of 204°32′44″ is 85°56′07″, what are the azimuth and zenith angles corrected for deviation of the vertical?

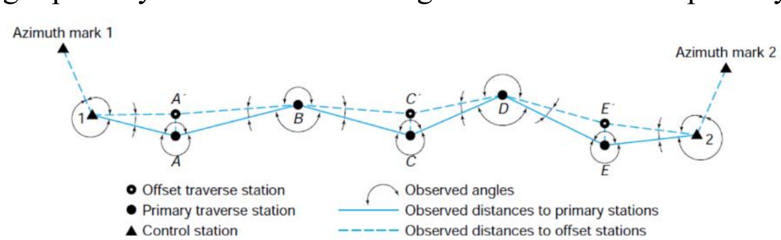
**z = 85°56'00.1"; ∀ = 204°32'47.3"** by Equations (19.36) and (19.34), respectively.

**19.32** At the same observation station as for Problem 19.31, the observed zenith angle on a course with an azimuth of 154°00′59″ is 42°22′21″, what are the azimuth and zenith angles corrected for deviation of the vertical?

**z = 42°22′14.1″; ∀ = 154°01′05.6″** by Equations (19.36) and (19.34), respectively.

**19.33** Describe how a control traverse can be strengthened.

See Section 19.13.2: "Control traverses can be strengthened to provide additional checks in the data by establishing "offset stations" such as *A*′, *C*′, and *E*′ of figure below. An offset station is set near every-other primary traverse station. In performing the field observations, instrument setups are made only at the primary traverse stations. All possible angles are observed with horizon closures at each station; thus, four angles are determined at interior single primary stations and two angles are observed at primary stations with nearby offset stations."



**19.34** What is the orthometric height of a point? … geodetic height of a point?

The orthometric height is the perpendicular distance a point is above the geoid. The geodetic height is the perpendicular distance a point is above the ellipsoid. They differ in value by the geoid height at the point.

**19.35** Compute the collimation correction factor *C* for the following field data, taken in accordance with the example and sketch in the field notes of Figure 19.18. With the instrument at station 1, high, middle, and low cross-hair readings were 6.334, 5.501, and 4.668 ft on station *A* and 4.978, 3.728, and 2.476 ft on station *B*. With the instrument at station 2, high, middle, and low readings were 7.304, 6.053, and 4.803 ft on *A* and 5.111, 4.279, and 3.446 ft on *B*.

**<u>0.0022 ft/ft</u>**

| | r1 | i1 | R1 | I1 | | |
|---|---|---|---|---|---|---|
| 1 | 6.334 | | 4.978 | | | |
| | 5.501 | 0.833 | 3.728 | 1.250 | C | −0.000598 |
| | 4.668 | 0.833 | 2.476 | 1.252 | | |
| | 5.501 | 1.666 | 3.727 | 2.502 | | |
| | R2 | I2 | r2 | i2 | | |
| 2 | 7.304 | | 5.111 | | | |
| | 6.053 | 1.251 | 4.279 | 0.832 | | |
| | 4.803 | 1.250 | 3.446 | 0.833 | | |
| | 6.053 | 2.501 | 4.279 | 1.665 | | |

**19.36** A leveling instrument having a collimation factor of 0.00005 m/m of interval was used to run a section of three-wire differential levels from BM *A* to BM *B*. Sums of backsights and foresights for the section were 2506.837 m and 2538.464 m, respectively. Backsight stadia intervals totaled 3408.9, while the sum of foresight intervals was 2486.7. What is the corrected elevation difference from BM *A* to BM *B*?

**<u>−31.581 m</u>** by Equation (19.19): 2506.837 – 2538.464 + 0.00005(3408.9 – 2486.7)

**19.37** The relative error of the difference in elevation between two benchmarks directly connected in a level circuit and located 70 km apart is ±0.006 m. What order and class of leveling does this represent?

**<u>Second order, Class I</u>**; $c = 0.72$ mm

**19.38** Discuss the advantages of breaking a long leveling line into shorter loops.

From Section 19.14, paragraph 9: "For long lines, one procedure used to help isolate mistakes and minimize field time is to run small loops with approximately five setups between temporary benchmarks. In this procedure as each loop is completed, it is checked for acceptable closure before proceeding forward to the next loop. This procedure increases the number of observations, but helps minimize the amount of time that is required to uncover mistakes."

**19.39** The baseline components of a GPS baseline vector observed at a station $A$ in meters are (1514.650, 643.816, 730.323). The geodetic coordinates of the first base station are 43°15′48.05796″ N latitude and 89°23′04.64831″ W longitude. What are the changes in the local geodetic coordinate system of ($\Delta n$, $\Delta e$, $\Delta u$)?

**(961.896, 1521.477, 43.568)**

| R | | | neu |
|---|---|---|---|
| -0.00736 | 0.685313 | 0.728211 | 961.896 |
| 0.999942 | 0.01074 | 0 | 1521.477 |
| 0.007821 | -0.72817 | 0.685353 | 43.568 |

**19.40** In Problem 19.39, what are the slant distance, zenith angle, and azimuth for the baseline vector?

**$S$ = 1800.565, Az = 57°41′54.4″, $z$ = 88°36′48.6″** by Equation (19.52)

**19.41** If the slant distance between two stations is 798.273 m, the zenith angle between them is 94°36′12″ and the azimuth of the line is 163°15′47″, what are the changes in the local geodetic coordinates?

**(−18.450, 229.144, −64.067)** by Equation (19.52)

# 20 STATE PLANE COORDINATES AND OTHER MAP PROJECTIONS

Asterisks (*) indicate problems that have answers given in Appendix G.

**20.1** Discuss the advantages of placing surveys on state plane coordinate systems.

From Section 20.1, paragraphs 1&2: It allows computations to be made using simple coordinate geometry formulas, and “A state plane coordinate system is a map projection that provides a common datum of reference for horizontal control of all surveys in a large area, just as the geoid furnishes a single datum for vertical control. It eliminates having individual surveys based on different assumed coordinates, unrelated to those used in other adjacent work.”

**20.2** What is a developable surfaces?

From Section 20.2, paragraph 1: “… points are projected mathematically from the ellipsoid to some imaginary *developable surface*—a surface that can conceptually be developed or “unrolled and laid out flat” without distortion of shape or size.”

**20.3** Which developable surfaces are typically used in the state plane coordinate system.

**Cylinder and cone**, from Section 20.2, paragraph 2: “The Lambert conformal conic projection utilizes an imaginary cone as its developable surface and the Transverse Mercator employs a fictitious cylinder.”

**20.4** What three map projections are used in state plane coordinates?

From Section 20.2, paragraph 2 and Section 20.12.2: **Lambert conformal conic, Transverse Mercator, and Oblique Mercator.**

**20.5** What surveying observations are distorted by a conformal map projection?

From Section 20.2, paragraph 5: “…points couldn’t be projected from the ellipsoid to developable surfaces without introducing distortions in the **lengths of lines or the shapes of areas**.”

**20.6** What are the defining parameters for the Lambert conformal conic map projection?

From Section 20.6.1, paragraph 1: The defining parameters for the ellipsoid ($a, f$), north and south standard parallels ($\varphi_S, \varphi_N$), grid origin ($\varphi_0, \lambda_0$), false northing ($N_b$) and easting ($E_0$).

**20.7** Develop a table of SPCS83 elevation factors for geodetic heights ranging from 0 to 1000 m. Use increments of 100 m and an average radius for the Earth of 6,371,000 m.

| Height (m) | Scale |
|---|---|

| | |
|---|---|
| 0 | 1.00000000 |
| 100 | 0.99998430 |
| 200 | 0.99996861 |
| 300 | 0.99995291 |
| 400 | 0.99993722 |
| 500 | 0.99992153 |
| 600 | 0.99990583 |
| 700 | 0.99989014 |
| 800 | 0.99987445 |
| 900 | 0.99985875 |
| 1000 | 0.99984306 |

**20.8** Define the direct problem in state plane coordinates.

From Section 20.6.2, paragraph 2: "The coordinates *E* and *N* of point *P*, whose geodetic latitude $\phi_P$, and geodetic longitude $\lambda_P$, are known, are to be determined."

**20.9** Define the inverse problem in state plane coordinates.

From Section 20.6.3, paragraph 1: "The inverse problem in state plane coordinate computations is the determination of the geodetic latitude and longitude of a station based on its state plane coordinates and zone."

**20.10** Develop a table similar to Table 20.1 for a range of latitudes from 40°30' N to 40°35' N in the Pennsylvania North Zone with standard parallels of 40°53' N and 41°57' N, and a grid origin at (40°10' N, 77°45' W).

| Latitude | R (m) | Tab. Diff. | k |
|---|---|---|---|
| 40°30' | 7342329.667 | 30.84819 | 1.000083949 |
| 40°31' | 7340478.776 | 30.84814 | 1.000079382 |
| 40°32' | 7338627.887 | 30.84809 | 1.000074899 |
| 40°33' | 7336777.002 | 30.84805 | 1.000070499 |
| 40°34' | 7334926.119 | 30.84800 | 1.000066182 |
| 40°35' | 7333075.239 | 30.84796 | 1.000061949 |

***20.11** The Pennsylvania North Zone SPCS83 state plane coordinates of points *A* and *B* are as follows:

| Point | E(m) | N(m) |
|---|---|---|
| *A* | 541,983.399 | 115,702.804 |
| *B* | 541,457.526 | 115,430.257 |

Calculate the grid length and grid azimuth of line *AB*.

<u>**592.304 m, 242°36′12″**</u>

**20.12** Similar to Problem 20.11, except points *A* and *B* have the following New Jersey SPCS83 state plane coordinates:

| Point | E(m) | N(m) |
|---|---|---|
| *A* | 131,124.094 | 264,920.458 |
| *B* | 131,391.924 | 264,523.316 |

**479.014 m, 146°00′16″**

**20.13** What are the SPCS83 coordinates (in sft) and convergence angle for a station in the North zone of Pennsylvania with geodetic coordinates of 41°14′14.22063″ N and 76°43′34.06012″ W?

From WolfPack: **X = 2,250,089.72 sft Y = 391,741.75 sft**

Scale = 0.9999617584 ; Radius : 7,260,452.567 m

Convergence angle = **0°40′38.4″**

**20.14*** Similar to Problem 20.13 except that the station's geodetic coordinates are 41°13'20.03582" N and 75°58'46.28764" W. Give coordinates in meters.

From WolfPack:

**X = 2,455,513.33 sft Y = 389,571.28 sft**

Scale = 0.9999626142 Radius : 7,262,124.0805

Convergence angle = **1°10'16.46"**

**20.15** What is the scale factor for the station in Problem 20.13? **k = 0.9999 6176**

**20.16** What is the scale factor for the station in Problem 20.14? **k = 0.9999 6261**

**20.17*** What are the SPCS83 coordinates in meters for a station in New Jersey with geodetic coordinates of 40°44′32.73687″ N and 74°10′45.47356″ W?

**(177,084.383, 211,996.039)**

**20.18** Similar to Problem 20.17 except that the geodetic coordinates of the station are 40°44′34.70026″ N and 74°10′44.92986″ W.

**(177,096.916, 212,056.644)**

**20.19** What are the convergence angle and scale factor at the station in Problem 20.17?

**0°12′33.5″ and 0.9999 0903**

**20.20** What are the convergence angle and scale factor at the station in Problem 20.18?

**0°12′33.9″ and 0.9999 0903**

**20.21** What is the grid azimuth and grid distance in meters between the points in Problems 20.17 and 20.18?

**11°41′02.3″ and 61.887 m**

**20.22** If the average geodetic height of the line between the points in Problems 20.17 and 20.18 is 100 ft, what is the combined factor for the line, geodetic azimuth, and ground distance in meters? (Use an average radius for the Earth of 6,371,000 m)

**<u>0.99990425, 11°53′29.5″ and 61.893 m</u>**

$$EF = \frac{6{,}371{,}000}{6{,}371{,}000+30.480} = 0.9999\,9522,$$

$$CF = 0.99990903(0.99999522) = 0.99990425, L_m = \frac{61.887}{0.99990425} = 61.893$$

α = 11°41′02.3″ + 0°12′33.5″ = 11°53′29.5″

**20.23*** What are the geodetic coordinates for a point *A* in Problem 20.11?

*X* = 541983.399 m, *Y* = 115702.804 m

**<u>(41°12'23.2037"N, 78°26'30.3340"W)</u>**

**20.24** Similar to Problem 20.23 except for point *B* in Problem 20.11?

*X* = 541457.526 m, *Y* = 115430.257 m

**<u>(41°12'14.2321"N, 78°26'52.8116"W)</u>**

**20.25*** What are the geodetic coordinates for a point *A* in Problem 20.12?

X = 131,124.094 m Y = 264,920.458 m

**<u>Latitude = 41°13′09.36026″ N, Longitude = 74°43′30.43981″ W</u>**

Scale = 0.9999 0438
Convergence angle =−0°10'19.5713"

**20.26** Similar to Problem 20.25 except for point *B* in Problem 20.12.

X = 131,391.924 m Y = 264,523.316 m

**<u>Latitude = 41°12′56.50785″ N, Longitude = 74°43′18.89706″ W</u>**

Scale = 0.9999 0426 ; Convergence angle = −0°08′46.39″

**20.27** In computing state plane coordinates for a project area whose mean orthometric height is 234 m, an average scale factor of 0.99996871 was used. The average geoid height for the area is −31.284 m. The given distances between points in this project area were computed from SPCS83 state plane coordinates. What horizontal length would have to be observed to lay off these lines on the ground? (Use 6,371,000 m for an average radius for the Earth.)

Elevation factor = 0.99996818; Combined factor = 0.9993689

**(a)*** 2834.79 ft **<u>2834.97 ft</u>**

**(b)** 608.803 m **<u>608.841 m</u>**

**(c)** 1013.25 ft **<u>1013.31 ft</u>**

**20.28** Similar to Problem 20.27, except that the mean project area elevation was 100.997 m, the geoidal separation −22.663 m, the scale factor 0.99994959, and the computed lengths of lines from SPCS83 were:

Elevation factor = 0.9999 8771; Combined factor = 0.9999 3730

**(a)** 558.028 m **<u>558.063 m</u>**

**(b)** 1202.39 ft **<u>1202.46 ft</u>**

**(c)** 610.803 m **<u>610.841 m</u>**

**20.29** The horizontal ground lengths of a three-sided closed polygon traverse were measured in feet as follows: $AB$ = 416.04, $BC$ = 372.22, and $CA$ = 531.55 ft. If the average scale factor is 0.99990643, orthometric height of the area is 6843.68 ft, and the average geoid height is −31.273 m, calculate grid lengths of the lines suitable for use in computing SPCS83 coordinates. (Use 6,371,000 m for an average radius for the Earth.)

Elevation factor = 0.9996 7760

$AB$ = **<u>415.867 ft</u>**

$BC$ = **<u>372.065 ft</u>**

$CA$ = **<u>531.329 ft</u>**

**20.30** For the traverse of Problem 20.29, the grid azimuth of a line from $A$ to a nearby azimuth mark was 309°22′06″ and the clockwise angle measured at $A$ from the azimuth mark to $B$, 14°26′18″. The measured interior angles were $A$= 44°11′41″, $B$ = 84°36′49″, and $C$ = 51°11′20″. Balance the angles and compute grid azimuths, latitudes and departures balanced latitude and departures, linear misclosure, and relative precision for the traverse. (Note: Line BC bears southwesterly.)

**From Wolfpack**

Angle Summary

| Station | Unadj. Angle | Adj. Angle |
|---|---|---|
| A | 44°11'41.0" | 44°11'44.3" |
| B | 84°36'49.0" | 84°36'52.3" |
| C | 51°11'20.0" | 51°11'23.3" |

Angular misclosure (sec): -10"

| Course | Length | Azimuth | Unbalanced Dep | Unbalanced Lat |
|---|---|---|---|---|
| AB | 415.867 | 323°48'24.0" | -245.5744 | 335.6167 |
| BC | 372.065 | 228°25'16.3" | -278.3209 | -246.9207 |
| CA | 531.329 | 99°36'39.7" | 523.8712 | -88.7098 |
| | ---------- | | --------- | --------- |

```
    Sum =  1,319.261                          -0.0240    -0.0138

   Linear misclosure  = 0.0277
   Relative Precision = 1 in 47,600
          Balanced                               Coordinates
      Dep          Lat      Point          X                Y
-----------------------------------------------------------------
   -245.5668   335.6211      A    774,924.101  1,525,719.119
   -278.3141  -246.9168      B    774,678.534  1,526,054.740
    523.8809   -88.7043      C    774,400.220  1,525,807.823
```

**20.31** Using grid lengths and azimuths of Problem 20.30 and the fact that the (N, E) coordinates in meters for Station A are (465,040.178, 236,197.338), calculate coordinates of the stations in feet.

From WolfPack (see 20.30)

| Station | N (ft) | E (ft) |
|---|---|---|
| *A* | 1,525,719.12 | 774,924.10 |
| *B* | 1,526,054.74 | 774,678.53 |
| *C* | 1,525,807.82 | 774,400.22 |

**20.32*** What is the combined factor for the traverse of Problem 20.30, and what distance precision does this yield when grid distances are compared to ground distances?

**<u>0.9995 8406</u>** $\mathbf{1}{:}\ \boldsymbol{Precision} = \mathbf{1}{:}\frac{\mathbf{1}}{\mathbf{1-0.99958406}} = \mathbf{1{:}\,2400}$

**20.33** What is the ground versus grid problem and what two methods can be used to solve this problem?

From Section 20.13, paragraph 1: "As discussed in Section 10.10 surveyors often determine the length of a line by inversing coordinates. However when using map projection coordinates such as state plane coordinates, the inversing coordinates results in what is known as a grid length, which is a distance on the map projection surface between the two points inversed. Since the typical distance needed by surveyors is a horizontal ground length, Equation (20.53) must be rearranged to convert the grid distance to a ground distance as

ground distance = (grid distance)/CF

where CF is the combined factor for the line. However when this operation is performed in the field, the appropriate combined factor for the line may not be known. This is known as the *ground versus grid problem*."

Paragraph 2: "One solution to this problem is to design a map projection that is near the surface of the Earth, and thus provide grid distances that very closely match ground distances. These are known as *low-distortion projections* (LDP). Another solution is to provide the survey controller with an average combined factor, which is known as a *project factor*, for the entire project site."

**20.34** What scale factor should be used with the Lambert conformal conic map projection when creating an LDP that is secant to the Earth at the height of $h_{avg}$? …for the Transverse Mercator map projection?

For the Lambert conformal conic: $k_{LDP} = 1 + \frac{h_{avg}}{R_e}$

For the Transverse Mercator: $k_{LDP} = 0.99999\left(1 + \frac{h_{avg}}{R_e}\right)$

**20.35** The average geodetic height in a project area is 6892.76 ft. Using an average radius of the Earth of 6,371,000 m, what is the appropriate scale factor for an LDP using the Transverse Mercator projection in an LDP?

By Equation (20.78), $k_{LDP} = 0.99999\left[1 + \frac{6892.76\left(\frac{12}{39.37}\right)}{6{,}371{,}000}\right] = \mathbf{1.00107188}$

**20.36** The traverse in Problems 10.9 through 10.11 was performed in the Pennsylvania North Zone of SPCS83. The average elevation for the area was 505.87 m and the average geoidal height was −31.56 m. Using the data in Table 20.1 and a mean radius for the Earth, compute a project factor, reduce the observations to grid, and adjust the traverse. Compare this solution with that obtained in Chapter 10. (Use 6,371,000 m for an average radius of the Earth.)

Using the initial coordinates (in meters) from Chapter 10, the scale factor at each station is (from WolfPack):

| Station | X | Y | Scale |
|---|---|---|---|
| A | 310,630.892 m | 121,311.411 m | 0.9999635428 |
| B | 310,544.945 m | 121,105.619 m | 0.9999636633 |
| C | 310,676.353 m | 120,999.637 m | 0.9999637217 |
| D | 310,823.019 m | 121,107.996 m | 0.9999636555 |
| E | 310,837.294 m | 121,336.597 m | 0.9999635237 |

Their average is 0.99996361;
The average elevation factor is $\frac{6{,}371{,}000}{6{,}371{,}000+505.87-31.56} = 0.99992556$
The project factor is 0.99988917
Reduced distances are:

| Course | Obs. Dist | Grid. Dist. |
|---|---|---|
| *AB* | 383.846 | 383.803 |
| *BC* | 360.256 | 360.216 |
| *CD* | 342.244 | 342.206 |
| *DE* | 336.228 | 336.191 |
| *EA* | 267.550 | 267.520 |

Traverse Adjustment from WolfPack:

```
Angle Summary
Station   Unadj. Angle     Adj. Angle
---------------------------------------
    1      57°00'34.0"      57°00'36.0"
    2      88°24'40.0"      88°24'42.0"
    3     126°37'20.0"     126°37'22.0"
    4      46°03'46.0"      46°03'48.0"
    5     221°53'30.0"     221°53'32.0"

Angular misclosure (sec): -10"
```

```
                                           Unbalanced
Course          Length       Azimuth         Dep         Lat
------------------------------------------------------------
       1-2       383.803     35°09'32.0"  221.0114   313.7813
       2-3       360.216    303°34'14.0" -300.1340   199.1863
       3-4       342.206    250°11'36.0" -321.9616  -115.9556
       4-5       336.191    116°15'24.0"  301.5032  -148.7286
       5-1       267.520    158°08'56.0"   99.5699  -248.2998
               ----------                ---------  ---------
        Sum =  1,689.936                   -0.0110    -0.0164
```

```
        Balanced                         Coordinates
     Dep          Lat     Point          X              Y
------------------------------------------------------------
     221.0139   313.7850      1    310,630.892    121,311.411
    -300.1316   199.1898      2    310,851.906    121,625.196
    -321.9593  -115.9523      3    310,551.774    121,824.386
     301.5054  -148.7253      4    310,229.815    121,708.434
      99.5716  -248.2972      5    310,531.320    121,559.708

     Linear misclosure  = 0.0198
     Relative Precision = 1 in 85,600

     Area: 126,200 sq. ft.
            2.898 acres {if distance units are feet}
```

```
Adjusted Observations
---------------------
      Course     Distance    Azimuth        Point        Angle
--------------------------------------------------------------
        1-2      383.807     35°09'32"         1     57°00'38"
        2-3      360.216    303°34'16"         2     88°24'44"
        3-4      342.203    250°11'37"         3    126°37'21"
        4-5      336.192    116°15'22"         4     46°03'44"
        5-6      267.518    158°08'54"         5    221°53'32"
```

Note: The adjusted coordinates from Chapter 10 and here are provided below. The same linear misclosure and relative precision was achieved even though the coordinates vary as shown since the traverse was scaled incorrectly in Chapter 10.

| | Initial Coordinates | | Grid Coordinates | |
|---|---|---|---|---|
| Sta | E (m) | N (m) | E (m) | N (m) |
| *A* | 310,630.892 | 121,311.411 | 310,630.892 | 121,311.411 |

| | | | | |
|---|---|---|---|---|
| ***B*** | **310,851.931** | **121,625.231** | **310,851.906** | **121,625.196** |
| ***C*** | **310,551.766** | **121,824.443** | **310,551.774** | **121,824.386** |
| ***D*** | **310,229.771** | **121,708.478** | **310,229.815** | **121,708.434** |
| ***E*** | **310,531.309** | **121,559.736** | **310,531.320** | **121,559.708** |

**20.37** The traverse in Problems 10.12 through 10.14 was performed in the New Jersey zone of SPCS83. The average elevation for the area was 134.93 m and the average geoidal separation was −32.86 m. Using the data in Table 20.3 and 20.4, and a mean radius for the earth, compute a project factor, reduce the observations to grid, and adjust the traverse. Compare this solution with that obtained in Chapter 10.

Using the initial coordinates from Chapter 10, the scale factor at each station is:

| Sta | E (m) | N (m) | k |
|---|---|---|---|
| *A* | 243,605.596 | 25,393.201 | 1.00000786 |
| *B* | 243,919.416 | 25,172.162 | 1.00000858 |
| *C* | 244,118.628 | 25,472.327 | 1.00000904 |
| *D* | 244,002.663 | 25,794.322 | 1.00000877 |
| *E* | 243,853.921 | 25,492.784 | 1.00000843 |

Their average is 1.00000854

The average elevation factor is $\frac{6{,}371{,}000}{6{,}371{,}000+134.93-32.86} = 1.00000854$

The project factor is 0.99999252

The reduced distances are:

| Course | Obs. Dist. | Grid Dist. |
|---|---|---|
| *AB* | 383.846 | 383.8489 |
| *BC* | 360.256 | 360.2587 |
| *CD* | 342.244 | 342.2466 |
| *DE* | 336.228 | 336.2305 |
| *EA* | 267.550 | 267.5520 |

The adjusted traverse from WolfPack:

```
Station   Unadj. Angle     Adj. Angle
---------------------------------------
   1       57°00'34.0"      57°00'36.0"
   2       88°24'40.0"      88°24'42.0"
   3      126°37'20.0"     126°37'22.0"
   4       46°03'46.0"      46°03'48.0"
   5      221°53'30.0"     221°53'32.0"

Angular misclosure (sec): -10"
```

```
                                             Unbalanced
Course          Length       Azimuth          Dep          Lat
------------------------------------------------------------
       1-2        383.849    125°09'32.0"  313.8189  -221.0378
       2-3        360.259     33°34'14.0"  199.2099   300.1695
       3-4        342.247    340°11'36.0" -115.9694   321.9998
       4-5        336.231    206°15'24.0" -148.7460  -301.5387
       5-1        267.552    248°08'56.0" -248.3295   -99.5818
                 ----------                ---------  ---------
        Sum =  1,690.137                    -0.0162     0.0111

            Balanced                        Coordinates
        Dep          Lat      Point          X              Y
------------------------------------------------------------
     313.8225  -221.0403      1     243,605.596     25,393.201
     199.2133   300.1672      2     243,919.419     25,172.161
    -115.9661   321.9975      3     244,118.632     25,472.328
    -148.7428  -301.5409      4     244,002.666     25,794.325
    -248.3269   -99.5835      5     243,853.923     25,492.785

     Linear misclosure  = 0.0196
     Relative Precision = 1 in 86,300

     Area: 126,300 sq. ft.
            2.899 acres {if distance units are feet}

Adjusted Observations
---------------------
      Course     Distance    Azimuth        Point         Angle
--------------------------------------------------------------
        1-2      383.853   125°09'32"         1     57°00'38"
        2-3      360.259    33°34'16"         2     88°24'44"
        3-4      342.243   340°11'37"         3    126°37'21"
        4-5      336.231   206°15'22"         4     46°03'44"
        5-6      267.550   248°08'54"         5    221°53'32"
```

Note: The adjusted coordinates from Chapter 10 and here are provided below. The same linear misclosure and relative precision was achieved even though the coordinates vary as shown since the traverse was scaled incorrectly in Chapter 10.

| | 10's Solution | | Current Solution | |
|---|---|---|---|---|
| Sta | E (m) | N (m) | E (m) | N (m) |
| *A* | 243,605.596 | 25,393.201 | 243,605.596 | 25,393.201 |
| ***B*** | **243,919.416** | **25,172.162** | **243,919.419** | **25,172.161** |
| ***C*** | **244,118.628** | **25,472.327** | **244,118.632** | **25,472.328** |
| ***D*** | **244,002.663** | **25,794.322** | **244,002.666** | **25,794.325** |
| ***E*** | **243,853.921** | **25,492.784** | **243,853.923** | **25,492.785** |

**20.38** The traverse in Problem 10.21 was performed in the Pennsylvania SPCS 1983 north zone. The average elevation of the area was 85.78 m and the average geoid height was −31.85 m. Using 6,371,000 m for the mean radius of the earth, compute a project factor, reduce the observations to grid, and adjust the traverse using the compass rule. Compare this solution with that obtained in Problem 10.22.

Using the first and last station coordinates, the average k is

The elevation factor is 0.99999138

The reduced distances are:

| Course | Obs. Dist. | kavg | CF | Grid Dist. |
|---|---|---|---|---|
| *AB* | 224.111 | 0.99992411 | 0.99991549 | 224.0921 |
| *BC* | 116.738 | 0.99992399 | 0.99991537 | 116.7281 |
| *CD* | 231.566 | 0.99992383 | 0.99991520 | 231.5464 |
| *DE* | 97.217 | 0.99992368 | 0.99991505 | 97.2087 |

The traverse computations are:

```
Angle Summary
Station   Unadj. Angle      Adj. Angle
--------------------------------------
   1      253°03'38.0"      253°03'40.0"
   2       91°32'06.0"       91°32'08.0"
   3      242°25'54.0"      242°25'56.0"
   4      111°12'02.0"      111°12'04.0"
   5      295°31'13.0"      295°31'15.0"

Angular misclosure (sec): -10"
```

```
                                              Unbalanced
Course       Length       Azimuth          Dep          Lat
-------------------------------------------------------------
 1-2        224.092    324°01'03.0" -131.6627   181.3345
 2-3        116.728    235°33'11.0"  -96.2599   -66.0264
 3-4        231.546    297°59'07.0" -204.4713   108.6519
 4-5         97.209    229°11'11.0"  -73.5714   -63.5356
         ----------                 ---------   ---------
  Sum =     669.575                 -505.9652   160.4244

Misclosure in Departure = -505.9652 - -505.9400 = -0.0252
Misclosure in Latitude  = 160.4244 - 160.4030 = 0.0214
```

```
          Balanced                          Coordinates
      Dep          Lat       Point         X               Y
-------------------------------------------------------------
    -131.6542   181.3274      A    194,325.090      25,353.988
     -96.2555   -66.0301      B    194,193.436      25,535.315
    -204.4625   108.6445      C    194,097.180      25,469.285
     -73.5678   -63.5388      D    193,892.718      25,577.930
                              E    193,819.150      25,514.391

     Linear misclosure  = 0.0330
     Relative Precision = 1 in 20,300
```

Adjusted Observations

```
---------------------
    Course     Distance    Azimuth      Point      Angle
-------------------------------------------6,371,000-----
       AB      224.081 324°01'05.4"     A  253°03'42.4"
       BC      116.727 235°33'01.2"     B    91°31'55.8"
       CD      231.535 297°59'04.8"     C  242°26'03.6"
       DE       97.208 229°11'00.9"     D  111°11'56.1"
                                        E  295°31'15.0"
```

**Note:** That solution from Chapter 10 and here are different since this is a link traverse. The relative precision went from 1:8,800 to 1:20,300. The coordinates vary as shown below since the traverse was scaled incorrectly in Chapter 10.

| | Initial coordinates | | Final coordinates | |
|---|---|---|---|---|
| Sta | E (m) | N (m) | E (m) | N (m) |
| *A* | 194,325.090 | 25,353.988 | 194,325.090 | 25,353.988 |
| *B* | 194,193.440 | 25,535.326 | 194,193.436 | 25,535.315 |
| *C* | 194,097.180 | 25,469.288 | 194,097.180 | 25,469.285 |
| *D* | 193,892.720 | 25,577.937 | 193,892.718 | 25,577.930 |
| *E* | 193,819.150 | 25,514.391 | 193,819.150 | 25,514.391 |

**20.39*** If the geodetic azimuth of a line is 205°06'36.2" the convergence angle is −0°42'26.1" and the arc-to-chord correction is +0.8" what is the equivalent grid azimuth for the line?

**<u>205°39'03.1"</u>** = 205°06'36.1" + 0°42'36.2" + 0.8"

**20.40** If the geodetic azimuth of a line is 243°06′34.5″ the convergence angle is 0°46′44.2″ and the arc-to-chord correction is −0.9″, what is the equivalent grid azimuth for the line?

**<u>242°19′49.4″</u>** = 243°06′34.5″ − 0°46′44.2″ − 0.9″

**20.41** Using the values given in Problems 20.39 and 20.40, what is the acute grid angle between the two azimuths?

**<u>36°30′46.3″</u>**

**20.42** The grid azimuth of a line is 158°13′26″. If the convergence angle at the endpoint of the azimuth is −1°58′02.9″ and the arc-to-chord correction is +1.5″, what is the geodetic azimuth of the line?

**<u>156°15′21.6″</u>** = 158°13′26″ − 1°58′02.9″ − 1.5″

**20.43** Similar to Problem 20.42, except the convergence angle is +2°16′32.7″ and the arc-to-chord correction is +1.6″.

**<u>160°29′57.1″</u>** = 158°13′26″ + 2°16'32.7" − 1.6"

**20.44** A project is bounded by the geodetic coordinates in the southwest corner of (32°15′37″ N, 106°48′49″W, 1180 m) and in the northeast corner of (32°21′43″ N, 106°43′56″W, 1290 m). Should the transverse Mercator or the Lambert conformal conic map projection be used in the design of an LDP for the region?

**Transverse Mercator** since the region is 366″ in the north-south direction and only 293″ in width. Thus the transverse Mercator map projection works best since scale changes going east to west.

**20.45** In Problem 20.44 what scale factor ($k_{LDP}$) be used in the design of the LDP? (Use an average radius for the Earth.)

**<u>1.000183845</u>** $= 0.99999\left(1+\frac{1235}{6{,}371{,}000}\right)$

**20.46** The region in problem 20.44 is surrounds Las Cruces, NM, which is in the New Mexico Central Zone of the SPCS 1983. What is the appropriate project factor for this region?

**<u>1.00012372</u>**; The midpoint of the zone is at 32°18′10″N, 106°46′22.5″W, 1235 m. Thus the average scale factor, *k*, is 0.999929889. The elevation factor based on the average geodetic height is 1.000193847 yielding a project factor for the region of 1.000123723.

# 21 BOUNDARY SURVEYS

Asterisks (*) indicate problems that have answers given in Appendix G.

**21.1** Define the following terms:

(a) Aliquot part
From Section 21.1, paragraph 4:

"… smaller subdivisions of the United States *Public Land Survey System* (*PLSS*) commonly referred to as the *aliquot part."*

(b) Subdivision survey
From Section 21.2, paragraph 1:

"... *subdivision surveys* to establish new smaller parcels of land within lands already surveyed.

(c) Practical location

From Section 21.11, paragraph 1: "The term *practical location* is used by the legal profession to describe an agreement, either explicit or implied, in which two adjoining property owners mark out an ambiguous boundary, or settle a boundary dispute."

(d) Adverse possession
From Section 21.2, paragraph 3:

"Adverse rights can generally be applied to gain ownership of property by occupying a parcel of land for a period of years specified by state law, and performing certain acts. To claim land or rights to it by adverse possession, its occupation or use must be (1) actual, (2) exclusive, (3) open and notorious, (4) hostile, and (5) continuous. It may also be necessary for the property to be held under color of title (a claim to a parcel of real property based on some written instrument, though a defective one). In some states all taxes must be paid. The time required to establish a claim of adverse possession varies from a minimum of 5 years in California to a maximum of 60 years for urban property in New York. The customary period is 20 years."

**21.2** What is the primary responsibility of a professional surveyor?

From Section 21.2, paragraph 4: "The responsibility of a professional surveyor is to weigh all evidence and try to establish the originally intended boundary between the parties involved in any property-line dispute, although without legal authority to force a compromise or settlement."

**21.3** Visit your county courthouse and obtain a copy of a metes-and-bounds property description. Write a critique of the description, with suggestions on how the description could have been improved.

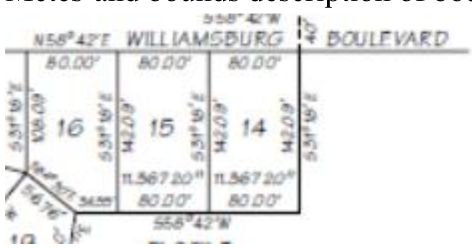
Student critique required.

**21.4** In a description by metes and bounds, what purpose may be served by the phrase "more or less" following the acreage?

From Section 21.4, paragraph 5: “The expression ‘more or less,’ which may follow a computed area, allows for minor errors, and avoids nuisance suits for insignificant variations.”

**21.5** Write a metes-and-bounds description for the exterior boundary of lot 23 in Figure 21.2.

Metes-and bounds description of boundary for lot 15 in Figure 21.2.




**21.6** Write a metes-and-bounds description for the house and lot where you live. Draw a map of the property.

Student metes-and-bounds description of personal lot.

**21.7** What are the essential elements required when writing a deed description by coordinates?

From Section 21.6: The date of the survey and the reference datum.

**21.8** From the metes-and-bounds description of the lot in the Town of Little Wolf, described in Section 21.4, compute the lot’s misclosure.

From WolfPack: **<u>0.003 ft</u>**

Title: Figure 21.1 Type: Polygon traverse

| Course | Length | Bearing | Unbalanced Dep | Lat |
|---|---|---|---|---|
| 1-2 | 424.26 | N90°00'00.0"E | 424.260 | 0.000 |
| 2-3 | 150.00 | S45°00'00.0"E | 106.066 | -106.066 |
| 3-4 | 200.00 | S45°00'00.0"W | -141.421 | -141.421 |
| 4-5 | 141.42 | S90°00'00.0"W | -141.420 | -0.000 |
| 5-1 | 350.00 | N45°00'00.0"W | -247.487 | 247.487 |

```
           ----------                 ---------  ---------
 Sum =   1,265.68                      -0.003     -0.000

            Balanced                            Coordinates
        Dep         Lat      Point         X              Y
------------------------------------------------------------
      424.261       0.000      1       5,000.00       5,000.00
      106.066    -106.066      2       5,424.26       5,000.00
     -141.421    -141.421      3       5,530.33       4,893.93
     -141.420      -0.000      4       5,388.91       4,752.51
     -247.487     247.487      5       5,247.49       4,752.51

     Linear misclosure  = 0.003
     Relative Precision = 1 in 466,600

     Area: 85,000 sq. ft.
            1.951 acres {if distance units are feet}
```

**21.9** What is the point of commencement in a property description?

From Section 21.2, paragraphs 6:
"1. *Point of commencement (POC)*. This is an established reference point such as a corner of the PLSS or NSRS monument to which the property description is tied or referenced. It serves as the starting point for the description."

**21.10** What is the point of beginning in a property description?

From Section 21.2, paragraph 7:
"2. *Point of beginning (POB)*. This point must be identifiable, permanent, well referenced, and one of the property corners. Coordinates, preferably state plane, should be given if known or computable. Note that a POB is no more important than others and a called for monument in place at the next corner establishes its position, even though bearing and distance calls to it may not agree."

**21.11** What is the primary objective in performing a retracement surveys?

From Section 21.7, paragraph 1:
"Retracement surveys are run for the purpose of relocating or reestablishing previously surveyed boundary lines."

**21.12*** List in their order of importance the following types of evidence when conducting retracement surveys: (a) measurements, (b) call for a survey, (c) intent of the parties, (d) monuments, and (e) senior rights.

From Section 21.7, paragraph 3: (e) Senior rights, (c) intent of parties, (b) call for survey, (d) monuments, (a) measurements

**21.13** In performing retracement surveys, list in their order of importance, the four different types of measurements called for in a description for your state.

From Section 21.7, paragraph 3: In most states, it is (**1**) distance, (**2**) direction, (**3**) area, and (**4**) coordinates. However in some states, such as Pennsylvania it is (**1**) direction, (**2**) distance, (**3**) area, and (**4**) coordinates.

**21.14** List in order the steps that must be performed in making subdivision surveys?

From Section 21.8, paragraph 7:
(1) Exterior survey; (2) Interior survey, design, and layout

**21.15** When writing a legal description using coordinates, what information must be included in the description?

From Section 21.6, paragraph 3: "In preparing coordinate descriptions nowadays, the datum upon which the coordinates are based should be identified as being either in NAD27, NAD83 (xxxx), or as appropriate, along with the epoch date of the coordinates to avoid any later confusion."

**21.16** Why are lot-and-block descriptions not subject to junior and senior rights?

From Section 21.5, paragraph 3: "Lot-and-block descriptions typically are created simultaneously and thus are not subject to junior and senior rights."

**21.17** Two disputing neighbors employ a surveyor to check their boundary line. Discuss the surveyor's authority if (a) the line established is agreeable to both clients, and (b) the line is not accepted by one or both of them.

(a) Obtain written agreement, record the accepted lines on the original plat, and have corrected deeds recorded.
(b) From Section 21.4, paragraph 4: Marks can be set on the ground, but their acceptance cannot be forced by a surveyor – it must be resolved in the courts.

**21.18** What is the purpose of retracement surveys?

From Section 21.7, paragraph 1. "Retracement surveys are run for the purpose of relocating or reestablishing previously surveyed boundary lines."

**21.19** Compute the misclosure of lot 19 in Figure 21.2. On the basis of your findings, would this plat be acceptable for recording? Explain.

Lot 19

| Course | Distance | Azimuth | Departure | Latitude |
|---|---|---|---|---|
| *AB* | 56.76 | 95°30' | 56.50 | –5.44 |
| *BC* | 113.30 | 156°17'40" | 45.55 | –103.74 |

| | | | | |
|---|---|---|---|---|
| *CD* | 10.00 | 246°17'40" | –9.16 | –4.02 |
| *DE* | 70.41 | 260°53'50" | –69.52 | –11.14 |
| *EF* | 36.16 | 275°30' | –35.99 | 3.47 |
| *FA* | 123.27 | 5°30' | 11.82 | 122.70 |
| | Σ = 409.90 | | Σ = –0.80 | Σ = 1.83 |

Linear misclosure: 2.00

Relative precision: 1:200

The lot does not close mathematically. It has a relative precision of only 1:200, which is not acceptable for a property survey.

**21.20*** Compute the area of lot 19 of Figure 21.2.

**9720 sq. ft.**

| Lot 19 X | Lot 19 Y | Double area | Double area |
|---|---|---|---|
| 56.50 | −5.44 | | −247.8 |
| 45.55 | −103.74 | −5,861.3 | 950.3 |
| −9.16 | −4.02 | −183.1 | 279.5 |
| −69.52 | -11.14 | 102.0 | 400.9 |
| −35.99 | 3.47 | −241.2 | 41.0 |
| 11.82 | 122.70 | −4,416.0 | 6,932.6 |
| 56.50 | −5.44 | −64.3 | |
| | | −10,663.9 | 8,356.4 |

Lot 19:

Area of segment = ½ × $139.62^2$[29°12'20"×π/180° – sin(29°12'20")] = 212 sq. ft.

Area: ½|10,663.9 + 8356.4| + 212 = **9720 sq. ft.**

**21.21** Determine the misclosure of lot 50 of Figure 21.2, and compute its area.

**Computer solution:**

```
Title: Problem 21-21 Type: Polygon traverse
                                          Unbalanced
Course       Length         Azimuth         Dep          Lat
~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
 1-2          30.38       8°30'00.0"      4.490       30.046
 2-3         123.00      35°00'00.0"     70.550      100.756
 3-4          90.77       9°30'00.0"     14.981       89.525
 4-5          90.11     143°00'00.0"     54.230      -71.965
 5-6          95.50     148°42'00.0"     49.614      -81.601
 6-7          19.67     238°42'00.0"    -16.807      -10.219
```

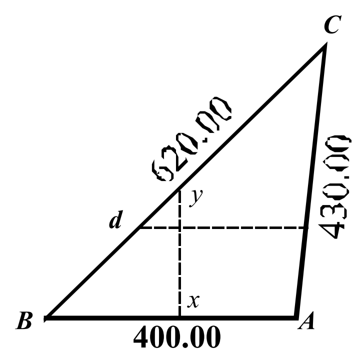
```
 7-8        133.19    245°00'30.0"  -120.719    -56.271
 8-1         62.59    251°19'00.0"   -59.292    -20.050
        ----------                 ---------  ---------
  Sum =     645.21                    -2.953    -19.779

            Balanced                          Coordinates
        Dep         Lat      Point          X              Y
~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
         4.629      30.978      1        1,000.00       1,000.00
        71.113     104.526      2        1,004.63       1,030.98
        15.397      92.308      3        1,075.74       1,135.50
        54.642     -69.203      4        1,091.14       1,227.81
        50.051     -78.673      5        1,145.78       1,158.61
       -16.717      -9.616      6        1,195.83       1,079.94
      -120.110     -52.188      7        1,179.12       1,070.32
       -59.005     -18.131      8        1,059.01       1,018.13

     Linear misclosure  = 19.998
     Relative Precision = 1 in 0

     Area: 18,200 sq. ft.
            0.418 acres {if distance units are feet}
```

**21.22** For the accompanying figure; using a line perpendicular to *AB* through *x*, divide the parcel into two equal parts, and determine lengths *xy* and *By*.




a = 620 s – a = 105
b = 430 s – b = 295
c = <u>400</u> s – c = 325
Σ = 1450 $Area = \sqrt{725(105)(295)(325)} = 85,431\ \text{ft}^2$

2/5 Area = 34,172 ft$^2$

$$B = \cos^{-1}\left(\frac{620^2 + 400^2 - 430^2}{2 \times 620 \times 400}\right) = 43°32'53"$$

Area BXY = ½(BX)(XY) = 34,172
XY = BX tan B = 0.950555 BX
34,172 = ½×0.950555 BX$^2$; so

BX = 268.14 ft
XY = 268.14 (0.950555) = 254.88 ft

$$BY = \sqrt{266.14^2 + 254.88^2} = 368.50ft$$

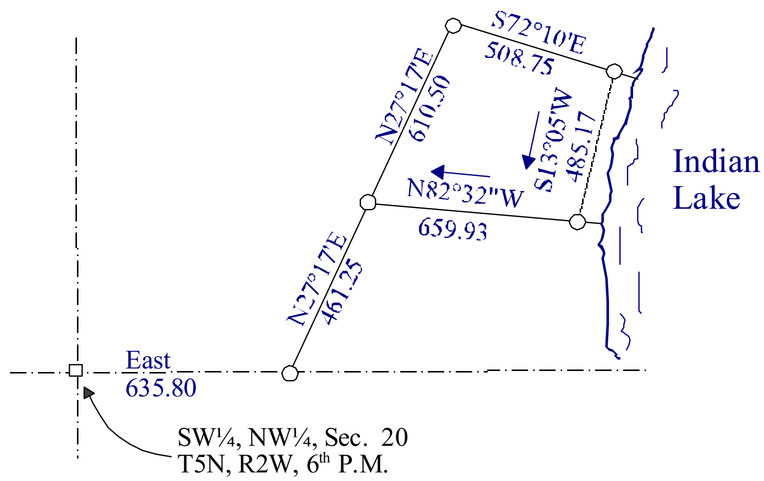
**21.23** For the figure of Problem 21.22, calculate the length of line *de*, parallel to *BA*, which will divide the tract into two equal parts. Give lengths *Bd*, *de* and *eA*.

From Problem 22.22, Area = 85,431 sq. ft.
Required Area = 85,431 – 34,172 = 51,260 sq. ft.
51,260 = [400 + *de*/2]*h*; *de* = 400 – *h*/tan *B*;
where from Problem 22-22, *B* = 43°32’53”
*C* = sin$^{-1}$[2(85,431)/(620.0×43.0)] = 39°51’31”

$A = 180° – 43°32'53" – 39°51'31" = 96°35'36"$
$51{,}260 = [400/2 + 400/2 – h/(2 \tan B) + (h/2) \tan (A+90°)]h$
$51{,}260 = 400\, h – 0.4682\, h^2$
$h = 157.00$ ft

$de = 400 – 157.00/\tan(43°32'53") + 157.00 \tan(6°35'36") = 252.98$ ft
$Bd = 157.00/\sin(43°32'53") = 227.88$ ft
Departure of $Ae = 227.88 \sin 43°32'53" + 252.98 – 400 = 9.98$ ft
$Ae = \sqrt{9.98^2 + 157.00^2} = 1576.31$ ft

**21.24** Prepare a metes-and-bound description for the parcel shown. Assume all corners are marked with 1-in. diameter steel rods, and a 20 ft. meander line setback from Indian Lake.

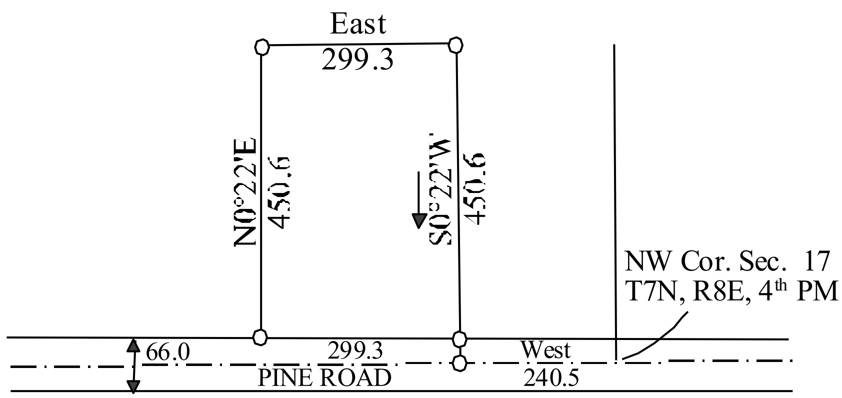



Commencing at a 1-in. diameter steel rod at the SW¼ , NW¼ , Sec. 20, T5N, R2W, 6th P.M.; thence east, 635.80 ft. to a 1-in. diameter steel rod; then N27°17′E, 461.25 ft. to a 1-in. diameter steel rod which is the point of beginning of this parcel; thence N27°17′E, 610.50 ft. to a 1-in. diameter steel rod; thence S72°10′E, 508.75 ft. to a 1-in. diameter steel rod, located N72°10′E, 20 ft. more or less from Indian Lake; said pipe being the beginning of a meander line along said Indian Lake; thence S13°05′W, 485.17 ft. along the meander line to a 1-in. diameter steel rod at the end of the meander line, said pipe being N82°32′W, 659.93 ft. to a 1-in. diameter steel rod at the point of beginning, including all lands lying between the meander line and the westerly shore of Indian Lake which lie between the extensions of the northerly and southerly boundaries of the parcel herein described; said parcel containing 7.39 acres, more or less.

**21.25** Draw a plat map of the parcel in Problem 21.24 at a convenient scale. Label all monuments and the lengths and directions of each boundary line on the drawing. Include a title, scale, North arrow, and legend.

Scaled drawing required.

**21.26** Prepare a metes-and-bounds description for the property shown. Assume all corners are marked with 2-in. diameter iron pipes.




Commencing at a 2-in. diameter iron pipe at the NW corner, Sec 17, T7N, R8E, 4th PM; thence West, 240.5 ft. along the centerline of Pine Road to a 2-in. diameter iron pipe; thence North, 33.0 ft. to a 2-in. diameter iron pipe which is the point of beginning for this parcel; thence West, 299.3 ft. along the northerly right of way line of Pine Road to a 2-in. diameter iron pipe; thence N0°22′E, 450.6 ft. to a 2-in. diameter iron pipe; thence East, 299.3 ft. to a 2-in diameter iron pipe; thence S0°22′W, 450.6 ft. to a 2-in. diameter iron pipe at the point of beginning, said parcel containing 3.10 acres, more or less.

**21.27** Create a 1.25-acre tract on the westerly side of the parcel in Problem 21.26 with a line parallel to the westerly property line. Give the lengths and bearings of all lines for both new parcels.

Required area = 1.25(43,560) = 54,450 sq. ft. = 450.6 $x$ where $x$ = 120.84 ft

**West Lot**

| Course | Length | Bearing |
|---|---|---|
| BC | 450.60 | N0°22′E |
| CF | 145.01 | East |
| FE | 450.06 | S0°22′W |
| EB | 145.01 | West |

**East Lot: AE = 299.13 – 120.84 = 178.46 ft**

| Course | Length | Bearing |
|---|---|---|
| EF | 450.60 | N0°22′E |
| FD | 120.84 | East |

| | | |
|---|---|---|
| DA | 450.06 | S0°22′W |
| AE | 120.84 | West |

**21.28** Discuss the ownership limits of a condominium unit.

Typically, to the center of the walls, floors and ceilings of the dwelling.

**21.29** Define *common elements* and *limited common elements* in relation to condominiums. Given examples of each.

From Section 21.12, paragraph 2:

Common elements are elements jointly owned and used by all units such as sidewalks, stairways, swimming pool, tennis courts, etc.

Limited common elements are elements reserved for the exclusive use of a particular unit such as a designated parking space.

**21.30** What types of measurements are typically made by surveyors in performing work for condominium developments?

As-built surveys.

**21.31** What is the objective of a resurvey of an existing property?

From 21.7, paragraph 1: "The objective of resurveys therefore is to restore boundary markers to their original locations, not to correct them, and this should guide all of the surveyor's actions."

**21.32** List in order the steps that must be performed in making subdivision surveys.

From 21.8: Exterior survey and an interior survey including design and layout.

# 22 PUBLIC LAND SURVEYS

**22.1*** Convert 41.3 chains to feet.

**<u>2726 ft</u>** = 41.3(66) = 2725.8

**22.2** What is meant by the term public lands?

From Section 22.1, paragraph 1: "The term *public lands* is applied broadly to the areas that have been subject to administration, survey, and transfer of title to private owners under the public lands laws of the United States since 1785. These lands include those turned over to the federal government by the colonial states, and the larger areas acquired by purchase from (or treaty with) the Native Americans or foreign powers that had previously exercised sovereignty."

**22.3** In what states are public land surveys not applicable?

From Section 22.1, paragraph 2: "Title to the vacant lands, and therefore direction over the surveys within their own boundaries, was retained by the colonial states, the other New England and Atlantic coast states (except Florida), and later by the states of West Virginia, Kentucky, Tennessee, Texas, and Hawaii."

**22.4** Define aliquot part.

Smaller subdivisions of Sections in the U.S. Public Land Survey System (PLSS).

**22.5** Why are the boundaries of the public lands established by duly appointed surveyors unchangeable, even though incorrectly set in the original surveys?

From Section 22.2, paragraph 8: "Correcting mistakes or errors now would disrupt too many accepted property lines and result in an unmanageable number of lawsuits."

**22.6** What is the convergence in feet of meridians for the following conditions:

**(a)*** 24 mi apart, extended 24 mi, at mean latitude 45°20′ N.

**<u>777.0 ft</u>** By Equation 22.2: $c = \frac{4}{3}24(24)\tan(45°20') = 776.99$ ft

**(b)** 6 mi apart, extended 12 mi, at mean latitude 29°45′N

**<u>54.9 ft</u>** By Equation 22.2: $c = \frac{4}{3}6(12)\tan 29°45' = 54.87$ ft

**22.7** What is the angular convergence, in seconds, for the two meridians defining a township exterior at mean latitude of:

(a) 64°56′N **11′09″** By Equation (22.1): $\theta = 52.13(6)\tan 64°56' = 668.7''$

(b) 29°39′N **2′58″** By Equation (22.1): $\theta = 52.13(6)\tan 29°39' = 178.0''$

**22.8** What is the nominal distance in miles between the following?

**(a)*** First Guide Meridian East, and the west Range Line of R8E.

**30 mi** = 9(6) − 24

**(b)** SE corner of Sec. 19, T6S, R5E, Indian PM, and the NE corner of Sec. 10, T6S, R5E, Indian PM.

**4.24 mi** $3\sqrt{2} = 4.24$ mi

**22.9** Define a range in the public land survey system.

From Section 22.9, paragraph 1: "North and south rows of townships are called *ranges*..."

Sketch and label pertinent lines and legal distances, and compute nominal areas of the parcels described in Problems 22.10 through 22.12.

**22.10** N1/2, SE1/4, Sec. 23, T1N, R2W, Boise PM. **80 ac with sketch**

**22.11** SE1/4, NW1/4, Sec. 13, T1N, R2E, Fairbanks PM. **40 ac with sketch**

**22.12** NE1/4, SE1/4, SE1/4, Sec. 30, T1S, R4E, 6th PM. **10 ac with sketch**

**22.13** What are the nominal dimensions and acreages of the following parcels:

**(a)** SE 1/4, NW 1/4, Sec. 22. **20 ch by 20 ch; 40 ac**

**(b)** N 1/2, NE 1/4, Sec. 1. **20 ch by 40 ch; 160 ac**

**(c)** SW 1/4, NW 1/4, NE 1/4, Sec. 14. **10 ch by 10 ch; 10 ac**

**22.14** How many rods of fence are required to enclose the following:

**(a)*** A parcel including the NE 1/4, NE 1/4, Sec. 32, and the NW 1/4, NW 1/4, Sec. 33, T 2 N, R 3 E?

**240 rods**

**(b)** A parcel consisting of Secs. 22, 23, 26, and 27 of T2N, R1W?

**2560 rods** $8(80)4 = 2560$

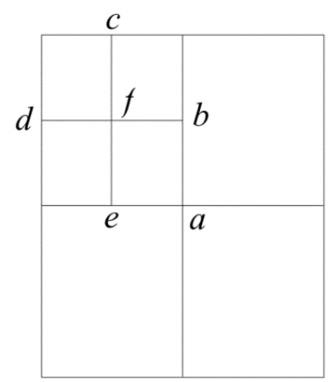
**22.15** What lines of the U.S. public-land system were run as random lines?

From Sections 221.10 and 221.11: All east-west section lines and those section lines from the 5-mi line north, except to the standard parallels. Those laid out parallel are section lines both north-south and east-west.

**22.16** In subdividing a township, which section line is run first? Which last?

From Section 22.11: The first is the north-south line between Sections 35 and 36. The last is the north-south line between sections 5 and 6.

**22.17** Corners of the SE 1/4 of the NW 1/4 of Sec. 22 are to be monumented. If all section and quarter-section corners originally set are in place, explain the procedure to follow, and sketch all lines to be run and corners set.

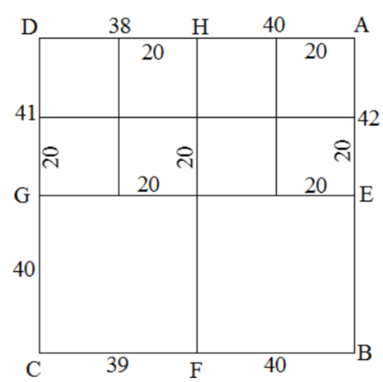



Check the section and quarter corners that are in place. Connect opposite quarter corners and set the center corner *a*. Split the distances between the four corners of the NW 1/4, and set the quarter-quarter corners *b, c, d,* and *e*. Connect opposite quarter-quarter corners and set *f* the center of the NW 1/4. The SE 1/4 is shown as a solid area.

**22.18** The quarter-section corner between Secs. 15 and 16 is found to be 40.28 ch from the corner common to Secs. 9, 10, 15, and 16. Where should the quarter-quarter-section corner be set along this line in subdividing Sec. 15?

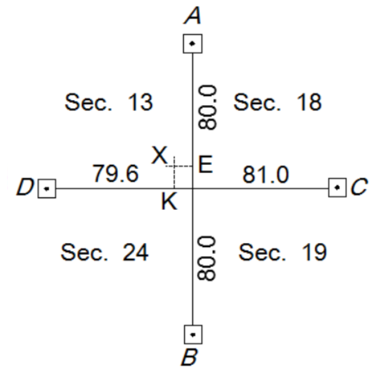
Set the quarter-quarter corner at 40.28 ch/2, which is **20.14** ch or **1329** ft on the section line from the NW corner of Section 15.

**22.19** As shown in the figure, in a normal township the exterior dimensions of Sec. 6 on the west, north, east, and south sides are 81, 78, 82, and 79 ch, respectively. Explain with a sketch how to divide the section into quarter sections.




Assuming all the section corners are in place, set the center of section at the intersection of EG and HF. Set the quarter-quarter corners 20 ch north from E, B, F, C, and G, and 20 ch west from A, B, F, and H. Connect the quarter-quarter corners thus throwing all the discrepancies to the north and west.

**22.20** The problem figure shows original record distances. Corners *A, B, C*, and *D* are found, but corner *E* is lost. Measured distances are $AB$ = 7035.6 ft and $CD$ = 10,718.4 ft. Explain how to establish corner *E*.




$AE$ = **<u>5367.90 ft</u>**; $DK$ = **<u>5345.58 ft</u>**

$$AE = \frac{80.0}{160.0}7035.6 = 3517.80$$

$$DK = \frac{79.6}{160.6}10718.4 = 5312.48 \text{ ft}$$

Establish *K* at 5312.48 ft from *D*. Establish *E* at 3517.80 ft from *A* or *B* . Run east-west line through *E* and north-south line through *K*. Their intersection at *X* is the re-established section corner.

To restore the corners in Problems 22.21 through 22.24, which method is used, single proportion or double proportion?

**22.21*** Township corners on guide meridians; section corners on range lines

**<u>single; single</u>**

**22.22** Section corners on section lines; township corners on township lines.

**<u>double; double</u>**

**22.23** Quarter-section corners on range lines.

**<u>single</u>**

**22.24** Quarter-quarter-section corners on section lines.

**<u>single</u>**

**22.25** What is a closing corner?

From Section 22.7, paragraph 2: "Because meridians converge, a closing corner (CC) is set at the intersection of each guide meridian and standard parallel or baseline (see Figure 22.3)."

**22.26** What is a quadrangle?

From Section 22.7, paragraph 3: "Correction lines and guide meridians, established according to instructions, created quadrangles (or tracts) whose nominal dimensions are 24 mi on a side." They are the tracts of land contained between adjacent guide meridians and standard parallels.

**22.27** Why are meander lines not accepted as the boundaries defining the ownership of lands adjacent to a stream or

From Section 22.18, paragraph 3: "Meander lines follow the mean high-water mark and are used only for plotting and protraction of the area. They are not boundaries defining the limits of property adjacent to the water" because they do not follow the shorelines exactly.

**22.28** The southern boundary of a township lies on a standard parallel at latitude 38°30′N. What is the theoretical length of its northern boundary?

**38.3 ft** Latitude increases about 1.15′ per statute mile so mean latitude is about 38°30′ + 3(1.15′) = 38°33′27″. By Eq. (221.2): $c = 4/3(6)(6)\tan(38°33'27'') = 38.26$ ft.

**22.29** Why are the areas of many public-lands sections smaller than the nominal size?

Convergence of the meridians and errors and mistakes in the original surveys that allowed low precisions.

**22.30** Visit the NILS web site and briefly describe the four components of NILS.

The four major components are survey management, measurement management, parcel management, and the Geocommunicator. The students should briefly describe each of these.

**22.31** Visit the BLM website at http://www.blm.gov/wo/st/en/prog/more/nils.html, and prepare a paper on the NILS project.

Individual report.

# 23 CONSTRUCTION SURVEYING

Asterisks (*) indicate problems that have partial answers given in Appendix G.

**23.1** What are the first requirements in a construction survey?

From Section 23.1, paragraph 2: "An accurate **control, topographic survey, and site map** are the first requirements in designing streets, sewer and water lines, and structures."

**23.2** List the common steps for surveying engineers in a construction project.

From Section 23.1, paragraph 5: "Common steps for surveying engineers in any construction project consist of (1) placement of horizontal and vertical control, (2) a topographic survey used in the creation of an existing surface map, (3) staking of the design, which may include physically staking the design on the ground to guide the equipment operators or calibration of the equipment and uploading and maintenance of the multiple design surfaces into a machine control system, (4) periodic checks on the layout of the design determination of quantities moved or placed during the construction process, and (5) a final as-built survey of the project."

**23.3** Describe how a plumbing level can be used to ensure verticality in the construction of a tall building.

From Section 231.2.1: "The instrument shown in Figure 231.2 projects a visible laser beam a distance of 5 m below and 100 m above the instrument along the plumb line. These instruments are useful for alignment of objects in vertical structures. A similar type of single-beam laser projects a visible laser-beam at a selected grade–a device that is especially useful in aligning pipelines." The instrument can be set under precisely laid out holes in each floor to ensure that verticality is maintained in the building.

**23.4** What details must the surveyor and contractor discuss before the start of a pipeline project?

From Section 23.4, paragraph 3: "Prior to staking a pipeline, the surveyor and contractor should discuss details of the project. An understanding must be reached concerning the planned trench width, where the installation equipment will be placed, and how and where the excavated material will be stockpiled."

**23.5** Discuss how a laser is used in pipeline layout.

From Section 23.4, paragraph 7: "If laser devices are used for laying pipes, the beam is oriented along the pipe's planned horizontal alignment and grade, and the trench opened. Then with the beam set at some even number of feet above the pipe's planned invert, measurements can be made using a story pole to set the pipe segments. Thus, the laser beam is equivalent to a batter board string line. On some jobs that have a deep wide cut, the laser instrument is set up in the trench to give line and grade for laying pipes. And, if the pipe is large enough, the laser beam can be oriented inside it."

**23.6** What is a story pole and how is it used in pipeline layout?

From Section 23.4, paragraph 5: "A graduated pole or special rod, often called a *story pole,* is used to measure the required distance from the string to the pipe invert."

**23.7** Where should stakes be set closer on the stake out of a pipeline and why?

From Section 231.4, paragraph 4: "Marks should be closer together on horizontal and vertical curves than on straight segments." Placing stakes closer together aids in defining the curve by reducing chord distances.

**23.8** What information is typically conveyed to the contractor on stakes for laying a pipeline?

From Section 23.5, paragraph 2: "Information conveyed to the contractor on stakes for laying pipelines usually consists of two parts: (1) giving the depth of cut (or fill), normally only to the nearest 0.1 ft, to enable a rough trench to be excavated; and (2) providing precise grade information, generally to the nearest 0.01 ft, to guide in the actual placement of the pipe invert at its planned elevation."

**23.9** A sewer pipe is to be laid from station 10+00 to station 12+50 on a −0.50% grade, starting with invert elevation 3344.23 ft at 10+00 Calculate invert elevations at each 50-ft station along the line.

| Station | Elevation (ft) |
|---|---|
| 10+00 | 3344.23 |
| 10+50 | 3343.98 |
| 11+00 | 3343.73 |
| 11+50 | 3343.48 |
| 12+00 | 3343.23 |
| 12+50 | 3342.98 |

**23.10*** A sewer pipe must be laid from a starting invert elevation of 784.58 ft at station 9+25 to an ending invert elevation 782.48 ft at station 12+75. Determine the uniform grade needed, and calculate invert elevations at each 50-ft station.

| Station | Elevation (ft) | grade |
|---|---|---|
| 925 | 784.58 | −0.60% |
| 950 | 784.43 | |
| 1000 | 784.13 | |
| 1050 | 783.83 | |
| 1100 | 783.53 | |
| 1150 | 783.23 | |
| 1200 | 782.93 | |

| | |
|---|---|
| 1250 | 782.63 |
| 1275 | 782.48 |

**23.11** Grade stakes for a pipeline running between stations 0+00 and 5+64 are to be set at each full station. Elevations of the pipe invert must be 868.25 ft at station 0+00 and 862.05 ft at 5+64, with a uniform grade between. After staking an offset centerline, an instrument is set up nearby, and a plus sight of 4.06 taken on BM *A* (elevation 873.25 ft). The following minus sights are taken with the rod held on ground at each stake: (0 + 00, 5.51); (1 + 00, 5.67); (2 + 00, 5.03); (3 + 00, 7.16); (4 + 00, 7.92); (5 + 00, 8.80); (5+64, 9.10) and (*A*, 4.06). Prepare a set of suitable field notes for this project (see Plate B.6 in Appendix B) and compute the cut required at each stake. Close the level circuit on the benchmark.

| | | | | | Grade: | −1.10% |
|---|---|---|---|---|---|---|
| Station | +Sight | HI | −Sight | Ground Elevation | Pipe Invert | Cut/Fill |
| A | 4.06 | 877.31 | | 873.25 | | |
| 0+00 | | | 5.51 | 871.80 | 868.25 | C3.55 |
| 1+00 | | | 5.67 | 871.64 | 867.15 | C4.49 |
| 2+00 | | | 5.03 | 872.28 | 866.05 | C6.23 |
| 3+00 | | | 7.16 | 870.15 | 864.95 | C5.20 |
| 4+00 | | | 7.92 | 869.39 | 863.85 | C5.54 |
| 5+00 | | | 8.80 | 868.51 | 862.75 | C5.76 |
| 5+64 | | | 9.10 | 868.21 | 862.05 | C6.16 |
| A | | | 4.06 | 873.25 | | |

**23.12** If batter boards are to be set exactly 8.00 ft above the pipe invert at each station on the project of Problem 23.11, calculate the necessary rod readings for placing the batter boards. Assume the instrument has the same HI as in Problem 23.11.

| Station | +Sight | HI | Rod Reading |
|---|---|---|---|
| A | 4.06 | 877.31 | |
| 0+00 | | | 1.06 |
| 1+00 | | | 2.16 |
| 2+00 | | | 3.26 |
| 3+00 | | | 4.36 |
| 4+00 | | | 5.46 |
| 5+00 | | | 6.56 |
| 5+64 | | | 7.26 |

**23.13** Assume there is a pipe where a change in direction of 20° occurs 30 ft from the beginning of an 80-ft pipe. At this same location, the grade changes from +0.5% to +15%. What is the appropriate bend angle in the pipe to the nearest decimal degree to accommodate the change in horizontal direction and change in grade?

**19.9°**

| Vector | *i* | *j* | *k* |
|---|---|---|---|
| V1 | 0 | 30.00 | 0.15 |
| V2 | 17.101 | 46.98 | 7.5 |

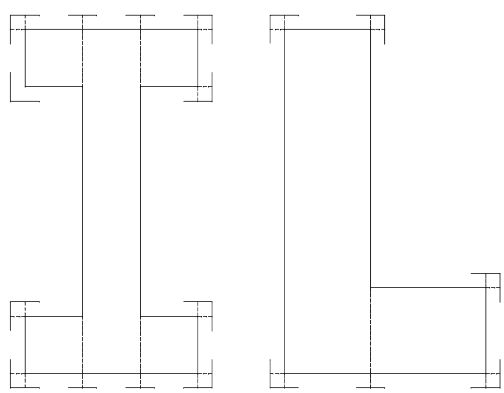
$$\theta = \cos^{-1}\frac{0(17.10)+30(46.98)+0.15(7.5)}{30(50)} = 19.87°$$

**23.14** What are the requirements for the placement of horizontal and vertical control in a project?

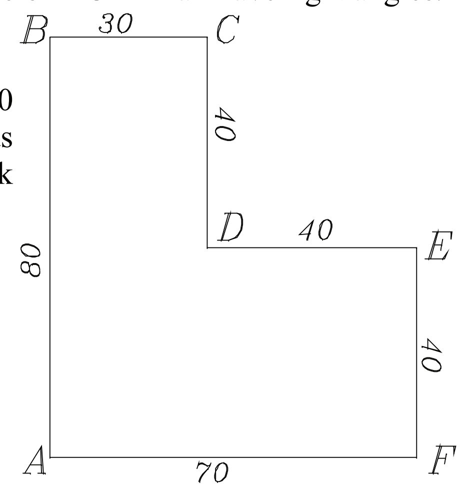
From Section 23.3, paragraph 3: "The control points must be:

1. Convenient for use, that is, located sufficiently close to the item being built so that work is minimized and accuracy enhanced in transferring alignment and grade.
2. Far enough from the actual construction to ensure working room for the contractor and to avoid possible destruction of stakes.
3. Clearly marked and understood by the contractor in the absence of a surveyor.
4. Supplemented by guard stakes to deter removal, and referenced to facilitate restoring them. Contracts usually require the owner to pay the cost of setting initial control points and the contractor to replace damaged or removed ones.
5. Suitable for securing the accuracy agreed on for construction layout (which may be to only the nearest foot for a manhole, 0.01 ft for an anchor bolt, or 0.001 ft for a critical feature)."

**23.15** By means of a sketch, show how and where batter boards should be located: (**a**) for an I-shaped building (**b**) For an L-shaped structure.



**23.16** A building in the shape of an L must be staked. Corners *ABCDEF* all have right angles. Proceeding clockwise around the building, the required outside dimensions are *AB* = 80.00 ft, *BC* = 30.00 ft, *CD* = 40.00 ft, *DE* = 40.00 ft, *EF* = 40.00 ft, and *FA* = 70.00 ft. After staking the batter boards for this building and stretching string lines taut, check measurements of the diagonals should be made. What should be the values of *AC, AD, AE, FB, FC, FD,* and *BD?*




$$AC = \sqrt{80^2 + 30^2} = 85.44\,\text{ft}$$

$$AD = \sqrt{40^2 + 30^2} = 50.00\,\text{ft}$$

$$AE = \sqrt{40^2 + 70^2} = 80.62\,\text{ft}$$

$$FB = \sqrt{80^2 + 70^2} = 106.30\,\text{ft}$$

$$FC = \sqrt{80^2 + 40^2} = 89.44\,\text{ft}$$

$$FD = \sqrt{40^2 + 40^2} = 56.57\,\text{ft}$$

$$BD = \sqrt{40^2 + 30^2} = 50.00\,\text{ft}$$

**23.17*** Compute the floor area of the building in Problem 23.16.

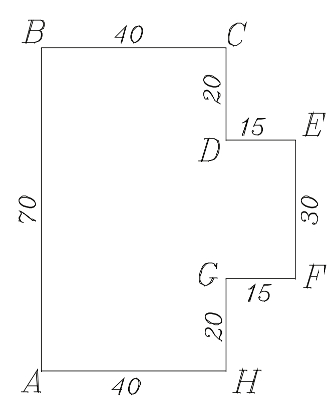
Area: = 30(80) + 40(40) = **4000 ft$^2$**

**23.18*** The design floor elevation for a building to be constructed is 1069.31 ft. An instrument is set up nearby, leveled, and a plus sight of 5.34 ft taken on BM *A* whose elevation is 1070.58 ft. If batter boards are placed exactly 1.00 ft above floor elevation, what rod readings are necessary on the batter board tops to set them properly?

**5.61 ft**

| Station | +Sight | HI | −Sight | Elev |
|---|---|---|---|---|
| A | 5.34 | 1075.92 | | 1070.58 |
| | | | 5.61 | 1069.31 |

**23.19** Compute the diagonals necessary to check the stakeout of the building in Figure 23.8.




$$AC = BH = \sqrt{40^2 + 70^2} = 80.62\,\text{ft}$$

$$BE = AF = \sqrt{55^2 + 20^2} = 58.52\,\text{ft}$$

$$BF = AE = \sqrt{50^2 + 55^2} = 74.33\,\text{ft}$$

$$BD = AG = \sqrt{40^2 + 20^2} = 44.72\,\text{ft}$$

**23.20** Why is it necessary to design a street with a grade that is at least 0.50% between block corners?

From Section 23.7, paragraph 16: "Streets need a minimum 0.50% grade for drainage from intersection to intersection, or from midblock both ways to the corners. They are also crowned to provide for lateral flow to gutters. Drainage profiles, prepared to verify or construct drainage cross sections, can be used to locate drainage structures and easements accurately. An experienced engineer when asked a question regarding the three most important items in highway work, thoughtfully replied "drainage, drainage, and drainage."

**23.21** Where is the invert of a pipe measured?

From Section 23.4, paragraph 5: "the *invert* (flow line or lower inside surface) of the pipe"

**23.22** What is meant by stakeless construction?

From Section 23.12, paragraph 1: "In recent years, research has led to *stakeless* construction where GNSS receivers, robotic total stations, and laser levels are used to guide earth-moving equipment in real time."

**23.23** How can horizontal orientation be achieved with a total station on a construction site?

From Section 23.10, paragraph 3: 'With total station instruments, three methods are commonly used for horizontal orientation: (1) azimuth, (2) coordinates, and (3) resection."

**23.24** How can vertical orientation be achieved with a total station on a construction site?

From Section 23.10, paragraph 7: "Vertical orientation of a total station (i.e., determining its *HI*) can be achieved using one of two procedures. The simplest case occurs if the elevation of the occupied station is known, as then it is only necessary to carefully measure and add the *hi* (height of instrument above the point) to the elevation of the point. If the occupied station's elevation is unknown, then another station of known elevation must be sighted. The situation is illustrated in Figure 23.1415, where the instrument is located at station A of unknown elevation, and station B whose elevation is known is sighted. From slope distance *S* and zenith angle *z* the instrument computes *V*. Then its *HI* is

$$HI = elev_B + h_r - V \tag{23.1}$$

where $h_r$ is the reflector height above station *B*."

**23.25** Discuss the advantages of combining digital elevation models with design templates in staking out highway alignments with a data collector.

From Section 23.7, paragraph 14: "Total station instruments, with their ability to automatically reduce measured slope distances to horizontal and vertical components, speed slope staking significantly, especially in rugged terrain where slope intercept elevations differ greatly from centerline grade. When the data collector allows the user to input the design template (see Section 26.3), it can rapidly determine the positions of the slope stakes using field observed data."

**23.26** The base receiver in a GNSS survey is started in autonomous mode. What must occur before the roving receivers can be used to stakeout points?

From Section 231.11, paragraph 2: “. The localization process discussed in Sections 15.9 and 19.7 transforms this set of low-accuracy GNSS coordinates into the project control reference frame eliminating the inaccuracies of the autonomous base station coordinates.”

**23.27** What is the surveyor’s role on a construction site using machine guidance or control?

From Section 23.12, paragraph 2: “Using machine guidance or control, the surveyor’s role in construction surveying shifts to tasks such as establishing the project reference coordinate systems and control, creating a DEM (see Section 18.14) of the existing surface for the design and grading work, managing the electronic design on the job site, calibrating the surveying equipment with respect to the construction site, providing for the calibration of the cutting surfaces of the heavy equipment with respect to the surveying control, and developing the necessary digital data for the system operation”

**23.28** A highway centerline subgrade elevation is 660.67 ft at station 12+00 and 665.80 ft at 17+00 with a smooth grade in between. To set blue tops for this portion of the centerline, a level is setup in the area and a plus sight of 5.19 ft taken on a benchmark whose elevation is 665.96 ft. From that HI, what rod readings will be necessary to set the blue tops for the full stations from 12+00 through 17+00?

HI = 671.15 $g$ = 1.03%

| Station | Subgrade Elevation | Rod Reading |
|---|---|---|
| 12+00 | 660.67 | 10.48 |
| 13+00 | 661.70 | 9.45 |
| 14+00 | 662.72 | 8.43 |
| 15+00 | 663.75 | 7.40 |
| 16+00 | 664.77 | 6.38 |
| 17+00 | 665.80 | 5.35 |

**23.29*** Similar to Problem 23.27, except the elevations at stations 12+00 and 17+00 are 405.65 and 410.63 ft, respectively, the BM elevation is 406.36 ft, and the backsight is 5.66 ft.

HI = 412.02 $g$ = 0.80%

| Station | Subgrade Elevation | Rod Reading |
|---|---|---|
| 12+00 | 406.65 | 5.37 |
| 13+00 | 407.45 | 4.57 |
| 14+00 | 408.24 | 3.78 |
| 15+00 | 409.04 | 2.98 |
| 16+00 | 409.83 | 2.19 |
| 17+00 | 410.63 | 1.39 |

**23.30** Discuss the checks that should be made when laying out a building using coordinates.

From Section 23.6, paragraph 3: "Measuring the distances between adjacent points, and also the diagonals checks the layout."

**23.31** What are the advantages of an as-built survey using a laser scanner?

From Section 23.13: "However, in projects that involve extensive detail, danger to instrument operator, or interruption of daily commerce, laser scanning can provide superior results in a fraction of the time. … A traditional survey would have either lacked the detail provided by the three-dimensional laser-scanned image or cost considerably more to locate all the existing elements. Using laser-scanning technology in these projects saved thousands of dollars and provided safe conditions for the field crews."

**23.32** What procedures may be used to ensure verticality on a multistory building?

From Section 23.7, paragraph 7: "On multistory buildings, care is required to ensure vertical alignment in the construction of walls, columns, elevator shafts, structural steel, and so on. One method of checking plumbness of constructed members is to carefully aim a total station's line of sight on a reference mark at the base of the member. The line of sight is then raised to its top. For an instrument that has been carefully leveled and that is in proper adjustment, the line of sight will define a vertical plane as it is raised. It should not be assumed that the instrument is in good adjustment; therefore, the line should be raised in both the direct and reversed positions. It is necessary to check plumbness in two perpendicular directions when using this procedure. To guide construction of vertical members in real time, two instruments can be set up with their lines of sight oriented perpendicular to each other, and verticality monitored as construction progresses. Alternatively, lasers can be used to guide and monitor vertical construction."

**23.33** How should finished grades be established in machine control projects?

From Section 23.11, paragraph 4: "However in finished grading, a robotic total station or laser level is required. As previously mentioned, one manufacturer has combined a laser level with the GNSS receiver to provide millimeter accuracies in both horizontal and vertical location."

**23.34** What is the minimum number of horizontal control points needed to establish finish-grades using a robotic total station on a machine-controlled project that is 2 mi in length?

**11control stations**; $\frac{2(5280)}{1000} = 10.56$

**23.35** What is the minimum number of control points needed to guide machines using a GNSS receiver on a machine-guidance project that is 6 mi in length?

**1 control station**; $\frac{6(5280)\left(\frac{12}{39.37}\right)}{10000} = 0.97$

**23.36** Do an article review on an application of machine control.

Independent study.

# 24 HORIZONTAL CURVES

Asterisks (*) indicate problems that have partial answers given in Appendix *G*.

**24.1** How is the effect of centrifugal force counteracted on a horizontal curve?

From Section 24.1, paragraph 4: "The effect of centrifugal force on a vehicle passing around a curve can be balanced by *superelevation*, which raises the outer rail of a track or outer edge of a highway pavement."

**24.2** What is the difference between a curve defined by the arc definition and one defined by the chord definition?

From Section 24.2, paragraph 2: "There are two different designations for degree of curve, the arc definition and the chord definition, both of which are defined using the English system of units. By the arc definition, degree of curve is the central angle subtended by a circular arc of 100 ft [see Figure 24.3(a)]. This definition is preferred for highway work. By the chord definition, degree of curve is the angle at the center of a circular arc subtended by a chord of 100 ft [see Figure 24.3(b)]."

**24.3** For the following circular curves having a radius *R*, what is their degree of curvature by (1) arc definition and (2) chord definition?

| | | | |
|---|---|---|---|
| ***(a)** | 500.00 ft | (1) **11°27′33″** | (2) **11°28′42″** |
| **(b)** | 1500.00 ft | (1) **3°49′11″** | (2) **3°49′14″** |
| **(c)** | 2800.00 ft | (1) **2°02′47″** | (2) **2°02′47″** |

Compute *L, T, E, M, LC, R*, and stations of the PC and PT for the circular curves in Problems 24.4 through 24.6. Use the chord definition for the railroad curve and the arc definition for the highway curves.

**24.4*** Railroad curve with $D_c = 4°00'$, $I = 24°00'$, and PI station = 36 + 45.00 ft.

**24.5** Highway curve with $D_a = 3°15'$, $I = 15°30'$, and PI station = 32 + 55.00 ft.

**24.6** Highway curve with $R = 500.000$ m, $I = 22°30'$, and PI station = 6+517.500 m.

| | **24.4** | **24.5** | **24.6** |
|---|---|---|---|
| PI | 36+45.00 | 24+65.00 | 6+517.500 |
| $D_c$ | 4 | | |
| $D_a$ | | 3.25 | |
| $R$ | 1432.68 | 1762.95 | 500 |
| $I$ | 24 | 15.5 | 22.5 |

| | | | |
|---|---|---|---|
| *L* | 600 | 476.92 | 196.350 |
| *T* | 304.53 | 239.93 | 99.456 |
| *E* | 32.01 | 16.25 | 9.796 |
| *M* | 31.31 | 16.10 | 9.607 |
| *LC* | 595.74 | 475.47 | 195.090 |
| PC | 33+40.47 | 22+25.07 | 6+418.044 |
| $PT_{Back}$ | 42+45.00 | 29+41.92 | 6+713.850 |
| $PT_{Forward}$ | 39+49.53 | 27+04.93 | 6+616.956 |

Tabulate *R* or *D, T, L, E, M,* PC, PT, deflection angles, and incremental chords to lay out the circular curves at full stations (100 ft or 30 m) in Problems 24.7 through 24.14.

**24.7** Highway curve with $D_a$ = 2°30′, $I$ = 10°30′, and PI station = 36 +44.50 ft.

```
                 Intersection Angle = 10°30'00"
                Degree of Curvature = 2°30'00"
                             Radius = 2,291.83
               Circular Curve Length = 420.00
                   Tangent Distance = 210.59
           Circular Curve Long Chord = 419.41
                    Middle Ordinate = 9.61
                           External = 9.65

                      PI Stationing = 36+44.50
                      38+53.91 Back = 38+55.09 Ahead

        Station |       Chord |   Defl. Increment |      Defl. Angle |
=====================================================================
       38+53.91 |       53.91 |          0°40'26" |          5°15'00" |
       38+00.00 |       99.99 |          1°15'00" |          4°34'34" |
       37+00.00 |       99.99 |          1°15'00" |          3°19'34" |
       36+00.00 |       99.99 |          1°15'00" |          2°04'34" |
       35+00.00 |       66.09 |          0°49'34" |          0°49'34" |
       34+33.91
=====================================================================
```

**24.8** Railroad curve with $D_c$ = 2°00′, $I$ = 10°00′, and PI station = 24 + 50.50 ft.

```
                 Intersection Angle = 10°30'00"
                Degree of Curvature = 2°30'00"
                             Radius = 2,291.83
               Circular Curve Length = 420.00
                   Tangent Distance = 210.59
           Circular Curve Long Chord = 419.41
                    Middle Ordinate = 9.61
                           External = 9.65

                      PI Stationing = 36+44.50
                      38+53.91 Back = 38+55.09 Ahead

     Station |       Chord |   Defl. Increment |       Defl. Angle |
==================================================================
```

| | | | |
|---|---|---|---|
| 38+53.91 | 53.91 | 0°40'26" | 5°15'00" |
| 38+00.00 | 99.99 | 1°15'00" | 4°34'34" |
| 37+00.00 | 99.99 | 1°15'00" | 3°19'34" |
| 36+00.00 | 99.99 | 1°15'00" | 2°04'34" |
| 35+00.00 | 66.09 | 0°49'34" | 0°49'34" |
| 34+33.91 | | | |

**24.9** Highway curve with $R = 550$ m, $I = 5°00'$, and PI station = 3 + 290.600 m.

```
     Intersection Angle = 5°00'00"
   Degree of Curvature = 3°10'31"
                Radius = 550.000
  Circular Curve Length = 47.997
      Tangent Distance = 24.014
Circular Curve Long Chord = 47.981
       Middle Ordinate = 0.523
              External = 0.524

        PI Stationing = 3+290.600
   3+314.583 Back = 3+314.614 Ahead
```

| Station | Chord | Defl. Increment | Defl. Angle |
|---|---|---|---|
| 3+314.583 | 14.583 | 0°45'35" | 2°30'00" |
| 3+300.000 | 29.996 | 1°33'45" | 1°44'25" |
| 3+270.000 | 3.414 | 0°10'40" | 0°10'40" |
| 3+266.586 | | | |

**24.10** Highway curve with $R = 900$ m, $I = 12°30'$, and PI station = 4+200.600 m.

```
     Intersection Angle = 12°30'00"
   Degree of Curvature = 1°56'26"
                Radius = 900.000
  Circular Curve Length = 196.350
      Tangent Distance = 98.566
Circular Curve Long Chord = 195.960
       Middle Ordinate = 5.349
              External = 5.381

        PI Stationing = 4+200.600
   4+298.384 Back = 4+299.166 Ahead
```

| Station | Chord | Defl. Increment | Defl. Angle |
|---|---|---|---|
| 4+298.384 | 8.383 | 0°16'01" | 6°15'00" |
| 4+290.000 | 29.999 | 0°57'18" | 5°58'59" |
| 4+260.000 | 29.999 | 0°57'18" | 5°01'42" |
| 4+230.000 | 29.999 | 0°57'18" | 4°04'24" |
| 4+200.000 | 29.999 | 0°57'18" | 3°07'06" |
| 4+170.000 | 29.999 | 0°57'18" | 2°09'48" |
| 4+140.000 | 29.999 | 0°57'18" | 1°12'31" |
| 4+110.000 | 7.966 | 0°15'13" | 0°15'13" |
| 4+102.034 | | | |

**24.11** Highway curve with $R$ = 1200 ft, $I$ = 30°00′, and PI station = 45 + 50.00 ft.

```
     Intersection Angle = 30°00'00"
    Degree of Curvature = 4°46'29"
                 Radius = 1,200.00
  Circular Curve Length = 628.32
       Tangent Distance = 321.54
Circular Curve Long Chord = 621.17
        Middle Ordinate = 40.89
               External = 42.33

         PI Stationing = 45+50.00
         48+56.78 Back = 48+71.54 Ahead
```

| Station | Chord | Defl. Increment | Defl. Angle |
|---|---|---|---|
| 48+56.78 | 56.77 | 1°21'20" | 15°00'00" |
| 48+00.00 | 99.97 | 2°23'14" | 13°38'40" |
| 47+00.00 | 99.97 | 2°23'14" | 11°15'26" |
| 46+00.00 | 99.97 | 2°23'14" | 8°52'11" |
| 45+00.00 | 99.97 | 2°23'14" | 6°28'57" |
| 44+00.00 | 99.97 | 2°23'14" | 4°05'43" |
| 43+00.00 | 71.53 | 1°42'28" | 1°42'28" |
| 42+28.46 | | | |

**24.12** Highway curve with $L$ = 270 m, $R$ = 600 m, and PI station = 4 + 350.000 m.

```
     Intersection Angle = 25°46'59"
    Degree of Curvature = 2°54'38"
                 Radius = 600.000
  Circular Curve Length = 270.000
       Tangent Distance = 137.325
Circular Curve Long Chord = 267.728
        Middle Ordinate = 15.124
               External = 15.515
          PI Stationing = 4+350.000
         4+482.675 Back = 4+487.325 Ahead
```

| Station | Chord | Defl. Increment | Defl. Angle |
|---|---|---|---|
| 4+482.675 | 12.675 | 0°36'19" | 12°53'30" |
| 4+470.000 | 29.997 | 1°25'57" | 12°17'11" |
| 4+440.000 | 29.997 | 1°25'57" | 10°51'14" |
| 4+410.000 | 29.997 | 1°25'57" | 9°25'18" |
| 4+380.000 | 29.997 | 1°25'57" | 7°59'21" |
| 4+350.000 | 29.997 | 1°25'57" | 6°33'24" |
| 4+320.000 | 29.997 | 1°25'57" | 5°07'28" |
| 4+290.000 | 29.997 | 1°25'57" | 3°41'31" |
| 4+260.000 | 29.997 | 1°25'57" | 2°15'35" |
| 4+230.000 | 17.325 | 0°49'38" | 0°49'38" |
| 4+212.675 | | | |

**24.13** Highway curve with $T$ = 131.65 ft, $R$ = 1200 ft, and PI station = 67 + 50.00 ft.

$$I = 2\,\text{atan}\left(\frac{131.65}{1000}\right) = 14°59'59"$$

```
      Intersection Angle = 14°59'59"
     Degree of Curvature = 4°46'29"
                  Radius = 1,200.00
   Circular Curve Length = 314.15
       Tangent Distance = 157.98
Circular Curve Long Chord = 313.26
        Middle Ordinate = 10.27
               External = 10.35

          PI Stationing = 67+50.00
          69+06.17 Back = 69+07.98 Ahead
```

| Station | Chord | Defl. Increment | Defl. Angle |
|---|---|---|---|
| 69+06.17 | 6.17 | 0°08'51" | 7°30'00" |
| 69+00.00 | 99.97 | 2°23'14" | 7°21'09" |
| 68+00.00 | 99.97 | 2°23'14" | 4°57'55" |
| 67+00.00 | 99.97 | 2°23'14" | 2°34'40" |
| 66+00.00 | 7.98 | 0°11'26" | 0°11'26" |
| 65+92.02 | | | |

**24.14** Railroad curve with $T = 300.00$ ft, $D_C = 2°30'$, and PI station = 48 + 00.00 ft.

$$R = 50/\sin\left(\frac{2°30'}{2}\right) = 2292.01\text{ ft}$$

$$I = 2\,\text{atan}\left(\frac{300}{2292.01}\right) = 14°54'51"$$

```
      Intersection Angle = 14°54'51"
     Degree of Curvature = 2°30'00"
                  Radius = 2,292.01
   Circular Curve Length = 596.57
       Tangent Distance = 300.00
Circular Curve Long Chord = 594.93
        Middle Ordinate = 19.39
               External = 19.55

          PI Stationing = 48+00.00
          50+96.56 Back = 51+00.00 Ahead
```

| Station | Chord | Defl. Increment | Defl. Angle |
|---|---|---|---|
| 50+96.56 | 96.56 | 1°12'25" | 7°27'25" |
| 50+00.00 | 100.00 | 1°15'00" | 6°15'00" |
| 49+00.00 | 100.00 | 1°15'00" | 5°00'00" |
| 48+00.00 | 100.00 | 1°15'00" | 3°45'00" |
| 47+00.00 | 100.00 | 1°15'00" | 2°30'00" |
| 46+00.00 | 100.00 | 1°15'00" | 1°15'00" |
| 45+00.00 | 0.00 | 0°00'00" | 0°00'00" |
| 44+100.00 | | | |

In Problems 24.15 through 24.18 tabulate the curve data, deflection angles, and total chords needed to lay out the following circular curves at full-station increments using a total station instrument set up at the PC.

**24.15** The curve of Problem 24.7.

```
             Intersection Angle = 10°30'00"
            Degree of Curvature = 2°30'00"
                         Radius = 2,291.83
          Circular Curve Length = 420.00
              Tangent Distance = 210.59
      Circular Curve Long Chord = 419.41
               Middle Ordinate = 9.61
                      External = 9.65

              PI Stationing = 36+44.50
              38+53.91 Back = 38+55.09 Ahead
```

| Station | Chord | Defl. Increment | Defl. Angle |
|---|---|---|---|
| 38+53.91 | 419.41 | 0°40'26" | 5°15'00" |
| 38+00.00 | 365.70 | 1°15'00" | 4°34'34" |
| 37+00.00 | 265.94 | 1°15'00" | 3°19'34" |
| 36+00.00 | 166.05 | 1°15'00" | 2°04'34" |
| 35+00.00 | 66.09 | 0°49'34" | 0°49'34" |
| 34+33.91 | | | |

**24.16** The curve of Problem 24.8

```
             Intersection Angle = 10°00'00"
            Degree of Curvature = 2°00'00"
                         Radius = 2,864.93
          Circular Curve Length = 500.00
              Tangent Distance = 250.65
      Circular Curve Long Chord = 499.39
               Middle Ordinate = 10.90
                      External = 10.94

              PI Stationing = 24+50.50
              26+99.85 Back = 27+01.15 Ahead
```

| Station | Chord | Defl. Increment | Defl. Angle |
|---|---|---|---|
| 26+99.85 | 499.39 | 0°59'55" | 5°00'00" |
| 26+00.00 | 399.84 | 1°00'00" | 4°00'05" |
| 25+00.00 | 300.03 | 1°00'00" | 3°00'05" |
| 24+00.00 | 200.12 | 1°00'00" | 2°00'05" |
| 23+00.00 | 100.15 | 1°00'00" | 1°00'05" |
| 22+00.00 | 0.15 | 0°00'05" | 0°00'05" |
| 21+99.85 | | | |

**24.17** The curve of Problem 24.9

```
             Intersection Angle = 5°00'00"
```

Degree of Curvature = 3°10'31"
Radius = 550.000
Circular Curve Length = 47.997
Tangent Distance = 24.014
Circular Curve Long Chord = 47.981
Middle Ordinate = 0.523
External = 0.524

PI Stationing = 3+290.600
3+314.583 Back = 3+314.614 Ahead

| Station | Chord | Defl. Increment | Defl. Angle |
|---|---|---|---|
| 3+314.583 | 47.981 | 0°45'35" | 2°30'00" |
| 3+300.000 | 33.408 | 1°33'45" | 1°44'25" |
| 3+270.000 | 3.414 | 0°10'40" | 0°10'40" |
| 3+266.586 | | | |

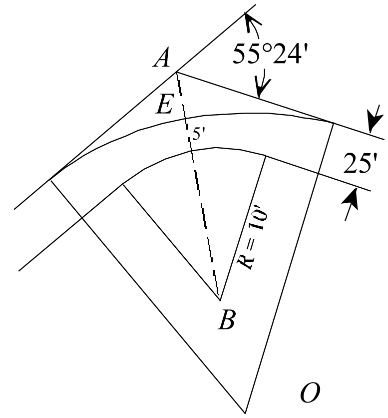
**24.18** The curve of Problem 24.10

Intersection Angle = 12°30'00"
Degree of Curvature = 1°56'26"
Radius = 900.000
Circular Curve Length = 196.350
Tangent Distance = 98.566
Circular Curve Long Chord = 195.960
Middle Ordinate = 5.349
External = 5.381

PI Stationing = 4+200.600
4+298.384 Back = 4+299.166 Ahead

| Station | Chord | Defl. Increment | Defl. Angle |
|---|---|---|---|
| 4+298.384 | 195.960 | 0°16'01" | 6°15'00" |
| 4+290.000 | 187.625 | 0°57'18" | 5°58'59" |
| 4+260.000 | 157.763 | 0°57'18" | 5°01'42" |
| 4+230.000 | 127.858 | 0°57'18" | 4°04'24" |
| 4+200.000 | 97.918 | 0°57'18" | 3°07'06" |
| 4+170.000 | 67.950 | 0°57'18" | 2°09'48" |
| 4+140.000 | 37.963 | 0°57'18" | 1°12'31" |
| 4+110.000 | 7.966 | 0°15'13" | 0°15'13" |
| 4+102.034 | | | |

**24.19** A rail line on the center of a 80-ft street makes a 55°24′ turn into another street of equal width. The corner curb line has $R$ = 10 ft. What is the largest R that can be given a circular curve for the track centerline if the law requires it to be at least 5 ft from the curb?

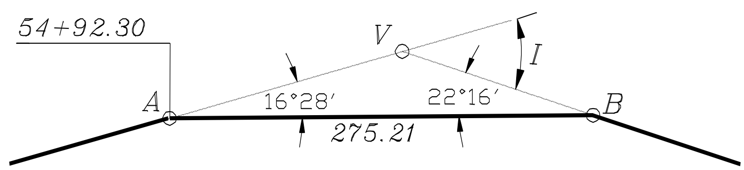



$$AB = (25 + 10)/\cos\left(\frac{55°24'}{2}\right) = 39.530 \text{ ft}$$

$$AE = 39.53 - (10+5) = 24.53 \text{ ft}$$

$$R = \frac{24.53}{1/\cos\left(\frac{55°24'}{2}\right) - 1} = 189.51 \text{ ft}$$

**24.20** A highway survey PI falls in a pond, so a cut off line $AB$ = 275.21 ft is run between the tangents. In the triangle formed by points A, B, and PI, the angle at $A$ = 16°28′ and at $B$ = 22°16′. The station of A is 54+92.30 ft. Calculate and tabulate curve notes to run, by deflection angles and incremental chords, a 4°30′ (arc definition) circular curve at full-station increments to connect the tangents.




$$I = 16°28' + 22°16' = 38°44'$$

$$AV = \frac{275.21\sin(22°16')}{\sin(180° - 38°44')} = 166.665 \text{ ft}$$

PI = 54 + 92.30 + 166.66 = 56 + 58.96

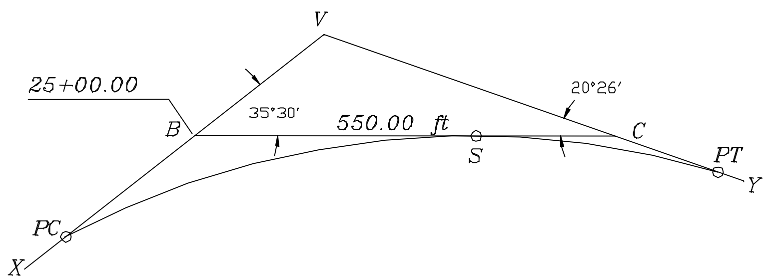
```
   Intersection Angle = 38°44'00"
 Degree of Curvature = 4°30'00"
               Radius = 1,273.24
 Circular Curve Length = 860.74
     Tangent Distance = 447.55
Circular Curve Long Chord = 844.44
     Middle Ordinate = 72.05
            External = 76.37

       PI Stationing = 56+58.96
       60+72.15 Back = 61+06.51 Ahead
```

| Station | Chord | Defl. Increment | Defl. Angle |
|---|---|---|---|
| 60+72.15 | 72.15 | 1°37'25" | 19°22'00" |
| 60+00.00 | 99.97 | 2°15'00" | 17°44'35" |
| 59+00.00 | 99.97 | 2°15'00" | 15°29'35" |
| 58+00.00 | 99.97 | 2°15'00" | 13°14'35" |

| | | | |
|---|---|---|---|
| 57+00.00 | 99.97 | 2°15'00" | 10°59'35" |
| 56+00.00 | 99.97 | 2°15'00" | 8°44'35" |
| 55+00.00 | 99.97 | 2°15'00" | 6°29'35" |
| 54+00.00 | 99.97 | 2°15'00" | 4°14'35" |
| 53+00.00 | 88.57 | 1°59'35" | 1°59'35" |
| 52+11.41 | | | |

**24.21** In the figure, a single circular highway curve (arc definition) will join tangents *XV* and *VY* and also be tangent to *BC*. Calculate *R, L*, and the stations of the PC and PT.




$$I = 35°30' + 20°26' = 55°56'$$

$$R = \frac{550}{\tan(35°30'/2) + \tan(20°20'/2)} = 1099.27 \text{ ft}$$

$$L = 1099.27I = 1073.13 \text{ ft}$$

$$T = 1099.27\tan(55°56'/2) = 583.67 \text{ ft}$$

$$BV = \frac{550.00\sin(20°26')}{\sin(55°56')} = 231.79 \text{ ft}$$

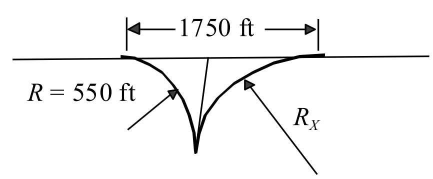
PI = 2500 + 231.79 = 27 + 31.79

PC = 2731.79 – 583.67 = 21 + 48.12

$\text{PT}_{Back}$ = 2148.12 + 1073.13 = 32 + 21.25

$\text{PT}_{Forward}$ = 2731.79 + 583.67 = 33 + 15.46

**24.22*** Compute $R_x$ to fit requirements of the figure and make the tangent distances of the two curves equal.




**1392.04 ft**

$$T_x = 1750/2 = 875$$

$$I = 2\tan^{-1}(875/550) = 115°41'43.5''$$

$$I_x = 180° - I = 64°18'16.5''$$

$$R_x = \frac{875}{\tan(64°18'16.5'')} = 1392.04 \text{ ft}$$

**24.23** After a backsight on the PC with $0°00'$ set on the instrument, what is the deflection angle to the following circular curve points?

**(a)*** Setup at curve midpoint, deflection to the PT. ***I*/2**

**(b)** Instrument at curve midpoint, deflection to the 3/4 point. **3/8*I***

**(c)** Setup at 1/4 point of curve, deflection to 3/4 point. **3/8*I***

**24.24** In surveying a construction alignment, why should the *I* angle be measured using both faces of the instrument?

To account for possible instrumental errors, to increase the precision of the observation, and to check for possible mistakes.

**24.25** A highway curve (arc definition) to the right, having $R = 500$ m and $I = 15°30'$, will be laid out by coordinates with a total station instrument setup at the PI. The PI station is 3 + 855.200 m, and its coordinates are $X = 75{,}428.863$ m and $Y = 36{,}007.434$ m. The azimuth (from north) of the back tangent proceeding toward the PI is 45°00′00″. To orient the total station, a backsight will be made on a POT on the back tangent. Compute lengths and azimuths necessary to stake the curve at 30-m stations.

| Station | δ | Total δ | Chord | Chord Azimuth |
|---|---|---|---|---|
| 3+922.416 | 1°17'04" | 7°45'00" | 134.851 | 52°45'1" |
| 3+900.000 | 1°43'08" | 6°27'56" | 112.608 | 51°27'57" |
| 3+870.000 | 1°43'08" | 4°44'48" | 82.752 | 49°44'49" |
| 3+840.000 | 1°43'08" | 3°01'40" | 52.822 | 48°1'41" |
| 3+810.000 | 1°18'33" | 1°18'33" | 22.845 | 46°18'33" |
| 3+787.153 | | | | 45°0'0" |

**24.26** In Problem 24.25, compute the *XY* coordinates at 30-m stations.

| Station | Azimuth | Chord | *X* | *Y* |
|---|---|---|---|---|
| 3+922.416 | 52°45'01" | 134.851 | 75,488.089 | 36,040.941 |
| 3+900.000 | 51°27'57" | 112.608 | 75,468.833 | 36,029.470 |
| 3+870.000 | 49°44'49" | 82.752 | 75,443.903 | 36,012.789 |
| 3+840.000 | 48°01'41" | 52.822 | 75,420.018 | 35,994.643 |
| 3+810.000 | 46°18'33" | 22.845 | 75,397.265 | 35,975.098 |
| 3+787.153 | 45°00'00" | | 75,380.746 | 35,959.317 |

**24.27** An exercise track must consist of two semicircles and two tangents, and be exactly 1000 m along its centerline. The two tangent sections are 200 m each. Calculate the radius for the curves.

Curves = 1000 − 400 = 600 m

$R = 600/(2\pi)$ = **<u>95.493 m</u>**

What sight distance is available if there is an obstruction on a radial line through the PI inside the curves in Problems 24.28 and 24.29?

**24.28*** For Problem 24.7, obstacle 15 ft from curve.

By Equation 24.24: $C = \sqrt{8(15)2291.83} = \mathbf{524\ ft}$

**24.29** For Problem 24.9, obstacle 10 m from curve.

By Equation 24.24: $C = \sqrt{8(10)550} = \mathbf{209\ m}$

**24.30** If the misclosure for the curve of Problem 24.7, computed as described in Section 24.8, is 0.05 ft, what is the field layout precision?

Precision = 0.05/[2(210.59) + 420.00], or 1:16,800

**24.31** Assume that a 100-ft entry spiral will be used with the curve of Problem 24.7. Compute and tabulate curve notes to stake out the alignment from the TS to ST at full stations using a total station and the deflection-angle, total chord method.

```
                 Spiral Angle: 1°15'00"
                 Spiral Throw: 0.18
          Spiral Long Tangent: 66.67
         Spiral Short Tangent: 33.33
                Spiral Length: 100.00
    Spiral Long Chord Length: 100.00

    Exit spiral notes for layout from ST to CS
           with tangent as backsight.
```

| | Station | Chord | Defl. Angle |
|---|---|---|---|
| ST | 39+03.89 | | |
| | 39+00.00 | 3.89 | 0°00'02" |
| CS | 38+03.89 | 100.00 | 0°25'00" |

```
       Intersection Angle = 10°30'00" (Back to Forward Tangent)
Circular Curve Intersection Angle = 8°00'00"
              Degree of Curvature = 2°30'00"
                           Radius = 2,291.83
            Circular Curve Length = 320.00
          Tangent Distance (TS-PI) = 260.61
        Circular Curve Long Chord = 319.74
              Long Chord (TS - ST) = 519.02
                         External = 10.39
  Circular Curve Tangent Distance = 160.26

                    PI Stationing = 36+44.50
              39+03.89 Back = 39+05.11 Ahead
```

| Station | Chord | Defl. Increment | Defl. Angle |
|---|---|---|---|
| 38+03.89 | 319.74 | 0°02'55" | 4°00'00" |
| 38+00.00 | 315.86 | 1°15'00" | 3°57'05" |
| 37+00.00 | 216.03 | 1°15'00" | 2°42'05" |
| 36+00.00 | 116.09 | 1°15'00" | 1°27'05" |
| 35+00.00 | 16.11 | 0°12'05" | 0°12'05" |
| 34+83.89 | | | |

```
*************************************
*******  Spiral Staking Notes  *******
*************************************
```

| | Station | Chord | Defl. Angle |
|---|---|---|---|
| SC | 34+83.89 | 100.00 | 0°25'00" |
| | 34+00.00 | 16.11 | 0°00'39" |
| TS | 33+83.89 | | |

**24.32** Same as Problem 24.31, except use a 200-ft spiral for the curve of Problem 24.8.

```
              Spiral Angle: 2°00'00"
              Spiral Throw: 0.58
       Spiral Long Tangent: 133.34
      Spiral Short Tangent: 66.67
             Spiral Length: 200.00
  Spiral Long Chord Length: 199.99
```

Exit spiral notes for layout from ST to CS
with tangent as backsight.

| | Station | Chord | Defl. Angle |
|---|---|---|---|
| ST | 27+99.83 | | |
| | 27+00.00 | 99.83 | 0°09'58" |
| | 26+00.00 | 199.82 | 0°39'56" |
| CS | 25+99.83 | 199.99 | 0°40'00" |

```
                 Defining Curve Parameters
  Curve to the right of the back tangent as viewed from the PC.
            Intersection Angle = 10°00'00" (Back to Forward
Tangent)
      Circular Curve Intersection Angle = 6°00'01"
                  Degree of Curvature = 2°00'00"
                               Radius = 2,864.93
                Circular Curve Length = 300.03
             Tangent Distance (TS-PI) = 350.70
            Circular Curve Long Chord = 299.89
                 Long Chord (TS - ST) = 698.72
                             External = 13.28
        Circular Curve Tangent Distance = 150.15

                      PI Stationing = 24+50.50
                      27+99.83 Back = 28+01.20 Ahead
```

| Station | Chord | Defl. Increment | Defl. Angle |
|---|---|---|---|
| 25+99.83 | 299.89 | 0°59'54" | 3°00'00" |
| 25+00.00 | 200.15 | 1°00'00" | 2°00'07" |
| 24+00.00 | 100.18 | 1°00'00" | 1°00'07" |
| 23+00.00 | 0.20 | 0°00'07" | 0°00'07" |
| 22+99.80 | | | |

```
                    ***************************************
                    *******  Spiral Staking Notes  *******
                    ***************************************

             Station |      Chord |       Defl. Angle |
          =============================================
SC        22+99.80 |     199.99 |          0°40'00" |
          22+00.00 |     100.20 |          0°10'02" |
          21+00.00 |       0.20 |          0°00'00" |
TS        20+99.80 |            |                   |
```

**24.33** Same as Problem 24.31, except for the curve of Problem 24.10, with a 50-m entry spiral using stationing of 30 m and a total station instrument.

```
                       Spiral Angle: 0°20'50"
                       Spiral Throw: 0.025
                Spiral Long Tangent: 33.333
               Spiral Short Tangent: 16.667
                      Spiral Length: 50.000
           Spiral Long Chord Length: 50.000

          Exit spiral notes for layout from ST to CS
                  with tangent as backsight.

             Station |      Chord |       Defl. Angle |
          =============================================
ST        46+73.804 |            |                   |
          46+50.000 |     23.804 |          0°01'34" |
CS        46+23.804 |     50.000 |          0°06'57" |

                   Defining Curve Parameters
     Curve to the right of the back tangent as viewed from the PC.
               Intersection Angle = 12°30'00" (Back to Forward
Tangent)
      Circular Curve Intersection Angle = 11°48'20"
                   Degree of Curvature = 1°23'20"
                                Radius = 4,125.296
                 Circular Curve Length = 850.000
              Tangent Distance (TS-PI) = 476.796
             Circular Curve Long Chord = 848.497
                  Long Chord (TS - ST) = 947.924
                              External = 24.768
     Circular Curve Tangent Distance = 426.510

                      PI Stationing = 42+00.600
                    46+73.804 Back = 46+77.396 Ahead

      Station |      Chord |   Defl. Increment |       Defl. Angle |
======================================================================
    46+23.804 |    848.497 |          0°01'35" |         5°54'010" |
    46+20.000 |    844.713 |          0°12'30" |          5°52'35" |
    45+90.000 |    814.866 |          0°12'30" |          5°40'05" |
    45+60.000 |    785.007 |          0°12'30" |          5°27'35" |
    45+30.000 |    755.138 |          0°12'30" |          5°15'05" |
    45+00.000 |    725.259 |          0°12'30" |          5°02'35" |
    44+70.000 |    695.370 |          0°12'30" |          4°50'05" |
```

| | | | |
|---|---|---|---|
| 44+40.000 | 665.472 | 0°12'30" | 4°37'35" |
| 44+10.000 | 635.566 | 0°12'30" | 4°25'05" |
| 43+80.000 | 605.651 | 0°12'30" | 4°12'35" |
| 43+50.000 | 575.728 | 0°12'30" | 4°00'05" |
| 43+20.000 | 545.797 | 0°12'30" | 3°47'35" |
| 42+90.000 | 515.859 | 0°12'30" | 3°35'05" |
| 42+60.000 | 485.915 | 0°12'30" | 3°22'35" |
| 42+30.000 | 455.964 | 0°12'30" | 3°10'05" |
| 42+00.000 | 426.007 | 0°12'30" | 2°57'35" |
| 41+70.000 | 396.044 | 0°12'30" | 2°45'05" |
| 41+40.000 | 366.076 | 0°12'30" | 2°32'35" |
| 41+10.000 | 336.103 | 0°12'30" | 2°20'05" |
| 40+80.000 | 306.126 | 0°12'30" | 2°07'35" |
| 40+50.000 | 276.145 | 0°12'30" | 1°55'05" |
| 40+20.000 | 246.160 | 0°12'30" | 1°42'35" |
| 39+90.000 | 216.171 | 0°12'30" | 1°30'05" |
| 39+60.000 | 186.180 | 0°12'30" | 1°17'35" |
| 39+30.000 | 156.187 | 0°12'30" | 1°05'05" |
| 39+00.000 | 126.191 | 0°12'30" | 0°52'35" |
| 38+70.000 | 96.194 | 0°12'30" | 0°40'05" |
| 38+40.000 | 66.195 | 0°12'30" | 0°27'35" |
| 38+10.000 | 36.196 | 0°12'30" | 0°15'05" |
| 37+80.000 | 6.196 | 0°02'35" | 0°02'35" |
| 37+73.804 | | | |

******* Spiral Staking Notes *******

| | Station | Chord | Defl. Angle |
|---|---|---|---|
| SC | 37+73.804 | 50.000 | 0°06'57" |
| | 37+50.000 | 26.196 | 0°01'54" |
| TS | 37+23.804 | | |

**24.34** Compute the area bounded by the two arcs and tangent in Problem 24.22.

**<u>306,460 ft$^2$</u>**

Area of parallelogram; = 1,699,285

Area of Sector R = 305,414

Area of Sector $R_x$ = 1,087,408

Area = 1699285 − 305,414 - 1,087,408 = 306,460

**24.35** In an as-built survey, the XY coordinates in meters of three points on the centerline of a highway curve are determined to be A: (3770.52, 4913.84); B: (3580.80, 4876.37); C: (3399.27, 4809.35). What are the radius, and coordinates for the center of the curve in meters?

```
Center of Circle at: X = 3,911.375
                     Y = 3,701.621
With A Radius of   : 1,220.375
```

Matrix setup:

$$\begin{bmatrix} 754104 & 982768 & -1 \\ 716160 & 975274 & -1 \\ 679854 & 961870 & -1 \end{bmatrix} \begin{bmatrix} X_O \\ Y_O \\ f \end{bmatrix} = - \begin{bmatrix} 377052^2 + 491384^2 \\ 358080^2 + 487637^2 \\ 339927^2 + 480935^2 \end{bmatrix} = - \begin{bmatrix} 38,362,644616 \\ 36,601,130169 \\ 34,684,839554 \end{bmatrix}$$

Solution: X = A$^{-1}$L = $\begin{bmatrix} -3911375 \\ -3701621 \\ 27,511,534994 \end{bmatrix}$

By Eq. (24.35): $R = \sqrt{(-3911.375)^2 + (-3701.621)^2 - 27{,}511{,}534.994} = 1220.375$

**24.36** In Problem 24.35, if the (x, y) coordinates in meters of two points on the centerline of the tangents are (3042.28, 4616.77) and (4435.66, 4911.19), what are the coordinates of the PC, PT, and the curve parameters L, T, and I?

**PC: (3324.32, 4771.52)**
**PT: (3937.16, 4921.72)**
***I* = 29°57′52″**
***L* = 638.23**
***T* = 326.59**

$$O1 = \sqrt{(304228 - 391138)^2 + (461677 - 370162)^2} = 126207$$

$$Az_{O1} = \tan^{-1}\left(\frac{-869.10}{915.15}\right) + 360° = 316°28'43"$$

$$O2 = \sqrt{(443566 - 391138)^2 + (491119 - 370162)^2} = 131831$$

$$Az_{O2} = \tan^{-1}\left(\frac{524.28}{1209.57}\right) + 0° = 23°26'03.2"$$

Solve triangle O-1-PC: $1 - PC = \sqrt{126207^2 - 122038^2} = 321.70$

∠1-O-PC = acos(1220.38/122.07) = 14°46′04″

Solve triangle O-2-PT: $2 - PT = \sqrt{131831^2 - 122038^2} = 498.61$

∠PT-O-2 = acos(1220.38/1318.31) = 22°13′25″

$Az_{O1}$ = 316°28’43” + 14°46’04” = 331°14’47”
$Az_{O2}$ = 23°26′03″ – 22°13′25″ = 1°12′38″

$X_{PC}$ = 3911.38 + 1220.38 sin(331°14′47″) = 3324.32
$Y_{PC}$ = 3701.62 + 1220.38 cos(331°14′47″) = 4771.52
$X_{PT}$ = 3911.38 + 1220.38 sin(1°12′38″) = 3937.16
$Y_{PT}$ = 3701.62 + 1220.38 cos(1°12′38″) = 4921.72

*I* = 360° + 1°12′38″ – 331°14′47″ = 29°57′52″
*L* = (1220.38)(29°57′52″)(π/180°) = 638.23

$$T = 1220.38 \tan(29°57'52''/2) = 326.59$$

# 25 VERTICAL CURVES

Asterisks (*) indicate problems that have partial answers given in Appendix G.

**25.1** Why are vertical curves needed on the grade lines for highways and railroads?

From Section 25.1, paragraph 1: "Curves are needed to provide smooth transitions between straight segments (tangents) of grade lines for highways and railroads."

**25.2** Identify the type of vertical curve (sag or crest) by the following rates of change in grade.

***(a)** −2.3% **Crest**

**(b)** +4.3% **Sag**

**(c)** −0.5% **Crest**

**25.3** What factors must be taken into account when designing a grade line on any highway or railroad?

From Section 25.1, paragraph 3: "There are several factors that must be taken into account when designing a grade line of tangents and curves on any highway or railroad project. In addition, the curves must be designed to (a) fit the grade lines they connect, (b) have lengths sufficient to meet specifications covering a maximum rate of change of grade (which affects the comfort of vehicle occupants), and (c) provide sufficient sight distance for safe vehicle operation (see Section 25.11)."

Tabulate station elevations for an equal-tangent parabolic curve for the data given in Problems 25.4 through 25.9, and check the table by second differences when working in English units.

**25.4*** A −2.50% grade meets a +1.75% grade at station 44+25 and elevation 3386.96 ft, 400-ft curve, stakeout at half stations.

```
  BVC Station = 42+25.00
BVC Elevation = 3391.96
```

| Station | x (Sta) | g1*x | r/2*x*x | Elevation | 1st | 2nd |
|---|---|---|---|---|---|---|
| 46+25.00 | 4.00 | -10.00 | 8.50 | 3,390.46 | | |
| 46+00.00 | 3.75 | -9.38 | 7.47 | 3,390.06 | | |
| 45+50.00 | 3.25 | -8.13 | 5.61 | 3,389.45 | 0.96 | |
| 45+00.00 | 2.75 | -6.88 | 4.02 | 3,389.10 | | 1.07 |
| 44+50.00 | 2.25 | -5.63 | 2.69 | 3,389.02 | -0.11 | |
| 44+00.00 | 1.75 | -4.38 | 1.63 | 3,389.21 | | 1.06 |
| 43+50.00 | 1.25 | -3.13 | 0.83 | 3,389.67 | -1.17 | |
| 43+00.00 | 0.75 | -1.88 | 0.30 | 3,390.38 | | |

```
  42+50.00      0.25   -0.63     0.03     3,391.37
  42+25.00      0      0         0       3,391.96
 ===============================================================
    Minimum elevation = 3,389.02 @ station 44+60.29
```

**25.5** A +4.5% grade meets a +1.50% grade at station 4 + 200 and elevation 605.568 m, 180-m curve, stakeout at 30-m increments.

```
            BVC Station = 4+110.000
            BVC Elevation = 601.518

Station     x (Sta)   g1*x   r/2*x*x   Elevation
================================================
4+290.000   1.800    8.100   -2.700     606.918
4+260.000   1.500    6.750   -1.875     606.393
4+230.000   1.200    5.400   -1.200     605.718
4+200.000   0.900    4.050   -0.675     604.893
4+170.000   0.600    2.700   -0.300     603.918
4+140.000   0.300    1.350   -0.075     602.793
4+110.000   0.000    0.000   -0.000     601.518
===============================================
```

**25.6** A 455-ft curve, grades of $g_1$ = +2.00% and $g_2$ = −1.50%, VPI at station 78 + 60, and elevation 7855.35 ft, stakeout at full stations.

```
BVC Station = 76+32.500
BVC Elevation = 7850.800

Station     x (Sta)   g1*x   r/2*x*x  Elevation   1st     2nd
===============================================================
80+87.50    4.550     9.10  -7.962    7,851.938
80+00.00    3.675     7.35  -5.194    7,852.956
79+00.00    2.675     5.35  -2.752    7,853.398  -0.442  -0.769
78+00.00    1.675     3.35  -1.079    7,853.071   0.327  -0.769
77+00.00    0.675     1.35  -0.175    7,851.975   1.096
76+32.50        0        0       0    7,850.800
===============================================================
          Maximum elevation = 7,853.400 @ station 78+92.50
```

**25.7** A 500-ft curve, grades of $g_1$ = −3.00% and $g_2$ = −1.25%, VPI at station 38 + 00, and elevation 560.00 ft, stakeout at full stations.

BVC Station = 35+50.000
BVC Elevation = 567.5

| Station | x (Sta) | g1*x | r/2*x² | Elevation | 1st Diff | 2nd Diff |
|---|---|---|---|---|---|---|
| 40+50.00 | 5.00 | −15.00 | 4.375 | 556.875 | | |
| 40+00.00 | 4.50 | −13.50 | 3.544 | 557.544 | −1.60 | 0.35 |
| 39+00.00 | 3.50 | −10.50 | 2.144 | 559.144 | −1.95 | 0.35 |
| 38+00.00 | 2.50 | −7.50 | 1.094 | 561.094 | −2.30 | 0.35 |
| 37+00.00 | 1.50 | −4.50 | 0.394 | 563.394 | −2.65 | |
| 36+00.00 | 0.50 | −1.50 | 0.044 | 566.044 | | |
| 35+50.00 | 0.00 | 0.00 | 0 | 567.500 | | |

**25.8** A 600-ft curve, $g_1 = -1.50\%$, $g_2 = +4.50\%$, VPI station = 46 + 00, VPI elevation = 1395.00 ft, stakeout at full stations.

BVC Station = 43+00.00
BVC Elevation = 1399.50

| Station | x (Sta) | g1*x | r/2*x*x | Elevation | 1st Diff | 2nd Diff |
|---|---|---|---|---|---|---|
| 49+00.00 | 6.00 | −9.00 | 18.00 | 1,408.50 | | |
| 48+00.00 | 5.00 | −7.50 | 12.50 | 1,404.50 | 4.00 | 1.00 |
| 47+00.00 | 4.00 | −6.00 | 8.00 | 1,401.50 | 3.00 | 1.00 |
| 46+00.00 | 3.00 | −4.50 | 4.50 | 1,399.50 | 2.00 | 1.00 |
| 45+00.00 | 2.00 | −3.00 | 2.00 | 1,398.50 | 1.00 | 1.00 |
| 44+00.00 | 1.00 | −1.50 | 0.50 | 1,398.50 | 0.00 | |
| 43+00.00 | 0.00 | 0.00 | 0.00 | 1,399.50 | | |

Minimum elevation = 1,398.375 @ station 44+50.00

**25.9** A 60-m curve, $g_1 = -1.500\%$, $g_2 = 2.600\%$, VPI station = 12 + 280, VPI elevation = 155.600 m, stakeout at 10-m increments.

BVC Station = 12+250.000
BVC Elevation = 156.050

| Station | x (Sta) | g1*x | r/2*x*x | Elevation |
|---|---|---|---|---|
| 12+310.000 | 0.600 | −0.900 | 1.230 | 156.380 |
| 12+300.000 | 0.500 | −0.750 | 0.854 | 156.154 |
| 12+290.000 | 0.400 | −0.600 | 0.547 | 155.997 |
| 12+280.000 | 0.300 | −.450 | 0.308 | 155.907 |
| 12+270.000 | 0.200 | −0.300 | 0.137 | 155.887 |
| 12+260.000 | 0.100 | −0.150 | 0.034 | 155.934 |
| 12+250.000 | 0.000 | 0.000 | 0.000 | 156.050 |

Minimum elevation = 155.885 @ Sta 12+271.951

Field conditions require a highway curve to pass through a fixed point. Compute a suitable equal-tangent vertical curve and full-station elevations for Problems 25.10 through 25.12.

**25.10** Grades of $g_1 = -2.50\%$ and $g_2 = +1.00\%$, VPI elevation 750.00 ft at station 30 + 00. Fixed elevation 753.00 ft at station 30 + 00.

L = 685.714 ft   reduced Equation (25.3): $0.4375L = 3$

```
BVC Station = 26+57.143
BVC Elevation = 758.571
```

| Station | x (Sta) | g1*x | r/2*x*x | Elevation |
|---|---|---|---|---|
| 33+42.857 | 6.857 | -17.143 | 12.000 | 753.429 |
| 33+00.000 | 6.429 | -16.071 | 10.547 | 753.047 |
| 32+00.000 | 5.429 | -13.571 | 7.521 | 752.521 |
| 31+00.000 | 4.429 | -11.071 | 5.005 | 752.505 |
| **30+00.000** | **3.429** | **-8.571** | **3.000** | **753.000** |

```
29+00.000        2.429       -6.071        1.505         754.005
28+00.000        1.429       -3.571        0.521         755.521
27+00.000        0.429       -1.071        0.047         757.547
26+57.143            0            0            0         758.571
====================================================================
Minimum elevation = 752.449 @ station 31+46.939
```

**25.11** Grades of $g_1 = -1.50\%$ and $g_2 = +1.75\%$, VPI elevation 1430.00 ft at station 15 + 00 Fixed elevation 1436.50 ft at station 14 + 00.

***L* = 1605.86 ft** from quadratic: $0.40625L^2 - 6.625L + 1.625 = 0$

BVC Station = 6+97.07
BVC Elevation = 1442.044

| Station | x (Sta) | g1*x | r/2*x*x | Elevation | 1st Diff | 2nd Diff |
|---|---|---|---|---|---|---|
| 23+02.930 | 16.059 | -24.088 | 26.095 | 1444.051 | | |
| 23+00.000 | 16.029 | -24.044 | 26.000 | 1444.000 | | |
| 22+00.000 | 15.029 | -22.544 | 22.857 | 1442.357 | 1.643 | |
| 21+00.000 | 14.029 | -21.044 | 19.917 | 1440.917 | 1.440 | 0.203 |
| 20+00.000 | 13.029 | -19.544 | 17.179 | 1439.679 | 1.238 | 0.202 |
| 19+00.000 | 12.029 | -18.044 | 14.643 | 1438.643 | 1.036 | 0.202 |
| 18+00.000 | 11.029 | -16.544 | 12.310 | 1437.810 | 0.833 | 0.203 |
| 17+00.000 | 10.03 | -15.044 | 10.179 | 1437.179 | 0.631 | 0.202 |
| 16+00.000 | 9.03 | -13.544 | 8.250 | 1436.750 | 0.429 | 0.202 |
| 15+00.000 | 8.03 | -12.044 | 6.524 | 1436.524 | 0.226 | 0.203 |
| **14+00.000** | **7.03** | **-10.544** | **5.000** | **1436.500** | **0.024** | **0.202** |
| 13+00.000 | 6.03 | -9.044 | 3.679 | 1436.679 | -0.179 | 0.203 |
| 12+00.000 | 5.029 | -7.544 | 2.560 | 1437.060 | -0.381 | 0.202 |
| 11+00.000 | 4.029 | -6.044 | 1.643 | 1437.643 | -0.583 | 0.202 |
| 10+00.000 | 3.029 | -4.544 | 0.929 | 1438.429 | -0.786 | 0.203 |
| 9+00.000 | 2.029 | -3.044 | 0.417 | 1439.417 | -0.988 | 0.202 |
| 8+00.000 | 1.029 | -1.544 | 0.107 | 1440.607 | -1.190 | 0.202 |
| 7+00.000 | 0.029 | -0.044 | 0.000 | 1442.000 | -1.393 | 0.203 |
| 6+97.070 | 0 | 0 | 0.000 | 1442.044 | | |

Minimum elevation = 1436.485 @ 14+38.236

**25.12** Grades of $g_1 = +4.500\%$ and $g_2 = +2.000\%$ VPI station 6+300 and elevation 205.930 m. Fixed elevation 205.620 m at station 6+400. (Use 100-m stationing)

**L = 1102.933 m**
BVC Station = 5+748.533
BVC Elevation = 181.114

| Station | x (Sta) | g1*x | r/2*x*x | Elevation |
|---|---|---|---|---|
| 6+851.466 | 11.029 | 49.632 | -13.787 | 216.959 |
| 6+800.000 | 10.515 | 47.316 | -12.530 | 215.900 |
| 6+700.000 | 9.515 | 42.816 | -10.260 | 213.670 |
| 6+600.000 | 8.515 | 38.316 | -8.217 | 211.213 |

| | | | | |
|---|---|---|---|---|
| 6+500.000 | 7.515 | 33.816 | -6.400 | 208.530 |
| **6+400.000** | **6.515** | **29.316** | **-4.810** | **205.620** |
| 6+300.000 | 5.515 | 24.816 | -3.447 | 202.483 |
| 6+200.000 | 4.515 | 20.316 | -2.310 | 199.120 |
| 6+100.000 | 3.515 | 15.816 | -1.400 | 195.530 |
| 6+00.000 | 2.515 | 11.316 | -0.717 | 191.713 |
| 5+900.000 | 1.515 | 6.816 | -0.260 | 187.670 |
| 5+800.000 | 0.515 | 2.316 | -0.030 | 183.400 |
| 5+748.533 | 0.000 | 0.000 | 0.000 | 181.114 |

**25.13** A −1.10% grade meets a +0.60% grade at station 36 + 00 and elevation 800.00 ft. The +0.60% grade then joins a +2.40% grade at station 39 + 00. Compute and tabulate the notes for an equal-tangent vertical curve, at half-stations, that passes through the midpoint of the 0.60% grade.

Midpoint = (3600 + 3900)/2 = 37+50
Elevation @ 37+50 = 800.00 + 1.5*0.6 = 800.90
Find station and elevation of P (VPI of g1 and g2)
Along g1: E1 = –1.10x; Y x = –E1/(1.10) (a)
Along g3: E2 = 2.40(3 – x) (b)
Along g2: E1 + E2 = 0.60´3 = 1.80 (c)

Substitute (a) into (b):
E2 = 2.40[ 3 + E1/(1.10)] (d)

Substitute (d) into (c) and solve for E1
2.40[3 + E1/(1.10)] = 1.80 – E1
E1 = –1.69714
From (a): x = 1.69714/1.10 = 1.54286 sta = 154.29
Sta of P = 3600 + 154.29 = 3754.29
Elev of P = 800 + E1 = 798.30

Find L:
800.90 = 798.30+1.10[L/2–(L/2–0.0429)]+(3.50/2L)(L/2–0.0429)2
L2 – 6.00653L + 0.007347 = 0

```
BVC Station = 34+54.025
BVC Elevation = 801.603
```

| Station | x (Sta) | g1*x | r/2*x*x | Elevation |
|---|---|---|---|---|
| 40+54.555 | 6.005 | -6.606 | 10.509 | 805.506 |
| 40+50.000 | 5.960 | -6.556 | 10.350 | 805.398 |
| 40+00.000 | 5.460 | -6.006 | 8.687 | 804.284 |
| 39+50.000 | 4.960 | -5.456 | 7.168 | 803.316 |
| 39+00.000 | 4.460 | -4.906 | 5.796 | 802.493 |
| 38+50.000 | 3.960 | -4.356 | 4.569 | 801.816 |
| 38+00.000 | 3.460 | -3.806 | 3.488 | 801.285 |
| 37+50.000 | 2.960 | -3.256 | 2.553 | 800.900 |

```
37+00.000     2.460      -2.706      1.763       800.660
36+50.000     1.960      -2.156      1.119       800.566
36+00.000     1.460      -1.606      0.621       800.618
35+50.000     0.960      -1.056      0.268       800.816
35+00.000     0.460      -0.506      0.062       801.159
34+54.025         0           0          0       801.603
==========================================================
Minimum elevation = 800.565 @ station 36+42.763
```

**25.14** When is it advantageous to use an unequal-tangent vertical curve instead of an equal-tangent one?

From Section 25.8, paragraph 1: They are used to enable the vertical curve to closely fit the ground conditions, which is used to minimize excessive cut or fill quantities.

Compute and tabulate full-station elevations for an unequal-tangent vertical curve to fit the requirements in Problems 25.15 through 25.18.

**25.15** A +2.50% grade meets a −1.25% grade at station 60+00 and elevation 3310.00 ft. Length of first curve 600 ft, second curve 400 ft.

BVC Station = 54+00.00
BVC Elevation = 3295.00

| Station | x (Sta) | g1*x | r/2*x² | Elevation |
|---|---|---|---|---|
| 64+00.00 | 4.00 | 4.00 | −4.50 | 3305.000 |
| 63+00.00 | 3.00 | 3.00 | −2.531 | 3305.969 |
| 62+00.00 | 2.00 | 2.00 | −1.125 | 3306.375 |
| 61+00.00 | 1.00 | 1.00 | −0.281 | 3306.219 |
| 60+00.00 | 0.00 | 0.00 | 0.000 | 3305.500 |
| CVC | | | | |
| 60+00.00 | 6.00 | 15.00 | −4.500 | 3305.500 |
| 59+00.00 | 5.00 | 12.50 | −3.125 | 3304.375 |
| 58+00.00 | 4.00 | 10.00 | −2.000 | 3303.000 |
| 57+00.00 | 3.00 | 7.50 | −1.125 | 3301.375 |
| 56+00.00 | 2.00 | 5.00 | −0.500 | 3299.500 |
| 54+00.00 | 0.00 | 0.00 | 0.000 | 3295.000 |

**25.16** Grade $g_1$ = +1.25%, $g_1$ = +3.75%, VPI at station 62+00 and elevation 1053.95 ft, $L_1$ = 500 ft and $L_2$ = 600 ft.

BVC Station = 57+00.00
BVC Elevation = 1047.70

| Station | x (Sta) | g1*x | r/2*x² | Elevation | 1st Diff | 2nd Diff |
|---|---|---|---|---|---|---|

| | | | | | | |
|---|---|---|---|---|---|---|
| 68+00.00 | 6.00 | 15.68 | 3.41 | 1076.450 | | |
| 67+00.00 | 5.00 | 13.07 | 2.367 | 1072.795 | 3.655 | 0.189 |
| 66+00.00 | 4.00 | 10.46 | 1.515 | 1069.329 | 3.466 | 0.189 |
| 65+00.00 | 3.00 | 7.84 | 0.852 | 1066.052 | 3.277 | 0.190 |
| 64+00.00 | 2.00 | 5.23 | 0.379 | 1062.965 | 3.087 | 0.189 |
| 63+00.00 | 1.00 | 2.61 | 0.095 | 1060.067 | 2.898 | 0.190 |
| 62+00.00 | 0.00 | 0.00 | 0.000 | 1057.359 | 2.708 | |
| CVC | | | | | | |
| 62+00.00 | 5.00 | 6.25 | 3.409 | 1057.359 | | |
| 61+00.00 | 4.00 | 5.00 | 2.182 | 1054.882 | 2.477 | 0.272 |
| 60+00.00 | 3.00 | 3.75 | 1.227 | 1052.677 | 2.205 | 0.273 |
| 59+00.00 | 2.00 | 2.50 | 0.545 | 1050.745 | 1.932 | |
| 57+00.00 | 0.00 | 0 | 0.000 | 1047.700 | 3.045 | |

**25.17** Grades $g_1$ of +3.00% and $g_2$ of −1.00% meet at the VPI at station 4+300 and elevation 268.473 m. Lengths of curves are 200 m and 350 m. (Use 40-m stationing.)

BVC Station = 4+100.000
BVC Elevation = 262.473

| Station | x (Sta) | g1*x | r/2*x² | Elevation |
|---|---|---|---|---|
| 4+650.000 | 2.750 | 1.591 | -2.55 | 264.973 |
| 4+640.000 | 3.400 | 1.545 | -2.402 | 265.071 |
| 4+600.000 | 3.000 | 1.364 | -1.870 | 265.421 |
| 4+560.000 | 2.600 | 1.182 | -1.405 | 265.705 |
| 4+520.000 | 2.200 | 1.000 | -1.006 | 265.922 |
| 4+480.000 | 1.800 | 0.818 | -0.673 | 266.072 |
| 4+440.000 | 1.400 | 0.636 | -0.407 | 266.157 |
| 4+400.000 | 1.000 | 0.455 | -0.208 | 266.174 |
| 4+360.000 | 0.600 | 0.273 | -0.075 | 266.125 |
| 4+320.000 | 0.200 | 0.091 | -0.008 | 266.010 |
| CVC | | | | |
| 4+300.000 | 2.000 | 6.000 | -2.545 | 265.928 |
| 4+240.000 | 1.400 | 4.200 | -1.247 | 265.426 |
| 4+200.000 | 1.000 | 3.000 | -0.636 | 264.837 |
| 4+160.000 | 0.600 | 1.800 | -0.229 | 264.044 |
| 4+100.000 | 0.000 | 0.000 | 0.000 | 262.473 |

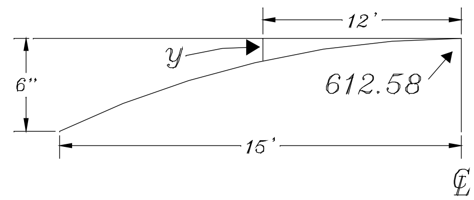
**25.18** A −1.40% grade meets a +2.75% grade at station 89 + 00 and elevation 6320.64 ft. Length of first curve is 300 ft, of second curve, 400 ft.

BVC Station = 86+00.00

BVC Elevation = 6324.84

| Station | x (Sta) | g1*x | r/2*x² | Elevation | 1st Diff | 2nd Diff |
|---|---|---|---|---|---|---|
| 93+00.00 | 4.00 | 3.89 | 3.56 | 6,331.64 | | |
| 92+00.00 | 3.00 | 2.91 | 2.00 | 6,329.11 | 2.528 | 0.445 |
| 91+00.00 | 2.00 | 1.943 | 0.89 | 6327.029 | 2.083 | 0.445 |
| 90+00.00 | 1.00 | 0.971 | 0.222 | 6325.391 | 1.638 | 0.444 |
| 89+00.00 | 0.00 | 0.000 | 0.000 | 6324.197 | 1.194 | |
| CVC | | | | | | |
| 89+00.00 | 3.00 | -4.200 | 3.557 | 6324.197 | | |
| 88+00.00 | 2.00 | -2.800 | 1.581 | 6323.621 | 0.576 | 0.790 |
| 87+00.00 | 1.00 | -1.400 | 0.395 | 6323.835 | -0.214 | 0.791 |
| 86+00.00 | 0.00 | 0.000 | 0.000 | 6324.840 | -1.005 | |

**25.19*** A manhole is 12 ft from the centerline of a 30-ft wide street that has a 6-in. parabolic crown. The street center at the station of the manhole is at elevation 612.58 ft. What is the elevation of the manhole cover?




Elev = **612.26 ft**

$$y = \left(\frac{12}{15}\right)^2 6 = 3.84\,\text{in.} = \ 0.32\ \text{ft}$$

**25.20** A 40-ft wide street has an average parabolic crown from the center to each edge of 1/4 in./ft. How much does the surface drop from the street center to a point 4 ft from the edge?

**0.33 ft** y = (20 − 4)(1/4 in.) = 4.00 in. = 0.333 ft

**25.21** Determine the station and elevation at the low point of the curve in Problem 25.4.

**3389.02 ft @ station 44+60.29**

**25.22*** Calculate the station and elevation at the high point of the curve in Problem 25.6.

**7853.40 ft @ station 78+92.50**

**25.23** Compute the station and elevation at the low point of the curve of Problem 25.9.

**1398.375 ft @ station 44+50.00**

**25.24** What are the station and elevation of the low point of the curve of Problem 25.13?

**800.595 @ station 36+41.763**

**25.25** What are the requirements for sight distances on a vertical curve?

From Section 25.12, paragraph 1: "The vertical alignments of highways should provide ample sight distance for safe vehicular operation. Two types of sight distances are involved: (1) stopping sight distance (the distance required, for a given "design speed,"1 to safely stop a vehicle thus avoiding a collision with an unexpected stationary object in the roadway ahead) and (2) passing sight distance (the distance required for a given design speed, on two-lane two-way highways to safely overtake a slower moving vehicle, pass it, and return to the proper lane of travel leaving suitable clearance for an oncoming vehicle in the opposing lane)."

**25.26*** Compute the passing sight distance available in Problem 25.6. (Assume $h_1$ = 3.50 ft and $h_2$ = =4.25 ft.)

**669.32 ft** By Eq. (251.10): $S = 0.5\left[4.55 + 2\frac{\left(\sqrt{3.5}+\sqrt{4.25}\right)^2}{2.0+1.50}\right] = 6.6932 \text{ sta}$

So $S > L$ is satisfied

**25.27** Similar to Problem 25.26, except for a stopping sight distance with an $h_2$ = 2.00 ft.

**535.83 ft** By Eq. (251.10): $S = 0.5\left[4.55 + 2\frac{\left(\sqrt{3.5}+\sqrt{2}\right)^2}{2.0+1.5}\right] = 5.3583 \text{ sta}$

So $S > L$ is satisfied

**25.28** What is the minimum required length of curve in Problem 25.7 assuming a design speed of 70 mph.

**10,149 ft** From Table 25.4, $S$ = 24.80 sta; $L$ = 5.00 sta; So $S > L$

By Eq. (25.12): $L = 2(24.8) - \frac{4+3.5(24.80)}{-3{,}00+1.25} = 101.4857 \text{ sta}$

**25.29** In determining sight distances on vertical curves, how does the designer determine whether the cars or objects are on the curve or tangent?

Try either formula in Section 25.11 and compare the derived sight distance with the length of the curve. If the derived sight distance does not fit the requirements of $S > L$ or $S < L$, then use the other formula.

What is the minimum length of a vertical curve to provide a required sight distance for the conditions given in Problems 25.30 through 25.32?

**25.30*** Grades of +3.00% and −2.00%, sight distance 600 ft, $h_1$ = 3.50 ft and $h_2$ = 2.00 ft.

**833.99 ft** By Eq. (25.9): $L = \frac{6^2(3+2)}{2\left(\sqrt{3.5}+\sqrt{2}\right)^2} = 8.3399 \text{ sta}$

So $S < L$ is satisfied

**25.31** A crest curve with grades of +2.50% and −1.00% sight distance 500 ft, $h_1$ = 3.50 ft and $h_2$ = 4.25 ft.

**116.36 ft** By Eq. (25.10): $L = 2(5) - \frac{2(\sqrt{3.5}+\sqrt{4.25})^2}{2.5+1} = 1.1636 \text{ sta } (S > L)$

**25.32*** A backsight of 4.86 ft is taken on a benchmark whose elevation is 33.86 ft. What rod reading is needed at that HI to set a blue top at grade elevation of 34.80 ft?

**3.92 ft** = 33.86 + 4.86 – 34.80

**25.33** A backsight of 4.52 ft is taken on a benchmark whose elevation is 658.28 ft. A foresight of 5.04 ft and a backsight of 6.04 ft are then taken in turn on $TP_1$ to establish a HI. What rod reading will be necessary to set a blue top at a grade elevation of 657.96 ft?

**5.84 ft** = 658.28 + 4.52 – 5.04 + 6.04 – 657.96

# 26 VOLUMES

Asterisks (*) indicate problems that have partial answers given in Appendix G.

**26.1** What are the principle methods used to determine volumes in surveying?

From Section 26.2, paragraph 1: Three principal systems are used: (1) the cross-section method, (2) the unit-area (or borrow-pit) method, and (3) the contour-area method.

**26.2** Prepare a table of end areas versus depths of fill from 0 to 20 ft by increments of 2 ft for level sections, a 40-ft-wide level roadbed with side slopes of 2:1.

Area = $2h^2 + 40h$

| Fill Depth (ft) | End Area ($ft^2$) |
|---|---|
| 0 | 0 |
| 2 | 88 |
| 4 | 192 |
| 6 | 312 |
| 8 | 448 |
| 10 | 600 |
| 12 | 768 |
| 14 | 952 |
| 16 | 1152 |
| 18 | 1368 |
| 20 | 1600 |

**26.3** Similar to Problem 26.2, except use side slopes of 1:1.

Area = $1h^2 + 40h$

| Fill Depth (ft) | End Area ($ft^2$) |
|---|---|
| 0 | 0 |
| 2 | 84 |
| 4 | 176 |
| 6 | 276 |
| 8 | 384 |
| 10 | 500 |
| 12 | 624 |
| 14 | 756 |
| 16 | 896 |

| | |
|---|---|
| 18 | 1044 |
| 20 | 1200 |

Draw the cross sections and compute $V_e$ for the data given in Problems 26.4 through 26.7.

**26.4*** Two level sections 75 ft apart with center heights 4.8 and 7.2 ft in fill, base width 30 ft, side slopes 2:1.

**708 yd$^3$ = 19,116 ft$^3$**

End areas = 190.1 and 319.7 ft$^2$ computed as Area = $2h^2 + 30h$

**26.5** Two level sections of 30-m stations with center heights of 1.44 and 1.58 m. in cut, base width 10 m, side slopes 1-1/2:1.

**555.8 m$^3$**

End areas = 17.5 m$^2$ and 19.5 m$^2$ computed as Area = $1.5h^2 + 10h$.

**26.6** The end area at station 36+00 is 265 ft$^2$. Notes giving distance from centerline and cut ordinates for station 36+80 are C 3.8/16.6; C 4.9/0; C 5.6/10.2. Base is 20 ft.

**707.6 yd$^3$ = 19,110 ft$^3$**

| x | y | − | + |
|---|---|---|---|
| −10 | 0 | | 0 |
| −16.6 | 3.8 | −38 | 0 |
| 0 | 4.9 | −81.34 | 49.98 |
| 10.2 | 5.6 | 0 | 56 |
| 10 | 0 | 0 | 0 |
| −10 | 0 | 0 | |
| | | −119.34 | 105.98 |

End Area @ 36 + 80 = 112.66 ft$^2$

**26.7** An irrigation ditch with $b$ = 10 ft and side slopes of 2:1. Notes giving distances from centerline and cut ordinates for stations 52+00 and 53+00 are C 2.4/10.8; C 2.8; C 4.5/10.4; and C 2.1/12.2; C 3.5; C 3.7/11.2.

**290.3 yd$^3$ = 7839 ft$^3$**

| x | y | − | + |
|---|---|---|---|
| -5 | 0 | | 0 |
| 10.8 | 2.4 | -12 | 0 |
| 0 | 2.8 | 30.24 | 29.12 |
| 10.4 | 4.5 | 0 | 22.5 |
| 5 | 0 | 0 | 0 |
| -5 | 0 | 0 | |
| | | 18.24 | 51.62 |

| x | y | − | + |
|---|---|---|---|
| -5 | 0 | | 0 |
| 12.2 | 2.1 | -10.5 | 0 |
| 0 | 3.5 | 42.7 | 39.2 |
| 11.2 | 3.7 | 0 | 18.5 |
| 5 | 0 | 0 | 0 |
| -5 | 0 | 0 | |
| | | 32.2 | 57.7 |

End areas: 52 + 00 = 75.8 ft$^2$ and 53 + 00 = 81.0 ft$^2$

**26.8** Why must cut and fill volumes be totaled separately?

From Section 26.2, last paragraph and Section 26.9, paragraph 3: To balance cuts and fills volumes so that materials is kept on site as much as possible and since contractors are generally paid for cuts only.

**26.9*** For the data tabulated, calculate the volume of excavation in cubic yards between stations 10 + 00 and 15 + 00.

| Station | Cut End Area (ft$^2$) |
|---|---|
| 10 + 00 | 263 |
| 11 + 00 | 358 |
| 12 + 00 | 446 |
| 13 + 00 | 402 |
| 14 + 00 | 274 |
| 15 + 00 | 108 |

**4237.0 yd$^3$**

| Station | Cut End Area (ft$^2$) | Volume |
|---|---|---|
| 10 + 00 | 156 | |
| 11 + 00 | 208 | 674.1 |
| 12 + 00 | 342 | 1018.5 |
| 13 + 00 | 240 | 1077.8 |
| 14 + 00 | 198 | 811.1 |
| 15 + 00 | 156 | 655.6 |
| | | 4237.0 |

**26.10** For the data listed, tabulate cut, fill, and cumulative volumes in cubic yards between stations 10 + 00 and 20 + 00. Use an expansion factor of 1.25 for fills.

| Station | End Area | |
|---|---|---|
| | Cut | Fill |
| | 0 | |
| | 108 | |
| | 238 | |
| | 201 | |
| | 106 | |
| 14 + 60 | 0 | 0 |
| | | 102 |
| 16 + 00 | | 138 |
| 17 + 00 | | 205 |
| 18 + 00 | | 147 |

| | | |
|---|---|---|
| 19 + 00 | | 138 |
| 20 + 00 | | 106 |

| Station | End Area (ft²) Cut | Fill | Volumes Cut (yd3) | Fill (yd3) | 1.25Fill (yd3) | Cumulative (yd3) |
|---|---|---|---|---|---|---|
| 10 + 00 | 0 | | | | | |
| 11 + 00 | 108 | | 200.0 | | | 200.0 |
| 12 + 00 | 238 | | 640.7 | | | 840.7 |
| 13 + 00 | 201 | | 813.0 | | | 1653.7 |
| 14 + 00 | 106 | | 568.5 | | | 2222.2 |
| 14 + 60 | 0 | 0 | 117.8 | | | 2340.0 |
| 15 + 00 | | 102 | | 75.6 | 94.4 | 2245.6 |
| 16 + 00 | | 138 | | 444.4 | 555.6 | 1690.0 |
| 17 + 00 | | 205 | | 635.2 | 794.0 | 896.0 |
| 18 + 00 | | 147 | | 651.9 | 814.8 | 81.2 |
| 19 + 00 | | 138 | | 527.8 | 659.7 | -578.5 |
| 20 + 00 | | 106 | | 451.9 | 564.8 | -1143.3 |

**26.11** Calculate the section areas in Problem 26.4 by the coordinate method.

Areas: **190.1 ft²** and **319.7 ft²**

| x | y | − | + |
|---|---|---|---|
| −-15 | 0 | | 0 |
| −24.6 | 4.8 | −72.0 | 118.1 |
| 24.6 | 4.8 | −118.1 | 72.0 |
| 15 | 0 | 0 | 0 |
| −15 | 0 | 0 | |
| | | −190.1 | 190.1 |

| x | y | − | + |
|---|---|---|---|
| −15 | 0 | | 0 |
| −29.4 | 7.2 | −108.0 | 211.7 |
| 29.4 | 7.2 | −211.7 | 108.0 |
| 15 | 0 | 0 | 0 |
| −15 | 0 | 0 | |
| | | −319.7 | 319.7 |

**26.12** Compute the section areas in Problem 26.5 by the coordinate method.

Areas: **17.5 m²** and **19.5 m²**

| x | y | − | + |
|---|---|---|---|
| −5 | 0 | | 0 |
| −7.16 | 1.44 | −7.2 | 10.3 |
| 7.16 | 1.44 | −10.3 | 7.2 |
| 5 | 0 | 0 | 0 |
| −5 | 0 | 0 | |
| | | −17.5 | 17.5 |

| x | y | − | + |
|---|---|---|---|
| −5 | 0 | | 0 |
| −7.37 | 1.58 | −7.9 | 11.6 |
| 7.37 | 1.58 | −11.6 | 7.9 |
| 5 | 0 | 0 | 0 |
| −5 | 0 | 0 | |
| | | −19.5 | 19.5 |

**26.13** Determine the section areas in Problem 26.7 by the coordinate method.

See solution to problem 26.7

**26.14*** Compute $C_P$ and $V_P$ for Problem 26.4. Is $C_P$ significant?

**$C_P$ = 3 yd³, $V_P$ = 705 yd³**; represents on 0.3% of volume, so not significant.

By Equation (26.4): $C_p = \frac{75}{12(27)}(4.8-7.2)(39.6-44.4) = 2.7\ \text{yd}^3$

**26.15** Calculate $C_P$ and $V_P$ for Problem 26.7. Would $C_P$ be significant in rock cut?

**$C_P$ = 0.5 yd³, $V_P$ = 54 yd³**; represents on 0.9% of volume, so not significant.

By Equation (26.4): $C_p = \frac{100}{12(27)}(2.8-3.5)(21.2-23.4) = 0.48\ \text{yd}^3$

**26.16** From the following excerpt of field notes, plot the cross section on graph paper and superimpose on it a design template for a 30-ft wide level roadbed with fill slopes of 1-1/2:1 and a subgrade elevation at centerline of 3250.26 ft. Determine the end area graphically by counting squares.

| **HI = 3246.99** | | | | | | |
|---|---|---|---|---|---|---|
| 20 + 00 Lt | $\frac{5.2}{50}$ | $\frac{4.8}{22}$ | $\frac{6.6}{0}$ | $\frac{5.9}{12}$ | $\frac{7.0}{30}$ | $\frac{8.1}{50}$ |

**404.1 ft²**

| | | | | | | |
|---|---|---|---|---|---|---|
| -27.22 | -22 | 0 | 12 | 30 | 30.44 | 15 | -15 |
| 3242.12 | 3242.19 | 3240.39 | 3241.09 | 3239.99 | 3239.97 | 3250.26 | 3250.26 |

| x | y | − | + |
|---|---|---|---|
| 0 | 3240.4 | | 38884.7 |
| 12 | 3241.1 | 0 | 97232.7 |
| 30 | 3240.0 | 38879.9 | 98629.9 |
| 30.44 | 3240.0 | 97199 | 48599.5 |
| 15 | 3250.26 | 98942.5 | −48754 |
| −15 | 3250.26 | 48753.9 | −88462 |
| −27.22 | 3242.1 | −48632 | −71327 |
| −22 | 3242.19 | −88242 | 0 |
| 0 | 3240.4 | −71289 | |
| | | 75613.0 | 74804.7 |

Area = 0.5|75,613.0 − 74,804.7| = 404.14

**26.17** For the data of Problem 26.16, determine the end area by plotting the points in a CAD package, and listing the area.

**404.1 ft²**

**26.18** For the data of Problem 26.16, calculate slope intercepts, and determine the end area by the coordinate method.

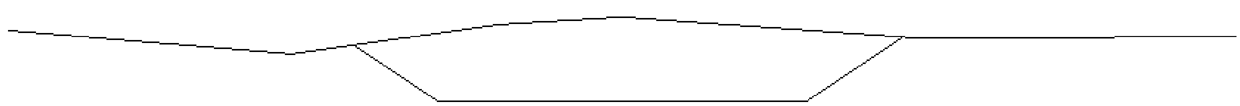
**Slope intercepts at (-27.22, 3242.12) and (30.44, 3239.99)**

**Area = 404.1 ft$^2$**; See Problem 26.16

**26.19** From the following excerpt of field notes, plot the cross section on graph or in a CAD program and superimpose on it a design template for a 30-ft wide level roadbed with cut slopes of 1-1/2:1 and a subgrade elevation of 240.88 ft. Determine the end area graphically by counting squares.

| HI = 252.66 ft | | | | | | |
|---|---|---|---|---|---|---|
| 46 + 00 Lt | $\frac{6.0}{50}$ | $\frac{7.9}{27}$ | $\frac{5.5}{10}$ | $\frac{4.9}{0}$ | $\frac{6.6}{24}$ | $\frac{6.5}{50}$ |

**232.8 ft$^2$**



**Left side intercept**

Grade slope: $\frac{247.16-244.76}{17-27} = -0.141176$

Grade elevation @ −15 is $247.16 - 0.141176(5) = 246.454$ ft

Ordinate is = 246.454 – 240.88 = 5.574

Horizontal distance from −15 is $\frac{5.574}{0.66667+0.141176} = 6.90$ ft, or at –21.90 ft

Elevation @ −21.90 ft = 240.88 + 0.66667(6.90) = 245.48 ft

Left intercept coordinates are (−21.90, 245.48)

**Right side intercept**

Grade slope: $\frac{246.06-247.76}{24-0} = -0.070833$

Grade elevation @ +15 is $247.76 - 0.070833(15) = 246.698$ ft

Ordinate is = 246.698 – 240.88 = 5.818

Horizontal distance from −15 is $\frac{5.818}{0.66667+0.141176} = 7.89$ ft, or at +22.89 ft

Elevation @ 22.89 ft = 247.76 − 0.141176(22.89) = 246.14 ft

Right intercept coordinates are (22.89, 246.14)

Area computed by coordinates = 232.8

**26.20** For the data of Problem 26.19, calculate slope intercepts and determine the end area by the coordinate method.

**Slope intercepts are (-21.90, 245.48) and (22.89, 246.14)**

**Area = 232.8 ft$^2$**

From WOLFPACK or see work in Problem 26.19

```
Station: 46+00   Roadbed elevation:    240.880
```

```
-21.90     -10.00       0.00      22.89      15.00     -15.00
245.48     247.16     247.76     246.14     240.88     240.88
Cut End Area = 232.8
```

**26.21*** Complete the following notes and compute $V_e$ and $V_P$. The roadbed is level, the base is 30 ft.

| | | | |
|---|---|---|---|
| Station 89 + 00 | $\frac{C3.1}{24.3}$ | $\frac{C4.9}{0}$ | $\frac{C4.3}{35.2}$ |
| Station 88 + 00 | $\frac{C6.4}{34.2}$ | $\frac{C3.6}{0}$ | $\frac{C5.7}{32.1}$ |

$V_e$ = **761.8 yd³ = 20,568.3 ft³**; $V_p$ = **759.1 yd³**

89 + 00

| x | y | − | + |
|---|---|---|---|
| 0 | 4.9 | | 172.5 |
| 35.2 | 4.3 | 0 | 64.5 |
| 15 | 0 | 0 | 0.0 |
| −15 | 0 | 0 | 0.0 |
| −24.3 | 3.1 | −46.5 | 0.0 |
| 0 | 4.9 | −119.1 | |
| | | −165.6 | 237.0 |

88 + 00

| x | y | − | + |
|---|---|---|---|
| 0 | 3.6 | | 115.6 |
| 32.1 | 5.7 | 0.0 | 85.5 |
| 15 | 0 | 0.0 | 0.0 |
| −15 | 0 | 0.0 | 0.0 |
| −34.2 | 6.4 | −96.0 | 0.0 |
| 0 | 3.6 | −123.1 | |
| | | −219.1 | 201.1 |

Area of 88 + 00 = 210.1 ft²   Area of 89 + 00 = 201.3 ft²

$$C_p = \frac{100}{12(27)}(4.9 - 3.6)(59.5 - 66.3) = -2.7 \text{ yd}^3$$

**26.22** Similar to Problem 26.21, except the base is 24 ft.

**$V_e$ = 707.6 yd³; $V_p$ = 704.9 yd³**

89 + 00   Base 24

| x | y | − | + |
|---|---|---|---|
| 0 | 4.9 | | 172.5 |
| 35.2 | 4.3 | 0 | 51.6 |
| 12 | 0 | 0 | 0.0 |
| -12 | 0 | 0 | 0.0 |
| -24.3 | 3.1 | −37.2 | 0.0 |
| 0 | 4.9 | −119.07 | |
| | | −56.27 | 224.1 |
| w1 | 59.5 | | |
| | Area | 190.2 | |

88 + 00

| x | y | − | + |
|---|---|---|---|
| 0 | 3.6 | | 115.6 |
| 32.1 | 5.7 | 0.0 | 68.4 |
| 12 | 0 | 0.0 | 0.0 |
| -12 | 0 | 0.0 | 0.0 |
| -34.2 | 6.4 | −76.8 | 0.0 |
| 0 | 3.6 | −123.1 | |
| | | −199.9 | 184.0 |
| w2 | 66.3 | | |
| | Area | 191.9 | |

$V_e = 19{,}105.8\ ft^3 = 707.6\ yd^3$

$$C_p = \frac{100}{12(27)}(4.9 - 3.6)(59.5 - 66.3) = -2.7 \text{ yd}^3$$

**26.23** Calculate $V_e$ and $V_P$ for the following notes. Base is 24 ft.

| | | | |
|---|---|---|---|
| 12 + 90 | $\frac{C6.4}{43.6}$ | $\frac{C3.6}{0}$ | $\frac{C5.7}{40.8}$ |
| 12 + 30 | $\frac{C3.1}{30.4}$ | $\frac{C4.9}{0}$ | $\frac{C4.3}{35.2}$ |

**$V_e = 477.4$ yd³; $V_p = 472.9$ yd³**

12 + 90

| x | y | − | + |
|---|---|---|---|
| 0 | 3.6 | | 146.9 |
| 40.8 | 5.7 | 0 | 68.4 |
| 12 | 0 | 0 | 0.0 |
| −12 | 0 | 0 | 0.0 |
| −43.6 | 6.4 | −76.8 | 0.0 |
| 0 | 3.6 | −156.96 | |
| | | −233.76 | 215.3 |
| | w1 | 84.4 | |
| | Area | 224.5 | |

12 + 30

| x | y | − | + |
|---|---|---|---|
| 0 | 4.9 | | 172.5 |
| 35.2 | 4.3 | 0.0 | 51.6 |
| 12 | 0 | 0.0 | 0.0 |
| −12 | 0 | 0.0 | 0.0 |
| −30.4 | 3.1 | −37.2 | 0.0 |
| 0 | 4.9 | −149.0 | |
| | | −186.2 | 224.1 |
| | w2 | 65.6 | |
| | Area | 205.1 | |

$V_e = 12{,}889.2 \text{ ft}^3 = 477.4 \text{ yd}^3$

$$C_P = \frac{60}{12(27)}(4.9 - 3.6)(65.6 - 84.4) = -4.5 \text{ yd}^3$$

**26.24** Calculate $V_e$, $C_P$, and $V_P$ for the following notes. The base in fill is 20 ft and base in cut is 30 ft.

| | | | | |
|---|---|---|---|---|
| 46 + 00 | $\frac{C3.4}{20.1}$ | $\frac{C2.0}{0}$ | $\frac{C0.0}{6.0}$ | $\frac{F2.0}{13.0}$ |
| 45 + 00 | $\frac{C2.2}{18.3}$ | $\frac{0.0}{0}$ | $\frac{F3.0}{14.5}$ | |

Cut: **$V_e = 401.1$ yd³; $C_p = 1.1$ yd³; $V_p = 400$ yd³**

Fill: **$V_e = 35.2$ yd³; $C_p = -1.0$ yd³; $V_p = 36.2$ yd³**

46 + 00 cut

| x | y | − | + |
|---|---|---|---|
| 0 | 2 | | 12.0 |
| 6 | 0 | 0 | 0.0 |

45 + 00 cut

| x | y | − | + |
|---|---|---|---|
| 0 | 0 | | 0.0 |
| −15 | 0 | 0.0 | 0.0 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| −15 | 0 | 0 | 0.0 | | −18.3 | 22 | −330.0 | 0.0 |
| −20.1 | 3.4 | −51 | 0.0 | | 0 | 0 | 0.0 | 0.0 |
| 0 | 2 | −40.2 | | | | | −330.0 | 0.0 |
| | | −91.2 | 12.0 | | | | | |

| | | | | |
|---|---|---|---|---|
| Cut area | 51.6 | | Cut area | 165 |
| Fill area | 4 | | Fill area | 15 |

| 46 + 00 | fill | | | | 45 + 00 | fill | | |
|---|---|---|---|---|---|---|---|---|
| 6 | 0 | | 0.0 | | 0 | 0 | | 0 |
| 13 | 2 | 12 | 20.0 | | 14.5 | 3 | 0 | 30 |
| 10 | 0 | 0 | 0.0 | | 10 | 0 | 0 | 0 |
| 6 | 0 | 0 | | | 0 | 0 | 0 | |
| | | 12 | 20.0 | | | | 0 | 30 |

| | $ft^3$ | $yd^3$ |
|---|---|---|
| Cut Volume | 10830 | 401.1 |
| Fill Volume | 950 | 35.2 |

$$\text{Cut: } C_P = \frac{100}{12(27)}(0-2.0)(18.3-20.1) = 1.1 \text{ yd}^3$$

$$\text{Fill: } C_P = \frac{100}{12(27)}(0-2.0)(14.5-13) = -1.0 \text{ yd}^3$$

For Problems 26.25 and 26.26, compute the reservoir capacity (in acre-ft) between highest and lowest contours for areas on a topographic map.

**26.25***

| Elevation (ft) | 860 | 870 | 880 | 890 | 900 | 910 |
|---|---|---|---|---|---|---|
| Area ($ft^2$) | 1370 | 1660 | 2293 | 2950 | 3550 | 4850 |

**3.1136 ac-ft**

| Contour | Area | Volume ($ft^3$) |
|---|---|---|
| 860 | 1370 | |
| 870 | 1660 | 0.34780 |
| 880 | 2293 | 0.45374 |
| 890 | 2950 | 0.60181 |
| 900 | 3550 | 0.74610 |
| 910 | 4850 | 0.96419 |
| | | 3.11364 |

**26.26**

| Elevation (ft) | 1015 | 1020 | 1025 | 1030 | 1035 | 1040 |
|---|---|---|---|---|---|---|
| Area (ft$^2$) | 1815 | 2097 | 2391 | 2246 | 2363 | 2649 |

**2.6001 ac-ft**

| Contour | Area | Volume (ft$^3$) |
|---|---|---|
| 1015 | 1815 | |
| 1020 | 2097 | 0.44904 |
| 1025 | 2391 | 0.51515 |
| 1030 | 2246 | 0.53225 |
| 1035 | 2363 | 0.52904 |
| 1040 | 2649 | 0.57530 |
| | | 2.60078 |

**26.27** State two situations where prismoidal corrections are most significant.

From Section 26.8, paragraph 1: When paying for expensive cuts, such as in rock.

**26.28*** Distances (ft) from the left bank, corresponding depths (ft), and velocities (ft/sec), respectively, are given for a river discharge measurement. What is the volume in ft$^3$/sec? 0, 1.0, 0; 10, 2.3, 1.30; 20, 3.0, 1.54; 30, 2.7, 1.90; 40, 2.4, 1.95; 50, 3.0, 1.60; 60, 3.1, 1.70; 74, 3.0, 1.70; 80, 2.8, 1.54; 90, 3.3, 1.24; 100, 2.0, 0.58; 108, 2.2, 0.28; 116, 1.5, 0.

**419.3 ft$^3$/s**

| Distance | Depth | Area | Velocity | $V_{avg}$ | Discharge |
|---|---|---|---|---|---|
| 0 | 1.0 | | 0.00 | | |
| 10 | 2.3 | 16.5 | 1.30 | 0.65 | 10.73 |
| 20 | 3.0 | 26.5 | 1.54 | 1.42 | 37.63 |
| 30 | 2.7 | 28.5 | 1.90 | 1.72 | 49.02 |
| 40 | 2.4 | 25.5 | 1.95 | 1.93 | 49.09 |
| 50 | 3.0 | 27.0 | 1.60 | 1.78 | 47.93 |
| 60 | 3.1 | 30.5 | 1.70 | 1.65 | 50.33 |
| 70 | 3.0 | 30.5 | 1.70 | 1.70 | 51.85 |
| 80 | 2.8 | 29.0 | 1.54 | 1.62 | 46.98 |
| 90 | 3.3 | 30.5 | 1.24 | 1.39 | 42.40 |
| 100 | 2.0 | 26.5 | 0.58 | 0.91 | 24.12 |
| 108 | 2.2 | 16.8 | 0.28 | 0.43 | 7.22 |
| 116 | 1.5 | 14.8 | 0.00 | 0.14 | 2.07 |
| | | | | | 419.3 |

## 27 PHOTOGRAMMETRY

Asterisks (*) indicate problems that have partial answers given in Appendix G.

**27.1** Describe the difference between vertical, low oblique, and high oblique aerial photos.

From Section 27.4, Paragraph 1:
Aerial photographs exposed with single-lens frame cameras are classified as vertical (taken with the camera axis aimed vertically downward, or as nearly vertical as possible) and oblique (made with the camera axis intentionally inclined at an angle between the horizontal and vertical). Oblique photographs are further classified as high if the horizon shows on the picture, and low if it does not.

**27.2** Define the term interpretative photogrammetry.

From Section 27.1, Paragraph 1: "*Interpretative photogrammetry* involves recognizing objects from their photographic images and judging their significance. Critical factors considered in identifying objects are the shapes, sizes, patterns, shadows, tones, and textures of their images." This area is now called remote sensing.

**27.3** Define the term metric photogrammetry.

(a) From Section 27.1, Paragraph 4: "Metrical photogrammetry is accomplished in different ways depending upon project requirements and the type of equipment available. Simple analyses and computations can be made by making measurements on paper prints of aerial photos using engineer's scales, and assuming that the photos are "truly vertical," that is, the camera axis coincided with a plumb line at the time of photography. These methods produce results of lower order, but they are suitable for a variety of applications. Other more advanced techniques, including *analog*, *analytical*, and *softcopy* methods, do not assume vertical photos and provide more accurate determinations of the spatial locations of objects. The analog procedure relies on precise optical and mechanical devices to create models of the terrain that can be measured and mapped. The analytical method is based upon precise measurements of the photographic positions of the images of objects of interest, followed by a mathematical solution for their locations. Softcopy instruments utilize digital images in computerized procedures that are highly automated."

**27.4** Describe briefly an unmanned aerial system.

From Section 27.1, Paragraph 5: "Unmanned aerial systems (UAS) combine small unmanned aerial vehicles such as those shown in Figure 1.1 with satellite positioning and digital cameras."

**27.5** The distance between two points on a vertical photograph is *ab* and the corresponding ground distance is *AB*. For the following data, compute the average photographic scale along the line ab.

*(**a**) *ab* = 2.41 in.; *AB* = 4820 ft. 2.41/4820 = **1/2000 in./ft.**

(**b**) *ab* = 5.47 in.; *AB* = 16,410 ft. 5.47/16,410 = **1/3000 in./ft.**

(**c**) *ab* = 56.48 mm; *AB* = 282.4 m. 0.05648/282.4 = **1:5000**

**27.6** On a vertical photograph of flat terrain, section corners appear a distance *d* apart. If the camera focal length is *f* compute flying height above average ground in feet for the following data:

(**a**) *d* = 4.507 in.; *f* = 3-½ in. *H* = 3.5(5280/4.507) = **4100 ft.**

(**b**) *d* = 67.056 mm; *f* = 152.4 mm *H*= 152.4(5280/50.8) = **12,000 ft.**

**27.7** On a vertical photograph of flat terrain, the scaled distance between two points is *ab*. Find the average photographic scale along *ab* if the measured length between the same line is *AB* on a map plotted at a scale of $S_{map}$ for the following data.

(**a**) *ab* = 1.456 in.; *AB* = 4.55 in.; $S_{map}$ = 1:9600
*AB* = (4.55/12) 9600 = 3640 ft.
*S* = 3640/1.456 = **1 in./2500 ft.**

(**b**) *ab* = 41.53 mm; *AB* = 6.23 mm; $S_{map}$ = 1:20,000
AB = (0.00623)20,000 = 124.6 m
*S* = 124.6/0.04153 = **1:3000**

**27.8** What are the average scales of vertical photographs for the following data, given flying height above sea level, *H*, camera focal length, *f*, and average ground elevation *h*?

*(**a**) *H* = 7300 ft.; *f* = 152.4 mm; *h* = 1240 ft

$$S = \frac{f}{H - h_A} = \frac{\frac{152.4}{25.4}}{7300 - 1240} = \mathbf{1\ in./1010\ ft}$$

(**b**) *H* = 6980 ft; *f* = 6.000 in.; *h* = 986 ft
*S* = 6.000 / (69800 – 986) = **1 in./999 ft.**

(**c**) *H* = 3524 m; *f* = 88.90 mm; *h* = 857 m
*S* = 0.0889 / (3524 – 857) = **1:30,000**

**27.9** The length of a football field from goal post to goal post scales 49.15 mm on a vertical photograph. Find the approximate dimensions (in meters) of a large rectangular building that also appears on this photo and whose sides measure 21.5 mm by 14.0 mm. (Hint: Football goal post are 120 yards apart.)

120 yds = 360 ft = 109.728 m.

0.04915 / 109.728 = 1:2232.5
0.0215 (2232.5) = 47.999 m.
0.0068 (2232.5) = 31.255 m.

**27.10*** Compute the area in acres of a triangular parcel of land whose sides measure 48.78 mm, 84.05 mm, and 69.36 mm on a vertical photograph taken from 6050 ft above average ground with a 152.4 mm focal length camera.

S = [(152.4 / 25.4) / 12] / 6050 = 39.69816 ft/mm
a = 48.78 (39.69816) = 1936.48 ft
b = 84.05 (39.69816) = 3336.63 ft
c = 69.36 (39.69816) = 2653.46 ft
s = 2446.499 / 2 = 1223.250
area = sqrt[s(s-a)(s-b)(s-c)] = 2,665,548 $ft^2$ = **69.19 ac.**

**27.11** Calculate the flight height above average terrain that is required to obtain vertical photographs at an average scale of *S* if the camera focal length is *f* for the following data:

(a) $S$ = 1:6000; $f$ = 152.40 mm
H = 0.5 (6000) = **3000 ft**

(b) $S$ = 1:9600; $f$ = 88.90 mm
H = 0.0889(39.37/12) (9600) = **2800 ft**

**27.12** Determine the horizontal distance between two points *A* and *B* whose elevations above datum are $h_A$ = 1410 ft. and $h_B$ = 990 ft and whose images *a* and *b* on a vertical photograph have photo coordinates $x_a$ = 2.95 in., $y_a$ = 2.32 in., $x_b$ = −1.64 in., and $y_b$ = −2.66 in. The camera focal length was 152.4 mm and the flying height above datum 7500 ft.

$S_A$ = 6/(7500 – 1410) = 1 in./1015 ft
$X_A$ = 2.95(1015) = 2994 ft. $Y_A$ = 2.32 (1015) = 2355 ft.
$S_B$ = 6/(7500 – 990) = 1 in./1085 ft.
$X_B$ = −1.64 (1085) = −1779 ft. $Y_B$ = −2.66 (1085) = −2886 ft.
Distance = $\sqrt{(2994+1779)^2+(2355+2886)^2} = \mathbf{7089\ ft}$

**27.13*** Similar to Problem 27.12, except that the camera focal length was 3-1/2 in., the flying height above datum 4075 ft, and elevations $h_A$ and $h_b$ 983 ft and 1079 ft, respectively. Photo coordinates of images *a* and *b* were $x_a$ = 108.81 mm, $y_a$ = −73.73 mm, $x_b$ = −87.05 mm, and $y_b$ = 52.14 mm.

$S_A$ = 3.5 / (4075 – 983) = 1 in. / 883.4 ft.
$X_A$ = (108.81 / 25.4) (883.4) = 3784.48 ft.
$Y_A$ = (−73.73 / 25.4) (883.4) = −2564.38 ft.
$S_B$ = 3.5 / (4075 − 1079) = 1 in. / 856 ft.
$X_B$ = (−87.05 / 25.4) (856) = −2933.65 ft.
$Y_B$ = (52.14/25.4) (856) = 1757.16 ft.
Distance = **7988 ft**

**27.14** On the photograph of Problem 27.12, the image $c$ of a third point $C$ appears. Its elevation $h_C$ = 1350 ft and its photo coordinates are $x_c$ = 3.20 in. and $y_c$ = −2.66 in. Compute the horizontal angles in triangle $ABC$.

$S_C$ = 6 / (7500 – 1350) = 1 in. / 1025 ft.
$X_C$ = 3.20 (1025) = 3280 ft.  $Y_C$ = −2.66 (1025) = −2726 ft.
**Angle A = 45°32′50″**  **Angle B = 45°51′52″**  **Angle C = 88°35′17″**

**27.15** On the photograph of Problem 27.12, the image $d$ of a third point $D$ appears. Its elevation is $h_D$ = 1170 ft and its photo coordinates are $x_d$ = 2.72 in. and $y_d$ = 3.09 in. Calculate the area, in acres, of triangle $ABD$.

$S_D$ = 6/(7500 – 1170) = 1 in. / 1055 ft.
$X_D$ = 2.72 (1055) = 2867 ft.
$Y_D$ = 3.09 (1055) = 3261 ft.

Dist.$_{AB}$ = 7089 ft.  Dist.$_{BD}$ = 7706 ft.  Dist.$_{DA}$ = 915 ft.
s = 7855
area = sqrt [s(s-a)(s-b)(s-c)] = 2,497,846 ft$^2$ = 57.34 ac.

**27.16** Determine the height of a radio tower, which appears on a vertical photograph for the following conditions of flying height above the tower base H, distance on the photograph from principal point to tower base $r_d$ and distance from principal point to tower top $r_t$

*(a) $H$ = 2425 ft.; $r_b$ = 3.18 in.; $r_t$ = 3.34 in.

$$d = \frac{rh}{H} \qquad h = \frac{dH}{r} \qquad h = \frac{(3.34 - 3.18)2425}{3.34} = 116.17 \text{ ft.}$$

**(b)** $H$ = 6400 ft.; $r_b$ =96.28 mm; $r_t$ = 97.67 mm.

$$h = \frac{(97.67 - 96.28)6400}{97.67} = \mathbf{91.1\ ft}$$

**27.17** On a vertical photograph, images $a$ and $b$ of ground points $A$ and $B$ have photographic coordinates $x_a$ = 3.27 in., $y_a$ = 2.28 in., $x_b$ = −1.95 in. and $y_b$ = −2.50 in. The horizontal distance between $A$ and $B$ is 5350 ft, and the elevations of $A$ and $B$ above datum are 652 ft and 785 ft, respectively. Using Equation (27.9), calculate the flying height above datum for a camera having a focal length of 152.4 mm.

$$L^2 = \left[\frac{(H-h_b)x_b - (H-h_a)x_a}{f}\right]^2 + \left[\frac{(H-h_b)y_b - (H-h_a)y_a}{f}\right]^2$$

$$5350^2 = \left[\frac{-1.95(H-785) - 3.27(H-652)}{6}\right]^2 + \left[\frac{-2.50(H-785) - 2.28(H-652)}{6}\right]^2$$

$$0 = 1.39158H^2 - 1978.12614H - 27{,}919{,}387.625897$$

H = 5246 ft. = **5250 ft**

**27.18** Similar to Problem 27.17, except $x_a$ = −52.53 mm, $y_a$ = 69.67 mm, $x_b$ = 26.30 mm,

$y_b = -59.29$ mm line length $AB = 4695$ ft. and elevations of points $A$ and $B$ are 925 and 875 ft, respectively.

$$4695^2 = \left[\frac{26.30(H-875)+52.53(H-925)}{152.4}\right]^2 + \left[\frac{-59.29(H-875)-69.67(H-925)}{152.4}\right]^2$$

$$0 = 0.9835997H^2 - 1777.81255H - 21{,}239{,}688.082$$

H = 5638 ft = **5640 ft**

**27.19*** An air base of 3205 ft exists for a pair of overlapping vertical photographs taken at a flying height of 5500 ft above MSL with a camera having a focal length of 152.4 mm. Photo coordinates of points $A$ and $B$ on the left photograph are $x_a$ = 40.50 mm, $y_a$ = 42.80 mm, $x_b$ = 23.59 mm, and $y_b$ = −59.15 mm. The $x$ photo coordinates on the right photograph are $x_a$ = −60.68 mm and $x_b$ = −70.29 mm. Using the parallax equations, calculate horizontal length $AB$.

$$p_a = x_a - x_{1a} = 40.50 - (-60.68) = 101.18 \text{ mm.}$$

$$p_b = x_b - x_{1b} = 23.59 - (-70.29) = 93.88 \text{ mm.}$$

$$X_A = \frac{B}{p_a}x_a = \frac{3205}{101.18}40.50 = 1282.89 \text{ ft.}$$

$$X_B = \frac{B}{p_b}x_b = \frac{3205}{93.88}23.59 = 805.35 \text{ ft.}$$

$$Y_A = \frac{B}{p_a}y_a = \frac{3205}{101.18}42.80 = 1355.74 \text{ ft.}$$

$$Y_B = \frac{B}{p_a}y_b = \frac{3205}{93.88}(-59.15) = -2019.34 \text{ ft.}$$

$$\text{Distance}_{AB} = \sqrt{(805.35-1282.89)^2 + (-2019.34-1355.74)^2} = 3408.70 \text{ ft.}$$

**27.20** Similar to Problem 27.19, except the air base is 6940 ft, the flying height above mean sea level is 12,520 ft, the $x$ and $y$ photo coordinates on the left photo are $x_a$ = 37.98 mm, $y_a$ = 50.45 mm, $x_b$ = 24.60 mm, and $y_b$ = −46.89 mm, and the $x$ photo coordinates on the right photo are $x_a$ = −52.17 mm and $x_b$ = −63.88 mm.

$$p_a = x_a - x'_a = 37.98 - (-52.17) = 90.15 \text{ mm.}$$

$$p_b = x_b - x'_b = 24.60 - (-63.88) = 88.48 \text{ mm.}$$

$$X_A = \frac{B}{p_a}x_a = \frac{6940}{90.15}37.98 = 2923.81 \text{ ft}$$

$$Y_A = \frac{B}{p_a}y_a = \frac{6940}{90.15}50.45 = 3883.78 \text{ ft}$$

$$X_B = \frac{B}{p_b}x_b = \frac{6940}{88.48}24.60 = 1893.78 \text{ ft}$$

$$Y_B = \frac{B}{p_b}y_b = \frac{6940}{88.48}(-46.89) = -3609.72 \text{ ft}$$

$$\text{Distance}_{AB} = \sqrt{(2923.81 - 1893.78)^2 + (3883.78 + 3609.72)^2} = \mathbf{7564\ ft}$$

**27.21*** Calculate the elevations of points *A* and *B* in Problem 27.19.

$$h_A = H - \frac{Bf}{p_a} = 5500 - \frac{3205(152.4)}{101.18} = 672.54 \text{ ft.}$$

$$h_B = H - \frac{Bf}{p_b} = 5500 - \frac{3205(152.4)}{93.88} = 297.17 \text{ ft.}$$

**27.22** Compute the elevations of points *A* and *B* in Problem 27.20.

$$h_A = H - \frac{Bf}{p_a} = 12{,}520 - \frac{6940(152.4)}{90.15} = \mathbf{787.8} \text{ ft}$$

$$h_B = H - \frac{Bf}{p_b} = 12{,}520 - \frac{6940(152.4)}{88.48} = \mathbf{566.4} \text{ ft}$$

**27.23** List the four different categories of stereoscopic plotting instruments.

From Section 27.14:
Stereoplotters can be classified into four different categories: (1) optical projection, (2) mechanical projection, (3) analytical, and (4) digital or "softcopy" systems.

**27.24** Name the three stages in stereoplotter orientation, and briefly explain the objectives of each.

From Section 27.14, Paragraphs 5 – 7: "A stereoplotter operator, preparing to measure or map a stereomodel, must go through a three-stage orientation process consisting of *interior orientation*, *relative orientation*, and *absolute orientation*."

**Interior orientation** ensures that the projected light rays are geometrically correct, i.e., angles $\theta_1'$ and $\theta_2'$ of Figure 27.14(b), (i.e., the angles between the projected light rays and the axis of the projector lens), must be identical to corresponding angles $\theta_1$ and $\theta_2$ respectively, in Figure 27.14(a), (i.e., the angles between the incoming light rays and the camera axis). Preparing the diapositives to exacting specifications, and centering them carefully in the projectors accomplish this.

After the diapositives have been placed in the projectors and the lights turned on, corresponding light rays will not intersect to form a clear model because of tilts in the photographs and unequal flying heights. To achieve intersections of corresponding light rays, the projectors are moved linearly along the X, Y, and Z-axes and also rotated about these axes until they duplicate the relative tilts and flying heights that existed when the photographs were taken. This process is called relative orientation, and when accomplished, parallactic angle of Figure 27.14(b) for each corresponding pair of light rays will be identical to its corresponding parallactic angle $\phi'$ of Figure 27.14(a), and a perfect three-dimensional model will be formed.

The model is brought to required scale by making the rays of at least two, but preferably three, ground control points intersect at their positions plotted on a manuscript map prepared at the desired scale. It is leveled by adjusting the projectors so the counter reads the correct elevations for each of a minimum of three, but preferably four, corner ground control points when the floating mark is set on them. **Absolute orientation** is a term applied to the processes of scaling and leveling the model.

**27.25** How can the operator's left and right eyes restricted to the left and right images, respectively?

From Section 27.14.1, Paragraph 3: "Plotter viewing systems must provide a stereoscopic view and hence be designed so that the left and right eyes see only projected images of the corresponding left and right diapositives. One method of accomplishing this is to project one image with a blue filter and the other with a red filter. The operator wears a pair of spectacles with corresponding blue and red lenses. Another method of separating the left and right images is to project them on half of the screen. The operator must then look through a viewing system similar to a stereoscope to restrict the left eye to view the left image only and the right eye the right. These systems of viewing stereoscopically are called the *anaglyphic* method. Other viewing systems are known as *passive* and *active*. In the passive viewing system, left and right images are projected in opposite polarity. The operator wears polarized spectacles with corresponding polarity in the lenses so that the left can only see the left image and the right eye can only see the right image. As shown in Figure 27.15, the active viewing system also projects the left and right images in opposite polarity. However, in the active system, the images are rapidly swapped in the projection system also. The operator's viewing spectacles are synchronized so that the left and right eyes can see the corresponding left and right images only."

**27.26** What kind of images do softcopy stereoplotters require? Describe two different ways they can be obtained.

From Section 27.14.2, Paragraph 1: "The softcopy systems utilize digital or "softcopy" images. These images can be acquired by using a digital camera of the type described in Section 27.3 or by scanning the negatives of images taken with film cameras."

**27.27** Compare an orthophoto with a conventional line and symbol map.

From Section 27.15, Paragraph 4: "Orthophotos combine the advantages of both aerial photos and line maps. Like photos, they show features by their actual images rather than as lines and symbols, thus making them more easily interpreted and understood. Like maps, orthophotos show the features in their true planimetric positions. Therefore true distances, angles, and areas can be scaled directly from them."

**27.28** Discuss the advantages of orthophotos as compared to maps.

From Section 27.15, Paragraph 4: "Orthophotos combine the advantages of both aerial photos and line maps. Like photos, they show features by their actual images rather than as lines and symbols, thus making them more easily interpreted and understood. Like maps, orthophotos show the features in their true planimetric positions. Therefore true distances, angles, and areas can be scaled directly from them."

Aerial photography is to be taken of a tract of land that is *X*-mi square. Flying height will be *H* ft above average terrain, and the camera has focal length *f*. If the focal plane opening is 9 × 9 in. and minimum sidelap is 30%, how many flight lines will be needed to cover the tract for the data given in Problems 27.29 and 27.30?

**27.29*** $X = 8$; $H = 4000$ ft; $f = 152.40$ mm
$S = 6 / 4000 = 1$ in./666.67 ft.
$d_S = 9(S)(1 - 0.3) = 4200$ ft.
# of Flight Lines = [8 (5280)]/4200 = 10.05 + 1 = 11 so choose **12**

**27.30** $X = 10$; $H = 6000$ ft; $f = 6.000$ in.
$S = 6/6000 = 1$ in./1000 ft.
$d_S = 9(1000)(1 - 0.3) = 6300$ ft.
# of Flight Lines = [10(5280)]/6300 = 8.4 + 1 = 9.4 so choose **10**

Aerial photography was taken at a flying height *H* ft above average terrain. If the camera focal plane dimensions are 9 × 9 in. the focal length is *f* and the spacing between adjacent flight lines is *X* ft, what is the percent sidelap for the data given in Problems 27.31 and 27.32?

**27.31*** $H = 4500$; $f = 152.4$ mm; $X = 4700$
S = 6 / 4500 = 1 in. / 750 ft.
$4700 = 9(750)x$ $x = 0.696 = 69.6 =$ **30.4%** sidelap

**27.32** $H = 7000$; $f = 88.9$ mm, $X = 13{,}500$
$S = 3.5/7000 = 1$ in./2000 ft.
$13{,}500 = 9(2000)x$ $x = 0.667 =$ **33.3%** sidelap

Photographs at a scale of *S* are required to cover an area *X* mi square. The camera has a focal length *f* and focal plane dimensions of 9 × 9 in. If endlap is 60% and sidelap 30%, how many photos will be required to cover the area for the data given in Problems 27.33 and 27.34?

**27.33** $S = 1{:}6000$; $X = 6$; $f = 152.4$ mm
$de = (9 / 12)(6000)(1 - 0.6) = 1800$ ft.
$ds = (9 / 12)(6000)(1 - 0.3) = 3150$ ft.
# of Flight Lines = 6(5280) / 3150 = 10.057 +1 = 11 so choose **12**
# of Photos per Line = 6(5280) / 1800 + 2 + 2 + 1 = 22.6 = **23**
Total # of photos = 12(23) = **276 photos**

**27.34** $S = 1{:}14{,}400$; $X = 40$; $f = 3.5$ in.
$de = (9/12)(14{,}400)(1 - 0.6) = 4320$ ft.
$ds = (9/12)(14{,}400)(1 - 0.3) = 7560$ ft.

# of Flight Lines = 40(5280) / 7560 = 27.9 +1 = **28**
# of Photos per Line = 40(5280) / 4320 + 2 + 2 + 1 = 53.9 = **54**
Total # of photos = 28(54) = **1566 photos**

**27.35** Describe a system that employs GNSS and that can reduce or eliminate ground control surveys in photogrammetry?

From Section 27.16, Paragraph 3: "Currently GNSS is being used for real-time positioning of the camera at the instant each photograph is exposed. The kinematic GNSS surveying procedure is being employed (see Chapter 15), which requires two GNSS receivers. One unit is stationed at a ground control point; the other is placed within the aircraft carrying the camera. The integer ambiguity problem is resolved using on-the-fly techniques (see Section 15.2). During the flight, camera positions are continuously determined at time intervals of a few seconds using the GNSS units and precise timing of each photo exposure is also recorded. From this information, the precise location of each exposure station, in the ground coordinate system, can be calculated. Many projects have been completed using these methods and they have produced highly accurate results, especially when supplemented with only a few ground control points. It is now possible to complete photogrammetric projects with only a few ground photo control points used for checking purposes."

**27.36** To what wavelengths of electromagnetic energy is the human eye sensitive? What wavelengths produce the colors blue, green, and red?

From Section 27.19, Paragraph 4: "Within the wavelengths of visible light, the human eye is able to distinguish different colors. The primary colors (blue, green, and red) consist of wavelengths in the ranges of 0.4–0.5, 0.5–0.6, and 0.6–0.7 $\mu$m, respectively. All other hues are combinations of the primary colors. To the human eye, an object appears a certain color because it reflects energy of wavelengths producing that color. If an object reflects all wavelengths of energy in the visible range, it will appear white, and if it absorbs all wavelengths, it will be black. If an object absorbs all green and red energy but reflects blue, that object will appear blue."

**27.37** Discuss the uses and advantages of satellite imagery.
From Section 27.19: "Satellite imagery is unique because it affords a practical means of monitoring our entire planet on a regular basis. Images of this type have been applied for land-use mapping; measuring and monitoring various agricultural crops; mapping soils; detecting diseased crops and trees; locating forest fires; studying wildlife; mapping the effects of natural disasters such as tornadoes, floods, and earthquakes; analyzing population growth and distribution; determining the locations and extent of oil spills; monitoring water quality and detecting the presence of pollutants; and accomplishing numerous other tasks over large areas for the benefit of humankind."

Problems 27.38 through 27.42 involve using WolfPack with images 5 and 6 on the CD that accompany this book. The ground coordinates of the paneled points are listed in the file "ground.crd." The coordinates of the fiducials are listed in the file "camera.fid." To do these problems, digitize the eight fiducials and paneled points 21002, 4, 41, GYM, WIL1A, WIL1B, and RD on both images. After digitizing the points, perform an interior orientation to compute photo coordinates for the points on images 5 and 6. The focal length of the camera is 153.742 mm.

*Responses will be affected by the quality of the observations. Approximate values for the problem are supplied herewith.*

**27.38** Using photo coordinates for points 4 and GYM on image 5, determine the scale of the photo.

| | |
|---|---|
| Photo Coordinates: | 4: (–1.187, 70.338) mm |
| | GYM:(78.889, –14.642) mm |
| Ground Coordinates: | 4: (745143.093, 128206.079) m. |
| | GYM: (745413.425, 127875.820) m. |
| ab = 116.771 mm | AB = 426.791 m. |

S = 0.116771 / 426.791 = 1:3655

**27.39** Using photo coordinates for points 4 and GYM on image 5, determine the flying height of the camera at the time of exposure.

H = 938 m. or -220 m.

**27.40** Using photo coordinates for points 4 and GYM on image 5 and 6, determine the ground coordinates of points WIL1A and WIL1B using Eq. (27.12) and Eq. (27.13).

Distance = 423.354 m.

**27.41** Using the exterior orientation option in WolfPack, determine the exterior orientation elements for image 5.

*Responses will be affected by the quality of the observations.*

**27.42** Using the exterior orientation option in WolfPack, determine the exterior orientation elements for image 6.

*Responses will be affected by the quality of the observations.*

## *28 INTRODUCTION TO GEOGRAPHIC INFORMATION SYSTEMS*

Asterisks (*) indicate problems that have partial answers given in Appendix G.

**28.1** Describe the concept of layers in a geographic information system.

From Section 28.1, Paragraph 4:
A generalized concept of how data of different types or "layers" are collected and overlaid in a GIS is illustrated in Figure 28.1. In that figure, maps A through G represent some of the different layers of spatially related information that can be digitally recorded and incorporated into a GIS database, and include parcels of different land ownership A, zoning B, floodplains C, wetlands D, land cover E, and soil types F. Map G is the geodetic reference framework, consisting of the network of survey control points in the area. Note that these control points are found in each of the other layers thereby providing the means for spatially locating all data in a common reference system. Thus composite maps that merge two or more different data sets can be accurately created. For example in Figure 28.1, bottom map H is the composite of all layers

**28.2** Discuss the role of a geographic reference framework in a GIS.

From Section 28.1, Paragraph 4: "It allows the user to relate information on different layers of the GIS."

**28.3** List the fundamental components of a GIS.

From Section 28.1, Paragraph 2:
A more detailed definition (Hanigan, 1988) describes a GIS as "any information management system that can:

1. Collect, store, and retrieve information based on its spatial location;
2. Identify locations within a targeted environment that meet specific criteria;
3. Explore relationships among data sets within that environment;
4. Analyze the related data spatially as an aid to making decisions about that environment;
5. Facilitate selecting and passing data to application-specific analytical models capable of assessing the impact of alternatives on the chosen environment; and
6. Display the selected environment both graphically and numerically either before or after analysis."

**28.4** List the fields within surveying and mapping that are fundamental to the development and implementation of GISs.

From Section 28.1, Paragraph 7: "Virtually every aspect of surveying, and thus all material presented in the preceding chapters of this book, bear upon GIS development, management, and use."

**28.5** Discuss the importance of metadata to a GIS.

From Section 28.8, Paragraph 1:
Once created, data can travel almost instantaneously through a network and be transformed, modified, and used for many different kinds of spatial analyses. It can then be re-transmitted to another user, and then to another, etc.2 It is important that each change made to any data set be documented by updating its associated metadata.

**28.6** Name and describe the different simple spatial objects used for representing graphic data in digital form. Which objects are used in raster format representations?

From Section 28.4.1:
The simple spatial objects most commonly used in spatially locating data are illustrated in Figure 28.2 and described as follows:

1. Points define single geometric locations. They are used to locate features such as houses, wells, mines, or bridges [see Figure 28.2(a)]. Their coordinates give the spatial locations of points, commonly in state plane or UTM systems (see Chapter 20).
2. Lines and strings are obtained by connecting points. A line connects two points, and a string is a sequence of two or more connected lines. Lines and strings are used to represent and locate roads, streams, fences, property lines, etc. [see Figure 28.2(b)].
3. Interior areas consist of the continuous space within three or more connected lines or strings that form a closed loop [see Figure 28.2(c)]. For example, interior areas are used to represent and locate the limits of governmental jurisdictions, parcels of land ownership, different types of land cover, or large buildings.
4. Pixels are usually tiny squares that represent the smallest elements into which a digital image is divided [see Figure 28.2(d)]. Continuous arrays of pixels, arranged in rows and columns, are used to enter data from aerial photos, orthophotos, satellite images, etc. Assigning a numerical value to each pixel specifies the distributions of colors or tones throughout the image. Pixel size can be varied, and is usually specified by the number of dots per inch (dpi). As an example, 100 dpi would correspond to squares having dimensions of 1/100 in. on each side. Thus 100 dpi yields 10,000 pixels per square inch.
5. Grid cells are single elements, usually square, within a continuous geographic variable. Similar to pixels, their sizes can be varied, with smaller cells yielding improved resolution. Grid cells may be used to represent slopes, soil types, land cover, water table depths, land values, population density, and so on. The distribution of a given data type within an area is indicated by assigning a numerical value to each cell; for example, showing soil types in an area using the number 2 to represent sand, 5 for loam, and 9 for clay, as illustrated in Figure 28.2(e).

**28.7** What are the primary differences between a GIS and LIS?

From Section 28.2, Paragraph 1: "The distinguishing characteristic between the two is that a LIS has its focus directed primarily toward land records data."

**28.8** How many pixels are required to convert the following documents to raster form for the conditions given:

*(a) A 384-in. square map scanned at 200 dpi. $384^2(200)^2$ = **5,898,240,000**

**(b)** A 9-in. square aerial photo scanned at 600 dpi. $9^2(600)^2$ = **29,160,000**

**(c)** An orthophoto of 11×17 in. dimensions scanned at 200 dpi $11(17)(200)^2$ = **7,480,000**

**28.9** Explain how data can be converted from:

**(a)** Vector to raster format See Section 28.6.1
**(b)** Raster to vector format See Section 28.6.2

**28.10** For what types of data is the vector format best suited?

From Section 28.4.2, Paragraph 7: "Examples include aerial photos, orthophotos, and satellite images."

**28.11** Discuss the compromising relationships between grid cell size and resolution in raster data representation.

From Section 28.4.2, Paragraph 6: "It is important to note, that as grid resolution increases, so does the volume of data (number of grid cells) required to enter the data."

**28.12** Define the term topology and discuss its importance in a GIS.

From Section 28.4.3, Paragraph 1: "Topology is a branch of mathematics that describes how spatial objects are related to each other. The unique sizes, dimensions, and shapes of the individual objects are not addressed by topology. Rather, it is only their relative relationships that are specified."

From Section 28.4.3, Last Paragraph: "The relationships expressed through the identifiers for points, lines, and areas of Table 28.1, and the topology in Table 28.2, conceptually yield a "map." With these types of information available to the computer, the analysis and query processes of a GIS are made possible."

**28.13** Develop identifier and topology tables similar to those of Tables 28.1 and 28.2 in the text for the vector representation of (see the following figures):

**(a)** Problem 28.13(a)

| Identifier | Coordinates | Line Identifier | Points | Area Identifier | Lines |
|---|---|---|---|---|---|
| 1 | $x_1,y_1$ | a | 1,2 | I | a,g,f,e |
| 2 | $x_2,y_2$ | b | 2,3 | II | b,h,g |

| | | | | | |
|---|---|---|---|---|---|
| 3 | $x_3,y_3$ | c | 3,4 | III | c,d,f,h |
| 4 | $x_4,y_4$ | d | 4,5 | | |
| 5 | $x_5,y_5$ | e | 5,1 | | |
| | | f | 5,6 | | |
| | | g | 6,2 | | |
| | | h | 6,3 | | |

| Connectivity | | Direction | | | Adjacency | | |
|---|---|---|---|---|---|---|---|
| Nodes | Chains | Chain | From Node | To Node | Chain | Left Polygon | Right Polygon |
| 1-2 | a | a | 1 | 2 | a | 0 | I |
| 2-3 | b | b | 2 | 3 | b | 0 | II |
| 3-4 | c | c | 3 | 4 | c | 0 | III |
| 4-5 | d | d | 4 | 5 | d | 0 | III |
| 5-1 | e | e | 5 | 1 | e | 0 | I |
| 5-6 | f | f | 5 | 6 | f | I | III |
| 6-2 | g | g | 6 | 2 | g | I | II |
| 6-3 | h | h | 6 | 3 | h | II | III |

**(b)** Problem 28.13(b)

| Identifier | Coordinates | Line Identifier | Points | Area Identifier | Lines |
|---|---|---|---|---|---|
| 1 | $x_1,y_1$ | a | 1,2 | I | b,c,q,l |
| 2 | $x_2,y_2$ | b | 3,4 | II | d,r,m,q |
| 3 | $x_3,y_3$ | c | 4,5 | III | e,f,n,r |
| 4 | $x_4,y_4$ | d | 5,6 | IV | g,k,l,m,n,o,p |
| 5 | $x_5,y_5$ | e | 6,7 | V | a,o,s,j |
| 6 | $x_6,y_6$ | f | 7,8 | VI | h,i,s,p |
| 7 | $x_7,y_7$ | g | 8,9 | | |
| 8 | $x_8,y_8$ | h | 9,10 | | |
| 9 | $x_9,y_9$ | i | 10,11 | | |
| 10 | $x_{10},y_{10}$ | j | 11,1 | | |
| 11 | $x_{11},y_{11}$ | k | 2,3 | | |
| 12 | $x_{12},y_{12}$ | l | 3,13 | | |
| 13 | $x_{13},y_{13}$ | m | 13,14 | | |
| 14 | $x_{14},y_{14}$ | n | 14,8 | | |
| 15 | $x_{15},y_{15}$ | o | 2,12 | | |
| 16 | $x_{16},y_{16}$ | p | 12,9 | | |
| 17 | $x_{17},y_{17}$ | q | 5,13 | | |
| 18 | $x_{19},y_{18}$ | r | 6,14 | | |
| 19 | $x_{19},y_{19}$ | s | 11,12 | | |

| Connectivity | | Direction | | | Adjacency | | | Nestedness | |
|---|---|---|---|---|---|---|---|---|---|
| Nodes | Chains | Chain | From Node | To Node | Chain | Left Polygon | Right Polygon | Polygon | Nested Node |
| 1,2 | a | a | 1 | 2 | A | 0 | V | III | d |
| 3,4 | b | b | 3 | 4 | B | 0 | I | IV | b,c |
| 4,5 | c | c | 4 | 5 | C | 0 | I | V | a |
| 5,6 | d | d | 5 | 6 | D | 0 | II | | |
| 6,7 | e | e | 6 | 7 | E | 0 | II | | |
| 7,8 | f | f | 7 | 8 | F | 0 | III | | |
| 8,9 | g | g | 8 | 9 | G | 0 | IV | | |
| 9,10 | h | h | 9 | 10 | H | 0 | VI | | |
| 10,11 | i | i | 10 | 11 | I | 0 | VI | | |
| 1,11 | j | j | 1 | 11 | J | 0 | V | | |
| 2,3 | k | k | 2 | 3 | K | 0 | IV | | |
| 3,13 | l | l | 3 | 13 | L | I | IV | | |
| 13,14 | m | m | 13 | 14 | M | II | IV | | |
| 8,14 | n | n | 8 | 14 | N | III | IV | | |
| 2,12 | o | o | 2 | 12 | O | IV | V | | |
| 9,12 | p | p | 9 | 12 | P | IV | VI | | |
| 5,13 | q | q | 5 | 13 | Q | I | II | | |
| 6,14 | r | r | 6 | 14 | R | II | II | | |
| 11,12 | s | s | 11 | 12 | S | V | VI | | |

**28.14** Compile a list of linear features for which the topological relationship of adjacency would be important.

Streets, railroads, rivers, streams, transmission lines, bus routes, mail routes, electric circuits, water mains, and so on.

**28.15** Prepare a raster (grid cell) representation of the sample map of:

**(a)** Problem 28.15(a), using a cell size of 0.10-in. square (see accompanying figure).

**(b)** Problem 28.15(b), using a cell size of 0.20-in. square (see accompanying figure).

**28.16** Discuss the advantages and disadvantages of using the following equipment for converting maps and other graphic data to digital form:

***(a)** tablet digitizers

From Section 28.7.3, Last Paragraph: "Data files generated in this manner can be obtained quickly and relatively inexpensively. Of course the accuracy of the resultant data can be no better than the accuracy of the document being digitized, and its accuracy is further diminished by differential shrinkages or expansions of the paper or materials upon which the document is printed and by inaccuracies in the digitizer and the digitizing process."

**(b)** scanners.

From Section 28.7.6, Last Paragraph: "Accuracy of the raster file obtained from scanning depends somewhat on the instrument's precision, but pixel size or resolution is generally the major factor. A smaller pixel size will normally yield superior resolution. However there are certain tradeoffs that must be considered. Whereas a large pixel size will result in a coarse representation of the original, it will require less scanning time and computer storage. Conversely, a fine resolution, which generates a precise depiction of the original, requires more scanning time and computer storage. An additional problem is that at very fine resolution, the scanner will record too much "noise," that is, impurities such as specks of dirt. For these reasons and others, this is the least preferred method of capturing data in a GIS."

**28.17** Explain the concepts of the following terms in GIS spatial analysis, and give an example illustrating the beneficial application of each: (a) adjacency; and (b) connectivity.

From Section 28.9.2: "Adjacency and connectivity are two important boundary operations that often assist significantly in management and decision-making. An example of adjacency is illustrated in Figure 28.10(d) and relates to a zoning change requested by the owner of parcel A. Before taking action on the request, the jurisdiction's zoning administrators are required to notify all owners of adjacent properties B through H. If the GIS database includes the parcel descriptions with topology and other appropriate attributes, an adjacency analysis will identify the abutting properties and provide the names and addresses of the owners.

Connectivity involves analyses of the intersections or connections of linear features. The need to repair a city water main serves as an example to illustrate its value. Suppose that the decision has been made that these repairs will take place between the hours of 1:00 and 4:00 P.M. on a certain date. If infrastructure data are stored within the city's GIS database, all customers connected to this line whose water service will be interrupted by the repairs can be identified and their names and addresses tabulated. The GIS can even print a letter and address labels to facilitate a mailing announcing details of the planned interruption to all affected customers."

**28.18** If data were being represented in vector format, what simple spatial objects would be associated with each of the following topological properties?

| | |
|---|---|
| (a) Connectivity | Points and Lines |
| (b) Direction | Points and Lines |
| (c) Adjacency | Interior Areas |
| (d) Nestedness | Interior Areas and Points |

**28.19** Prepare a transparency having a 0.10-in grid, overlay it onto Figure 28.4(a), and indicate the grid cells that define the stream. Now convert this raster representation to vector using the method described in Section 28.6.2. Repeat the process using a 0.20-in grid. Compare the two resulting vector representations of the stream and explain any differences.

*Suggestion: If you have access to a scanner, scan Figure 28.4(a) and have the students import it into their CAD package. Then set both the grid and snap to 0.1 and 0.2. and* *have the student trace the image in raster format. They will quickly realize the number of problems that occur when scanning linear features such as streams and edges of features.*

**28.20** Discuss how spatial and non-spatial data are related in a GIS.

From Section 28.5, Paragraph 2: "In general, spatial data will have related nonspatial attributes and thus some form of linkage must be established between these two different types of information. Usually this is achieved with a common identifier that is stored with both the graphic and the nongraphic data. Identifiers such as a unique parcel identification number, a grid cell label, or the specific mile point along a particular highway may be used."

**28.21** What are the actual ground dimensions of a pixel for the following conditions:

**(a)** A 1:10,000 scale, 9 in. square orthophoto scanned at 300 dpi?

$(9 \times 300)/10{,}000 =$ **0.27 in.**

***(b)** A 500 in. square, 1:24,000 map, scanned at 300 dpi?

$(500 \times 300)/24{,}000 =$ **6.25 in.**

**28.22** Describe the following GIS functions, and give two examples where each would be valuable in analysis:

**(a)** line buffering, and

From Section 28.9.1, Paragraph 2: "Line buffering, illustrated in Figure 28.10(b), creates new polygons along established lines such as streams and roads. To illustrate the use of line buffering, assume that to preserve the natural stream bank and prevent erosion, a zoning commission has set the construction setback distance from a certain stream at D. Line buffering can quickly identify the areas within this zone."

**(b)** spatial joins

From Section 28.9.3, Paragraph 2: "Having these various data sets available in spatially related layers makes the overlay function possible. Its employment in a GIS can be compared to using a collection of Mylar overlays in traditional mapping. However, much greater efficiency and flexibility are possible when operating in the computer environment of a GIS, and not only can graphic data be overlaid, but attribute information can be combined as well."

**28.23** Go to the PASDA web site or a similar web site in your state and download an example of:

**(a)** An orthophoto
**(b)** Zoning
**(c)** Floodplains and wetlands
**(d)** Soil types

**28.24** Compile a list of data layers and attributes that would likely be included in an LIS.

Control to establish a common basis of reference coordinates; political boundaries; U.S. Public Land System, legal descriptions in metes and bounds, block and lot deeds; easements; improvements; parcel ownership, parcel values, hydrography, and so on.

**28.25** Compile a list of data layers and attributes that would likely be included in a GIS for:

**(a)** Selecting the optimum corridor for constructing a new rapid-transit system to connect two major cities

Control; state, county, and municipal boundaries; U.S. Public Land Survey System; legal descriptions and parcels; easements; parcel ownership; parcel values; existing transportation routes; topography; hydrography; existing land use; zoning; soil types; depth to bedrock or underground mines; depth of water table; utilities; and so on.

**(b)** Choosing the best location for a new airport in a large metropolitan area.

All layers in (a) plus building locations with heights; tower locations and heights; and other vertical obstructions; quiet zones; and so on.

**(c)** Routing a fleet of school buses

Control; state, county, and municipal boundaries; existing transportation routes with data on widths, grades, pavement type/condition, and speed limits; load and height restrictions on highways and bridges; construction activity; detours; and so on.

**(d)** Selecting the fastest routes for reaching locations of fires from various fire stations in a large city

Control; municipal boundaries; ward boundaries; fire station locations; transportation routes with grades, lighting, and traffic signals; traffic counts; accident records; construction activity; detours; and so on.

**28.26** In Section 28.9.3, a flood-warning example is given to illustrate the value of simultaneously applying more than one GIS analytical function. Describe another example.

Answers will vary

**28.27** Consult the literature on GISs and, based on your research, describe an example that gives an application of a GIS in:

**(a)** Natural resource management
**(b)** Agriculture
**(c)** Engineering
**(d)** Forestry

Independent Project

# Part II:
# Sample Introductory Course

## Sample Syllabus

15-Week, 3 Credit, Semester Course

Required Text: Ghilani, Charles D. and Paul R. Wolf. 2014. **Elementary Surveying (An Introduction to Geomatics), 14th Ed**. Prentice Hall, Upper Saddle River, NJ.

Materials:

- Safety vest
- Field book and 3H or 4H pencil
- Computation note pad
- Scientific calculator with 10-digit display
- Engineer's Scale

Grading:

- Homework.... 15%
- Practical exercises20%
- Quizzes (5)..... 5%
- Hour exams (3)30%
- Article reviews (3) 6%
- Portfolio ......... 4%
- Final exam.... 20%

Lecture Schedule

| Week | Lecture No. | Subject | Reading | Homework |
|---|---|---|---|---|
| 1 | 1 | Class policies, Introduction | Chapter 1 | – |
| | 2 | Introduction | Chapter 1 | – |
| 2 | 3 | Units Significant Figures | 2.1 to 5 | 2.1, 3, 5, 10, 14 |
| | 4 | Field Notes | 2.6 to 15 | – |
| 3 | 5 | Errors - mean, standard deviation, probable error | 3.1 to 16 | 3.3, 6, 11, 16 |
| | 6 | Error propagation | 3.17 to 21 | 3.19, 21, 27, 30(a) |
| 4 | 7 | Leveling - Theory and Methods | Chapter 4, Part 1 | 4.1, 4, 13, 16, 18 |

| | | | | |
|---|---|---|---|---|
| | 8 | Leveling - Equipment | Chapter 4, Part 2 | 4.19, 20, 24, 28 |
| 5 | – | Exam 1 | – | – |
| 5,6 | 9–10 | Leveling - Field Procedures | Chapter 5 | 5.1, 3, 9, 13, 23 |
| 6 | 11 | Taping | Chapter 6,Part 1 | 6.6, 9, 11, 19, 22 |
| 7 | 12 | EDM | Chapter 6, Part 2 | 6.25, 28(b), 33, 38, 43 |
| | 13 | Angles, Azimuths, and Bearings | 7.1 to 9 | 7.3, 6, 11, 24, 39 |
| 8 | 14 | The Compass and Magnetic Declination | 7.10 to 16 | 7.31, 37 |
| | 15 | Total Station Instruments | Chapter 8, Part 1 | – |
| 9 | 16 | Angle Measurements | Chapter 8, Part 2 | 8.2, 4, 12, 21, 35 |
| | 17 | Traversing | Chapter 9 | 9.5, 10, 12, 13, 24 |
| 10 | – | Exam 2 | | |
| | 18 | Traverse Computations | Chapter 10 | |
| 11 | 19–20 | Traverse Computations | Chapter 10 | 10.12, 13, 14, 15, 24, 25 |
| 12 | 21–22 | COGO – Intersections | 11.1 to 6 | 11.4, 10, 12, 16, 17 |
| 13 | 23–24 | COGO – Resection and Coordinate Transformations | 11.7 to 11 | 11.20, 22, 23, 24, 27 |
| 14 | 25 | Area by Simple Figures | 12. 1 to 12.4 | – |
| | – | Exam 3 | – | – |
| 15 | 26–27 | Area by Coordinates | 12.4 to 11 | 12.2, 4, 13, 24, 26 |

**Article Reviews**

A short review of journal articles will be due in the following weeks of the course. Possible sources for articles are listed at the end of each chapter in the book. Papers will be graded on completeness of thought, grammar, spelling, and punctuation. All reviews should be word processed and contain the following items.

**Citation**: See examples of proper citations in the bibliography at the end of each chapter.

**Author's thesis**: A brief statement or two on the main focus of the article.

**Author's argument**: A review of the article stating how the author supported the thesis.

**Reviewer's opinion**: Not all that is written is correct. Write a brief paragraph on why you agree or disagree with the author's thesis and how this article relates to this class.

| Week | Subject |
|---|---|
| 2 | Problems 1.20 or 21 |
| 7 | Write an article review on one of articles listed in the bibliography for Chapter 4. |
| 12 | Write an article review on one of articles listed in the bibliography for Chapter 8. |

**Practical Exercises** (Refer to the list of Sample Practical Exercises on the following pages.)

| Week | Practical Exercise |
|---|---|
| 1 – 2 | A |
| 3 | B |
| 4 | D |
| 5 – 6 | E |
| 7 | F |
| 8 | G |
| 9 – 10 | H |
| 11 | I |
| 12 | J, Problem 11.18 |
| 13 | J, Problem 11.37 |
| 14 | K |
| 15 | Review for final |

## Sample Practical Exercises

To fully understand and appreciate the theory discussed in Elementary Surveying, a student should be exposed to a series of practical, hands-on exercises. This section covers a sample set of exercises for your consideration. Some exercises assume that the instructor has assigned a set of traverse stations to the students for leveling, distance and angle observations.

| Chapter | Number | Exercise |
|---|---|---|
| 2 | A | Students should read the manual for their survey controller and determine the proper procedure for setting up a project. |
| 3 | B | Outdoor lab: Develop a pacing lab. In this lab layout a 100-yard, -meter, line on a level section of ground. Have students pace the line 10 times estimating the length of the last pace. Following this, have the students pace the traverse that will be assigned to them for distance measurement in Chapter 6.<br><br>Students should develop a report giving the length of their pace and the standard deviation. They should compute the length of the lines of the traverse in feet or meters along with the estimated error in the length.<br><br>$$E = E_{pace}\sqrt{n}$$<br><br>where *n* is the number of paces. |
| 3 | C | Inside lab: Hang a plumb bob from the ceiling of your room. Have the students measure the length of the string from support to the tip of mass center of the bob. Now measure the period of the plumb bob using a stopwatch. Repeat this procedure ten times.<br><br>Student should develop a report providing the average period (T) of the pendulum, and its standard deviation. They should then compute the “approximate” value for gravity using the formula<br><br>$$g = \frac{\pi^2 \ell \left(1 + \frac{h}{8\ell}\right)^2}{T^2}$$<br><br>where *l* is the length of the string, *h* is the height the pendulum falls during a half oscillation. Note the pendulum string is not weightless, nor the pivot frictionless, so do not assume this to be an accurate value for gravity. |

4 D Have the students perform a collimation test of their automatic/digital level following the method discussed in Section 4.15.5. Have the student report on the collimation error in their instrument and discuss how this error will be removed when using the instrument for differential leveling. They should also compute the maximum allowable difference in plus and minus sight distances if this error is to be kept under on-half of their reading. For example 0.005 ft if the minimum reported elevation is to 0.01 ft.

5 E Using a nearby bench mark as control, the students should run a leveling loop from the bench mark, over their stations, and back to the bench mark meeting Third Order leveling specifications.

The report should contain a listing of the final adjusted elevations for each station, discuss any problems encountered in the field, include a copy of the final field notes, and provide the misclosure in the loop. If the exercise for Chapter 4 was performed, then collimation error should be removed from each elevation.

6 F Using a tape, measure the length of each course in the assigned traverse. The line should be measured twice and a precision computed.

The report should contain a copy of the field notes, and discuss any problems encountered.

6 G Using a EDM, determine the horizontal length of each course in a line. The line should be measured from two stations.

The report should contain a copy of the field notes, the average length for each line, and discuss any problems that may have occurred in the field.

8 H Using a theodolite or total station, the students should close the angular horizon about each of their stations turning each angle two times with each face of the instrument (2DR). Using this information, the students should determine the horizon misclosure, adjust the angles at each station, and then adjust the interior angles of the traverse.

The report should contain the original field notes, list the horizon misclosure at each station, adjusted angles, traverse misclosure, and the correct geometric sum of each angle. Students should make sure that all angles are geometrically closed.

| | | |
|---|---|---|
| 10 | I | Using the distances observed in Chapter 8 and the angles observed in Chapter 8, and an assigned or assumed azimuth for one course of their traverse, students should perform a compass rule adjustment of the traverse. Using starting coordinates of (1000.000, 5000.00), the report should contain the linear precision, relative precision, the adjusted latitudes and departures, coordinates for each station, and adjusted observations. |
| 11 | J | Do problem 11.18, 11.37, 11.38, or 11.39. |
| 12 | K | Compute the area of the traverse from the exercise for Chapter 10. |
| 13 | L | Do Problem 13.35 and 13.36. |
| 14 | M | Do Problem 14.40. |
| 14 | N | Perform a rapid static survey of your traverse. Adjust the baselines and the network. Report on the adjusted baseline vector components, the loop closures as discussed in Section 14.5.4. |
| 15 | O | Have students perform an kinematic mapping surveying of a local area. |
| 16 | P | Do one of the problems from 16.41 to 16.45. |
| 17 | Q | Have students collect radial data to map an assigned area around their traverse. If a controller is available, the students should use the codes discussed in Section 17.11 that are appropriate for your software. |
| 18 | R | Have students create a map of the data collected in exercise Q. |
| 19 | S | Have students create the program for Problem 19.43 or 19.44. |
| 20 | T | Have students create the program for Problem 20.47 or 20.48. |
| 21 | U | Research the deeds for you school or an assigned parcel and perform a boundary survey. In the report, note the survey procedures used, their closures, found monuments in agreement with the deed, monuments that do not agree, and monuments not found. |

| | | |
|---|---|---|
| 22 | V | Layout a township at a 1/10th scale following the procedures discussed in Chapter 22. |
| 23 | W | Perform a profile level courses for the traverse from Chapter 10 using 25-ft stationing. |
| 24 | X | Compute the stakeout notes for a horizontal curve with an intersection angle of 60° and length of 300 ft or 100 m. If you are using English units, use 25-ft stationing. If you are using metric units, use 10-m stationing. Stake the curve in the field using the incremental chord method.<br><br>If you have a data collector, use WOLFPACK to compute coordinates for the given horizontal curve and stake it out using the controller's stake out functions. |
| 25 | Y | After profile-leveling the horizontal curve staked out in the previous exercise, compute a vertical alignment that minimizes excavation. |
| 26 | Z | Do Problem 26.31 or 26.32. |
| 27 | AA | Have students do either Problem 27.38, 27.39, 27.40, or 27.41. |
| 28 | BB | Using a GIS software package and the shape files provided by the NGS at http://www.ngs.noaa.gov/cgi-bin/datasheet.prl develop a GIS that allows the user to find NGS control stations in your county and sort by type and quality. |

## Sample Quizzes

### Quiz 1

1. One acre equal _____________ square feet and ____________ square Gunter's chains.
2. Give the answer of the following problems rounded to the correct number of significant figures:
   a. Sum of 0.0237, 30.05, 254.0 _____________
   b. Product of 31.75 × 4.0 _____________
   c. Quotient of 793.82 ÷ 71 _____________

### Quiz 2

1. For the following ten repeated EDM observations what are
   325.686, 325.685, 325.687, 325.681, 325.691, 325.686, 325.681, 325.686, 325.690, and 325.689

   a. Most probable value _____________
   b. Standard error in a single observation _____________
   c. 95 per cent probable error ___________

## Quiz 3

1. For the following sequential minus (FS) and plus (BS) sights observed on a closed level circuit, set up the left-side of a standard set of level notes and give an adjusted elevation for BM A.

BM X (elev = 850.25): 3.87

TP 1: FS = 3.73, BS = 6.80

TP 2: FS = 7.04, BS = 6.22

BM A: FS = 9.16, BS = 3.49

TP 3: FS = 7.20, BS = 8.65

TP 4: FS = 6.38, BS = 6.00

BM X FS = 1.58

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## Quiz 4

1. A 100-ft tape is calibrated at 68° F, fully supported with 15 lbs of tension and found to be 99.987 ft long. Is this tape is used fully supported with 15 lbs of tension at 86° F to measure a distance that is recorded as 136.48 ft, what is the corrected length of the line?

   ______________

2. If a certain EDM has a centering error of 3 mm and a scalar error of 3 ppm, what is the uncertainty in a observed distance of 1380.25 ft?

   _____________

## Quiz 5

1. In 1895 when the magnetic declination was 6°45′ East, line AB had a magnetic bearing of S 7 30 E.
   a. What is the magnetic bearing of AB today if the current magnetic declinations is 2 30 W?

   ______________

   b. What are the true bearing and true azimuth of this line?

   _____________

## Quiz 6

1. If the slope of a line is 0.3258, what is the azimuth of the line? _____________
2. What is the area in square units of a polygon with coordinates at its vertices of (103.45, 214.87), (250.34, 567.98), and (185.02, 386.94) _____________

# Sample Exams

## Exam 1

(1 point each)

**True – False** [Fill in the circle indicating whether the statement is true (T) or false (F).]

○ T ○ F 1. The current definition of the meter is 39.37 inches is equivalent to one meter.

○ T ○ F 2. The length of 1429.75 m is equivalent to 4690.72 survey feet to the correct number of significant figures.

○ T ○ F 3. Random errors may be mathematically computed and removed from observations.

○ T ○ F 4. The number 1.0020 has five significant figures.

○ T ○ F 5. A set of precise observations is always accurate.

○ T ○ F 6. National representation of surveying interest is the principal interest of the American Congress on Surveying and Mapping.

○ T ○ F 7. One acre is 43, 560 square feet.

○ T ○ F 8. A Gunter's chain is 100 ft long.

○ T ○ F 9. The correctly round sum of 46.328 + 1.03 + 375.1 is 422.4.

○ T ○ F 10. The National Geodetic Survey is responsible for establishing a network of survey control monuments.

○ T ○ F 11. The arrangement of a field book is a matter of personal preference.

○ T ○ F 12. It is best to only enter a minimum amount of data into a field book.

○ T ○ F 13. A new page should be started in the field book for each new day of work.

○ T ○ F 14. The geoid is an equipotential surface.

○ T ○ F 15. Earth curvature always causes rod readings to be too high.

○ T ○ F 16. Parallax exists when the focal point of the objective lens does not coincide with the focal point of the eyepiece lens.

○ T ○ F 17. The NAVD 88 datum is based on the average elevation of 26 tide gage stations.

○ T ○ F 18. A page check in differential leveling only provides an arithmetic check of the notes.

○ T ○ F 19. Automatic levels guarantee a horizontal line of sight at each setup.

○ T ○ F 20. The statistical term used to express the precision of a data set is called standard deviation.

**Problems/Short answers**

(5 points)

A. Discuss why the term geomatics is being used to identify the profession of surveying.

(10 points)

B. State the number of significant figures in each of the following values.

______ 0.0024 ______ 7620 ______ 0.0007621 _______ 1050.130 ______ 750.

(10 points)

C. Convert the following observations as indicated.
   (a) 164.803 m to U.S. Survey feet ____________
   (b) 215.648 grads to degrees-minutes-seconds ____________
   (c) 12 ch 7 lks to survey feet ____________
   (d) 123,600 sq. ft. to acres ____________
   (e) 153 26 14 to radians ____________

(15 points)

D. Compute the most probable value, standard deviation, and 95% probable error for the following set of angle observations.

$116°13'46'', 116°13'46'', 116°13'48'', 116°13'44'', 116°13'50''$

MPV = ______________

σ = _____________

$E_{95\%}$ = ______________

(5 points)

E. Compute the combined Earth curvature and refraction on a 3000-ft site. ______________

(20 points)

F. A differential leveling circuit is starts at bench mark Hydrant (Elevation 430.330 m) and ends at bench mark Post (Elevation 430.002 m). The readings (in meters) list in the order taken are 0.983 on Hydrant, 5.467 and 4.086 pm TP1, 0.952 and 3.905 on Mark; and 2.886 on Post. Use this information to complete the left-side of differential leveling notes.

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# Exam 2

(1 point each)

**True – False** [Fill in the circle indicating whether the statement is true (T) or false (F).]

○ T ○ F 1. EDM's are unaffected by refraction.

○ T ○ F 2. The length correction in taping is an example of a random error.

○ T ○ F 3. The velocity of an electromagnetic wave does not change when passing through atmosphere.

○ T ○ F 4. A cut tape is graduated with an extra foot beyond the zero mark.

○ T ○ F 5. The NGS as specifications for "third-class leveling."

○ T ○ F 6. A collimation test checks if the line of sight in a leveling instrument is horizontal.

○ T ○ F 7. When measuring distance with an EDM, the line of sight should never be within 1 m anywhere along its path.

○ T ○ F 8. A rod level will increase both the accuracy and speed in the field.

○ T ○ F 9. In general, humidity is irrelevant when measuring distances with a near-infrared EDM.

○ T ○ F 10. Magnetic declination is the difference between geodetic azimuth and magnetic azimuth.

○ T ○ F 11. A total station is in adjustment if its line-of-sight axis is perpendicular to its vertical axis.

○ T ○ F 12. One-second of arc is about 0.05 ft in 10,000 ft.

○ T ○ F 13. The DIN 18723 standard is based on the observation of a single direction.

○ T ○ F 14. The "principle of reversion" is used when adjusting level bubbles.

○ T ○ F 15. In practice, instruments should always be kept in good adjustment, but used as though they might not be.

(1 point each)

**Fill in the blank**

The kinds of horizontal angles most commonly observed in surveying are:

(1) ____________________, (2) __________________, and (3) _____________________.

A. Azimuths may be (1) ______________________, (2) _____________________,

(3) _____________________, (4) _____________________, (5) _____________________, and

(6) _____________________.

**Short Answer Problems**

(10 points)

B. A distance is measured with an EDM having the instrument/reflector offset constant set to 0 mm. The slope distance is reported as 2435.672 m with a zenith angle reading of $93°34'05''$. The offset is later determined to be 23 mm. What is the correct horizontal distance for this observation?

_______________

(10 points)

C. A 100-ft steel tape has a length 99.987 ft when fully supported at a temperature of 68° F and tension of 10-lbs. What is the corrected length of a measured by this fully-supported tape if the recorded length, temperature and tension are 83.05 ft, 33° F, and 25 lbs of tension, respectively.

(10 points)

D. A 867.89 ft distance is measured with an EDM that has a manufacturer's specified accuracy of 3 mm + 3 ppm. Both the instrument and target miscentering errors are assumed to be ±0.005 ft. What is the uncertainty in this observation?

_______________

(10 points)

E. The magnetic bearing of a line in 1884 was N 23 15'W . The magnetic declination at this times was 5 12 W. What is the true bearing of this line?

_____________

(10 points)

F. A zenith angle was measured twice direct giving values of 92 14 26 and 92 14 28 , and twice reversed yielding readings of 267 45 30 both times. What is the mean zenith angle, and the indexing error?

______________

(10 points)

G. Discuss the field procedure used to prolong a line of sight.

## Exam 3

(1 point each)

**True – False** [Fill in the circle indicating whether the statement is true (T) or false (F).]

○ T ○ F 1. A closed traverse begins and ends at a station of known coordinates.

○ T ○ F 2. If the azimuth of a line is 272°15′26″, then the bearing of the same line is S92°15′26″W.

○ T ○ F 3. Excepts for deflection angles, surveyors should always turn angles clockwise.

○ T ○ F 4. Angles to the right are clockwise angles with backsights at the "rearward" station and the foresights on the "forward" station.

○ T ○ F 5. To avoid ambiguity, only two reference ties should be used to reference a station.

○ T ○ F 6. Adjustment of angles is dependent on the size of the angle.

○ T ○ F 7. The departure of a course is the change in the *x* coordinate.

○ T ○ F 8. Open traverses should only be used as a last resort in surveying.

○ T ○ F 9. A single angular mistake can be identified by extending the perpendicular bisector of the linear closure line.

○ T ○ F 10. The intersection of two lines with known lengths always results in two possible solutions.

(5 points)

A. What is the geometric closure on a closed polygon traverse with 18 sides? ______________

(10 points)

B. What is the azimuth of line CD if the azimuth of AB is 105°39′12″, angle ABC is 67°35′08″, and angle BCD is 275°10′15″?

______________

(10 points)

C. If line AB has an azimuth of 156°14′34″ and line BC has an azimuth of 41°56′42″, what is angle ABC?

______________

(5 points)

D. What is the angular misclosure on a five-sided traverse with observed interior angles of 83°07′23″, 105°23′01″, 124°56′48″, 111°51′31″, and 114°41′27″?

______________

(15 points)

E. Fill in the missing parts of the closed traverse table below.

| Azimuth | Distance | Departure | Latitude |
|---|---|---|---|
| 45°32′15″ | 415.76 | | |
| 101°56′35″ | | | −112.85 |
| | 644.65 | −502.27 | |
| 209°23′00″ | 668.46 | | |

(10 points)

F. Station A has *xy* coordinates (in feet) of (42992.36, 14354.37) and station B has *xy* coordinates of (43476.79, 15110.90). What are the course length and azimuth?

________________

________________

(10 points)

G. If the sum of the departures in a closed polygon traverse having a total perimeter of 3911.05 ft is 0.22 ft, what is the correction to a course of length 1007.38 ft have a departure of 726.76 ft?

________________

(10 points)

H. What is the linear misclosure and relative precision of a traverse of 2169.91ft if the misclosure in departure and latitude are −0.017 ft and −0.086 ft, respectively?

________________

________________

(10 points)

I. The azimuth of a line in an assumed coordinate system is 242°15′26″. The azimuth of the same line in a datum is 168°38′22″. What is the rotational angle needed to perform a two-dimensional conformal coordinate transformation?

________________

(5 points)

J. If the standard error for each angle measurement of a traverse is ±3″, what is the estimated error in the geometric closure in the sum of the angles for a 12-sided traverse?

________________

## Final Exam

**Equation Sheet**

$\pm\sqrt{E_a^2 + E_b^2 + E_c^2 + \ldots}$

$\bar{M} - M$

$\pm\sqrt{\frac{\Sigma v^2}{n-1}}$

$\pm E\sqrt{n}$

$\left(\frac{l - l'}{l'}\right)L$

$L \cos \alpha$

$0.65 \times 10^{-5}(T_1 - T)L$

$(P_1 - P)\frac{L}{\left(29 \times 10^6\right)A}$

$-\frac{w^2 L_S^3}{24 P_1^2}$

$R\theta$

$\frac{2 \times E_{DIN}}{\sqrt{n}}$

$\alpha \tan(v)$

$0.574\, M^2 = 0.0206\, F^2$

$0.0675\, K^2$

$S \sin \alpha$

$hi + V - r$

$\frac{R_B - r_A - r_B + R_A}{2}$

*Elev + BS − FS*

$(n-2)180°$

$L \sin \alpha$

$L \cos \alpha$

$-\frac{\text{(total departure misclosure)}}{\text{traverse perimeter}} \times \text{length of } AB$

$-\frac{\text{(total latitude misclosure)}}{\text{traverse perimeter}} \times \text{length of } AB$

$$X_A + \text{departure } AB$$
$$Y_A + \text{latitude } AB$$

$$\sqrt{(\text{departure } AB)^2 + (\text{latitude } AB)^2}$$

$$\tan^{-1}\left(\frac{\Delta X}{\Delta Y}\right) + C$$

$$\sqrt{s(s-a)(s-b)(s-c)}$$

$$b\left(\frac{h_0}{2} + h_1 + h_2 + \cdots + \frac{h_n}{2}\right)$$

$$S \times \sigma_S \sqrt{2}$$

$$\frac{1}{2}[X_A(Y_E - Y_B) + X_B(Y_A - Y_C) + X_C(Y_B - Y_D) + X_D(Y_C - Y_E) + X_E(Y_D - Y_A)]$$

$$\text{meridian distance of } AB + \frac{1}{2}\text{departure of } AB + \frac{1}{2}\text{departure of } BC$$

$$\pi R^2 \times (\theta/360°)$$

$$\pm\sqrt{A^2E_b^2 + B^2E_a^2}$$

$$\frac{E}{\sqrt{n}}$$

## Final Exam

(1 point each)

**TRUE - FALSE** [Fill in the circle indicating whether the statement is true (T) or false (F).]

❍T ❍F 1. The number 768,000 has six significant digits.

❍T ❍F 2. There are 45,360 square feet in one acre.

❍T ❍F 3. Using the survey foot definition, one meter equals 39.37 inches.

❍T ❍F 4. 24 times 360.01 equals 8640 to the correct number of significant digits.

❍T ❍F 5. 78.149 is equal to 78.2 when round to the tenths place.

❍T ❍F 6. Field notes should be discarded when a project is complete.

❍T ❍F 7. Sketches in a field book should be drawn to an accurate scale.

❍T ❍F 8. A zenith angle is measured in the horizontal plane.

❍T ❍F 9. The sensitivity of a level vial is determined by its radius of curvature.

❍T ❍F 10. In leveling, balancing plus sight and minus sight distances corrects for instrument collimation errors.

❍T ❍F 11. Refraction always causes the line of sight to appear to be too high.

❍T ❍F 12. "*Accuracy*" denotes absolute nearness to the truth.

❍T ❍F 13. Bringing the bubble *halfway back* to center compensates for the fact that the vertical axis of a total station is not perpendicular to the axis of the plate bubble.

❍T ❍F 14. Measuring angles both direct and reverse compensates for the fact that the vertical axis in a total station may not be perpendicular to the horizontal axis.

❍T ❍F 15. Under a fixed set of conditions, random errors have the same magnitude and sign.

❍T ❍F 16. Waving the rod during leveling compensates for curvature and refraction.

❍T ❍F 17. A slope distance measured by EDM must be corrected for atmospheric pressure and temperature.

❍T ❍F 18. Three repeated measurements for a distance are 395.28, 395.27, and 395.29 ft., respectively. These measurements are precise when the true value of the measurement is 395.95 ft.

❍T ❍F 19. Vertical lines at all locations are parallel.

❍T ❍F 20. A ten second level vial is more sensitive than a two second vial.

❍T ❍F 20. A prism causes all electronically measured distances to appear to be too long.

❍T ❍F 21. The geoid is an equipotential surface.

❍T ❍F 22. A zenith angle of 88°15′ is equivalent to a vertical angle of 1°45′.

❍T ❍F 23. The sum of the interior angles of a seven-side closed polygon traverse should be 900°.

❍T ❍F 24. The compass rule adjustment is known as an arbitrary adjustment technique.

❍T ❍F 25. Angles of larger magnitude should always receive the largest corrections.

❍T ❍F 26. A precision of 1:5000 means that there can be 0.5 foot of error in every 2500 ft.

❍T ❍F 27. Surveying plats show slope distances recorded between points.

❍T ❍F 28. When the tape is only supported at its ends, the recorded distance is always too long.

❍T ❍F 29. A link traverse is an example of an open traverse.

❍T ❍F 30. 4129.57 m is equal to 2.56599 mi to the correct number of places.

**PROBLEMS**

The following are six repeated measurements of a taped distance.

429.35 , 429.34, 429.37, 429.32, 429.39, 429.33

{5 points}

**A.** What is the most probable value of the measurement? (nearest 0.01) MPV = ______________

{5 points}

**B.** What is the standard deviation? (nearest 0.001) σ = _____________

The following questions apply to measurements using a steel tape which was calibrated to be 99.890-ft long when fully supported at 68°F and 10-lbs. pull. Its cross-sectional area was 0.0050 square inches.

{5 points}

**C.** A line AB was measured on flat ground with the tape fully supported using 10 lbs. of pull and recorded to be 275.20-ft. long when the temperature was 43°F. What is its corrected horizontal length? (nearest 0.001 ft)

L = ___________________

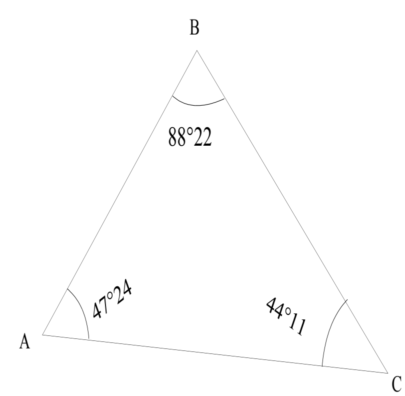
{5 points}

**D.** A horizontal distance *DE* exactly 275.20 ft. is required to be laid out according to a blue print. If this distance will be laid out on flat ground using the tape fully supported with a 20-lb pull, and the temperature is 85°F what distance must be measured using this tape. (nearest 0.01-ft)

L = ___________________

{10 points}

**E.** For the measured angles given on the figure below, and assuming the fixed azimuth of the line AB is 47°45′, calculate the adjusted angles and azimuths of lines BC and CA and show a check. (nearest 1′)




∠A = _______°_______'

∠B = _______°_______'

∠C = _______°_______'

$Az_{BC}$ = _______°_______'

$Az_{CA}$ = _______°_______'

{10 points}

**F.** What is the area of the five-sided parcel below to the nearest square foot? ... nearest 0.001 acre?

| Station | X (ft) | Y (ft) |
|---|---|---|
| *A* | 0.00 | 591.64 |
| *B* | 125.66 | 847.60 |
| *C* | 716.31 | 294.07 |
| *D* | 523.62 | 0.00 |
| *E* | 517.55 | 202.97 |

AREA = ___________________ ft$^2$

AREA = _________________ acres

(10 points)

**G.** The coordinates of A and B are (23451.23, 10034.56) and (22678.93, 12387.43), respectively. What are the azimuth and distance of the line AB. (nearest 1", nearest 0.01 ft)

AB = ____________________ ft

$Az_{AB}$ = ______________°______'______"

(10 points)

**H** Two measurements that presented unusual difficulty in the field were omitted in the survey of a boundary, as shown in the following field notes. Compute the missing distance and azimuth for line *EA*. (nearest 1"; nearest 0.01 ft)

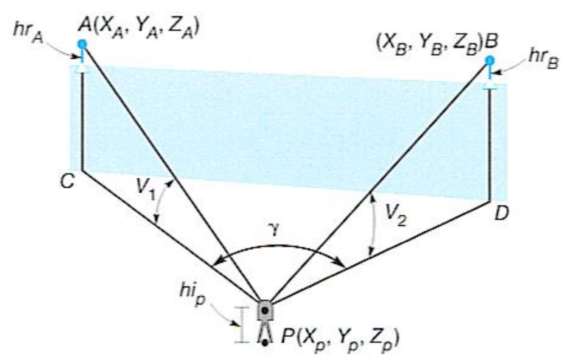
| Line | Distance | Azimuth | Departure | Latitude |
|---|---|---|---|---|

| | | | | |
|---|---|---|---|---|
| *AB* | 671.07 | 88°00′00″ | | |
| *BC* | 436.45 | 91°00′00″ | | |
| *CD* | 510.67 | 206°00′00″ | | |
| *DE* | 778.05 | 318°00′00″ | | |
| *EA* | | | | |

*EA* = ____________________ ft

$Az_{EA}$ = _________________°_______'________"

(10 points)




I In the figure to the right, the *X*, *Y*, and *Z* coordinates (in feet) of station *A* are 3860.83, 4819.98, and 154.06, respectively, and those at *B* are 6865.48, 5007.21, and 135.69, respectively. Determine the three-dimensional of a total station at point *P* base upon the following observations.

$v_1$ = 0°29′06″ *PA* = 2423.67 ft

$v_2$ = −0°03′04″ *PB* = 2413.00 ft

γ = 76°59′20″

$hr_A$ = 5.20 ft $hr_B$ = 5.20 ft $hi_P$ = 5.20 ft

## Preface

The Instructor's Manual has been prepared as a convenience for instructors who adopt, for use in their classes, the textbook ELEMENTARY SURVEYING (AN INTRODUCTION TO GEOMATICS), 15th Edition, by Charles D. Ghilani. As a benefit to the instructor, each problem consists of the book question and a derived solution. For most questions, a reference to the equations, section, and/or paragraph containing the answer has been included together with a copy of the relevant material from the text. This is provided so that useful feedback and references to the appropriate parts of the book can be easily provided to the students. Every attempt has been made to provide correct responses to the questions, but, most assuredly, a few mistakes may have occurred. Please accept our apologies for them and correct them in your printouts. Any erratum that is found is the book will be graciously accepted by the author to update subsequent printings of this and future editions.

The companion website for this the book at http://www.pearsonhighered.com/ghilani contains the software WOLFPACK, MATRIX, and STATS. These software packages are discussed throughout the book where appropriate and can be used to check many of the solutions. These software packages are freeware, and can be freely downloaded and installed on your computer systems. However, they are only to be used for educational purposes and no support for using these packages is implied or given. Discussions in the book and the accompanying help file system should be sufficient for determining the proper procedures for using the software.

These software packages can be used by students to check their responses to numerical questions in the book, or to do some of the more numerically intensive problems in the book that would otherwise be extremely time-consuming. Additionally there is a software package called PROBLEM GENERATOR, which uses a normal random number generator and statistics to generate realistic data sets for a single observation, multiple observations, horizontal and vertical networks, traverse problems, GNSS networks, and two-dimensional conformal coordinate transformations. These software packages often only require coordinates of stations and the desired observations to be determined. The generated observations are randomly perturbed using a normal random number generator to create realistic data sets. This software can be used to create instructor-generated additional problems for the practice and examination.

Also on the companion website is a Mathcad® electronic book (e-book) to accompany most of the chapters in the book. Additionally, Mathcad worksheets have been developed for several map projections mentioned in the book, but not thoroughly discussed. For those who do not have Mathcad®, html files of these worksheets have also been placed on the companion website. Additionally, several MS Excel® spreadsheets which demonstrate these computations are on the companion website. All of the software on the companion website requires Windows

95 or later operating system and have accompanying help files, which discuss their use. Mathcad requires version 14.0 or higher.

Additionally, short videos of the solutions to some problems are available on the companion website for this book. There are also instrument videos on field procedures. These videos include the PowerPoint presentations with them, which you can use in your classes. The calibration videos discuss methods to check for instrumental errors, field compensation procedures when appropriate, and how the instruments are adjusted with strong statements about most should be done only by a qualified technician. However, checking the tripod and adjusting the level vials do suggest that the surveyor can and should make these adjustments when necessary. The video presentations for the instrumentation include:

- Adjusting the Level Vials
- Checking the Tripod
- Checking the Optical/Laser Plummet
- Checking the Cross Hairs
- Checking the Perpendicularity of the Horizontal Axis versus the Line of Sight Axis
- Checking the Perpendicularity of the Horizontal Axis versus the Vertical Axis
- Checking the Instrument/Reflector Constant in a EDM
- Checking the Vertical Indexing Error
- Centering the Instrument over a Point
- Leveling an Instrument
- Reading a Level Rod
- Differential Leveling
- Precise Leveling
- Removing Parallax
- Observing an Angle

Charles D. Ghilani

# TABLE OF CONTENTS