1 Introduction

1.1 DEFINITION OF SURVEYING

Surveying, which has recently also been interchangeably called *geomatics* (see Section 1.2), has traditionally been defined as the science, art, and technology of determining the relative positions of points above, on, or beneath the Earth's surface, or of establishing such points. In a more general sense, however, surveying (geomatics) can be regarded as that discipline which encompasses all methods for measuring and collecting information about the physical earth and our environment, processing that information, and disseminating a variety of resulting products to a wide range of clients. Surveying has been important since the beginning of civilization. Its earliest applications were in measuring and marking boundaries of property ownership. Throughout the years its importance has steadily increased with the growing demand for a variety of maps and other spatially related types of information and the expanding need for establishing accurate line and grade to guide construction operations.

Today the importance of measuring and monitoring our environment is becoming increasingly critical as our population expands, land values appreciate, our natural resources dwindle, and human activities continue to stress the quality of our land, water, and air. Using modern ground, aerial, and satellite technologies, and computers for data processing, contemporary surveyors are now able to measure and monitor the Earth and its natural resources on literally a global basis. Never before has so much information been available for assessing current conditions, making sound planning decisions, and formulating policy in a host of land-use, resource development, and environmental preservation applications.

Recognizing the increasing breadth and importance of the practice of surveying, the *International Federation of Surveyors* (see Section 1.11) adopted the following definition:

"A surveyor is a professional person with the academic qualifications and technical expertise to conduct one, or more, of the following activities;

- to determine, measure and represent the land, three-dimensional objects, point-fields, and trajectories;
- to assemble and interpret land and geographically related information;
- to use that information for the planning and efficient administration of the land, the sea and any structures thereon; and
- to conduct research into the above practices and to develop them.

Detailed Functions

The surveyor's professional tasks may involve one or more of the following activities, which may occur either on, above, or below the surface of the land or the sea and may be carried out in association with other professionals.

1. The determination of the size and shape of the earth and the measurements of all data needed to define the size, position, shape and contour of any part of the earth and monitoring any change therein.
2. The positioning of objects in space and time as well as the positioning and monitoring of physical features, structures and engineering works on, above or below the surface of the earth.
3. The development, testing and calibration of sensors, instruments and systems for the above-mentioned purposes and for other surveying purposes.
4. The acquisition and use of spatial information from close range, aerial and satellite imagery and the automation of these processes.
5. The determination of the position of the boundaries of public or private land, including national and international boundaries, and the registration of those lands with the appropriate authorities.
6. The design, establishment and administration of geographic information systems (GIS) and the collection, storage, analysis, management, display and dissemination of data.
7. The analysis, interpretation and integration of spatial objects and phenomena in GIS, including the visualization and communication of such data in maps, models and mobile digital devices.
8. The study of the natural and social environment, the measurement of land and marine resources and the use of such data in the planning of development in urban, rural and regional areas.
9. The planning, development and redevelopment of property, whether urban or rural and whether land or buildings.
10. The assessment of value and the management of property, whether urban or rural and whether land or buildings.
11. The planning, measurement and management of construction works, including the estimation of costs.

In application of the foregoing activities surveyors take into account the relevant legal, economic, environmental, and social aspects affecting each project."

The breadth and diversity of the practice of surveying (geomatics), as well as its importance in modern civilization, are readily apparent from this definition.

1.2 GEOMATICS

As noted in Section 1.1, geomatics is a relatively new term that is now commonly being applied to encompass the areas of practice formerly identified as surveying. The name has gained widespread acceptance in the United States, as well as in other English-speaking countries of the world, especially in Canada, the United Kingdom, and Australia. In the United States, the *Surveying Engineering Division* of The American Society of Civil Engineers changed its name to the *Geomatics Division*. Many college and university programs in the United States that were formerly identified as "Surveying" or "Surveying Engineering" are now called "Geomatics" or "Geomatics Engineering."

The principal reason cited for making the name change is that the manner and scope of practice in surveying have changed dramatically in recent years. This has occurred in part because of recent technological developments that have provided surveyors with new tools for measuring and/or collecting information, for computing, and for displaying and disseminating information. It has also been driven by increasing concerns about the environment locally, regionally, and globally, which have greatly exacerbated efforts in monitoring, managing, and regulating the use of our land, water, air, and other natural resources. These circumstances, and others, have brought about a vast increase in demands for new spatially related information.

Historically surveyors made their measurements using ground-based methods and until rather recently the transit and tape[1] were their primary instruments. Computations, analyses, and the reports, plats, and maps they delivered to their clients were prepared (in hard copy form) through tedious manual processes. Today the modern surveyor's arsenal of tools for measuring and collecting environmental information includes electronic instruments for automatically measuring distances and angles, satellite surveying systems for quickly obtaining precise positions of widely spaced points, and modern aerial digital imaging and laser-scanning systems for quickly mapping and collecting other forms of data about the earth upon which we live. In addition, computer systems are available that can process the measured data and automatically produce plats, maps, and other products at speeds unheard of a few years ago. Furthermore, these products can be prepared in electronic formats and be transmitted to remote locations via telecommunication systems.

Concurrent with the development of these new data collection and processing technologies, *geographic information systems* (GISs) have emerged and matured. These computer-based systems enable virtually any type of spatially related information about the environment to be integrated, analyzed,

[1]These instruments are described in Appendix A and Chapter 6, respectively.

displayed, and disseminated.[2] The key to successfully operating geographic information systems is spatially related data of high quality, and the collection and processing of this data placing great new demands upon the surveying community.

As a result of these new developments noted above, and others, many feel that the name surveying no longer adequately reflects the expanded and changing role of their profession. Hence the new term geomatics has emerged. In this text, the terms surveying and geomatics are both used, although the former is used more frequently. Nevertheless students should understand that the two terms are synonymous as discussed above.

1.3 HISTORY OF SURVEYING

The oldest historical records in existence today that bear directly on the subject of surveying state that this science began in Egypt. Herodotus recorded that Sesostris (about 1400 B.C.) divided the land of Egypt into plots for the purpose of taxation. Annual floods of the Nile River swept away portions of these plots, and surveyors were appointed to replace the boundaries. These early surveyors were called *rope-stretchers*, since their measurements were made with ropes having markers at unit distances.

As a consequence of this work, early Greek thinkers developed the science of geometry. Their advance, however, was chiefly along the lines of pure science. Heron stands out prominently for applying science to surveying in about 120 B.C. He was the author of several important treatises of interest to surveyors, including *The Dioptra*, which related the methods of surveying a field, drawing a plan, and making related calculations. It also described one of the first pieces of surveying equipment recorded, the *diopter* [Figure 1.1(a)]. For many years Heron's work was the most authoritative among Greek and Egyptian surveyors.

Significant development in the art of surveying came from the practical-minded Romans, whose best-known writing on surveying was by Frontinus. Although the original manuscript disappeared, copied portions of his work have been preserved. This noted Roman engineer and surveyor, who lived in the first century, was a pioneer in the field, and his essay remained the standard for many years. The engineering ability of the Romans was demonstrated by their extensive construction work throughout the empire. Surveying necessary for this construction resulted in the organization of a surveyors' guild. Ingenious instruments were developed and used. Among these were the *groma* [Figure 1.1(b)], used for sighting; the *libella*, an A-frame with a plumb bob, for leveling; and the *chorobates*, a horizontal straightedge about 20 ft long with supporting legs and a groove on top for water to serve as a level.

One of the oldest Latin manuscripts in existence is the *Codex Acerianus*, written in about the sixth century. It contains an account of surveying as practiced by the Romans and includes several pages from Frontinus's treatise. The

[2] Geographic information systems are briefly introduced in Section 1.9, and then described in greater detail in Chapter 28.

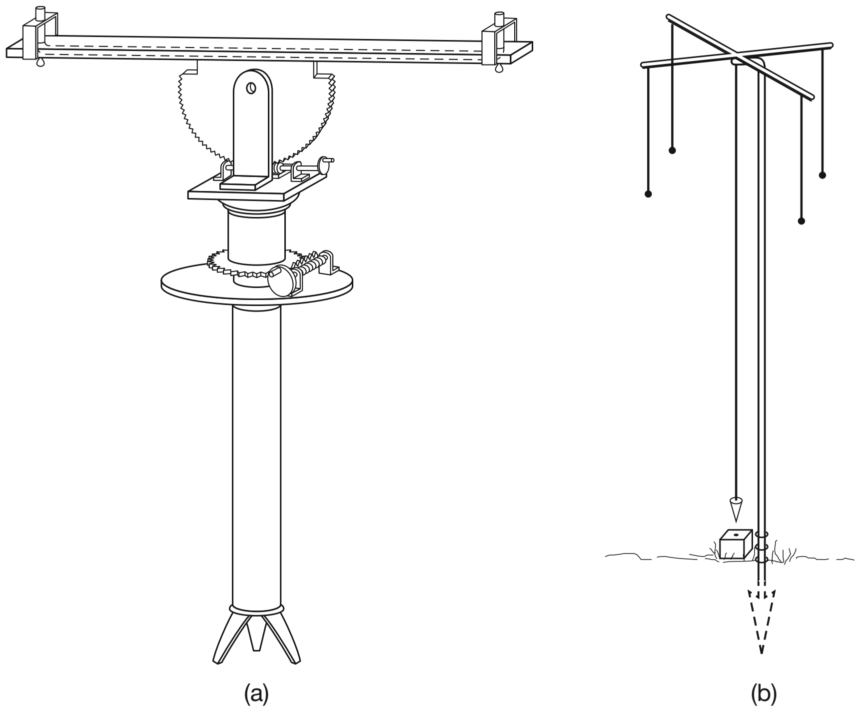

Figure 1.1 Historical surveying instruments: (a) the diopter, (b) the groma.

manuscript was found in the 10th century by Gerbert and served as the basis for his text on geometry, which was largely devoted to surveying.

During the Middle Ages, the Arabs kept Greek and Roman science alive. Little progress was made in the art of surveying, and the only writings pertaining to it were called "practical geometry."

In the 13th century, Von Piso wrote *Practica Geometria*, which contained instructions on surveying. He also authored *Liber Quadratorum*, dealing chiefly with the *quadrans*, a square brass frame having a 90° angle and other graduated scales. A movable pointer was used for sighting. Other instruments of the period were the *astrolabe*, a metal circle with a pointer hinged at its center and held by a ring at the top, and the *cross staff*, a wooden rod about 4 ft long with an adjustable crossarm at right angles to it. The known lengths of the arms of the cross staff permitted distances to be measured by proportion and angles.

Early civilizations assumed the Earth to be a flat surface, but by noting the Earth's circular shadow on the moon during lunar eclipses and watching ships gradually disappear as they sailed toward the horizon, it was slowly deduced that the planet actually curved in all directions.

Determining the true size and shape of the Earth has intrigued humans for centuries. History records that a Greek named Eratosthenes was among the first to compute its dimensions. His procedure, which occurred about 200 B.C., is illustrated in Figure 1.2. Eratosthenes had concluded that the Egyptian cities of Alexandria and Syene were located approximately on the same meridian, and he had also observed that at noon on the summer solstice, the sun was directly overhead at Syene. (This was apparent because at that time of that day, the image of

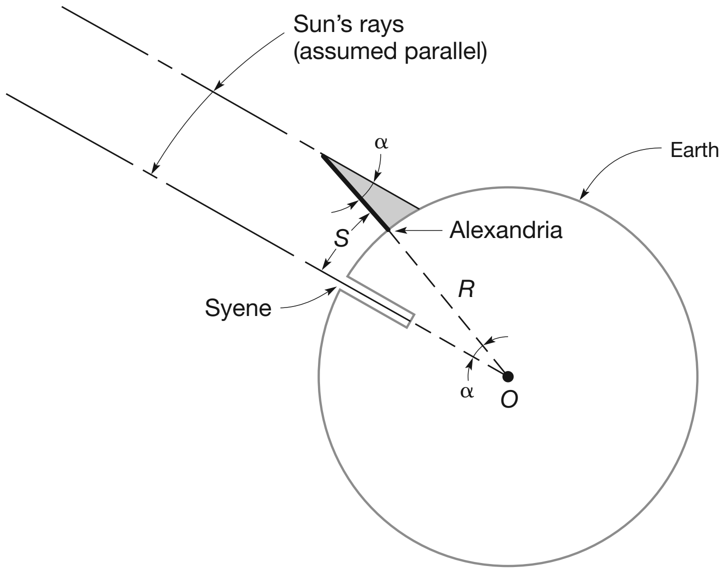

Figure 1.2 Geometry of the procedure used by Eratosthenes to determine the Earth's circumference.

the sun could be seen reflecting from the bottom of a deep vertical well there.) He reasoned that at that moment, the sun, Syene, and Alexandria were in a common meridian plane, and if he could measure the arc length between the two cities, and the angle it subtended at the Earth's center, he could compute the Earth's circumference. He determined the angle by measuring the length of the shadow cast at Alexandria from a vertical staff of known length. The arc length was found from multiplying the number of caravan days between Syene and Alexandria by the average daily distance traveled. From these measurements, Eratosthenes calculated the Earth's circumference to be about 25,000 mi. Subsequent precise geodetic measurements using better instruments, but techniques similar geometrically to Eratosthenes', have shown his value, though slightly too large, to be amazingly close to the currently accepted one. (Actually, as explained in Chapter 19, the Earth approximates an oblate spheroid having an equatorial radius about 13.5 mi longer than the polar radius.)

In the 18th and 19th centuries, the art of surveying advanced more rapidly. The need for maps and locations of national boundaries caused England and France to make extensive surveys requiring accurate triangulation; thus, geodetic surveying began. The U.S. Coast Survey (now the National Geodetic Survey of the U.S. Department of Commerce) was established by an act of Congress in 1807. Initially its charge was to perform hydrographic surveys and prepare nautical charts. Later its activities were expanded to include establishment of reference monuments of precisely known positions throughout the country.

Increased land values and the importance of precise boundaries, along with the demand for public improvements in the canal, railroad, and turnpike eras, brought surveying into a prominent position. More recently, the large volume of general construction, numerous land subdivisions that require precise records, and demands posed by the fields of exploration and ecology have entailed an augmented surveying program. Surveying is still the sign of progress in the development, use, and preservation of the Earth's resources.

In addition to meeting a host of growing civilian needs, surveying has always played an important role in our nation's defense activities. World Wars I and II, the Korean and Vietnam conflicts, and the more recent conflicts in the Middle East and Europe have created staggering demands for precise measurements and accurate maps. These military operations also provided the stimulus for improving instruments and methods to meet these needs. Surveying also contributed to, and benefited from, the space program where new equipment and systems were needed to provide precise control for missile alignment and for mapping and charting portions of the moon and nearby planets.

Developments in surveying and mapping equipment have now evolved to the point where the traditional instruments that were used until about the 1960s or 1970s—the transit, theodolite, dumpy level, and steel tape—have now been almost completely replaced by an array of new "high-tech" instruments. These include electronic *total station instruments*, which can be used to automatically measure and record horizontal and vertical distances, and horizontal and vertical angles; and *global navigation satellite systems* (GNSS) such as the *global positioning system* (GPS) that can provide precise location information for virtually any type of survey. Laser-scanning instruments combine automatic distance and angle measurements to compute dense grids of coordinated points. Also new aerial cameras and remote sensing instruments have been developed, which provide images in digital form, and these images can be processed to obtain spatial information and maps using new *digital photogrammetric restitution instruments* (also called *softcopy plotters*). Figure 1.3, 1.4, 1.5, and 1.6, respectively, show a total station instrument, 3D mobile mapping system, laser-scanning instrument, and modern softcopy plotter. The 3D mobile mapping system in Figure 1.4 is an integrated system consisting of scanners, GNSS receiver, inertial measurement unit, and a high-quality hemispherical digital camera that can map all items within 30 m of the vehicle as the vehicle travels at highway speeds. The system can capture 1.3 million data points per second providing the end user with high-quality, georeferenced coordinates on all items visible in the images.

Figure 1.3 LEICA TPS 1100 total station instrument. (Courtesy Leica Geosystems AG.)

Figure 1.4
The IP-S2 3D mobile mapping system. (Courtesy Topcon Positioning Systems.)

Figure 1.5
LEICA HDS 3000 laser scanner. (Courtesy of Christopher Gibbons, Leica Geosystems AG.)

Figure 1.6 Intergraph Image Station Z softcopy plotter. (From *Elements of Photogrammetry: With Applications in GIS*, by Wolf and Dewitt, 2000, Courtesy Intergraph, Inc., and the McGraw-Hill Companies.)

■ 1.4 GEODETIC AND PLANE SURVEYS

Two general classifications of surveys are *geodetic* and *plane*. They differ principally in the assumptions on which the computations are based, although field measurements for geodetic surveys are usually performed to a higher order of accuracy than those for plane surveys.

In geodetic surveying, the curved surface of the Earth is considered by performing the computations on an *ellipsoid* (curved surface approximating the size and shape of the Earth—see Chapter 19). It is now becoming common to do geodetic computations in a three-dimensional, *Earth-centered, Earth-fixed* (ECEF) Cartesian coordinate system. The calculations involve solving equations derived from solid geometry and calculus. Geodetic methods are employed to determine relative positions of widely spaced monuments and to compute lengths and directions of the long lines between them. These monuments serve as the basis for referencing other subordinate surveys of lesser extents.

In early geodetic surveys, painstaking efforts were employed to accurately observe angles and distances. The angles were measured using precise ground-based theodolites, and the distances were measured using special tapes made from metal having a low coefficient of thermal expansion. From these basic measurements, the relative positions of the monuments were computed. Later, electronic instruments were used for observing the angles and distances. Although these latter types of instruments are still sometimes used on geodetic surveys, satellite positioning has now almost completely replaced other instruments for these types of surveys. Satellite positioning can provide the needed positions with much greater accuracy, speed, and economy. GNSS receivers enable ground stations to be located precisely by observing distances to satellites operating in known positions along their orbits. GNSS surveys are being used in all forms of surveying including geodetic, hydrographic, construction, and boundary surveying. The principles of operation of the global positioning system are given in Chapter 13, field and office procedures used in static GNSS surveys are discussed in Chapter 14, and the methods used in kinematic GNSS surveys are discussed in Chapter 15.

In plane surveying, except for leveling, the reference base for fieldwork and computations is assumed to be a flat horizontal surface. The direction of a plumb line (and thus gravity) is considered parallel throughout the survey region, and all observed angles are presumed to be plane angles. For areas of limited size, the surface of our vast ellipsoid is actually nearly flat. On a line 5 mi long, the ellipsoid arc and chord lengths differ by only about 0.02 ft. A plane surface tangent to the ellipsoid departs only about 0.7 ft at 1 mi from the point of tangency. In a triangle having an area of 75 square miles, the difference between the sum of the three ellipsoidal angles and three plane angles is only about 1 sec. Therefore, it is evident that except in surveys covering extensive areas, the Earth's surface can be approximated as a plane, thus simplifying computations and techniques. In general, algebra, plane and analytical geometry, and plane trigonometry are used in plane-surveying calculations. Even for very large areas, map projections, such as those described in Chapter 20, allow plane-surveying computations to be used. This book concentrates primarily on methods of plane surveying, an approach that satisfies the requirements of most projects.

■ 1.5 IMPORTANCE OF SURVEYING

Surveying is one of the world's oldest and most important arts because, as noted previously, from the earliest times it has been necessary to mark boundaries and divide land. Surveying has now become indispensable to our modern way of life. The results of today's surveys are used to (1) map the Earth above and below sea level; (2) prepare navigational charts for use in the air, on land, and at sea; (3) establish property boundaries of private and public lands; (4) develop data banks of land-use and natural resource information that aid in managing our environment; (5) determine facts on the size, shape, gravity, and magnetic fields of the earth; and (6) prepare charts of our moon and planets.

Surveying continues to play an extremely important role in many branches of engineering. For example, surveys are required to plan, construct, and maintain highways, railroads, rapid-transit systems, buildings, bridges, missile ranges, launching sites, tracking stations, tunnels, canals, irrigation ditches, dams, drainage works, urban land subdivisions, water supply and sewage systems, pipelines, and mine shafts. Surveying methods are commonly employed in laying out industrial assembly lines and jigs.[3] These methods are also used for guiding the fabrication of large equipment, such as airplanes and ships, where separate pieces that have been assembled at different locations must ultimately be connected as a

[3]See footnote 1.

unit. Surveying is important in many related tasks in agronomy, archeology, astronomy, forestry, geography, geology, geophysics, landscape architecture, meteorology, paleontology, and seismology, but particularly in military and civil engineering.

All engineers must know the limits of accuracy possible in construction, plant design and layout, and manufacturing processes, even though someone else may do the actual surveying. In particular, surveyors and civil engineers who are called on to design and plan surveys must have a thorough understanding of the methods and instruments used, including their capabilities and limitations. This knowledge is best obtained by making observations with the kinds of equipment used in practice to get a true concept of the theory of errors and the small but recognizable differences that occur in observed quantities.

In addition to stressing the need for reasonable limits of accuracy, surveying emphasizes the value of significant figures. Surveyors and engineers must know when to work to hundredths of a foot instead of to tenths or thousandths, or perhaps the nearest foot, and what precision in field data is necessary to justify carrying out computations to the desired number of decimal places. With experience, they learn how available equipment and personnel govern procedures and results.

Neat sketches and computations are the mark of an orderly mind, which in turn is an index of sound engineering background and competence. Taking field notes under all sorts of conditions is excellent preparation for the kind of recording and sketching expected of all engineers. Performing later office computations based on the notes underscores their importance. Additional training that has a carryover value is obtained in arranging computations in an organized manner.

Engineers who design buildings, bridges, equipment, and so on are fortunate if their estimates of loads to be carried are correct within 5%. Then a factor of safety of 2 or more is often applied. But except for some topographic work, only exceedingly small errors can be tolerated in surveying, and there is no factor of safety. Traditionally, therefore, both manual and computational precision are stressed in surveying.

■ 1.6 SPECIALIZED TYPES OF SURVEYS

Many types of surveys are so specialized that a person proficient in a particular discipline may have little contact with the other areas. Persons seeking careers in surveying and mapping, however, should be knowledgeable in every phase, since all are closely related in modern practice. Some important classifications are described briefly here.

Control surveys establish a network of horizontal and vertical monuments that serve as a reference framework for initiating other surveys. Many control surveys performed today are done using techniques discussed in Chapter 14 with GNSS instruments.

Topographic surveys determine locations of natural and artificial features and elevations used in map making.

Land, boundary, and *cadastral surveys* establish property lines and property corner markers. The term cadastral is now generally applied to surveys of the public lands systems. There are three major categories: *original surveys* to establish new section corners in unsurveyed areas that still exist in Alaska and several western states; *retracement surveys* to recover previously established boundary lines; and *subdivision surveys* to establish monuments and delineate new parcels of ownership. *Condominium surveys*, which provide a legal record of ownership, are a type of boundary survey.

Hydrographic surveys define shorelines and depths of lakes, streams, oceans, reservoirs, and other bodies of water. *Sea surveying* is associated with port and offshore industries and the marine environment, including measurements and marine investigations made by shipborne personnel.

Alignment surveys are made to plan, design, and construct highways, railroads, pipelines, and other linear projects. They normally begin at one control point and progress to another in the most direct manner permitted by field conditions.

Construction surveys provide line, grade, control elevations, horizontal positions, dimensions, and configurations for construction operations. They also secure essential data for computing construction pay quantities.

As-built surveys document the precise final locations and layouts of engineering works and record any design changes that may have been incorporated into the construction. These are particularly important when underground facilities are constructed, so their locations are accurately known for maintenance purposes, and so that unexpected damage to them can be avoided during later installation of other underground utilities.

Mine surveys are performed above and below ground to guide tunneling and other operations associated with mining. This classification also includes geophysical surveys for mineral and energy resource exploration.

Solar surveys map property boundaries, solar easements, obstructions according to sun angles, and meet other requirements of zoning boards and title insurance companies.

Optical tooling (also referred to as *industrial surveying* or *optical alignment*) is a method of making extremely accurate measurements for manufacturing processes where small tolerances are required.

Except for control surveys, most other types described are usually performed using plane-surveying procedures, but geodetic methods may be employed on the others if a survey covers an extensive area or requires extreme accuracy.

Ground, aerial, and *satellite surveys* are broad classifications sometimes used. Ground surveys utilize measurements made with ground-based equipment such as automatic levels and total station instruments. Aerial surveys are accomplished using either *photogrammetry* or *remote sensing*. Photogrammetry uses cameras that are carried usually in airplanes to obtain images, whereas remote sensing employs cameras and other types of sensors that can be transported in either aircraft or satellites. Procedures for analyzing and reducing the image data are described in Chapter 27. Aerial methods have been used in all the specialized types of surveys listed, except for optical tooling, and in this area *terrestrial* (ground-based) photographs are often used. Satellite surveys include the determination of ground locations from measurements made to satellites using GNSS receivers, or the use of satellite images for mapping and monitoring large regions of the Earth.

■ 1.7 SURVEYING SAFETY

Surveyors (geomatics engineers) generally are involved in both field and office work. The fieldwork consists in making observations with various types of instruments to either (a) determine the relative locations of points or (b) to set out stakes in accordance with planned locations to guide building and construction operations. The office work involves (1) conducting research and analysis in preparing for surveys, (2) computing and processing the data obtained from field measurements, and (3) preparing maps, plats, charts, reports, and other documents according to client specifications. Sometimes the fieldwork must be performed in hostile or dangerous environments, and thus it is very important to be aware of the need to practice safety precautions.

Among the most dangerous of circumstances within which surveyors must sometimes work are job sites that are either on or near highways or railroads, or that cross such facilities. Job sites in construction zones where heavy machinery is operating are also hazardous, and the dangers are often exacerbated by poor hearing conditions from the excessive noise, and poor visibility caused by obstructions and dust, both of which are created by the construction activity. In these situations, whenever possible, the surveys should be removed from the danger areas through careful planning and/or the use of *offset* lines. If the work must be done in these hazardous areas, then certain safety precautions should be followed. Safety vests of fluorescent yellow color should always be worn in these situations, and flagging materials of the same color can be attached to the surveying equipment to make it more visible. Depending on the circumstances, signs can be placed in advance of work areas to warn drivers of the presence of a survey party ahead, cones and/or barricades can be placed to deflect traffic around surveying activities, and flaggers can be assigned to warn drivers, or to slow or even stop them, if necessary. The *Occupational Safety and Health Administration* (OSHA), of the U.S. Department of Labor,[4] has developed safety standards and guidelines that apply to the various conditions and situations that can be encountered.

Besides the hazards described above, depending on the location of the survey and the time of year, other dangers can also be encountered in conducting field surveys. These include problems related to weather such as frostbite and overexposure to the sun's rays, which can cause skin cancers, sunburns, and heat stroke. To help prevent these problems, plenty of fluids should be drunk, large-brimmed hats and sunscreen can be worn, and on extremely hot days surveying should commence at dawn and terminate at midday or early afternoon. Outside work should not be done on extremely cold days, but if it is necessary, warm clothing should be worn and skin areas should not be exposed. Other hazards that can be encountered during field surveys include wild animals, poisonous snakes, bees, spiders, wood ticks, deer ticks (which can carry lyme disease), poison

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[4]The mission of OSHA is to save lives, prevent injuries, and protect the health of America's workers. Its staff establishes protective standards, enforces those standards, and reaches out to employers and employees through technical assistance and consultation programs. For more information about OSHA and its safety standards, consult its website http://www.osha.gov.

ivy, and poison oak. Surveyors should be knowledgeable about the types of hazards that can be expected in any local area, and always be alert and on the lookout for them. To help prevent injury from these sources, protective boots and clothing should be worn and insect sprays can be used. Certain tools can also be dangerous, such as chain saws, axes, and machetes that are sometimes necessary for clearing lines of sight. These must always be handled with care. Also, care must be exercised in handling certain surveying instruments, like long-range poles and level rods, especially when working around overhead wires, to prevent accidental electrocutions.

Many other hazards, in addition to those cited above can be encountered when surveying in the field. Thus, it is essential that surveyors always exercise caution in their work, and know and follow accepted safety standards. In addition, a first-aid kit should always accompany a survey party in the field, and it should include all of the necessary antiseptics, ointments, bandage materials, and other equipment needed to render first aid for minor accidents. The survey party should also be equipped with cell phones for more serious situations, and telephone numbers to call in emergencies should be written down and readily accessible.

1.8 LAND AND GEOGRAPHIC INFORMATION SYSTEMS

Land Information Systems (LISs) and *Geographic Information Systems* (GISs) are areas of activity that have rapidly assumed positions of major prominence in surveying. These computer-based systems enable storing, integrating, manipulating, analyzing, and displaying virtually any type of spatially related information about our environment. LISs and GISs are being used at all levels of government, and by businesses, private industry, and public utilities to assist in management and decision making. Specific applications have occurred in many diverse areas and include natural resource management, facilities siting and management, land records modernization, demographic and market analysis, emergency response and fleet operations, infrastructure management, and regional, national, and global environmental monitoring. Data stored within LISs and GISs may be both natural and cultural, and be derived from new surveys, or from existing sources such as maps, charts, aerial and satellite photos, tabulated data and statistics, and other documents. However, in most situations, the needed information either does not exist, or it is unsatisfactory because of age, scale, or other reasons. Thus, new measurements, maps, photos, or other data must be obtained.

Specific types of information (also called *themes* or *layers* of information) needed for land and geographic information systems may include political boundaries, individual property ownership, population distribution, locations of natural resources, transportation networks, utilities, zoning, hydrography, soil types, land use, vegetation types, wetlands, and many, many more. An essential ingredient of all information entered into LIS and GIS databases is that it be spatially related, that is, located in a common geographic reference framework. Only then are the different layers of information physically relatable so they can be analyzed using computers to support decision making. This geographic positional requirement will place a heavy demand upon surveyors (geomatics engineers) in the future, who will play key roles in designing, implementing, and managing these systems. Surveyors from virtually all of the specialized areas described in Section 1.6 will be involved in developing the needed databases. Their work will include establishing the required basic control framework; conducting boundary surveys and preparing legal descriptions of property ownership; performing topographic and hydrographic surveys by ground, aerial, and satellite methods; compiling and digitizing maps; and assembling a variety of other digital data files.

The last chapter of this book, Chapter 28, is devoted to the topic of land and geographic information systems. This subject seems appropriately covered at the end, after each of the other types of surveys needed to support these systems has been discussed.

■ 1.9 FEDERAL SURVEYING AND MAPPING AGENCIES

Several agencies of the U.S. government perform extensive surveying and mapping. Three of the major ones are:

1. The National Geodetic Survey (NGS), formerly the Coast and Geodetic Survey, was originally organized to map the coast. Its activities have included control surveys to establish a network of reference monuments throughout the United States that serve as points for originating local surveys, preparation of nautical and aeronautical charts, photogrammetric surveys, tide and current studies, collection of magnetic data, gravimetric surveys, and worldwide control survey operations. The NGS now plays a major role in coordinating and assisting in activities related to upgrading the national network of reference control monuments, and to the development, storage, and dissemination of data used in modern LISs and GISs.
2. The U.S. Geological Survey (USGS), established in 1879, has as its mission the mapping of our nation and the survey of its resources. It provides a wide variety of maps, from topographic maps showing the geographic relief and natural and cultural features, to thematic maps that display the geology and water resources of the United States, to special maps of the moon and planets. The National Mapping Division of the USGS has the responsibility of producing topographic maps. It currently has nearly 70,000 different topographic maps available, and it distributes approximately 10 million copies annually. In recent years, the USGS has been engaged in a comprehensive program to develop a national digital cartographic database, which consists of map data in computer-readable formats.
3. The Bureau of Land Management (BLM), originally established in 1812 as the General Land Office, is responsible for managing the public lands. These lands, which total approximately 264 million acres and comprise about one eighth of the land in the United States, exist mostly in the western states and Alaska. The BLM is responsible for surveying the land and managing its natural resources, which include minerals, timber, fish and

wildlife, historical sites, and other natural heritage areas. Surveys of most public lands in the conterminous United States have been completed, but much work remains in Alaska.

In addition to these three federal agencies, units of the U.S. Army Corps of Engineers have made extensive surveys for emergency and military purposes. Some of these surveys provide data for engineering projects, such as those connected with flood control. Surveys of wide extent have also been conducted for special purposes by nearly 40 other federal agencies, including the Forest Service, National Park Service, International Boundary Commission, Bureau of Reclamation, Tennessee Valley Authority, Mississippi River Commission, U.S. Lake Survey, and Department of Transportation.

All states have a surveying and mapping section for purposes of generating topographic information upon which highways are planned and designed. Likewise, many counties and cities also have surveying programs, as have various utilities.

■ 1.10 THE SURVEYING PROFESSION

The personal qualifications of surveyors are as important as their technical ability in dealing with the public. They must be patient and tactful with clients and their sometimes-hostile neighbors. Few people are aware of the painstaking research of old records required before fieldwork is started. Diligent, time-consuming effort may be needed to locate corners on nearby tracts for checking purposes as well as to find corners for the property in question.

Land or boundary surveying is classified as a learned profession because the modern practitioner needs a wide background of technical training and experience, and must exercise a considerable amount of independent judgment. Registered (licensed) professional surveyors must have a thorough knowledge of mathematics (particularly geometry, trigonometry, and calculus); competence with computers; a solid understanding of surveying theory, instruments, and methods in the areas of geodesy, photogrammetry, remote sensing, and cartography; some competence in economics (including office management), geography, geology, astronomy, and dendrology; and a familiarity with laws pertaining to land and boundaries. They should be knowledgeable in both field operations and office computations. Above all, they are governed by a professional code of ethics and are expected to charge professional-level fees for their work.

Permission to trespass on private property or to cut obstructing tree branches and shrubbery must be obtained through a proper approach. Such privileges are not conveyed by a surveying license or by employment in a state highway department or other agency (but a court order can be secured if a landowner objects to necessary surveys).

All 50 states, Guam, and Puerto Rico have registration laws for professional surveyors and engineers (as do the provinces of Canada). In general, a surveyor's license is required to make property surveys, but not for construction, topographic, or route work, unless boundary corners are set.

To qualify for registration as either a professional land surveyor (PLS) or a professional engineer (PE), it is necessary to have an appropriate college degree, although some states allow relevant experience in lieu of formal education. In addition, candidates must acquire two or more years of mentored practical experience and must also pass a two-day comprehensive written examination. In most states, common national examinations covering fundamentals and principles and practice of land surveying are now used. However, usually two hours of the principles and practice exam are devoted to local legal customs and aspects. As a result, transfer of registration from one state to another has become easier.

Some states also require continuing education units (CEUs) for registration renewal, and many more are considering legislation that would add this requirement. Typical state laws require that a licensed land surveyor sign all plats, assume responsibility for any liability claims, and take an *active part* in the fieldwork.

■ 1.11 PROFESSIONAL SURVEYING ORGANIZATIONS

There are many professional organizations in the United States and worldwide that serve the interests of surveying and mapping. Generally the objectives of these organizations are the advancement of knowledge in the field, encouragement of communication among surveyors, and upgrading of standards and ethics in surveying practice. The *American Congress on Surveying and Mapping* (ACSM) is the foremost professional surveying organization in the United States. Founded in 1941, ACSM regularly sponsors technical meetings at various locations throughout the country. These meetings bring together large numbers of surveyors for presentation of papers, discussion of new ideas and problems, and exhibition of the latest in surveying equipment. ACSM publishes a quarterly journal, *Surveying and Land Information Science*, and also regularly publishes its newsletter, *The ACSM Bulletin*.

As noted in the preceding section, all states require persons who perform boundary surveys to be licensed. Most states also have professional surveyor societies or organizations with full membership open only to licensed surveyors. These state societies are generally affiliated with ACSM and offer benefits similar to those of ACSM, except that they concentrate on matters of state and local concern.

The *American Society for Photogrammetry and Remote Sensing* (ASPRS) is a sister organization of ACSM. Like ACSM, this organization is also devoted to the advancement of the fields of measurement and mapping, although its major interests are directed toward the use of aerial and satellite imagery for achieving these goals. ASPRS has been cosponsor of many technical meetings with ACSM, and its monthly journal *Photogrammetric Engineering and Remote Sensing* regularly features surveying and mapping articles.

The *Geomatics Division* of the *American Society of Civil Engineers* (ASCE) is also dedicated to professional matters related to surveying and publishes quarterly the *Journal of Surveying Engineering*.

The *Surveying and Geomatics Educators Society* (SAGES) holds pedagogical conferences on the instruction of surveying/geomatics in higher educational institutions.

Another organization in the United States, the *Urban and Regional Information Systems Association* (URISA), also supports the profession of surveying and mapping. This organization uses information technology to solve problems in planning, public works, the environment, emergency services, and utilities. Its *URISA Journal* is published quarterly.

The *Canadian Institute of Geomatics* (CIG) is the foremost professional organization in Canada concerned with surveying. Its objectives parallel those of ACSM. This organization, formerly the *Canadian Institute of Surveying and Mapping* (CISM), disseminates information to its members through its *CIG Journal*.

The *International Federation of Surveyors* (FIG), founded in 1878, fosters the exchange of ideas and information among surveyors worldwide. The acronym *FIG* stems from its French name, *Fédération Internationale des Géométres*. FIG membership consists of professional surveying organizations from many countries throughout the world. ACSM has been a member since 1959. FIG is organized into nine technical commissions, each concerned with a specialized area of surveying. The organization sponsors international conferences, usually at four-year intervals, and its commissions also hold periodic symposia where delegates gather for the presentation of papers on subjects of international interest.

■ 1.12 SURVEYING ON THE INTERNET

The explosion of available information on the Internet has had a significant impact on the field of surveying (geomatics). The Internet enables the instantaneous electronic transfer of documents to any location where the necessary computer equipment is available. It brings resources directly into the office or home, where previously it was necessary to travel to obtain the information or wait for its transfer by mail. Software, educational materials, technical documents, standards, and much more useful information are available on the Internet. As an example of how surveyors can take advantage of the Internet, data from a *Continuously Operating Reference Station* (CORS) can be downloaded from the NGS website for use in a GNSS survey (see Section 14.3.5).

Many agencies and institutions maintain websites that provide data free of charge on the Internet. Additionally, some educational institutions now place credit and noncredit courses on the Internet so that distance education can be more easily achieved. With a web browser, it is possible to research almost any topic from a convenient location, and names, addresses, and phone numbers of goods or services providers in a specific area can be identified. As an example, if it was desired to find companies offering mapping services in a certain region, a web search engine could be used to locate web pages that mention this service. Such a search may result in over a million pages if a very general term such as “mapping services” is used to search, but using more specific terms can narrow the search.

Unfortunately the addresses of particular pages and entire sites, given by their *Universal Resource Locators* (URLs), tend to change with time. However, at the risk of publishing URLs that may no longer be correct, a short list of important websites related to surveying is presented in Table 1.1.

TABLE 1.1 UNIVERSAL RESOURCE LOCATOR ADDRESSES FOR SOME SURVEYING RELATED SITES

Universal Resource Locator	Owner of Site
http://www.ngs.noaa.gov	National Geodetic Survey
http://www.usgs.gov	U.S. Geological Survey
http://www.blm.gov	Bureau of Land Management
http://www.navcen.uscg.mil	U.S. Coast Guard Navigation Center
http://www.usno.navy.mil	U.S. Naval Observatory
http://www.acsm.net	American Congress on Surveying and Mapping
http://www.asprs.org	American Society for Photogrammetry and Remote Sensing
http://www.asce.org	American Society of Civil Engineers
http://www.pearsonhighered.com/ghilani	Companion website for this book

1.13 FUTURE CHALLENGES IN SURVEYING

Surveying is currently in the midst of a revolution in the way data are measured, recorded, processed, stored, retrieved, and shared. This is in large part because of developments in computers and computer-related technologies. Concurrent with technological advancements, society continues to demand more data, with increasingly higher standards of accuracy, than ever before. Consequently, in a few years the demands on surveying engineers (geomatics engineers) will likely be very different from what they are now.

In the future, the National Spatial Reference System, a network of horizontal and vertical control points, must be maintained and supplemented to meet requirements of increasingly higher-order surveys. New topographic maps with larger scales as well as digital map products are necessary for better planning. Existing maps of our rapidly expanding urban areas need revision and updating to reflect changes, and more and better map products are needed of the older parts of our cities to support urban renewal programs and infrastructure maintenance and modernization. Large quantities of data will be needed to plan and design new rapid-transit systems to connect our major cities, and surveyors will face new challenges in meeting the precise standards required in staking alignments and grades for these systems.

In the future, assessment of environmental impacts of proposed construction projects will call for more and better maps and other data. GISs and LISs that contain a variety of land-related data such as ownership, location, acreage, soil types, land uses, and natural resources must be designed, developed, and maintained. Cadastral surveys of the yet unsurveyed public lands are essential. Monuments set years ago by the original surveyors have to be recovered and remonumented for preservation of property boundaries. Appropriate surveys with very demanding accuracies will be necessary to position drilling rigs as mineral and oil explorations press further offshore. Other future challenges include making precise deformation surveys for monitoring existing structures such as dams, bridges, and skyscrapers to detect imperceptible movements that could be precursors to catastrophes caused by their failure. Timely measurements and maps of the effects of natural disasters such as earthquakes, floods, and hurricanes will be needed so that effective relief and assistance efforts can be planned and implemented. In the space program, the desire for maps of neighboring planets will continue. And we must increase our activities in measuring and monitoring natural and human-caused global changes (glacial growth and retreat, volcanic activity, large-scale deforestation, and so on) that can potentially affect our land, water, atmosphere, energy supply, and even our climate.

These and other opportunities offer professionally rewarding indoor or outdoor (or both) careers for numerous people with suitable training in the various branches of surveying.

PROBLEMS

NOTE: Answers for some of these problems, and some in later chapters, can be obtained by consulting the bibliographies, later chapters, websites, or professional surveyors.

1.1 Develop your personal definition for the practice of surveying.
1.2 Explain the difference between geodetic and plane surveys.
1.3 Describe some surveying applications in:
(a) Archeology **(b)** Mining **(c)** Agriculture
1.4 List 10 uses for surveying other than property and construction surveying.
1.5 Why is it important to make accurate surveys of underground utilities?
1.6 Discuss the uses for topographic surveys.
1.7 What are hydrographic surveys, and why are they important?
1.8 Name and briefly describe three different surveying instruments used by early Roman engineers.
1.9 Briefly explain the procedure used by Eratosthenes in determining the Earth's circumference.
1.10 Describe the steps a land surveyor would need to do when performing a boundary survey.
1.11 Do laws in your state specify the accuracy required for surveys made to lay out a subdivision? If so, what limits are set?
1.12 What organizations in your state will furnish maps and reference data to surveyors and engineers?
1.13 List the legal requirements for registration as a land surveyor in your state.
1.14 Briefly describe the European Galileo system and discuss its similarities and differences with GPS.
1.15 List at least five nonsurveying uses for GPS.
1.16 Explain how aerial photographs and satellite images can be valuable in surveying.
1.17 Search the Internet and define a VLBI station. Discuss why these stations are important to the surveying community.
1.18 Describe how a GIS can be used in flood emergency planning.
1.19 Visit one of the surveying websites listed in Table 1.1, and write a brief summary of its contents. Briefly explain the value of the available information to surveyors.

1.20 Read one of the articles cited in the bibliography for this chapter, or another of your choosing, that describes an application where GPS was used. Write a brief summary of the article.

1.21 Same as Problem 1.20, except the article should be on safety as related to surveying.

BIBLIOGRAPHY

Binge, M. L. 2009. "Surveying GIS Using GIS as a Business Tool." *Point of Beginning* 34 (No. 12): 34.

Buhler, D. A. 2006. "Cadastral Survey Activities in the United States." *Surveying and Land Information Science* 66 (No. 2): 115.

Dahn, R. E. and R. Lumos. 2006. "National Society of Professional Surveyors." *Surveying and Land Information Science* 66 (No. 2): 111.

Grahls, C. L. 2009. "Risky Exposure." *Point of Beginning* 34 (No. 10): 22.

Greenfeld, J. 2006. "The Geographic and Land Information Society and GIS/LIS Activities in the United States." *Surveying and Land Information Science* 66 (No. 2): 119.

Harris, C. 2007. "Whole New Ball Game." *Professional Surveyor* 27 (No. 2): 26.

Hohner, L. N. 2007. "Positioning Your Future." *Point of Beginning* 32 (No. 4): 18.

Jeffress, G. 2006. "Two Perspectives of GIS/LIS Education in the United States." *Surveying and Land Information Science* 66 (No. 2): 123.

Koon, R. 2009. "Safety Sense." *Point of Beginning* 35 (No. 1): 45.

———. 2009. "Safety Sense: Field Vehicle Safety." *Point of Beginning* 34 (No. 9): 37.

———. 2007. "Safety Sense: Stepping Out Safely." *Point of Beginning* 32 (No. 11): 52.

Lathrop, W. and D. Martin. 2006. "The American Association for Geodetic Surveying: Its Continuing Role in Shaping the Profession." *Surveying and Land Information Science* 66 (No. 2): 97.

Schultz, R. 2006. "Education in Surveying: Fundamentals of Surveying Exam." *Professional Surveyor* 26 (No. 3): 38.

Taland, D. 2009. "A Golden Image." *Point of Beginning* 35 (No. 2): 14.

Wagner, M. J. 2009. "Scanning the Horizon." Point of Beginning 35 (No. 2): 24.

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2 Units, Significant Figures, and Field Notes

PART I • UNITS AND SIGNIFICANT FIGURES

■ 2.1 INTRODUCTION

Five types of observations illustrated in Figure 2.1 form the basis of traditional plane surveying: (1) horizontal angles, (2) horizontal distances, (3) vertical (or zenith) angles, (4) vertical distances, and (5) slope distances. In the figure, *OAB* and *ECD* are horizontal planes, and *OACE* and *ABDC* are vertical planes. Then as illustrated, horizontal angles, such as angle *AOB*, and horizontal distances, *OA* and *OB*, are measured in horizontal planes; vertical angles, such as *AOC*, are measured in vertical planes; zenith angles, such as *EOC*, are also measured in vertical planes; vertical lines, such as *AC* and *BD*, are measured vertically (in the direction of gravity); and slope distances, such as *OC*, are determined along inclined planes. By using combinations of these basic observations, it is possible to compute relative positions between any points. Equipment and procedures for making each of these basic kinds of observations are described in later chapters of this book.

■ 2.2 UNITS OF MEASUREMENT

Magnitudes of measurements (or of values derived from observations) must be given in terms of specific units. In surveying, the most commonly employed units are for *length*, *area*, *volume*, and *angle*. Two different systems are in use for specifying units of observed quantities, the *English* and *metric* systems. Because of its widespread adoption, the metric system is called the *International System of Units*, and abbreviated *SI*.

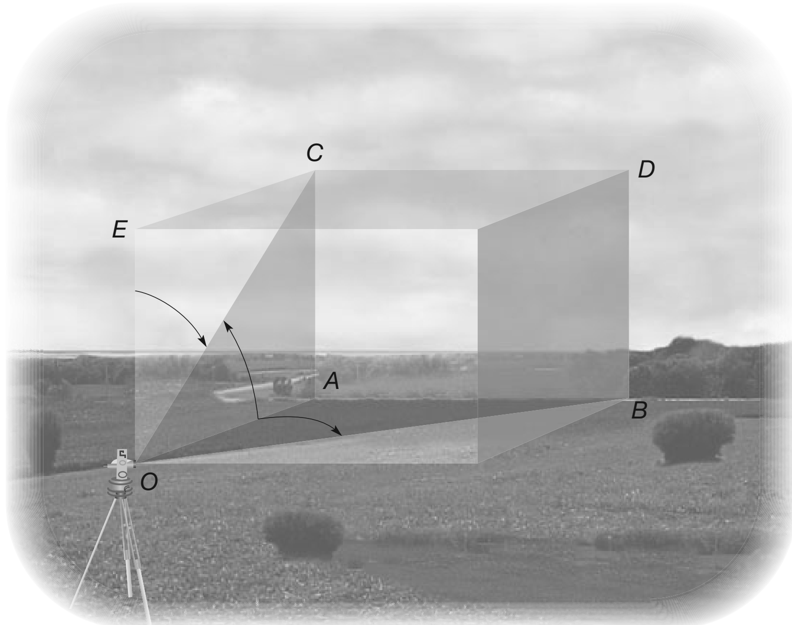


Figure 2.1 Kinds of measurements in surveying.

The basic unit employed for length measurements in the English system is the foot, whereas the meter is used in the metric system. In the past, two different definitions have been used to relate the foot and meter. Although they differ slightly, their distinction must be made clear in surveying. In 1893, the United States officially adopted a standard in which 39.37 in. was exactly equivalent to 1 m. Under this standard, the foot was approximately equal to 0.3048006 m. In 1959, a new standard was officially adopted in which the inch was equal to exactly 2.54 cm. Under this standard, 1 ft equals exactly 0.3048 m. This current unit, known as the international foot, differs from the previous one by about 1 part in 500,000, or approximately 1 foot per 100 miles. This small difference is thus important for very precise surveys conducted over long distances, and for conversions of high elevations or large coordinate values such as those used in State Plane Coordinate Systems as discussed in Chapter 20. Because of the vast number of surveys performed prior to 1959, it would have been extremely difficult and confusing to change all related documents and maps that already existed. Thus the old standard, now called the *U.S. survey foot*, is still used. Individual states have the option of officially adopting either standard. The National Geodetic Survey uses the meter in its distance measurements; thus, it is unnecessary to specify the foot unit. However, those making conversions from metric units must know the adopted standard for their state and use the appropriate conversion factor.

Because the English system has long been the officially adopted standard for measurements in the United States, except for geodetic surveys, the linear units of feet and *decimals* of a foot are most commonly used by surveyors. In construction, feet and inches are often used. Because surveyors perform all types of surveys including geodetic, and they also provide measurements for developing construction plans and guiding building operations, they must understand all the various systems of units and be capable of making conversions between them.

Caution must always be exercised to ensure that observations are recorded in their proper units, and conversions are correctly made.

A summary of the length units used in past and present surveys in the United States includes the following:

1 foot = 12 inches
1 yard = 3 feet
1 inch = 2.54 centimeters (basis of international foot)
1 meter = 39.37 inches (basis of U.S. survey foot)
1 rod = 1 pole = 1 perch = 16.5 feet
1 vara = approximately 33 inches (old Spanish unit often encountered in the southwestern United States)
1 Gunter's chain (ch) = 66 feet = 100 links (lk) = 4 rods
1 mile = 5280 feet = 80 Gunter's chains
1 nautical mile = 6076.10 feet (nominal length of a minute of latitude, or of longitude at the equator)
1 fathom = 6 feet.

In the English system, areas are given in *square feet* or *square yards*. The most common unit for large areas is the *acre*. Ten square chains (Gunter's) equal 1 acre. Thus an acre contains 43,560 ft^2, which is the product of 10 and 66^2. The *arpent* (equal to approximately 0.85 acre, but varying somewhat in different states) was used in land grants of the French crown. When employed as a linear term, it refers to the length of a side of 1 square arpent.

Volumes in the English system can be given in *cubic feet* or *cubic yards*. For very large volumes, for example, the quantity of water in a reservoir, the *acre-foot* unit is used. It is equivalent to the area of an acre having a depth of 1 ft, and thus is 43,560 ft^3.

The unit of angle used in surveying is the *degree*, defined as 1/360 of a circle. One degree (1°) equals 60 min, and 1 min equals 60 sec. Divisions of seconds are given in tenths, hundredths, and thousandths. Other methods are also used to subdivide a circle, for example, 400 *grads* (with 100 *centesimal min*/grad and 100 *centesimal sec*/min. Another term, *gons*, is now used interchangeably with grads. The military services use *mils* to subdivide a circle into 6400 units.

A *radian* is the angle subtended by an arc of a circle having a length equal to the radius of the circle. Therefore, 2π rad = 360°, 1 rad ≈ 57°17′44.8″ ≈ 57.2958°, and 0.01745 rad ≈ 1°.

■ 2.3 INTERNATIONAL SYSTEM OF UNITS (SI)

As noted previously, the meter is the basic unit for length in the metric or SI system. Subdivisions of the meter (m) are the *millimeter* (mm), *centimeter* (cm), and *decimeter* (dm), equal to 0.001, 0.01, and 0.1 m, respectively. A kilometer (km) equals 1000 m, which is approximately five eighths of a mile.

Areas in the metric system are specified using the *square meter* (m^2). Large areas, for example, tracts of land, are given in *hectares* (ha), where one hectare is equivalent to a square having sides of 100 m. Thus, there are 10,000 m^2, or about

2.471 acres per hectare. The *cubic meter* (m^3) is used for volumes in the SI system. Degrees, minutes, and seconds, or the radian, are accepted SI units for angles.

The metric system was originally developed in the 1790s in France. Although other definitions were suggested at that time, the French Academy of Sciences chose to define the meter as 1/10,000,000 of the length of the Earth's meridian through Paris from the equator to the pole. The actual length that was adopted for the meter was based on observations that had been made up to that time to determine the Earth's size and shape. Although later measurements revealed that the initially adopted value was approximately 0.2 mm short of its intended definition related to the meridional quadrant, still the originally adopted length became the standard.

Shortly after the metric system was introduced to the world, Thomas Jefferson who was the then secretary of state, recommended that the United States adopt it, but the proposal lost by one vote in the Congress! When the metric system was finally legalized for use (but not officially adopted) in the United States in 1866, a meter was defined as the interval under certain physical conditions between lines on an international prototype bar made of 90% platinum and 10 percent iridium, and accepted as equal to exactly 39.37 inches. A copy of this bar was held in Washington, D.C. and compared periodically with the international standard held in Paris. In 1960, at the General Conference on Weights and Measures (CGPM), the United States and 35 other nations agreed to redefine the meter as the length of 1,650,763.73 waves of the orange-red light produced by burning the element krypton (Kr-86). That definition permitted industries to make more accurate measurements and to check their own instruments without recourse to the standard meter-bar in Washington. The wavelength of this light is a true constant, whereas there is a risk of instability in the metal meter-bar. The CGPM met again in 1983 and established the current definition of the meter as the length of the path traveled by light in a vacuum during a time interval of 1/299,792,458 sec. Obviously, with this definition, the speed of light in a vacuum becomes exactly 299,792,458 m/sec. The advantage of this latest standard is that the meter is more accurately defined, since it is in terms of time, the most accurate of our basic measurements.

During the 1960s and 1970s, significant efforts were made toward promoting adoption of SI as the legal system for weights and measures in the United States. However, costs and frustrations associated with making the change generated substantial resistance, and the efforts were temporarily stalled. Recognizing the importance to the United States of using the metric system in order to compete in the rapidly developing global economy, in 1988 the Congress enacted the *Omnibus Trade and Competitiveness Act*. It designated the metric system as the *preferred* system of weights and measures for U.S. trade and commerce. The Act, together with a subsequent *Executive Order* issued in 1991, required all federal agencies to develop definite metric conversion plans and to use SI standards in their procurements, grants, and other business-related activities to the extent economically feasible. As an example of one agency's response, the Federal Highway Administration adopted a plan calling for (1) use of metric units in all publications and correspondence after September 30, 1992 and (2) use of metric units on all plans and contracts for federal highways after September 30, 1996. Although the Act and Executive Order did not mandate states, counties, cities, or industries to convert to metric, strong incentives were provided, for example, if SI directives were not complied with, certain federal matching funds could be withheld. In light of these developments, it appeared that the metric system would soon become the official system for use in the United States. However, again much resistance was encountered, not only from individuals but also from agencies of some state, county, and town and city governments, as well as from certain businesses. As a result, the SI still has not been adopted officially in the United States.

Besides the obvious advantage of being better able to compete in the global economy, another significant advantage that would be realized in adopting the SI standard would be the elimination of the confusion that exists in making conversions between the English System and the SI. The 1999 crash of the Mars Orbiter underscores costs and frustrations associated with this confusion. This $125 million satellite was supposed to monitor the Martian atmosphere, but instead it crashed into the planet because its contractor used English units while NASA's Jet Propulsion Laboratory was giving it data in the metric system. For these reasons and others, such as the decimal simplicity of the metric system, surveyors who are presently burdened with unit conversions and awkward computations involving yard, foot, and inch units should welcome official adoption of the SI. However, since this adoption has not yet occurred, this book uses both English and SI units in discussion and example problems.

■ 2.4 SIGNIFICANT FIGURES

In recording observations, an indication of the accuracy attained is the number of digits (significant figures) recorded. By definition, the number of significant figures in any observed value includes the positive (certain) digits plus one (*only one*) digit that is estimated or rounded off, and therefore questionable. For example, a distance measured with a tape whose smallest graduation is 0.01 ft, and recorded as 73.52 ft, is said to have four significant figures; in this case the first three digits are certain, and the last is rounded off and therefore questionable but still significant.

To be consistent with the theory of errors discussed in Chapter 3, it is essential that data be recorded with the correct number of significant figures. If a significant figure is dropped in recording a value, the time spent in acquiring certain precision has been wasted. On the other hand, if data are recorded with more figures than those that are significant, false precision will be implied. The number of significant figures is often confused with the number of decimal places. Decimal places may have to be used to maintain the correct number of significant figures, but in themselves they do not indicate significant figures. Some examples follow:

Two significant figures: 24, 2.4, 0.24, 0.0024, 0.020
Three significant figures: 364, 36.4, 0.000364, 0.0240
Four significant figures: 7621, 76.21, 0.0007621, 24.00.

Zeros at the end of an integer value may cause difficulty because they may or may not be significant. In a value expressed as 2400, for example, it is not known how many figures are significant; there may be two, three, or four, and therefore definite rules must be followed to eliminate the ambiguity. The preferred method of eliminating this uncertainty is to express the value in terms of powers of 10. The significant figures in the measurement are then written in scientific notation as a number between 1 and 10 with the correct number of zeros and power of 10. As an example, 2400 becomes 2.400×10^3 if both zeros are significant, 2.40×10^3 if one is, and 2.4×10^3 if there are only two significant figures. Alternatively, a bar may be placed over the last significant figure, as $240\bar{0}$, $24\bar{0}0$, and $2\bar{4}00$ for 4, 3, and 2 significant figures, respectively.

When observed values are used in the mathematical processes of addition, subtraction, multiplication, and division, it is imperative that the number of significant figures given in answers be consistent with the number of significant figures in the data used. The following three steps will achieve this for addition or subtraction: (1) identify the column containing the rightmost significant digit in each number being added or subtracted, (2) perform the addition or subtraction, and (3) round the answer so that its rightmost significant digit occurs in the leftmost column identified in step (1). Two examples illustrate the procedure.

(a)

$$\begin{array}{r} 46.7418 \\ +\quad 1.03 \\ \underline{+375.0} \\ 422.7718 \end{array}$$

(answer 422.8)

(b)

$$\begin{array}{r} 378. \\ \underline{-2.1} \\ 375.9 \end{array}$$

(answer 376.)

In (**a**), the digits 8, 3, and 0 are the rightmost significant ones in the numbers 46.7418, 1.03, and 375.0, respectively. Of these, the 0 in 375.0 is leftmost with respect to the decimal. Thus, the answer 422.7718 obtained on adding the numbers is rounded to 422.8, with its rightmost significant digit occurring in the same column as the 0 in 375.0. In (**b**), the digits 8 and 1 are rightmost, and of these the 8 is leftmost. Thus, the answer 375.9 is rounded to 376.

In multiplication, the number of significant figures in the answer is equal to the least number of significant figures in any of the factors. For example, $362.56 \times 2.13 = 7721.2528$ when multiplied, but the answer is correctly given as 772. Its three significant figures are governed by the three significant digits in 2.13. Likewise, in division the quotient should be rounded off to contain only as many significant figures as the least number of significant figures in either the divisor or the dividend. These rules for significant figures in computations stem from error propagation theory, which is discussed further in Section 3.17.

On the companion website for this book at http://www.pearsonhighered.com/ghilani are instructional videos that can be downloaded. The icon in the margin indicates the availability of such videos. The video *significant figures.mp4* discusses the rules applied to significant figures and rounding, which is covered in the following section.

In surveying, four specific types of problems relating to significant figures are encountered and must be understood.

1. Field measurements are given to some specific number of significant figures, thus dictating the number of significant figures in answers derived when the measurements are used in computations. In an intermediate calculation, it is

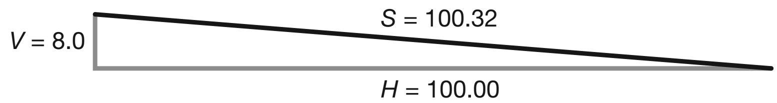


Figure 2.2
Slope correction.

a common practice to carry at least one more digit than required, and then round off the final answer to the correct number of significant figures.

2. There may be an implied number of significant figures. For instance, the length of a football field might be specified as 100 yd. But in laying out the field, such a distance would probably be measured to the nearest hundredth of a foot, not the nearest half-yard.
3. Each factor may not cause an equal variation. For example, if a steel tape 100.00 ft long is to be corrected for a change in temperature of 15°F, one of these numbers has five significant figures while the other has only two. However, a 15° variation in temperature changes the tape length by only 0.01 ft. Therefore, an adjusted tape length to five significant figures is warranted for this type of data. Another example is the computation of a slope distance from horizontal and vertical distances, as in Figure 2.2. The vertical distance V is given to two significant figures, and the horizontal distance H is measured to five significant figures. From these data, the slope distance S can be computed to five significant figures. For small angles of slope, a considerable change in the vertical distance produces a relatively small change in the difference between slope and horizontal distances.
4. Observations are recorded in one system of units but may have to be converted to another. A good rule to follow in making these conversions is to retain in the answer a number of significant figures equal to those in the observed value. As an example, to convert 178 ft 6-3/8 in. to meters, the number of significant figures in the measured value would first be determined by expressing it in its smallest units. In this case, 1/8th in. is the smallest unit and there are $(178 \times 12 \times 8) + (6 \times 8) + 3 = 17{,}139$ of these units in the value. Thus, the measurement contains five significant figures, and the answer is $17{,}139 \div (8 \times 39.37 \text{ in./m}) = 54.416$ m, properly expressed with five significant figures. (Note that 39.37 used in the conversion is an exact constant and does not limit the number of significant figures.)

2.5 ROUNDING OFF NUMBERS

Rounding off a number is the process of dropping one or more digits so the answer contains only those digits that are significant. In rounding off numbers to any required degree of precision in this text, the following procedures will be observed:

1. When the digit to be dropped is lower than 5, the number is written without the digit. Thus, 78.374 becomes 78.37. Also 78.3749 rounded to four figures becomes 78.37.
2. When the digit to be dropped is exactly 5, the nearest even number is used for the preceding digit. Thus, 78.375 becomes 78.38 and 78.385 is also rounded to 78.38.

3. When the digit to be dropped is greater than 5, the number is written with the preceding digit increased by 1. Thus, 78.386 becomes 78.39.

Procedures 1 and 3 are standard practice. When rounding the value 78.375 in procedure 2, however, some people always take the next higher hundredth, whereas others invariably use the next lower hundredth. However, using the nearest even digit establishes a uniform procedure and produces better-balanced results in a series of computations. It is an improper procedure to perform two-stage rounding where, for example, in rounding 78.3749 to four digits it would be first rounded to five figures, yielding 78.375, and then rounded again to 78.38. The correct answer in rounding 78.3749 to four figures is 78.37.

It is important to recognize that rounding should only occur with the final answer. Intermediate computations should be done without rounding to avoid problems that can be caused by rounding too early. Example (**a**) of Section 2.4 is repeated below to illustrate this point. The sum of 46.7418, 1.03, and 375.0 is rounded to 422.8 as shown in the "correct" column. If the individual values are rounded prior to the addition as shown in the "incorrect" column, the incorrect result of 422.7 is obtained.

Correct	**Incorrect**
46.7418	46.7
+ 1.03	+ 1.0
+ 375.0	+ 375.0
422.7718	422.7
(answer 422.8)	(answer 422.7)

PART II • FIELD NOTES

2.6 FIELD NOTES

Field notes are the records of work done in the field. They typically contain measurements, sketches, descriptions, and many other items of miscellaneous information. In the past, field notes were prepared exclusively by hand lettering in field books or special note pads as the work progressed and data were gathered. However, automatic data collectors, also known as electronic field book and survey controllers, have been introduced that can interface with many different modern surveying instruments. As the work progresses, they create computer files containing a record of observed data. Data collectors are rapidly gaining popularity, but when used, manually prepared sketches and descriptions often supplement the numerical data they generate. Regardless of the manner or form in which the notes are taken, they are extremely important.

Whether prepared manually, created by a data collector, or a combination of these forms, surveying field notes are the only permanent records of work done in the field. If the data are incomplete, incorrect, lost, or destroyed, much or all of the time and money invested in making the measurements and records have been wasted. Hence, the job of data recording is frequently the most important and difficult one in a surveying party. Field books and computer files containing information gathered over a period of weeks are worth many thousands of dollars because of the costs of maintaining personnel and equipment in the field.

Recorded field data are used in the office to perform computations, make drawings, or both. The office personnel using the data are usually not the same people who took the notes in the field. Accordingly, it is essential that without verbal explanations notes be intelligible to anyone.

Property surveys are subject to court review under some conditions, so field notes become an important factor in litigation. Also, because they may be used as references in land transactions for generations, it is necessary to index and preserve them properly. The salable "goodwill" of a surveyor's business depends largely on the office library of field books. Cash receipts may be kept in an unlocked desk drawer, but field books are stored in a fireproof safe!

■ 2.7 GENERAL REQUIREMENTS OF HANDWRITTEN FIELD NOTES

The following points are considered in appraising a set of field notes:

Accuracy. This is the most important quality in all surveying operations.

Integrity. A single omitted measurement or detail can nullify use of the notes for computing or plotting. If the project was far from the office, it is time-consuming and expensive to return for a missing measurement. Notes should be checked carefully for completeness before leaving the survey site and never "fudged" to improve closures.

Legibility. Notes can be used only if they are legible. A professional-looking set of notes is likely to be professional in quality.

Arrangement. Note forms appropriate to a particular survey contribute to accuracy, integrity, and legibility.

Clarity. Advance planning and proper field procedures are necessary to ensure clarity of sketches and tabulations, and to minimize the possibility of mistakes and omissions. Avoid crowding notes; paper is relatively cheap. Costly mistakes in computing and drafting are the end results of ambiguous notes.

Appendix B contains examples of handwritten field notes for a variety of surveying operations. Their plate number identifies each. Other example note forms are given at selected locations within the chapters that follow. These notes have been prepared keeping the above points in mind.

In addition to the items stressed in the foregoing, certain other guidelines must be followed to produce acceptable handwritten field notes. The notes should be lettered with a sharp pencil of at least 3H hardness so that an indentation is made in the paper. Books so prepared will withstand damp weather in the field (or even a soaking) and still be legible, whereas graphite from a soft pencil, or ink from a pen or ballpoint, leaves an undecipherable smudge under such circumstances.

Erasures of recorded data are not permitted in field books. If a number has been entered incorrectly, a line is run through it without destroying the number's legibility, and the proper value is noted above it (see Figure 5.5). If a partial or entire page is to be deleted, diagonal lines are drawn through opposite corners, and **VOID** is lettered prominently on the page, giving the reasons.

Field notes are presumed to be "original" unless marked otherwise. Original notes are those taken at the same time the observations are being made. If the original notes are copied, they must be so marked (see Figure 5.11). Copied notes may not be accepted in court because they are open to question concerning possible mistakes, such as interchanging numbers, and omissions. The value of a distance or an angle placed in the field book from memory 10 min after the observation is definitely unreliable. Students are tempted to scribble notes on scrap sheets of paper for later transfer in a neater form to the field book. This practice may result in the loss of some or all of the original data and defeats one purpose of a surveying course—to provide experience in taking notes under actual field conditions. In a real job situation, a surveyor is not likely to spend any time at night transcribing scribbled notes. Certainly, an employer will not pay for this evidence of incompetence.

2.8 TYPES OF FIELD BOOKS

Since field books contain valuable data, suffer hard wear, and must be permanent in nature, only the best should be used for practical work. Various kinds of field books as shown in Figure 2.3 are available, but bound and loose-leaf types are most common. The bound book, a standard for many years, has a sewed binding, a hard cover of leatherette, polyethylene, or covered hardboard, and contains 80 leaves. Its use ensures maximum testimony acceptability for property survey records in courtrooms. Bound duplicating books enable copies of the original notes to be made through carbon paper in the field. The alternate duplicate pages are perforated to enable their easy removal for advance shipment to the office.

Loose-leaf books have come into wide use because of many advantages, which include (1) assurance of a flat working surface, (2) simplicity of filing individual project notes, (3) ready transfer of partial sets of notes between field and office, (4) provision for holding pages of printed tables, diagrams, formulas, and

Figure 2.3
Field books.
(Courtesy Topcon Positioning Systems.)

sample forms, (5) the possibility of using different rulings in the same book, and (6) a saving in sheets and thus cost since none are wasted by filing partially filled books. A disadvantage is the possibility of losing sheets.

Stapled or spiral-bound books are not suitable for practical work. However, they may be satisfactory for abbreviated surveying courses that have only a few field periods, because of limited service required and low cost. Special column and page rulings provide for particular needs in leveling, angle measurement, topographic surveying, cross-sectioning, and so on.

A camera is a helpful note-keeping "instrument." Moderately priced, reliable, lightweight cameras can be used to document monuments set or found and to provide records of other valuable information or admissible field evidence. Recorded images can become part of the final record of survey. Tape recorders can also be used in certain circumstances, particularly where lengthy written explanations would be needed to document conditions or provide detailed descriptions.

■ 2.9 KINDS OF NOTES

Four types of notes are kept in practice: (1) sketches, (2) tabulations, (3) descriptions, and (4) combinations of these. The most common type is a combination form, but an experienced recorder selects the version best fitted to the job at hand. The note forms in Appendix B illustrate some of these types and apply to field problems described in this text. Other examples are included within the text at appropriate locations. Sketches often greatly increase the efficiency with which notes can be taken. They are especially valuable to persons in the office who must interpret the notes without the benefit of the notekeeper's presence. The proverb about one picture being worth a thousand words might well have been intended for notekeepers!

For a simple survey, such as measuring the distances between points on a series of lines, a sketch showing the lengths is sufficient. In measuring the length of a line forward and backward, a sketch together with tabulations properly arranged in columns is adequate, as in Plate B.1 in Appendix B. The location of a reference point may be difficult to identify without a sketch, but often a few lines of description are enough. Photos may be taken to record the location of permanent stations and the surrounding locale. The combination of a sketch with dimensions and photographic images can be invaluable in later station relocation. Benchmarks are usually briefly described, as in Figure 5.5.

In notekeeping this axiom is always pertinent: *When in doubt about the need for any information, include it and make a sketch. It is better to have too much data than not enough.*

■ 2.10 ARRANGEMENTS OF NOTES

Note styles and arrangements depend on departmental standards and individual preference. Highway departments, mapping agencies, and other organizations engaged in surveying furnish their field personnel with sample note forms, similar to those in Appendix B, to aid in preparing uniform and complete records that can be checked quickly.

It is desirable for students to have as guides, predesigned sample sets of note forms covering their first fieldwork to set high standards and save time. The note forms shown in Appendix B are composites of several models. They stress the open style, especially helpful for beginners, in which some lines or spaces are skipped for clarity. Thus, angles observed at a point A (see Plate B.4) are placed opposite A on the page, but distances observed between A and B on the ground are recorded on the line between A and B in the field book.

Left- and right-hand pages are practically always used in pairs and therefore carry the same page number. A complete title should be lettered across the top of the left page and may be extended over the right one. Titles may be abbreviated on succeeding pages for the same survey project. Location and type of work are placed beneath the title. Some surveyors prefer to confine the title on the left page and keep the top of the right one free for date, party, weather, and other items. This design is revised if the entire right page has to be reserved for sketches and benchmark descriptions. Arrangements shown in Appendix B demonstrate the flexibility of note forms. The left page is generally ruled in six columns designed for tabulation only. Column headings are placed between the first two horizontal lines at the page top and follow from left to right in the anticipated order of reading and recording. The upper part of the left or right page must contain the following items:

1. *Project name, location, date, time of day* (A.M. *or* P.M.), *and starting and finishing times*. These entries are necessary to document the notes and furnish a timetable as well as to correlate different surveys. Precision, troubles encountered, and other facts may be gleaned from the time required for a survey.
2. *Weather*. Wind velocity, temperature, and adverse weather conditions such as rain, snow, sunshine, and fog have a decided effect on accuracy in surveying operations. Surveyors are unlikely to do their best possible work at temperatures of 15°F or with rain pouring down their necks. Hence, weather details are important in reviewing field notes, in applying corrections to observations due to temperature variations, and for other purposes.
3. *Party*. The names and initials of party members and their duties are required for documentation and future reference. Jobs can be described by symbols, such as ⊼ for instrument operator, ϕ for rod person, and N for notekeeper. The party chief is frequently the notekeeper.
4. *Instrument type and number*. The type of instrument used (with its make and serial number) and its degree of adjustment affects the accuracy of a survey. Identification of the specific equipment employed may aid in isolating some errors—for example, a particular tape with an actual length that is later found to disagree with the distance recorded between its end graduations.

To permit ready location of desired data, each field book must have a table of contents that is kept current daily. In practice, surveyors cross-index their notes on days when field work is impossible.

■ 2.11 SUGGESTIONS FOR RECORDING NOTES

Observing the suggestions given in preceding sections, together with those listed here, will eliminate some common mistakes in recording notes.

1. Letter the notebook owner's name and address on the cover and the first inside page using permanent ink. Number all field books for record purposes.
2. Begin a new day's work on a new page. For property surveys having complicated sketches, this rule may be waived.
3. Employ any orderly, standard, familiar note form type, but, if necessary, design a special arrangement to fit the project.
4. Include explanatory statements, details, and additional observations if they might clarify the notes for field and office personnel.
5. Record what is read without performing any mental arithmetic. Write down what you read!
6. Run notes down the page, except in route surveys, where they usually progress upward to conform with sketches made while looking in the forward direction. (See Plate B.5 in Appendix B.)
7. Use sketches instead of tabulations when in doubt. Carry a straightedge for ruling lines and a small protractor to lay off angles.
8. Make drawings to general proportions rather than to exact scale, and recognize that the usual preliminary estimate of space required is too small. Lettering parallel with or perpendicular to the appropriate features, showing clearly to what they apply.
9. Exaggerate details on sketches if clarity is thereby improved, or prepare separate diagrams.
10. Line up descriptions and drawings with corresponding numerical data. For example, a benchmark description should be placed on the right-hand page opposite its elevation, as in Figure 5.5.
11. Avoid crowding. If it is helpful to do so, use several right-hand pages of descriptions and sketches for a single left-hand sheet of tabulation. Similarly, use any number of pages of tabulation for a single drawing. Paper is cheap compared with the value of time that might be wasted by office personnel in misinterpreting compressed field notes, or by requiring a party to return to the field for clarification.
12. Use explanatory notes when they are pertinent, always keeping in mind the purpose of the survey and needs of the office force. Put these notes in open spaces to avoid conflict with other parts of the sketch.
13. Employ conventional symbols and signs for compactness.
14. A meridian arrow is vital for all sketches. Have north at the top and on the left side of sketches if possible.
15. Keep tabulated figures inside of and off column rulings, with decimal points and digits in line vertically.
16. Make a mental estimate of all measurements before receiving and recording them in order to eliminate large mistakes.
17. Repeat aloud values given for recording. For example, before writing down a distance of 124.68, call out "one, two, four, point six, eight" for verification by the person who submitted the measurement.

18. Place a zero before the decimal point for numbers smaller than 1; that is, record 0.37 instead of .37.
19. Show the precision of observations by means of significant figures. For example, record 3.80 instead of 3.8 only if the reading was actually determined to hundredths.
20. Do not superimpose one number over another or on lines of sketches, and do not try to change one figure to another, as a 3 to a 5.
21. Make all possible arithmetic checks on the notes and record them before leaving the field.
22. Compare all misclosures and error ratios while in the field. On large projects where daily assignments are made for several parties, completed work is shown by satisfactory closures.
23. Arrange essential computations made in the field so they can be checked later.
24. Title, index, and cross-reference each new job or continuation of a previous one by client's organization, property owner, and description.
25. Sign surname and initials in the lower right-hand corner of the right page on all original notes. This places responsibility just as signing a check does.

■ 2.12 INTRODUCTION TO DATA COLLECTORS

Advances in computer technology in recent years have led to the development of sophisticated automatic data collection systems for taking field notes. These devices are about the size of a pocket calculator and are produced by a number of different manufacturers. They are available with a variety of features and capabilities. Figure 2.4 illustrates two different data collectors.

Data collectors can be interfaced with modern surveying instruments, and when operated in that mode they can automatically receive and store data in

(a) (b)

Figure 2.4 Various data collectors that are used in the field: (a) Trimble TSC2 data collector (Courtesy of Trimble) and (b) Carlson Explorer data collector.

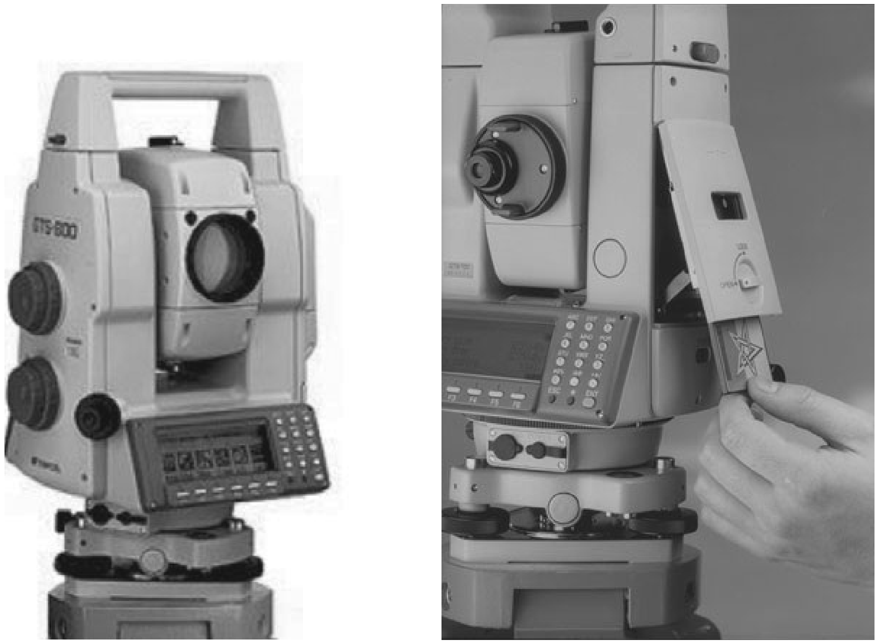
computer compatible files as observations are taken. Control of the measurement and storage operations is maintained through the data collector's keyboard. For clarification of the notes, the operator inputs point identifiers and other descriptive information along with the measurements as they are being recorded automatically. When a job is completed or at day's end, the files can be transferred directly to a computer for further processing.

In using automatic data collectors, the usual preliminary information such as date, party, weather, time, and instrument number is entered manually into the file through the keyboard. For a given type of survey, the data collector's internal microprocessor is programmed to follow a specific sequence of steps. The operator identifies the type of survey to be performed from a menu, or by means of a code, and then follows instructions that appear on the unit's screen. Step-by-step prompts will guide the operator to either (a) input "external" data (which may include station names, descriptions, or other information) or (b) press a key to initiate the automatic recording of observed values. Since data collectors require the users to follow specific steps when performing a survey, they are often referred to as *survey controllers*.

Data collectors store information in either binary or ASCII (American Standard Code for Information Interchange) format. Binary storage is faster and more compact, but usually the data must be translated to ASCII before they can be read or edited. Most data collectors enable an operator to scroll through stored data, displaying them on the screen for review and editing while still at the job site. The organizational structures used by different data collectors in storing information vary considerably from one manufacturer to the next. They all follow specific rules, and once they are understood, the data can be readily interpreted by both field and office personnel. The disadvantage of having varied data structures from different manufacturers is that a new system must be learned with each instrument of different make. Efforts have been made toward standardizing the data structures. The *Survey Data Management System* (SDMS), for example, has been adopted by the *American Association of State Highway and Transportation Officials* (AASHTO) and is recommended for all surveys involving highway work. The example field notes for a radial survey given in Table 17.1 of Section 17.9 are in the SDMS format.

Most manufacturers of modern surveying equipment have developed data collectors specifically to be interfaced with their own instruments, but some are flexible. The Trimble TSC2 survey controller shown in Figure 2.4(a), for example, can be interfaced with Trimble instruments, but it can also be used with others. In addition to serving as a data collector, the TSC2 is able to perform a variety of timesaving calculations directly in the field. It has a Windows CE operating system and thus can run a variety of Windows software programs. Additionally, it has Bluetooth technology so that it can communicate with instruments without using cables and has WiFi capabilities for connecting to the Internet.

Some automatic data collectors can also be operated as electronic field books. In the electronic field book mode, the data collector is not interfaced with a surveying instrument. Instead of handwriting the data in a field book, the notekeeper enters observations into the data collector manually by means of keyboard strokes after readings are taken. This has the advantage of enabling field notes to be recorded directly in a computer format ready for further processing, even though the surveying instruments being used may be older and not



(a) (b)

Figure 2.5 The Topcon GTS 800 total station with internal data collector. (Courtesy Topcon Positioning Systems.)

compatible for direct interfacing with data collectors. However, data collectors provide the utmost in efficiency when they are interfaced with surveying instruments such as total stations that have automatic readout capabilities.

The touch screen of the Carlson Explorer data collector shown in Figure 2.4(b) is a so-called third-party unit; that is, it is made by an independent company to be interfaced with instruments manufactured by others. It also utilizes a Windows CE operating system and has both Bluetooth and WiFi capabilities. It can be either operated in the electronic field book mode or interfaced with a variety of instruments for automatic data collection.

Many instrument manufacturers incorporate data collection systems as internal components directly into their equipment. This incorporates all features of external data collectors, including the display panel, within the instrument. The Topcon GTS 800 shown in Figure 2.5 has an MS-DOS®-based operating system with the ability to run the TDS (Tripod Data System) Survey Pro Software® onboard. It comes standard with 2 MB of program memory and 2 MB of internal data memory. The instrument has a PCMCIA[1] port for use with external data cards to allow for transfer of data from the field to the office without the instrument.

Data collectors currently use the Windows® CE operating system. A pen and pad arrangement enables the user to point on menus and options to run software. The data collectors shown in Figure 2.4 and the Trimble TSC2 data collector shown in Figure 2.6 have this type of interface. A code-based GPS antenna can be inserted into a PCMCIA port of several data collectors to add code-based GPS capabilities to the unit. Most modern data collectors have the capability of

[1]A PCMCIA port conforms to the *Personal Computer Memory Card International Association* standards.

Figure 2.6 Trimble TSC2 with Bluetooth technology. (Courtesy of Trimble.)

running advanced computer software in the field. As one example of their utility, field crews can check their data before sending it to the office (Figure 2.7).

As each new series of data collectors is developed, more sophisticated user interfaces are being designed, and the software that accompanies the systems is being improved. These systems have resulted in increased efficiency and productivity, and have provided field personnel with new features, such as the ability to perform additional field checks. However, the increased complexity of operating surveying instruments with advanced data collectors also requires field personnel with higher levels of education and training.

■ 2.13 TRANSFER OF FILES FROM DATA COLLECTORS

At regular intervals, usually at lunchtime and at the end of a day's work, or when a survey has been completed, the information stored in files within a data collector is transferred to another device. This is a safety precaution to avoid accidentally losing substantial amounts of data. Ultimately, of course, the files are downloaded to a host computer, which will perform computations or generate maps and plots from the data. Depending on the peripheral equipment available, different procedures for data transfer can be used. In one method that is particularly convenient when surveying in remote locations, data can be returned to the home office via telephony technology using devices called *data modems*. (Modems convert computer data into audible tones for transmission via telephone systems.) Thus, office personnel can immediately begin using the data. In areas with cell phone coverage, this operation can be performed in the field. Another method of data transfer consists in downloading data straight into a computer by direct hookup via an RS-232 cable. This can be performed in the office, or it can be done in the field if a laptop computer is available. In areas with

Figure 2.7 Screen of a Trimble TSC2 survey controller. (Courtesy of Trimble.)

wireless Internet, data can be transferred to the office using wireless connections. Data collectors with WiFi capabilities allow field crews to communicate directly with office personnel, thus allowing data to be transferred, checked, and verified before the crews leave the field.

Some surveying instruments, for example, the Topcon GTS 800 Series total station shown in Figure 2.5, are capable of storing data externally on PCMCIA cards. These cards can, in turn, be taken to the office, where the files can be downloaded using a computer with a PCMCIA port. These ports are standard for most laptop computers, and thus allow field crews to download data from the PCMCIA card and external or internal data collector to storage devices on the computer at regular intervals in the field. With the inclusion of a modem, field crews can transfer files to an office computer over phone lines. Office personnel can check field data, or compute additional points to be staked, in the office and return the results to the field crews while they are still on the site.

From the preceding discussion, and as illustrated in Figure 2.8, automatic data collectors are central components of modern computerized surveying systems. In these systems, data flow automatically from the field instrument through the collector to the printer, computer, plotter, and other units in the system. The term

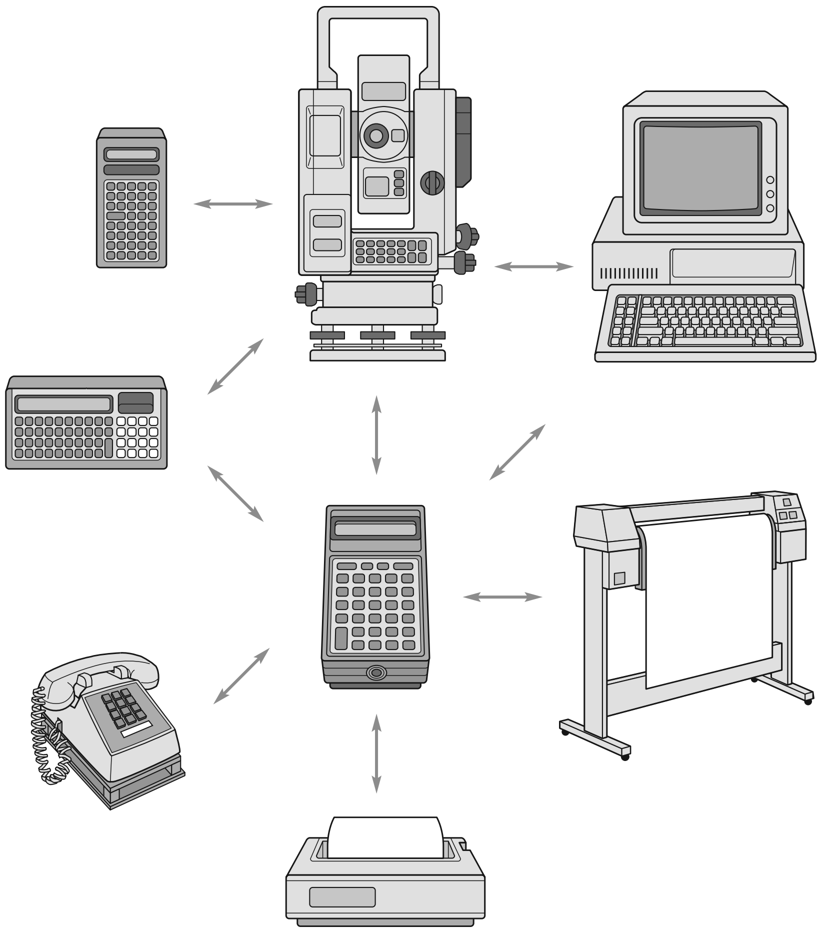


Figure 2.8 Automatic data collector—a central component in modern computerized surveying systems. (Reprinted with permission from Sokkia Corporation.)

"field-to-finish systems" is often applied when this form of instrumentation and software is utilized in surveying.

■ 2.14 DIGITAL DATA FILE MANAGEMENT

Once the observing process is completed in the field, the generated data files must be transferred (downloaded) from the data collector to another secure storage device. Typical information downloaded from a data collector includes a file of computed coordinates and a raw data file. Data collectors generally provide the option of saving these and other types of files. In this case, the coordinate file consists of computed coordinate values generated using the observations and any applied field corrections. Field corrections may include a scale factor, offsets, and Earth curvature and refraction corrections applied to distances. Field crews generally can edit and delete information from the computed file. However, the raw data file consists of the original unreduced observations and cannot be altered in the field. The necessity for each type of data file is dependent on the intended use of the survey. In most surveys, it would be prudent to save both the coordinate and raw files. As an example, for projects that require specific closures, or that are subject to legal review, the raw data file is an essential element of the survey. However, in topographic and GNSS surveys large quantities of data are often generated. In these types of projects, the raw data file can be eliminated to provide more storage space for coordinate files.

With data collectors and digital instruments, personnel in modern surveying offices deal with considerably more data than was customary in the past. This increased volume inevitably raises new concerns about data reliability and safe storage. Many methods can be used to provide backup of digital data. Some storage options include removable media disks and tapes. Since these tend to be magnetic, there is an inevitable danger that data could be lost due to the presence of external magnetic devices, or from the failure of the disk's surface. Because of this problem, it is wise to keep two copies of the files for all jobs. Another inexpensive solution to this problem is the use of compact disk (CD) and digital video disk (DVD) writers. These drives will write an optical image of a project's data on a portable disk media. Since CDs and DVDs are small but have large storage capabilities, entire projects, including drawings, can be recorded in a small space that is easily archived for future reference. However, these disks can fail when scratched. Thus, care must be taken in their handling and storage.

2.15 ADVANTAGES AND DISADVANTAGES OF DATA COLLECTORS

The major advantages of automatic data collection systems are that (1) mistakes in reading and manually recording observations in the field are precluded and (2) the time to process, display, and archive the field notes in the office is reduced significantly. Systems that incorporate computers can execute some programs in the field, which adds a significant advantage. As an example, the data for a survey can be corrected for systematic errors and misclosures computed, so verification that a survey meets closure requirements is made before the crew leaves a site.

Data collectors are most useful when large quantities of information must be recorded, for example, in topographic surveys or cross-sectioning. In Section 17.9, their use in topographic surveying is described, and an example set of notes taken for that purpose is presented and discussed.

Although data collectors have many advantages, they also present some dangers and problems. There is the slight chance, for example, the files could be accidentally erased through carelessness or lost because of malfunction or damage to the unit. Some difficulties are also created by the fact that sketches cannot be entered into the computer. However, this problem can be overcome by supplementing files with sketches made simultaneously with the observations that include field codes. These field codes can instruct the drafting software to draw a map of the data complete with lines, curves, and mapping symbols. The process of collecting field data with field codes that can be interpreted later by software is known as a *field-to-finish* survey. This greatly reduces the time needed to complete a project. Field-to-finish mapping surveys are discussed in more detail in Section 17.12. It is important to realize that not all information can be stored in digital form, and thus it is important to keep a traditional field book to enter sketches, comments, and additional notes when necessary. In any event, these devices should not be used for long-term storage. Rather the data should be downloaded and immediately saved to some permanent storage device such as a CD or DVD once the field collection for a project is complete.

Data collectors are available from numerous manufacturers. They must be capable of transferring data through various hardware in modern surveying systems such as that illustrated in Figure 2.8. Since equipment varies considerably, it is important when considering the purchase of a data collector to be certain it fits the equipment owned or perhaps needed in the future.

PROBLEMS

Asterisks (*) indicate problems that have partial answers given in Appendix G.

2.1 List the five types of measurements that form the basis of traditional plane surveying.

2.2 Give the basic units that are used in surveying for length, area, volume, and angles in
(a) The English system of units.
(b) The SI system of units.

2.3 Why was the survey foot definition maintained in the United States?

2.4 Convert the following distances given in meters to U.S. survey feet:
*(a) 4129.574 m **(b)** 738.296 m **(c)** 6048.083 m

2.5 Convert the following distances given in feet to meters:
*(a) 537.52 ft **(b)** 9364.87 ft **(c)** 4806.98 ft

2.6 Compute the lengths in feet corresponding to the following distances measured with a Gunter's chain:
*(a) 10 ch 13 lk **(b)** 6 ch 12 lk **(c)** 24 ch 8 lk

2.7 Express 95,748 ft^2 in:
*(a) acres **(b)** hectares **(c)** square Gunter's chains

2.8 Convert 5.6874 ha to:
(a) acres **(b)** square Gunter's chains

2.9 What are the lengths in feet and decimals for the following distances shown on a building blueprint?
(a) 30 ft 9-3/4 in. **(b)** 12 ft 6-1/32 in.

2.10 What is the area in acres of a rectangular parcel of land measured with a Gunter's chain if the recorded sides are as follows:
*(a) 9.17 ch and 10.64 ch **(b)** 12 ch 36 lk and 24 ch 28 lk

2.11 Compute the area in acres of triangular lots shown on a plat having the following recorded right-angle sides:
(a) 208.94 ft and 232.65 ft **(b)** 9 ch 25 lk and 6 ch 16 lk

2.12 A distance is expressed as 125,845.64 U.S. survey feet. What is the length in
*(a) international feet? **(b)** meters?

2.13 What are the radian and degree–minute–second equivalents for the following angles given in grads:
*(a) 136.0000 grads **(b)** 89.5478 grads **(c)** 68.1649 grads

2.14 Give answers to the following problems in the correct number of significant figures:
*(a) sum of 23.15, 0.984, 124, and 12.5
(b) sum of 36.15, 0.806, 22.4, and 196.458
(c) product of 276.75 and 33.7
(d) quotient of 4930.27 divided by 1.29

2.15 Express the value or answer in powers of 10 to the correct number of significant figures:
(a) 11,432
(b) 4520
(c) square of 11,293
(d) sum of (11.275 + 0.5 + 146.12) divided by 7.2

2.16 Convert the adjusted angles of a triangle to radians and show a computational check:
*__(a)__ 39°41′54″, 91°30′16″, and 48°47′50″
(b) 82°17′43″, 29°05′54″, and 68°36′23″
2.17 Why should a pen not be used in field notekeeping?
2.18 Explain why one number should not be superimposed over another or the lines of sketches.
2.19* Explain why data should always be entered directly into the field book at the time measurements are made, rather than on scrap paper for neat transfer to the field book later.
2.20 Why should a new day's work begin on a new page?
2.21 Explain the reason for item 18 in Section 2.11 when recording field notes.
2.22 Explain the reason for item 24 in Section 2.11 when recording field notes.
2.23 Explain the reason for item 27 in Section 2.11 when recording field notes.
2.24 When should sketches be made instead of just recording data?
2.25 Justify the requirement to list in a field book the makes and serial numbers of all instruments used on a survey.
2.26 Discuss the advantages of survey controllers that can communicate with several different types of instruments.
2.27 Discuss the advantages of survey controllers.
2.28 Search the Internet and find at least two sites related to
(a) Manufacturers of survey controllers.
(b) Manufacturers of total stations.
(c) Manufacturers of global navigation satellite system (GNSS) receivers.
2.29 What advantages are offered to field personnel if the survey controller provides a map of the survey?
2.30 Prepare a brief summary of an article from a professional journal related to the subject matter of this chapter.
2.31 Describe what is meant by the phrase "field-to-finish."
2.32 Why are sketches in field books not usually drawn to scale?
2.33 Create a computational program that solves Problem 2.16.

BIBLIOGRAPHY

Alder, K. 2002. *The Measure of All Things—The Seven-Year Odyssey and Hidden Error that Transformed the World*. New York, NY: The Free Press.
Bedini, S. A. 2001. "Roger Sherman's Field Survey Book." *Professional Surveyor Magazine* 21 (No. 4): 70.
Bennett, T. D. 2002. "From Operational Efficiency to Business Process Improvement." *Professional Surveyor* 22 (No. 2): 46.
Brown, L. 2003. "Building a Better Handheld." *Point of Beginning* 28 (No. 7): 24.
Durgiss, K. 2001. "Advancing Field Data Collection with Wearable Computers." *Professional Surveyor* 21 (No. 4): 14
Meade, M. E. 2007. "The International versus U.S. Survey Foot." *Point of Beginning* 33 (No. 1): 66.
Paiva, J. V. R. 2006. "The Evolution of the Data Collector." 32 (No. 2): 22.
Pepling, A. 2003. "TDS Recon." *Professional Surveyor* 23 (No. 9): 34.
Ghilani, C. D. 2010. *Adjustment Computations: Spatial Data Analysis*. New York, NY: Wiley.

3

Theory of Errors in Observations

■ 3.1 INTRODUCTION

Making observations (measurements), and subsequent computations and analyses using them, are fundamental tasks of surveyors. Good observations require a combination of human skill and mechanical equipment applied with the utmost judgment. However, no matter how carefully made, observations are never exact and will always contain errors. Surveyors (geomatics engineers), whose work must be performed to exacting standards, should therefore thoroughly understand the different kinds of errors, their sources and expected magnitudes under varying conditions, and their manner of propagation. Only then can they select instruments and procedures necessary to reduce error sizes to within tolerable limits.

Of equal importance, surveyors must be capable of assessing the magnitudes of errors in their observations so that either their acceptability can be verified or, if necessary, new ones made. The design of measurement systems is now practiced. Computers and sophisticated software are tools now commonly used by surveyors to plan measurement projects and to investigate and distribute errors after results have been obtained.

■ 3.2 DIRECT AND INDIRECT OBSERVATIONS

Observations may be made directly or indirectly. Examples of *direct observations* are applying a tape to a line, fitting a protractor to an angle, or turning an angle with a total station instrument.

An *indirect observation* is secured when it is not possible to apply a measuring instrument directly to the quantity to be observed. The answer is therefore determined by its relationship to some other observed value or values. As an example, we can find the distance across a river by observing the length of a line on one side of the river and the angle at each end of this line to a point on the other side, and then computing the distance by one of the standard trigonometric formulas. Many indirect observations are made in surveying, and since all measurements contain errors, it is inevitable that quantities computed from them will also contain errors. The manner by which errors in measurements combine to produce erroneous computed answers is called error propagation. This topic is discussed further in Section 3.17.

3.3 ERRORS IN MEASUREMENTS

By definition, an error is the difference between an observed value for a quantity and its true value, or

$$E = X - \overline{X} \tag{3.1}$$

where E is the error in an observation, X the observed value, and $\overline{X}$ its true value. It can be unconditionally stated that (1) no observation is exact, (2) every observation contains errors, (3) the true value of an observation is never known, and, therefore, (4) the exact error present is always unknown. These facts are demonstrated by the following. When a distance is observed with a scale divided into tenths of an inch, the distance can be read only to hundredths (by interpolation). However, if a better scale graduated in hundredths of an inch was available and read under magnification, the same distance might be estimated to thousandths of an inch. And with a scale graduated in thousandths of an inch, a reading to ten-thousandths might be possible. Obviously, accuracy of observations depends on the scale's division size, reliability of equipment used, and human limitations in estimating closer than about one tenth of a scale division. As better equipment is developed, observations more closely approach their true values, but they can never be exact. Note that observations, not counts (of cars, pennies, marbles, or other objects), are under consideration here.

3.4 MISTAKES

These are observer blunders and are usually caused by misunderstanding the problem, carelessness, fatigue, missed communication, or poor judgment. Examples include transposition of numbers, such as recording 73.96 instead of the correct value of 79.36; reading an angle counterclockwise, but indicating it as a clockwise angle in the field notes; sighting the wrong target; or recording a measured distance as 682.38 instead of 862.38. Large mistakes such as these are not considered in the succeeding discussion of errors. They must be detected by careful and systematic checking of all work, and eliminated by repeating some or all of the measurements. It is very difficult to detect small mistakes because they merge with errors. When not exposed, these small mistakes will therefore be incorrectly treated as errors.

■ 3.5 SOURCES OF ERRORS IN MAKING OBSERVATIONS

Errors in observations stem from three sources, and are classified accordingly.

Natural errors are caused by variations in wind, temperature, humidity, atmospheric pressure, atmospheric refraction, gravity, and magnetic declination. An example is a steel tape whose length varies with changes in temperature.

Instrumental errors result from any imperfection in the construction or adjustment of instruments and from the movement of individual parts. For example, the graduations on a scale may not be perfectly spaced, or the scale may be warped. The effect of many instrumental errors can be reduced, or even eliminated, by adopting proper surveying procedures or applying computed corrections.

Personal errors arise principally from limitations of the human senses of sight and touch. As an example, a small error occurs in the observed value of a horizontal angle if the vertical crosshair in a total station instrument is not aligned perfectly on the target, or if the target is the top of a rod that is being held slightly out of plumb.

■ 3.6 TYPES OF ERRORS

Errors in observations are of two types: *systematic* and *random*.

Systematic errors, also known as *biases*, result from factors that comprise the "measuring system" and include the environment, instrument, and observer. So long as system conditions remain constant, the systematic errors will likewise remain constant. If conditions change, the magnitudes of systematic errors also change. Because systematic errors tend to accumulate, they are sometimes called *cumulative errors*.

Conditions producing systematic errors conform to physical laws that can be modeled mathematically. Thus, if the conditions are known to exist and can be observed, a correction can be computed and applied to observed values. An example of a constant systematic error is the use of a 100-ft steel tape that has been calibrated and found to be 0.02 ft too long. It introduces a 0.02-ft error each time it is used, but applying a correction readily eliminates the error. An example of variable systematic error is the change in length of a steel tape resulting from temperature differentials that occur during the period of the tape's use. If the temperature changes are observed, length corrections can be computed by a simple formula, as explained in Chapter 6.

Random errors are those that remain in measured values after mistakes and systematic errors have been eliminated. They are caused by factors beyond the control of the observer, obey the laws of probability, and are sometimes called *accidental errors*. They are present in all surveying observations.

The magnitudes and algebraic signs of random errors are matters of chance. There is no absolute way to compute or eliminate them, but they can be estimated using adjustment procedures known as *least squares* (see Section 3.21 and Chapter 16). Random errors are also known as *compensating errors*, since they tend to partially cancel themselves in a series of observations. For example, a person interpolating to hundredths of a foot on a tape graduated only to tenths, or reading a level rod marked in hundredths, will presumably estimate too high on some values and too low on others. However, individual personal characteristics may nullify such partial compensation since some people are inclined to interpolate high, others interpolate low, and many favor certain digits—for example, 7 instead of 6 or 8, 3 instead of 2 or 4, and particularly 0 instead of 9 or 1.

■ 3.7 PRECISION AND ACCURACY

A *discrepancy* is the difference between two observed values of the same quantity. A small discrepancy indicates there are probably no mistakes and random errors are small. However, small discrepancies do not preclude the presence of systematic errors.

Precision refers to the degree of refinement or consistency of a group of observations and is evaluated on the basis of discrepancy size. If multiple observations are made of the same quantity and small discrepancies result, this indicates high precision. The degree of precision attainable is dependent on equipment sensitivity and observer skill.

Accuracy denotes the absolute nearness of observed quantities to their true values. The difference between precision and accuracy is perhaps best illustrated with reference to target shooting. In Figure 3.1(a), for example, all five shots exist in a small group, indicating a precise operation; that is, the shooter was able to repeat the procedure with a high degree of consistency. However, the shots are far from the bull's-eye and therefore not accurate. This probably results from misaligned rifle sights. Figure 3.1(b) shows randomly scattered shots that are neither precise nor accurate. In Figure 3.1(c), the closely spaced grouping, in the bull's-eye, represents both precision and accuracy. The shooter who obtained the results in (a) was perhaps able to produce the shots of (c) after aligning the rifle sights. In surveying, this would be equivalent to the calibration of observing instruments.

As with the shooting example, a survey can be precise without being accurate. To illustrate, if refined methods are employed and readings taken carefully, say to 0.001 ft, but there are instrumental errors in the measuring device and corrections are not made for them, the survey will not be accurate. As a numerical example, two observations of a distance with a tape assumed to be 100.000 ft long, that is actually 100.050 ft, might give results of 453.270 and 453.272 ft. These values are precise, but they are not accurate, since there is a systematic error of approximately $4.53 \times 0.050 = 0.23$ ft in each. The precision obtained would be expressed as $(453.272 - 453.270)/453.271 = 1/220{,}000$, which is excellent, but

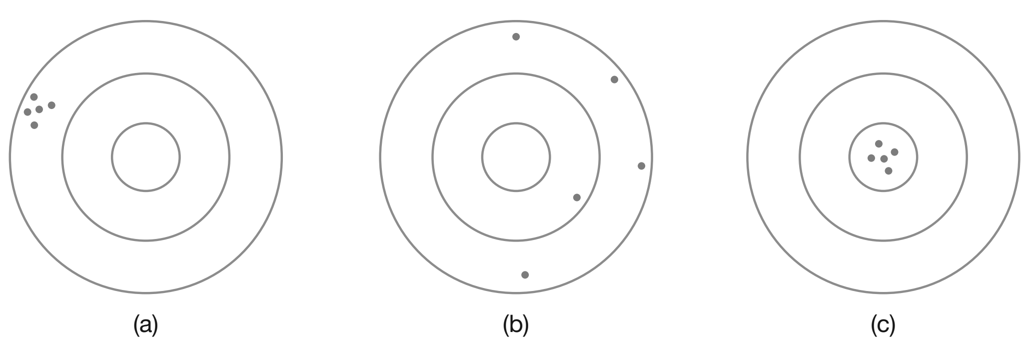


Figure 3.1 Examples of precision and accuracy. (a) Results are precise but not accurate. (b) Results are neither precise nor accurate. (c) Results are both precise and accurate.

accuracy of the distance is only $0.23/453.271 = 1$ part in 2000. Also, a survey may appear to be accurate when rough observations have been taken. For example, the angles of a triangle may be read with a compass to only the nearest 1/4 degree and yet produce a sum of exactly 180°, or a zero misclosure error. On good surveys, precision and accuracy are consistent throughout.

3.8 ELIMINATING MISTAKES AND SYSTEMATIC ERRORS

All field operations and office computations are governed by a constant effort to eliminate mistakes and systematic errors. Of course it would be preferable if mistakes never occurred, but because humans are fallible, this is not possible. In the field, experienced observers who alertly perform their observations using standardized repetitive procedures can minimize mistakes. Mistakes that do occur can be corrected only if discovered. Comparing several observations of the same quantity is one of the best ways to identify mistakes. Making a common sense estimate and analysis is another. Assume that five observations of a line are recorded as follows: 567.91, 576.95, 567.88, 567.90, and 567.93. The second value disagrees with the others, apparently because of a transposition of figures in reading or recording. Either casting out the doubtful value, or preferably repeating the observation can eradicate this mistake.

When a mistake is detected, it is usually best to repeat the observation. However, if a sufficient number of other observations of the quantity are available and in agreement, as in the foregoing example, the widely divergent result may be discarded. Serious consideration must be given to the effect on an average before discarding a value. It is seldom safe to change a recorded number, even though there appears to be a simple transposition in figures. Tampering with physical data is always a bad practice and will certainly cause trouble, even if done infrequently.

Systematic errors can be calculated and proper corrections applied to the observations. Procedures for making these corrections to all basic surveying observations are described in the chapters that follow. In some instances, it may be possible to adopt a field procedure that automatically eliminates systematic errors. For example, as explained in Chapter 5, a leveling instrument out of adjustment causes incorrect readings, but if all backsights and foresights are made the same length, the errors cancel in differential leveling.

3.9 PROBABILITY

At one time or another, everyone has had an experience with games of chance, such as coin flipping, card games, or dice, which involve probability. In basic mathematics courses, laws of combinations and permutations are introduced. It is shown that events that happen randomly or by chance are governed by mathematical principles referred to as probability.

Probability may be defined as the ratio of the number of times a result should occur to its total number of possibilities. For example, in the toss of a fair die there is a one-sixth probability that a 2 will come up. This simply means that there are six possibilities, and only one of them is a 2. In general, if a result may occur in m ways and fail to occur in n ways, then the probability of its occurrence is $m/(m + n)$. The probability that any result will occur is a fraction between 0 and 1; 0 indicating impossibility and 1 denoting absolute certainty. Since any result must either occur or fail, the sum of the probabilities of occurrence and failure is 1. Thus if 1/6 is the probability of throwing a 2 with one toss of a die, then $(1-1/6)$, or 5/6 is the probability that a 2 will not come up.

The theory of probability is applicable in many sociological and scientific observations. In Section 3.6, it was pointed out that random errors exist in all surveying work. This can perhaps be better appreciated by considering the measuring process, which generally involves executing several elementary tasks. Besides instrument selection and calibration, these tasks may include setting up, centering, aligning, or pointing the equipment; setting, matching, or comparing index marks; and reading or estimating values from graduated scales, dials, or gauges. Because of equipment and observer imperfections, exact observations cannot be made, so they will always contain random errors. The magnitudes of these errors, and the frequency with which errors of a given size occur, follow the laws of probability.

For convenience, the term error will be used to mean only random error for the remainder of this chapter. It will be assumed that all mistakes and systematic errors have been eliminated before random errors are considered.

■ 3.10 MOST PROBABLE VALUE

It has been stated earlier that in physical observations, the true value of any quantity is never known. However, its *most probable value* can be calculated if redundant observations have been made. *Redundant observations* are measurements in excess of the minimum needed to determine a quantity. For a single unknown, such as a line length that has been directly and independently observed a number of times using the same equipment and procedures,[1] the first observation establishes a value for the quantity and all additional observations are redundant. The most probable value in this case is simply the arithmetic mean, or

$$\overline{M} = \frac{\Sigma M}{n} \tag{3.2}$$

where $\overline{M}$ is the most probable value of the quantity, ΣM the sum of the individual measurements M, and n the total number of observations. Equation (3.2) can be derived using the principle of least squares, which is based on the theory of probability.

As discussed in Chapter 16, in more complicated problems, where the observations are not made with the same instruments and procedures, or if several interrelated quantities are being determined through indirect observations, most probable values are calculated by employing least-squares methods. The

[1]The significance of using the same equipment and procedures is that observations are of equal reliability or weight. The subject of unequal weights is discussed in Section 3.20.

treatment here relates to multiple direct observations of the same quantity using the same equipment and procedures.

■ 3.11 RESIDUALS

Having determined the most probable value of a quantity, it is possible to calculate *residuals*. A residual is simply the difference between the most probable value and any observed value of a quantity, which in equation form is

$$v = \overline{M} - M \tag{3.3}$$

where v is the residual in any observation M, and $\overline{M}$ is the most probable value for the quantity. Residuals are theoretically identical to errors, with the exception that residuals can be calculated whereas errors cannot because true values are never known. Thus, residuals rather than errors are the values actually used in the analysis and adjustment of survey data.

■ 3.12 OCCURRENCE OF RANDOM ERRORS

To analyze the manner in which random errors occur, consider the data of Table 3.1, which represents 100 repetitions of an angle observation made with a precise total station instrument (described in Chapter 8). Assume these observations are free from mistakes and systematic errors. For convenience in analyzing the data, except for the first value, only the seconds' portions of the observations are tabulated. The data have been rearranged in column (1) so that entries begin with the smallest observed value and are listed in increasing size. If a certain value was obtained more than once, the number of times it occurred, or its *frequency*, is tabulated in column (2).

From Table 3.1, it can be seen that the *dispersion* (range in observations from smallest to largest) is $30.8 - 19.5 = 11.3$ sec. However, it is difficult to analyze the distribution pattern of the observations by simply scanning the tabular values; that is, beyond assessing the dispersion and noticing a general trend for observations toward the middle of the range to occur with greater frequency. To assist in studying the data, a *histogram* can be prepared. This is simply a bar graph showing the sizes of the observations (or their residuals) versus their frequency of occurrence. It gives an immediate visual impression of the distribution pattern of the observations (or their residuals).

For the data of Table 3.1, a histogram showing the frequency of occurrence of the residuals has been developed and is plotted in Figure 3.2. To plot a histogram of residuals, it is first necessary to compute the most probable value for the observed angle. This has been done with Equation (3.2). As shown at the bottom of Table 3.1, its value is $27°43'24.9''$. Then using Equation (3.3), residuals for all observed values are computed. These are tabulated in column (3) of Table 3.1. The residuals vary from $5.4''$ to $-5.9''$. (The sum of the absolute value of these two extremes is the dispersion, or $11.3''$.)

To obtain a histogram with an appropriate number of bars for portraying the distribution of residuals adequately, the interval of residuals represented by

TABLE 3.1 ANGLE OBSERVATIONS FROM PRECISE TOTAL STATION INSTRUMENT

Observed Value (1)	No. (2)	Residual (Sec) (3)	Observed Value (1 Cont.)	No. (2. Cont.)	Residual (Sec) (3 Cont.)
27°43′19.5″	1	5.4	27°43′25.1″	3	−0.2
20.0	1	4.9	25.2	1	−0.3
20.5	1	4.4	25.4	1	−0.5
20.8	1	4.1	25.5	2	−0.6
21.2	1	3.7	25.7	3	−0.8
21.3	1	3.6	25.8	4	−0.9
21.5	1	3.4	25.9	2	−1.0
22.1	2	2.8	26.1	1	−1.2
22.3	1	2.6	26.2	2	−1.3
22.4	1	2.5	26.3	1	−1.4
22.5	2	2.4	26.5	1	−1.6
22.6	1	2.3	26.6	3	−1.7
22.8	2	2.1	26.7	1	−1.8
23.0	1	1.9	26.8	2	−1.9
23.1	2	1.8	26.9	1	−2.0
23.2	2	1.7	27.0	1	−2.1
23.3	3	1.6	27.1	3	−2.2
23.6	2	1.3	27.4	1	−2.5
23.7	2	1.2	27.5	2	−2.6
23.8	2	1.1	27.6	1	−2.7
23.9	3	1.0	27.7	2	−2.8
24.0	5	0.9	28.0	1	−3.1
24.1	3	0.8	28.6	2	−3.7
24.3	1	0.6	28.7	1	−3.8
24.5	2	0.4	29.0	1	−4.1
24.7	3	0.2	29.4	1	−4.5
24.8	3	0.1	29.7	1	−4.8
24.9	2	0.0	30.8	1	−5.9
25.0	2	−0.1	Σ = 2494.0	Σ = 100	

Mean = 2494.0/100 = 24.9″

Most Probable Value = 27°43′24.9″

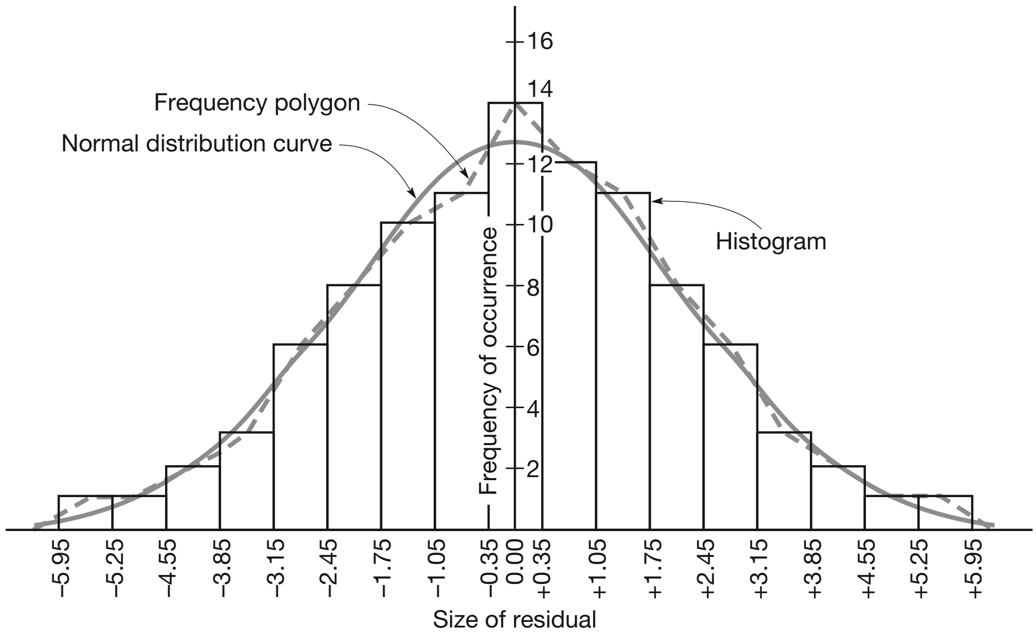


Figure 3.2 Histogram, frequency polygon, and normal distribution curve of residuals from angle measurements made with total station.

each bar, or the *class interval*, was chosen as 0.7″. This produced 17 bars on the graph. The range of residuals covered by each interval, and the number of residuals that occur within each interval, are listed in Table 3.2. By plotting class intervals on the abscissa against the number (frequency of occurrence) of residuals in each interval on the ordinate, the histogram of Figure 3.2 was obtained.

If the adjacent top center points of the histogram bars are connected with straight lines, the so-called *frequency polygon* is obtained. The frequency polygon for the data of Table 3.1 is superimposed as a heavy dashed blue line in Figure 3.2. It graphically displays essentially the same information as the histogram.

If the number of observations being considered in this analysis were increased progressively, and accordingly the histogram's class interval taken smaller and smaller, ultimately the frequency polygon would approach a smooth continuous curve, symmetrical about its center like the one shown with the heavy solid blue line in Figure 3.2. For clarity, this curve is shown separately in Figure 3.3. The curve's "bell shape" is characteristic of a normally distributed group of errors, and thus it is often referred to as the *normal distribution curve*. Statisticians frequently call it the *normal density curve*, since it shows the densities of errors having various sizes. In surveying, normal or very nearly normal error distributions are expected, and henceforth in this book that condition is assumed.

In practice, histograms and frequency polygons are seldom used to represent error distributions. Instead, normal distribution curves that approximate them are preferred. Note how closely the normal distribution curve superimposed on Figure 3.2 agrees with the histogram and the frequency polygon.

As demonstrated with the data of Table 3.1, the histogram for a set of observations shows the probability of occurrence of an error of a given size graphically by bar areas. For example, 14 of the 100 residuals (errors) in Figure 3.2 are between −0.35″ and +0.35″. This represents 14% of the errors, and the center histogram bar, which corresponds to this interval, is 14% of the total area of all

TABLE 3.2 RANGES OF CLASS INTERVALS AND NUMBER OF RESIDUALS IN EACH INTERVAL

Histogram Interval (Sec)	Number of Residuals in Interval
−5.95 to −5.25	1
−5.25 to −4.55	1
−4.55 to −3.85	2
−3.85 to −3.15	3
−3.15 to −2.45	6
−2.45 to −1.75	8
−1.75 to −1.05	10
−1.05 to −0.35	11
−0.35 to +0.35	14
+0.35 to +1.05	12
+1.05 to +1.75	11
+1.75 to +2.45	8
+2.45 to +3.15	6
+3.15 to +3.85	3
+3.85 to +4.55	2
+4.55 to +5.25	1
+5.25 to +5.95	1
	$\Sigma = 100$

bars. Likewise, the area between ordinates constructed at any two abscissas of a normal distribution curve represents the percent probability that an error of that size exists. Since the area sum of all bars of a histogram represents all errors, it therefore represents all probabilities, and thus its sum equals 1. Likewise, the total area beneath a normal distribution curve is also 1.

If the same observations of the preceding example had been taken using better equipment and more caution, smaller errors would be expected and the normal distribution curve would be similar to that in Figure 3.4(a). Compared to Figure 3.3, this curve is taller and narrower, showing that a greater percentage of values have smaller errors, and fewer observations contain big ones. For this comparison, the same ordinate and abscissa scales must be used for both curves. Thus, the observations of Figure 3.4(a) are more precise. For readings taken less precisely, the opposite effect is produced, as illustrated in Figure 3.4(b), which shows a shorter and wider curve. In all three cases, however, the curve maintained its characteristic symmetric bell shape.

From these examples, it is seen that relative precisions of groups of observations become readily apparent by comparing their normal distribution curves. The

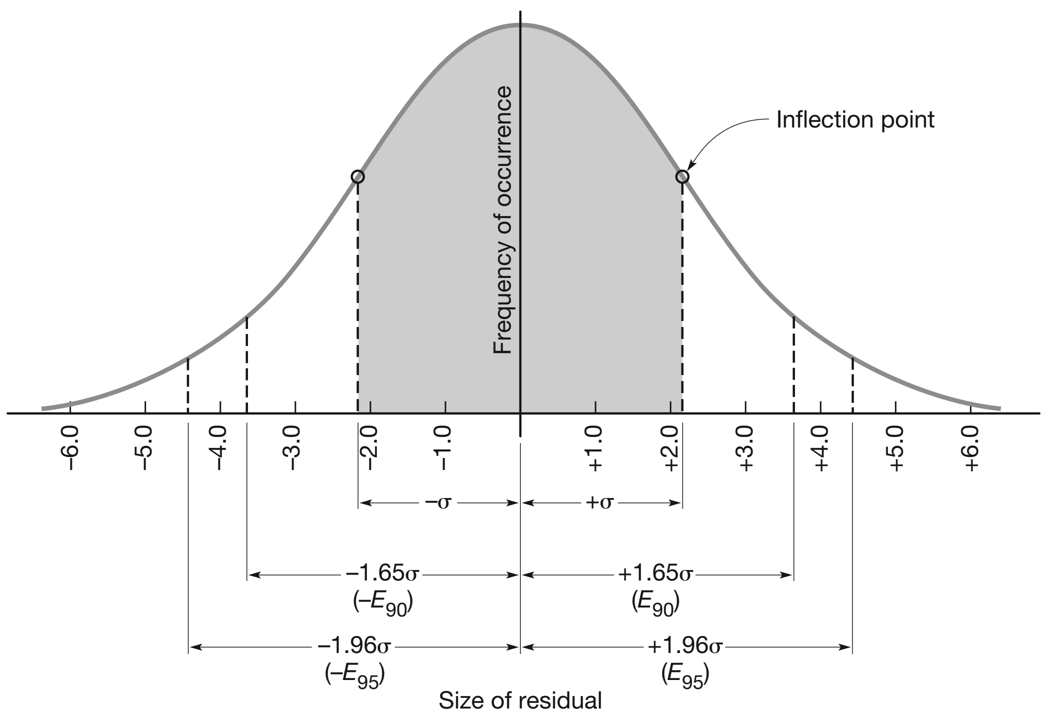


Figure 3.3 Normal distribution curve.

normal distribution curve for a set of observations can be computed using parameters derived from the residuals, but the procedure is beyond the scope of this text.

■ 3.13 GENERAL LAWS OF PROBABILITY

From an analysis of the data in the preceding section and the curves in Figures 3.2 through 3.4, some general laws of probability can be stated:

1. Small residuals (errors) occur more often than large ones; that is, they are more probable.
2. Large errors happen infrequently and are therefore less probable; for normally distributed errors, unusually large ones may be mistakes rather than random errors.
3. Positive and negative errors of the same size happen with equal frequency; that is, they are equally probable. [This enables an intuitive deduction of Equation (3.2) to be made: that is, the most probable value for a group of repeated observations, made with the same equipment and procedures, is the mean.]

■ 3.14 MEASURES OF PRECISION

As shown in Figures 3.3 and 3.4, although the curves have similar shapes, there are significant differences in their dispersions; that is, their abscissa widths differ. The magnitude of dispersion is an indication of the relative precisions of the observations. Other statistical terms more commonly used to express precisions of groups

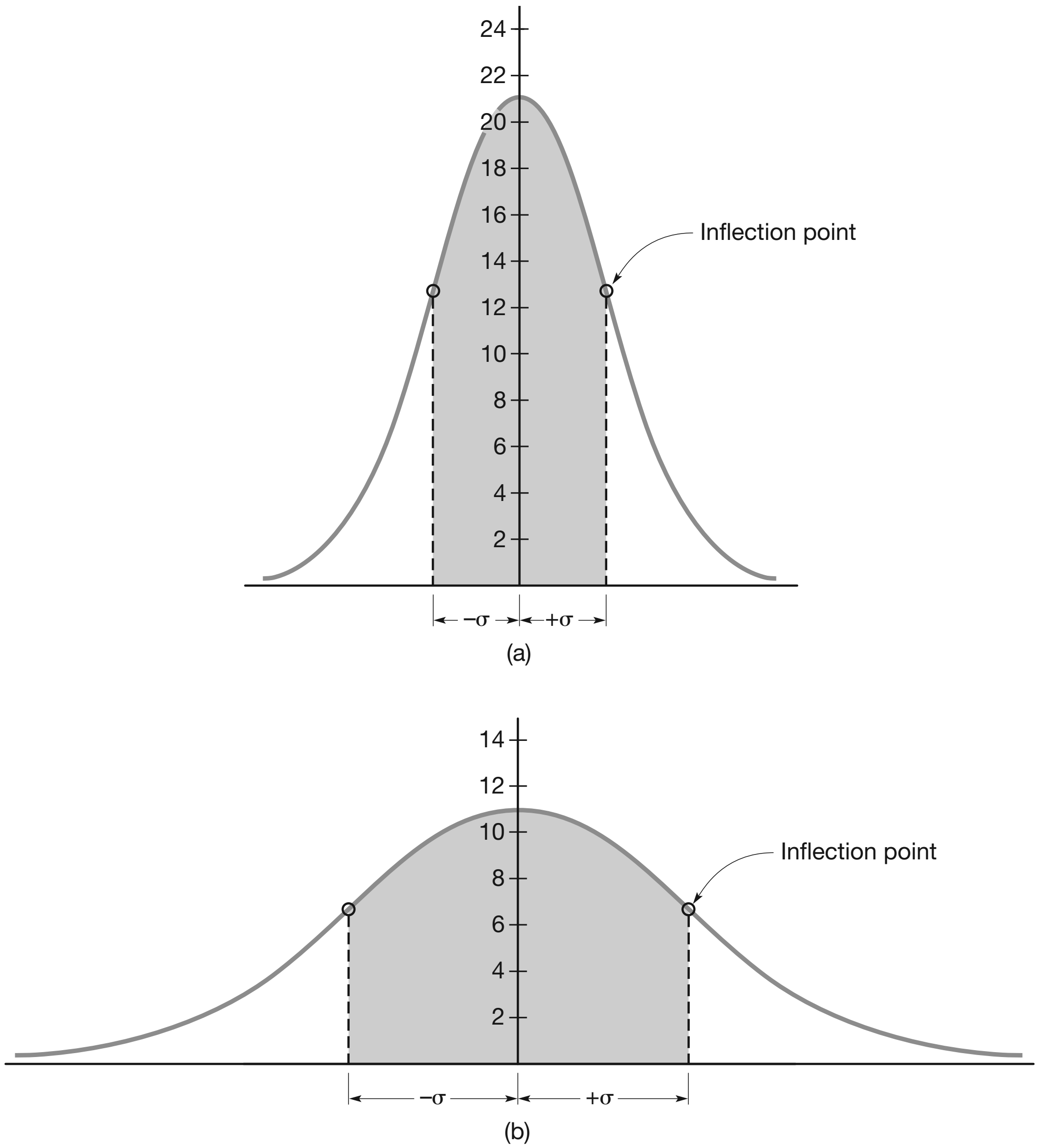


Figure 3.4
Normal distribution curves for:
(a) increased precision,
(b) decreased precision.

of observations are *standard deviation* and *variance*. The equation for the standard deviation is

$$\sigma = \pm\sqrt{\frac{\Sigma\nu^2}{n-1}} \tag{3.4}$$

where σ is the standard deviation of a group of observations of the same quantity, ν the residual of an individual observation, $\Sigma\nu^2$ the sum of squares of the individual residuals, and n the number of observations. *Variance* is equal to σ^2, the square of the standard deviation.

Note that in Equation (3.4), the standard deviation has both plus and minus values. On the normal distribution curve, the numerical value of the standard deviation is the abscissa at the inflection points (locations where the curvature changes from concave downward to concave upward). In Figures 3.3 and 3.4, these inflection points are shown. Note the closer spacing between them for the more precise observations of Figure 3.4(a) as compared to Figure 3.4(b).

Figure 3.5 is a graph showing the percentage of the total area under a normal distribution curve that exists between ranges of residuals (errors) having equal positive and negative values. The abscissa scale is shown in multiples of the standard deviation. From this curve, the area between residuals of $+\sigma$ and $-\sigma$ equals approximately 68.3% of the total area under the normal distribution curve. Hence, it gives the range of residuals that can be expected to occur 68.3% of the time. This relation is shown more clearly on the curves in Figures 3.3 and 3.4, where the areas between $\pm\sigma$ are shown shaded. The percentages shown in Figure 3.5 apply to all normal distributions; regardless of curve shape or the numerical value of the standard deviation.

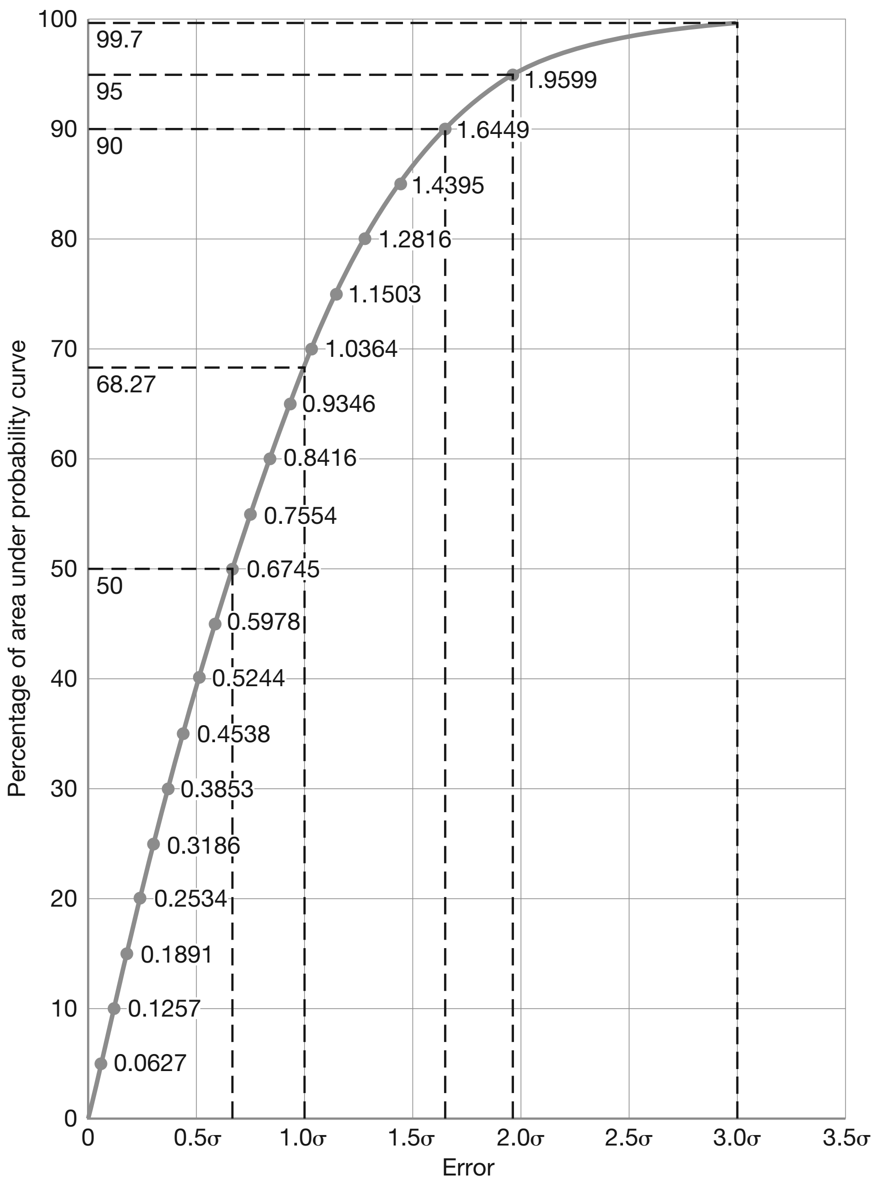


Figure 3.5 Relation between error and percentage of area under normal distribution curve.

3.15 INTERPRETATION OF STANDARD DEVIATION

It has been shown that the standard deviation establishes the limits within which observations are expected to fall 68.3% of the time. In other words, if an observation is repeated ten times, it will be expected that about seven of the results will fall within the limits established by the standard deviation, and conversely about three of them will fall anywhere outside these limits. Another interpretation is that one additional observation will have a 68.3% chance of falling within the limits set by the standard deviation.

When Equation (3.4) is applied to the data of Table 3.1, a standard deviation of ± 2.19 is obtained. In examining the residuals in the table, 70 of the 100 values, or 70%, are actually smaller than 2.19 sec. This illustrates that the theory of probability closely approximates reality.

3.16 THE 50, 90, AND 95 PERCENT ERRORS

From the data given in Figure 3.5, the probability of an error of any percentage likelihood can be determined. The general equation is

$$E_P = C_P \sigma \tag{3.5}$$

where E_P, is a certain percentage error and C_P, the corresponding numerical factor taken from Figure 3.5.

By Equation (3.5), after extracting appropriate multipliers from Figure 3.5, the following are expressions for errors that have a 50%, 90%, and 95% chance of occurring:

$$E_{50} = 0.6745\sigma \tag{3.6}$$

$$E_{90} = 1.6449\sigma \tag{3.7}$$

$$E_{95} = 1.9599\sigma \tag{3.8}$$

The 50 percent error, or E_{50}, is the so-called *probable error*. It establishes limits within which the observations should fall 50% of the time. In other words, an observation has the same chance of coming within these limits as it has of falling outside of them.

The 90 and 95 percent errors are commonly used to specify precisions required on surveying (geomatics) projects. Of these, the 95 percent error, also frequently called the *two-sigma* (2σ) *error*, is most often specified. As an example, a particular project may call for the 95 percent error to be less than or equal to a certain value for the work to be acceptable. For the data of Table 3.1, applying Equations (3.7) and (3.8), the 90 and 95 percent errors are ± 3.60 and ± 4.29 sec, respectively. These errors are shown graphically in Figure 3.3.

The so-called *three-sigma* (3σ) *error* is also often used as a criterion for rejecting individual observations from sets of data. From Figure 3.5, there is a 99.7% probability that an error will be less than this amount. Thus, within a group of observations, any value whose residual exceeds 3σ is considered to be a mistake, and either a new observation must be taken or the computations based on one less value.

The x-axis is an asymptote of the normal distribution curve, so the 100 percent error cannot be evaluated. This means that no matter what size error is found, a larger one is theoretically possible.

Example 3.1

To clarify definitions and use the equations given in Sections 3.10 through 3.16, suppose that a line has been observed 10 times using the same equipment and procedures. The results are shown in column (1) of the following table. It is assumed that no mistakes exist, and that the observations have already been corrected for all systematic errors. Compute the most probable value for the line length, its standard deviation, and errors having 50%, 90%, and 95% probability.

Length (ft)(1)	**Residual ν (ft)(2)**	**ν^2 (3)**
538.57	+0.12	0.0144
538.39	−0.06	0.0036
538.37	−0.08	0.0064
538.39	−0.06	0.0036
538.48	+0.03	0.0009
538.49	+0.04	0.0016
538.33	−0.12	0.0144
538.46	+0.01	0.0001
538.47	+0.02	0.0004
538.55	+0.10	0.0100
$\Sigma = 5384.50$	$\Sigma = 0.00$	$\Sigma\nu^2 = 0.0554$

Solution

By Equation (3.2), $\overline{M} = \dfrac{5384.50}{10} = 538.45 \text{ ft}$

By Equation (3.3), the residuals are calculated. These are tabulated in column (2) and their squares listed in column (3). Note that in column (2) the algebraic sum of residuals is zero. (For observations of equal reliability, except for round off, this column should always total zero and thus provide a computational check.)

By Equation (3.4), $\sigma = \pm\sqrt{\dfrac{\Sigma\nu^2}{n-1}} = \sqrt{\dfrac{0.0554}{9}} = \pm 0.078 = \pm 0.08 \text{ ft}.$

By Equation (3.6), $E_{50} = \pm 0.6745\sigma = \pm 0.6745(0.078) = \pm 0.05 \text{ ft}.$

By Equation (3.7), $E_{95} = \pm 1.6449(0.078) = \pm 0.13 \text{ ft}.$

By Equation (3.8), $E_{99} = \pm 1.9599(0.078) = \pm 0.15 \text{ ft}.$

The following conclusions can be drawn concerning this example.

1. The most probable line length is 538.45 ft.
2. The standard deviation of a single observation is ± 0.08 ft. Accordingly, the normal expectation is that 68% of the time a recorded length will lie between $538.45 - 0.08$ and $538.45 + 0.08$ or between 538.37 and 538.53 ft; that is, about seven values should lie within these limits. (Actually seven of them do.)
3. The probable error (E_{50}) is ± 0.05 ft. Therefore, it can be anticipated that half, or five of the observations, will fall in the interval 538.40 to 538.50 ft. (Four values do.)
4. The 90% error is ± 0.13 ft, and thus nine of the observed values can be expected to be within the range of 538.32 and 538.58 ft.
5. The 95% error is ± 0.15 ft, so the length can be expected to lie between 538.30 and 538.60, 95% of the time. (Note that all observations indeed are within the limits of both the 90 and 95 percent errors.)

3.17 ERROR PROPAGATION

It was stated earlier that because all observations contain errors, any quantities computed from them will likewise contain errors. The process of evaluating errors in quantities computed from observed values that contain errors is called *error propagation*. The propagation of random errors in mathematical formulas can be computed using the general law of the propagation of variances. Typically in surveying (geomatics), this formula can be simplified since the observations are usually mathematically independent. For example, let $a, b, c, \ldots, n$ be observed values containing errors $E_a, E_b, E_c, \ldots, E_n$, respectively. Also let Z be a quantity derived by computation using these observed quantities in a function f, such that

$$Z = f(a, b, c, \ldots, n) \tag{3.9}$$

Then assuming that $a, b, c, \ldots, n$ are independent observations, the error in the computed quantity Z is

$$E_Z = \pm\sqrt{\left(\frac{\partial f}{\partial a}E_a\right)^2 + \left(\frac{\partial f}{\partial b}E_b\right)^2 + \left(\frac{\partial f}{\partial c}E_c\right)^2 + \cdots + \left(\frac{\partial f}{\partial n}E_n\right)^2} \tag{3.10}$$

where the terms $\partial f/\partial a, \partial f/\partial b, \partial f/\partial c, \ldots, \partial f/\partial n$ are the partial derivatives of the function f with respect to the variables $a, b, c, \ldots, n$. In the subsections that follow, specific cases of error propagation common in surveying are discussed, and examples are presented.

3.17.1 Error of a Sum

Assume the sum of independently observed observations $a, b, c, \ldots$ is Z. The formula for the computed quantity Z is

$$Z = a + b + c + \cdots$$

The partial derivatives of Z with respect to each observed quantity are $\partial Z/\partial a = \partial Z/\partial b = \partial Z/\partial c = \cdots = 1$. Substituting these partial derivatives into Equation (3.10), the following formula is obtained, which gives the propagated error in the sum of quantities, each of which contains a different random error:

$$E_{Sum} = \pm\sqrt{E_a^2 + E_b^2 + E_c^2 + \cdots} \qquad \textbf{(3.11)}$$

where E represents any specified percentage error (such as σ, E_{50}, E_{90}, or E_{95}), and *a, b*, and *c* are the separate, independent observations.

The error of a sum can be used to explain the rules for addition and subtraction using significant figures. Recall the addition of 46.7418, 1.03, and 375.0 from Example (**a**) from Section 2.4. Significant figures indicate that there is uncertainty in the last digit of each number. Thus, assume estimated errors of ±0.0001, ±0.01, and ±0.1 respectively for each number. The error in the sum of these three numbers is $\sqrt{0.0001^2 + 0.01^2 + 0.1^2} = \pm 0.1$. The sum of three numbers is 422.7718, which was rounded, using the rules of significant figures, to 422.8. Its precision matches the estimated accuracy produced by the error in the sum of the three numbers. Note how the least accurate number controls the accuracy in the summation of the three values.

■ Example 3.2

Assume that a line is observed in three sections, with the individual parts equal to (753.81, ±0.012), (1238.40, ±0.028), and (1062.95, ±0.020) ft, respectively. Determine the line's total length and its anticipated standard deviation.

Solution

Total length = 753.81 + 1238.40 + 1062.95 = 3055.16 ft.

By Equation (3.11), $E_{Sum} = \pm\sqrt{0.012^2 + 0.028^2 + 0.020^2} = \pm 0.036$ ft

3.17.2 Error of a Series

Sometimes a series of similar quantities, such as the angles within a closed polygon, are read with each observation being in error by about the same amount. The total error in the sum of all observed quantities of such a series is called the *error of the series*, designated as E_{Series}. If the same error E in each observation is assumed and Equation (3.11) applied, the series error is

$$E_{Series} = \pm\sqrt{E^2 + E^2 + E^2 + \cdots} = \pm\sqrt{nE^2} = \pm E\sqrt{n} \qquad \textbf{(3.12)}$$

where E represents the error in each individual observation and n the number of observations.

This equation shows that when the same operation is repeated, random errors tend to balance out and the resulting error of a series is proportional to the square root of the number of observations. This equation has extensive use—for instance, to determine the allowable misclosure error for angles of a traverse, as discussed in Chapter 9.

Example 3.3

Assume that any distance of 100 ft can be taped with an error of ±0.02 ft if certain techniques are employed. Determine the error in taping 5000 ft using these skills.

Solution

Since the number of 100 ft lengths in 5000 ft is 50 then by Equation (3.12)

$$E_{Series} = \pm E\sqrt{n} = \pm 0.02\sqrt{50} = \pm 0.14 \text{ ft}$$

Example 3.4

A distance of 1000 ft is to be taped with an error of not more than ±0.10 ft. Determine how accurately each 100 ft length must be observed to ensure that the error will not exceed the permissible limit.

Solution

Since by Equation (3.12), $E_{Series} = \pm E\sqrt{n}$ and $n = 10$, the allowable error E in 100 ft is

$$E = \pm\frac{E_{Series}}{\sqrt{n}} \pm \frac{0.10}{\sqrt{10}} = \pm 0.03 \text{ ft}$$

Example 3.5

Suppose it is required to tape a length of 2500 ft with an error of not more than ±0.10 ft. How accurately must each tape length be observed?

Solution

Since 100 ft is again considered the unit length, $n = 25$, and by Equation (3.12), the allowable error E in 100 ft is

$$E = \pm\frac{0.10}{\sqrt{25}} = \pm 0.02 \text{ ft}$$

Analyzing Examples 3.4 and 3.5 shows that the larger the number of possibilities, the greater the chance for errors to cancel out.

3.17.3 Error of a Product

The equation for propagated AB, where E_a and E_b are the respective errors in A and B, is

$$E_{prod} = \pm\sqrt{A^2E_b^2 + B^2E_a^2} \tag{3.13}$$

The physical significance of the error propagation formula for a product is illustrated in Figure 3.6, where A and B are shown to be observed sides of a rectangular parcel of land with errors E_a and E_b respectively. The product AB is the parcel area. In Equation (3.13), $\sqrt{A^2E_b^2} = AE_b$ represents either of the longer (horizontal) crosshatched bars and is the error caused by either $-E_b$ or $+E_b$. The term $\sqrt{B^2E_a^2} = BE_a$ is represented by the shorter (vertical) crosshatched bars, which is the error resulting from either $-E_a$ or $+E_a$.

Example 3.6

For the rectangular lot illustrated in Figure 3.6, observations of sides A and B with their 95% errors are (252.46, ±0.053) and (605.08, ±0.072) ft, respectively. Calculate the parcel area and the expected 95% error in the area.

Solution

Area = 252.46 × 605.08 = 152,760 ft^2

By Equation (3.13),

$$E_{95} = \pm\sqrt{(252.46)^2(0.072)^2 + (605.08)^2(0.053)^2} = \pm 36.9 \text{ ft}^2$$

Example 3.6 can also be used to demonstrate the validity of one of the rules of significant figures in computation. The computed area is actually 152,758.4968 ft^2. However, the rule for significant figures in multiplication (see Section 2.4) states that there cannot be more significant figures in the answer

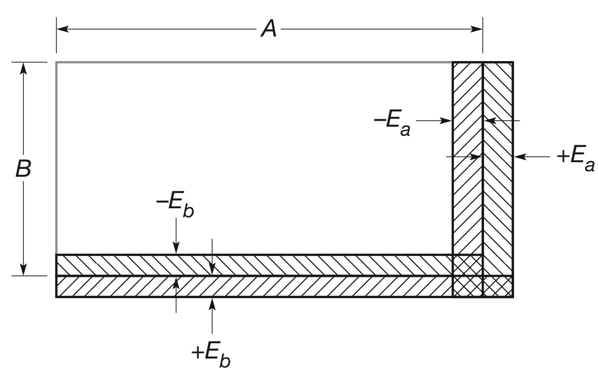


Figure 3.6
Error of area.

than in any of the individual factors used. Accordingly, the area should be rounded off to 152,760 (five significant figures). From Equation (3.13), with an error of ± 36.9 ft^2, the answer could be 152,758.4968 $\pm$ 36.9, or from 152,721.6 to 152,795.4 ft^2. Thus, the fifth digit in the answer is seen to be questionable, and hence the number of significant figures specified by the rule is verified.

3.17.4 Error of the Mean

Equation (3.2) stated that the most probable value of a group of repeated observations of equal weight is the arithmetic mean. Since the mean is computed from individual observed values, each of which contains an error, the mean is also subject to error. By applying Equation (3.12), it is possible to find the error for the sum of a series of observations where each one has the same error. Since the sum divided by the number of observations gives the mean, the error of the mean is found by the relation

$$E_m = \frac{E_{series}}{n}$$

Substituting Equation (3.12) for E_{series}

$$E_m = \frac{E\sqrt{n}}{n} = \frac{E}{\sqrt{n}} \tag{3.14}$$

where E is the specified percentage error of a single observation, E_m the corresponding percentage error of the mean, and n the number of observations.

The error of the mean at any percentage probability can be determined and applied to all criteria that have been developed. For example, the standard deviation of the mean, $(E_{68})_m$ or σ_m is

$$(E_{68})_m = \sigma_m = \frac{\sigma}{\sqrt{n}} = \pm\sqrt{\frac{\Sigma \nu^2}{n(n-1)}} \tag{3.15a}$$

and the 90 and 95 percent errors of the mean are

$$(E_{90})_m = \frac{E_{90}}{\sqrt{n}} = \pm 1.6449\sqrt{\frac{\Sigma \nu^2}{n(n-1)}} \tag{3.15b}$$

$$(E_{95})_m = \frac{E_{95}}{\sqrt{n}} = \pm 1.9599\sqrt{\frac{\Sigma \nu^2}{n(n-1)}}. \tag{3.15c}$$

These equations show that *the error of the mean varies inversely as the square root of the number of repetitions*. Thus, to double the accuracy—that is, to reduce the error by one half—four times as many observations must be made.

Example 3.7

Calculate the standard deviation of the mean and the 90% error of the mean for the observations of Example 3.1.

Solution

By Equation (3.15a), $\sigma_m = \dfrac{\sigma}{\sqrt{n}} = \pm\dfrac{0.078}{\sqrt{10}} = \pm 0.025 \text{ ft}$

Also, by Equation (3.15b), $(E_{90})_m = \pm 1.6449\,(0.025) = \pm 0.041 \text{ ft}$

These values show the error limits of 68% and 90% probability for the line's length. It can be said that the true line length has a 68% chance of being within ±0.025 of the mean, and a 90% likelihood of falling not farther than ±0.041 ft from the mean.

3.18 APPLICATIONS

The preceding example problems show that the equations of error probability are applied in two ways:

1. To analyze observations already made, for comparison with other results or with specification requirements.
2. To establish procedures and specifications in order that the required results will be obtained.

The application of the various error probability equations must be tempered with judgment and caution. Recall that they are based on the assumption that the errors conform to a smooth and continuous normal distribution curve, which in turn is based on the assumption of a large number of observations. Frequently in surveying only a few observations—often from two to eight—are taken. If these conform to a normal distribution, then the answer obtained using probability equations will be reliable; if they do not, the conclusions could be misleading. In the absence of knowledge to the contrary, however, an assumption that the errors are normally distributed is still the best available.

3.19 CONDITIONAL ADJUSTMENT OF OBSERVATIONS

In Section 3.3, it was emphasized that the true value of any observed quantity is never known. However, in some types of problems, the sum of several observations must equal a fixed value; for example, the sum of the three angles in a plane triangle has to total 180°. In practice, therefore, the observed angles are adjusted to make them add to the required amount. Correspondingly, distances—either horizontal or vertical—must often be adjusted to meet certain conditional requirements. The methods used will be explained in later chapters, where the operations are taken up in detail.

3.20 WEIGHTS OF OBSERVATIONS

It is evident that some observations are more precise than others because of better equipment, improved techniques, and superior field conditions. In making adjustments, it is consequently desirable to assign *relative weights* to individual observations. It can logically be concluded that if an observation is very precise, it will have a small standard deviation or variance, and thus should be weighted more heavily (held closer to its observed value) in an adjustment than an observation of lower precision. From this reasoning, it is deduced that weights of observations should bear an inverse relationship to precision. In fact, it can be shown that relative weights are inversely proportional to variances, or

$$W_a \propto \frac{1}{\sigma_a^2} \qquad \textbf{(3.16)}$$

where W_a is the weight of an observation a, which has a variance of σ_a^2. Thus, the higher the precision (the smaller the variance), the larger should be the relative weight of the observed value being adjusted. In some cases, variances are unknown originally, and weights must be assigned to observed values based on estimates of their relative precision. If a quantity is observed repeatedly and the individual observations have varying weights, the weighted mean can be computed from the expression

$$\overline{M}_W = \frac{\Sigma WM}{\Sigma W} \qquad \textbf{(3.17)}$$

where $\overline{M}_W$ is the weighted mean, ΣWM the sum of the individual weights times their corresponding observations, and ΣW the sum of the weights.

Example 3.8

Suppose four observations of a distance are recorded as 482.16, 482.17, 482.20, and 482.18 and given weights of 1, 2, 2, and 4, respectively, by the surveyor. Determine the weighted mean.

Solution

By Equation (3.17)

$$\overline{M}_W = \frac{482.16 + 482.17(2) + 482.20(2) + 482.14(4)}{1 + 2 + 2 + 4} = 482.16 \text{ ft}$$

In computing adjustments involving unequally weighted observations, corrections applied to observed values should be made *inversely proportional to the relative weights*.

Example 3.9

Assume the observed angles of a certain plane triangle, and their relative weights, are $A = 49°51'15''$, $W_a = 1$; $B = 60°32'08''$, $W_b = 2$; and $C = 69°36'33''$, $W_c = 3$. Compute the weighted mean of the angles.

Solution

The sum of the three angles is computed first and found to be 4″ less than the required geometrical condition of exactly 180°. The angles are therefore adjusted in inverse proportion to their relative weights, as illustrated in the accompanying tabulation. Angle *C* with the greatest weight (3) gets the smallest correction, $2x$; *B* receives $3x$; and *A*, $6x$.

	Observed Angle	Wt	Correction	Numerical Corr.	Rounded Corr.	Adjusted Angle
A	49°51′15″	1	$6x$	+2.18″	+2″	49°51′17″
B	60°32′08″	2	$3x$	+1.09″	+1″	60°32′09″
C	69°36′33″	3	$2x$	+0.73″	+1″	69°36′34″
Sum	179°59′56″	$\Sigma = 6$	$11x$	+4.00″	+4″	180°00′00″

$11x = 4''$ and $x = +0.36''$

It must be emphasized again that adjustment computations based on the theory of probability are valid only if systematic errors and employing proper procedures, equipment, and calculations eliminates mistakes.

3.21 LEAST-SQUARES ADJUSTMENT

As explained in Section 3.19, most surveying observations must conform to certain geometrical conditions. The amounts by which they fail to meet these conditions are called misclosures, and they indicate the presence of random errors. In Example 3.9, for example, the misclosure was 4″. Various procedures are used to distribute these misclosure errors to produce mathematically perfect geometrical conditions. Some simply apply corrections of the same size to all observed values, where each correction equals the total misclosure (with its algebraic sign changed), divided by the number of observations. Others introduce corrections in proportion to assigned weights. Still others employ rules of thumb, for example, the "compass rule" described in Chapter 10 for adjusting closed traverses.

Because random errors in surveying conform to the mathematical laws of probability and are "normally distributed," the most appropriate adjustment procedure should be based upon these laws. Least squares is such a method. It is not a new procedure, having been applied by the German mathematician Karl Gauss as early as the latter part of the 18th century. However, until the advent of computers, it was only used sparingly because of the lengthy calculations involved.

Least squares is suitable for adjusting any of the basic types of surveying observations described in Section 2.1, and is applicable to all of the commonly used surveying procedures. The method enforces the condition that *the sum of the weights of the observations times their corresponding squared residuals is minimized*. This fundamental condition, which is developed from the equation for the normal error distribution curve, provides most probable values for the adjusted quantities. In addition, it also (a) enables the computation of precisions of the adjusted values, (b) reveals the presence of mistakes so steps can be taken to eliminate them, and (c) makes possible the optimum design of survey procedures in the office before going to the field to take observations.

The basic assumptions that underlie least-squares theory are as follows: (1) mistakes and systematic errors have been eliminated so only random errors remain; (2) the number of observations being adjusted is large; and (3) the frequency distribution of errors is normal. Although these assumptions are not always met, the least-squares adjustment method still provides the most rigorous error treatment available, and hence it has become very popular and important in modern surveying. A more detailed discussion of the subject is presented in Chapter 16.

■ 3.22 USING SOFTWARE

Computations such as those in Table 3.1 can be long and tedious. Fortunately, spreadsheet software often has the capability of computing the mean and standard deviation of a group of observations. For example, in Microsoft Excel®, the mean of a set of observations can be determined using the average() function and the standard deviation can be determined using the stdev() function. Similarly, histograms of data can also be plotted once the data is organized into classes. The reader can download all of the Excel files for this book by downloading the file *Excel Spreadsheets.zip* from the companion website for this book at http://www.pearsonhighered.com/ghilani. The spreadsheet *c3.xls* demonstrates the use of the functions mentioned previously and also demonstrates the use of a spreadsheet to solve the example problems in this chapter. Also on the companion website for this book is the software STATS. This software can read a text file of data and compute the statistics demonstrated in this chapter. Furthermore, STATS will histogram the data using a user-specified number of class intervals. The help file that accompanies this software describes the file format for the data and the use of the software. For those having the software Mathcad® version 14.0 or higher, an accompanying e-book is available on the companion website. This e-book is in the file *Mathcad files.zip* on the companion website. If this book is decompressed in the Mathcad subdirectory *handbook*, the e-book will be available in the Mathcad help system. This e-book can also be accessed by selecting the file *elemsurv.hbk* in your Windows directory and has a worksheet that demonstrates the examples presented in this chapter. For those who do have Mathcad version 14.0 or higher, a set of hypertext markup language (html) files of the e-book are available on the companion website. These files can be accessed by opening the file *index.html* in your browser.

PROBLEMS

Asterisks (*) indicate problems that have partial answers given in Appendix G.

3.1 Explain the difference between *direct* and *indirect measurements* in surveying. Give two examples of each.

3.2 Define the term *systematic error*, and give two surveying examples of a systematic error.

3.3 Define the term *random error*, and give two surveying examples of a random error.

3.4 Explain the difference between accuracy and precision.

3.5 Discuss what is meant by the precision of an observation.

A distance AB is observed repeatedly using the same equipment and procedures, and the results, in meters, are listed in Problems 3.6 through 3.10. Calculate **(a)** the line's most probable length, **(b)** the standard deviation, and **(c)** the standard deviation of the mean for each set of results.

3.6* 65.401, 65.400, 65.402, 65.396, 65.406, 65.401, 65.396, 65.401, 65.405, and 65.404

3.7 Same as Problem 3.6, but discard one observation, 65.396.

3.8 Same as Problem 3.6, but discard two observations, 65.396 and 65.406.

3.9 Same as Problem 3.6, but include two additional observations, 65.398 and 65.408.

3.10 Same as Problem 3.6, but include three additional observations, 65.398, 65.408, and 65.406.

In Problems 3.11 through 3.14, determine the range within which observations should fall **(a)** 90% of the time and **(b)** 95% of the time. List the percentage of values that actually fall within these ranges.

3.11* For the data of Problem 3.6.

3.12 For the data of Problem 3.7.

3.13 For the data of Problem 3.8.

3.14 For the data of Problem 3.9.

In Problems 3.15 through 3.17, an angle is observed repeatedly using the same equipment and procedures. Calculate **(a)** the angle's most probable value, **(b)** the standard deviation, and **(c)** the standard deviation of the mean.

3.15* 23°30′00″, 23°29′40″, 23°30′15″, and 23°29′50″.

3.16 Same as Problem 3.15, but with three additional observations, 23°29′55″, 23°30′05″, and 23°30′20″.

3.17 Same as Problem 3.16, but with two additional observations, 23°30′05″ and 23°29′55″.

3.18* A field party is capable of making taping observations with a standard deviation of ±0.010 ft per 100 ft tape length. What standard deviation would be expected in a distance of 200 ft taped by this party?

3.19 Repeat Problem 3.18, except that the standard deviation per 30-m tape length is ±0.003 m and a distance of 120 m is taped. What is the expected 95% error in 120 m?

3.20 A distance of 200 ft must be taped in a manner to ensure a standard deviation smaller than ±0.04 ft. What must be the standard deviation per 100 ft tape length to achieve the desired precision?

3.21 Lines of levels were run requiring n instrument setups. If the rod reading for each backsight and foresight has a standard deviation σ, what is the standard deviation in each of the following level lines?

(a) $n = 26, \sigma = \pm 0.010$ ft

(b) $n = 36, \sigma = \pm 3$ mm

3.22 A line AC was observed in 2 sections AB and BC, with lengths and standard deviations listed below. What is the total length AC, and its standard deviation?
*(a) $AB = 60.00 \pm 0.015$ ft; $BC = 86.13 \pm 0.018$ ft
(b) $AB = 60.000 \pm 0.008$ m; 35.413 ± 0.005 m

3.23 Line AD is observed in three sections, AB, BC, and CD, with lengths and standard deviations as listed below. What is the total length AD and its standard deviation?
(a) $AB = 572.12 \pm 0.02$ ft; $BC = 1074.38 \pm 0.03$ ft; $CD = 1542.78 \pm 0.05$ ft
(b) $AB = 932.965 \pm 0.009$ m; $BC = 945.030$ m ± 0.010 m; $CD = 652.250$ m ± 0.008 m

3.24 A distance AB was observed four times as 236.39, 236.40, 236.36, and 236.38 ft. The observations were given weights of 2, 1, 3, and 2, respectively, by the observer. *(a) Calculate the weighted mean for distance AB. **(b)** What difference results if later judgment revises the weights to 2, 1, 2, and 3, respectively?

3.25 Determine the weighted mean for the following angles:
(a) 89°42′45″, wt 2; 89°42′42″, wt 1; 89°42′44″, wt 3
(b) 36°58′32″ ± 3″; 36°58′28″ ± 2″; 36°58′26″ ± 3″; 36°58′30″ ± 1″

3.26 Specifications for observing angles of an n-sided polygon limit the total angular misclosure to E. How accurately must each angle be observed for the following values of n and E?
(a) $n = 10$, $E = 8''$
(b) $n = 6$, $E = 14''$

3.27 What is the area of a rectangular field and its estimated error for the following recorded values:
*(a) 243.89 ± 0.05 ft, by 208.65 ± 0.04 ft
(b) 725.33 ± 0.08 ft by 664.21 ± 0.06 ft
(c) 128.526 ± 0.005 m, by 180.403 ± 0.007 m

3.28 Adjust the angles of triangle ABC for the following angular values and weights:
*(a) $A = 49°24'22''$, wt 2; $B = 39°02'16''$, wt 1; $C = 91°33'00''$, wt 3
(b) $A = 80°14'04''$, wt 2; $B = 38°37'47''$, wt 1; $C = 61°07'58''$, wt 3

3.29 Determine relative weights and perform a weighted adjustment (to the nearest second) for angles A, B, and C of a plane triangle, given the following four observations for each angle:

Angle A	Angle B	Angle C
38°47′58″	71°22′26″	69°50′04″
38°47′44″	71°22′22″	69°50′16″
38°48′12″	71°22′12″	69°50′30″
38°48′02″	71°22′12″	69°50′10″

3.30 A line of levels was run from benchmarks A to B, B to C, and C to D. The elevation differences obtained between benchmarks, with their standard deviations, are listed below. What is the difference in elevation from benchmark A to D and the standard deviation of that elevation difference?
(a) BM A to BM B = +34.65 ± 0.10 ft; BM B to BM C = −48.23 ± 0.08 ft; and BM C to BM D = −54.90 ± 0.09 ft
(b) BM A to BM B = +27.823 ± 0.015 m; BM B to BM C = +15.620 ± 0.008 m; and BM C to BM D = +33.210 ± 0.011 m
(c) BM A to BM B = −32.688 ± 0.015 m; BM B to BM C = +5.349 ± 0.022 m; and BM C to BM D = −15.608 ± 0.006 m

3.31 Create a computational program that solves Problem 3.9.
3.32 Create a computational program that solves Problem 3.17.
3.33 Create a computational program that solves Problem 3.29.

BIBLIOGRAPHY

Alder, K. 2002. *The Measure of All Things—The Seven-Year Odyssey and Hidden Error that Transformed the World*. New York: The Free Press.

Bell, J. 2001. "Hands On: TDS for Windows CE On the Ranger." *Professional Surveyor* 21 (No. 1): 33.

Buckner, R. B. 1997. "The Nature of Measurements: Part I—The Inexactness of Measurement—Counting vs. Measuring." *Professional Surveyor* 17 (No. 2).

________. 1997. "The Nature of Measurements: Part II—Mistakes and Errors." *Professional Surveyor* 17 (No. 3).

________. 1997. "The Nature of Measurements: Part III—Dealing With Errors." *Professional Surveyor* 17 (No. 4).

________. 1997. "The Nature of Measurements: Part IV—Precision and Accuracy." *Professional Surveyor* 17 (No. 5).

________. 1997. "The Nature of Measurements: Part V—On Property Corners and Measurement Science." *Professional Surveyor* 17 (No. 6).

________. 1997. "The Nature of Measurement: Part 6—Level of Certainty." *Professional Surveyor* 17 (No. 8).

________. 1998. "The Nature of Measurements: Part 7—Significant Figures in Measurements." *Professional Surveyor* 18 (No. 2).

________. 1998. "The Nature of Measurements: Part 8—Basic Statistical Analysis of Random Errors." *Professional Surveyor* 18 (No. 3).

Cummock, M. and G. Wagstaff. 1999. "Part 1: Measurements—A Roll of the Dice." *Point of Beginning* 24 (No. 6): 34.

Foster. R. 2003. "Uncertainty about Positional Uncertainty." *Point of Beginning* 28 (No. 11): 40.

Ghilani, C. D. and P. R. Wolf. 2010. *Adjustment Computations: Spatial Data Analysis*. New York: Wiley.

Ghilani, C. D. 2003. "Statistics and Adjustments Explained Part 1: Basic Concepts." *Surveying and Land Information Science* 63 (No. 2): 62.

________. 2003. "Statistics and Adjustments Explained Part 2: Sample Sets and Reliability." *Surveying and Land Information Science* 63 (No. 3): 141.

Uotila, U. A. 2006. "Useful Statistics for Land Surveyors." *Surveying and Land Information Science* 66 (No. 1): 7.