Introduction to the Technicalities of a Least Squares Adjustment for Field Operators

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Least Squares (Mathematically):

Is an adjustment where the sum of the weighted squares of the residuals is at a minimum.

How Does Least Squares Adjustments Work

- Least Squares Adjustments are used:
  - To adjust all the small random errors remaining after all the blunders and systematic errors have been removed.
  - When you want to more heavily weight some measurements more than others.
  - When you want to see the possible expected errors in your survey points.

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How Does Least Squares Adjustments Work (Continued)

- When you want to evaluate the quality of your measurements.
- When you want to include redundant measurements that cannot be included in conventional adjustments.

What Type of Measurements Can be Adjusted in a Least Squares Adjustment

- Traverse
- Levels
- GNSS
  - Traverse with Levels
  - Levels with GNSS vectors
  - Traverse with GNSS vectors
  - Traverse with Levels with GNSS vectors

What Type of Data is Needed for a Least Squares Adjustment

- Traverse observations
  - Horizontal angles
  - Distances
  - Vertical angles
  - HI's
  - Known coordinates
  - Error estimates

What Type of Data is Needed for a Least Squares Adjustment (Continued)

- Levels
  - Differences in elevation
  - Known elevations
  - Error estimates

What Type of Data is Needed for a Least Squares Adjustment (Continued)

- GNSS
  - GNSS vectors
  - Error Estimates
    - How are the errors estimated for GNSS

Why Do the Field Operators Need to Understand Error Estimates

- Field operators need to help the office understand the accuracy for all their measurements.
- Office and the field need coordination to be sure the field is not exceeding the errors the office expects and to be sure the office is not expecting more accuracy than the field can deliver.

What Determines the Accuracy of Survey Measurements

- Project requirements
- Density and accuracy of existing control
- Quality of the instruments you are using
- Tolerances for quality of measurements
- Types of procedures you use
- Equipment properly adjusted

Traverse Errors

EDM Errors With Care

• Nominal accuracy	±0.016 ft (5mm+5 ppm)
• Instrument Centering	±0.005 ft
• Reflector Centering	±0.005 ft
• Temperature & Pressure	±5 ppm
• Instrument Constant	±0.006 (2mm)
• Reflector Constant	±0.006 (2mm)

Traverse Errors (Continued)

$$\pm 3.0''(\text{DIN}) \times \sqrt{2}$$

Angles:

• Pointing	
• Collimation	1" - 30" Normal +
• Parallax	2" - 3" Normal +
• Inclination	1"- 3" Normal +
• Environmental	1" - 3" Normal +
• Tripod Stability	1" - 3" Normal +
• Centering Instrument & Targets	????

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Traverse Errors (Continued)

Accessories:

- Tribrachs
- Tripods
- Thermometer
- Barometer

Leveling Errors

- Balance
- Type of leveling turning point
- Sight length

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Leveling Tolerances

Order *Class*	*First* *I*	*First* *II*	*Second* *I*	*Second* *II*	*Third*
Section misclosures (backward and forward)					
One-Setup Section	±0.40mm	±1.00mm	-- -- -- --	-- -- -- --	-- -- -- --
Two runnings of a section less than 0.10 km in length	±0.95mm	±1.26mm	±1.90mm	±2.53mm	±3.79mm
Algebraic sum of all corrected section misclosures of a leveling line not to exceed	$3\sqrt{D}$	$4\sqrt{D}$	$6\sqrt{D}$	$8\sqrt{D}$	$12\sqrt{D}$
Section misclosure not to exceed (mm)	$3\sqrt{E}$	$4\sqrt{E}$	$6\sqrt{E}$	$8\sqrt{E}$	$12\sqrt{E}$
Loop misclosures					
Algebraic sum of all corrected misclosures not to exceed (mm)	$4\sqrt{F}$	$5\sqrt{F}$	$6\sqrt{F}$	$8\sqrt{F}$	$12\sqrt{F}$
Loop misclosure not to exceed (mm)	$4\sqrt{F}$	$5\sqrt{F}$	$6\sqrt{F}$	$8\sqrt{F}$	$12\sqrt{F}$
(D -- shortest length of leveling line (one-way) in km) (E -- shortest one-way length of section in km) (F -- length of loop in km)					

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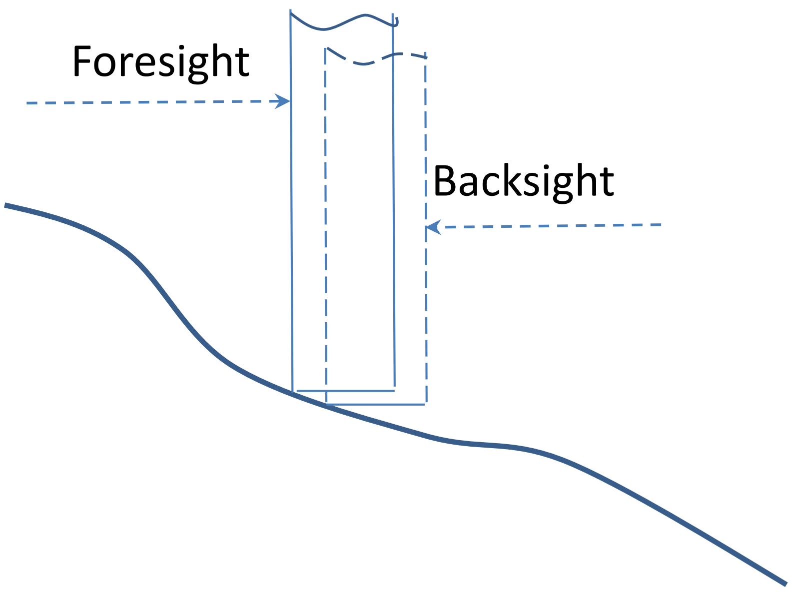
Leveling Without a Pin

- When leveling and the surface is not precisely flat, if the rod is not held at exactly the same location a systematic error will occur.
- Remember you are measuring very precisely especially with digital levels



Leveling Field Procedures

- Keep Sights Balanced

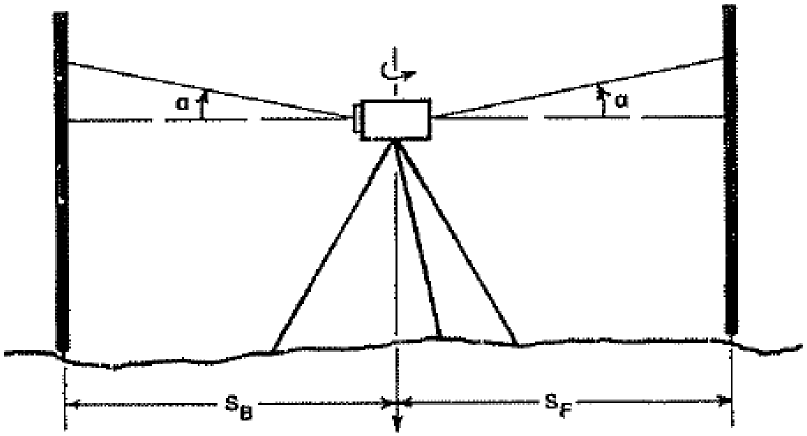


Figure 3-4.—Consistent collimation error cancels in a balanced setup since $s_B = s_F$.

- Requirements for lines of Sight and balance.

Lines of sight	First Order Class 1	First Order Class II	Second Order Class I	Second Order Class II	Third Order
Maximum sighting distance	160 Ft	195 Ft	195 Ft	230 Ft	295 Ft
Maximum imbalance					
Per Setup	±6Ft	±15Ft	±15Ft	±33Ft	±33Ft
Per Section	±13Ft	±33Ft	±33Ft	±33Ft	±33Ft

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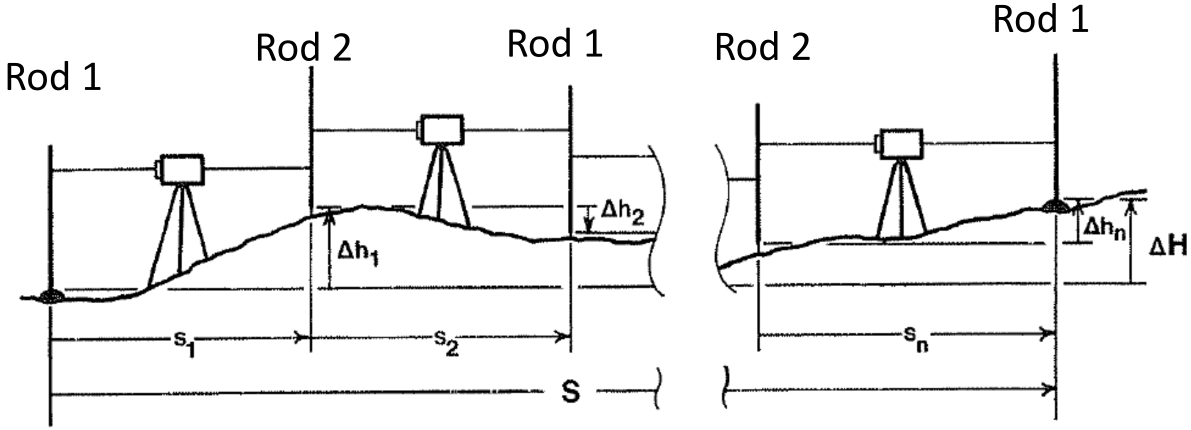
Leveling Field Procedures

- When using two rods label them “Rod 1” and “Rod 2”
- Come of your known bench mark with “Rod 1” and go into your known bench mark with “Rod 1”
- Leap frog your rods.
- This will eliminate any rod length errors.



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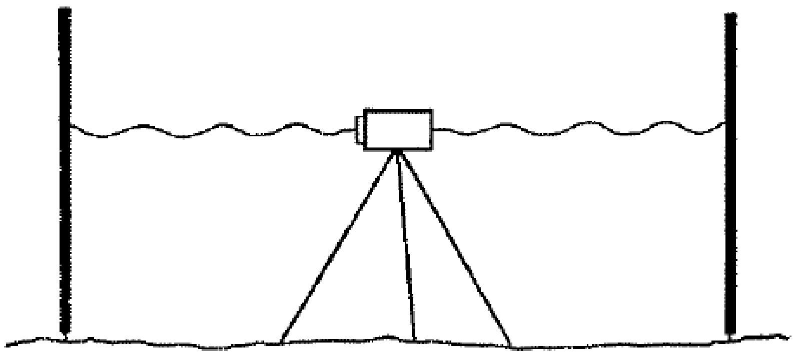
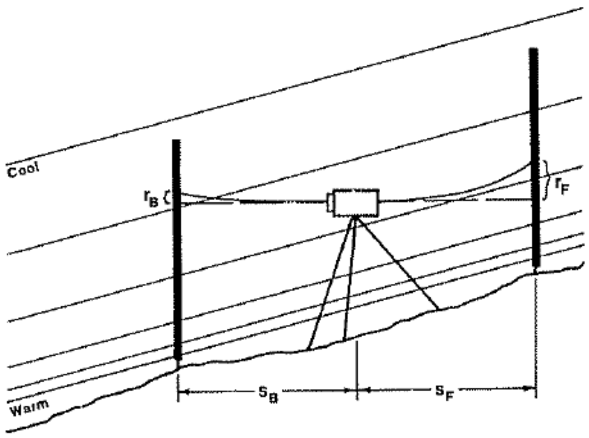
Leveling Field Procedures

- Heat Shimmer Usually Cancels in a Balanced Setup.



Shimmer.

- Refraction does not Cancel even with Balanced Setups. "No readings less Than 1.5 ft"



Refraction error, r, does not cancel on sloping terrain since $r_B \neq r_F$, even if $s_B = s_F$.

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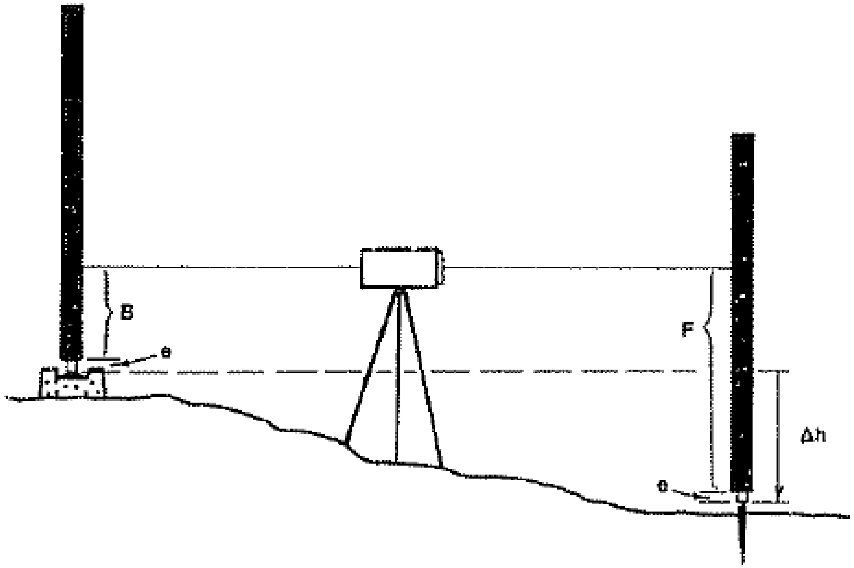
Leveling Field Procedures

- Using a Plug "Errors Cancel"



When using spacers the height, e, of each spacer cancels when the elevation difference, Δh, is computed: $\Delta h = (B + e) - (F + e) = B - F$.

GNSS Errors

- Tribrachs
- Range Pole Bubbles
- Range Pole Points
- HI’s
- Multipath
- Short Observations

GNSS Errors (Continued)

- First Four Can be Elliminated by Using Fixed Height Tripods.
- If You Use Them Properly

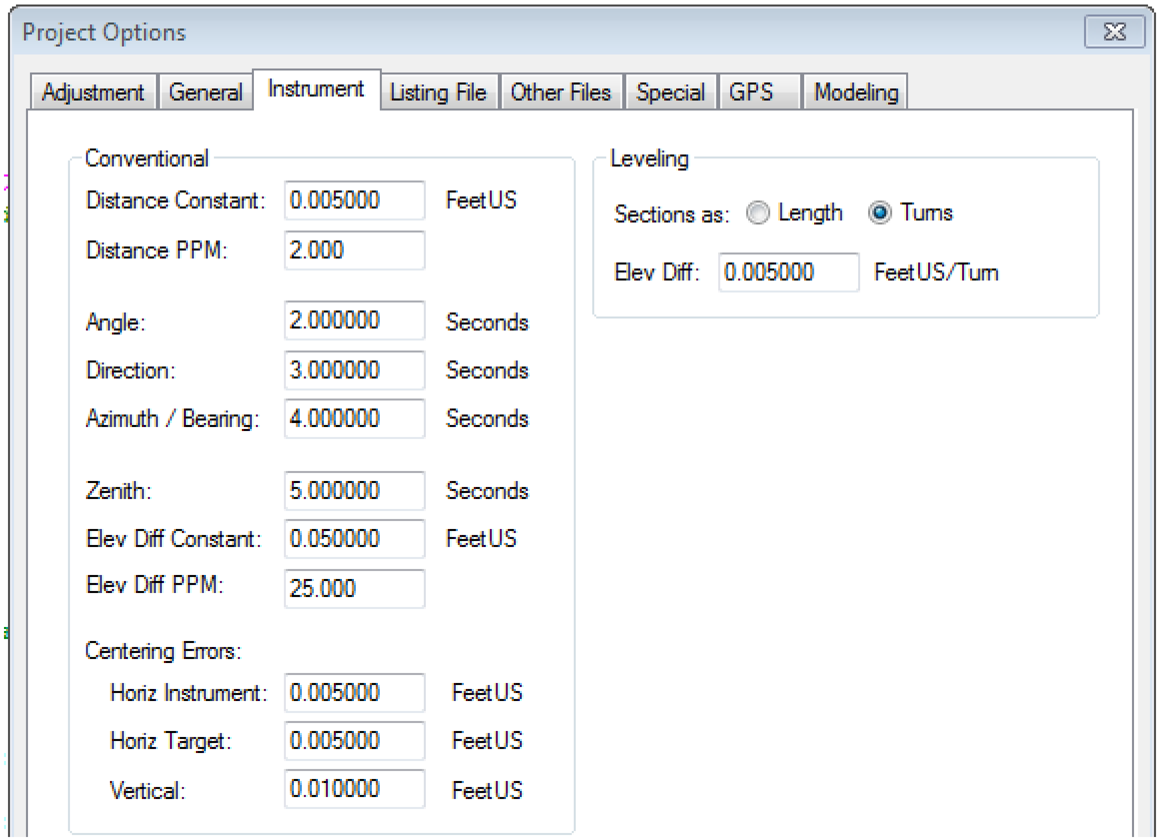
How Do You Get Your Data Into Your Adjustment Program

- Trimble
  - Imports Trimble data very well
  - Imports other types of data but not so well
  - Imports GPS data very well
  - Imports RINEX data well
- Leica
  - Imports Leica data very well
  - Imports other types of data but not so well
  - Imports GPS data very well
  - Imports RINEX data.

How Do You Get Your Data Into Your Adjustment Program (Continued)

- StarNet
  - Imports Trimble data and Leica data
    - 14 Different data sources
  - GPS Vector data
    - 17 Different data sources

Adjustment Setup



Preliminary Import Error Checking

- Mean turned angles
- Level closures
- Tollarances
- GPS vectors

What Are Our Steps to Do the Adjustment

- Step 1
  - – Sort Out the Errors From the Adjustment File
    - Eliminate Station Name Errors
    - Eliminate Possible Horizontal and Vertical Angle Errors
    - Eliminate Possible Distance Errors
    - Eliminate HI Errors

Preliminary Adjustment

- Make Corrections for Adjustment Errors
- Make Corrections for Any Blunder or Systematic Errors Discovered by the Adjustment.

Running the Adjustment

- Perform minimally constrained adjustment.
- Check for outliers.
  - Eliminate outliers.
- Set station weighting until Chi Square Test passes.
  - This sets your errors at the proper value from your initial estimate.
- Add one point at a time fixed horizontal.
- Check for outlines.
  - Eliminate outliers.

Running the Adjustment (Continued)

- Add one point at a time fixed vertical
- Check for outlines.
  - Eliminate outliers.
- If while you are locking down your horizontal and vertical control if your Chi-Square Test fails, do not rescale your errors. Doing this will push the errors into your survey control.

Evaluating Your Adjustment

- Reference Factor and Chi-Square Test

```
Adjustment Statistical Summary
==============================

       Iterations              =      2
       Number of Stations      =      7
       Number of Observations  =     21
       Number of Unknowns      =     15
       Number of Redundant Obs =      6

Observation   Count   Sum Squares       Error
                        of StdRes      Factor
Coordinates       6        6.036        1.876
     Angles       5        0.111        0.278
  Distances       5        0.054        0.194
    Zeniths       5        0.430        0.549

      Total      21        6.631        1.051

      The Chi-Square Test at 5.00% Level Passed

          Lower/Upper Bounds (0.454/1.552)
```

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Total Error Factor:

Also known as:

Reference Factor

or Variance of unit weight $\hat{\sigma}_o$

The relationship between the observation errors and the predicted error.

- When they are the same the $\hat{\sigma}_0 = 1$
- When the observation errors are less $\hat{\sigma}_o \langle 1$
- When the observation errors are more $\hat{\sigma}_o \rangle 1$

Chi-Square Test

- Tests the total error factor to see if it is significantly greater than 1.0 or less than 1.0

Residual

- The difference between any observed value and its adjusted value.
  - This should always be small.

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The Standardized Residual

The residual divided by the standard deviation

Observation	Residual	Std Error	Std Residual
1000.46 ft	.05 ft	.02 ft	2.50
100°12’23”	8”	12”	.66

The standardized residual is more sensitive to blunders than a residual, since it takes into consideration the geometry of the survey network.

- Standardized residuals that exceed 3 should be considered for rejection.

Residuals and Standardized Residuals In Your Adjustment

Adjusted Observations and Residuals
===================================

Adjusted Coordinate Observations (FeetUS)
(Stations with Partially Fixed Coordinate Components)

Station	Component	Adj Coordinate	Residual	StdErr	StdRes	File:Line
304	Elev	162.0998	-0.0936	1:19		
	E	1355489.3610	-0.0390	0.0500	0.8	
	N	524404.8372	-0.0307			
1001	N	524583.4989	0.0628	1:21		
	E	1355204.3408	0.0423	0.0500	0.8	
	Elev	162.0609	0.0343			

Adjusted Measured Angle Observations (DMS)

At	From	To	Angle	Residual	StdErr	StdRes	File:Line
302	301	1001	82-24-01.32	-0-00-02.68	8.51	0.3	1:34
303	302	304	347-59-38.93	-0-00-00.32	4.24	0.1	1:36
302	301	303	182-02-23.82	-0-00-00.68	9.10	0.1	1:32
303	302	306	103-41-50.00	0-00-00.00	25.67	0.0	1:40
304	303	305	83-02-34.25	-0-00-00.00	10.97	0.0	1:38

Adjusted Measured Distance Observations (FeetUS)

From	To	Distance	Residual	StdErr	StdRes	File:Line
302	303	384.2409	0.0015	0.0091	0.2	1:32
303	304	407.5308	-0.0014	0.0092	0.2	1:36
302	1001	254.0652	0.0003	0.0090	0.0	1:34
304	305	140.2624	-0.0000	0.0088	0.0	1:38
303	306	58.7463	-0.0000	0.0088	0.0	1:40

Adjusted Zenith Observations (DMS)

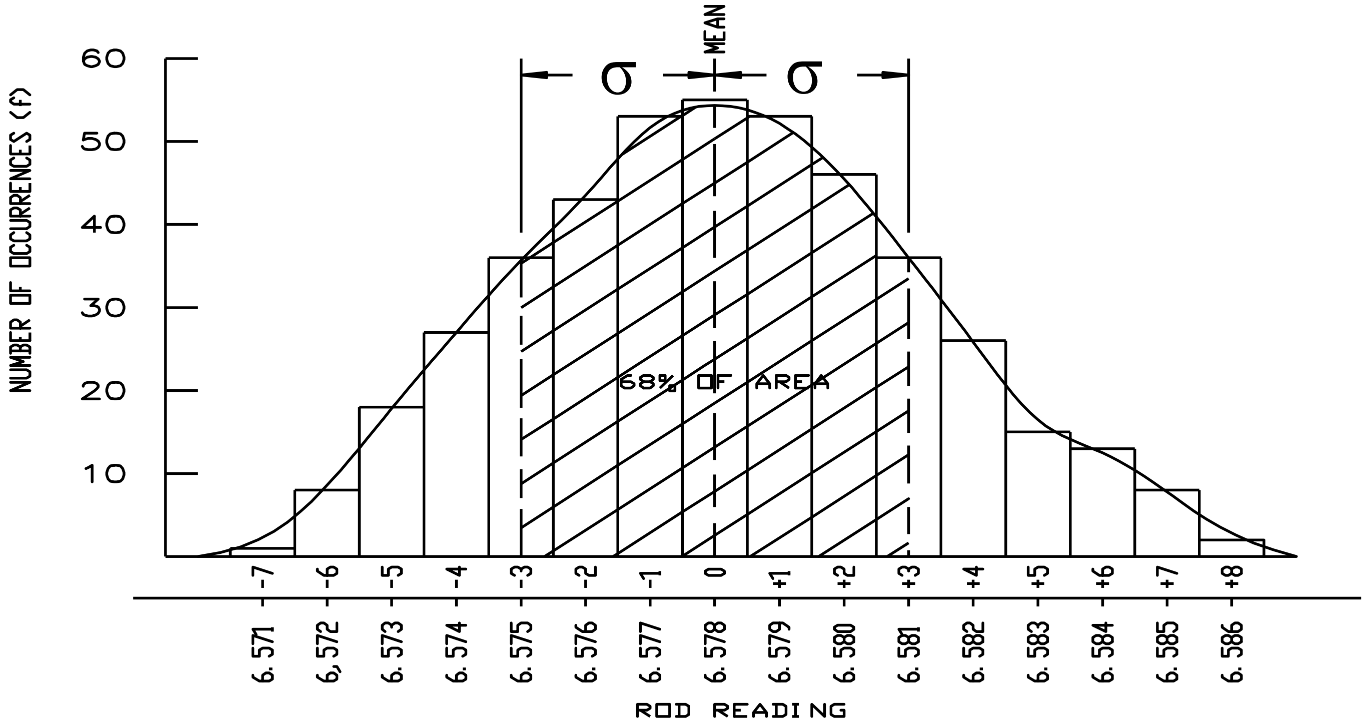
From	To	Zenith	Residual	StdErr	StdRes	File:Line
303	304	88-34-48.77	-0-00-03.98	8.73	0.5	1:36
302	303	91-30-53.17	-0-00-04.08	9.09	0.4	1:32
302	1001	90-08-50.59	0-00-01.84	12.52	0.1	1:34
304	305	90-36-06.75	-0-00-00.00	21.39	0.0	1:38
303	306	93-00-38.00	-0-00-00.00	49.86	0.0	1:40

Standardized Residual

Residuals

Standard Deviation σ_s

When blunders and systematic errors have been eliminated



The more measurements are repeated the more the frequency of reoccurring measurements plotted on a graph represent a bell shape curve clustered about the mean.

Standard Deviation

- Standard Deviation:
  - The name used to define the σ_S uncertainty of a single measurement of a set, to a defined level of confidence.

- Standard Deviation 1 sigma level:
  - 68.3% of a set of measurements fall σ_S This value from the arithmetic mean.

Standard Deviation

$$\sigma_s = \pm\sqrt{\frac{\sum v^2}{n-1}}$$

V = residual

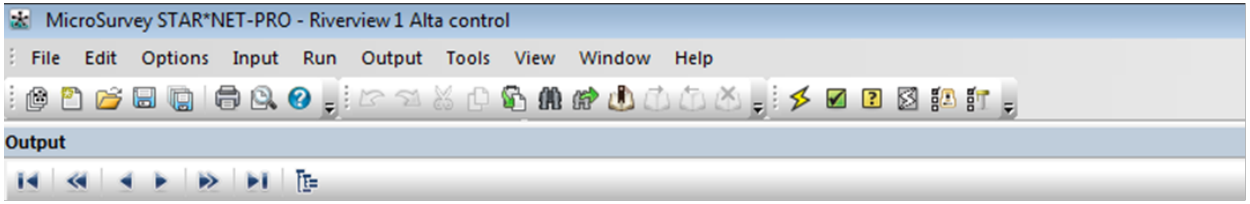
n = number of observations

Calculation of the Standard Deviation σ_s

Obs #	Reading	v (residual)	v^2
1	46.7"	-1.1"	1.21"
2	45.2"	-0.4"	0.16"
3	45.9"	+0.3"	0.09"
4	45.2"	-0.4"	0.16"
5	46.2"	+0.6"	0.36"
6	47.4"	+1.8"	3.24"
7	43.0"	-2.6"	6.76"
8	45.2"	-0.4"	0.16"
Mean:	45.6"	Total:	12.14"

$$\sigma_s = \pm\sqrt{\frac{12.14}{7}} \qquad \sigma_s = \pm 1.32''$$

Standard Deviation in Your Adjustment



Summary of Unadjusted Input Observations
==

Number of Measured Angle Observations (DMS) = 5

At	From	To	Angle	StdErr	t-T
302	301	303	182-02-24.50	9.10	0.01
302	301	1001	82-24-04.00	8.51	-0.00
303	302	304	347-59-39.25	4.24	[illegible].00
304	303	305	83-02-34.25	10.97	-0.00
303	302	306	103-41-50.00	25.67	0.00

Std Errors

Number of Measured Distance Observations (FeetUS) = 5

From	To	Distance	StdErr	HI	HT	Comb Grid	Type
302	303	384.2394	0.0091	5.730	5.320	0.[illegible]99556	S
302	1001	254.0649	0.0090	5.730	5.000	0.9999554	S
303	304	407.5322	0.0092	5.320	5.560	0.9999556	S
304	305	140.2624	0.0088	5.560	5.900	0.9999554	S
303	306	58.7463	0.0088	5.480	5.000	0.99995[illegible]	[illegible]

Number of Zenith Observations (DMS) = 5

From	To	Zenith	StdErr	HI	HT
302	303	91-30-57.25	9.09	5.730	[illegible]320
302	1001	90-08-48.75	12.52	5.730	5.000
303	304	88-34-52.75	8.73	5.320	5.560
304	305	90-36-06.75	21.39	5.560	5.900
303	306	93-00-38.00	49.86	5.480	5.000

Std Errors

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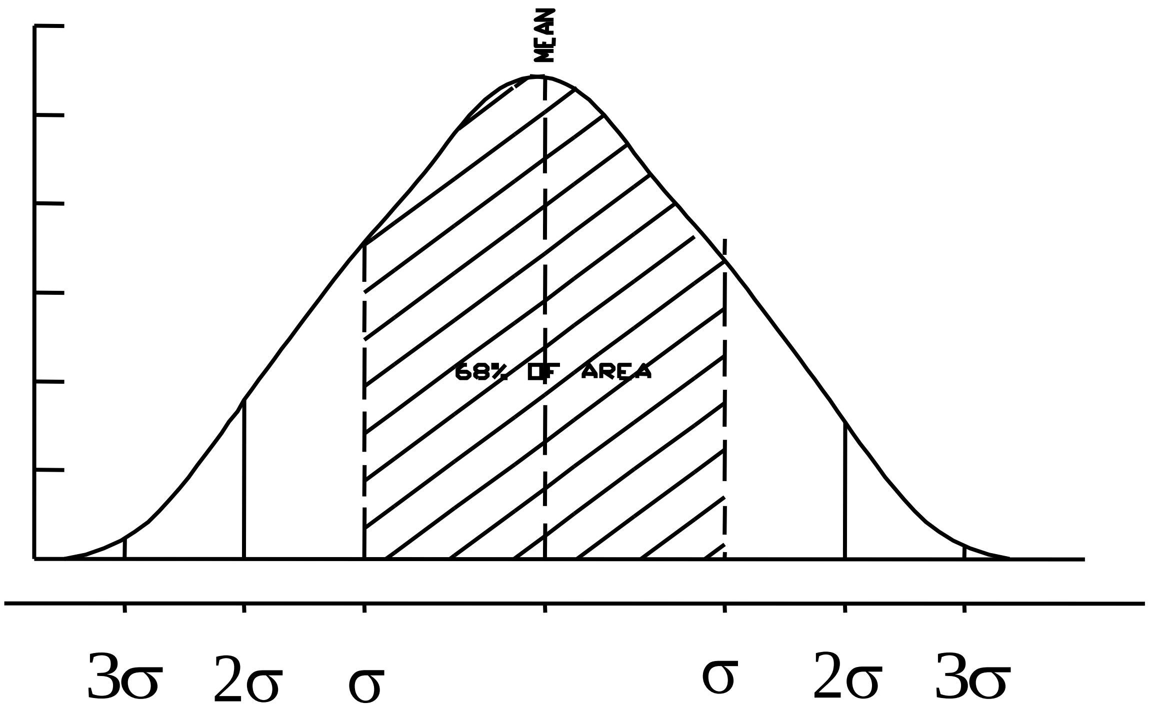
Levels of confidence

Name of error	Value	% Certainty
Probable or CEP	0.6745 σ	50%
Standard Deviation	1 σ	68.3
90% Error	1.6449 σ	90%
Two Sigma or 95% Error	2 σ	95%
99% Error	2.5 σ	99%
Three Sigma	3 σ	99.70%

(Note: Actually, the 95% error is closer to 1.96σ, but 2σ is often accepted as a convenient conversion.)

Standard Deviation with various levels of confidence



Standard Deviation of the Mean or Standard Error. σ_m

- Standard Deviation of the mean: σ_m
  - The ± uncertainty of the mean of a set of measurements, to a defined level of confidence.
  - The σ_m is relative to the true value for a set of measurements that have had all blunders and systematic errors removed.

$$\sigma_m = \frac{\sigma}{\sqrt{n}}$$

n= Number of observations

Standard Error σ_m

- There is a 68.3% chance of being within $\pm\ \sigma_m$ of the unknown true value.

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Confidence & Standard Error Relationships

- If you want more certainty that your measurements don't fall outside of your tolerance level, inflate the confidence level
- Doubling the certainty level will double the standard error
- To decrease the standard error, increase the accuracy, redundancy or known control.

Precision vs. Accuracy

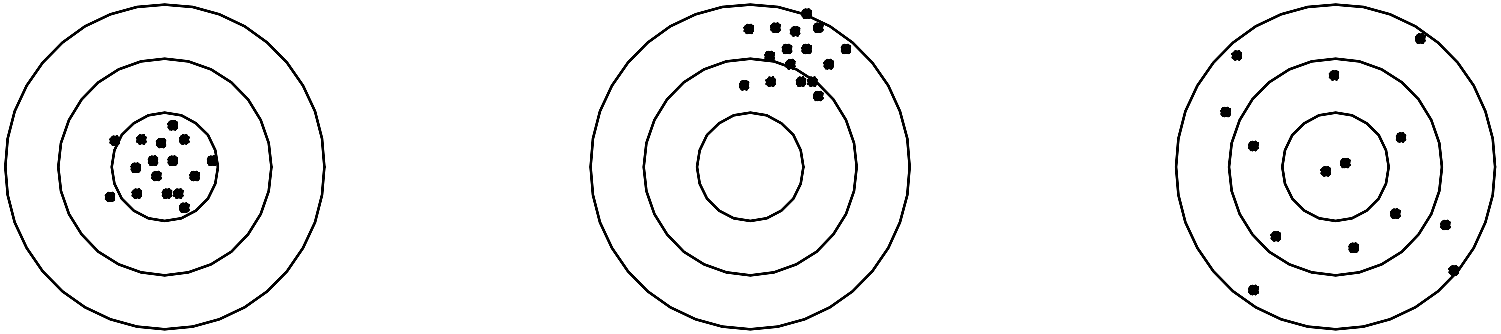


Fig #1 Fig #2 Fig #3

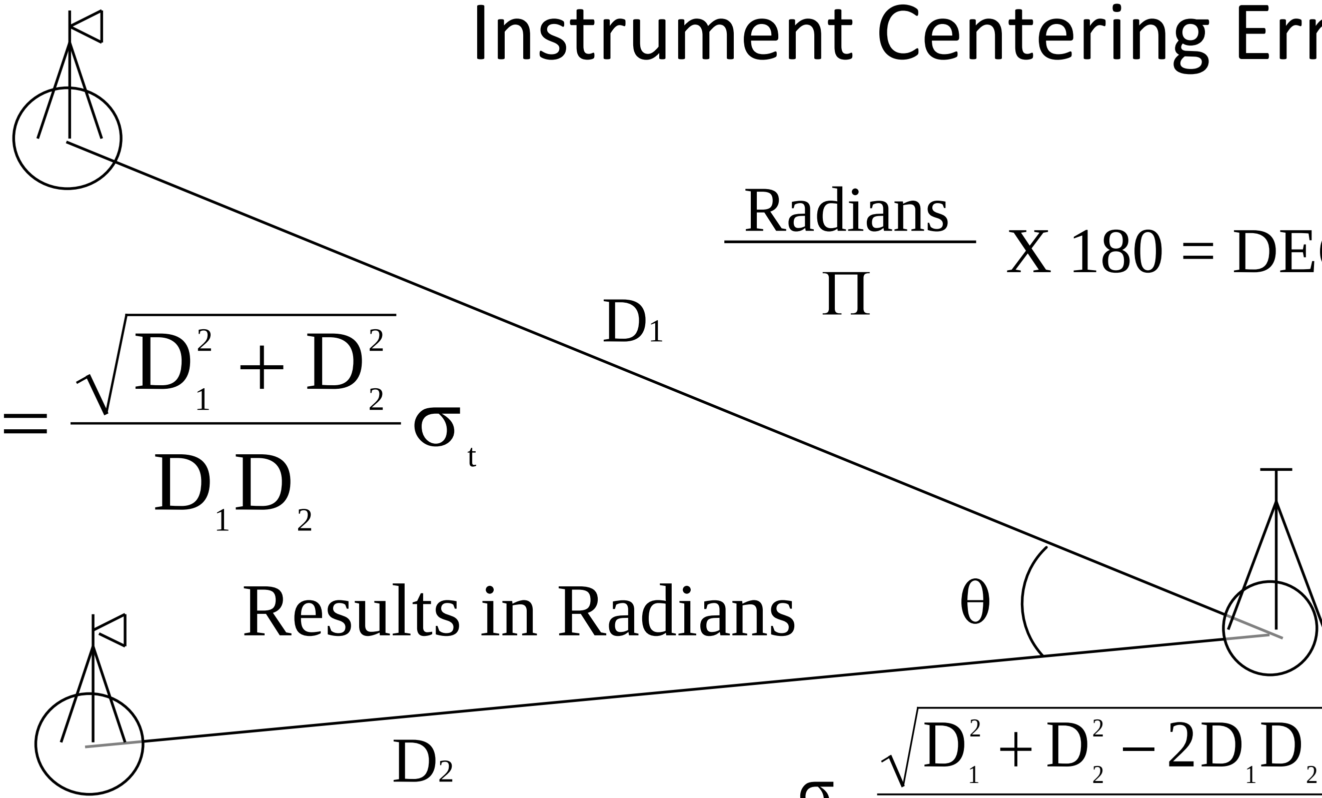
Grouping of rifle shots

- Figure #1 & #2 shots are precise.
- Figure #1 & #3 shots are very accurate.
- Figure #1 shots are precise and accurate.
- Figure #2 shots are precise not accurate.
- Figure #3 shots are accurate not precise.

Understanding Errors

Target Centering Error σ_{θ_t}

Instrument Centering Error $\sigma_{\theta c}$



$$\frac{\text{Radians}}{\Pi} \text{ X } 180 = \text{DEC DEG}$$

$$\sigma_{\theta_t} = \frac{\sqrt{D_1^2 + D_2^2}}{D_1 D_2}\sigma_t$$

Results in Radians

$$\sigma_{\theta_c} \frac{\sqrt{D_1^2 + D_2^2 - 2D_1D_2\cos\theta}}{D_1D_2}\sigma_c$$

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Error Propagation Due To Centering Errors for Instrument and Targets

Accuracy of Measuring Each Angle of a Traverse							
Horz Angle	$\theta =$	180	Deg				
Back Dist	$D_1 =$	600.000	Ft				
Fwd Dist	$D_2 =$	600.000	Ft				
Target Centering Error	$\sigma_t =$	0.010	Ft	$\sigma_{\theta t}$	=	4.9	"
Inst Centering Error	$\sigma_c =$	0.010	Ft	$\sigma_{\theta c}$	=	6.9	"
Number Positons	n=	2					
Pointing Accuracy	$\sigma_p =$	2.0	Sec				
Reading Accuracy	$\sigma_r =$	2.0	Sec				
Instrment Error				$\sigma_{\sigma p}$	=	1.4	"
Ponting Error				$\sigma_{\sigma r}$	=	1.4	"
	Dist	5280	Ft	σ_T	=	8.7	"
		206265					
		σ_T	=	1	/	23,832	=Precision
		Trav Dist					
		Precision	=	0.22	Ft	Trav Error in Feet	

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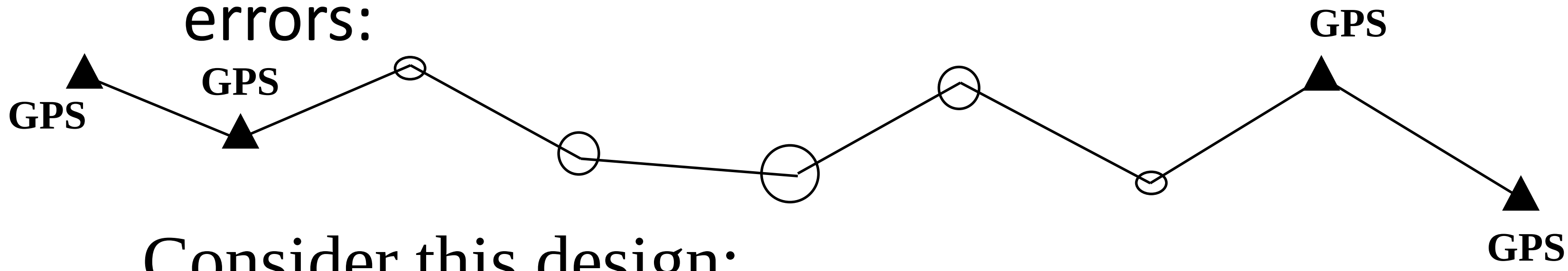
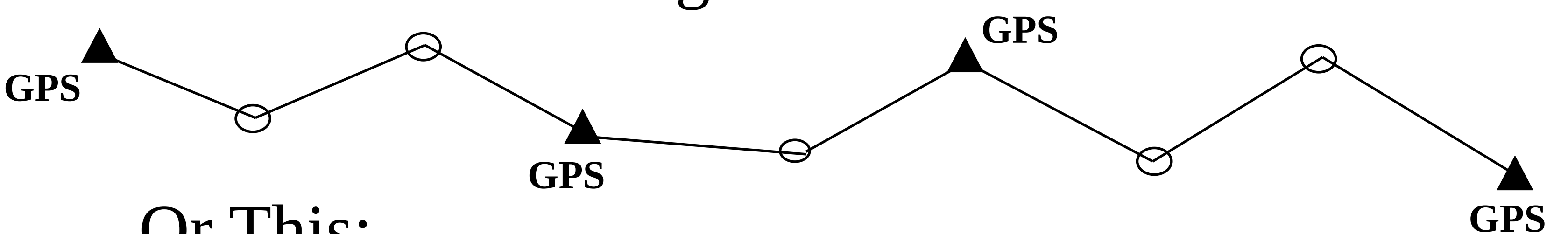
Planning A Traverse

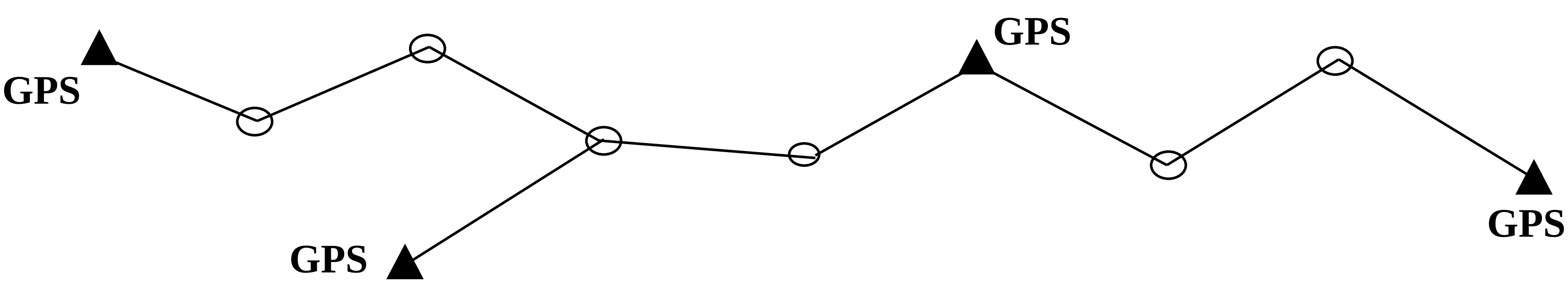
Instead of this design with its associated errors:



Consider this design:

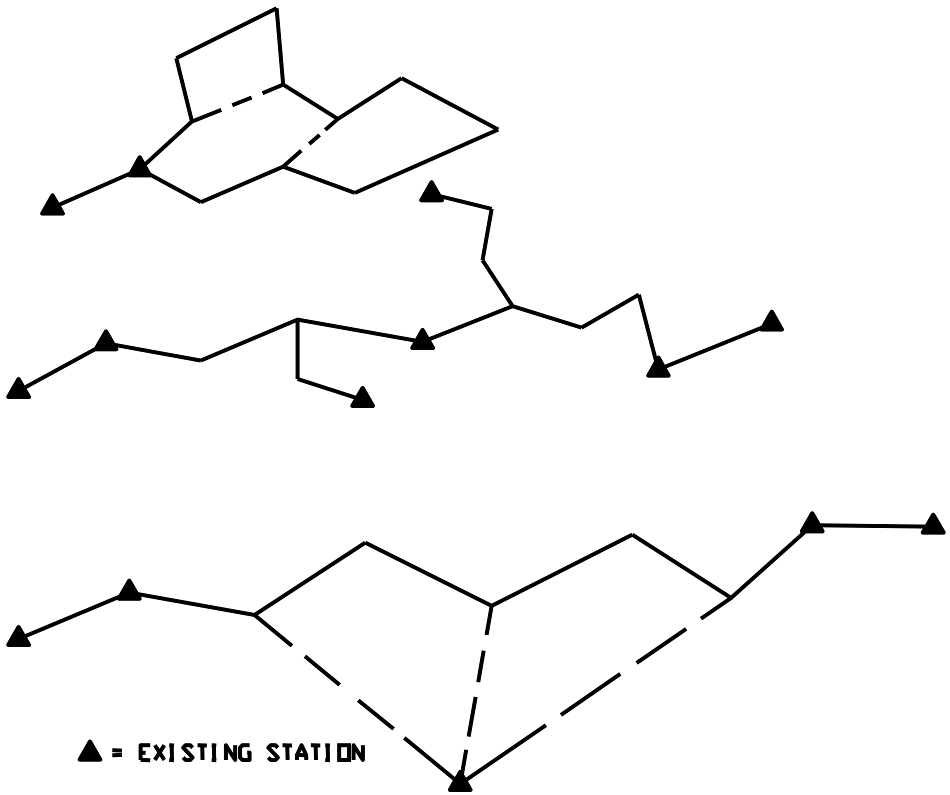


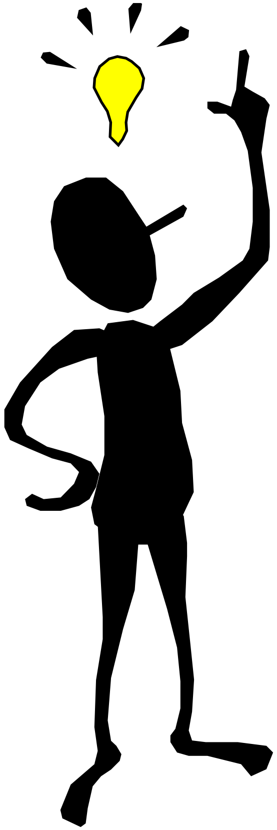
Or This:



Planning A Traverse (Continued)

Add redundancy where possible and practical.





QUESTIONS?

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