Ch 1 World of Foundations

Sunday, August 18, 2024 12:45 PM

Reading Assignment

- Course Information
- Ch. 1 Lecture Notes
- Sections 1.1 to 1.5 (Salgado)

Other Materials

- Introduction to Foundations Engineering (Power point Presentation)

Homework Assignment 1

- Problems 1-10 through 1-15 (Salgado)

Ch 1 World of Foundations - Learning Objectives

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1. Know and describe the branches of geotechnical engineering.

2. Know and describe other fields related to geotechnical engineering.

3. Know and understand the term: heterogeneous, anisotropic, nonconservative (i.e., inelastic) and nonlinear and how these terms are related to soils.

4. Understand how defects in the soil or rock (e.g., joints, fractures, weak layers and zones, etc.) can affect the behavior of the soil or rock and may lead to unacceptable performance.

5. Know and describe an example where such defects have led to a failure condition.

6. Understand the knowledge that is required to practice geotechnical engineering.

7. Know ways that you can develop/cultivate engineering judgment.

8. Understand the professional etiquette that will help may you a successful engineer.

Ch 1 Homework Assignment

Wednesday, August 17, 2011

1.7.2 Quantitative problems

Problem 1.10 Find the value of a pressure of 180 kPa in MPa, kgf/cm², tons per square foot (tsf), kilo-pounds per square foot (ksf), and pounds per square foot (psf).

Problem 1.11 Water has unit weight of 62.4 pcf. Starting from this number, obtain the unit weight of water in kN/m³.

Problem 1.12 A clay has unit weight of 15 kN/m³. What is its unit weight in pcf?

Problem 1.13 A load of 300 kN is applied on a square foundation element with side $B = 2$ m. Assuming a construction tolerance of 5 cm for the sides of the foundation element, what is the range of the average pressure acting on the base of the element?

Problem 1.14 The small-strain shear modulus of a certain sand is given by

$$G_0 = 600\frac{(\cancel{29} - e)^2}{1+e}\sigma_m'^{0.5} \qquad 2.9$$

for both G_0 and σ_m' in tsf. Find the equivalent dimensionless equation.

Problem 1.15 Calculate G_0 with the correct number of significant figures using the equation obtained in Problem 1.14 for a soil with $e = 0.59$ and $\sigma_m' = 350$ kPa.

System of Units

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Table 1-1 Absolute or LMT system

Quantity	US units	SI	CGS
L (length)	ft	m	cm
M (mass)	lb	kg	g
T (time)	s	s	s
F (force)	poundal	Newton (N)	dyne
g_c	1 (ft/s^2) (lb/poundal)	1 (m/s^2) (kg/N)	1 (cm/s^2)(g/dyne)

Table 1-4 Dimensions of the Quantities of Mechanics

Quantity	LMT system	LFT system
Length	L	L
Mass	M	$F\,L^{-1}\,T^2$
Time	T	T
Temperature	Q	Q
Force	$M\,L\,T^{-2}$	F
Mass density	$M\,L^{-3}$	$F\,L^{-4}\,T^2$
Unit weight	$M\,L^{-2}\,T^{-2}$	$F\,L^{-3}$
Stress	$M\,L^{-1}\,T^{-2}$	$F\,L^{-2}$
Velocity	$L\,T^{-1}$	$L\,T^{-1}$
Acceleration	$L\,T^{-2}$	$L\,T^{-2}$
Volumetric flow rate	$L^3\,T^{-1}$	$L^3\,T^{-1}$
Angle	dimensionless	dimensionless
Angular velocity	T^{-1}	T^{-1}
Angular acceleration	T^{-2}	T^{-2}
Work, energy	$M\,L^2\,T^{-2}$	$F\,L$
Power	$M\,L^2\,T^{-3}$	$F\,L\,T^{-1}$
Moment of force	$M\,L^2\,T^{-2}$	$F\,L$
Dynamic viscosity	$M\,L^{-1}\,T^{-1}$	$F\,L^{-2}\,T$
Kinematic viscosity	$L^2\,T^{-1}$	$L^2\,T^{-1}$
Surface tension	$M\,T^{-2}$	$F\,L^{-1}$

Useful conversions (http://www.onlineconversion.com)

1 kilonewton = 224.808 943 87 pound-force
100 kPa ~ 1 tsf = 2000 psf
1 m = 3.28 feet
1 inch = 2.54 cm
62.4 lb/ft^3 = 1 Mg / m^3 (unit weight of water)
1 kilogram = 2.2046226218 pound
1 kip = 1000 lbs

Unit Conversions

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Force

1 N = 0.2248089431 lbs

Stress or Pressure

	Pascal (Pa)	Bar (bar)	Technical atmosphere (at)	Atmosphere (atm)	Torr (Torr)	Pound-force per square inch (psi)
1 Pa	≡ 1 N/m^2	10^{-5}	1.0197×10^{-5}	9.8692×10^{-6}	7.5006×10^{-3}	145.04×10^{-6}
1 bar	10^5	≡ 10^6 dyn/cm^2	1.0197	0.98692	750.06	14.5037744
1 at	0.980665×10^5	0.980665	≡ 1 kgf/cm^2	0.96784	735.56	14.223
1 atm	1.01325×10^5	1.01325	1.0332	≡ 1 atm	760	14.696
1 Torr	133.322	1.3332×10^{-3}	1.3595×10^{-3}	1.3158×10^{-3}	≡ 1 Torr; ≈ 1 mmHg	19.337×10^{-3}
1 psi	6.895×10^3	68.948×10^{-3}	70.307×10^{-3}	68.046×10^{-3}	51.715	≡ 1 lbf/in^2

Pressure units

Pasted from <http://en.wikipedia.org/wiki/Technical_atmosphere>

Acceleration

1 g = 9.80665 m/s^2 or 35.30394 (km/h)/s (~32.174 ft/s^2)

For this course, it is sufficient to use 9.81 m/s^2 and 32.2ft/s^2)

Pasted from <http://en.wikipedia.org/wiki/Standard_gravity>

Unit Weight of Water

γ_W = 62.4 lb/ft^3 = 9.81 kN/m^3

Mass Density of Water

ρ_W = 1.940 slug/ft^3 = 1000 kg/m^3 or 1 Mg/m^3

Dimensionless Equations

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Dimensionless equations are used throughout the text. This practice avoids the awkward alternative of using equations specifically derived for a certain set of units. When unit-dependent equations are used and results are desired in a different set of units, the quantities in the equation need first to be converted to the desired units, which is often a source of error, confusion, or a waste of time.

All physical equations express relationships between certain quantities that should be fully independent of the way we decide to measure them. In order for this to be true, the expression relating the quantities must be dimensionally compatible or homogeneous. The dimensions on both sides of an equation must be the same, in terms of the fundamental dimensions.

- In a dimensionless equation, there is one or more variables that assume different values depending on the units in use.
- Try to write engineering equations in dimensionless form, whenever possible.
- The equation can be written in a dimensionless form by replacing the units in the equation with reference units.
    - Reference unit for stress = atmospheric pressure (p_a)
    - Reference unit for acceleration = gravitational constant (g)
    - Reference unit for unit weight = unit weight of water (γ_w)

Dimensionless Equations (cont.)

Thursday, August 31, 2017 2:27 PM

Example

The standard penetration blow count and the cone tip resistance can be related to each other for a clean sand by the following equation:

$N = q_{c\ (tsf)} / 4.5$

where: N = standard penetration in blows per foot, $q_{c\ tsf}$ = cone penetrometer tip resistance (tsf).

Note that the above equation is not dimensionless, because one of its variables, q_c is expressed in tsf (tons per square foot).

The unit tsf is a stress unit. The reference variable for stress is atmospheric pressure, which is abbreviated as p_A in our text book

The **standard atmosphere** (symbol: **atm = p_A**) is an international reference pressure defined as 101,325 Pa and formerly used as unit of pressure.[1] For practical purposes it has been replaced by the bar which is 100,000 Pa.[1] The difference of about 1% is not significant for many applications, and is within the error range of common pressure gauges. Pasted from <http://en.wikipedia.org/wiki/Atmosphere_(unit)>

Thus, the equation above can be written in it dimensionless form as:

$N = q_c$ * tons / ft^2 / 4.5 * (2000 lb / ton) * (ft^2 / 144 in^2) * (1 atm / 14.696 lb/in^2)

$N = q_c / p_A * 1/4.5 * 0.94507937$

$N = 0.21 * q_c / p_A$

Fields Related to Geotechnical Engineering

Thursday, March 11, 2010 11:43 AM

1. ***Geology*** is the study of the earth and other nearby planets. It is concerned with the materials that makeup the planet, the physical and chemical process that create and change these materials with time, and the history of the planet and the life that has formed and evolved.

2. ***Geophysics*** is a branch of experimental physics dealing with the earth, including it atmosphere and hydrosphere. It includes the sciences of dynamical geology and physical geography, and make use of geodesy, geology, seismology, meteorology, oceanography, magnetism, and other earth sciences in collecting and interpreting earth data. Applied geophysics applies methods of physics and engineering exploration by observation of seismic or electrical phenomena or of the earth's gravitational or magnetic fields or thermal distribution.

3. ***Geological Engineering / Engineering Geology*** are the application of the earth sciences to engineering practice for the purpose of assuring that the geologic factors affecting the location, design, construction, operation, and maintenance of engineering works are recognized and adequately addressed.

4. ***Seismology*** a geophysical science which is concerned with the study of earthquakes and how earthquake wave propagate through the earth and the measurement of the elastic properties of the earth.

5. ***Geoenvironmental Engineering*** a branch of civil/geotechnical engineering. Environmental concerns in relation to groundwater and waste disposal have spawned a new area of study called geoenvironmental engineering where biology and chemistry are important. This branch deals with waste contamination, clean-up, containment systems, etc.

Knowledge Req'd to Practice Geotechnical Engineering

Thursday, March 11, 2010 11:43 AM

(From "Application of Soil Mechanics in Practice" by Ralph Peck)

A. The **first area** of required knowledge is the **theoretical and experimental tools** that are often regarded as **soil mechanics proper**. Although the instances may be few in which elaborate theoretical calculations are justified, or in which elaborate testing programs of soil samples may be useful, the insight and judgment arising from an intimate knowledge of these matters cannot be overemphasized. *In spite of the fact that some of the more experienced practitioners of soil mechanics may rarely make a theoretical calculation, unconsciously they bring to focus on many a problem the fruit of years of theoretical studies and investigations that subsequently become an integral part of the engineering background.*

B. The **second foundation** of soil mechanics is **experience and judgment**. The *traditional knowledge of our predecessors, as well as a thorough knowledge of design and construction procedures and their consequences*, are utterly indispensable for successful practice.

1. **Empirical basis of judgment** - There was *a time when all engineering judgment was empirical*. Before the injection of science into engineering, the test of a design was often precedent. The builders of the great Gothic cathedrals were ignorant of stress analysis. There is considerable evidence that they consulted with the local designers and builders.

   No engineer can design successfully if he is not aware of what is practical to accomplish with the tools and equipment available at the time and place of his project. He needs detailed knowledge of what has to be done so that he can appreciate whether his proposed enterprise fall routinely among projects for which there is ample precedent or is in some respect unique. If he recognizes his enterprise as falling within the limits of precedent, he can test the results of all his calculations and assumptions against the accumulated experience of his fellow engineers and their predecessors. (Ralph Peck)

Empirical Basis of Judgement

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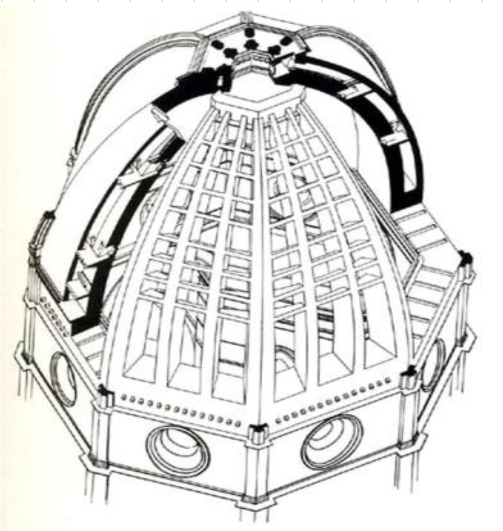
Dome of Santa Maria del Fiore

From <https://en.wikipedia.org/wiki/Florence_Cathedral>

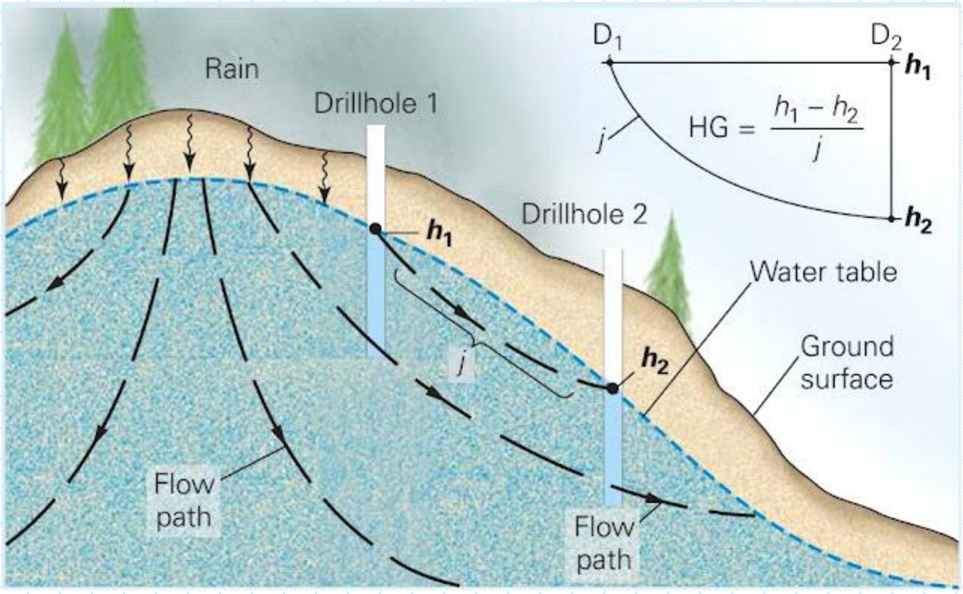
[How an Amateur Built the World's Biggest Dome]



2. **Theoretical basis of judgment** - The power of theoretical and analytical procedures in engineering is unquestioned. Computers not only enormously accelerate our thinking , they change the pattern of our thought. The rewards to be reaped from the computer seem almost limitless. Almost, but not quite.

Theory and calculations are not substitutes for judgment, but are the bases for sounder judgment. A theoretical framework into which the known empirical observations and facts can be accommodated permits us to extrapolate the new conditions with far greater confidence than we could justify by empiricism alone.

Theory, particularly with the aid of the electronic computer, permits us to carry out what we might call parametric exercises in which we can investigate the influence on the final design of variations in each of the factors affecting the design. (Ralph Peck)

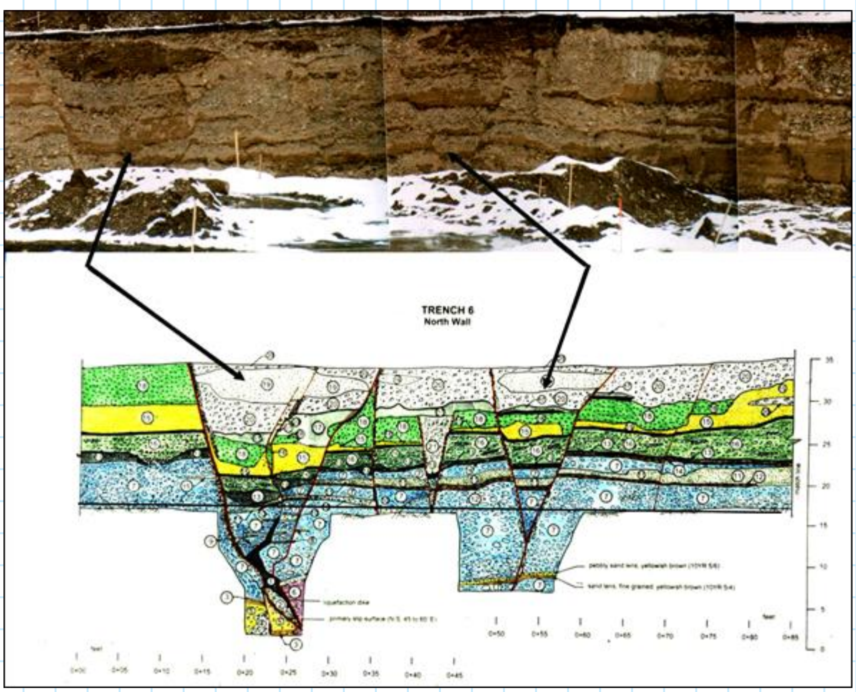


C **Sense of proportion** is one of the main facets of engineering judgment. Without it, an engineer cannot *test the results of a calculation against reasonableness*. Physical quantities, the size of things, could have not real meaning to him.

C. The third fundamental aspect of soil mechanics, and the one that has increased in significance in my mind over the past 20 years, **is geology**. Except for those projects dealing with earth as a construction material, all problems in applied soil mechanics are concerned with the behavior of natural materials in place. The history of formation and the anatomy of these deposits is the **domain of geology**.

<u>Listing of geology courses potentially useful to geotechnical engineer</u>

- Physical geology
- Historical geology
- Geomorphology
- Stratigraphy and Sedimentology
- Applied Geophysics
- Geologic Hazards
- Groundwater



Knowledge Req'd to Practice Geotechnical Engineering (cont.)

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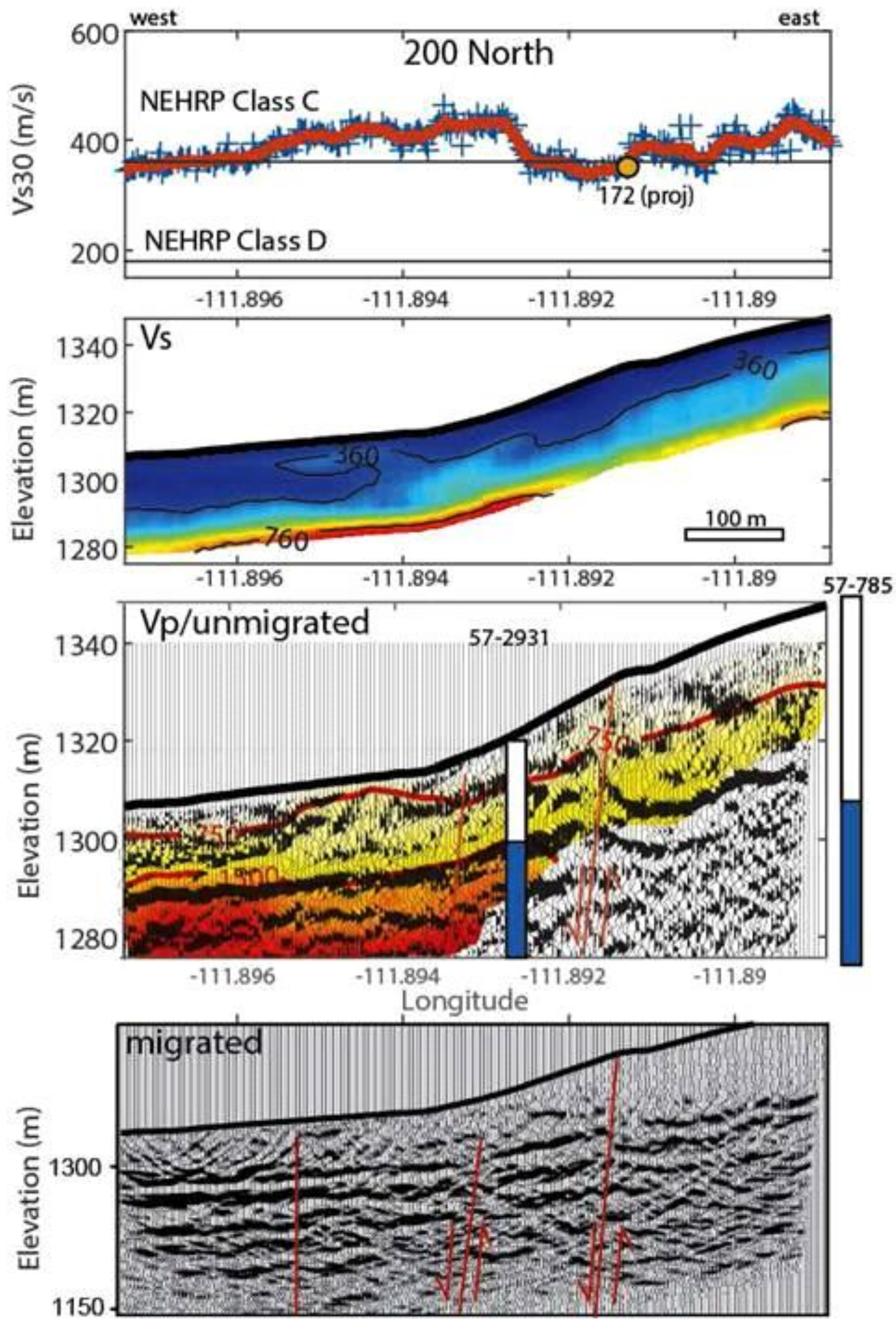

Figure 2-6 Seismic profile in seismic profile showing (a) Vs30 profile, (b) Vs profile for upper 30 m, (c) Vp map with unmigrated reflection image, and (d) migrated reflection image (Liberty 2016).

Ways to Develop Engineering Judgment

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1. Make the most of your educational experience by **devoting yourself to a systematic study** of your chosen subject and those related to it.

2. **Select your first job for the quality and kind of experience it can offer**. Plan a program of successive jobs with different experience during the first few years of your professional career. All too many graduates interested in soil mechanics and foundations find themselves working in firms whose principal endeavor is to obtain the logs of test borings, test the samples, and write reports containing the recommendations for types of foundations and for allowable soil or pile loads. *Without an opportunity to follow through on such projects, to see how the construction procedures work out and to learn how successfully the facilities performed, such experience is sterile*. There is no feed-back.

3. **Be involved with construction**, whenever possible. Learn how things are constructed and how design and construction must interact.

4. I would suggest that you not only **read carefully your professional magazines, but that you look closely at the advertisements**. A foundation engineer can profit greatly by reading the ads in magazines dealing with heavy construction. He gets a feeling for the tools of the trade, the problems being solved, and the general activity in the field.

5. Attend **specialty lectures** offered at the University and professional organizations.

6. **Keep a detailed notebook** about everything you do. The purpose is not so much as to make a record as to **develop the power of observation**. I also kept in that notebook the records of conversations with all sorts of people, including Terzaghi on his frequent visits.

7. **Read the Terzaghi Lectures (ASCE publication) and case histories of design and construction failures in geotechnical engineering literature**.

Professional Etiquette

Thursday, March 11, 2010 11:43 AM

A. Rules to Be Remembered (by Karl Terzaghi)

1. Engineering in a noble sport which calls for good sportsmanship. *Occasional blundering is part of the game*. Let it be your ambition to be the first to discover and announce your blunders. If somebody else gets ahead of you take it with a smile and thank him for his interest. Once you begin to feel tempted to deny your blunders in the face of reasonable evidence, you have ceased to be a good sport. You are already a crank or a grouch.

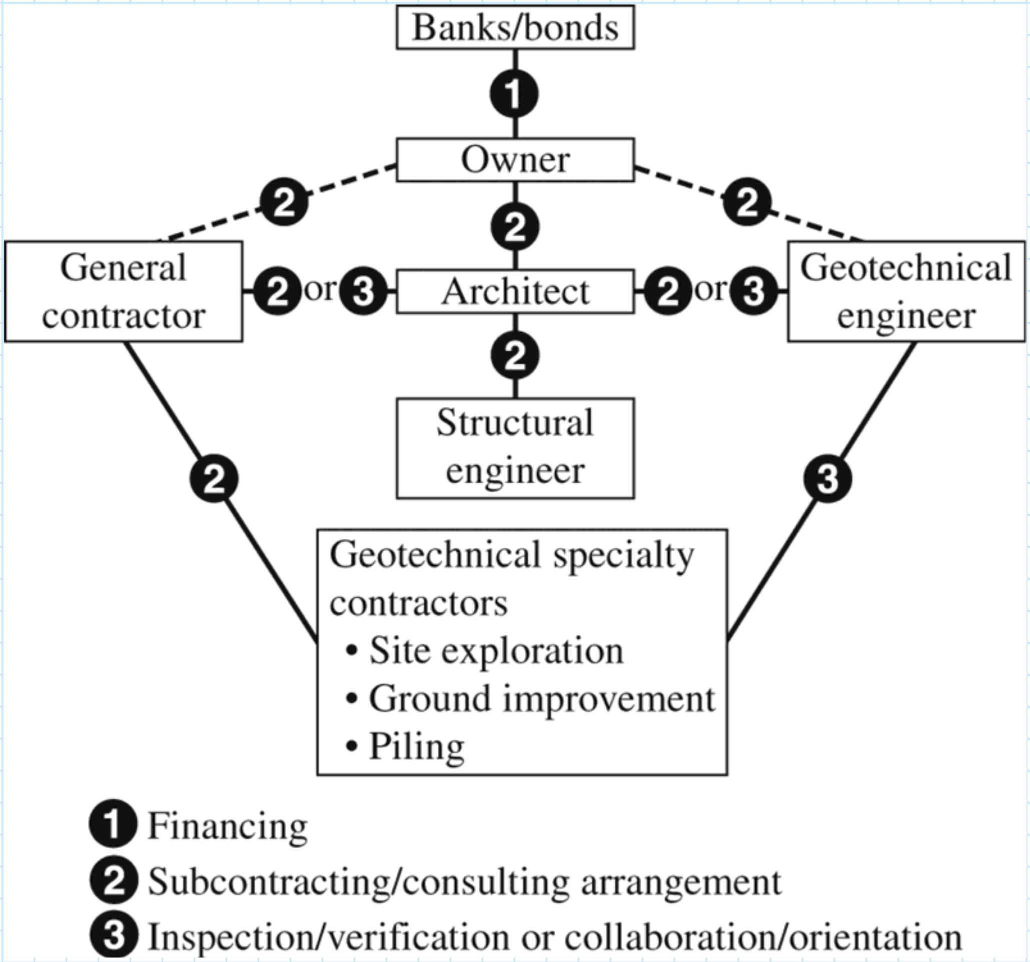
2. The worst habit you can possibly acquire is to become uncritical towards your own concepts and at the same time skeptical towards those of others. Once you arrive at that state you are in the grip of senility, regardless of your age.

3. When you commit one of your ideas to print, emphasize every controversial aspect of you thesis, which you can perceive. Thus you win the respect of your readers and it keeps you aware of the possibilities for further improvement. A departure for this role is the safest way to wreck you reputation and to paralyze your mental activities.

4. Very few people are either so dumb or so dishonest that you could not learn anything from them.

Practice of Engineering - Contracting Parties and Relationships

Monday, August 22, 2022 12:45 PM



Types of Contracts

- Lump Sum Contract - **a construction agreement in which the contractor agrees to complete the project for a predetermined, set price**
- Unit Price Contract - **a contractor is paid for the actual quantity of each line item performed as measured in the field during construction**
- Cost Plus Contract - a **contractor is paid for all of a project's expenses plus an additional fee for the job**
- Incentive Contracts - An incentive contract is a contract between two parties in which one party promises to grant an additional remuneration to another party for outstanding performances.
- Percentage of Construction Fee Contracts -
  Most construction managers will charge a fee of **three to five percent of the total project cost**. Some will also charge a five percent fee, yet mark their materials and labor up 10 percent, meaning you are actually paying them 15 percent.

Practice of Engineering - Profit Margins

Monday, August 22, 2022 12:45 PM

Architect and Engineering (A&E) Firms

There is a huge variation in the profit margins from one to another. While some firms struggle to hit 8% profit, others are sailing along with consistent profit margins over 30%, and in some cases, in excess of 40%!

What Profit Should You Be Making?

While these higher margins are much more expected and targeted in most other professional services industries, the average architecture, engineering, environmental and Geotech consulting firm is targeting 13% to 15%, and lower if 100% of their work is public. In fact, man**y firms that offer exclusively public work are averaging 6% to 8%** – hardly enough to counter the huge risk of these high-exposure projects.

From <https://aecbusiness.com/why-some-firms-make-more-money-than-others/>

Construction Profit Margins

Here's some 2013 data from the North American Industry Classification System. Below are the median pretax profit rates of various industries that fiscal year.

- New single-family residential buildings: **3.2%**
- Road, street, and bridge construction: **3.0%**
- Commercial & industrial buildings: **2.1%**
- Industrial buildings: **3.8%**
- Land subdivision: **8.7%**

From <https://www.freshbooks.com/hub/estimates/average-profit-margin-for-construction-industry>

Practice of Engineering - Civil Engineering Hierarchy

Monday, August 22, 2022 12:45 PM

Principal Engineer -

A principal engineer is an engineering professional with many years of experience in their field. They oversee projects and staff after working as engineers in a particular field. Different from other engineers, the role of principal engineer is a leadership role where they guide staff to ensure an engineering team completes projects on time and within budgets. They are professionally licensed engineers.
<https://www.indeed.com/career-advice/finding-a-job/principal-engineer-vs-senior-engineer>

Often principal engineers own founding stock or shares in the company.

Managing or Senior Engineer -

Senior engineers are engineering professionals who earn this title through experience. People in this role may handle several projects simultaneously, performing tasks of engineers and providing guidance for teams. These engineers are highly technical and ensure teams apply common engineering principles and concepts to their own teams' responsibilities. They are professionally licensed engineers.

From <https://www.indeed.com/career-advice/finding-a-job/principal-engineer-vs-senior-engineer>

Project Engineer - A project engineer is **responsible for the engineering and technical disciplines needed to complete a project**. The project engineer works to plan projects, establish project criteria, coordinate project reviews, and ensure the proper implementation of project elements. They are professionally licensed engineers.

From <https://www.google.com/search?q=project+engineer&rlz=1C1SQJL_enUS822US822&oq=project+engineer&aqs=chrome..69i57j69i65.2933j0j15&sourceid=chrome&ie=UTF-8>

Design Engineer -

Design engineers **identify complex design problems, conduct root-cause failure analyses, and anticipate production issues**. They then develop innovative design solutions, evaluate options, conduct tests, and implement solutions to meet timing, product cost and reliability targets. They are professionally licensed engineers.

Staff Engineer - Engineer In Training. Engineers graduating from an accredited engineering program that have passed the fundamentals of engineering examination.

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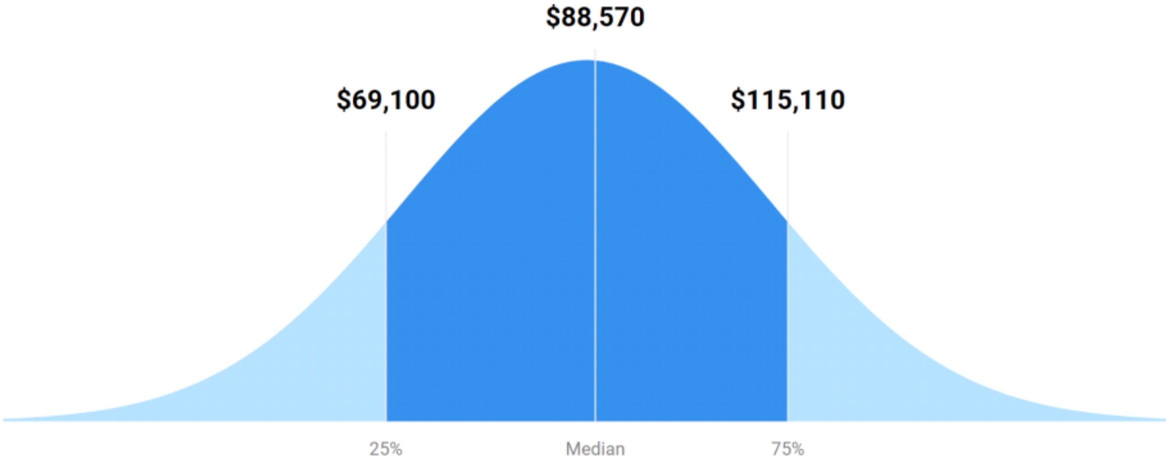
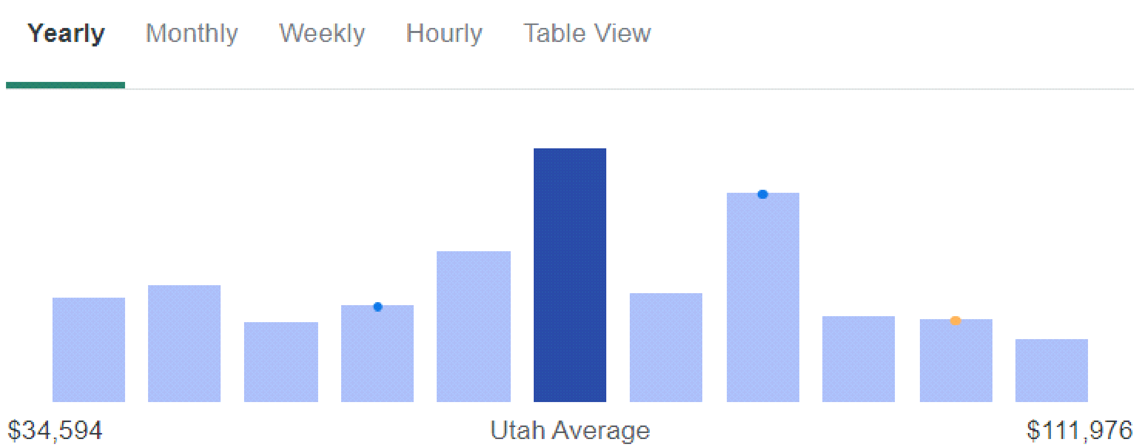
How Much Does a Civil Engineer Make?

Civil Engineers made a median salary of $88,570 in 2020. The best-paid 25 percent made $115,110 that year, while the lowest-paid 25 percent made $69,100.



https://money.usnews.com/careers/best-jobs/civil-engineer/salary

Civil Engineering Salary in Utah



$76,031 /year

$37 /hour

2024 Utah

https://www.ziprecruiter.com/Salaries/Civil-Engineering-Salary--in-Utah

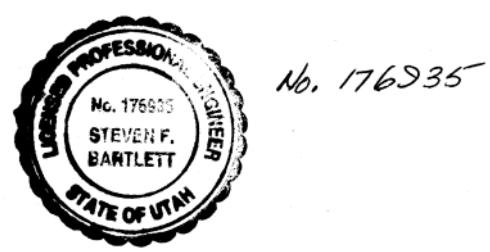
General Requirements for Engineering Calculations & Drawings

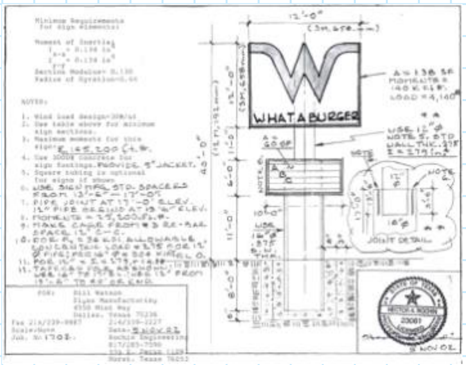
Sunday, February 17, 2019 5:48 AM

Civil Engineering Calculations and Drawings must be:

- Accurate
- Drawn to scale (can be used to obtain measurements)
- Dimensioned (shows dimensions of objects and features)
- Clear
- Complete
- Compliant with required codes and laws
- Preformed using the Standard of Care
- Reviewed
- Certified by professional engineer

These drawings are legal documents and professional engineers originating these drawings certify that they are correct.





This is an example of an engineer's stamp that is used to certify drawings and other engineering documents.

DIVISION OF OCCUPATIONAL & PROFESSIONAL LICENSING

Certificate of License Renewal

Control Number: 176935-2202-20190213

RENEWAL DATE: 02/13/2019

EXPIRATION DATE: Wed Mar 31 2021

ISSUED TO: Steven Floyd Bartlett

REFERENCE NUMBER(S), CLASSIFICATION(S) & DETAILS(S)

176935-2202 Professional Engineer

Please note that DOPL reserves the right to initiate action at any time against a licensee who did not meet the renewal/reinstatement requirements at the time this license was issued.

Professional Engineer License

Practice of Engineering - Standard of Care and Professional Ethics

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Standard of Care

The law provides that an engineer performing professional services for a client, owes the client the duty to have a degree of learning and skill ordinarily possessed by reputable civil engineers, practicing under similar circumstances. The engineer also has the duty to use the care and skill ordinarily used in like cases by reputable members of the profession practicing under similar circumstances. Also, the engineer has the duty to use reasonable diligence and best judgment in the exercise of skill and the application of learning. The failure to perform any one of these duties is defined as negligence.

Pasted from <http://www.asce-sf.org/index.php?option=com_content&task=view&id=378&Itemid=80>

Negligence

Failure to meet the standard of care is one of the elements of negligence. Performing in accordance with the applicable standard of care, however, can present conflicts in the practice of engineering. For example, compliance with the standard of care may require that a more extensive investigation be conducted than was originally anticipated in the original contract. A proper investigation will cost more money and therein lies the potential for conflict. “You get what you pay for” is not an absolute legal defense against failure to meet the standard of care.

Pasted from <http://www.asce-sf.org/index.php?option=com_content&task=view&id=378&Itemid=80>

Conservatism vs Economical Solutions

A cautious approach to the practice of engineering should protect the engineer from liability risks. However, it is also true that continuous use of the same methods and techniques stifles innovation, creativity and progress and may not lead to the best, most economical solutions. An experienced engineer should be able to find a balance between proven methods and techniques and adopting innovative ones. (Salgado p. 4)

Trends in Geotechnical / Foundation Engineering

Wednesday, August 17, 2011 12:45 PM

Methods of Analysis

- Empirical methods
  - Observations
  - Rules of Thumb
  - Long-established construction practices
- Analytical methods
  - Based on theory and analysis
  - Usually satisfy force equilibrium
  - Closed-form solutions
  - Simple to complex, depending on situation
  - Widely used in this course
- Numerical methods
  - Based on more advanced theory
  - Satisfy force equilibrium
  - Can obtain estimates of deformation
  - Involve computer modeling
    - Finite difference method
    - Finite element method

After many decades of research on analytical and numerical models for modeling geotechnical problems, user-friendly programs that take advantage of research results are now becoming available. Increasing use of computer analysis in geotechnical and foundation design, with a corresponding decline in reliance on rough rules of thumb and approximate methods, is likely to scale up in the coming years (Salgado, p. 9)

Introduction to Geotechnical Engineering

"Why You Should Study Geotechnical Engineering"

or

"Who Needs a Foundation Engineer Anyway???"

Steven F. Bartlett, Ph.D., P.E.
bartlett@civil.utah.edu

THE UNIVERSITY OF UTAH

What is Geotechnical Engineering?

- **Application of civil engineering to earth materials**
  - **Soil**
  - **Rock**
  - **Groundwater**

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Soil Behavior Introduction

"God did not create the earth according to ASTM Standards."

"Concrete and Steel are textbook material Soil is not."

ASTM = American Society of Testing and Materials

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The New York Times https://nyti.ms/3n4W9R3

GOOD QUESTION

What Makes Sand Soft?

Understanding how grains flow is vital for everything from landslide prediction to agricultural processing, and scientists aren't very good at it.

By Randall Munroe

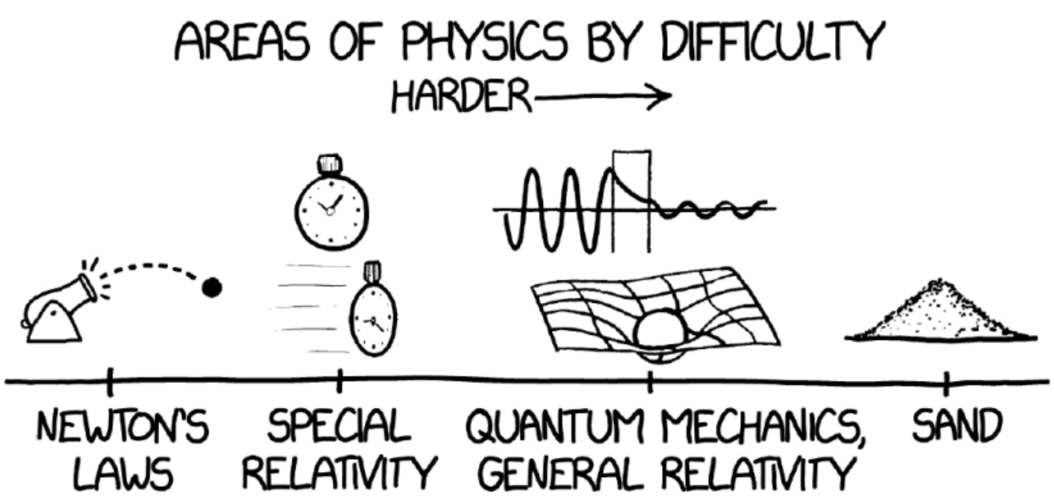
Nov. 9, 2020

What is the softest sand in the world? Why is some sand softer than others?

— Peter S., Brooklyn

We don't know. No one understands how sand works.

That may sound absurd, but it's sort of true. Understanding the flow of granular materials like sand is a major unsolved problem in physics.

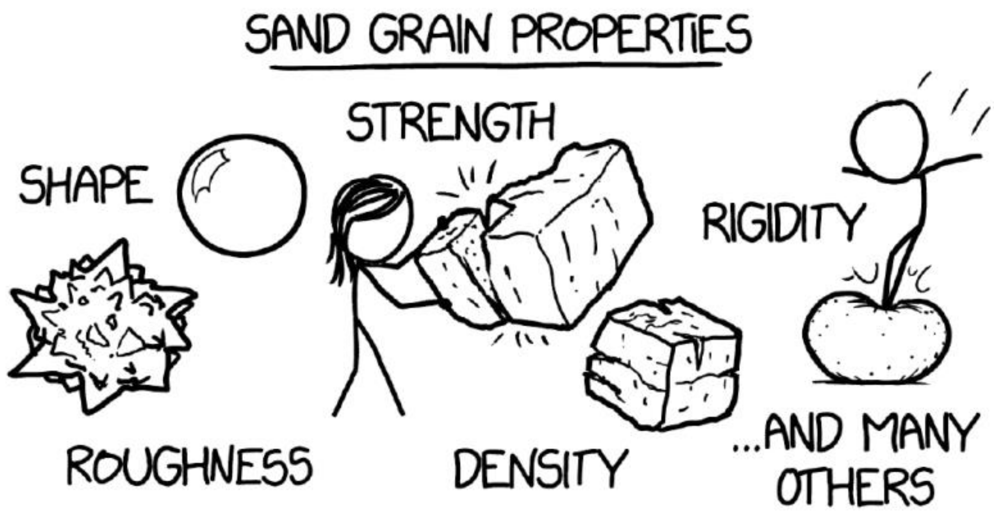


If you build an hourglass and fill it with sand grains with a known range of sizes and shapes, there is no formula to reliably predict how long the sand will take to flow through the hourglass, or whether it will flow at all. You have to just try it.

Soil Behavior (cont.)

Friday, January 04, 2013 2:31 PM

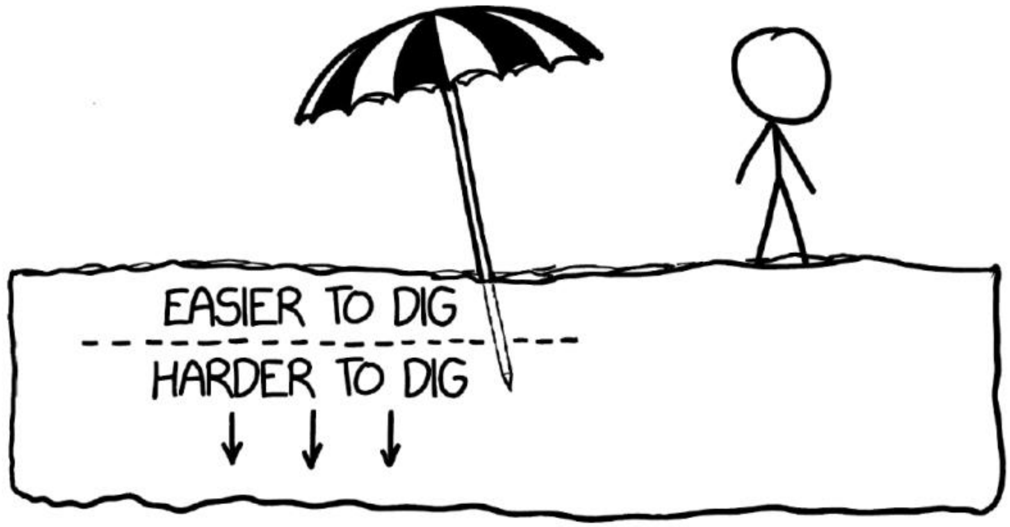
Karen Daniels, a physicist at North Carolina State University who studies sand and other granular materials — a field actually called "soft matter" — told me that sand is challenging in part because the grains have so many different properties, like size, shape, roughness and more: "One reason we don't have a general theory is that all of these properties matter."



But understanding individual grains is only the start. "You have to care not just about the properties of the particles, but how they're organized," Dr. Daniels said. Loosely packed grains might feel soft because they have room to flow around your hand, but when the same grains are packed together tightly, they don't have room to rearrange themselves to accommodate your hand, making them feel firm. This is part of why the surface layers of beach sand feel softer than the layers underneath: the grains in the deeper layers are pressed closer together.

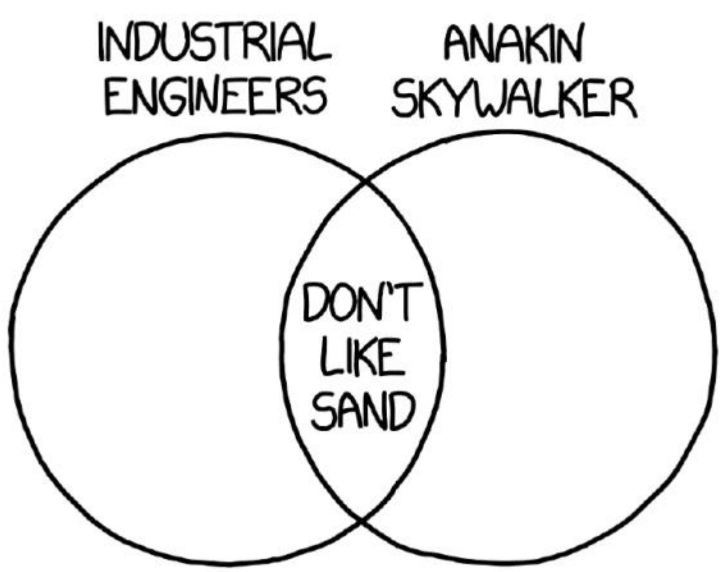
Soil Behavior (cont.)

Friday, January 04, 2013 2:31 PM



Our failure to find a general theory of sand isn't for lack of trying. For everything from agricultural processing to landslide prediction, understanding the flow of granular materials is extremely important, and we just aren't very good at it.

"People who work in particulate handling in chemical engineering factories can tell you that those machines spend a lot of time broken," Dr. Daniels said. "Anyone who's tried to fix an automatic coffee grinder knows they get stuck all the time. These are things that don't work very well."



Soil Behavior (cont.)

Friday, January 04, 2013 2:31 PM

Luckily, we're not totally in the dark, and can say a few things about what makes sand softer or harder.

Sand with rounder grains usually feels softer, because the grains slide past each other more easily. Smaller grains also don't produce the pinprick feeling of individual grains pressing into your skin. But if the grains are too small, moisture causes them to stick together, making the material feel clumpy and firm.

Dr. Daniels said that the softest granular material she had ever touched was a substance called Q-Cell, a silica powder used for filling dents in surfboards. The powder is made of hollow grains, so it feels extremely light, and the silica material stays dry, which keeps it from clumping. She compared the way it sloshes around to a bucket full of very fine, very dry beach sand.

A beach made of Q-Cell "sand" might be soft, but it wouldn't be very pleasant. Fine, dry powders are dust, not sand, and inhaling them can be extremely hazardous to your lungs. The ideal beach sand would probably have a grain size and shape that balanced softness, dustiness, clumping and a variety of other properties that make sand soft and nice to walk on. With so many subjective factors to consider, it's hard to say exactly what the ideal soft beach sand would be.

You'll just need to gather some experimental data.

Soil Behavior (cont.)

Thursday, March 11, 2010 11:43 AM

A. Most of the theories for the mechanic behavior of engineering materials *assume that the materials are homogeneous and isotropic*, and that they follow linear-stress strain law (e.g., steel and concrete).

B. Soils are **heterogeneous, anisotropic, nonconservative, nonlinear** materials.

- **heterogeneous** - material properties vary widely from point to point within the soil mass.
- **homogeneous** - material properties are the same from point to point within the soil mass.
- **anisotropic** - material properties are not the same in all directions
- **isotropic** - material properties are the same in all directions
- **conservative** - past history does not affect the current engineering behavior (i.e., memoryless)
- **nonconservative** - past history affects the current engineering behavior (i.e. soils have a memory of past stress history
- **nonlinear** - stress-strain curve is curved according the stress level
- **linear** - stress-strain curve is a straight line

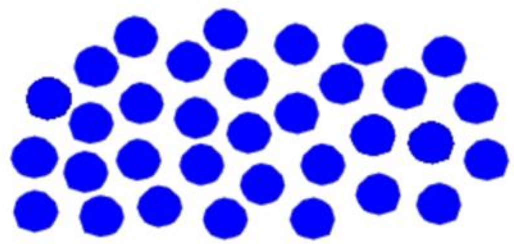
Because soils are **heterogeneous, anisotropic, nonconservative, nonlinear** materials, we must use more ***complex theory*** to describe their behavior, or apply large empirical corrections (***safety factors***) to our design to account for the real material behavior.

The behavior of soil and rock is often controlled by ***defect***s in the material (e.g., joints, fractures, weak layers and zones), yet ***laboratory tests and simplified methods often do not take into account*** such real characteristics.

Heterogeneity

Friday, January 04, 2013 2:31 PM

Soil is heterogeneous



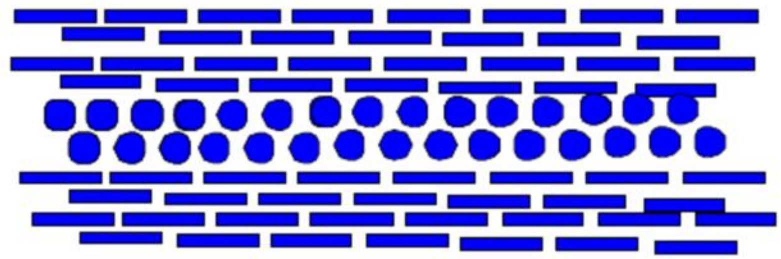
Homogeneous

Heterogeneous

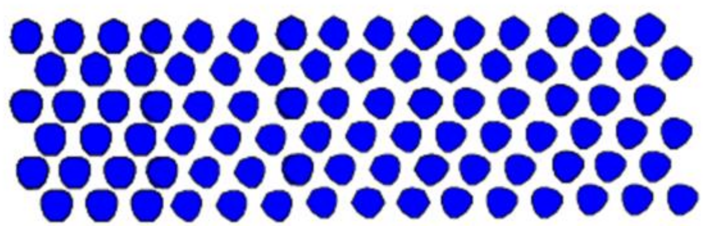
Anisotrophy

Friday, January 04, 2013 2:31 PM

Soil is anisotropic



Anisotropic



Isotropic

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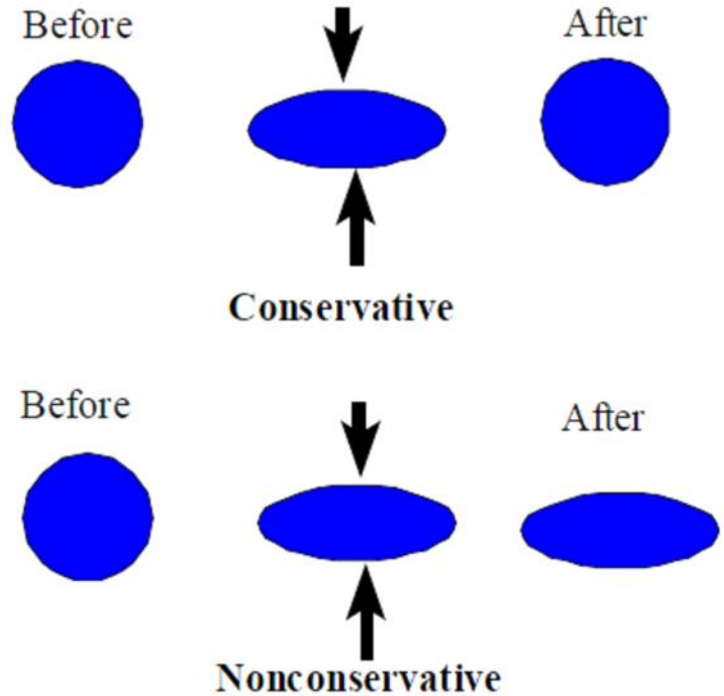
Nonconservative (Inelastic)

Friday, January 04, 2013 2:31 PM

Soil is nonconservative

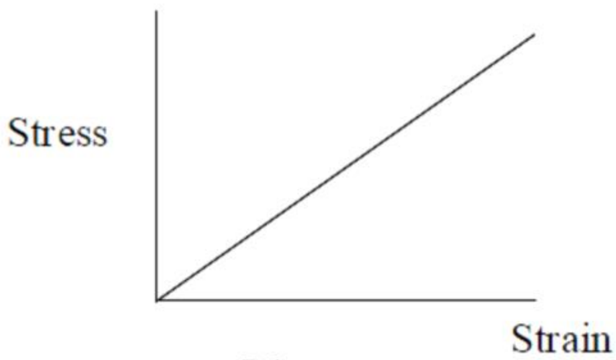


(but that doesn't mean it's liberal)

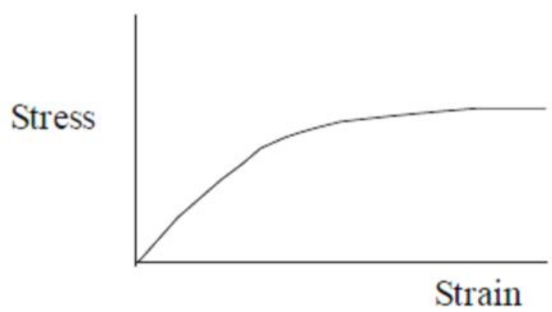
Nonlinearity

Friday, January 04, 2013 2:31 PM

Soil is nonlinear



Linear



Nonlinear

Examples of Geotechnical Projects
Chunnel

Location: Folkestone, England Sangatte, France

Completion Date: 1994

Cost: $21 billion

Length: 163,680 feet (31 miles)

Purpose: Railway

Setting: Underwater

From Building Big by David Macaulay

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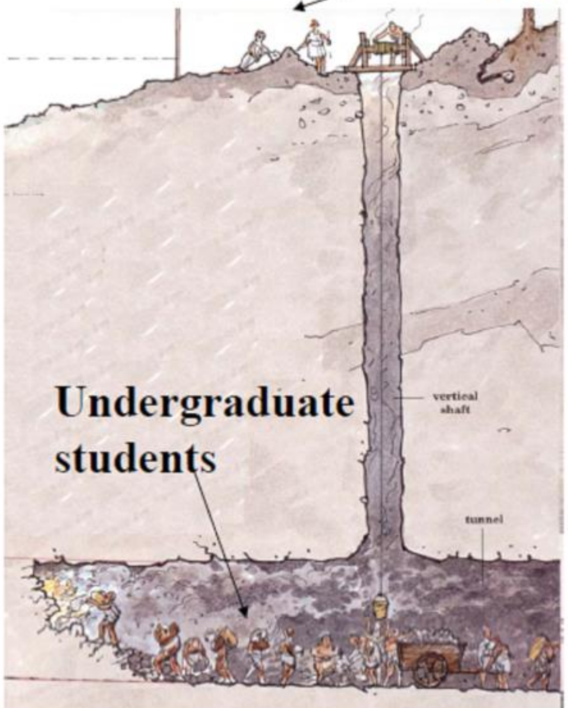
Tunnels

Old Way

Graduate Students

New Way



From <u>Building Big</u> by David Macaulay

Professors

Friday, January 04, 2013 2:31 PM

Examples of Geotechnical Projects
Golden Gate Bridge Foundations

Location: San Francisco and Sausalito, California, USA

Completion Date: 1937

Cost: $27 million

Length: 8,981 feet

Type: Suspension

From Building Big by David Macaulay

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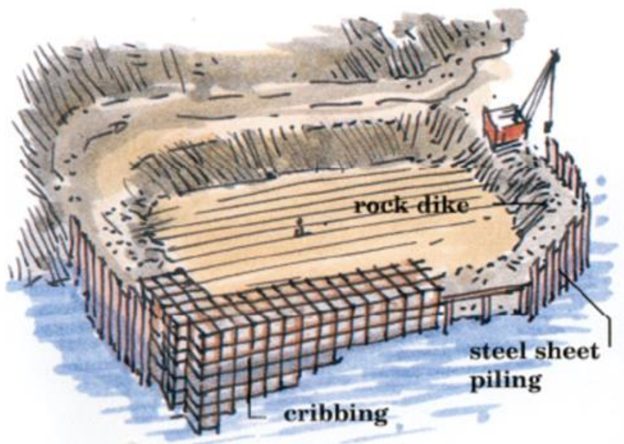
Golden Gate Bridge (Marin Pier)



Site of Marin Pier

From Building Big by David Macaulay

1. **Start rock dike (Coffer)**
2. **Crib dike part that is in water (timber box filled w/ rock and set in place).**
3. **Install sheet piling.**
4. **Pump area dry.**
5. **Construction foundation on rock surface exposed below water level.**

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Engineering Projects - Modern Sheet Piles with Retaining Ring

Friday, January 04, 2013 2:31 PM

Sheetpiles – Coffer Dam

Examples of Geotechnical "Structures"
Petronas Towers Foundations

Location: Kuala Lumpur, Malaysia

Completion Date: 1998

Cost: $1.6 billion

Height: 1,483 feet

Stories: 88

From Building Big by David Macaulay

Engineering Projects - Sheet Pile Coffer Dam with Dewating with Pumps

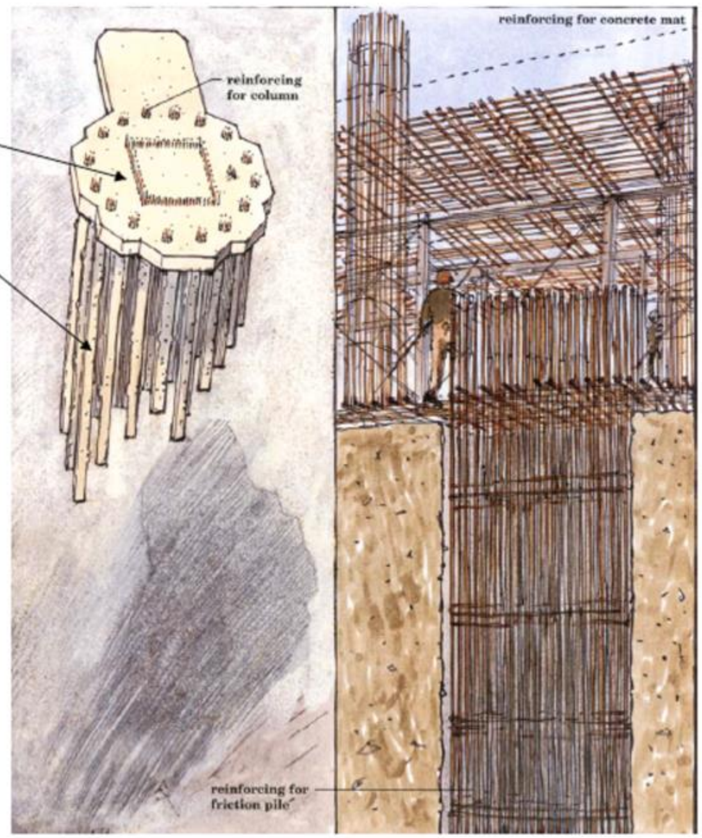
Friday, January 04, 2013 2:31 PM

Pumping inside Coffer Dams

Petronas Towers Foundations

Concrete mat

Pile Foundation



"Driven" Pile Foundations

Offshore Pile Foundations

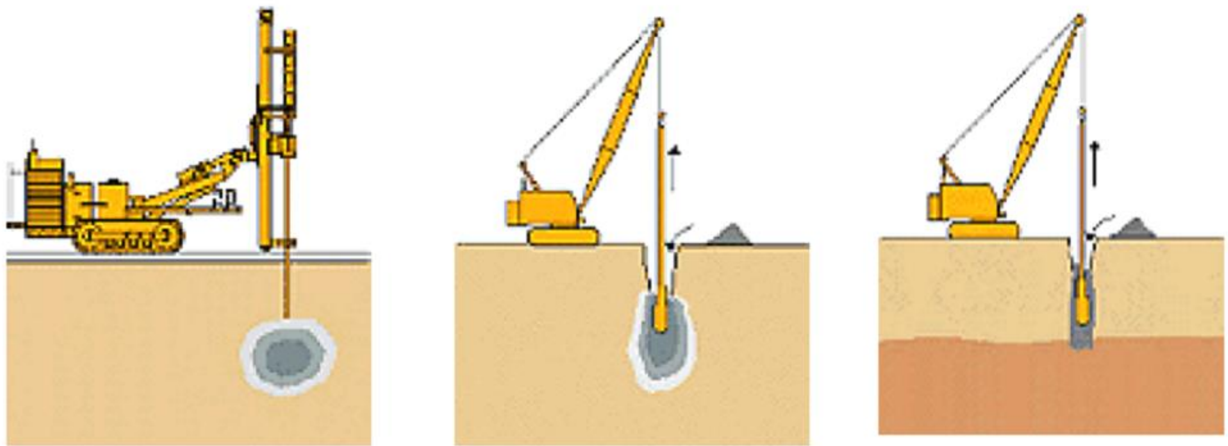
Friday, January 04, 2013 2:31 PM

Offshore Pile Foundations

Ground Improvement Examples

Friday, January 04, 2013 2:31 PM

Ground Modification
(Ground Improvement)



Compaction Grouting

Vibro-compaction

Vibro-replacement

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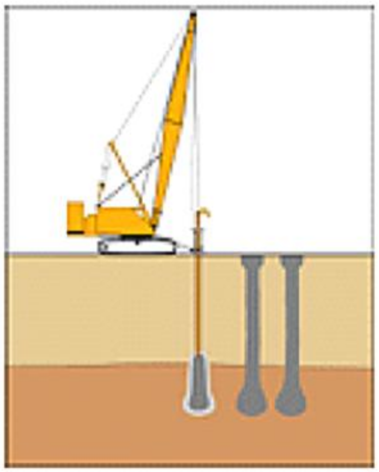
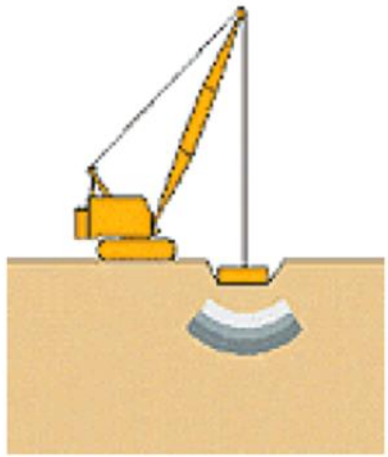
Ground Improvement Examples (cont.)

Friday, January 04, 2013 2:31 PM

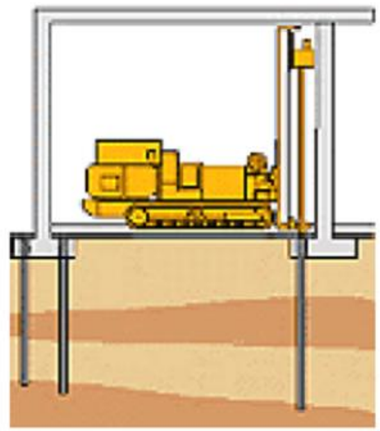
Ground Modification (cont)



Vibro-concrete



Dynamic Compaction

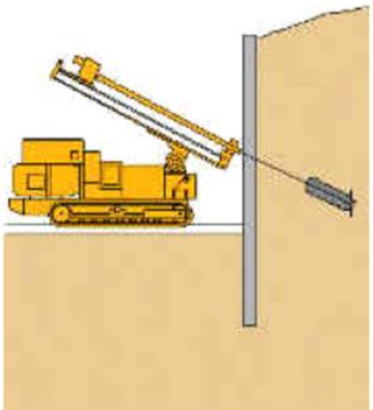


Mini piles

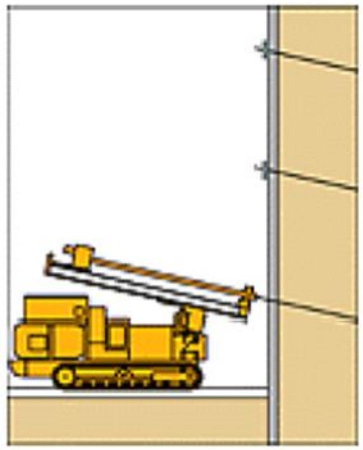
Ground Improvement Examples (cont.)

Friday, January 04, 2013 2:31 PM

Ground Modification (cont)



Rock Anchors



Soil Nails

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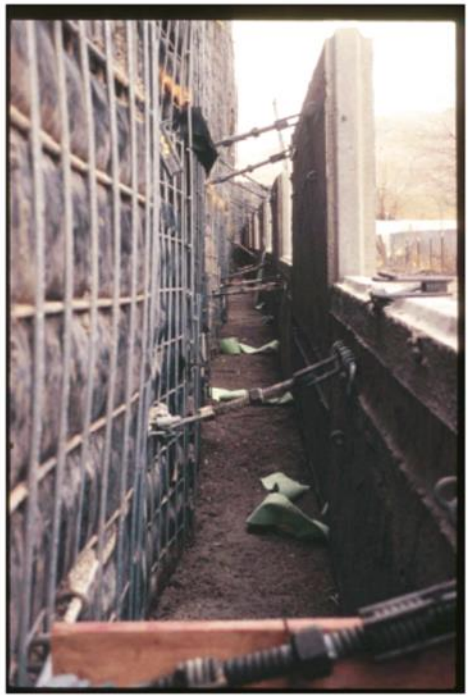
Mechanically Stabilized Earth (MSE) Retaining Wall

Friday, January 04, 2013 2:31 PM

Retaining Walls



I-15 Reconstruction Project
Salt Lake Valley

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Light Weight Embankments Using Geofoam (Expanded Polystyrene)

Friday, January 04, 2013 2:31 PM

Geofoam Embankments

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Geologic Hazards - Landslides

Friday, January 04, 2013 2:31 PM

Landslides
Thistle Slide – Spanish Fork Canyon

landslide
japan

Geologic Hazards - Debris Flow

Friday, January 04, 2013 2:31 PM

Debris Flow
(Mud Slide)

mudslide
brazil

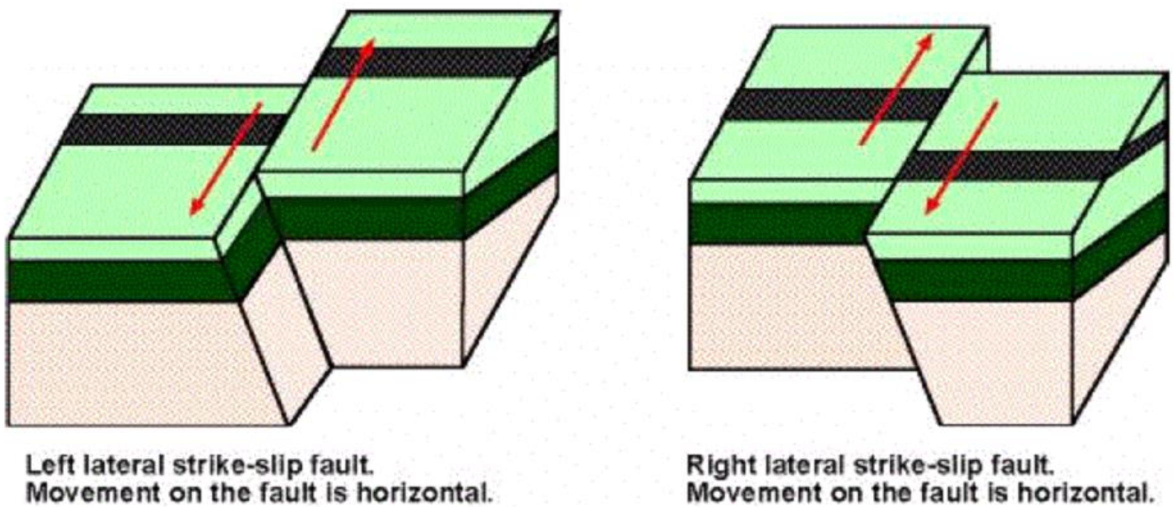
Geologic Hazards - Primary Types of Earthquake Hazard

Friday, January 04, 2013 2:31 PM

Earthquake Hazards

- Fault Rupture/Offset
- Strong Ground Motion
- Liquefaction
- Ground failure
- Tsunami

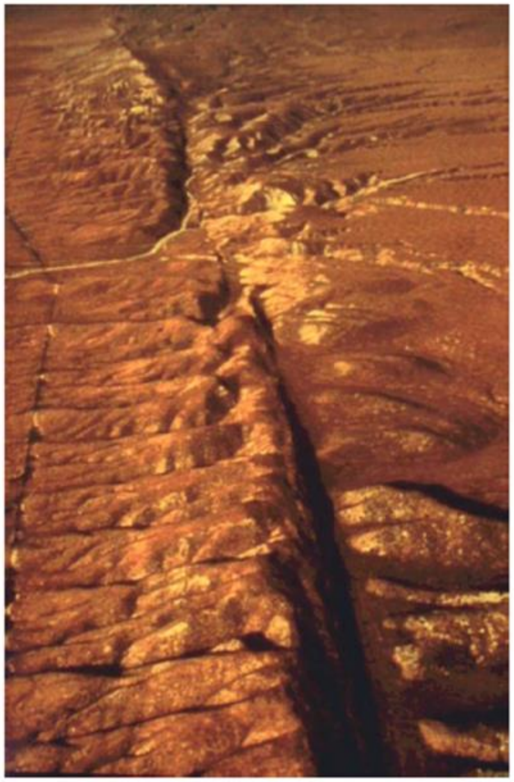
Earthquake Hazards
(San Andres Fault – A strike slip fault)



Geologic Hazards - Fault Offset - San Andres Fault

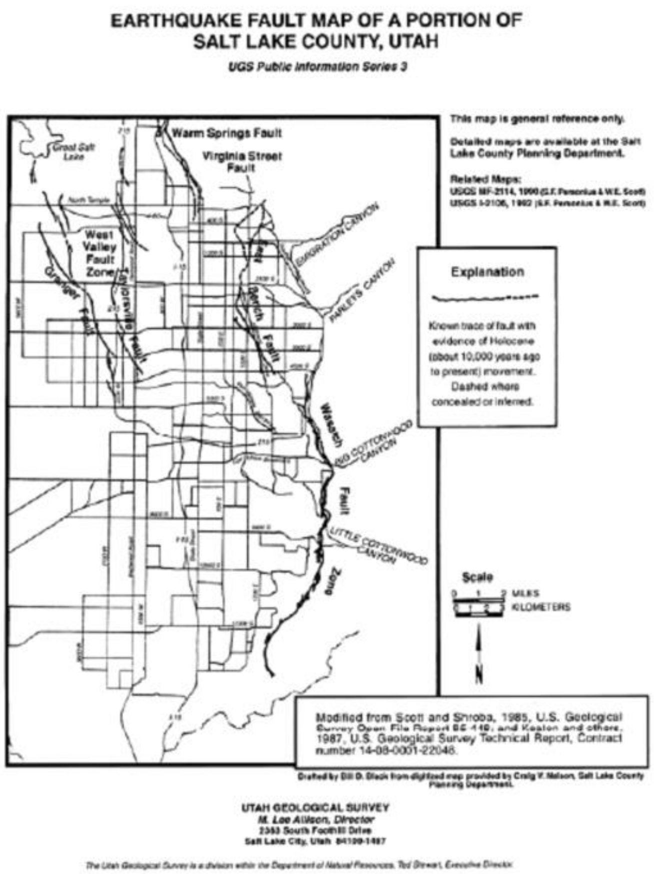
Friday, January 04, 2013 2:31 PM

Earthquake Hazards
(San Andres Fault)

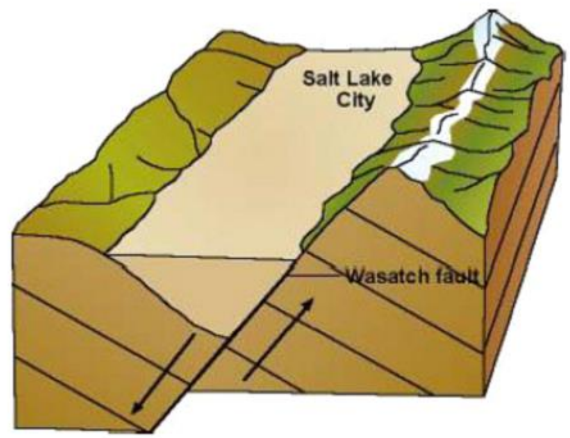


Friday, January 04, 2013 2:31 PM

Wasatch Fault (Is your house safe?)



Wasatch Fault is
A Normal Fault



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Geologic Hazards - Fault Offset - Wasatch Fault (cont.)

Friday, January 04, 2013 2:31 PM

Wasatch Fault (Its your Fault!)

Wasatch Fault – Little Cottonwood Canyon

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Fault Rupture Damage

1999 Chi-Chi Taiwan Earthquake

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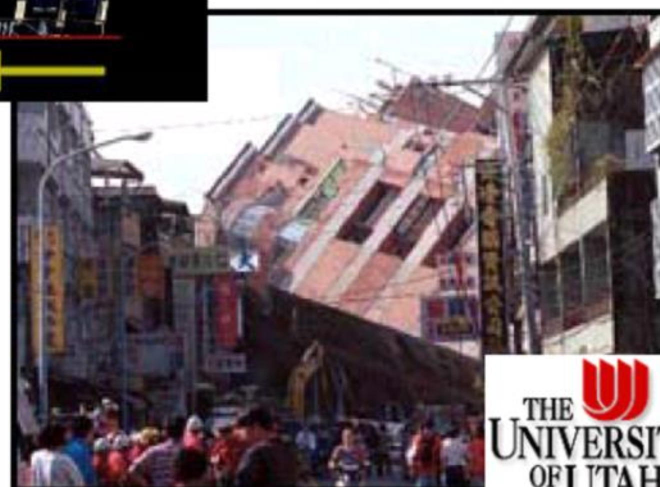
Geologic Hazards - Strong Ground Motion

Friday, January 04, 2013 2:31 PM

Strong Ground Motion

Collapsed Building, 1999 Chi-Chi Taiwan Earthquake



Strong Motion Video

1995 Kobe Japan Earthquake

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Strong Ground Motion
Building Collapse (1999 Turkey)

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Liquefaction Damage (1964 Niigata, Japan)

Liquefaction occurred causing the buildings to roll over on their side. People evacuated out the windows

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Geologic Hazards - Liquefaction (cont.)

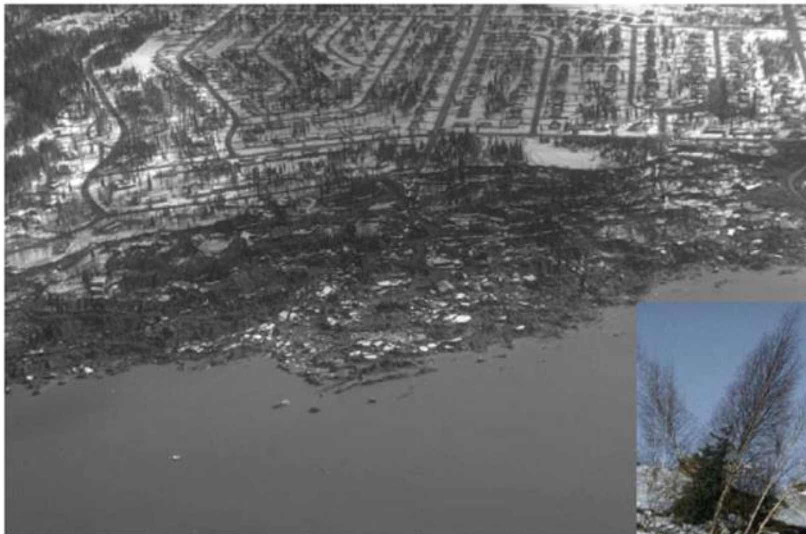
Friday, January 04, 2013 2:31 PM

Liquefaction Damage (1964 Niigata, Japan)

Geologic Hazards - Earthquake Induced Ground Failure

Friday, January 04, 2013 2:31 PM

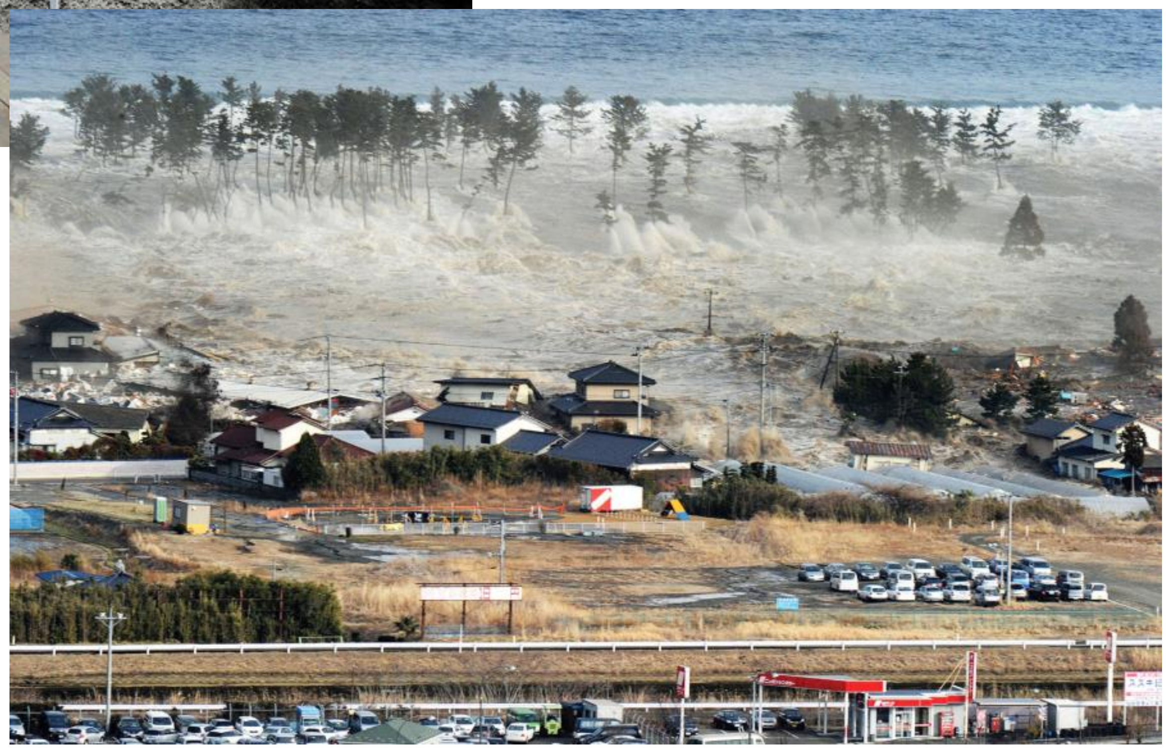
Ground Failure
(1964 Alaska Earthquake)



Geologic Hazards - Tsunami

Friday, January 04, 2013 2:31 PM

Japan Earthquake and Tsnumai, 2011



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Wednesday, August 17, 2011 12:45 PM

Ch 2 Foundation Design

Tuesday, August 27, 2024 12:45 PM

Reading Assignment

- Ch. 2 Lecture Notes
- Sections 2.1 to 2.7 (Salgado)

Other Materials

- none

Homework Assignment 2

- Problems 2-4 (5 points), 2-5 (10 points), 2-6 (10 points), 2-7 (10 points), 2-9 (10 points), 2-10 (15 points), 2-10 (20 points)

Problem 2.4 For a 30-story reinforced concrete building with a frame structure, estimate the range of column loads to be expected.

Problem 2.5 Two columns are supported each by a separate footing. The differential settlement between the two columns is 25 mm, and the distance between them is 4 m. Compute the angular distortion. Would this differential settlement be acceptable?

Problem 2.6 Considering a pair of footings with a span of 30 ft and a ratio of differential to total settlement of three-fourths, what is the maximum tolerable total settlement of each footing?

Problem 2.7 The loads to be supported by a foundation element are 1900 kN (dead load) and 1000 kN (live load). Write the ULS design equations according to both WSD and LRFD using the ACI load factors.

answer.

Problem 2.9 You are to design the foundations for a residential building with load-bearing brick walls resting on sand. The wall has a width of 12 m and a height of 3 m. What is the maximum tolerable settlement you will use in your verification of serviceability? Show how you arrived at your answer.

2.10

A foundation element has an estimated average strength of 200 kips with a coefficient of variation of 0.4. The design load on this foundation element has an average value of 80 kips with a coefficient of variation of 0.2. From this information, calculate the reliability index and the probability of failure.

Ch 2 Foundation Design - Learning Objectives

Tuesday, October 3, 2023 12:45 PM

1. Know the goals of an effective civil engineering design
2. Define the steps of the general engineering design process.
3. Know the steps of the foundation design process.
4. Know how to identify and the characteristics the two types of limit states in working state design.
5. Know the Limit States for Shallow Foundation Design
6. Know how to calculate the allowable load for working stress design
7. Know how to calculate the factor of safety given capacity and demand data.
8. Know how to apply LRFD design with its partial factors of safety and potential load combinations.
9. Given capacity and demand information, know how to calculate the reliability index and the probability of failure.
10. Know how to calculate the allowable differential settlement for a building system.

Case History - Tower of Pisa

Wednesday, August 17, 2011 12:45 PM

https://www.google.com/search?sca_esv=561038293&rlz=1C1SQJL_enUS822US822&q=fixing+the+leaning+tower+of+pisa&tbm=vid&source=lnms&sa=X&ved=2ahUKEwii4YXQooKBAxW-JUQIHW9JCpAQ0pQJegQICBAB&biw=1128&bih=525&dpr=3.41#fpstate=ive&vld=cid:2d40d52b,vid:XZ-r8a9C4fE

Submitted: 6 March 2009; Published: 01 July 2009
Reference: Burland J.B., Jamiolkowski M.B., Viggiani C., (2009). *Leaning Tower of Pisa: Behaviour after Stabilization Operations.* International Journal of Geoengineering Case histories, http://casehistories.geoengineer.org, Vol.1, Issue 3, p.156-169.

International Journal of Geoengineering Case Histories ©, Vol. 1, Issue 3, p.156
http://casehistories.geoengineer.org

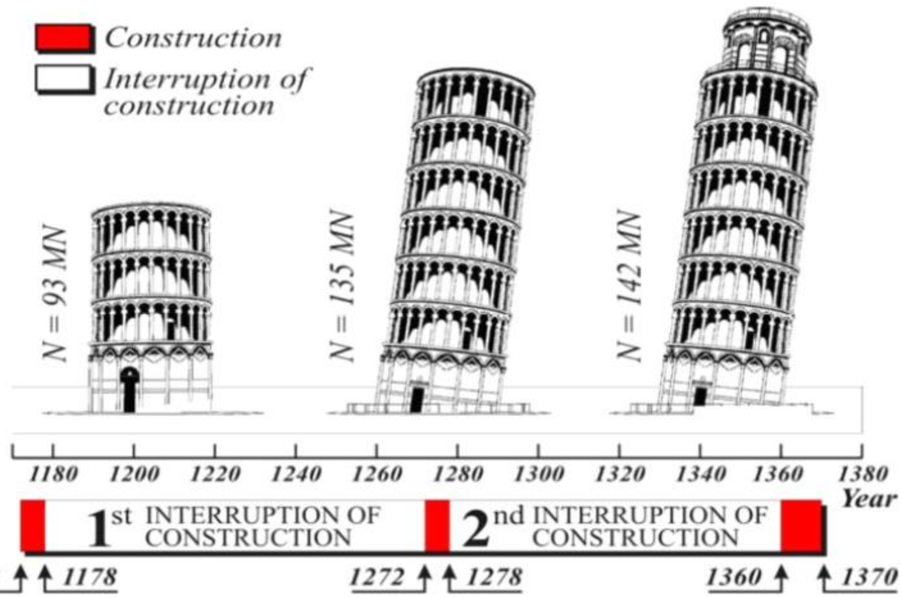


Figure 2. History of the Construction.

Its construction began in 1173 and continued (with two long interruptions) for about two hundred years as illustrated in Fig. 2. The Tower is built as a hollow masonry cylinder surrounded by six colonnades with columns and vaults rising from the base cylinder. The outer and inner walls are faced with competent San Giuliano marble, while the annular cavity between is filled with miscellaneous rock fragments and mortar, forming a typical medieval infill masonry structure. The Tower commenced leaning southwards during the second construction stage as shown in Fig. 2, and thereafter its inclination continued to increase. Fig. 3 shows the cross section through the Tower in the plane of maximum tilt as it was in 1993 before the stabilisation work commenced. **The average foundation pressure is 500 kPa and a detailed computer analysis (Burland and Potts, 1994), indicates that the pressure at the south edge was about 1000 kPa with the soil in a state of local yield, while the pressure at the north edge was close to zero.**

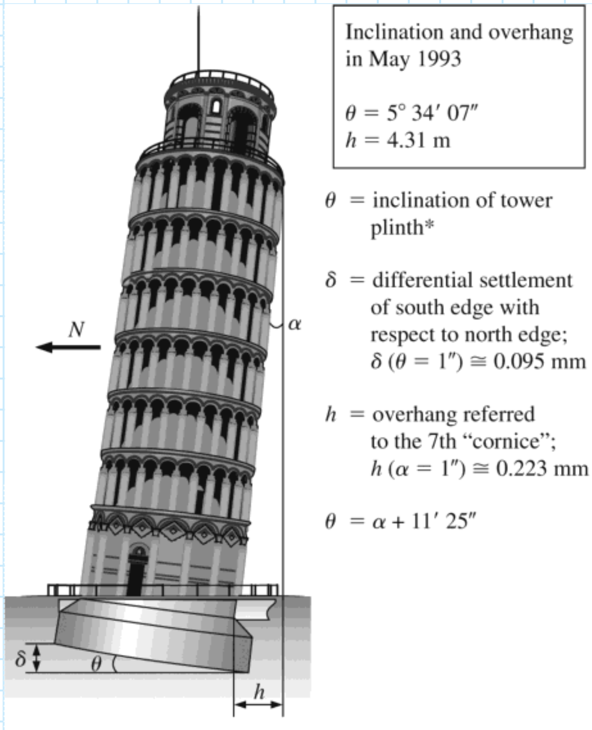
Application of the Design Process to Tower of Pisa

Wednesday, August 17, 2011 12:45 PM

1. What is the problem?
2. What are the constraints and design criteria?
3. What are the possible solutions?
4. What is the best or best solutions? (Can't answer this yet.)
5. How do we test the effectiveness of the solution?

Tower of Pisa (cont.)

Wednesday, August 17, 2011 12:45 PM



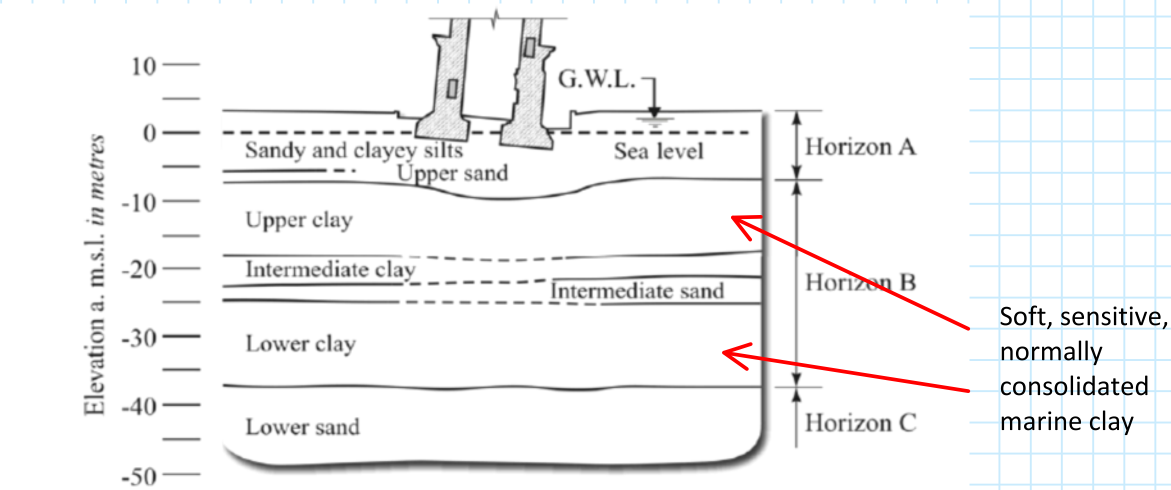
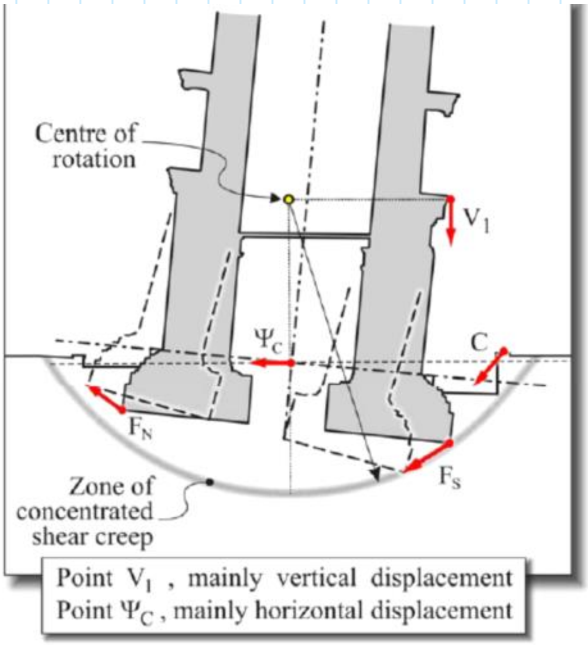


Figure 4. Soil profile.

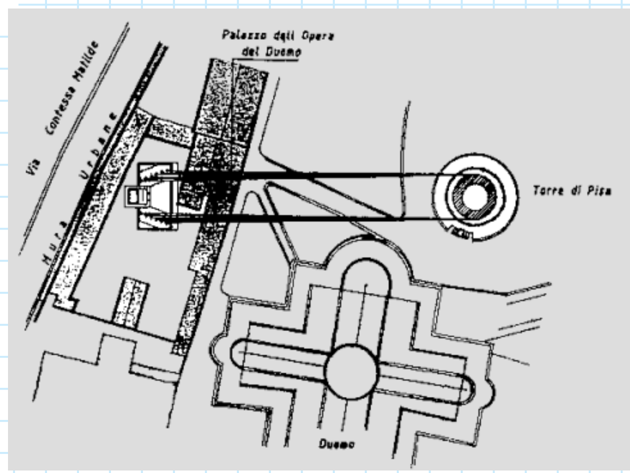
The ground underlying the Pisa Tower consists of three formations as shown in Fig. 4. Horizon A, about 10m thick, is composed of soft estuarine deposits of sandy and clayey silts laid down under tidal conditions. Horizon B consists of soft sensitive normally consolidated marine clay extending to a depth of about 40m. Because it is very sensitive, this material loses much of its strength if disturbed. Horizon C is dense marine sand extending to a depth of about 60m. An upper perched water table in Horizon A is encountered between 1m and 2m below the level of Piazza dei Miracoli corresponding to elev. +3.0 above m.s.l. The contact between Horizon A and the marine clay of Horizon B is dished beneath the Tower, indicating that it **experienced average settlements of between 3.0m and 3.5m**.

Tower of Pisa (cont.)

Wednesday, August 17, 2011 12:45 PM



Kinematically admissible mechanism of ground movement, curvilinear concentrated shear creep zone passing through more plastic layer of horizon A



Temporary stabilization of the foundation was achieved during the second half of 1993 by the application of **600 t of lead weights** on the north side of the foundations via a post-tensioned removable concrete ring cast around the base of the Tower at plinth level. This caused a reduction in inclination of about one minute of arc and, more importantly, **reduced the overturning moment by about ten percent**. In September 1995 the **load was increased to 900 t** in order to control the accelerating southward movements of the Tower during an unsuccessful attempt to replace the unsightly lead weights with temporary ground anchors. This difficult period during the Committee's activity has been called "Black September.

Tower of Pisa (cont.)

Wednesday, August 17, 2011 12:45 PM

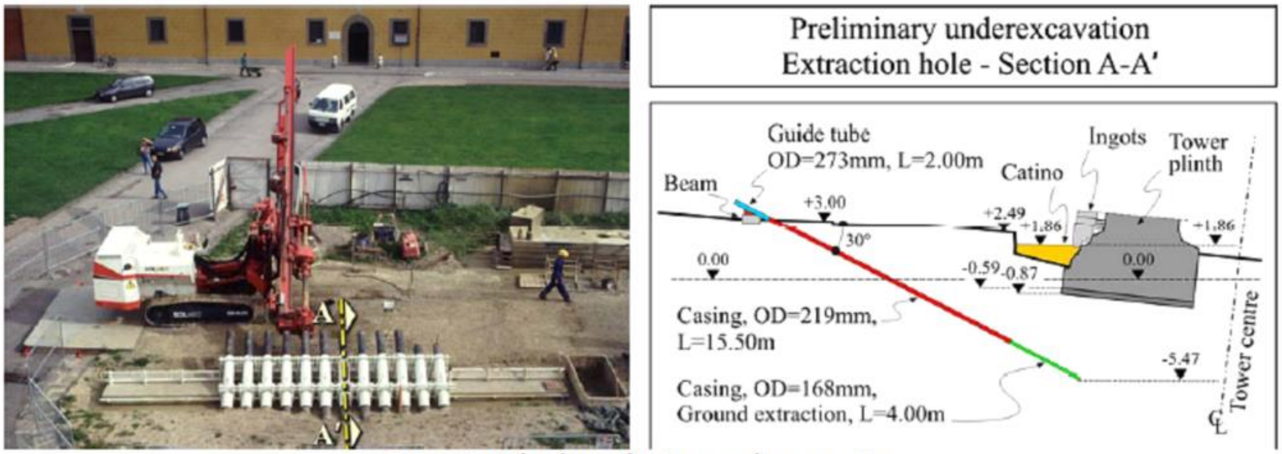


Figure 9. Holes for preliminary underexcavation.

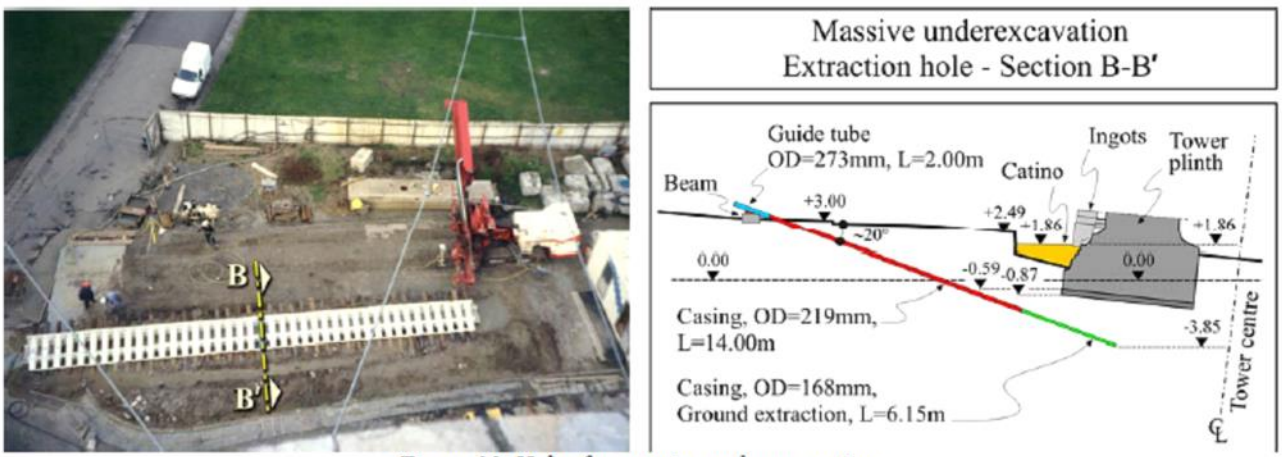


Figure 10. Holes for massive underexcavation.

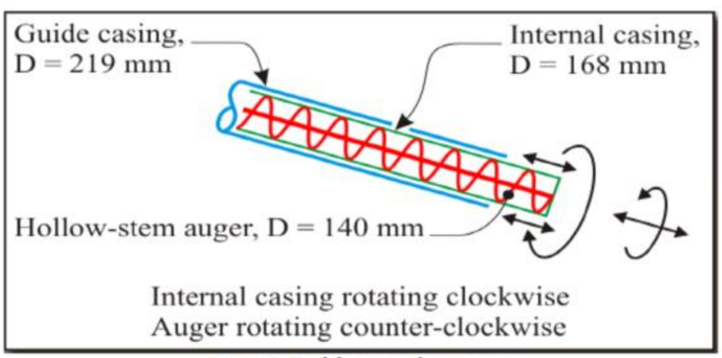


Figure 8. Tool for ground extraction.

Tower of Pisa (cont.)

Wednesday, August 17, 2011 12:45 PM

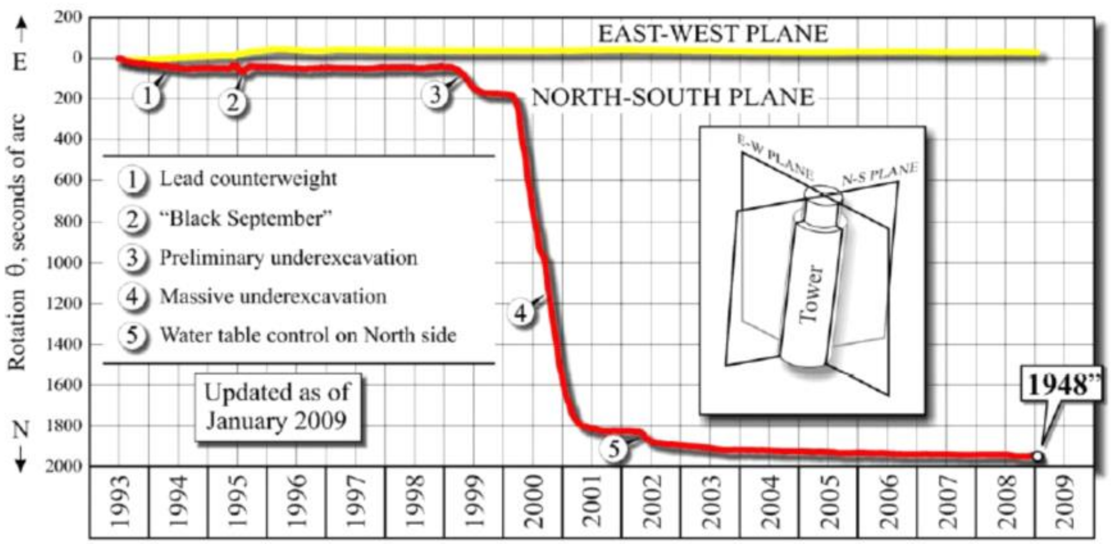


Figure 16. Rotation of Tower foundation as result of stabilization works.

1948/60/60=0.5411

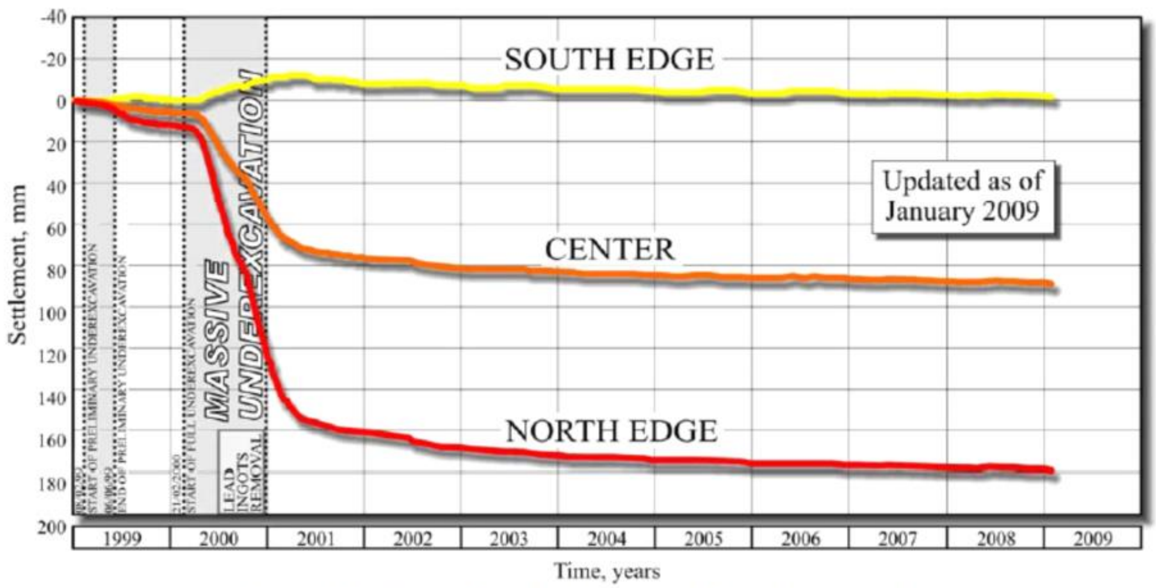


Figure 17. Settlement of Tower foundation as result of stabilization works.

General Design Goals and Process

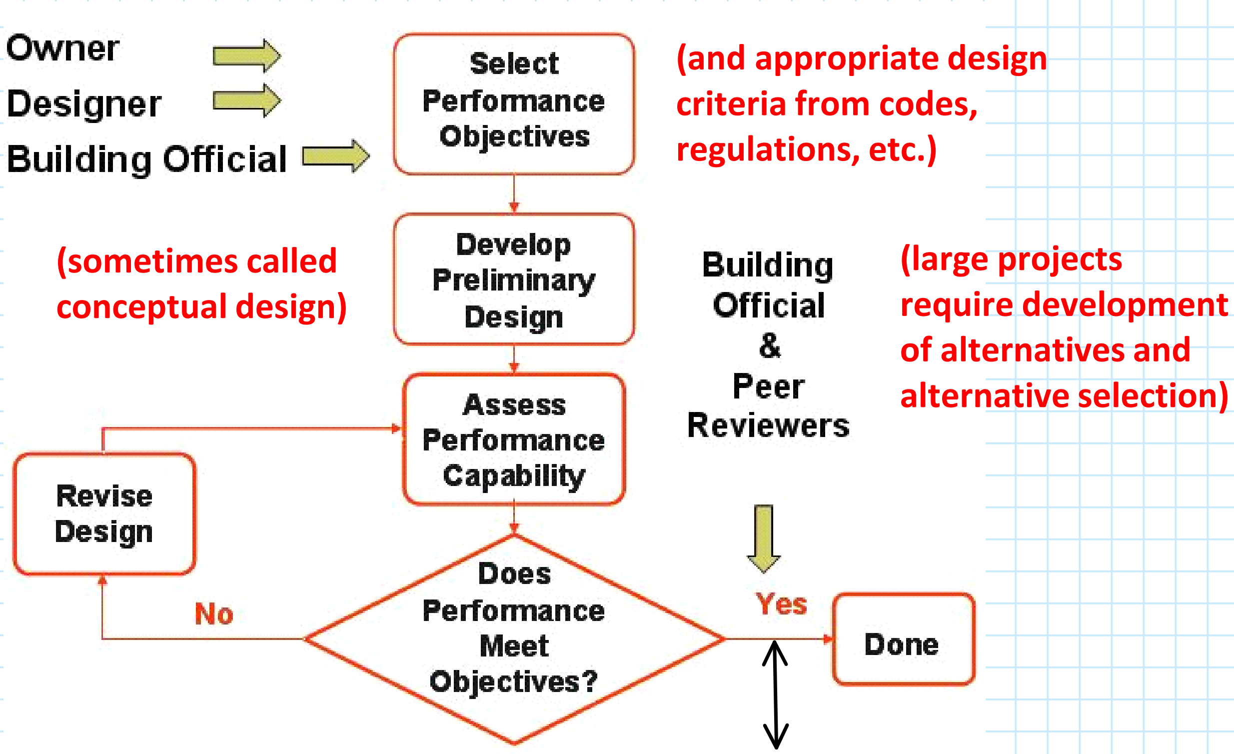
Tuesday, October 3, 2023 12:45 PM

Goals

- Safety
- Serviceability
- Economy

✓The optimum foundation solution transfers the superstructure loads to the ground in a way that minimizes cost (construction and maintenance) over the life of the structures without sacrificing safety or performance

Design Process



Value Engineering

Pasted from <http://www.ahm531.com/Images/pbsd.jpg>

Value engineering is usually done by an independent design firm. In this process, the proposed design is reviewed to see if it can be made more cost-effective, more efficient, constructed more quickly, or if value can be added without increasing the cost.

Foundation Design Process

Wednesday, August 17, 2011 12:45 PM

Foundation Design Process

1. Determination of Design Loads (in conjunction with structural engineer)
2. Subsurface Investigation (sufficient depth and extent for the foundation type)
3. Selection of Suitable Type of Foundation (based on loads and ground conditions)
4. Final Selection, Placement and Proportioning of Foundation Elements (optimization - calculations play a large role)
5. Construction

Determination of Design Loads

- Dead loads (i.e., loads from self-weight due to gravity)
- Live loads
  - People
  - Traffic
  - Equipment
- Wind
- Snow
- Water (Flood)
- Seismic
- Blast (as applicable)

Required Information

- Magnitude of load
- Direction (vertical, horizontal, inclined)
- Point of Application (centered or eccentric)

Table 2-1 Typical vertical loads for residential buildings with reinforced concrete frame and brick walls

Load type	Load per floor
Distributed load	12 kN/m^2
Minimum column load	100 kN
Average column load	200 kN
Maximum column load	300 kN

1 kN ≈ 225 lb

100 kN ≈ 225000 lb = 225 kips

1 kN/m^2 = 1 kPa ≈ 0.01 tsf = 20 psf

12 kN/m^2 ≈ 240 psf ≈ 0.25 ksf

Foundation Design Process (cont.)

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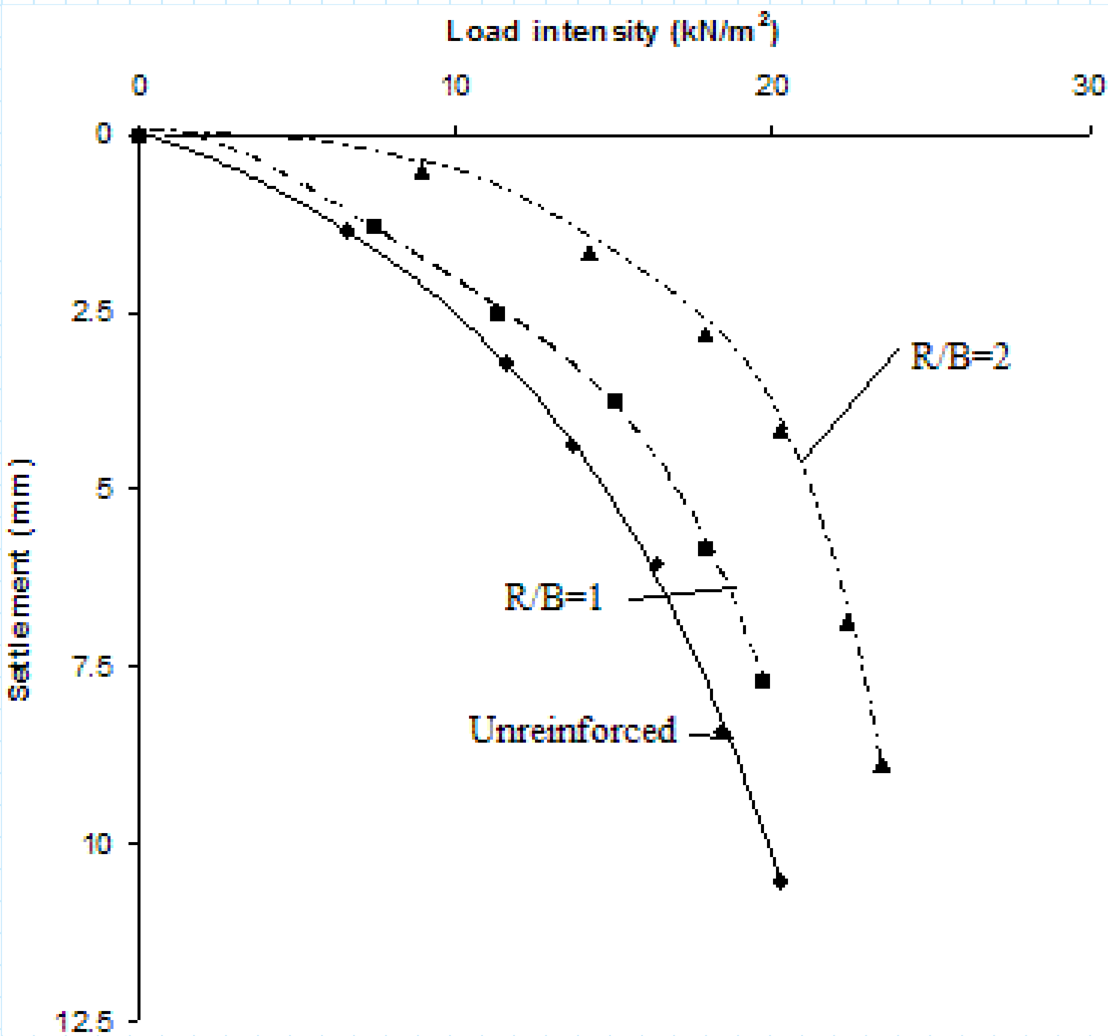
Subsurface Investigations (see also Ch. 7)

- Existing Performance Data (e.g., field monitoring, Load test data) (see below)
- Existing Subsurface Data (from previous or nearby geotechnical reports)
- Drilling, Trenching, Test Pits
- Sampling (Disturbed vs Undisturbed)
- In situ testing (e.g., SPT, CPT, borehole shear, pressure meter tests)
- Laboratory Testing (shear strength, consolidation, shrink, swell, collapse, etc.)

Existing Performance Data



Load test for a shallow foundation (B = footing width)

Foundation Design Process (cont.)

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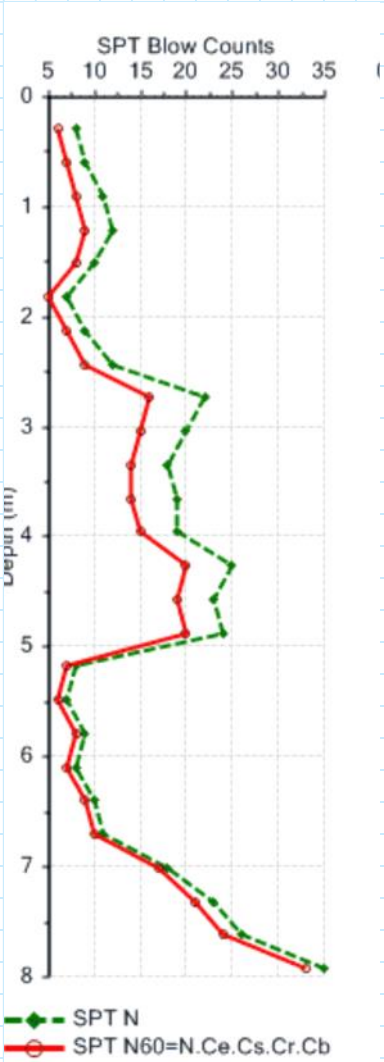
Existing Subsurface Data

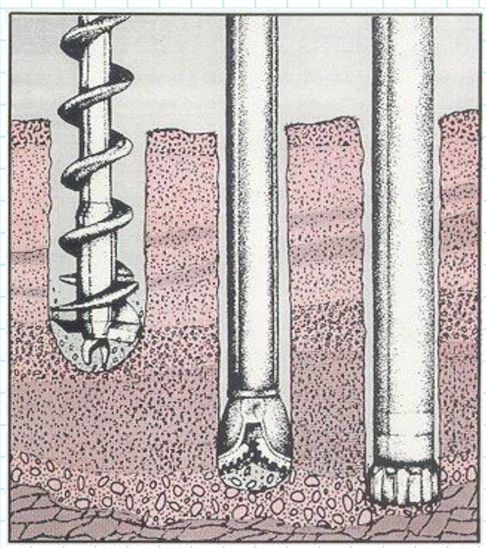
Drill log with SPT (Standard Penetration Test) Blow Count (i.e., N values)



Pasted from <http://www.geotechnicaldrillingontario.com/wp-content/uploads/2011/03/drillfocus3.jpg>

Drilling

- Auger (left)
- Rotary (middle)
- Coring (right)



Pasted from <http://www.thewaterexperts.com/goodsservices.htm>

Foundation Design Process (cont.)

Wednesday, August 17, 2011 12:45 PM

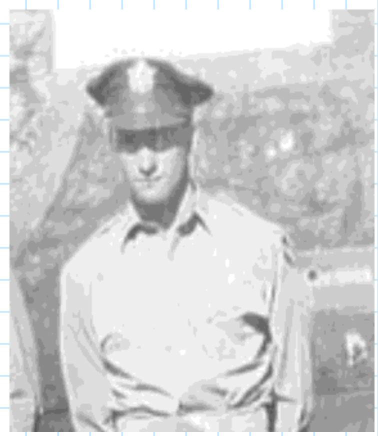
Trenching

Pasted from <http://www.uvu.edu/gel/data/region.html>

Trench safety

http://www.osha.gov/pls/oshaweb/owadisp.show_document?p_table=STANDARDS&p_id=10932

Don Hall, former Vernal man and well-known here during his youth and school days, who was killed in Salt Lake City Tuesday when a trench in which he was working caved in and suffocated him. He was a former bomber navigator in Italy. Funeral services will be held Monday at the Maeser Chapel.



Funeral services for Don Curtis Hall, 27, son of Mr. and Mrs. Thomas B. Hall and husband of Elizabeth Bartlett Hall, will be conducted at the Maeser Ward Chapel on Monday at 1 p.m.

Mr. Hall, a former air corps navigator, was accidently killed Tuesday at 1 p.m. in Salt Lake City when he was buried beneath the sandy loam of a trench in which he was working.

An apprentice plumber under the GI plan, Mr. Hall was completing a sewer trench when the cave-in occured. He suffocated before his fellow workmen could reach him. The trench had been dug from the house to the street and Mr. Hall was removing dirt at the street preparatory to connecting the sewer line with the main, according to witnesses.

Foundation Design Process (cont.)

Wednesday, August 17, 2011 12:45 PM

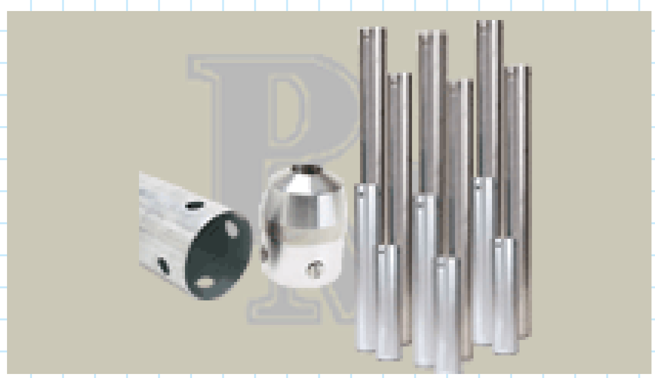
Test Pits

Pasted from <http://www.robertsongeoconsultants.com/index.php?page=imggalleries&id=22&library=2&title=field investigation>

Sampling

Disturbed

Pasted from <http://torquato.dthrotarydrilling.com/geotech-tools.html>



Undisturbed

Pasted from <http://www.premat.com.sg/dht/soilsamplingtool.html>

Foundation Design Process (cont.)

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In situ Testing (Common Types)

- Standard Penetration Test (SPT)
- Cone Penetrometer Test (CPT)
- Dilatometer Test (DMT)
- Borehole Shear Test (BST)
- Pressure meter Test (PMT)
- Vane Shear Test (VST)

Laboratory Testing (Common Types)

- Compaction Tests
- Consolidation
- Shear Strength
  - Unconfined compression test
  - Direct Shear Test
  - Triaxial Shear Test
- California Bearing Ratio (CBR)
- Permeability Test
  - Falling head
  - Constant head

Foundation Design Process (cont.)

Wednesday, August 17, 2011 12:45 PM

Selection of Suitable Type of Foundation

- Suitability
- Constructability
- Cost
- Expected Performance

Resources for Selection of Foundation Type

- Codes of practice
- Local practice
- Contractor's experience and capability
- Project specific factors and requirements

Final Selection, Placement and Proportioning of Foundation Elements

- Cost estimate for possible foundation types
  - Most economical
- Load-carrying capacity
- Depth of foundation elements
- Develop construction specifications

Construction

- Procurement of specialty contractor (sometimes required for deep foundations)
- Observation (inspection) of construction activities
- Foundation testing (load tests on piles, pull-out tests for anchors and soil nails, integrity tests on drilled piles and shafts)

Limit State or Working Stress Design or Allowable Stress Design

Wednesday, August 17, 2011 12:45 PM

Limit States

- Limit state is a condition to be avoided (e.g., collapse, unacceptable deformation or performance). A Limit State is a defined condition beyond which a structural component ceases to satisfy the provisions for which it is designed.

- Two types of limit states in working stress design
  - **Serviceability Limit State**
    - State where there is an inability of the system to perform its intended function safely or efficiently (e.g., excessive settlement, deformation or premature loss of operation).
  - **Ultimate Limit State**
    - State where there is collapse, catastrophic failure, or extreme deformation (e.g., structural collapse, bearing capacity failure, global failure of slope or retaining wall.

Serviceability Limit State

Excessive settlement of house has rendered part of the house uninhabitable.

Pasted from <http://www.structural-design-solutions.com/Foundation_Wall_Cracks.html>

Excessive deformation and buckling of welded-wire face of mechanically stabilized earth (MSE) wall. I-15 Reconstruction Project, Salt Lake City

Could the deformation of the wire face lead to premature corrosion and damage to wall system?

Limit State Design (cont.)

Wednesday, August 17, 2011 12:45 PM

Ultimate Limit State

Toppling of newly constructed apartment building in Shanghai, China

Pasted from <http://www.deseretnews.com/photo/gallery/slideshow/?storyId=695263278&photoId=690319141>

Failure of retaining wall at Judge Memorial High School, Salt Lake City Utah.

Pasted from <http://www.austin360.com/blogs/content/shared-gen/blogs/austin/blotter/entries/news/>

Retaining wall failure, San Antonio, Texas. Note that house above wall is threatened.

Ultimate Limit State

Earthquake (liquefaction) damage to bridge during 1964 Alaska Earthquake

Liquefaction damage to multi span bridge during 1964 Niigata, Japan Earthquake.

Limit State Design (cont.)

Wednesday, August 17, 2011 12:45 PM

Limit States for Shallow Foundation Design

Limit State	Nature of Limit State	Consequences
IA-1	Classical bearing capacity failure (ULS)	Excessive movement/collapse of foundation causes serious damage, partial collapse or complete collapse of structure
IA-2	Structural failure of foundation element (ULS)	Column is inadequately supported by foundation element, punching through it; this causes serious damage, partial or complete collapse of superstructure
IB	Excessive differential foundation settlement (ULS)	Excessive differential settlements create excessive additional loads in the structure, leading to structural damage
II	Excessive settlement (total or differential) (SLS)	Excessive settlements lead to serviceability problems, such as access problems, damage to architectural finishings, etc.
III	Stability failure of the whole foundation system or a subset thereof (ULS)	Collapse mechanism develops that encompass the foundations for the building or structure or a part of the foundations (a classical example would be stability failure of a slope on top of which is founded a building)

Design practice requires the engineer to check each possible limit state independently and show that none will be reached under the proposed design with the appropriate factor of safety.

Working Stress Design

Ultimate load

Design load

$$Q_a = \frac{Q_{ult}}{FS} \qquad Q_d \leq Q_a$$

Allowable load

Factor of Safety (FS)

All load and resistance uncertainities are lumped in this single factor

Ultimate load: the load that would lead to an ultimate limit state

Design or working load: the sum of the loads under consideration (unfactored)

Table 2-2 Factors of safety (modified after Vesic 1975)

Category	Typical structures	Observations	Soil exploration: Thorough	Soil exploration: Limited
A	Railway bridges Warehouses Blast furnaces Retaining walls Silos	Maximum design load likely to occur often Ultimate limit states with disastrous consequences	3	4
B	Highway bridges Light industrial and public buildings	Maximum design load may occur occasionally Ultimate limit states with serious consequences	2.5	3.5
C	Apartment buildings Office Buildings	Maximum design load unlikely to occur	2	3

This table is very general for foundation systems. For more specific information, see next page.

Limit State Design (cont.)

Wednesday, August 17, 2011 12:45 PM

Factors of Safety (cont.)

Foundation Analysis by Bowels has good recommendations for safety factors. He evaluates uncertainties and assigns a factor of safety by taking into account the following:

1. Magnitude of damages (loss of life and property damage)
2. Relative cost of increasing or decreasing the factor of safety
3. Relative change in probability of failure by changing the factor of safety
4. Reliability of soil data
5. Construction tolerances
6. Changes in soil properties due to construction operations
7. Accuracy (or approximations used) in developing design/ analysis methods

Pasted from <http://www.geotechnicalinfo.com/factor_of_safety.html>

Typical values of customary safety factors, F.S., as presented by Bowels.

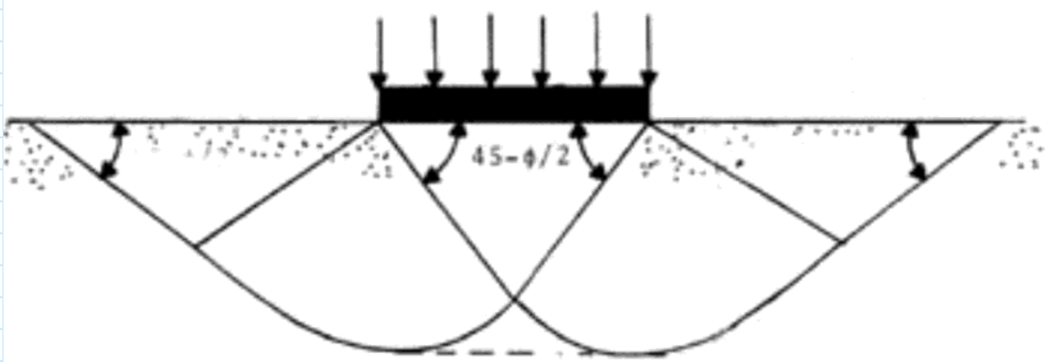
Failure Mode	**Foundation Type**	**F.S.**
Shear	Earthwork for Dams, Fills, etc.	1.2 - 1.6
Shear	Retaining Walls	1.5 - 2.0
Shear	Sheet piling, Cofferdams	1.2 - 1.6
Shear	Braced Excavations (Temporary)	1.2 - 1.5
Shear	Spread Footings	2 - 3
Shear	Mat Footings	1.7 - 2.5
Shear	Uplift for Footings	1.7 - 2.5
Seepage	Uplift, heaving	1.5 - 2.5
Seepage	Piping	3 - 5

Pasted from <http://www.geotechnicalinfo.com/factor_of_safety.html>

Limit State Design - Bearing Capacity Example

Wednesday, August 17, 2011 12:45 PM

100 kips



The above 2' x 2' footing is loaded with 100 kips. The ultimate bearing capacity, qu, of the footing is 50 kips per square foot. Calculate the factor of safety against bearing capacity failure.

FS = q ult / q

FS = 50 ksf / (100 kips / 2 ft x 2 ft) = 50 / 25

FS = 2

Note: The ultimate bearing capacity, q ult, is determined from an analytical solution (i.e., bearing capacity equation). We will learn how to use such equations later in the semester.

Load and Resistance Factor Design (LRFD)

Tuesday, August 29, 2017 6:08 AM

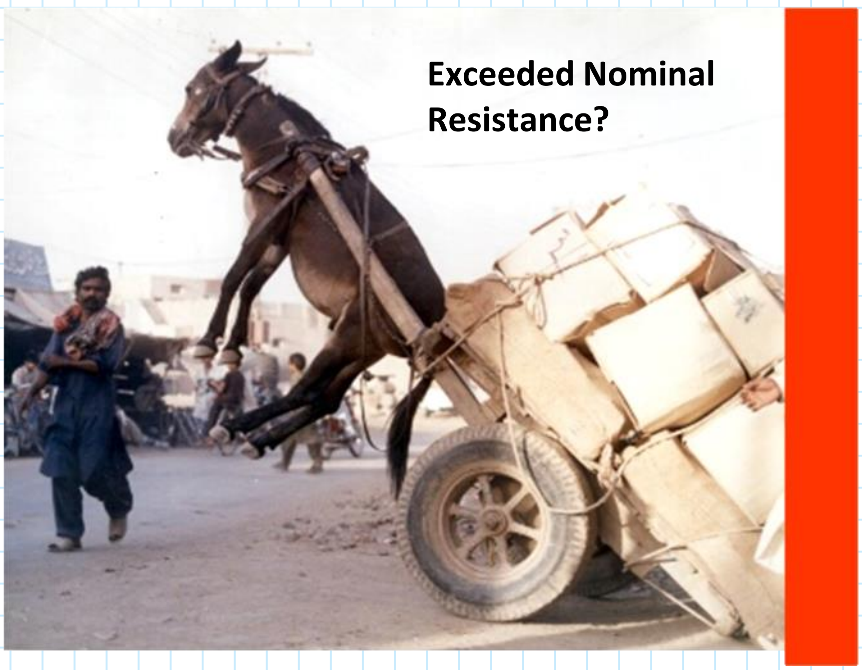
Background

- LRFD is an acronym for load and resistance factor design.
- LRFD is widely used as an alternative to working stress design (WSD) and is popular in structural and geotechnical engineering codes.
- LRFD has been adopted by:
  - ACI (American Concrete Institute)
  - AASHTO (American Association of Highway and Transportation Officials)
  - ASCE (American Society of Civil Engineers)
- Considers uncertainty in both the loads and the soils resistance
  - An important distinction between LRFD and WSD is the specific of uncertainty in LRFD to both the loads or resistances, whereas in WSD the uncertainty of both is blended into the factor of safety.

Basic Equation

$$\Sigma\, (LF)_i Q_{ni} \leq (RF) R_n$$

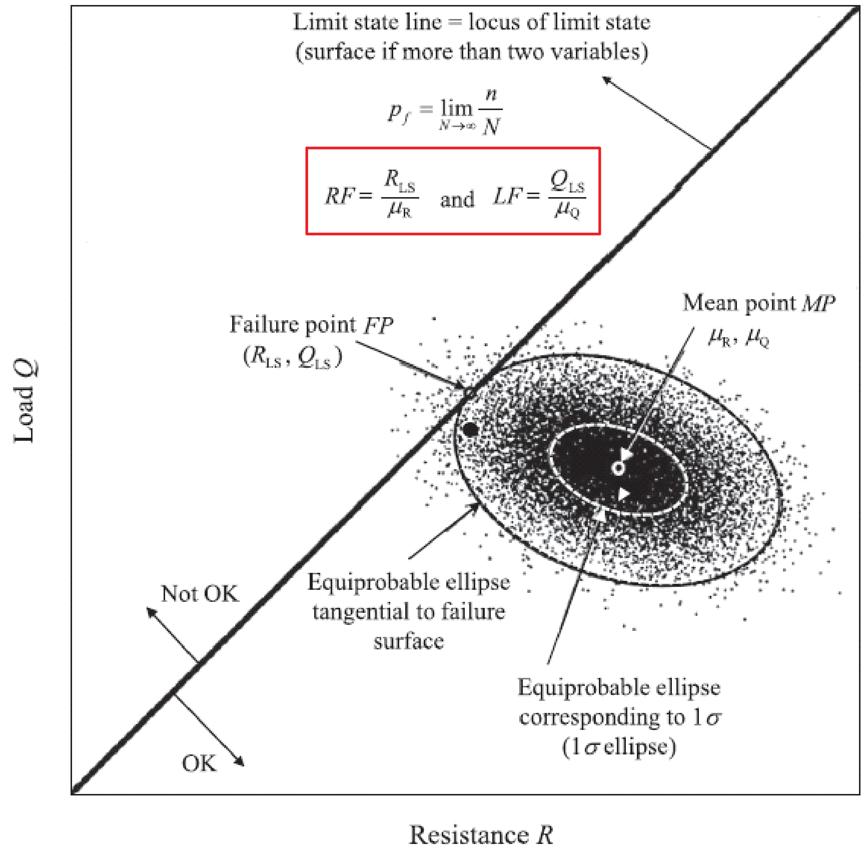
LF = load factors, Q_n = nominal loads, RF = resistance factors, R_n = nominal resistance



- Load factors are used to cover the uncertainty in the nominal loads to ensure that the resistance is not exceeded during the service life of the system.

LRFD (cont.)

Wednesday, August 28, 2024 12:45 PM



LRFD (cont.)

Wednesday, August 17, 2011 12:45 PM

Nominal Loads (Q_n) and Load Factors (LF)

- Nominal means the characteristic or unfactored load calculated for a particular loading case.
- The unfactored or nominal load in geotechnical engineering is calculated using the best estimate or the actual load using analytical or numerical techniques.
- A load factor is applied to the nominal load to account for the uncertainty in the nominal load.
- Load factors vary according to the specific code or agency (see below)

Table 2-3 Load factors

Loads	AASHTO (1998)	ACI (2002)	AISC (1994)	API (1993)	MOT (1992)	NRC (1995)
Dead	1.25–1.95 (0.65–0.9)	1.2 (0.9)	1.2–1.4 (0.9)	1.1–1.3 (0.9)	1.1–1.5 (0.65–0.95)	1.25 (0.85)
Live	1.35–1.75	1.6	1.6	1.1–1.5	1.15–1.4	1.5
Wind	1.4	1.3	1.3	1.2–1.35	1.3	1.5
Seismic	1.0	1.4	1.0	0.9	1.3	1.0

Note: Values in parentheses apply when the load effects tend to resist failure for a given load combination, that is, when the loads have a beneficial effect.

Resistance Factors (RF)

- Resistance is a quantifiable value that defines the point beyond which the particular limit state under investigation for a particular component will be exceeded.
- Resistance can be defined in terms of:
  - Load/Force (static/ dynamic, dead/ live)
  - Stress (normal, shear, torsional)
  - Strain or deformation
  - Number of cycles
  - Temperature
- A resistance factors is a partial factor of safety applied to the resistance of the component to account for uncertainty in the actual resistance or to place additional conservatism in the resistance value used in design.

LRFD (cont.)

Wednesday, August 17, 2011 12:45 PM

Limit States in LRFD

- Strength Limit State (similar to Ultimate Limit State in WSD)
- Service Limit State
- Extreme Event Limit State (wind, earthquake, flood, etc.)
- Fatigue Limit State

Strength Limit State

Failure of a retaining wall system resulting from shear failure within the backslope behind the retaining wall

Service Limit State

Rotation of a pedestal for a bridge girder. Although the pedestal has not reached an ultimate state, such rotation is undesirable and dangerous to safety.

LRFD (cont.)

Wednesday, August 17, 2011 12:45 PM

Extreme Event Limit State

Collapse of the I-880 double decker bridge structure during the 1989 Loma Prieta California Earthquake. Multiple fatalities resulted from this collapse. This collaspe was caused by shearing the supporting columns due to lateral earthquake loads.

Fatigue or Corrosion Limit State

Corrosion and fatigue of a bent for I-15 bridge at South Temple, Salt Lake City Utah.

This bridge and similar bridges were replaced by new construction in 1998 to 2001.

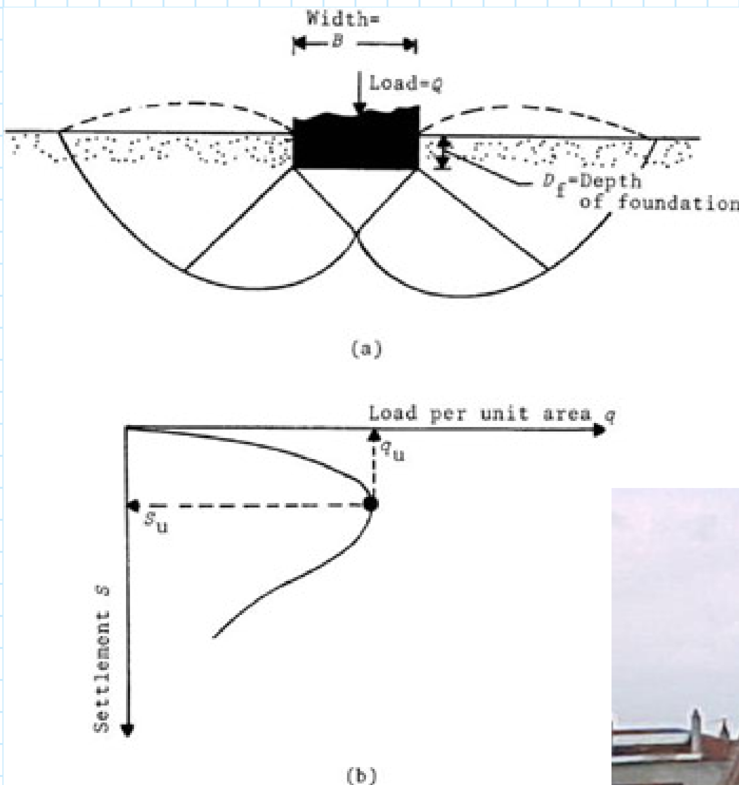
LRFD (cont.)

Wednesday, August 17, 2011 12:45 PM

Limit States for Shallow Foundations

- **LRFD Strength Limit State**
  - *Bearing Resistance*
  - *Sliding*
  - *Eccentricity Limits (Overturning)*
- **LRFD Service Limit State**
  - *Overall Stability*
  - *Vertical (Settlement) and Horizontal Movements*

Bearing Resistance and Bearing Capacity Failure



Bearing capacity failure of building resulting of liquefaction of foundation soil (Adapazari, Turkey)

Pasted from <http://www.geerassociation.org/GEER_Post%20EQ%20Reports/Duzce_1999/Adapazari.htm>

LRFD (cont.)

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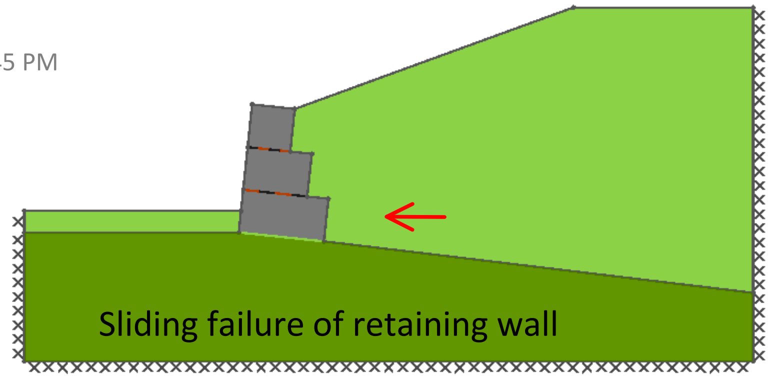
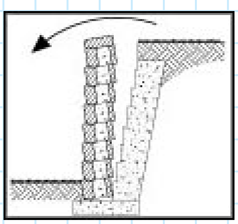
Sliding



Sliding of house atop foundation. If the foundation is not embedded enough, it can slide also.

Pasted from <http://www.ask.com/wiki/File:089srUSGSoffFoundation.jpg?qsrc=3044>

Overturning



Overturning failure of retaining wall

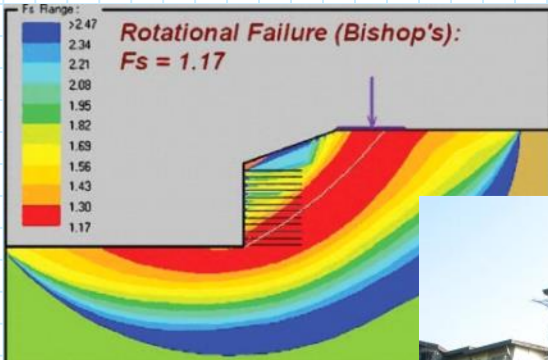
Pasted from <http://3.bp.blogspot.com/_dp6ybQVtWsw/TL-e5cAybKI/AAAAAAAAADo/WPZcPRM7qgw/s1600/building_collapse.bmp>

Overturning failure of apartment building on deep foundation

LRFD (cont.)

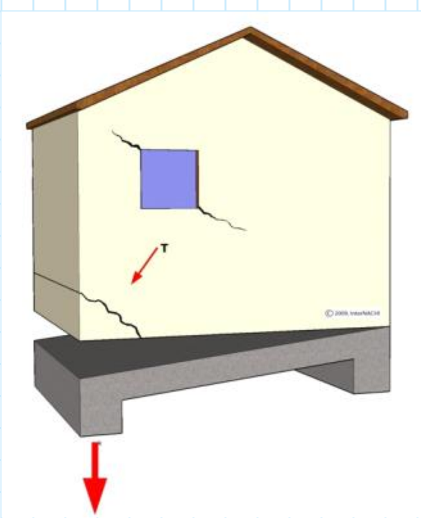
Wednesday, August 17, 2011 12:45 PM

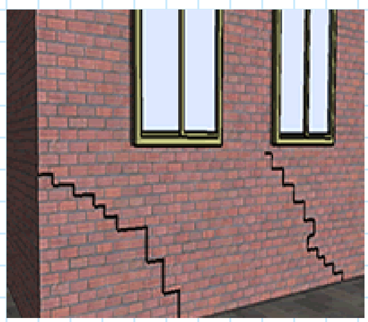
Overall (i.e., Global) Stability

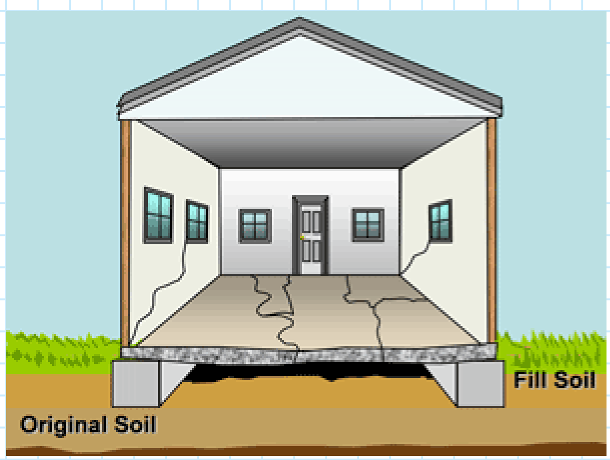


Global failure of MSE wall (Philippines)
Note that in a global failure the failure occurs behind and underneath the wall

Settlement

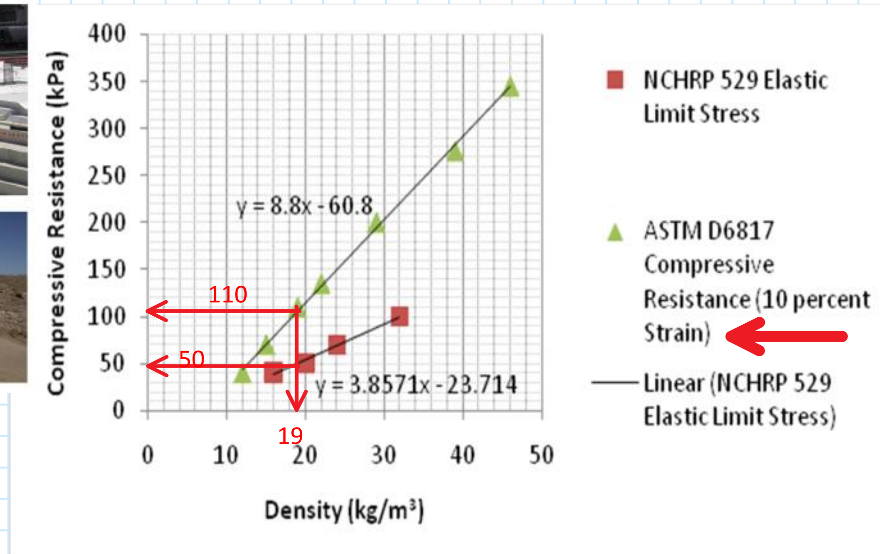






LRFD - EPS Geofoam Example

Wednesday, August 17, 2011 12:45 PM



For this example, EPS19 (19 kg/m^3) based on ASTM D6817 will be used. This density of EPS19 was selected for this example because it is commonly used in highways in both the U.S. and Europe and is the density used on the I-15 Reconstruction Project. The nominal compressive resistance at 10 percent vertical strain for EPS19 is 110 kPa from ASTM D6817. The estimated ***elastic limit stress for EPS19 as required by NCHRP 529 is calculated as 110 * 0.45 or about 50*** kPa based on the relations given in the above figure. **In other words, the D6817 value has been factored by a resistance factor of 0.45.**

Using NCHRP 529 and the dead load of 14.1 kPa and the live load of 15 kPa for this example produces:

DL+1.3LL $\leq 0.45\ \sigma_{r\ @10@strain}$

DL + LL = (14.1 kPa + 1.3*15.0)*1.2 = 40.32 kPa

where 14.1 is the dead load of the pavement system, 15 is the live load from traffic loading, and 1.3 is an impact load factor that is adjusted the live load for impact effects. In addition, another load factor of 1.2 is applied to the live and dead load, as required by NCHRP 529

40.32 < 0.45*110
40.32 < 49.5

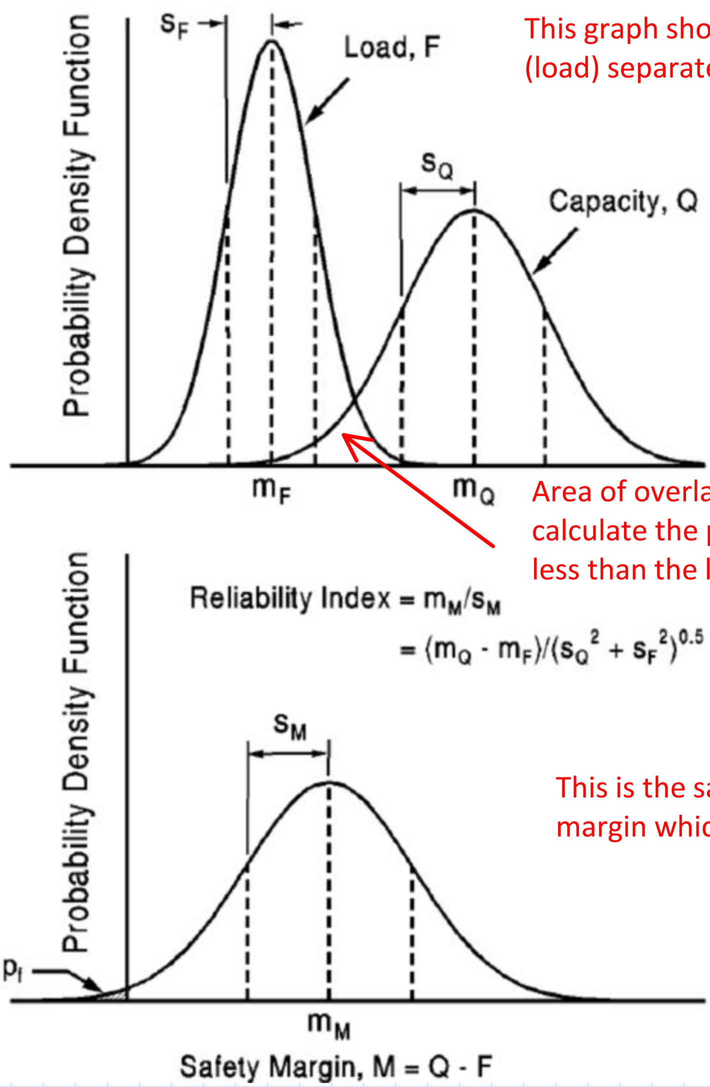
The serviceability limit state safety factor for this loading combination using NCHRP 529 is:
FS = 49.5/40.32=1.23

Reliability Based Design (cont.)

Wednesday, August 17, 2011 12:45 PM

Calculating probability of failure

Special lecture for Korean Geotechnical Society, Seoul, 9 July 2004 (KK Phoon)



This graph shows the capacity (resistance) and demand (load) separately.

Area of overlap of these distributions can be used to calculate the probability of failure because the capacity is less than the load or demand.

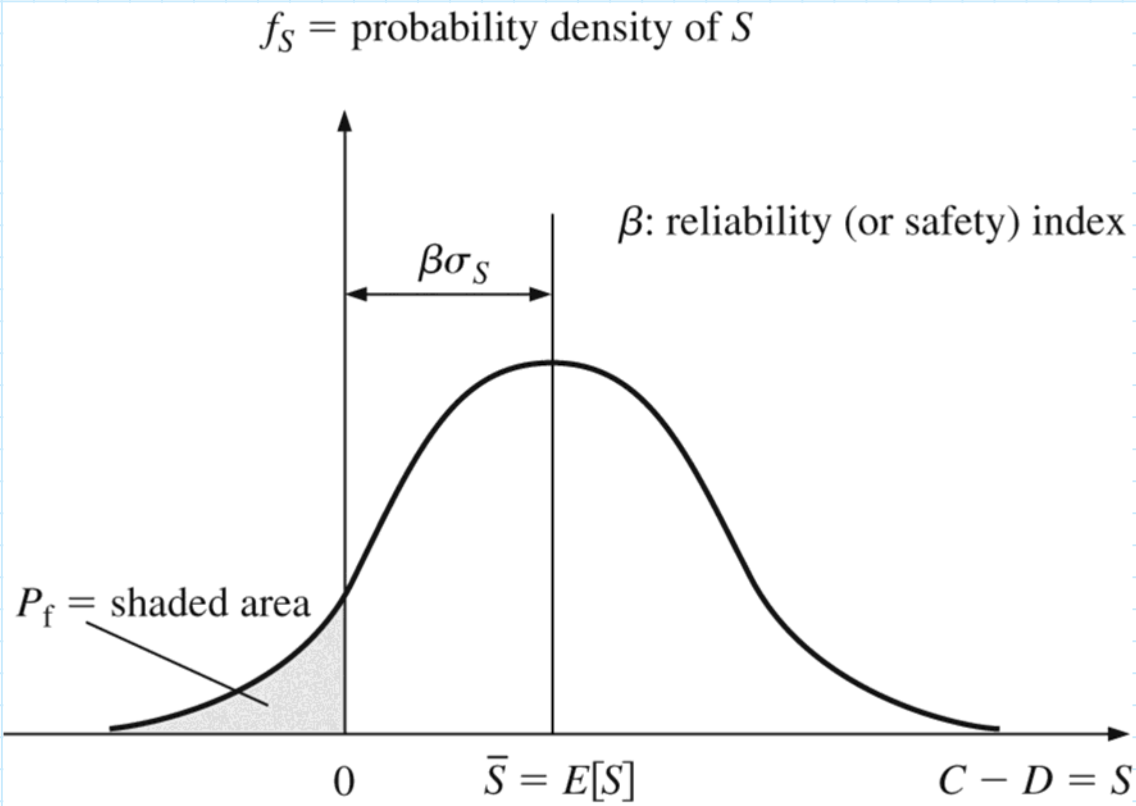
This is the same as Salgado where M is the safety margin which is called S.

- Reliability based design uses statistical methods to account for the uncertainty in both the loads (demand) and capacity (resistance).
- It can be used to express the reliability of the system (reliability index) and the probability of failure and is more rigorous than LRFD and WSD.
- It is often the basis for performance-based design and risk assessment
- $ Loss = ($ Value | % Loss) (% Loss | Event) (Event)

  Risk *Exposure* *Vulnerability* *Hazard*

Reliability Based Design (cont.)

Wednesday, August 17, 2011 12:45 PM



C = capacity
D = demand
S = safety margin = C - D

If S > 0, system has not failed, S < 0, system has failed
P_f = probability of failure = area of shaded area = p(C<D)

E[S] = Expected (mean) value of S = $C_{avg} - D_{avg}$
C_{avg} = Expected (mean) value of C
D_{avg} = Expected (mean) value of D

β = Reliability Index
$\beta = E[S]/\sigma_S = (C_{avg} - D_{avg})/(\sigma_C^2 + \sigma_D^2)^{0.5}$ (see previous page)

σ_S = standard deviation of S = standard deviation of C-D
σ_C = standard deviation of C
σ_D = standard deviation of D

Reliability Based Design - Probability of Failure

Tuesday, September 5, 2017 9:08 AM

$$P_f = \Phi(-\beta) \quad \text{or} \quad \beta = -\Phi^{-1}(P_f)\text{, with } \Phi(Z) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{Z} e^{-\frac{z^2}{2}}dz,$$

β	P_f	β	P_f	β	P_f
3.00	0.001358	2.00	0.022853	1.00	0.159127
2.95	0.001598	1.95	0.025702	0.95	0.171553
2.90	0.001877	1.90	0.028842	0.90	0.184581
2.85	0.002199	1.85	0.032294	0.85	0.198207
2.80	0.002570	1.80	0.036081	0.80	0.212424
2.75	0.002997	1.75	0.040224	0.75	0.227219
2.70	0.003486	1.70	0.044749	0.70	0.242578
2.65	0.004047	1.65	0.049668	0.65	0.258482
2.60	0.004687	1.60	0.055013	0.60	0.274910
2.55	0.005415	1.55	0.060802	0.55	0.291837
2.50	0.006243	1.50	0.067057	0.50	0.309233
2.45	0.007180	1.45	0.073798	0.45	0.327069
2.40	0.008240	1.40	0.081046	0.40	0.345308
2.35	0.009434	1.35	0.088818	0.35	0.363914
2.30	0.010778	1.30	0.097132	0.30	0.382846
2.25	0.012285	1.25	0.106004	0.25	0.402062
2.20	0.013971	1.20	0.115447	0.20	0.421519
2.15	0.015853	1.15	0.125472	0.15	0.441169
2.10	0.017948	1.10	0.136090	0.10	0.460964
2.05	0.020275	1.05	0.147307	0.05	0.480857
				0.00	0.500798

Reliability in Biomechanics, First Edition. Ghias Kharmanda and Abdelkhalak El Hami.

Reliability Based Design - Probability of Failure

Tuesday, September 5, 2017 9:09 AM

Special lecture for Korean Geotechnical Society, Seoul, 9 July 2004 (KK Phoon)

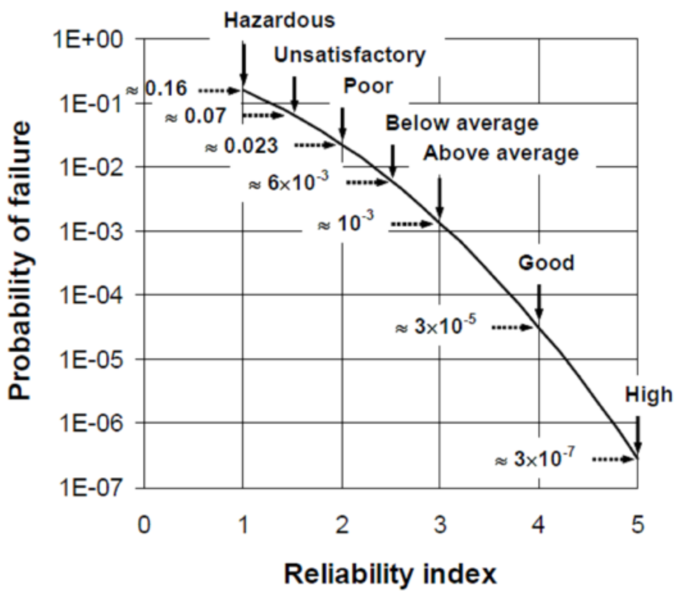


Figure 2. Relationship between reliability index (β) and probability of failure (p_f) (adapted from Table US Army Corps of Engineers 1997, Table B-1)

Based on the above chart, a reliability index, β, of about 3 or higher is desirable. The associated probability of failure (p_f) for $\beta = 3$ is about 1e-03 or 1 in 1000 (see table on next page).

The acceptable probability of failure is a function of the criticality of the structure and its societal importance. For example, nuclear facilities are designed with very low probabilities of failures because the consequences from failure have very high societal and environmental costs.

Reliability Based Design - Example Problem

Tuesday, September 5, 2017 12:45 PM

Given:

- The ultimate strength of a foundation member of 3000 psf (exact) in compression, on average.
- This coefficient of variation is 0.2 (exact) for this member.
- The member is loaded with a compressional stress that is equivalent to 1000 psf (exact), on average.
- The coefficient of variation for compressive stress is 0.5 (exact).
- Coefficient of variation = standard deviation / mean

Solution below (**Hidden**)

Required:
- Reliability Index, β
- Probability of Failure, P(f)

Assumptions:
- Failure occurs with the applied compressional stress exceeds the ultimate strength of the foundation member.
- The variation in strength and load are normally distributed.

Solution:

$\beta = E[S]/Q\sigma_S = (C_{avg} - D_{avg})/(\sigma_C^2 + \sigma_D^2)^{0.5}$ (see previous page)

σ_S = standard deviation of S = standard deviation of C-D

σ_C = standard deviation of C

σ_D = standard deviation of D

β = (3000-1000)/(((0.2x3000)^2+(0.5x1000)^2)^0.5)=2.5607 $\approx$ 2.56

P(f) for β = 2.55 is 0.005414 (table on previous page)

P(f) for β = 2.60 is 0.004687

P(f) for β = 2.56 using linear interpolation is:

P(f) = 0.005414+(0.005414-0.004687)/(2.60-2.55)*(2.55-2.56)=0.005269

<u>Summary of Answers</u>

β = 2.56

P(f) = 5.27 to x 10^{-3}

Reliability Based Design - Example

Wednesday, August 17, 2011 12:45 PM

Reliability Engineering and System Safety 47 (1995) 141-151

0951-8320/95/$9.50
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0951-8320(94)00063-8

Geotechnical system reliability of slopes

R. N. Chowdhury & D. W. Xu

Department of Civil and Mining Engineering, University of Wollongong, P.O. Box 1144, Wollongong, 2500 NSW, Australia

The geometry of the slope is as shown in Fig. 1 and this example is the well-known Congress Street cut. The clay deposit is divided into three layers. One set of mean values and standard deviations of the undrained shear strength are shown in Table 1(a). The set of slip surface tangential to bottom of clay 2 is denoted as set 2 and the set of slip surfaces tangent to bottom of clay 3 is denoted as set 3. The top layer of sand has negligible influence on stability because of zero cohesion and low normal stress and has, therefore, been neglected as in the analysis.

The results of analysis for the upper and lower hounds of the system failure probability are shown in Table 1(h) along with failure probability associated with the critical slip surface considering either slip surface set 2 or slip surface set 3 or both together i.e. the whole system. In this example, the results for set 2 and 3 considered separately show no difference between critical slip surface probability and system failure probability bounds. Even when the whole system is considered together, the lower bound is identical to the critical failure surface probability but the upper bound is higher by nearly 40%.

Reliability Based Design - Example

Wednesday, August 17, 2011 12:45 PM

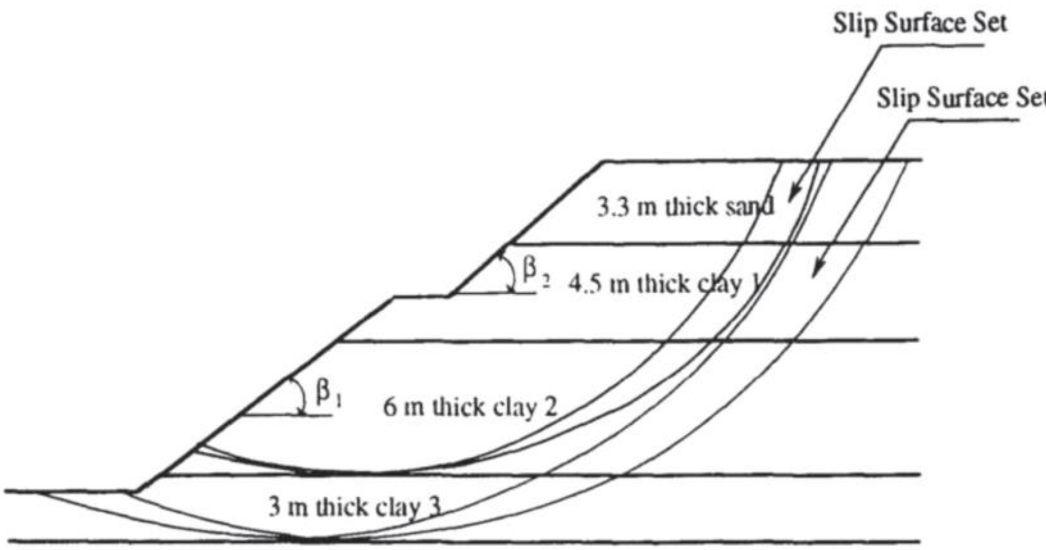

Fig. 1. Cross section of congress street cut.

Table 1. (a) Shear strength parameters for Example 1

	Clay 1 c_1	Clay 2 c_2	Clay 3 c_3
Mean (kPa)	55	43	56
Standard deviation (kPa)	20·4	8·2	13·2
Coefficient of variation	37%	19%	24%

The coefficient of variation has been expressed in percent here and not in decimal fraction.

(b) Computed failure probabilities for Example 1

Condition (1)	Failure probability (2)	Safety factor (3)
Layer 2 only (6 slip surfaces)		
Critical slip surface	0·265 92	1·117 8
Upper bound of system	0·265 92	—
Lower bound of system	0·265 92	—
Layer 3 only (6 slip surfaces)		
Critical slip surface	0·273 89	1·108 8
Upper bound of system	0·273 89	—
Lower bound of system	0·273 89	—
Layer 2 and 3 (12 slip surfaces)		
Critical slip surface	0·273 89	1·108 8
Upper bound of system	0·447 33	—
Lower bound of system	0·273 89	—

Settlement damage

Wednesday, August 17, 2011 12:45 PM

Settlement damage (cont.)

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Cracking and degree of damage

Table 2-4 Cracking width and the associated damage and serviceability/safety issues for residential, commercial, and industrial buildings (modified after Thorburn 1985)

Crack width (mm)	Degree of damage: Residential	Commercial	Industrial	Serviceability or safety issues
<0.1	None	None	None	None
0.1–1	Slight	Slight	Very slight	Cracks may be visible
1–2	Slight to moderate	Slight to moderate	Very slight	Possible penetration of humidity
2–3	Moderate	Moderate	Slight	Serviceability may be compromised
3–15	Moderate to severe	Moderate to severe	Moderate	Ultimate limit states may be reached
>15	Severe to dangerous	Moderate to dangerous	Severe to dangerous	Risk of collapse

Very slight: visible on close inspection; correctable with interior design/decoration tools.

Slight: external cracks may need to be filled for watertightness; doors and windows may jam slightly.

Moderate: replacement of small amount of brickwork needed; service pipes may be severed; jamming doors/windows.

Severe: replacement of portions of walls needed; window/door frames distorted; uneven floors; service pipes severed; leaning or bulging walls.

Dangerous: beams lose bearing; walls require shoring; windows broken by distortion; danger of instability.

Tolerable Angular Deflections for Frame Structures

$$\alpha = \begin{cases} \dfrac{1}{170} & \text{(ULS: frame cracking)} \\ \dfrac{1}{300} & \text{(SLS: wall craking)} \\ \boxed{\dfrac{1}{500} \text{(unlikely to lead to either SLS or ULS)}} \end{cases}$$

✓In design, **we use** α_{tol}= **1/500**. This value accounts for uncertainties in both the Skempton & McDonald's observations and the settlement calculations

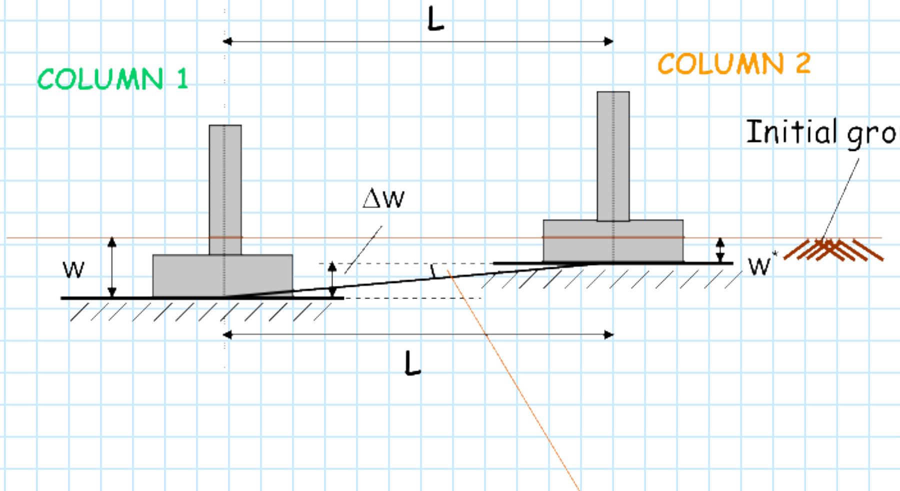
$$\alpha_{expected} \le \frac{1}{500} = 0.002$$

Differential Settlement and Angular Distortion

Wednesday, August 17, 2011 12:45 PM

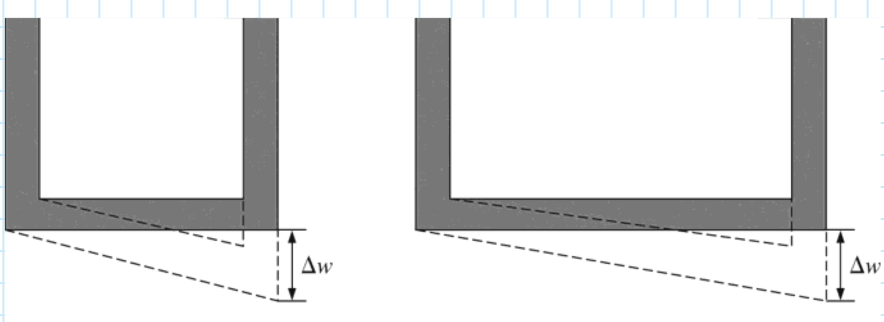
Quantification of Settlement

- Differential Settlement — Differential movement between columns
- Angular Distortion — Differential movement between columns divided by separation distance,



$$\alpha = \frac{\Delta w}{L}$$

Angular distortion of floor slab



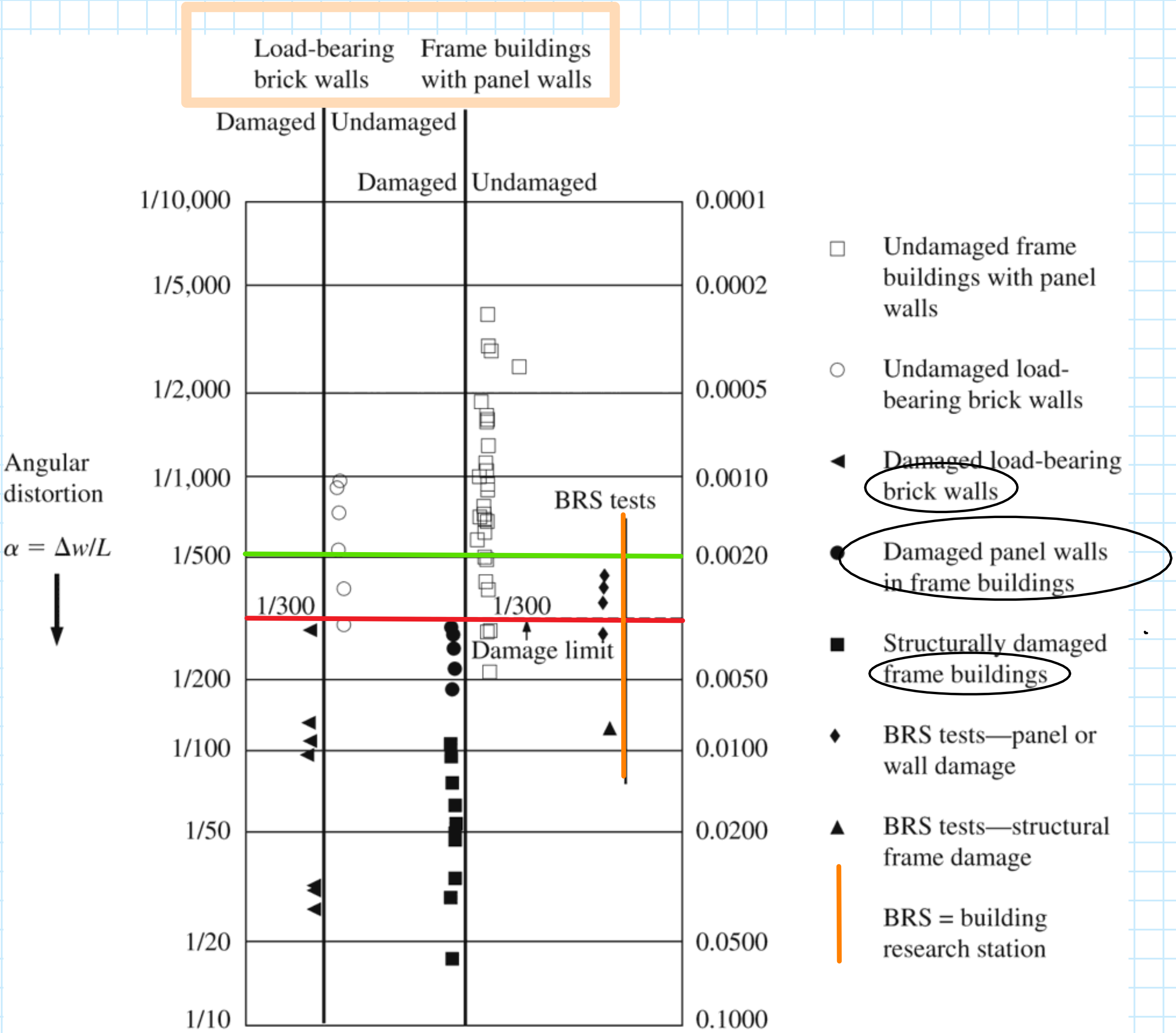
(a) (b)

Skempton and McDonald Criterion

Wednesday, August 17, 2011 12:45 PM

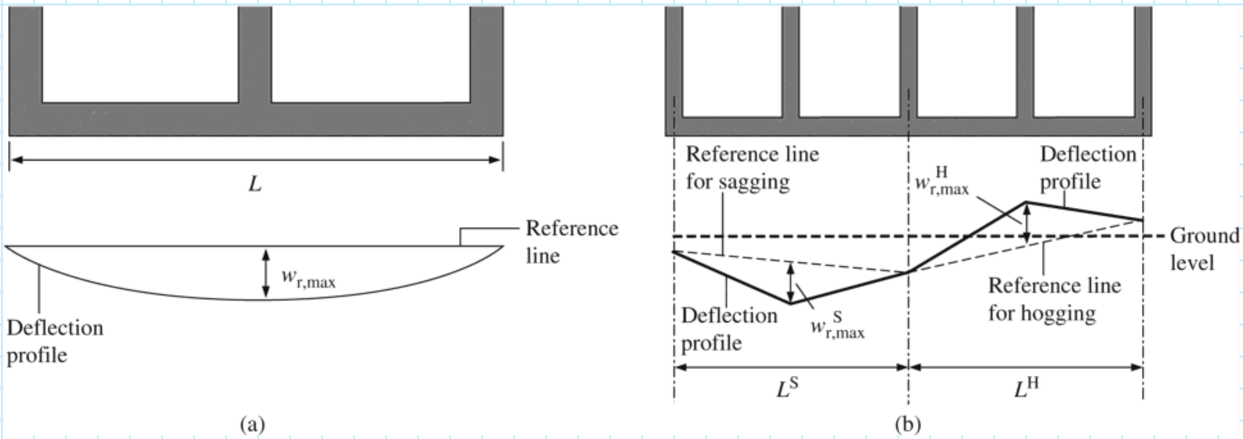
Tolerable Angular Distortion Criterion
Skempton and McDonald (1956)

- Valid only for frame buildings
- Based on the concept of angular distortion – the larger the distortion, the more likely that the building will undergo cracking

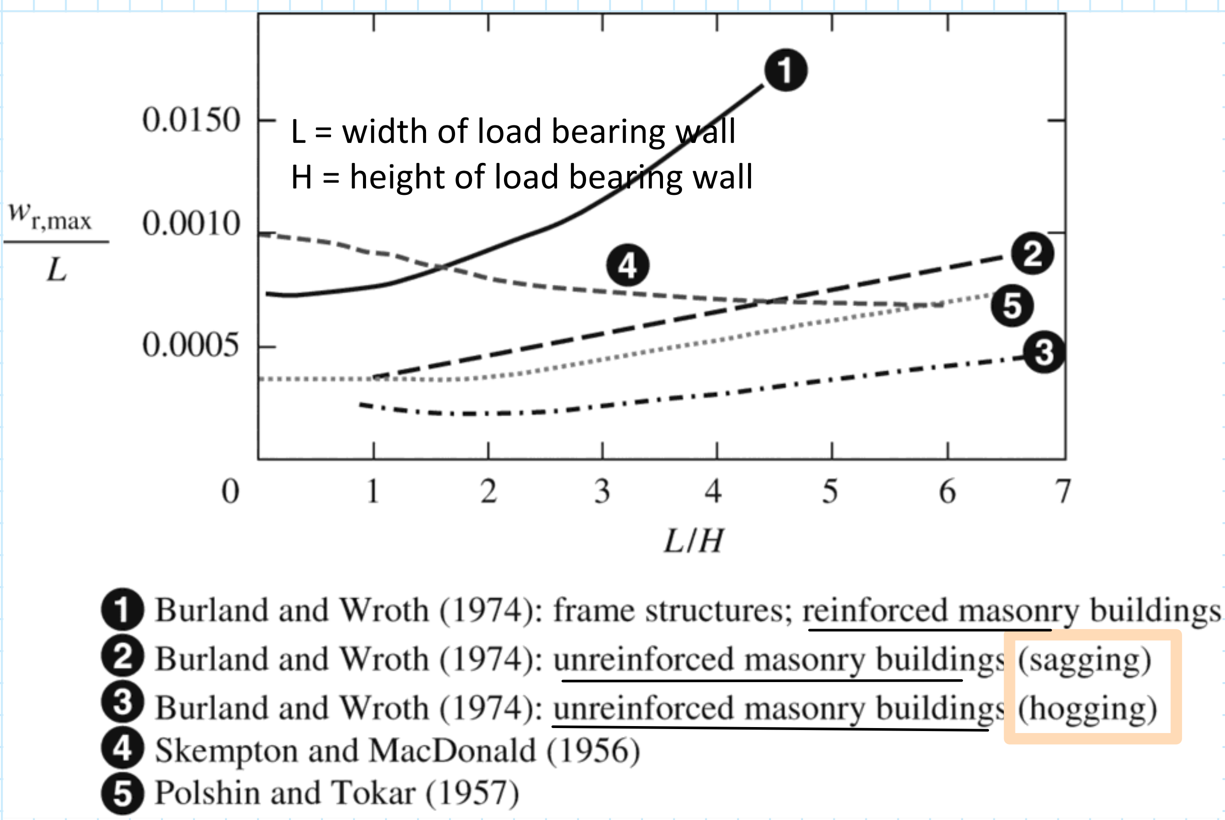


Maximum Relative Deflection, $w_{r,max}$

Wednesday, August 17, 2011 12:45 PM



Foundation configurations and maximum relative deformation, $w_{r,max}$ for (a) simple case of symmetric sagging and (b) more complex case where the foundation in the sagging mode and part in the hogging mode.



- Skempton and MacDonald works well for frame buildings.
- Polshin and Tokar works well for masonry buildings
- Burland and Wroth works reasonably well for both and has hogging criterion.

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Wednesday, August 17, 2011 12:45 PM

Ch 4a Stress, Strain and Shearing

Wednesday, August 17, 2011 12:45 PM

Reading Assignment

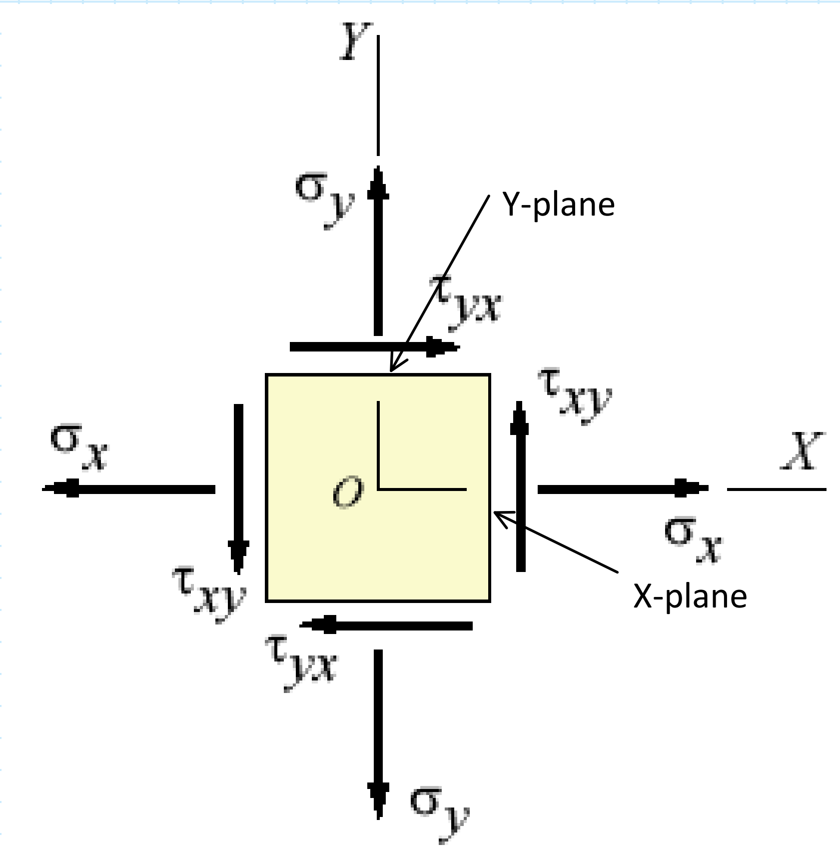
- Ch. 4a Lecture Notes
- Sections 4.1 - 4.3 (Salgado)

Other Materials

- Handout 4

Homework Assignment 3

- Problems 4-13 (15 points), 4-14 (25 points), 4-15 (25 points), 4-17 (30 points)



Ch 4a - Stress, Stress Transformation, Failure State Learning Objectives

Tuesday, October 3, 2023 12:45 PM

1. Know how to calculate normal and shear stresses on any plane

2. For a given general state of stress, know how to determine the magnitude and spatial orientation of the principal stresses using Mohr's circles or appropriate stress transformation equations.

3. Evaluate the peak shear strength given laboratory stress-strain data
   .
4. Determine Mohr's-Coulomb failure envelope from a set of stress-strain data.

5. Calculate the peak friction angle from the principal stresses at failure for a cohesionless material.

6. Know the general effects that shear generated excess porewater pressure has on the failure envelop.

Stress - Strain in Soils

Wednesday, August 17, 2011 12:45 PM

Introduction

- External forces on the soil mass induced internal stresses
- These stress can be categorized into:
    - **Normal stress** (stress that acts normal to plane)
    - **Shear stress** (stress that acts parallel to a plane)

- If the external stress is large, then a failure state may be reached
    - Soils usually reach a failure state by **shearing**
    - Mohr-Coulomb theory can be used to define the strength of the soil to resist shearing
        - Shearing can be **drained** (i.e., no effect of pore water)
            - Unsaturated, dry soil
        - Shearing can be **undrained** (pore water pressure has an effect)
            - Saturated soil
    - More advance soil models are also available, which are based on a better description of the soil behavior
        - These models are based on critical state soil mechanics which links shear strength theory with void ratio changes in the soil fabric

- If the state of stress (i.e., demand) is known on a soil element and the strength (i.e., capacity) of the soil is also known, then a safety margin, or factor of safety can be defined for that element.
    - FS = capacity/demand
    - FS = soil's shear strength along failure plane / shear stress on failure plane

- However, failure is complicated by the presence of water. Also, failure is often progressive soils, in that overstressing of the soil in one area can lead to redistribution of the shearing stress and concentration of stress in adjacent areas causing them to also reach a failure state.

Mohr's-Coulomb Strength Theory

Wednesday, August 17, 2011 12:45 PM

Mohr–Coulomb theory is a mathematical model (see yield surface) describing the response of brittle materials such as concrete, or rubble piles, to shear stress as well as normal stress. Most of the classical engineering materials somehow follow this rule in at least a portion of their shear failure envelope. Generally the theory applies to materials for which the compressive strength far exceeds the tensile strength.[1] In Geotechnical Engineering it is used to define shear strength of soils and rocks at different effective stresses. In structural engineering it is used to determine failure load as well as the angle of fracture of a displacement fracture in concrete and similar materials. Coulomb's friction hypothesis is used to determine the combination of shear and normal stress that will cause a fracture of the material. Mohr's circle is used to determine which principal stresses that will produce this combination of shear and normal stress, and the angle of the plane in which this will occur. According to the principle of normality the stress introduced at failure will be perpendicular to the line describing the fracture condition.

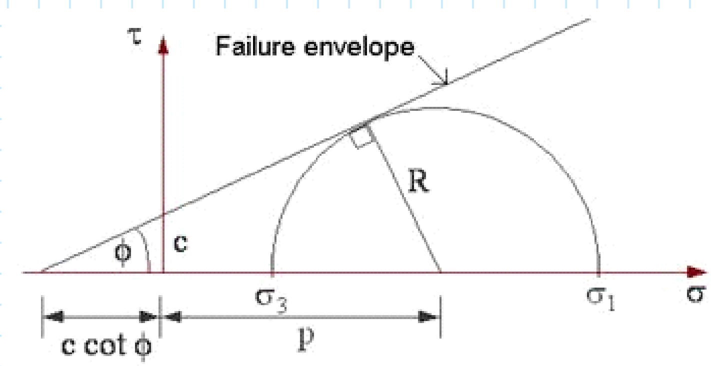
Pasted from <http://en.wikipedia.org/wiki/Mohr%E2%80%93Coulomb_theory>

The Mohr–Coulomb [5] failure criterion represents the linear envelope that is obtained from a plot of the shear strength of a material versus the applied normal stress. This relation is expressed as

$$\tau = \sigma \ \tan(\phi) + c$$

where τ is the shear strength, σ is the normal stress, c is the intercept of the failure envelope with the τ axis, and φ is the slope of the failure envelope. The quantity c is often called the **cohesion** and the angle φ is called the **angle of internal friction** . Compression is assumed to be positive in the following discussion

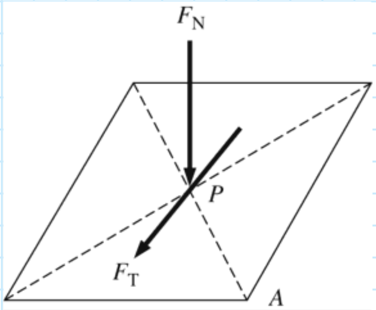
Pasted from <http://en.wikipedia.org/wiki/Mohr%E2%80%93Coulomb_theory>



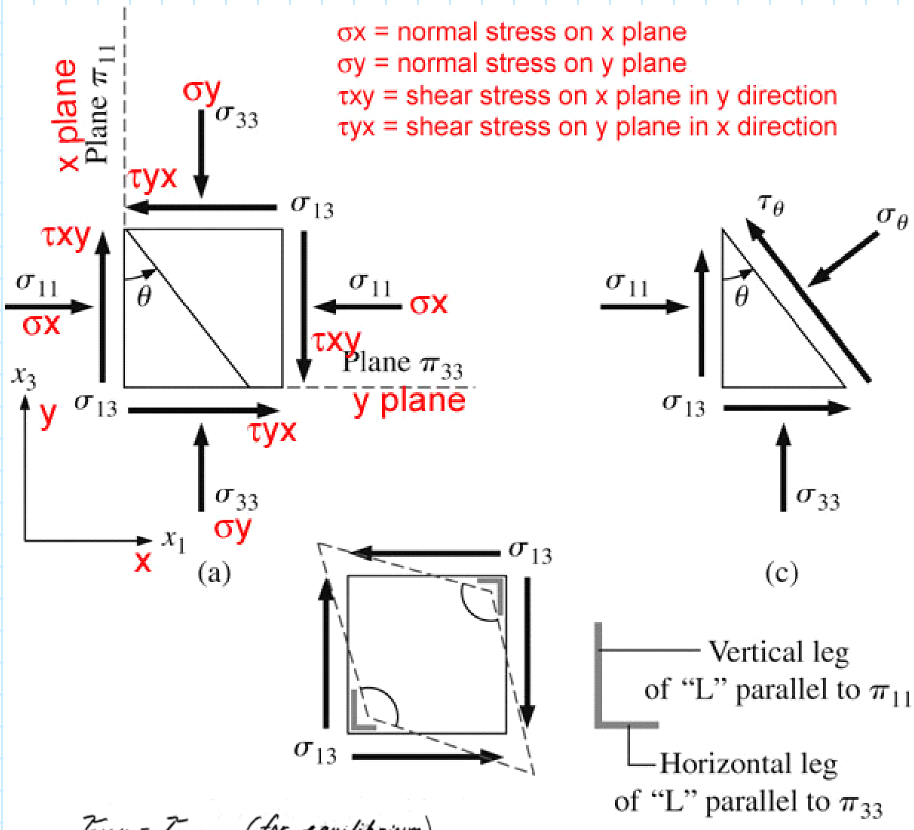
σ_1 and σ_3 are the principal stresses at failure

Definitions of Normal and Shear Stress on a Plane

Thursday, September 7, 2023 7:16 AM

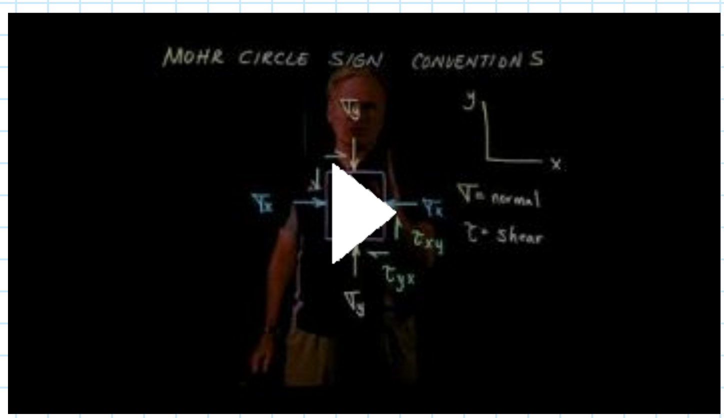


F_N = normal force
F_T = shear force
A = area of plane P
$\sigma = F_N / A$ (normal stress)
$\tau = F_T / A$ (shear stress)



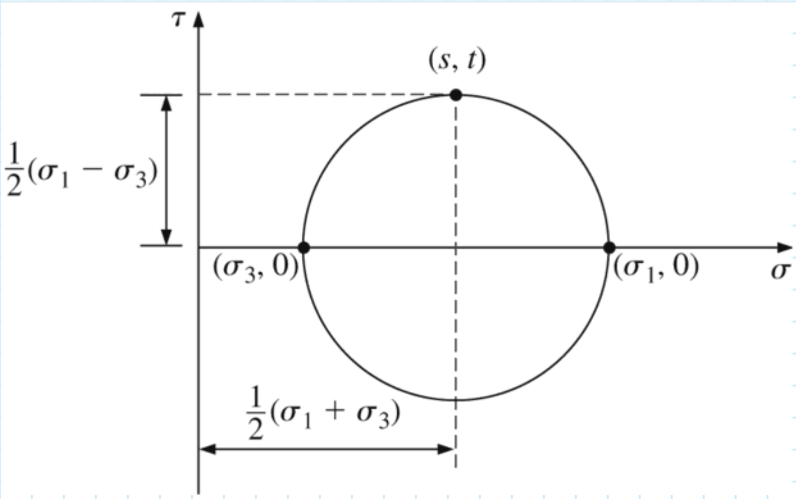
$\tau_{xy} = \tau_{yx}$ (for equilibrium)
σ = positive (compression)
σ = negative (tension)
α, θ = positive (CCW)
τ = positive by the right hand rule

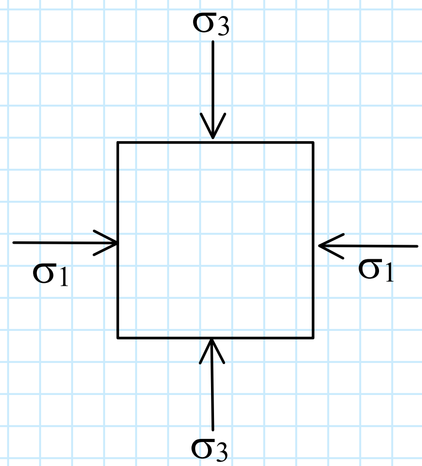
[CVEEN 5305 - Mohr Circles Part 1]



Geometry of Mohr's Circle

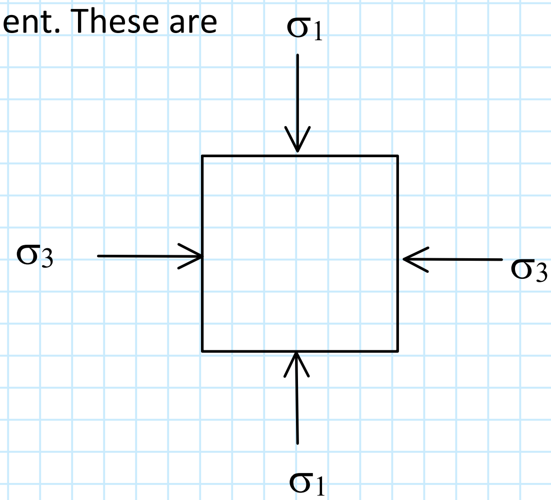
Thursday, September 7, 2023 12:45 PM



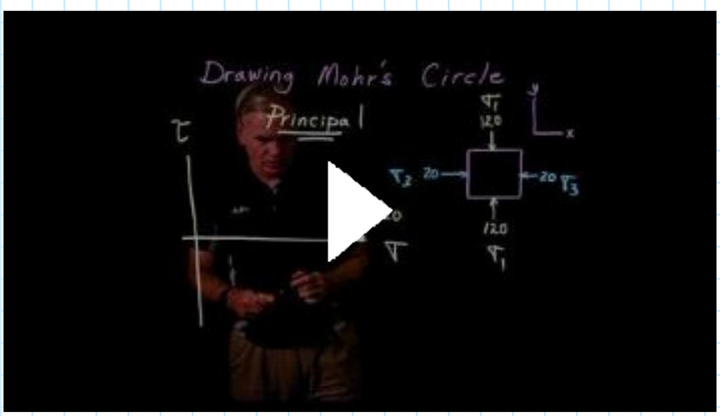


or

Note that no shear stress are acting on this element. These are **principal stresses**.



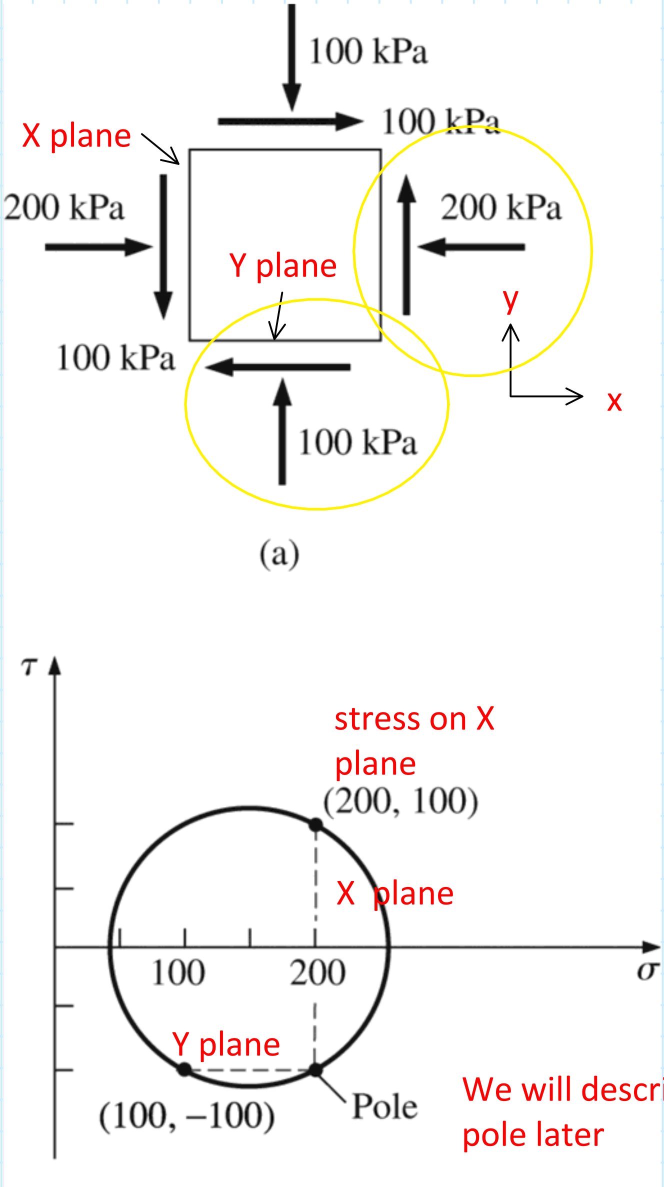
[CVEEN 5305 - Mohr Circles Part 2]



Note that the Mohr's circle does not inform us about the orientation of the principal planes.

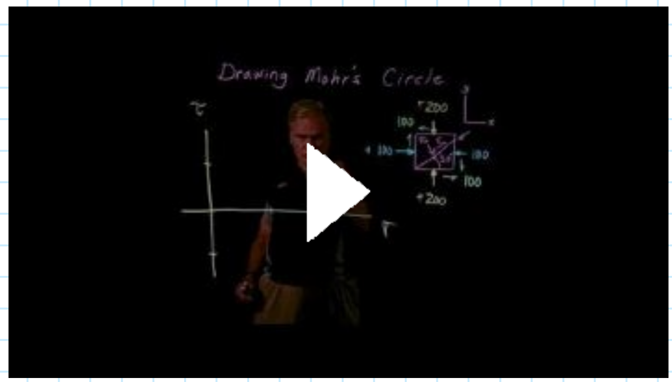
Mohr Circle of Stress

Wednesday, August 17, 2011 12:45 PM



Note the X plane is perpendicular to the x axis. Similarly the Y plane is perpendicular to the y axis

[CVEEN 5305 - Mohr Circles Part 3]



We will describe how to use the pole later

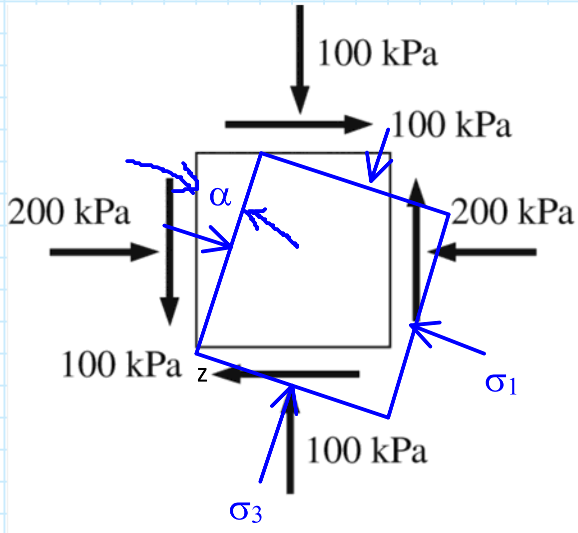
stress on y plane

Note that the Mohr's circle can be used to calculate state of stress on any plane. The first coordinate is the magnitude of the normal stress (σ) and the second coordinate is the magnitude of the shear stress (τ).

To construct a Mohr's circle, we need to know one of the following: (1) state of stress on 2 planes and the angle between those planes, or (2) state of stress on the principal planes, or (3) state of stress on the failure plane and the Mohr's-Coulomb properties of the material

Finding Major and Minor Principal Stresses - 2 alpha method

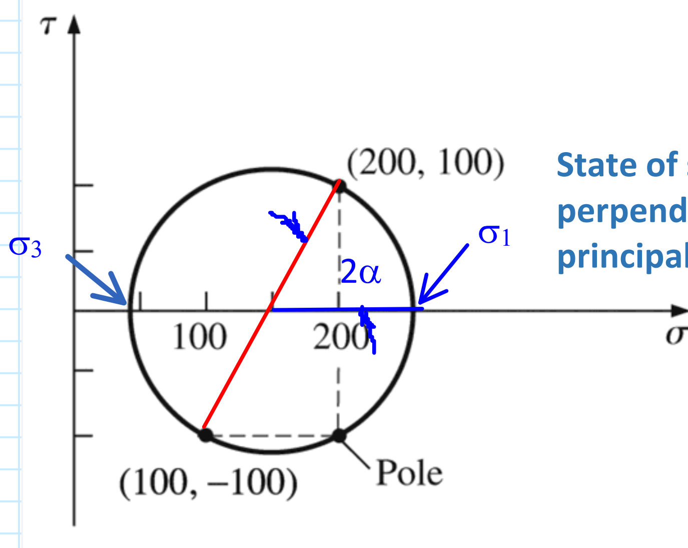
Wednesday, August 17, 2011 12:45 PM



(a)

In this example we want to find the location and magnitude of the principal stresses, σ_1 and σ_3

State of stress on plane perpendicular to minor principal stress



(b)

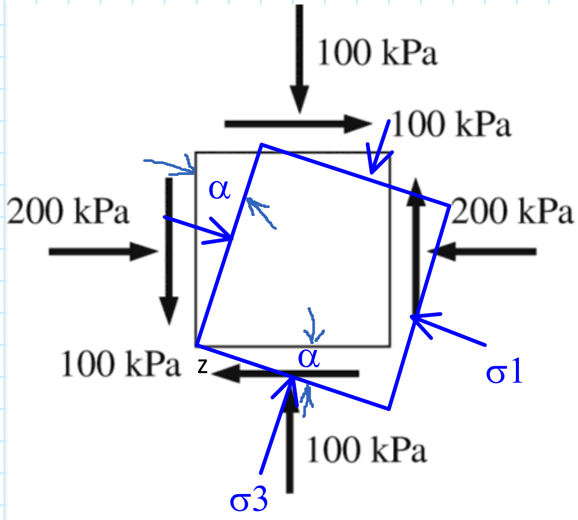
State of stress on plane perpendicular to major principal stress

Note that the major principal stress, σ_1, has the largest value of normal stress and the shear stress is zero. The plane upon which this stress acts is called the major principal plane.

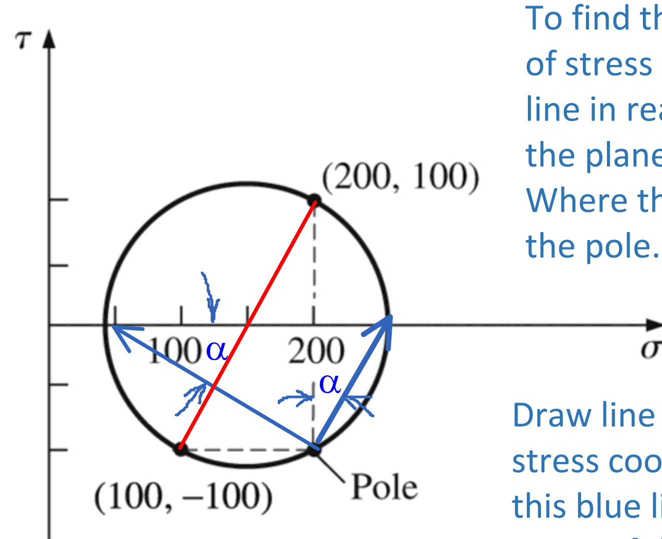
The minor principal stress, σ_3, has the smallest value of the normal stress and the shear stress is also zero. The plane upon which this stress acts is called the minor principal plane

Finding Major and Minor Principal Stresses - Pole method

Wednesday, August 17, 2011 12:45 PM



(a)



(b)

To find the pole, start at a known state of stress on the Mohr circle. Draw a line in real space that is a parallel to the plane upon which that stress acts. Where this line intersect the circle is the pole.

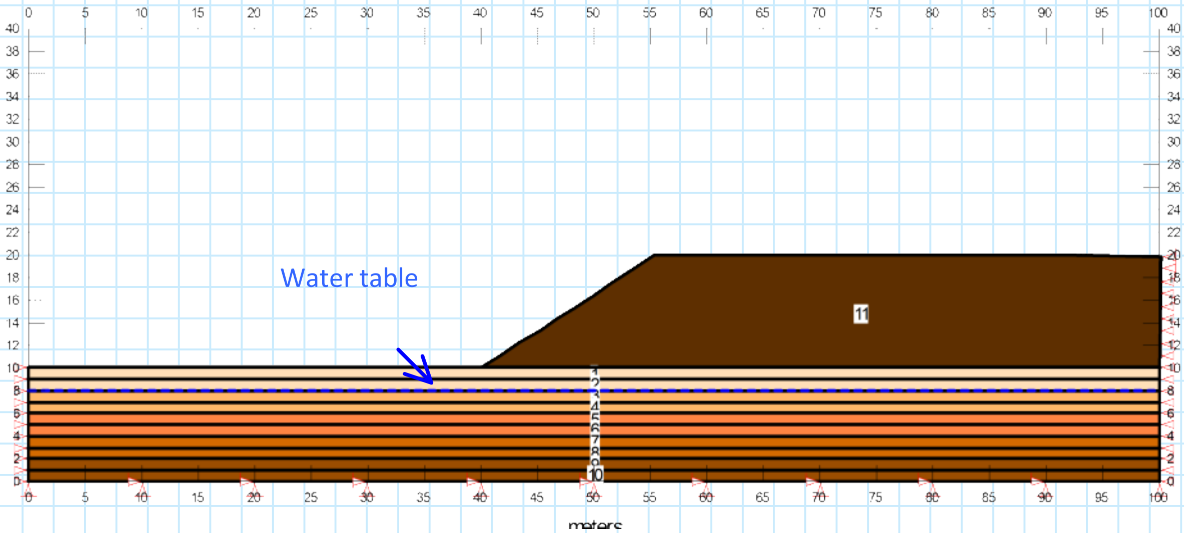
Draw line from pole to major principal stress coordinate. The orientation of this blue line is the orientation in true space of the plane upon which the major principal stress acts.

Note that the major principal stress, σ_1, has the largest value of normal stress and the shear stress is zero. The plane upon which this stress acts is called the major principal plane.

The minor principal stress, σ_3, has the smallest value of the normal stress and the shear stress is also zero. The plane upon which this stress acts is called the minor principal plane

State of Stress in Foundation Soil Example

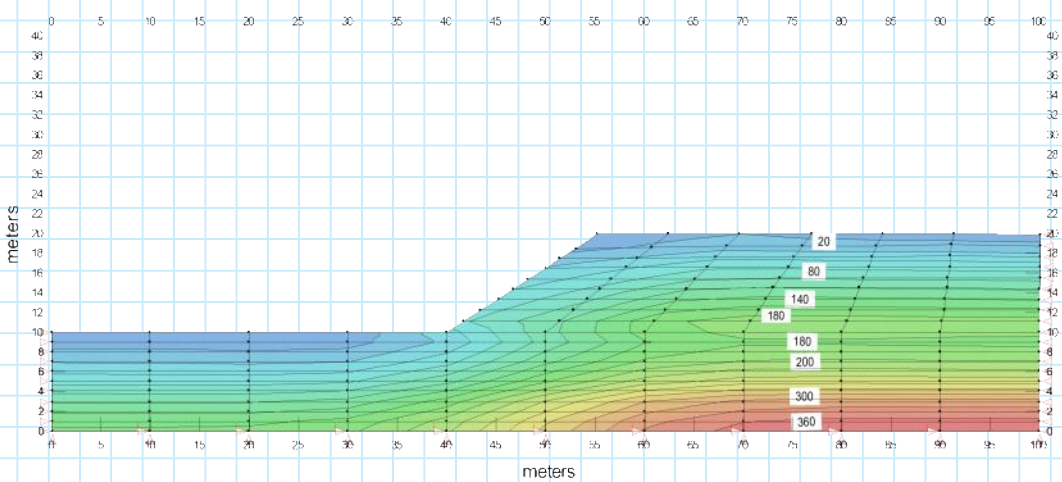
Wednesday, August 17, 2011 12:45 PM



Earthen embankment on soil foundation

Soil properties

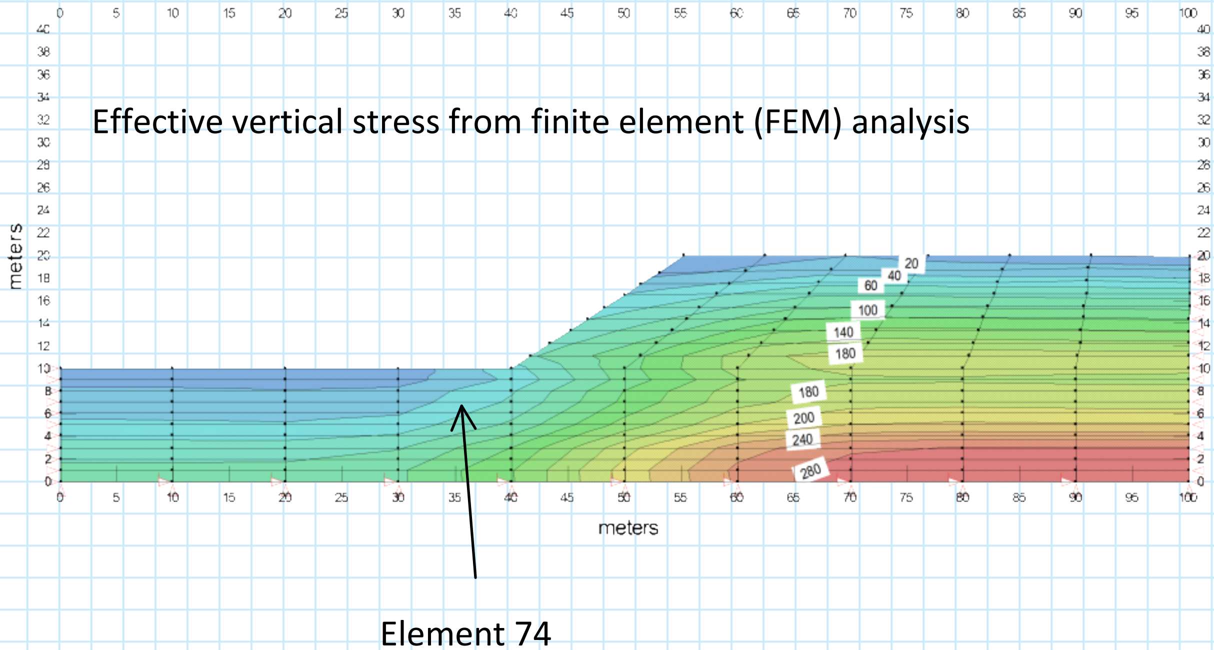
	γ (kN/m^3)	γ (lb/ft^3)	E (kPa)	v
Soil Layer 1	15.72	100	100000	0.37
Soil Layer 2	16.51	105	100000	0.37
Soil Layer 3	17.29	110	150000	0.35
Soil Layer 4	18.08	115	200000	0.3
Soil Layer 5	18.08	115	250000	0.3
Embankment	21.22	135	300000	0.3



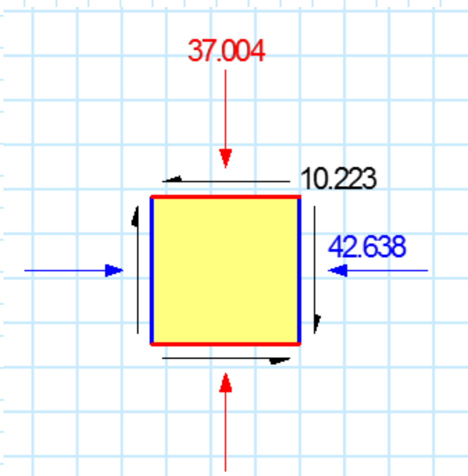
Vertical stress from finite element (FEM) analysis

State of Stress in Foundation Soil Example (cont.)

Wednesday, August 17, 2011 12:45 PM



Given: The normal and shear stress on element 74

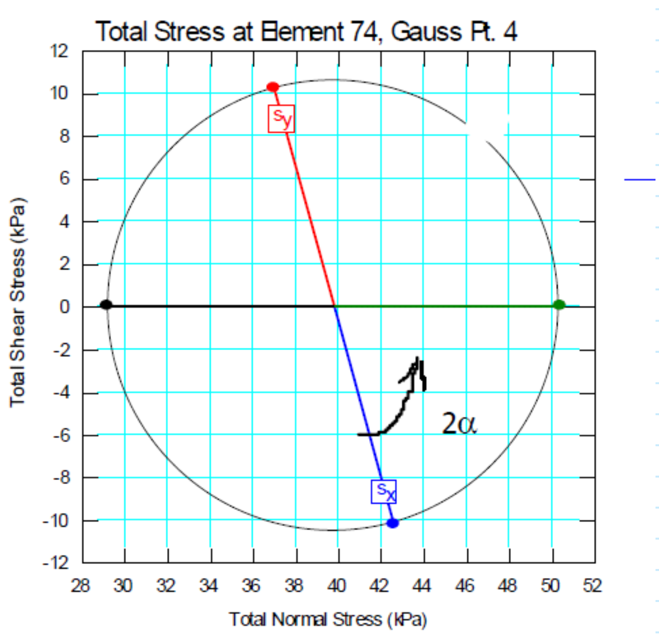


Find: The magnitudes and directions of the major and minor principal stresses

State of Stress in Foundation Soil Example (cont.)

Wednesday, August 17, 2011 12:45 PM

Solution: **(Hidden)**



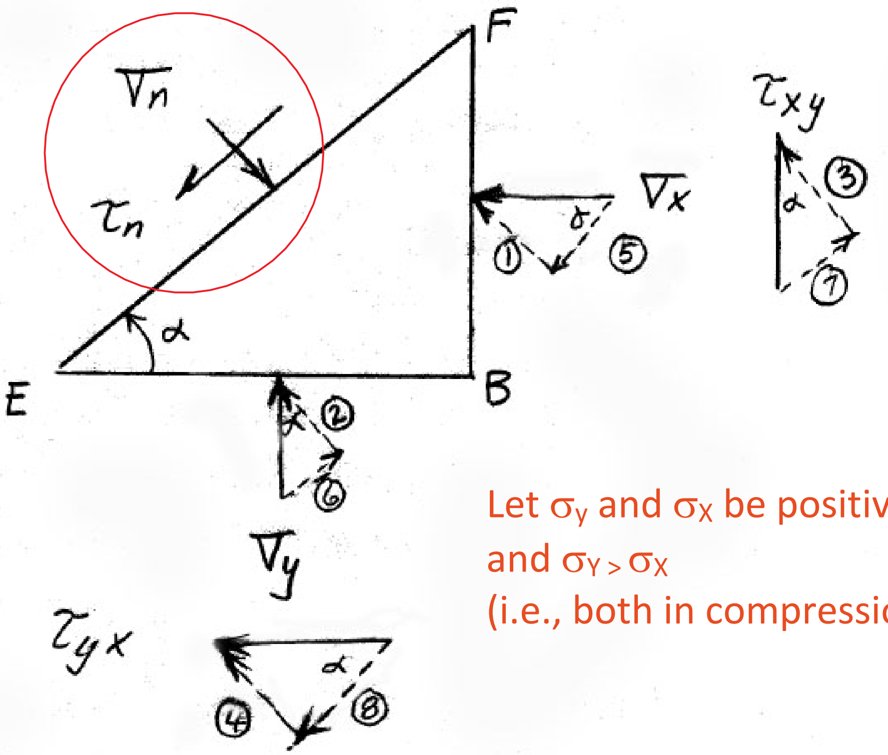
Approximate answer: **(Hidden)**

<u>Calculate FS for element 74 for the potential of shearing on a horizontal plane</u>
(Hidden)

Even though the F.S. is acceptable for this element does this mean the entire slope is stable?

Derivation of Formulas for Stresses on an Arbitrary Plane

Wednesday, August 17, 2011 12:45 PM



Let σ_y and σ_x be positive and $\sigma_Y > \sigma_X$ (i.e., both in compression)

- From FBD, determine σ_n

1) Let $\overline{EF} = 1$
2) $\overline{EB} = \overline{EF} \cos\alpha$
3) $\overline{FB} = \overline{EF} \sin\alpha$
4) Write σ_n in terms of force, not stress
5) Solve for σ_n

Step 4)

$$(\sigma_n)(\overline{EF}) = ① + ② + ③ + ④$$

$$= (\sigma_x)\underbrace{(\overline{EF})(\sin\alpha)}_{\text{Area of FB}}(\sin\alpha) +$$

Derivation of Formulas for Stresses on an Arbitrary Plane (cont.)

Wednesday, August 17, 2011

Step 4) (cont.)

$$(\sigma_y)\underbrace{(\overline{EF})(\cos\alpha)}_{\text{Area of EB}}(\cos\alpha) \; +$$

$$(\tau_{xy})\underbrace{(\overline{EF})(\sin\alpha)}_{\text{Area of FB}}(\cos\alpha) \; +$$

$$(\tau_{yx})\underbrace{(\overline{EF})(\cos\alpha)}_{\text{Area EB}}(\sin\alpha)$$

$$\sigma_n = \sigma_x \sin^2\alpha + \sigma_y \cos^2\alpha + 2\tau_{xy}\sin\alpha\cos\alpha$$

- With some trig identities, this can be rewritten to:

$$\boxed{\sigma_n = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\alpha + \tau_{xy}\sin 2\alpha}$$

For $\sigma_y > \sigma_x$, α referenced from Y-plane. For σ_1, let $\sigma_1 = \sigma_N$ and $\alpha = \theta_{P1}$ if you want to obtain σ_1

$$\boxed{\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\alpha + \tau_{xy}\sin 2\alpha}$$

For $\sigma_y > \sigma_x$, α referenced from X-plane. For σ_3, let $\sigma_3 = \sigma_N$ and $\alpha = \theta_{P3}$ if you want to obtain σ_3

- Step 5)

$$(\tau_n)(\overline{EF}) = 5 + 6 + 7 + 8$$

$$= -(\sigma_x)(\overline{EF})(\sin\alpha)(\cos\alpha)$$
$$+(\sigma_y)(\overline{EF})(\cos\alpha)(\sin\alpha)$$
$$+(\tau_{xy})(\overline{EF})(\sin\alpha)(\sin\alpha)$$
$$-(\tau_{xy})(\overline{EF})(\cos\alpha)(\cos\alpha)$$

$$\boxed{\tau_n = \frac{\sigma_y - \sigma_x}{2}\sin 2\alpha - \tau_{xy}\cos 2\alpha}$$

Derivation of Formulas for Stresses on an Arbitrary Plane (cont.)

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Let $\sigma_y = \sigma_1$ = major principal stress

σ_y = vertical stress (usually greatest)

$\sigma_x = \sigma_3$ = minor principal stress

σ_x = horizontal stress (usually less than σ_y, <u>but</u> not always)

$$\boxed{\sigma_n = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \cos 2\alpha}$$

Eq. 10-5, H & K

$$\boxed{\tau_n = \frac{\sigma_1 - \sigma_3}{2} \sin 2\alpha}$$

Eq. 10-6 H & K

Example Problem - Using Formulas

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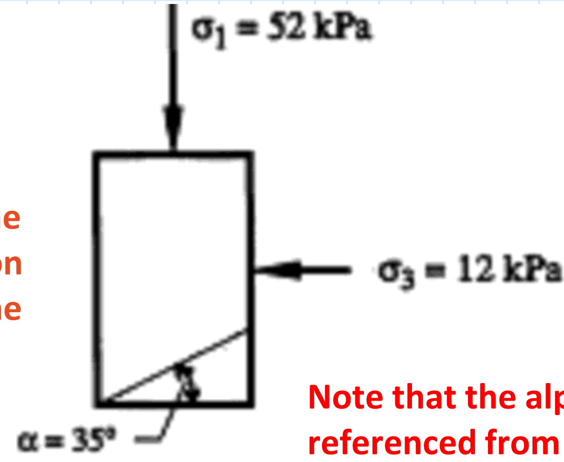
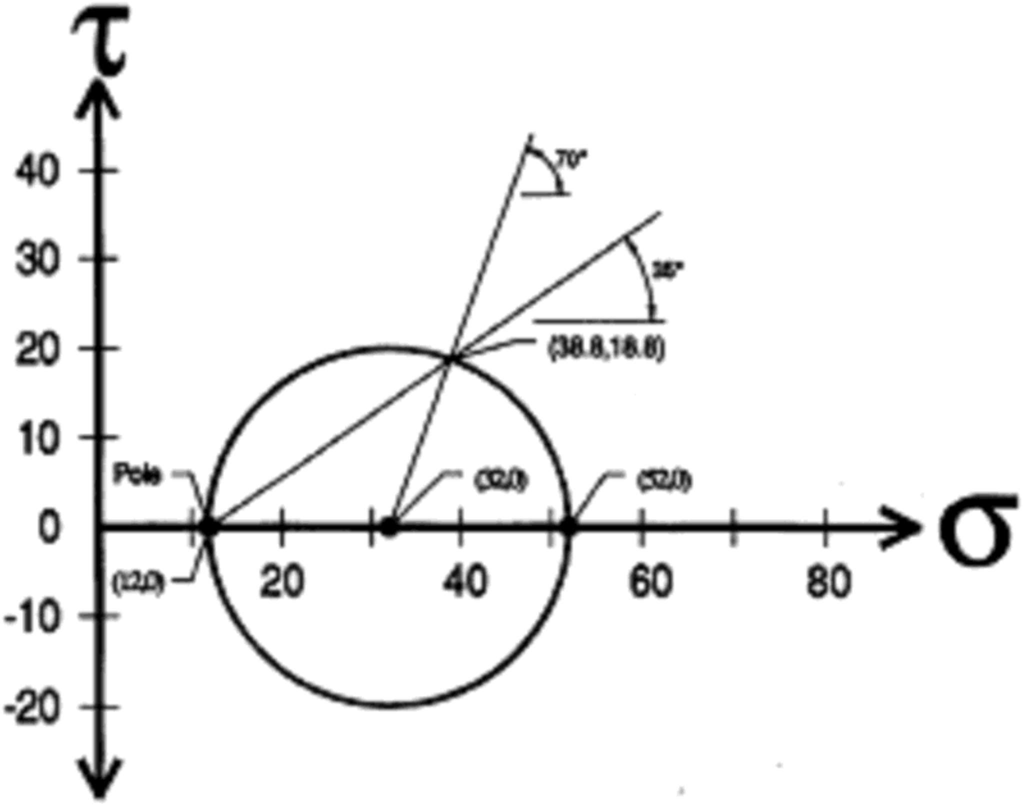
Note this example is for the case of finding the stress on an arbitrary plane given the principal stresses.



Note that the alpha angle is referenced from the plane upon which the principal stress acts.



From Eqs. 10-5 and 10-6, p. 436:

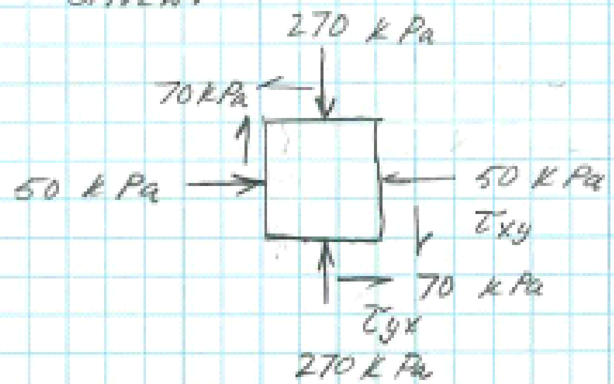
$$\sigma_\alpha = \frac{\sigma_1+\sigma_3}{2} + \frac{\sigma_1-\sigma_3}{2}\cdot\cos 2\alpha = \frac{52+12}{2} + \frac{52-12}{2}\cdot\cos(2\cdot 35^\circ) = 38.8\ kPa$$

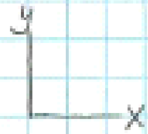
$$\tau_\alpha = \frac{\sigma_1-\sigma_3}{2}\cdot\sin 2\alpha = \frac{52-12}{2}\cdot\sin(2\cdot 35^\circ) = 18.8\ kPa$$

Another Example Problem

Wednesday, August 17, 2011 12:45 PM

GIVEN:





FIND: a) $\sigma_1 = ?$

b) $\sigma_3 = ?$

c) $\theta_p = ?$

<u>GRAPHICAL SOLUTION:</u>

a) see next sheet

$\sigma_1 = 290$ kPa (compression)

b) see next sheet

$\sigma_3 = 30$ kPa (compression)

c) θ_{p_1} = angle between horizontal plane on FBD and plane upon σ_1 acts.

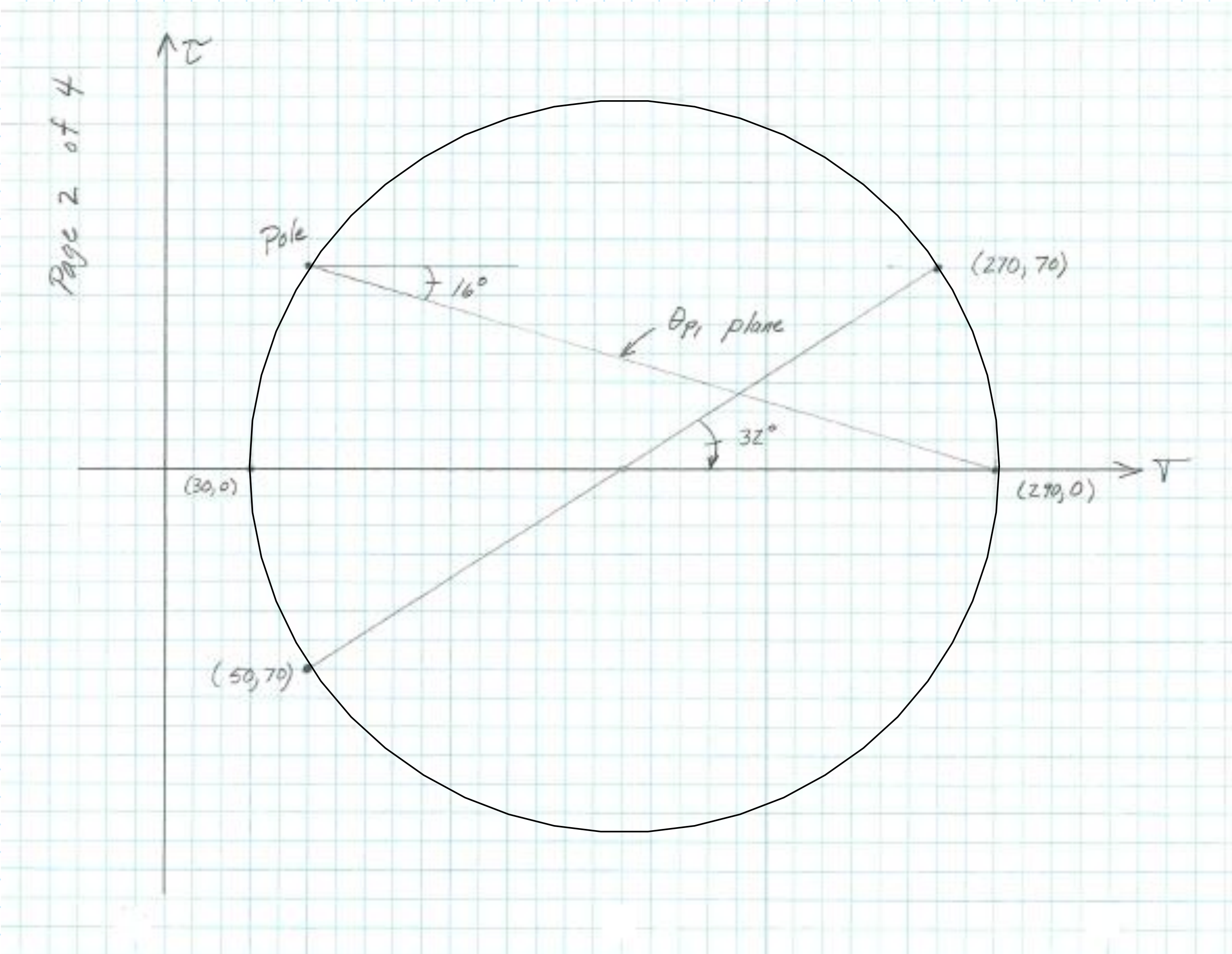
from graph $2\theta = 32°$ CW

$\theta_{p_1} = 16°$ CW ← from horizontal plane

also $\theta_{p_1} = 16°$ CW from horz. (pole method)

Another Example Problem (cont.)

Wednesday, August 17, 2011 12:45 PM



Another Example Problem (cont.)

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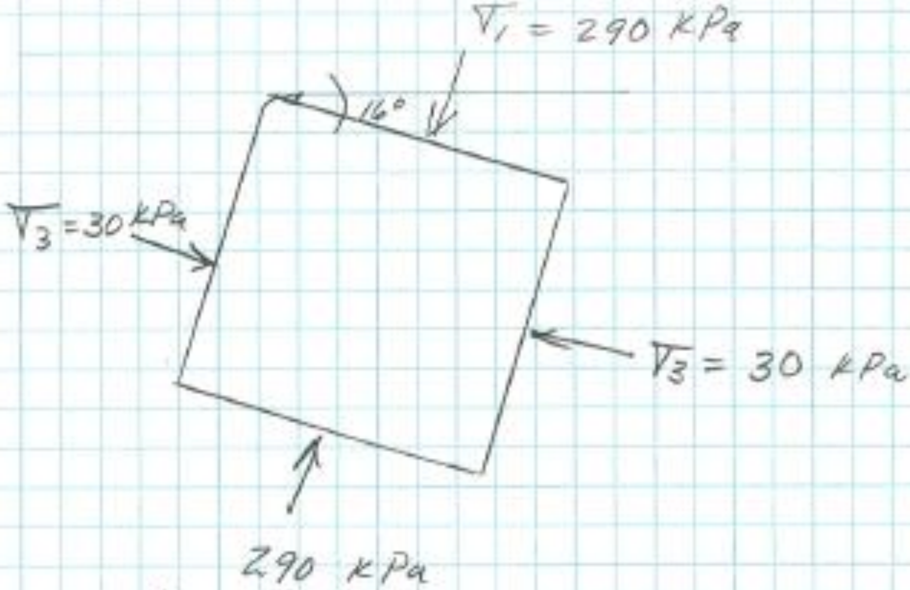
DRAW FBD



ANALYTICAL SOLUTION

$$\tan 2\theta_p = \frac{2\sigma_{13}}{\sigma_{11} - \sigma_{33}} = \frac{2\tau_{xy}}{\sigma_y - \sigma_x}$$

$$= \frac{2(-70)}{270-50}$$

$$= -0.63636\ldots$$

$$\tan^{-1}(2\theta_p) = -32.47^\circ$$

$$\theta_p = -16.3^\circ \leftarrow \text{(neg. means CW direction)}$$

Analytical Example Problem (cont.)

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ANALYTICAL SOLUTION (CONT.)

Note σ_y is the largest of the two normal stresses in this equation

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\sigma_n = \frac{50+270}{2} - \frac{50-270}{2} \cos[2(-16.3)] + -70 \sin[2(-16.3)]$$

$$\sigma_n = 160 - (-110)(0.842452397) + (37.71395495)$$

$$\sigma_n = 160 + 92.67 + 37.71$$

① $\sigma_1 = 290.4$ kPa ⟵ Checks w/ graphical solution

Note σ_y is the largest of the two normal stresses in this equation

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\sigma_n = \frac{50+270}{2} + \frac{50-270}{2} \cos(2(-16.3)) - 70 \sin(2(-16.3))$$

$$\sigma_n = 160 - 92.66976 - 37.71395$$

② $\sigma_3 = 29.6$ kPa ⟵ checks w/ graphical solution

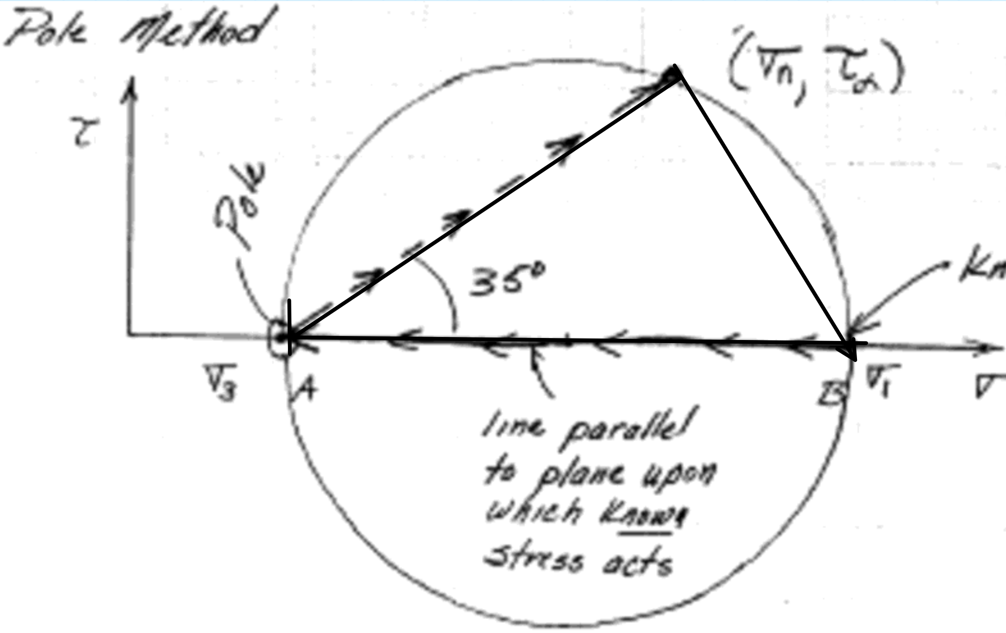
① Note the reference plane for θ for this calculation is from the <u>Y-plane</u> (i.e., horizontal plane)

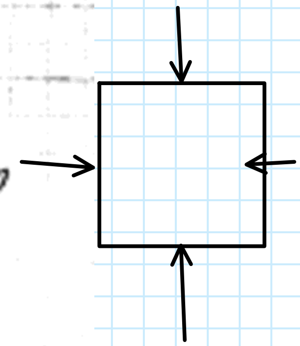
② Note the reference plane for θ for this calculation is from the <u>X-plane</u> (i.e., vertical plane

Pole Method - Principal Stress Known (e.g., Triaxial Test)

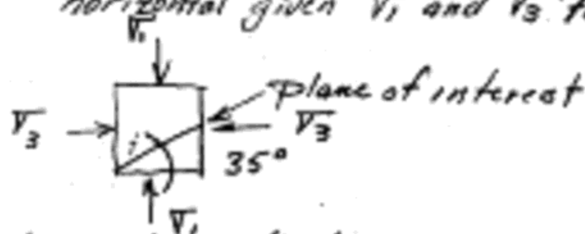
Wednesday, August 17, 2011 12:45 PM

- Pole Method





Example: Find the state of stress (σ_n, τ_α) for a plane inclined 35 degrees from the horizontal given σ_1 and σ_3 from a triaxial test.



Using the pole method

(1) Determine the location of the pole by drawing a straight line parallel to the plane that has the known state of stress, starting at the point with the known state of stress.

(2) Draw a line that has the same orientation (in real space) as the plane of interest

Pole Method Example - Normal and Shear Stress Known (Direct Shear)

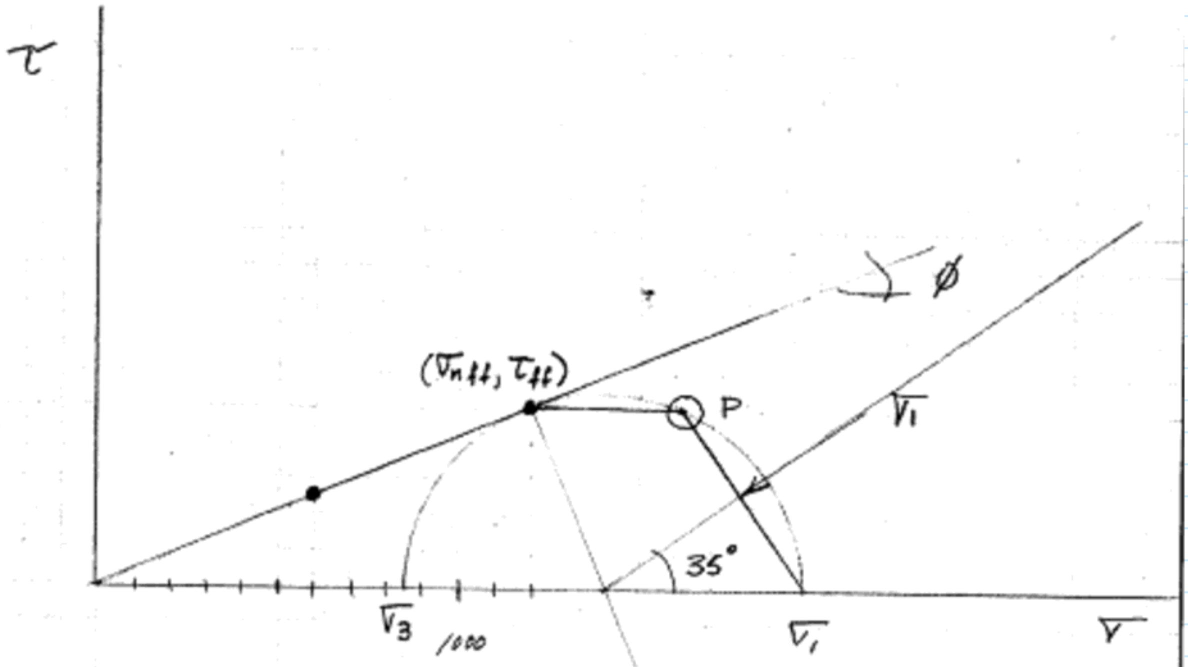
Wednesday, August 17, 2011 12:45 PM

From a direct shear test, the following is known

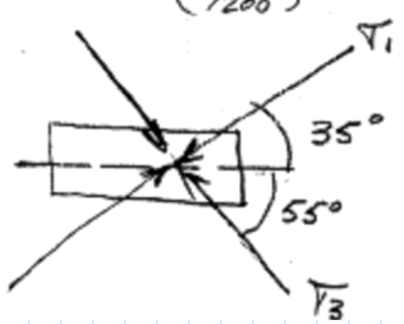
$\sigma_{nff} = 1200$ psf, $\tau_{ff} = 500$ psf } test 1

$\sigma_{nff} = 600$ psf, $\tau_{ff} = 250$ psf } test 2

Find ϕ and the orientation of σ_1 and σ_3 in relation to the horizontal shear plane for test 1.



$\phi = \arctan\left(\frac{500}{1200}\right) = 22.6°$

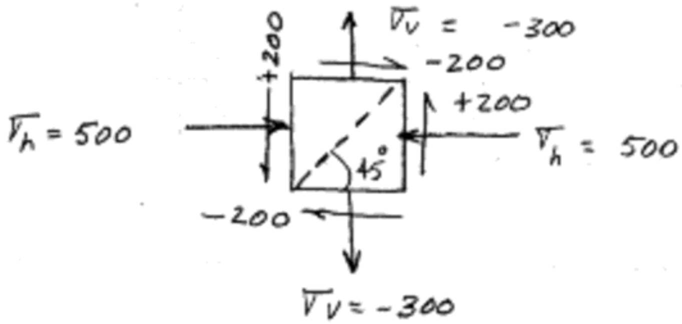


Thus, the principal stress σ_1 is not vertical during the direct shear test.

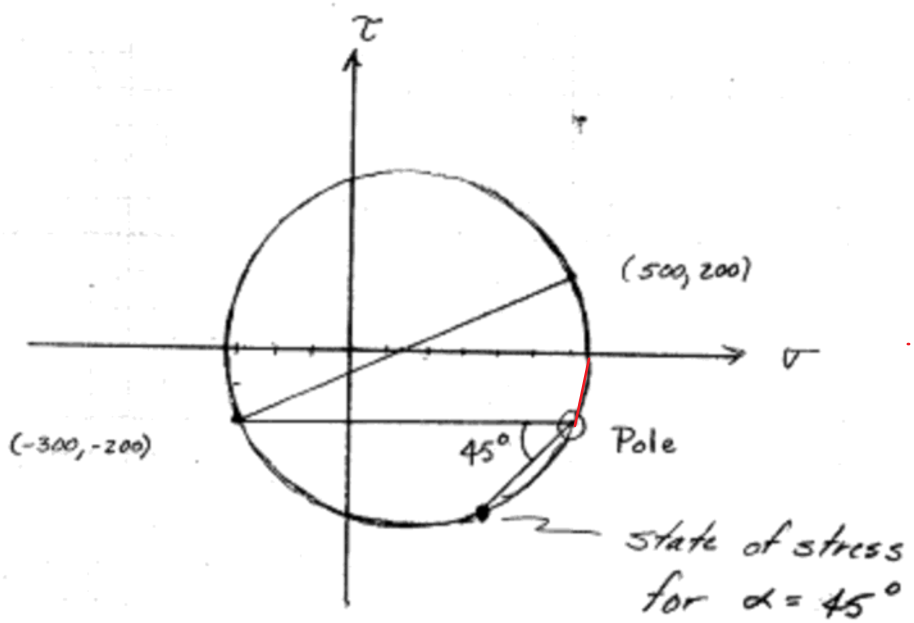
Pole Method Example

Wednesday, August 17, 2011 12:45 PM

The following stresses are acting upon an element as shown below. Find the state of stress on a plane that has $\alpha = 45°$



Draw Mohr Circle



Shear Failure and Maximum Shear Stress

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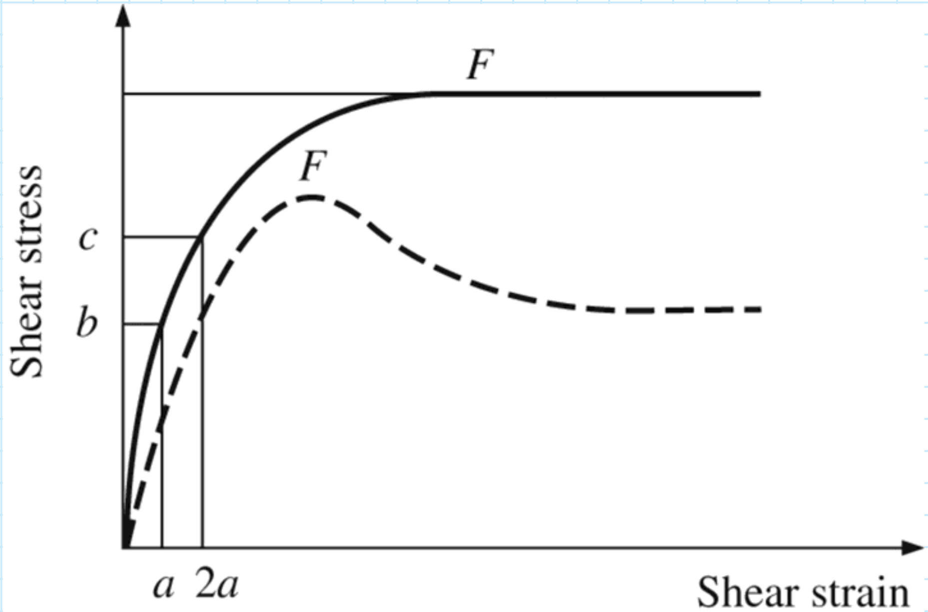
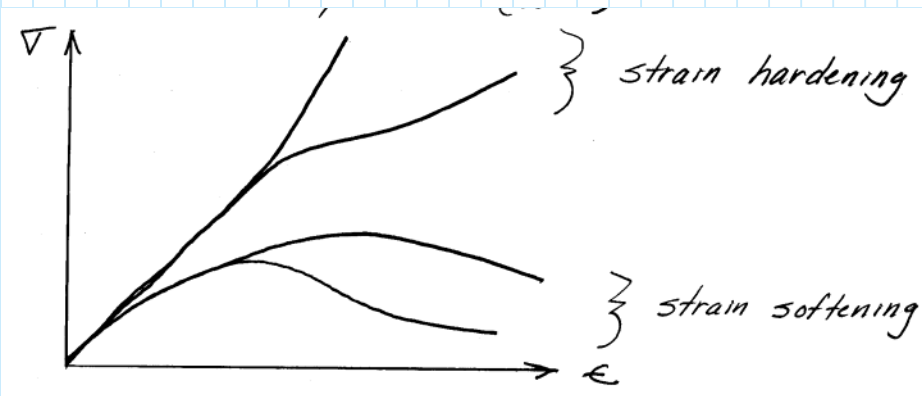
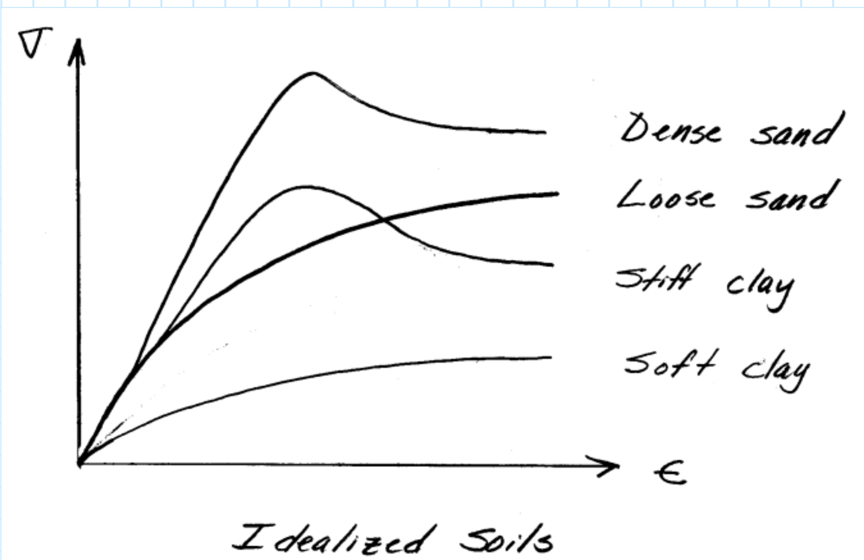


Ductile Failure

Strain Softening Failure

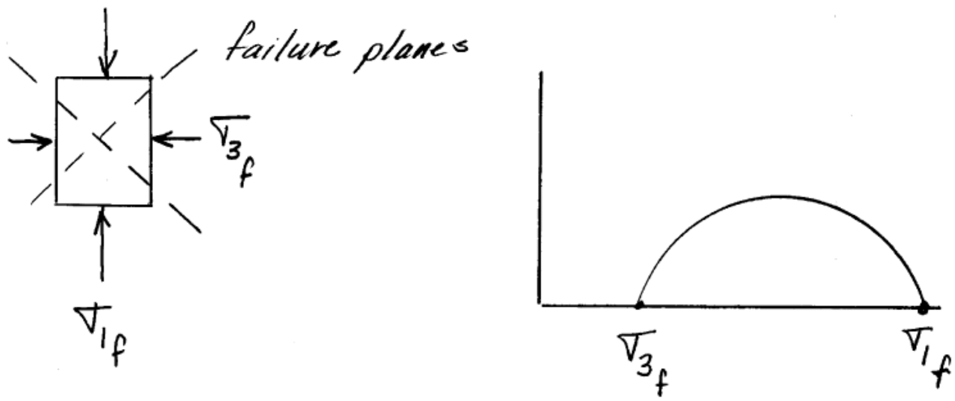




Developing the Failure Surface

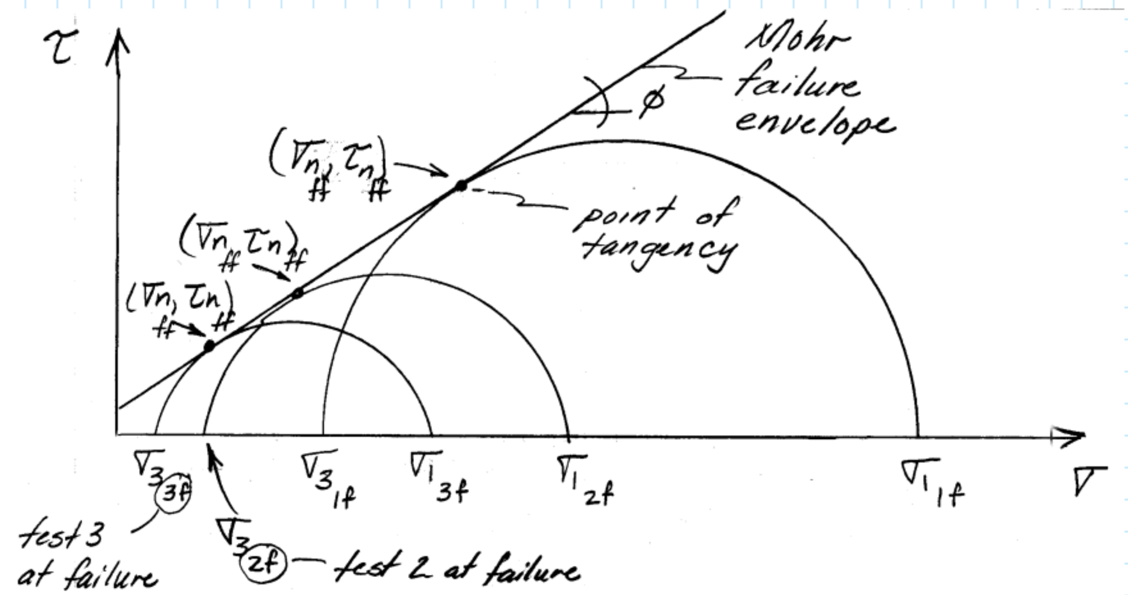
Wednesday, August 17, 2011 12:45 PM

- If we know σ_1 & σ_3 at failure, then we can draw the state of stress at failure for the soil element



- Repeat the above process for higher values of σ_3
  - $\sigma_{3_{3f}} < \sigma_{3_{2f}} < \sigma_{3_{1f}}$
  - $\sigma_{1_{3f}} < \sigma_{1_{2f}} < \sigma_{1_{1f}}$

Note that the triaxial shear device can be used to control values values of σ1 and σ3



Inclination of Failure Plane

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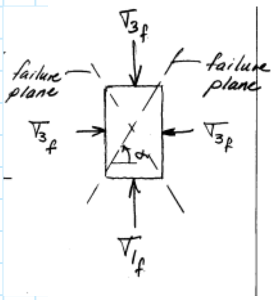
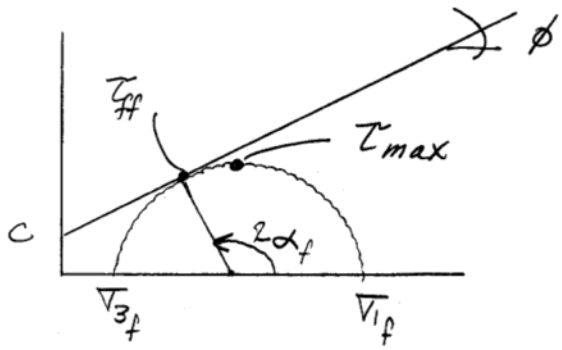
- Inclination of failure plane
  - Note that failure point is not the point of maximum shear stress





$$\tau_{max} = \frac{\sigma_{1f} - \sigma_{3f}}{2} > \tau_{ff}$$

- How do we estimate α_f for different values of ϕ?

$$\boxed{\alpha_f = 45 + \frac{\phi}{2}}$$

α_f is located CCW from the plane upon which the major principal stress acts

- Mohr-Coulomb and Principal Stresses (Obliquity relations)
  - For $c = 0$ (only)*

- $$\boxed{\sin\phi = \frac{(\sigma_{1f} - \sigma_{3f})}{(\sigma_{1f} + \sigma_{3f})}}$$

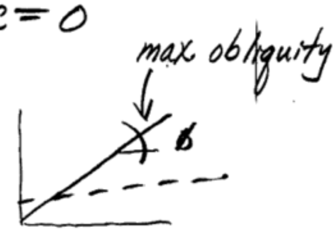
- $$\boxed{\frac{\sigma_1}{\sigma_3} = \frac{1 + \sin\phi}{1 - \sin\phi}}$$

- $$\boxed{\frac{\sigma_1}{\sigma_3} = \tan^2(45 + \phi/2)}$$

- $$\boxed{\frac{\sigma_3}{\sigma_1} = \tan^2(45 - \phi/2)}$$

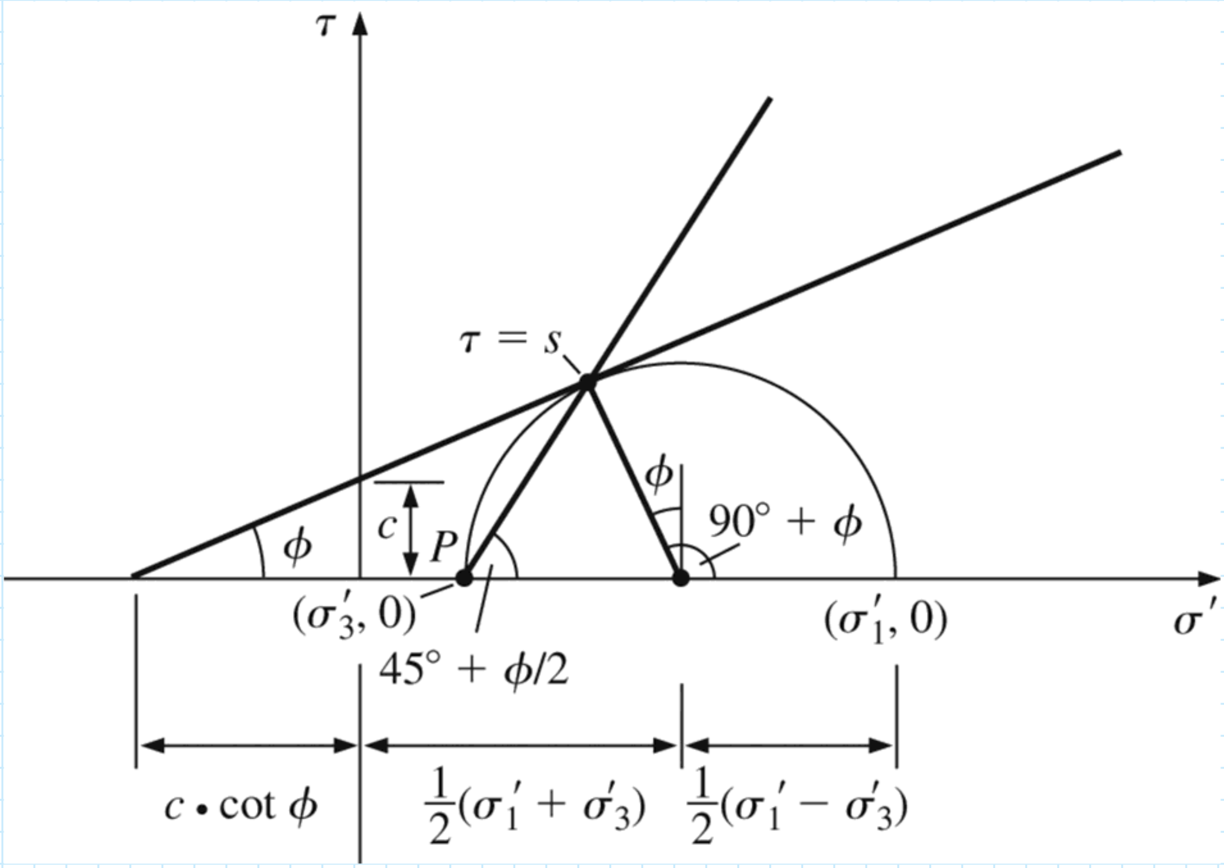
Note: These relationships are only valid when c = 0

* Maximum obliquity or inclination of envelope occurs when $c = 0$



Geometry of the Failure Envelope

Wednesday, August 17, 2011 12:45 PM



- Mohr Circle

Example Problem

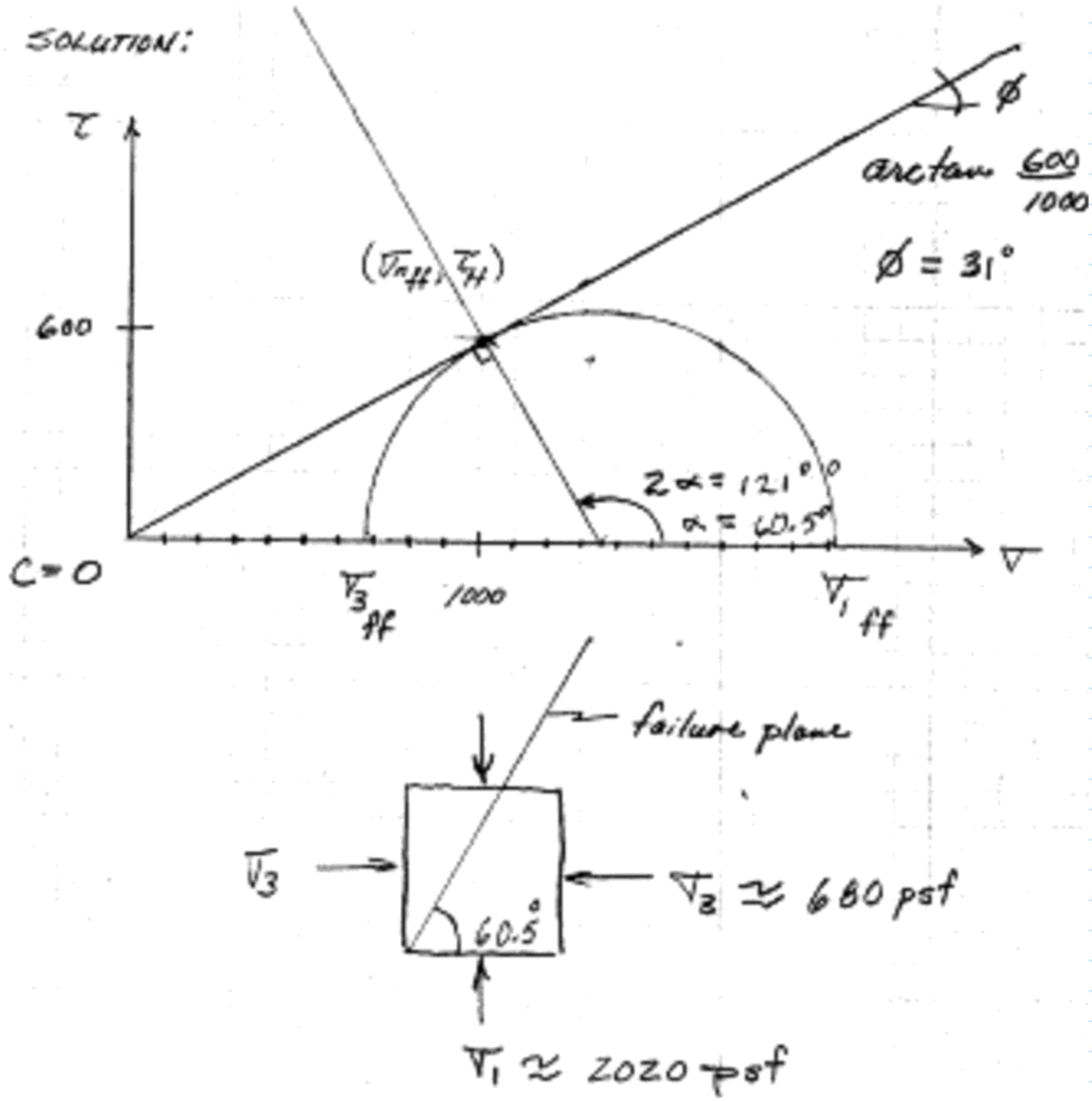
Given: σ_n at failure = 1000 psf

τ at failure = 600 psf

$c = 0$, $\tau_{ff} = \sigma_1 \tan\phi + c$

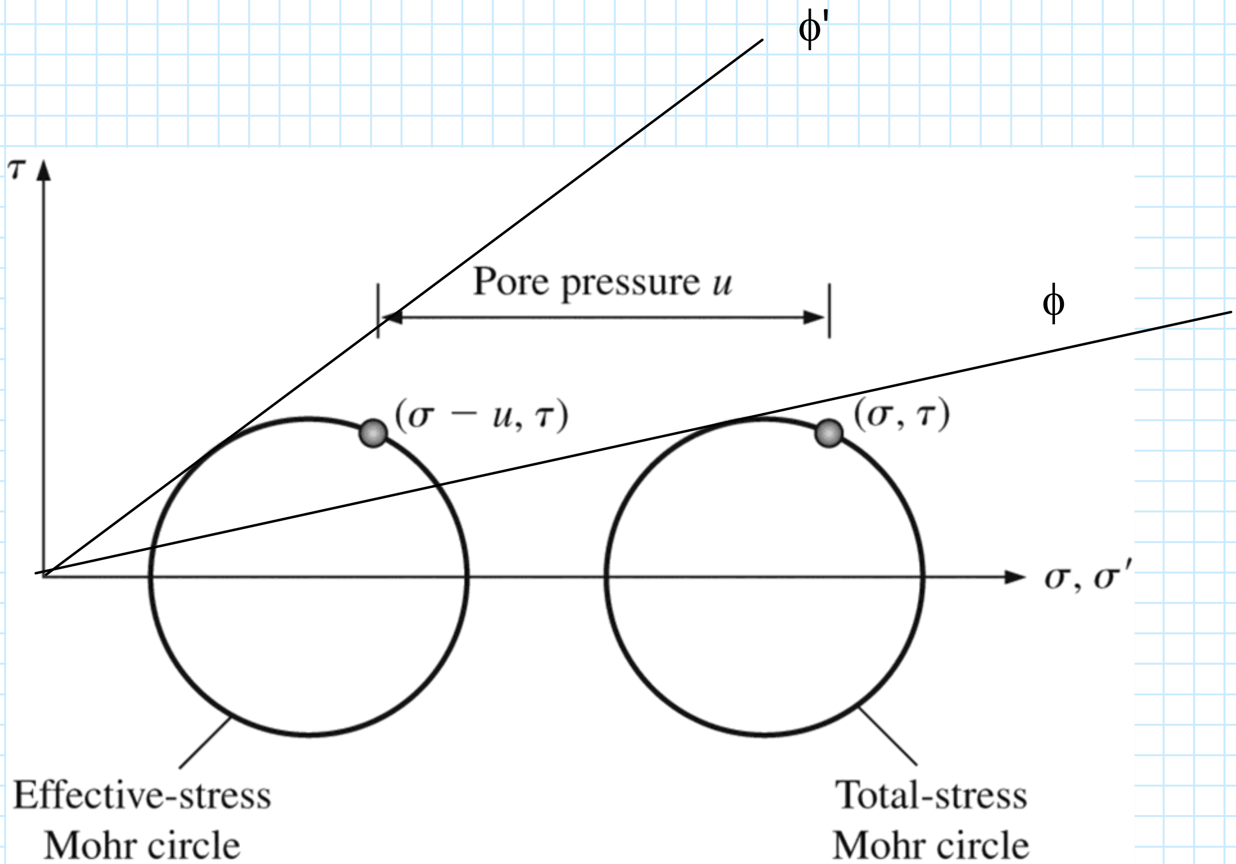
FIND: σ_1 & σ_3, α, ϕ = ?

SOLUTION:



Effects of pore water on failure envelope

Wednesday, August 17, 2011 12:45 PM

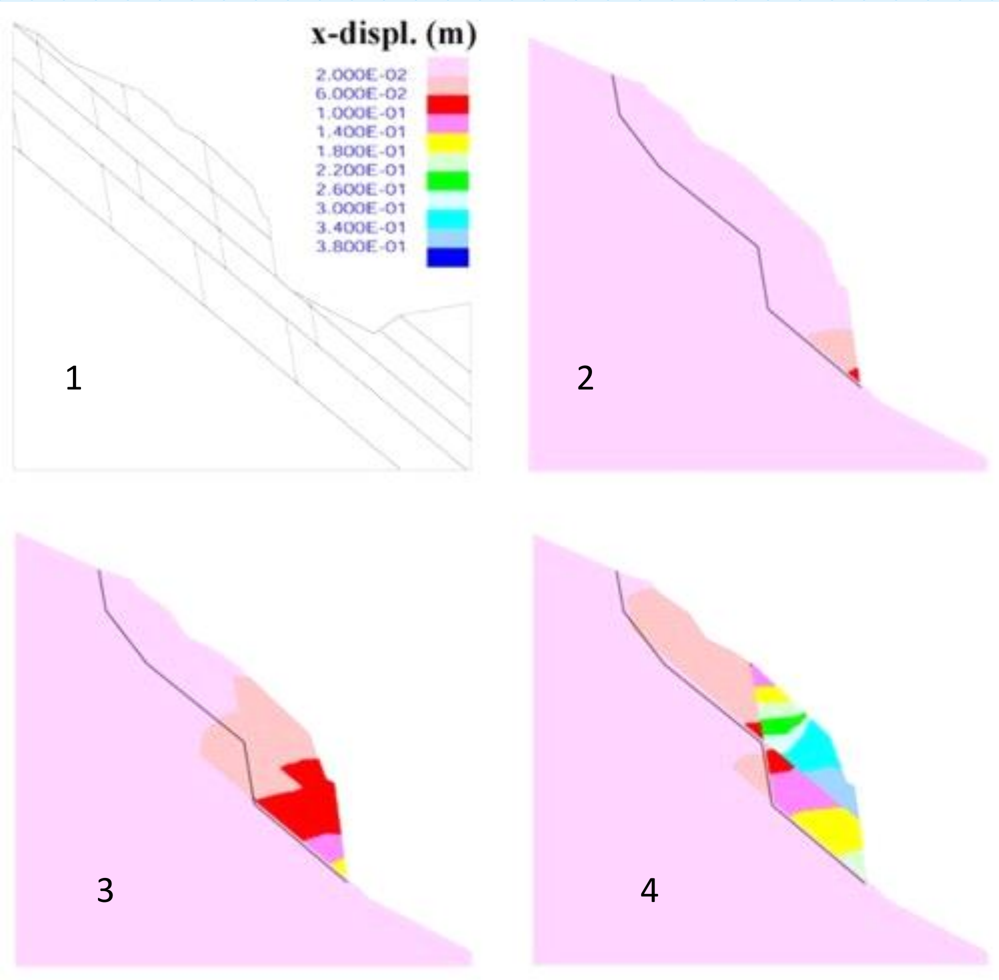


ϕ' = effective stress friction angle or drained friction angle

ϕ = total stress friction angle or undrained friction angle

Progressive Failure

Wednesday, August 17, 2011 12:45 PM

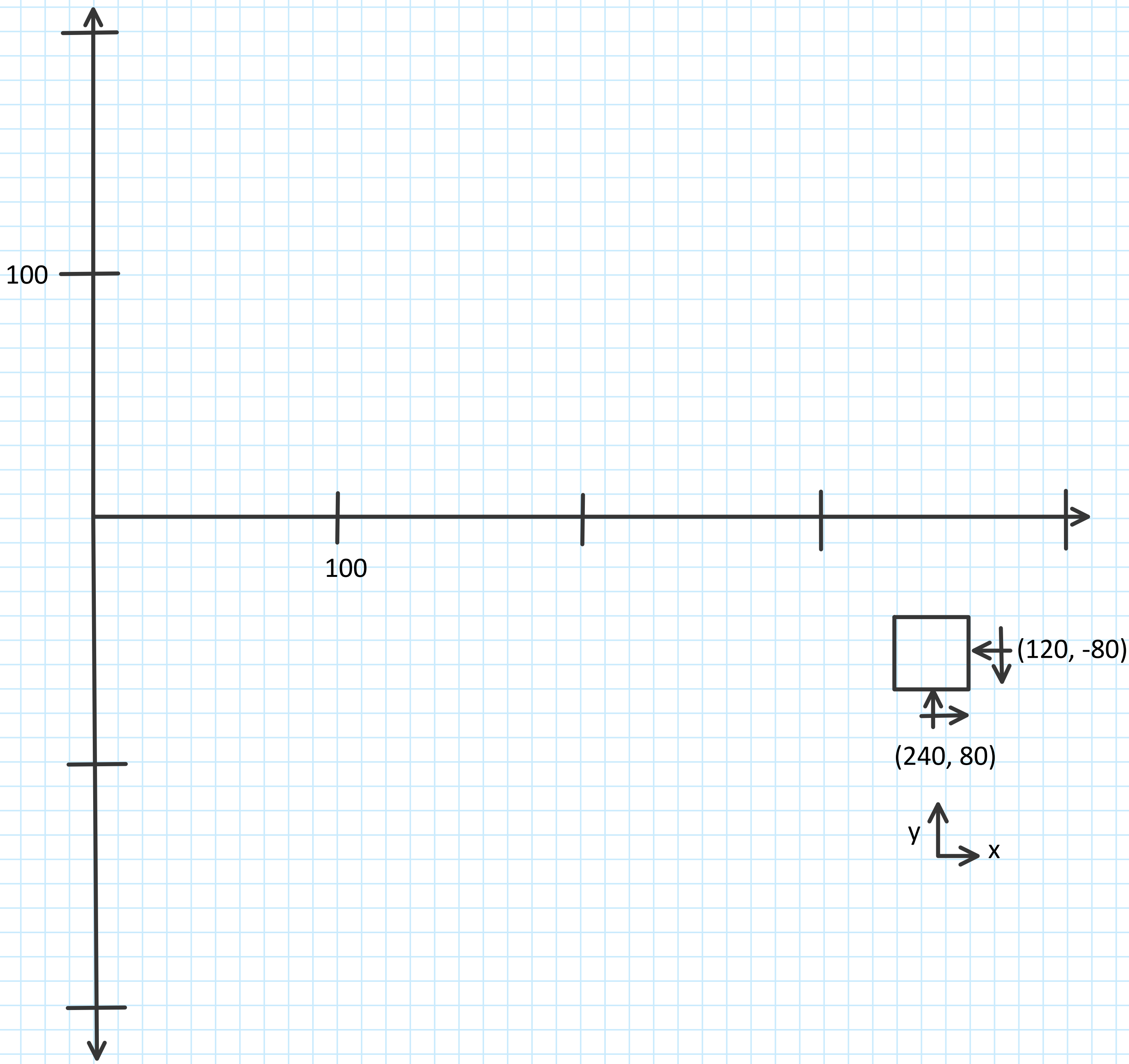


Pasted from <http://www.google.com/imgres?q=progressive+failure+of+slope&um=1&hl=en&safe=off&sa=N&rlz=1T4TSNF_enUS436US436&tbm=isch&tbnid=1fgzDk74as7DwM:&imgrefurl=http://www.rockslide.ethz.ch/Phase1/prelimnum&docid=cd5mWIfXvYgs2M&w=500&h=491&ei=SNxfTp2BBoStsAKluaQh&zoom=1&iact=rc&page=1&tbnh=120&tbnw=122&start=0&ndsp=23&ved=1t:429,r:14,s:0&tx=61&ty=95&biw=1024&bih=676>

1	Initial rock mass with no displacement
2	Failure of rock mass is initiated at red zone
3	Stresses are redistributed and additional failure is caused and propagates upslope
4	Rock mass continues to fail until it reaches a joint

Workout Problem 1

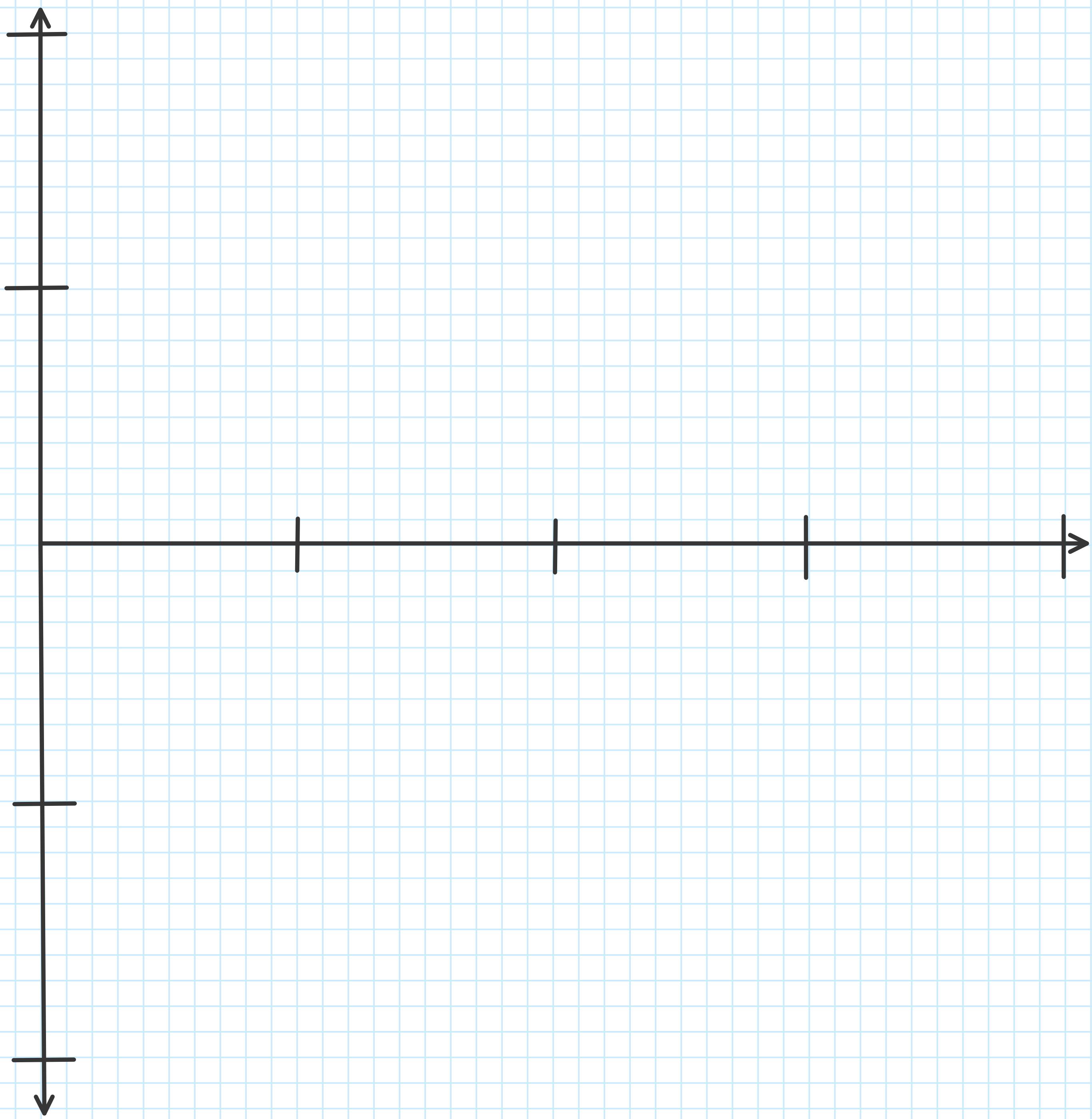
Thursday, September 7, 2023 12:45 PM



1) Find the magnitude and orientation of the principal stress
2) Find the magnitude of the stress on the X and Y planes

Graph Paper

Thursday, September 7, 2023 12:45 PM



Ch 4b Foundation Engineering

Wednesday, August 17, 2011 12:45 PM

Reading Assignment

- Ch. 4b Lecture Notes
- Sections 4.4 - 4.8 (Salgado)

Other Materials

- none

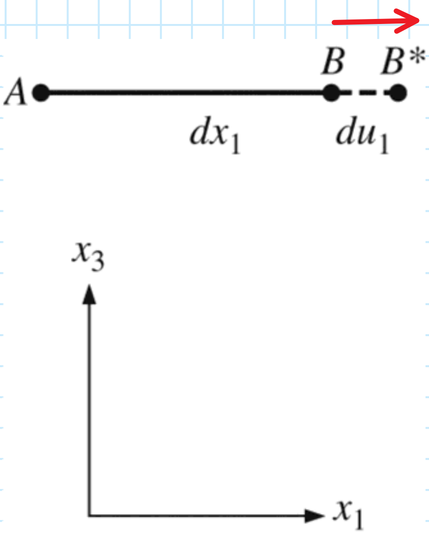
Homework Assignment 4

- Problems 4-18 (15 points), 4-19 (20 points), 4-21 (15 points), 4-22 (10 points), 4-23 (5 points) , 4-25 (30 points), 4-28 part c only (30 points)

Normal Strain

Wednesday, August 17, 2011 12:45 PM

Normal strain in a given direction (this case the x1 direction) quantifies the change in length (contraction or elongation) of an infinitesimal linear element (i.e., very small straight line) aligned with that direction.



AB is elongated to AB*

Definition of **normal strain increment:**

$$\varepsilon_{ii} = - \partial u_i \; / \; \partial x_i$$

For ***normal strain increment* is:**

$$\varepsilon_{ii} = - du_i \; / \; dx_i$$

where dx is the length of the original, undeformed element.

To be consistent with the sign convention for stresses, according to which ***tensile stresses are negative, the normal strain is negative for elongation****.*

Strain measures

Depending on the amount of strain, or local deformation, the analysis of deformation is subdivided into three deformation theories. We are using infinitesimal strain theory using small strain increments in Ch. 4 of Salgado.

- Infinitesimal strain theory, also called ***small strain theory***, *small deformation theory*, *small displacement theory*, or *small displacement-gradient theory,* where strains and rotations are both small. ***In this case, the undeformed and deformed configurations of the body can be assumed identical. The infinitesimal strain theory is used in the analysis of deformations of materials exhibiting elastic behavior, such as materials found in mechanical and civil engineering applications, e.g. concrete and ste el.***

- Finite strain theory also called ***large strain theory***, or *large deformation theory*, deals with deformations in which both rotations and strains are arbitrarily large. In this case, the undeformed and deformed configurations of the continuum are significantly different and a clear distinction has to be made between them.

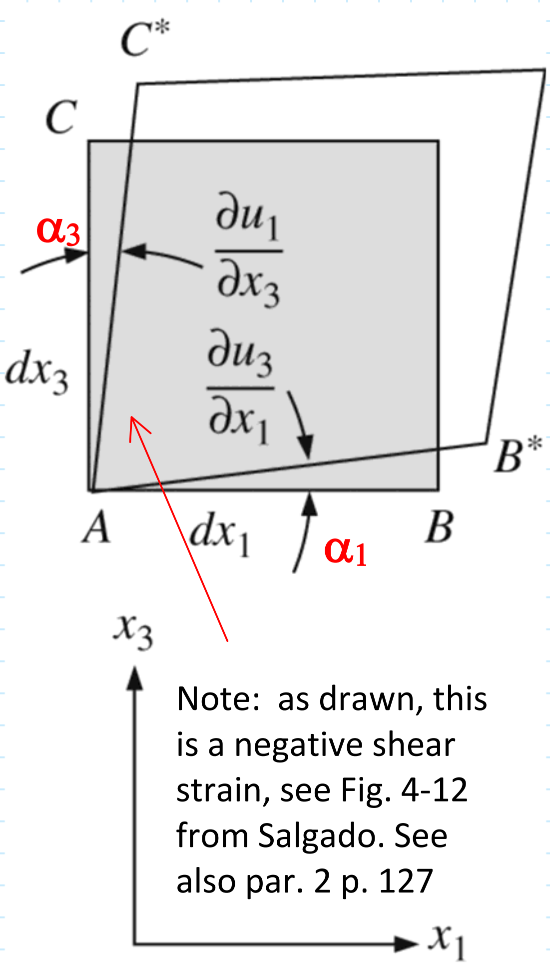
- *Large-displacement or large-rotation theory*, which assumes small strains but large rotations and displacements

Pasted from <http://en.wikipedia.org/wiki/Deformation_(mechanics)>

Shear Strain

Wednesday, August 17, 2011 12:45 PM

Shear strain is the distortion or change in shape of an element caused by shear stress acting on the edges of the element. It can be defined by the angular change of the element compared with its original shape.



Note: as drawn, this is a negative shear strain, see Fig. 4-12 from Salgado. See also par. 2 p. 127

α_1 = shear distortion angle (in radians)
α_3 = shear distortion angle in radians)

γ_{13} = shear strain of x_1 x_3 plane
$\gamma_{13} = -(\alpha_1 + \alpha_3)$
thus, the **shear strain increment** is:

$\gamma_{13} \approx -(\partial u_3/\partial x_1 + \partial u_1/\partial X_3)$ Eq. **(1)**

for small angles (small shear strain) tan α Is approximately equal to α

$\gamma_{13} \approx -(\tan \alpha_1 + \tan \alpha_3)$

Note that partial derivatives in Eq. 1 are required because we are exploring displacement that is occurring in a plane (i.e., two coordinate directions). $\partial u_3/\partial x_1$ means the change in position in the x_3 direction (y direction) with respect to the original length of the line segment dx_1

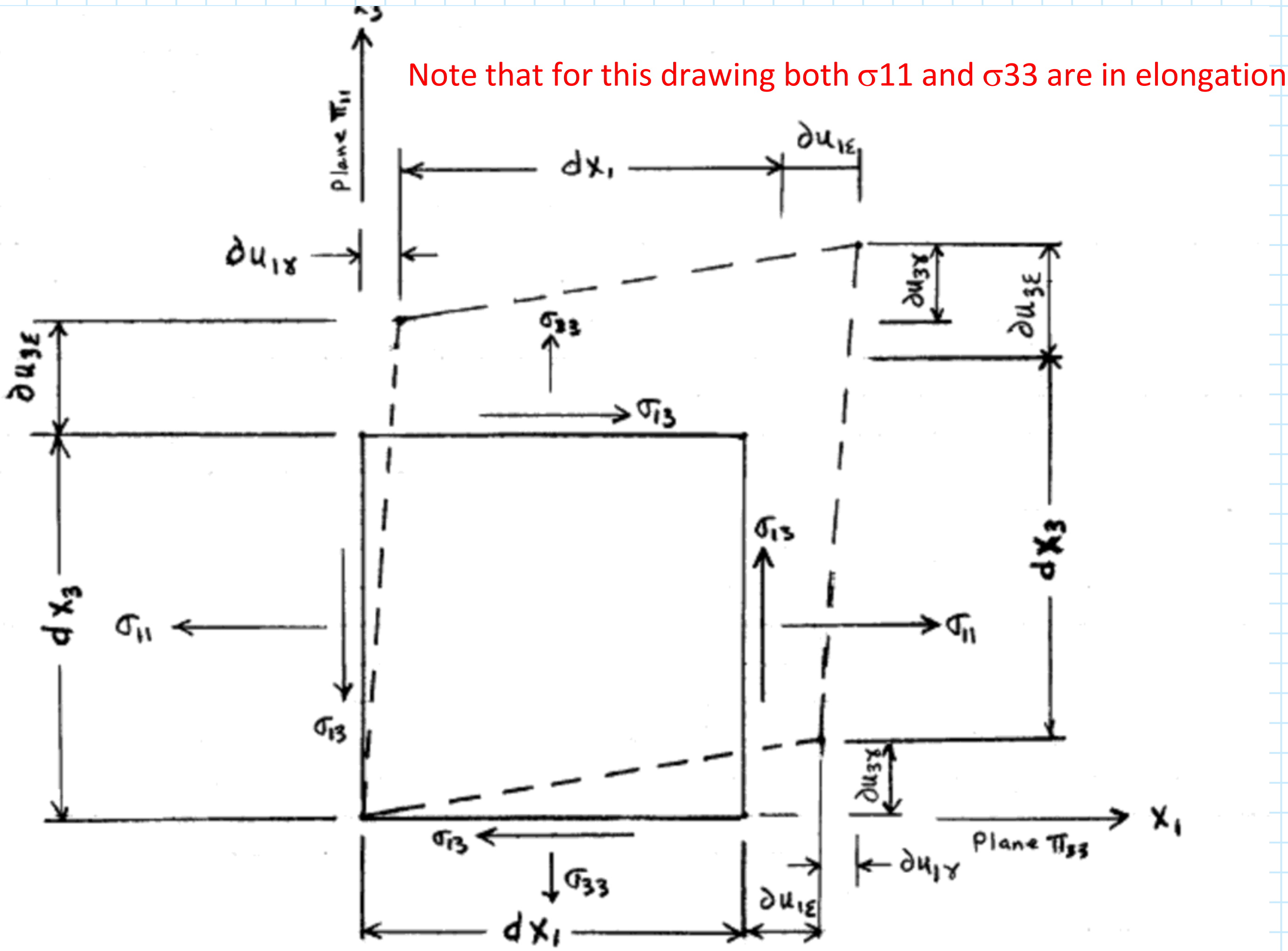
The shear strain, as defined in mechanics, ε_{Ij}, is based on small strain theory, where incremental strain are used; the shear strain for this case is one half the value of the engineering strain, γ_{13}.

$$\boxed{\varepsilon_{ij} = -1/2\gamma_{13} = -[\partial u_3/\partial x_1 + \partial u_1/\partial x_3]/2}$$

Mechanical shear strain (small strain) = 50 percent of engineering (large) shear strain

Nomenclature for Normal and Shearing Strain (Salgado)

Wednesday, August 17, 2011 12:45 PM



From Eq. 4.11, p. 125 of Salgado:

$$\varepsilon_{11} = -\frac{1}{2}\left(\frac{\partial u_{1E}}{\partial x_1} + \frac{\partial u_{1E}}{\partial x_1}\right) = -\frac{\partial u_{1E}}{\partial x_1}$$

$$\varepsilon_{33} = -\frac{1}{2}\left(\frac{\partial u_{3E}}{\partial x_3} + \frac{\partial u_{3E}}{\partial x_3}\right) = -\frac{\partial u_3}{\partial x_3}$$

$$\varepsilon_{13} = -\frac{1}{2}\left(\frac{\partial u_{1\gamma}}{\partial x_3} + \frac{\partial u_{3\gamma}}{\partial x_1}\right) = \varepsilon_{31} = -\frac{1}{2}\left(\frac{\partial u_{3\gamma}}{\partial x_1} + \frac{\partial u_{1\gamma}}{\partial x_3}\right)$$

Small strain theory

Mohr's Circle of Strain

Wednesday, September 14, 2022 5:35 PM

The sign convention we use for plotting the normal and shear strain increments are:

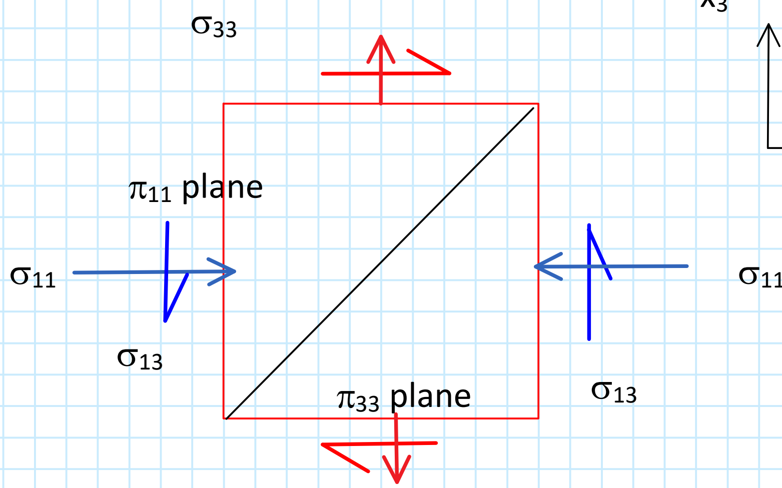
- Normal incremental strains (du_1/dx_1) are **positive for compression**
- Normal incremental strains are **negative for elongation**
- Incremental shear strains are **positive if the shear stress that acts on the plane is positive causes the right angle to increase (see below plot of strain)**

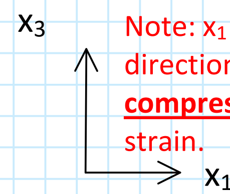
Plot of stress

Blue = positive

Red = negative

Note that for this drawing σ_{11} caused compression and σ_{33} causes elongation





Note: x_1 is in the direction of the greatest **compressive** normal strain.

Note: The π_{11} plane is normal to the x_1 direction and π_{33} plane is normal to the x_3 direction.

Plot of Strain

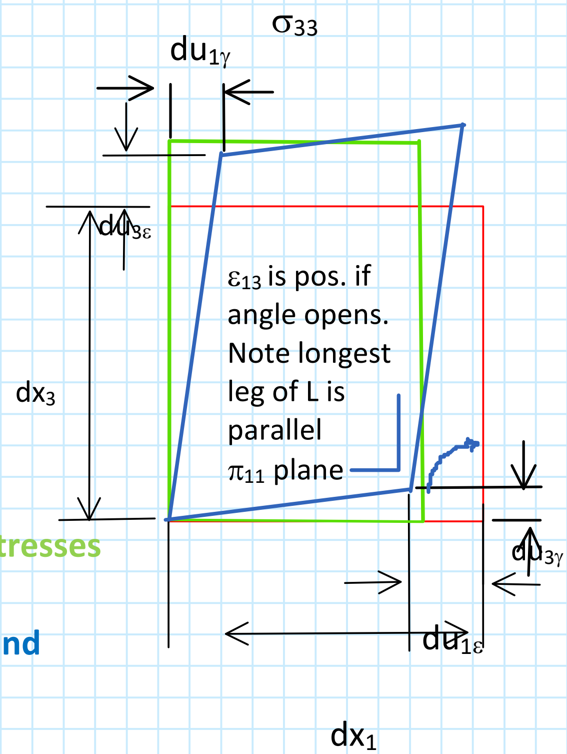
Normal strains

$\varepsilon_{11} = -du_{1\varepsilon}/dx_1$

$\varepsilon_{33} = -du_{3\varepsilon}/dx_3$

Shear strains

$\varepsilon_{13} = -1/2(du_{1\gamma}/dx_3 + du_{3\gamma}/dx_1)$



Red = original shape

Distortion due to normal stresses

Stresses combined

Distortion due to normal and

Draw the element with the normal and shear strains that caused the distortion to consistent with the normal and shear stresses.

Note ε_{13} is $1/2d\gamma$.
Note ε_{13} is the shear stress acting on the π_{11} plane in the x_3 direction.

Mohr's Circle of Strain - Example

Monday, September 12, 2022 5:35 PM

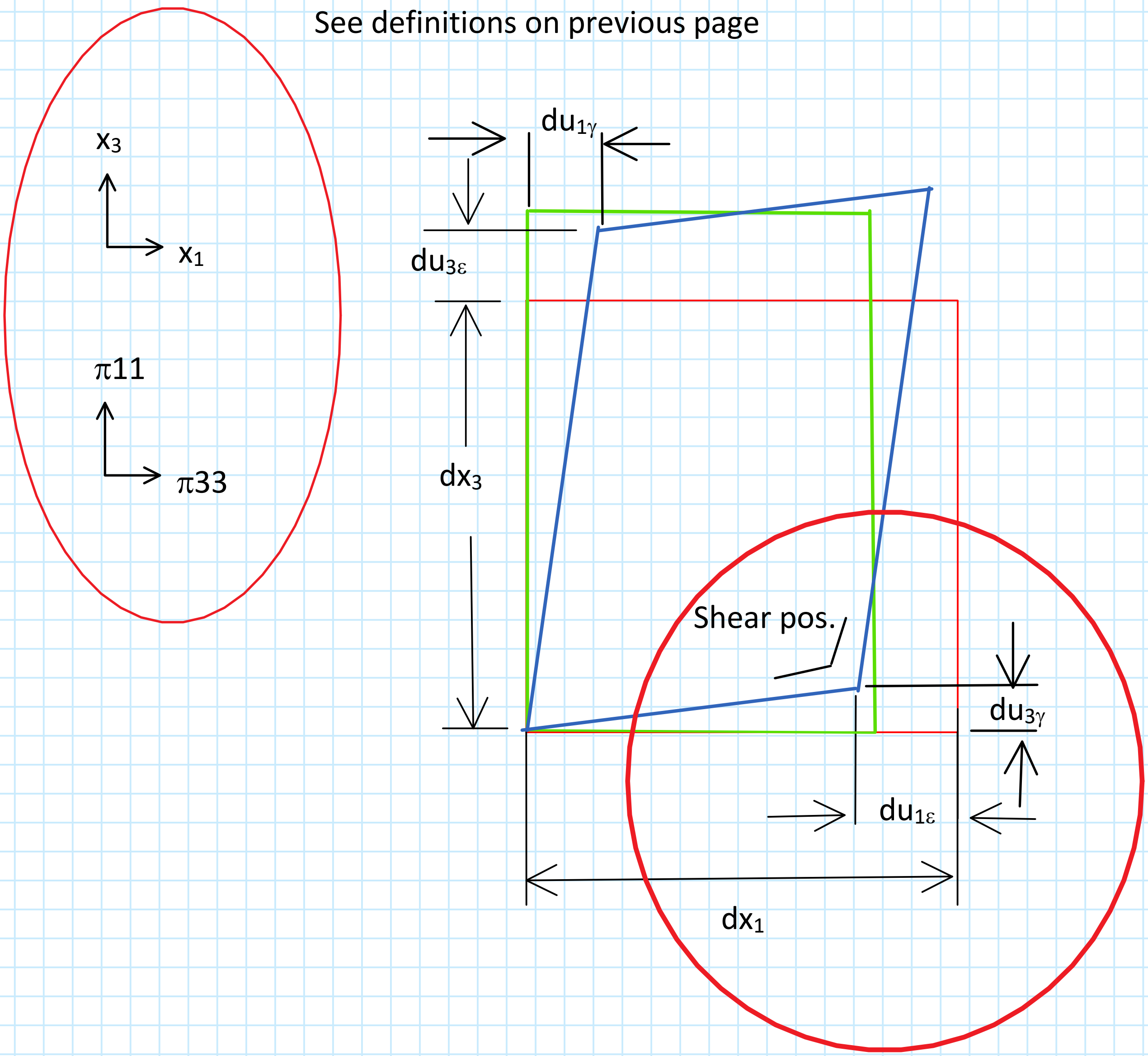
Example

Given: ε_{11} = 0.0206% (compression) ε_{33} = -0.0406% (elongation), ε_{13} = 0.0257%

Find: (a) magnitude of principal normal strain increments, $d\varepsilon_1$ and $d\varepsilon_3$

(b) orientation of the principle planes

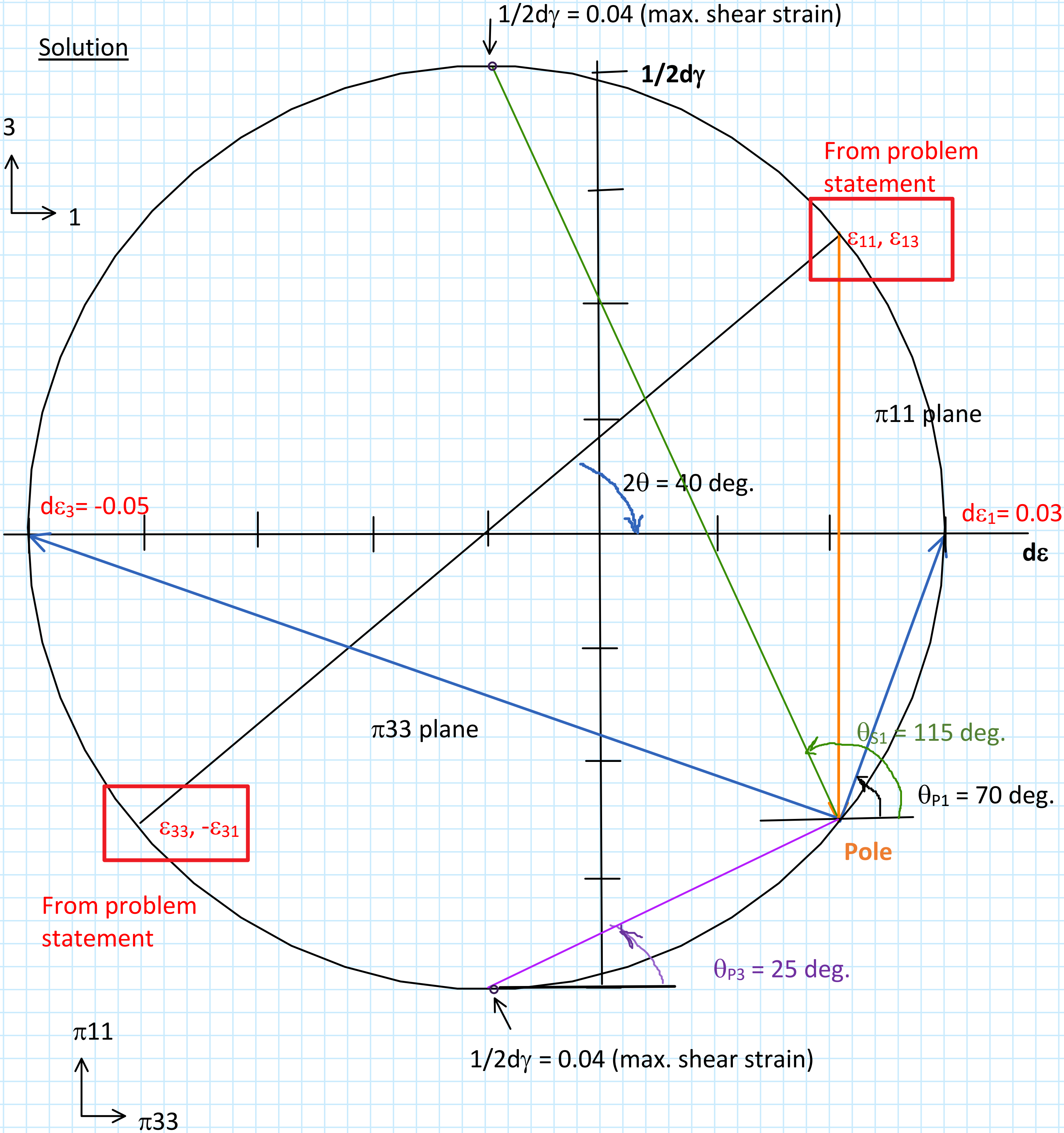
See definitions on previous page



Mohr's Circle of Strain - Example (cont.)

Friday, September 22, 2017 5:36 PM

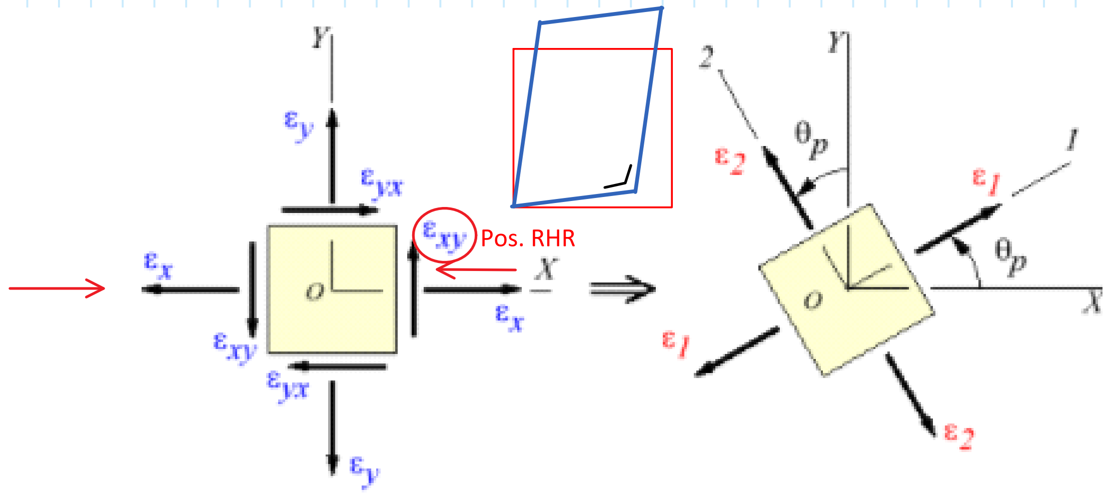
Solution



Mohr's Circle of Strain - Example (cont.)

Thursday, September 30, 2021 5:36 PM

Solution from strain calculator (see link below)



Strains in given coordinate system

Strains transformed to principal directions

Inputs

Normal Strain ε_x:	-0.0206	Compression negative using this solver
Normal Strain ε_y:	0.0406	Tension positive
Shear Strain ε_{xy}:	0.0257	Angle is greater than 90 deg.

Answers

Maximum Normal Stain ε_1:	0.0500	This will plot as a negative -0.05 on the Mohr's circle construction
Minimum Normal Strain ε_2:	-0.0300	This will plot as a positive 0.03
Principal Angle θ_p:	70.0 deg	CCW from the horizontal
Maximum Shear Strain ε_{xymax}:	0.0400	
Maximum Shear Angle θ_{s1}:	115 deg	CCW from horizontal plane
and θ_{s2}:	25.0 deg	CCW from horizontal plane

Calculate Again | Default Values

http://www.efunda.com/formulae/solid_mechanics/mat_mechanics/calc_principal_strain.cfm#calc

Principal Strain Increments

Wednesday, August 17, 2011 12:45 PM

You can use Eqs. 4.5 and 4.6 on p. 117 of the textbook to find strain increments for a plane of any orientation by substituting $d\varepsilon$ for σ and $\frac{1}{2}d\gamma$ for τ as follows:

$$d\varepsilon_\theta = \frac{1}{2}(d\varepsilon_{11} + d\varepsilon_{33}) + \frac{1}{2}(d\varepsilon_{11} - d\varepsilon_{33})\cos 2\theta + d\varepsilon_{13}\sin 2\theta$$

$$\frac{1}{2}d\gamma_\theta = \frac{1}{2}(d\varepsilon_{11} - d\varepsilon_{33})\sin 2\theta - d\varepsilon_{13}\cos 2\theta$$

The orientation of the major principal strain increment (θ_p) can be founded by substituting $d\varepsilon$ for σ in Eq. 4.7 as follows:

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{2d\varepsilon_{13}}{d\varepsilon_{11} - d\varepsilon_{33}}$$

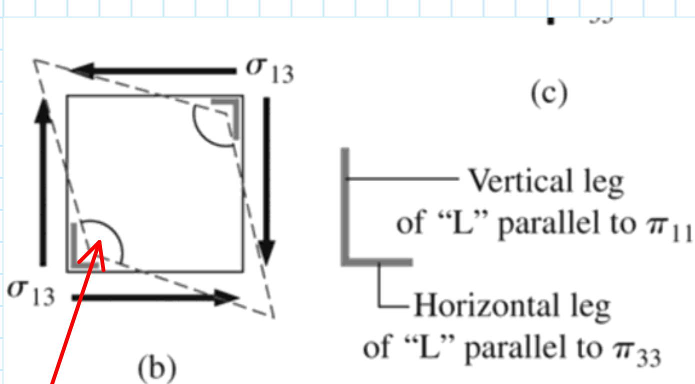
To use the first equation to solve for the principal strain increments, substitute θ_p for θ in the first equation above, as follows:

$$d\varepsilon_1 = \frac{1}{2}(d\varepsilon_{11} + d\varepsilon_{33}) + \frac{1}{2}(d\varepsilon_{11} - d\varepsilon_{33})\cos 2\theta_p + d\varepsilon_{13}\sin 2\theta_p$$

$$d\varepsilon_3 = \frac{1}{2}(d\varepsilon_{11} + d\varepsilon_{33}) + \frac{1}{2}(d\varepsilon_{11} - d\varepsilon_{33})\cos 2\theta_p + d\varepsilon_{13}\sin 2\theta_p$$

where $\tan(2\theta_p) = \frac{2d\varepsilon_{13}}{d\varepsilon_{11} + d\varepsilon_{33}}$

You must use sign conventions for all the terms that are consistent with Figure 4-2 on p. 116. Note that θ is referenced to the π_{11} plane and is positive if the plane on which you are determining the stresses or strains (the plane on which σ_θ and τ_θ occur for stress analysis, or the plane on which $d\varepsilon_\theta$ and $d\gamma_\theta$ occur for incremental strain analysis) is counterclockwise from the π_{11} plane. Therefore, when determining the major principal strain increment ($d\varepsilon_1$), θ_p is the angle from the π_{11} plane to the plane on which the major principal plane increment occurs; and when determining the minor principal strain increment ($d\varepsilon_3$), θ_p is the angle from the π_{11} plane to the plane on which the minor principal plane increment occurs.



Definition of shear strain for use in above equations.

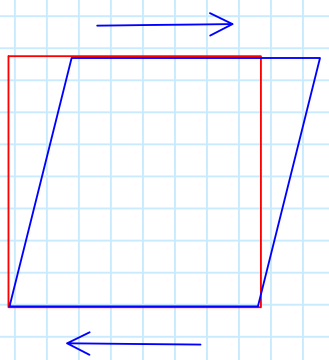
L is opening up = positive incremental shear strain

Dilatancy Angle

Wednesday, August 17, 2011 12:45 PM

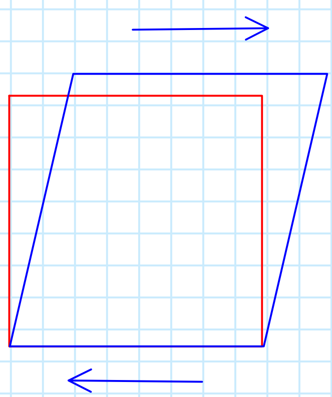
How does dilatancy affect the behavior of soil?

The **angle of dilation** controls an amount of plastic volumetric strain developed during plastic shearing and is assumed constant during plastic yielding. The value of $\psi=0$ corresponds to the volume preserving deformation while in shear.
Clays (regardless of overconsolidated layers) are characterized by a very low amount of dilation ($\psi\approx0$). As for sands, the angle of dilation depends on the angle of internal friction. For non-cohesive soils (sand, gravel) with the angle of internal friction $\varphi > 30°$ the value of dilation angle can be estimated as $\psi=\varphi-30°$. A negative value of dilation angle is acceptable only for rather loose sands.
Pasted from <http://www.finesoftware.eu/geotechnical-software/help/fem/angle-of-dilation/>



No dilatancy, dilatancy angle = 0. Note that the unit square has undergone distortion solely.

Sometimes this is called pure shear (i.e., no dilatancy is occurring)



Dilatancy during shear. Note that the unit square has undergone distortion and volumetric strain (change in volume).

Why is dilatancy important? How does it affect the behavior of the specimen during shear? How does it affect the pore pressures of a specimen during undrained shear?

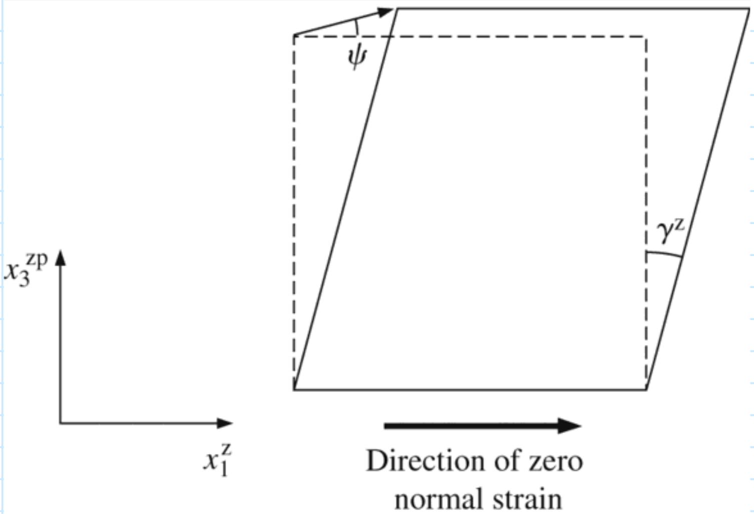
Dilatancy is **a significant contributor to shear strength in dense, coarse granular materials**, which is why proper compaction of engineered fill is so important. Increasing the density of the aggregate by compaction increases both dilatancy and internal friction and therefore the overall peak shear strength. Oct 1, 2020

From <https://www.google.com/search?q=dilatancy&rlz=1C1SQJL_enUS822US822&oq=dilatancy&aqs=chrome..69i57.5283j0j15&sourceid=chrome&ie=UTF-8>

Dilatancy Angle (cont.)

Wednesday, August 17, 2011 12:45 PM

Soils dilate (expand) or contract upon shearing and the degree of this dilatancy can be explained by the dilatancy angle, ψ.



This element is dilating during shear. **This is plastic behavior**.

The dilatancy angle can be calculated from the Mohr's circle of strain, see previous page. It can also be estimated from the following formulas.

Volumetric Strain

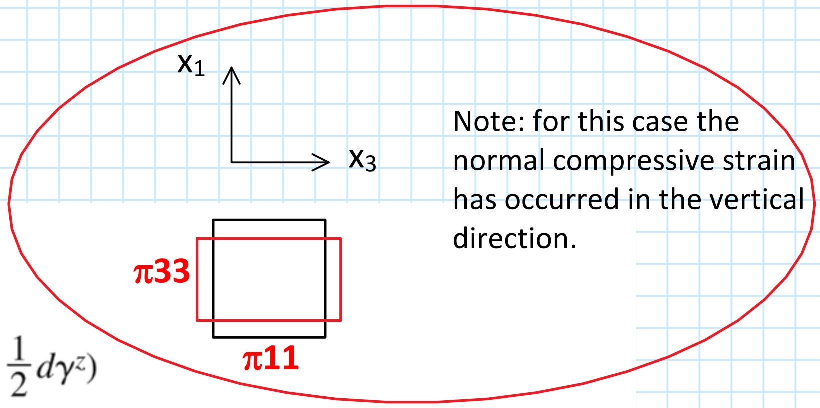
$$\sin\psi = \frac{OZ}{|OA_1|} = -\frac{\frac{1}{2}(d\varepsilon_1 + d\varepsilon_3)}{\frac{1}{2}(d\varepsilon_1 - d\varepsilon_3)} = -\frac{d\varepsilon_1 + d\varepsilon_3}{d\varepsilon_1 - d\varepsilon_3} = -\frac{d\varepsilon_v}{|d\gamma_{max}|}$$

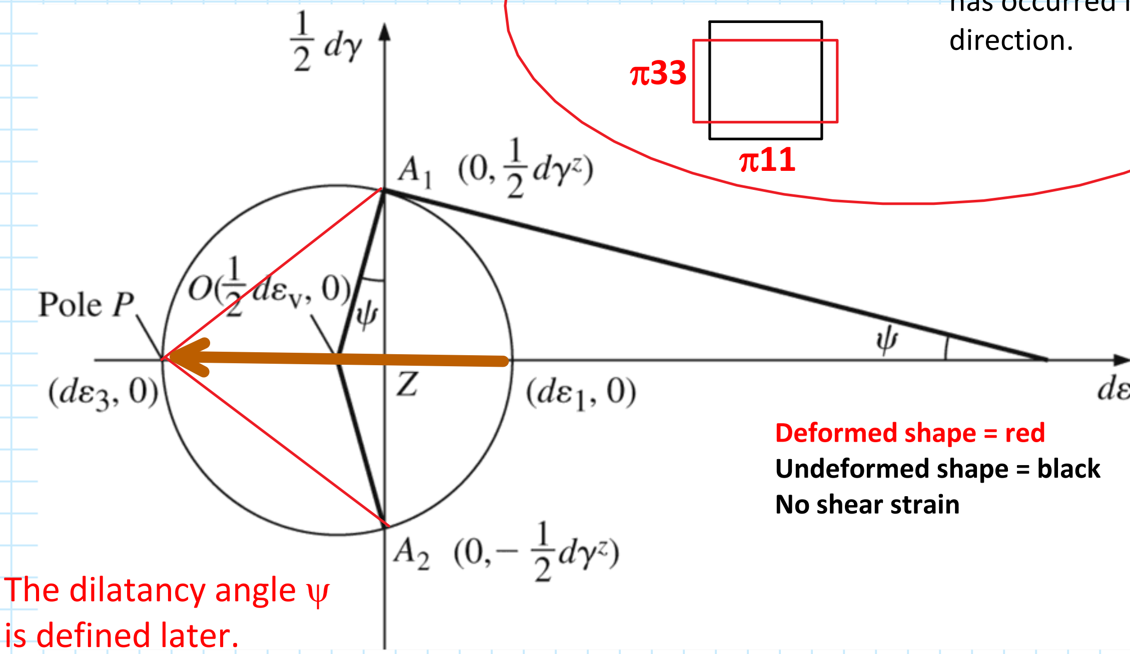
Shear Strain

$$\tan\psi = \frac{OZ}{|ZA_1|} = -\frac{\frac{1}{2}d\varepsilon_v}{|\frac{1}{2}d\gamma^z|} = -\frac{d\varepsilon_v}{|d\gamma^z|}$$

Dilatancy Angle (cont.)

Monday, September 12, 2022 12:03 PM





- The leftmost and rightmost points of the circle correspond to the magnitude of the **principal strain increments** $(d\varepsilon_1, 0)$ and $(d\varepsilon_3, 0)$
- The highest and lowest points on the circle correspond to the magnitude of the **maximum engineering shear strain increment** divided by 2.

The **pole method** can also be used in conjunction with the Mohr's circle of incremental strain to find the directions and magnitude of the incremental strain for other arbitrary planes. For the above use the following method to find the pole.

First, note that the principal normal strain increment, $d\varepsilon_1$, is caused by a stress acting on a **horizontal plane.** To find the pole, start at $(d\varepsilon_1, 0)$ on the Mohr's circle of strain and draw a horizontal line until you intersect the Mohr's circle once again at the point labeled Pole P. Once found, this pole can be used to locate other planes and find the magnitude of the normal and shear strains acting on these planes.

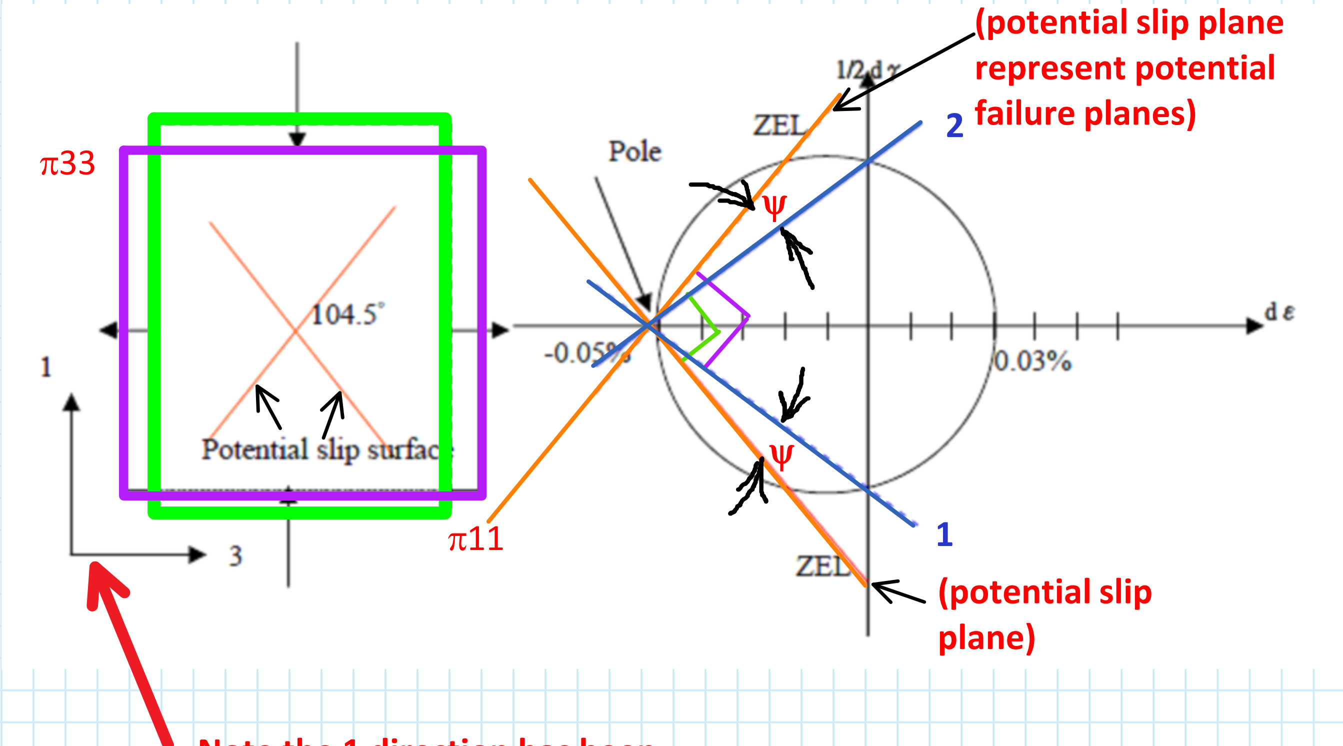
For a strain solver, see the following link. However, the sign conventions are different than those we have adopted.

http://www.efunda.com/formulae/solid_mechanics/mat_mechanics/calc_principal_strain.cfm

Zero Extension Line

Monday, September 12, 2022 12:45 PM

Green line = original shape
Purple line = deformed shape



Note the 1 direction has been changed to the vertical direction. Note that below ground, the largest stress is often in the vertical direction.

- The zero extension lines (ZEL) (i.e., potential slip surface) (red lines) are normal (90 degrees) to the planes where dε is zero (blue lines 1 & 2). Note that these dε = zero planes intersect the y-axis (i.e., 1/2dγ axis). One ZEL is found 90 deg (purple normal) to the dε = zero plane 1. The other ZEL is found 90 deg (green normal) to the dε = zero plane 2.

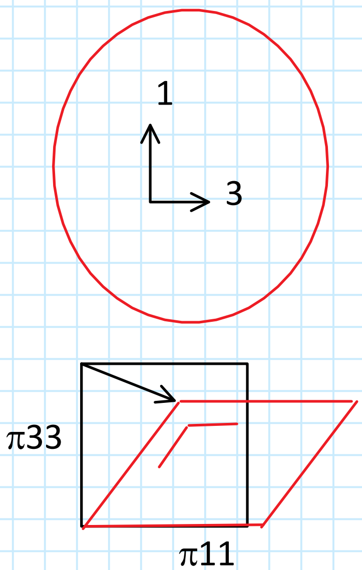
- The dilatancy angle, ψ, is the angle between the ZEL lines and the dε = zero planes

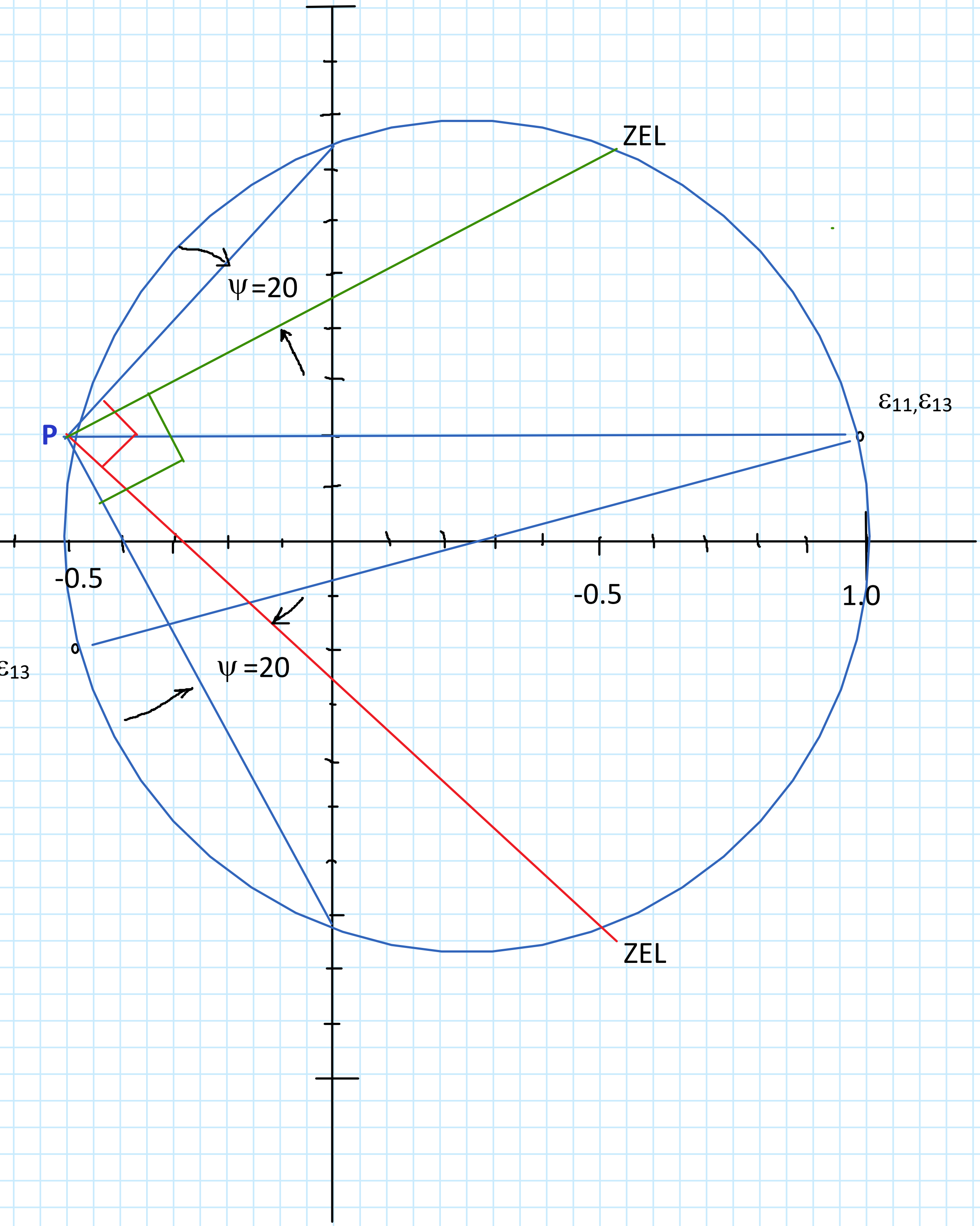
  Process to find slip planes: 1) draw Mohr's circle of strain, 2) find the pole, 3) using the pole find the orientation of blue lines 1 & 2 above. These represent the planes where the normal strains are zero. Find the potential slip planes (i.e., zero extension lines (ZELs) which are located 90 degrees from the blue lines. Note also the dilatancy angle ψ is the angle between the ZEL and the blue lines.

Zero Extension Line - Example

Wednesday, September 14, 2022 12:45 PM

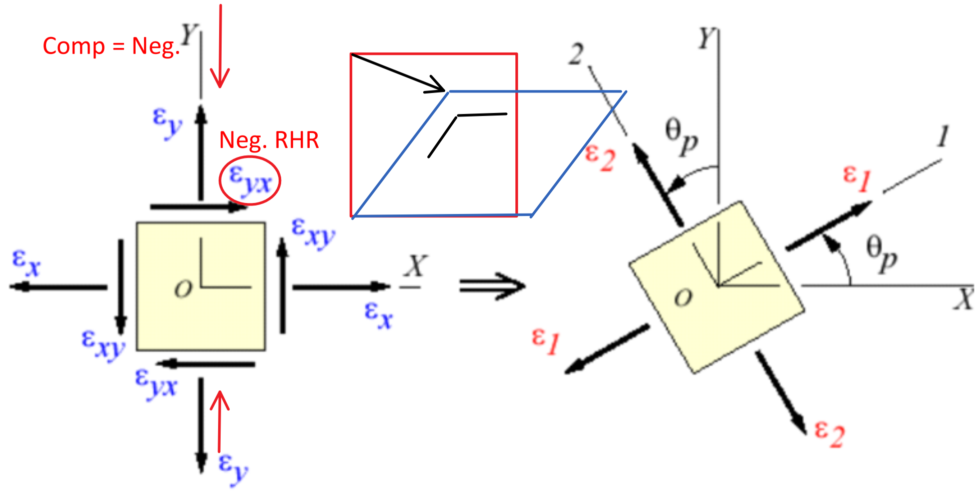
Given ε_{11}= 1.0 (compression positive), ε_{33}= -0.50(elongation negative), ε_{13} = 0.2
Fine ZELs, Find ψ





Zero Extension Line - Example

Thursday, September 30, 2021 12:45 PM



Inputs

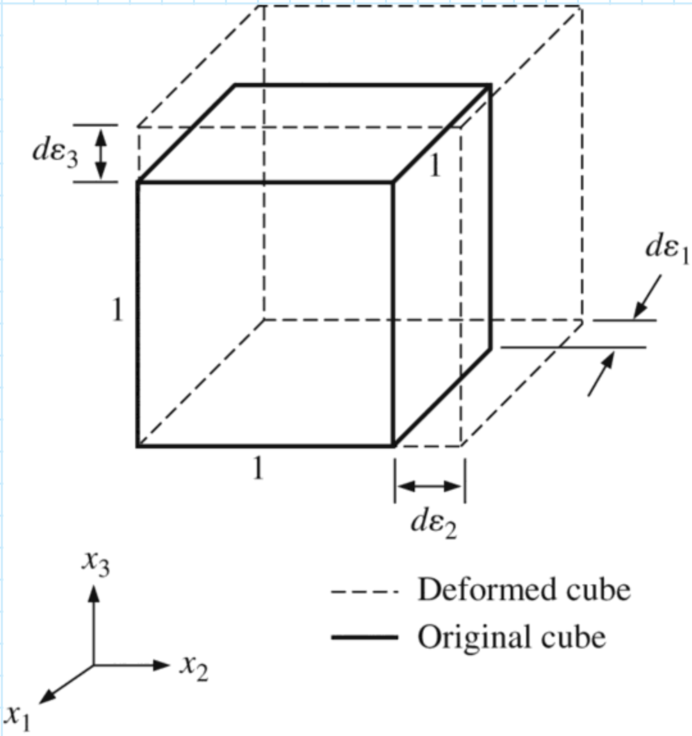
Normal Strain ε_x:	0.5	Pos. = elongation
Normal Strain ε_y:	-1	Neg. = compression
Shear Strain ε_{xy}:	-0.2	Neg. RHR

Answers

Maximum Normal Stain ε_1:	0.526	
Minimum Normal Strain ε_2:	-1.03	compression
Principal Angle θ_p:	-7.47 deg	
Maximum Shear Strain ε_{xymax}:	0.776	
Maximum Shear Angle θ_{s1}:	37.5 deg	CCW from horizontal
and θ_{s2}:	-52.5 deg	CW from horizontal

Volumetric Strain

Wednesday, August 17, 2011 12:45 PM



Note that no distortion of the cube is occurring. The only strain is compressional or extensional.

dV = change in volume for a unit cube

$dV = (1+du_1) * (1 + du_2) * (1 + du_3) - 1^3$

Volumetric strain increment (small strain theory)

$d\varepsilon_V = - dV/1 = 1 - (1-d\varepsilon_1)(1-d\varepsilon_2)(1-d\varepsilon_3)$
(neg. sign req'd to make contraction positive)

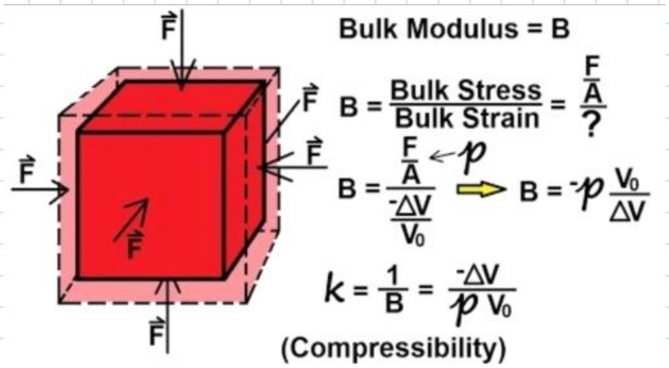
In small strain theory, the strain increment $d\varepsilon_V$ is very small and the second and third order terms are negligible, thus the above equation reduces to:
$d\varepsilon_V = d\varepsilon_1 + d\varepsilon_2 + d\varepsilon_3$

In terms of principal strains, then

$$\mathbf{d\varepsilon_V = d\varepsilon_{11} + d\varepsilon_{22} + d\varepsilon_{33}}$$

Bulk Modulus

Wednesday, August 17, 2011 12:45 PM

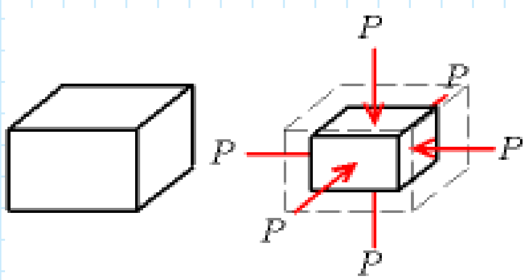


The bulk modulus (*K*) of a substance measures the substance's resistance to uniform compression. It is defined as the pressure increase needed to cause a given relative decrease in volume. Its base unit is that of pressure.
As an example, suppose an iron cannon ball with bulk modulus 160 GPa is to be reduced in volume by 0.5%. This requires a pressure increase of 0.005×160 GPa = 0.8 GPa (116,000 psi).

Definition

The bulk modulus *K* can be formally defined by the equation:

$$K = -V \frac{\partial P}{\partial V}$$



where *P* is pressure, *V* is volume, and $\partial P/\partial V$ denotes the partial derivative of pressure with respect to volume. The inverse of the bulk modulus gives a substance's compressibility.

Other moduli describe the material's response (strain) to other kinds of stress: the shear modulus describes the response to shear, and Young's modulus describes the response to linear strain. For a fluid, only the bulk modulus is meaningful. For an anisotropic solid such as wood or paper, these three moduli do not contain enough information to describe its behavior, and one must use the full generalized Hooke's law

Pasted from <http://en.wikipedia.org/wiki/Bulk_modulus>

Strains from 2D Plane Strain - Elastic Theory - Hooke's Law

Thursday, March 11, 2010 11:43 AM

Nonzero **stress**: $\sigma_x, \sigma_y, \sigma_z, \tau_{xy}$

Nonzero **strain** components: $\varepsilon_x, \varepsilon_y, \gamma_{xy}$

Isotropic linear elastic stress-strain law $\boxed{\underline{\sigma} = \underline{D}\,\underline{\varepsilon}}$ **Hooke's Law**

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \dfrac{1-2\nu}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

E = Young's Modulus or the Elastic Modulus

υ = Poisson's ratio

Note that for the plane strain case the normal stress in the z direction is not zero. However, since this stress is balanced, it produces no strain in this direction.

$$\sigma_z = \nu\left(\sigma_x + \sigma_y\right)$$

Hence, the D matrix for the **plane strain case** is

$$\underline{D} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \dfrac{1-2\nu}{2} \end{bmatrix}$$

Table 4-3 Relationship between the four most common elastic constants

	Elastic pair	
Elastic constant	***E, ν***	***K, G***
Young's modulus $E =$	E	$\dfrac{9KG}{3K + G}$
Poisson's ratio $\nu =$	ν	$\dfrac{3K - 2G}{6K + 2G}$
Shear modulus $G =$	$\dfrac{E}{2(1 + \nu)}$	G
Bulk modulus $K =$	$\dfrac{E}{3(1 - 2\nu)}$	K
Constrained modulus $M =$	$\dfrac{E(1 - \nu)}{(1 + \nu)(1 - 2\nu)}$	$\dfrac{3K + 4G}{3}$

Strains from 3D Elastic Theory

Wednesday, August 17, 2011 12:45 PM

Hooke's Law (3D)

$$\underline{\sigma} = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} \qquad \sigma := \begin{pmatrix} 60 \cdot 10^3 \\ 30 \cdot 10^3 \\ 30 \cdot 10^3 \\ 0 \cdot 10^3 \\ 0 \cdot 10^3 \\ 0 \cdot 10^3 \end{pmatrix} \qquad E := 5 \cdot 10^6 \qquad \nu := 0.1$$

$$\underline{D} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}$$

$$\Delta := \frac{E}{(1+\nu)(1-2\cdot\nu)} \cdot \begin{pmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2\cdot\nu}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\cdot\nu}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\cdot\nu}{2} \end{pmatrix}$$

$$\delta := \Delta^{-1} \qquad \underline{\varepsilon} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} \qquad \varepsilon := \delta \cdot \sigma \qquad \varepsilon = \begin{pmatrix} 0.011 \\ 4.2 \times 10^{-3} \\ 4.2 \times 10^{-3} \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Elastic Constants - Relationships

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	(λ, G)	(E, G)	(K, λ)	(K, G)	(λ, ν)
$K =$	$\lambda + \frac{2G}{3}$	$\frac{EG}{3(3G-E)}$			$\frac{\lambda(1+\nu)}{3\nu}$
$E =$	$\frac{G(3\lambda+2G)}{\lambda+G}$		$\frac{9K(K-\lambda)}{3K-\lambda}$	$\frac{9KG}{3K+G}$	$\frac{\lambda(1+\nu)(1-2\nu)}{\nu}$
$\lambda =$		$\frac{G(E-2G)}{3G-E}$		$K - \frac{2G}{3}$	
$G =$			$\frac{3(K-\lambda)}{2}$		$\frac{\lambda(1-2\nu)}{2\nu}$
$\nu =$	$\frac{\lambda}{2(\lambda+G)}$	$\frac{E}{2G} - 1$	$\frac{\lambda}{3K-\lambda}$	$\frac{3K-2G}{2(3K+G)}$	
$M =$	$\lambda + 2G$	$\frac{G(4G-E)}{3G-E}$	$3K - 2\lambda$	$K + \frac{4G}{3}$	$\frac{\lambda(1-\nu)}{\nu}$

	(G, ν)	(E, ν)	(K, ν)	(K, E)	(M, G)
$K =$	$\frac{2G(1+\nu)}{3(1-2\nu)}$	$\frac{E}{3(1-2\nu)}$			$M - \frac{4G}{3}$
$E =$	$2G(1+\nu)$		$3K(1-2\nu)$		$\frac{G(3M-4G)}{M-G}$
$\lambda =$	$\frac{2G\nu}{1-2\nu}$	$\frac{E\nu}{(1+\nu)(1-2\nu)}$	$\frac{3K\nu}{1+\nu}$	$\frac{3K(3K-E)}{9K-E}$	$M - 2G$
$G =$		$\frac{E}{2(1+\nu)}$	$\frac{3K(1-2\nu)}{2(1+\nu)}$	$\frac{3KE}{9K-E}$	
$\nu =$				$\frac{3K-E}{6K}$	$\frac{M-2G}{2M-2G}$
$M =$	$\frac{2G(1-\nu)}{1-2\nu}$	$\frac{E(1-\nu)}{(1+\nu)(1-2\nu)}$	$\frac{3K(1-\nu)}{1+\nu}$	$\frac{3K(3K+E)}{9K-E}$	

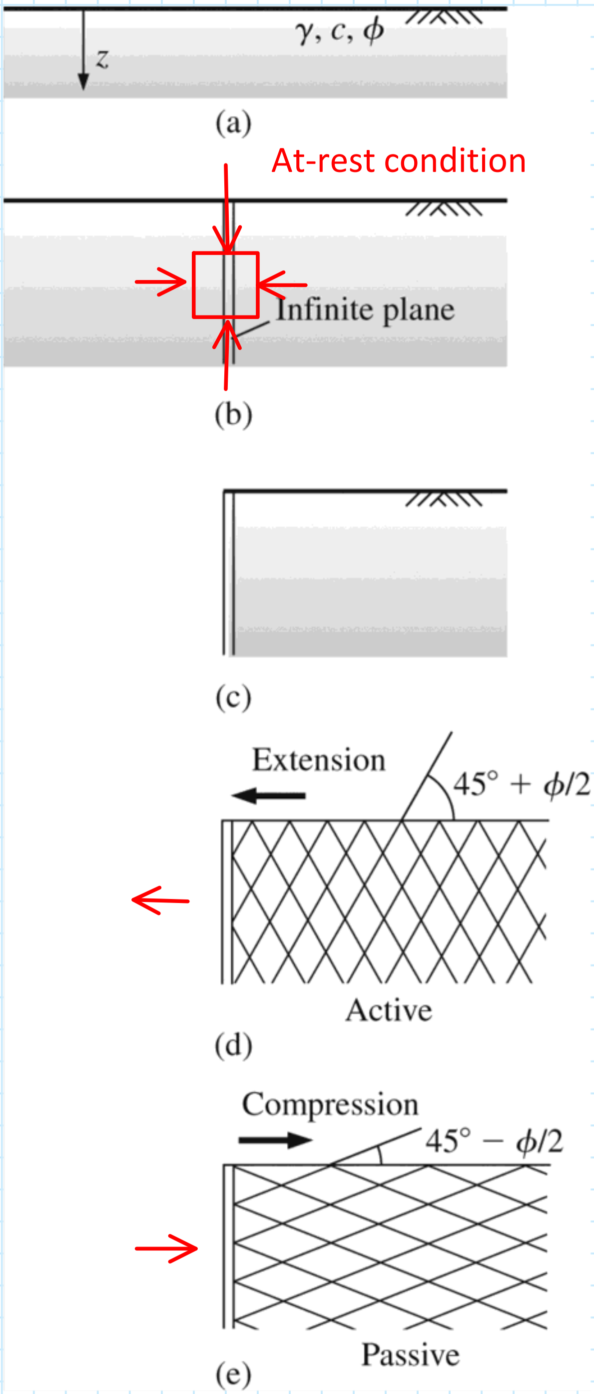
http://en.wikipedia.org/wiki/Bulk_modulus

Bulk modulus (K) • Young's modulus (E) • Lamé's first parameter (λ) •

Shear modulus (G) • Poisson's ratio (ν) • P-wave modulus (M)

At-Rest, Active and Passive Earth Pressure

Wednesday, August 17, 2011 12:45 PM



At-rest earth pressure:

a. Shear stress are zero.
b. $\sigma_V = \sigma_1$
c. $\sigma_H = \sigma_3$ $\quad K_o = \sigma_h/\sigma_v$
d. $\sigma_H = K_o\sigma_1$
e. $K_0 = 1 - \sin\phi$ (Normally consolidated
f. $K_0 = (1 - \sin\phi)\, OCR^{-1/2}$
g. $OCR = \sigma'_{vp}/\sigma'_v$
h. $K_0 = \nu / (1-\nu)$

Let us assume that:

a) wall is perfectly smooth (no shear stress develop on the interface between wall and the retained soil)
b) no sloping backfill
c) back of the wall is vertical
d) retained soil is a purely frictional material (c=0)

Active Case

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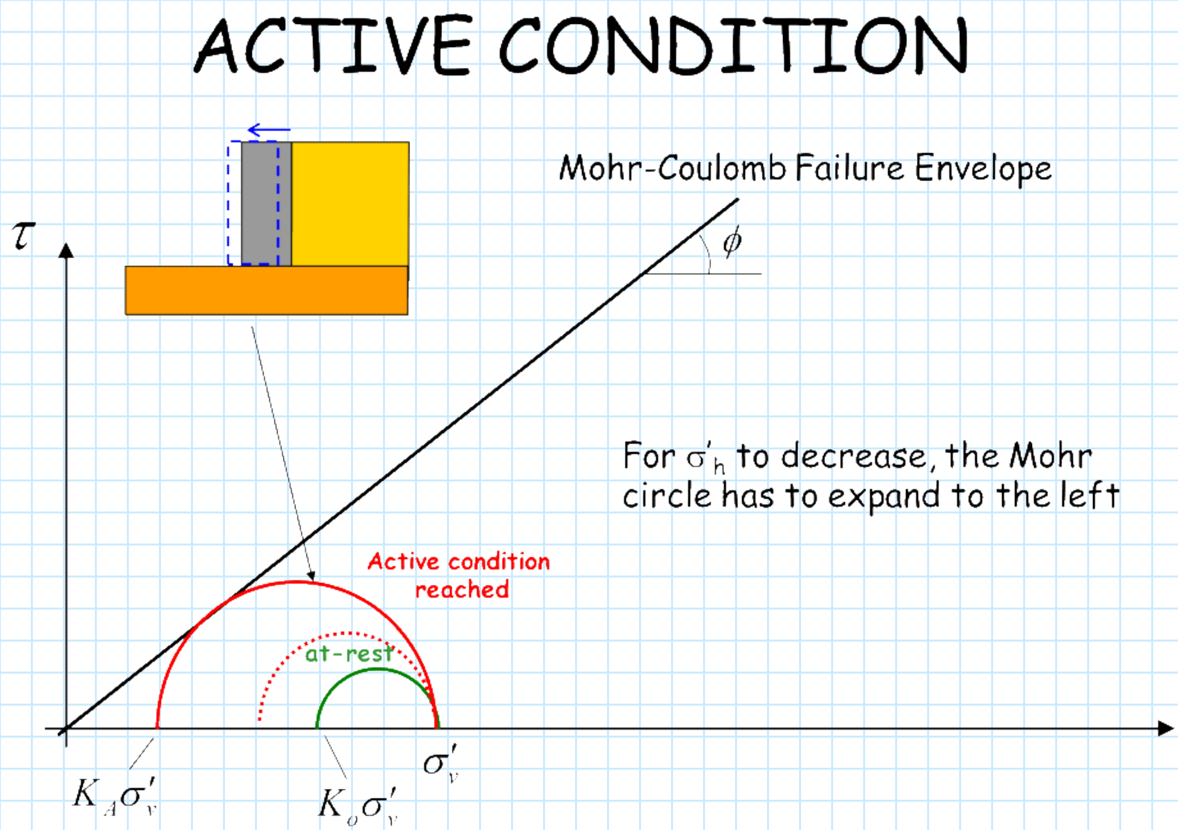
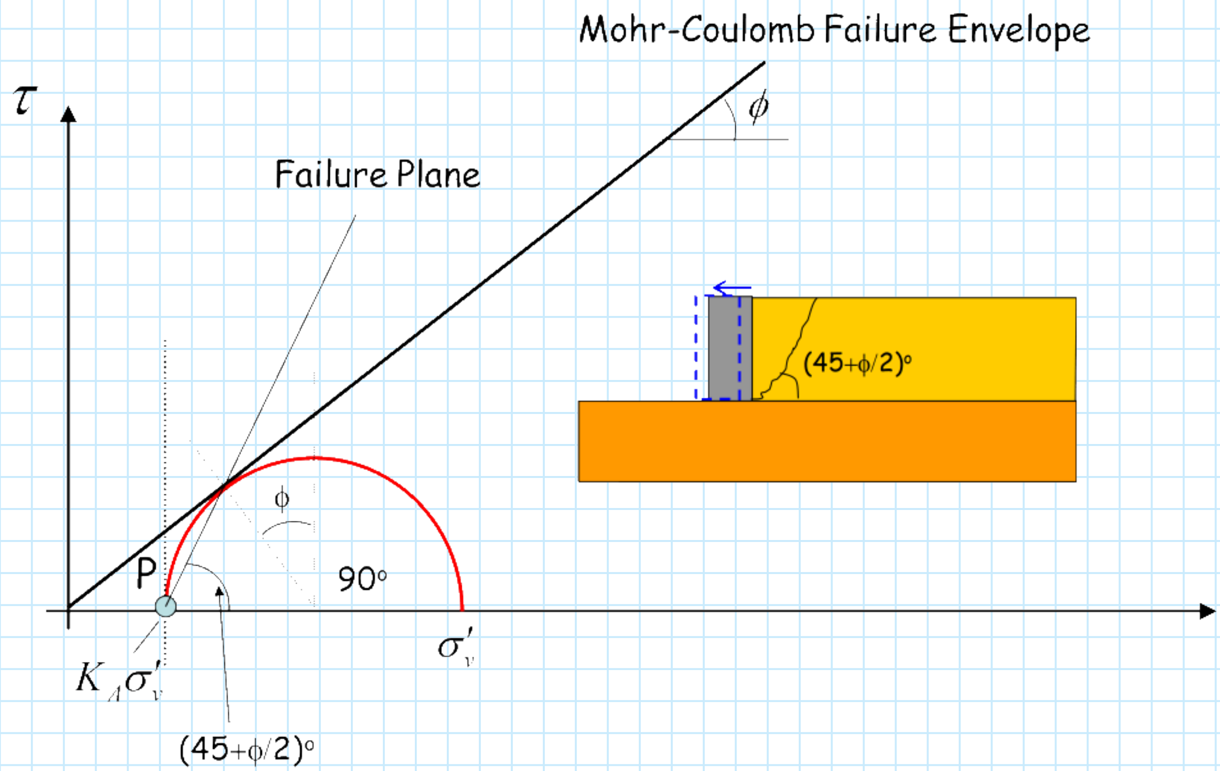
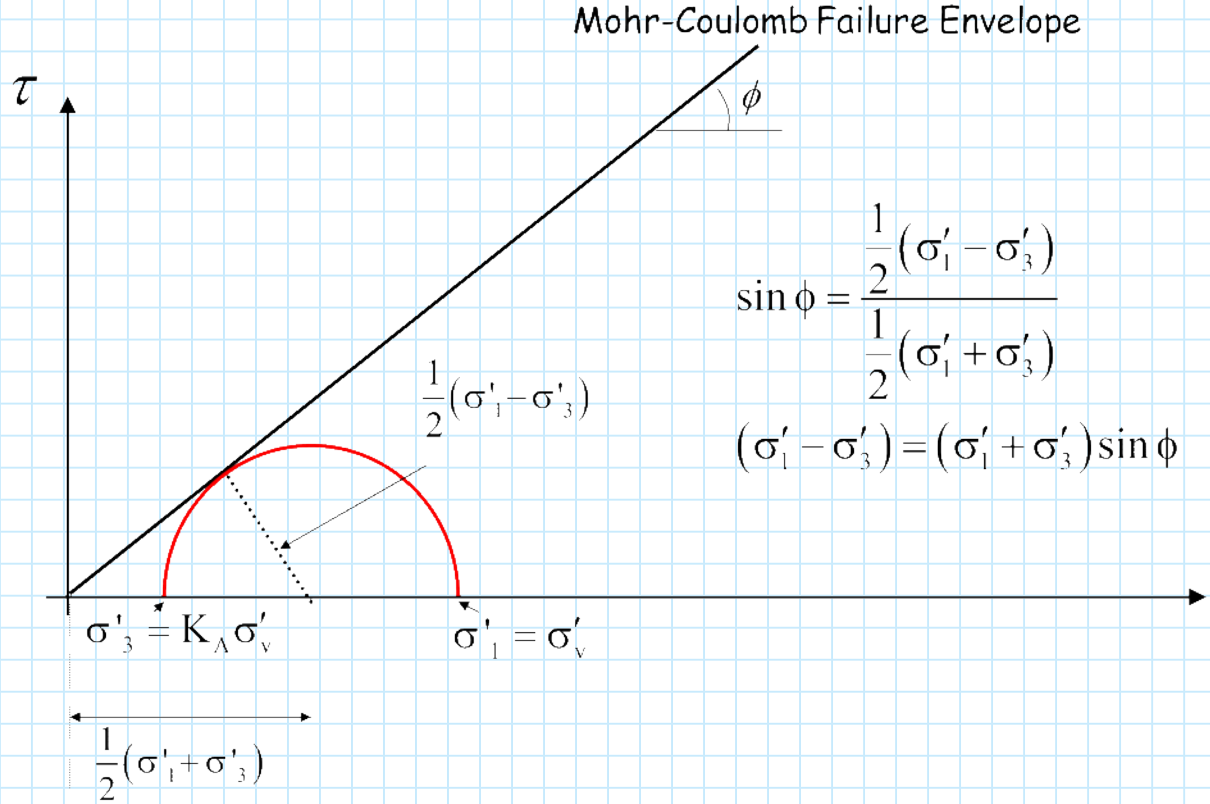




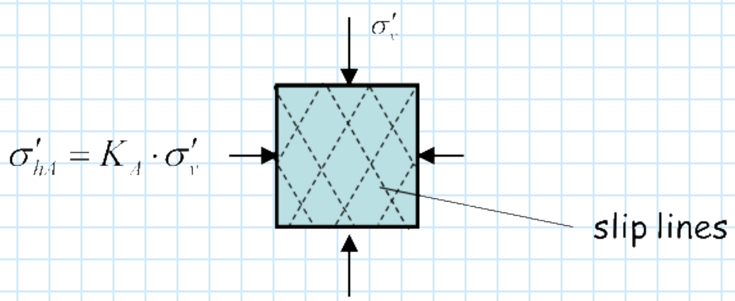
Active Case (cont.)

Wednesday, August 17, 2011 12:45 PM



$$\sin\phi = \frac{\frac{1}{2}(\sigma'_1-\sigma'_3)}{\frac{1}{2}(\sigma'_1+\sigma'_3)}$$

$$(\sigma'_1-\sigma'_3) = (\sigma'_1+\sigma'_3)\sin\phi$$



Mohr-Coulomb failure criterion $\sigma'_1-\sigma'_3 = (\sigma'_1+\sigma'_3)\cdot\sin\phi$

$$\sigma'_v-\sigma'_{hA} = (\sigma'_v+\sigma'_{hA})\cdot\sin\phi \qquad \Rightarrow \sigma'_{hA} = \sigma'_v\frac{1-\sin\phi}{1+\sin\phi}$$

$$\Rightarrow K_A\sigma'_v = \sigma'_v\frac{1-\sin\phi}{1+\sin\phi}$$

$$\Rightarrow K_A = \frac{1-\sin\phi}{1+\sin\phi} = \tan^2\left(45^o-\frac{\phi}{2}\right)$$

Passive Case

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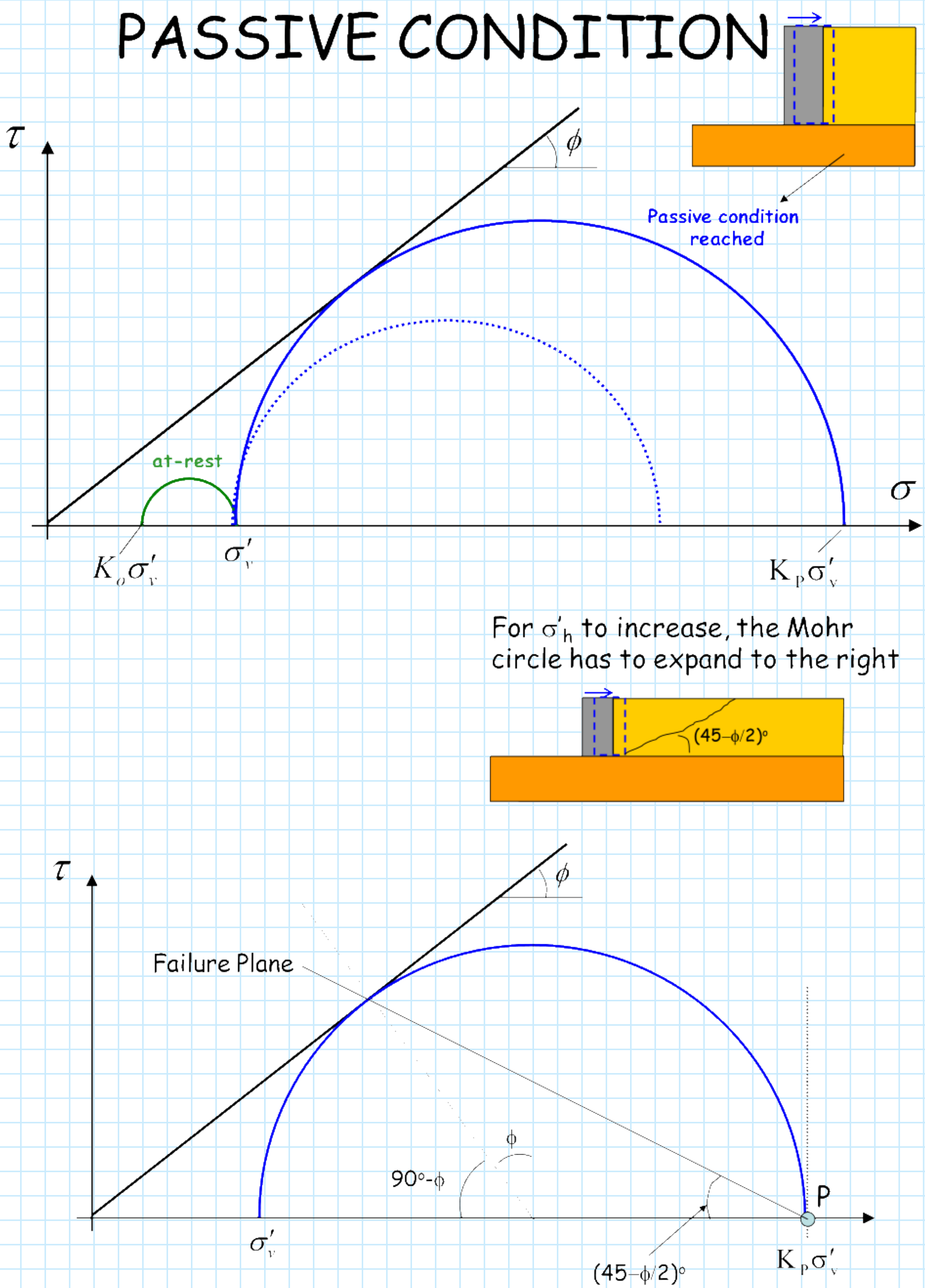

Vc b

Passive Case (cont.)

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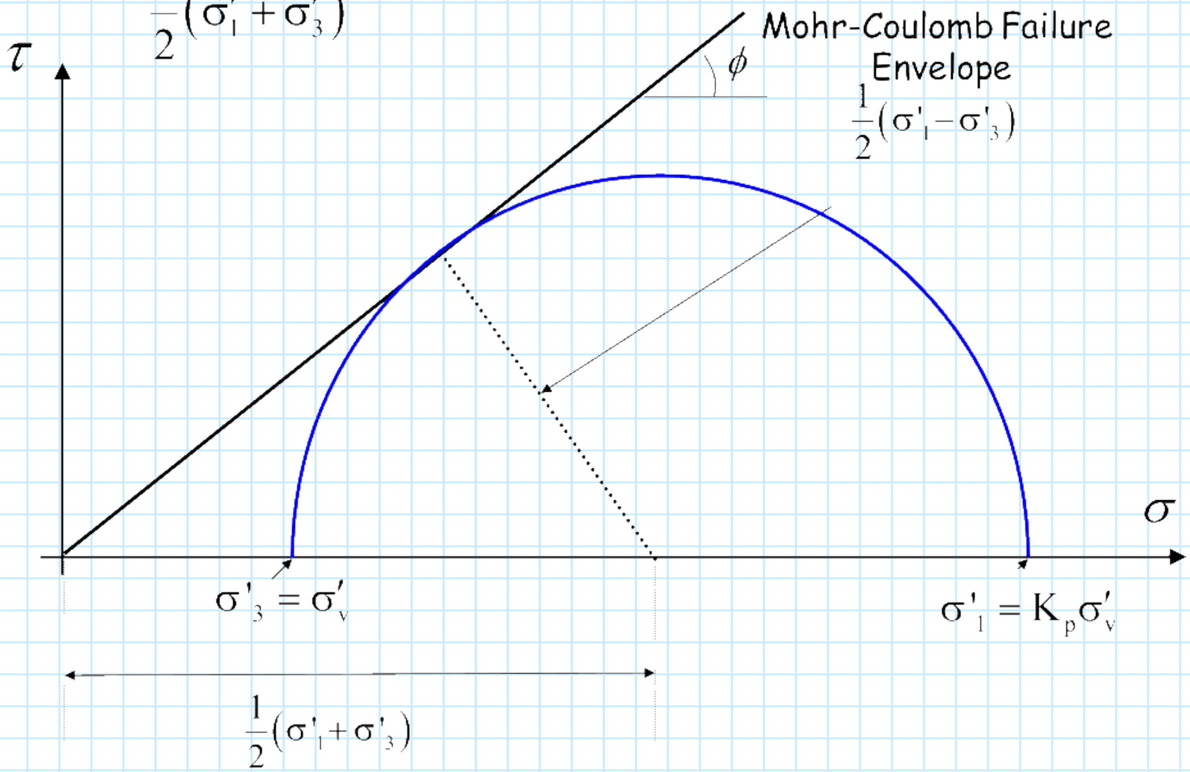
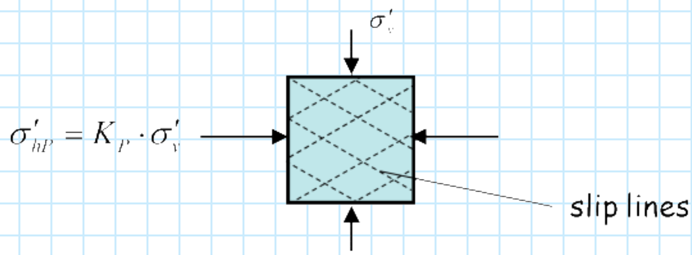
$$\sin\phi = \frac{\frac{1}{2}(\sigma_1' - \sigma_3')}{\frac{1}{2}(\sigma_1' + \sigma_3')} \Rightarrow (\sigma_1' - \sigma_3') = (\sigma_1' + \sigma_3')\sin\phi$$





Mohr-Coulomb failure criterion $\sigma_1' - \sigma_3' = (\sigma_1' + \sigma_3')\cdot\sin\phi$

$$\sigma'_{hP} - \sigma'_v = (\sigma'_{hP} + \sigma'_v)\cdot\sin\phi \quad \Rightarrow \sigma'_{hP} = \sigma'_v\frac{1+\sin\phi}{1-\sin\phi}$$

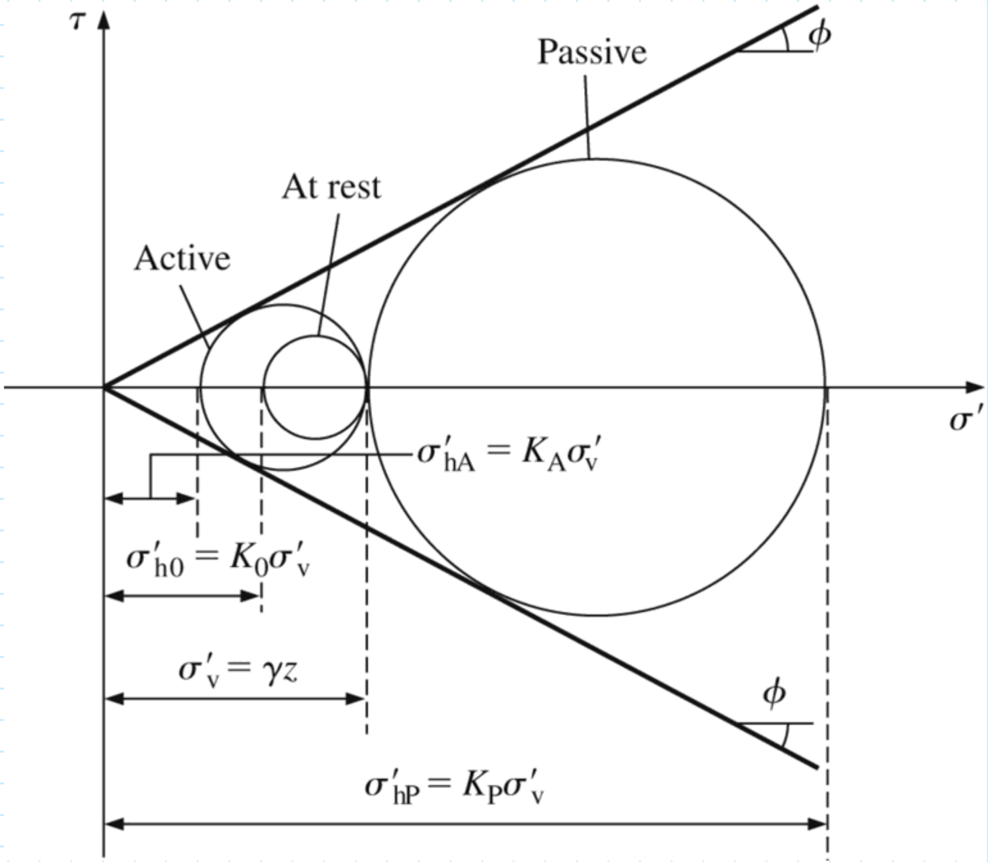
$$\Rightarrow K_P\sigma'_v = \sigma'_v\frac{1+\sin\phi}{1-\sin\phi}$$

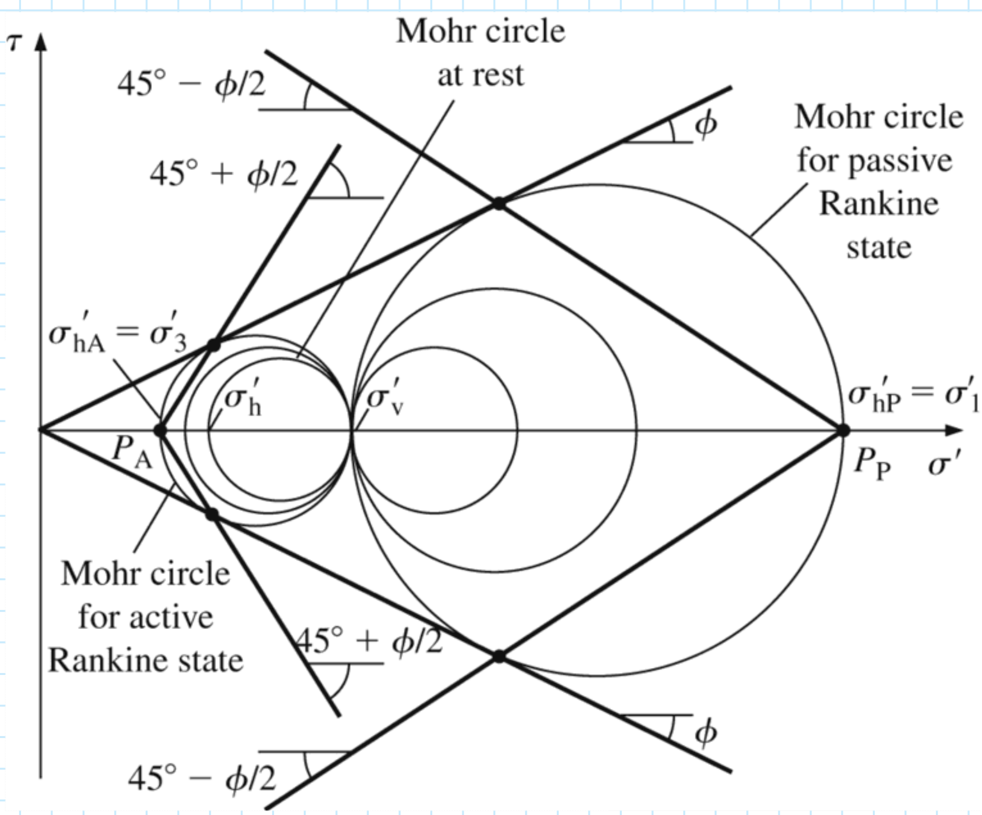
N= Flow Number

Coefficient of **passive earth pressure** $\Rightarrow K_P = \frac{1+\sin\phi}{1-\sin\phi} = \tan^2\left(45^o + \frac{\phi}{2}\right) = N$

Active and Passive State - Mohr's Circles

Wednesday, August 17, 2011 12:45 PM





Direct Shear Test

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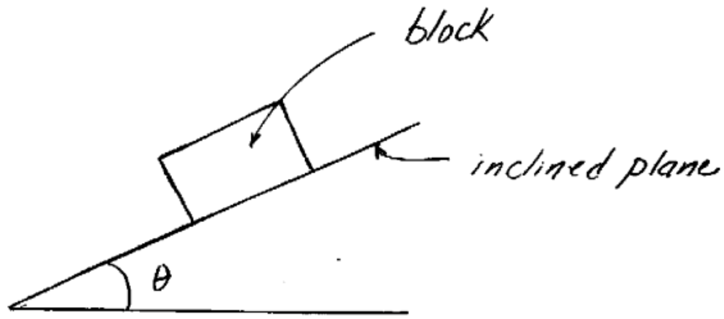
Video of direct shear test

https://www.youtube.com/watch?v=L1fWPypBP0g

- Advantages
  - inexpensive
  - fast
  - simple
  - good for determining sliding resistance
- Disadvantages
  - only suitable for drained conditions (i.e., granular soils or cohesive soils sheared slowly)
  - predetermined failure plane (not necessarily weakest)
  - stress concentrations at sample boundaries
  - uncontrolled rotation of principal stresses

- Sliding Block Analogy



- When component of gravity acting parallel to inclined plane exceeds the frictional force resisting sliding, then the block will slide.

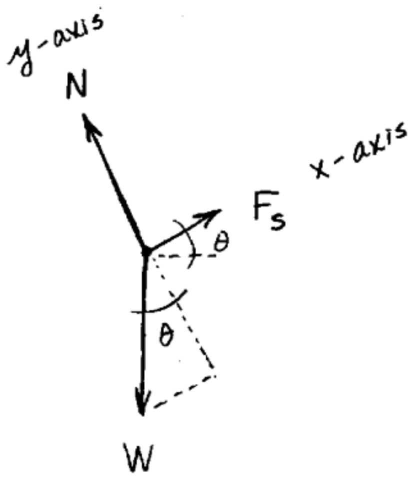
  - Force resisting sliding, F_s

    $F_s = \mu N$

    where μ = coefficient of sliding

    N = normal force (acts perpendicular to the inclined plane)

- Free Body Diagram



- Equilibrium
  - y-direction

$$N - W\cos\theta = 0$$
$$N = W\cos\theta$$

  - x-direction

$$F_S - W\sin\theta = 0$$
$$F_S = W\sin\theta$$
$$\mu N = W\sin\theta$$
$$\mu = \frac{W}{N}\sin\theta$$

$F_S = \mu N$ (previous page)

but $\dfrac{W}{N} = \dfrac{\text{hypot}}{\text{side adjacent}} = \dfrac{1}{\cos\theta}$

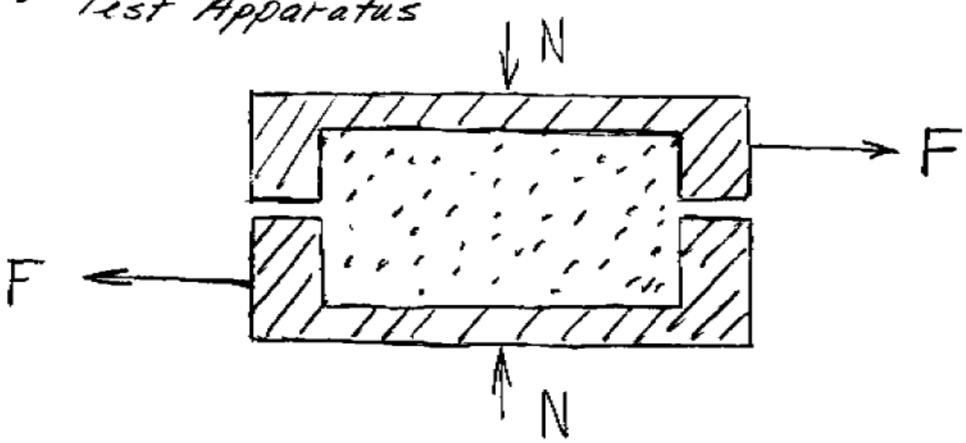
$$\mu = \frac{\sin\theta}{\cos\theta}$$

$$\boxed{\mu = \tan\theta}$$

- Direct Shear Test
  - Purpose
    - To determine μ for soil sliding upon soil
    - μ is also called ϕ, which is the internal angle of friction

$$\boxed{\mu = \tan\phi}$$

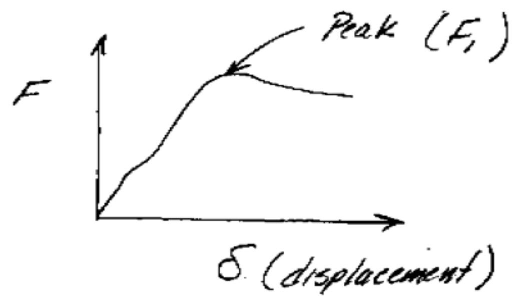
  - Test Apparatus



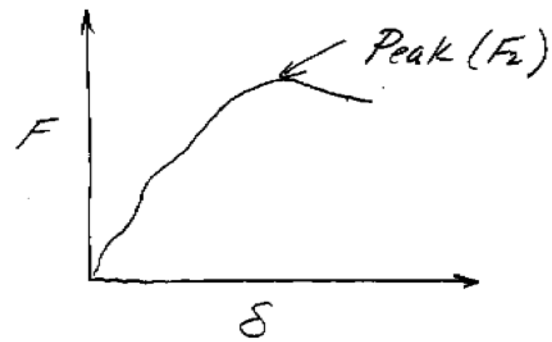
Direct Shear (cont.)

Wednesday, August 17, 2011 12:45 PM

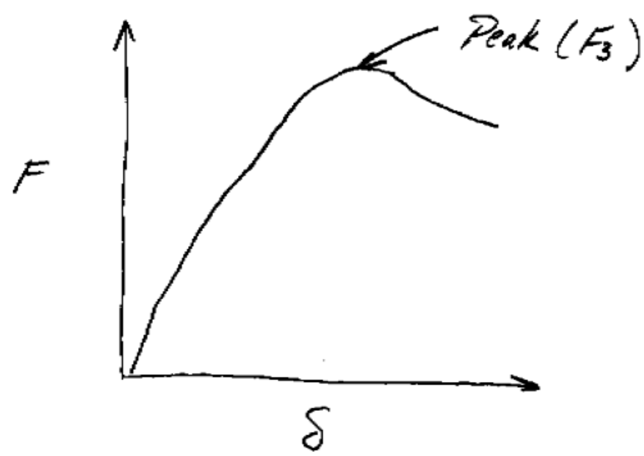
- Direct Shear Test (cont.)
  - Test Data
    - Test 1



- Test 2 ($N_2 > N_1$) (increase normal stress)



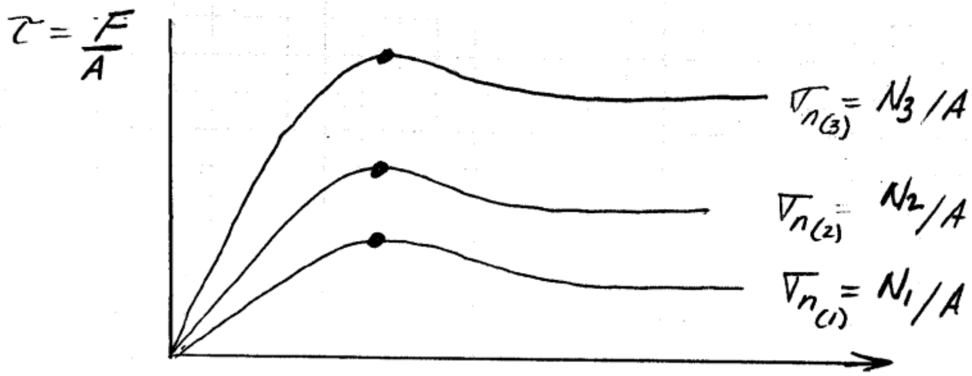
- Test 3 ($N_3 > N_2$)

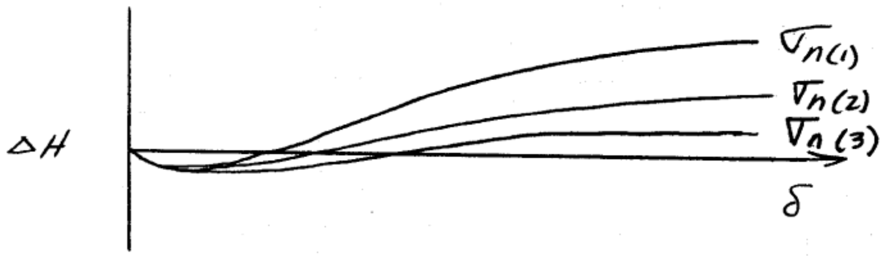


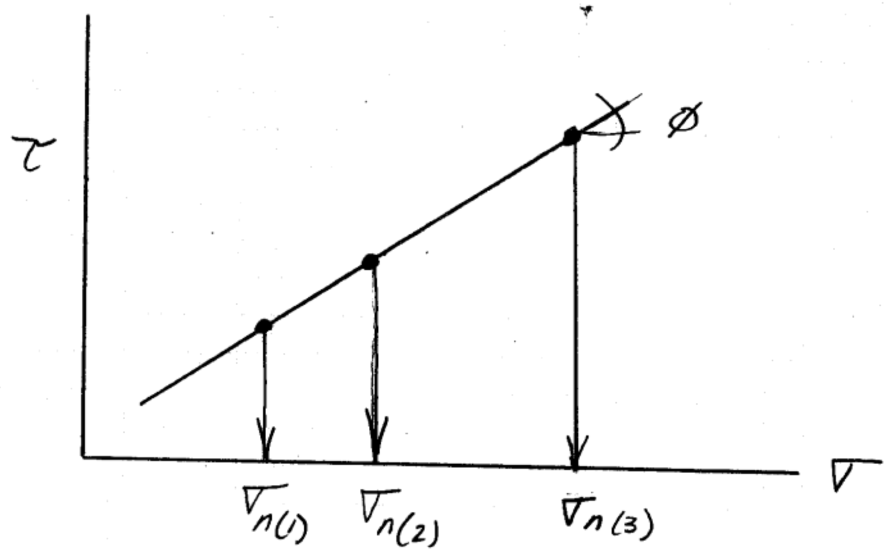
Direct Shear (cont.)

Wednesday, August 17, 2011 12:45 PM

- Direct Shear Test Results







Mohr Diagram

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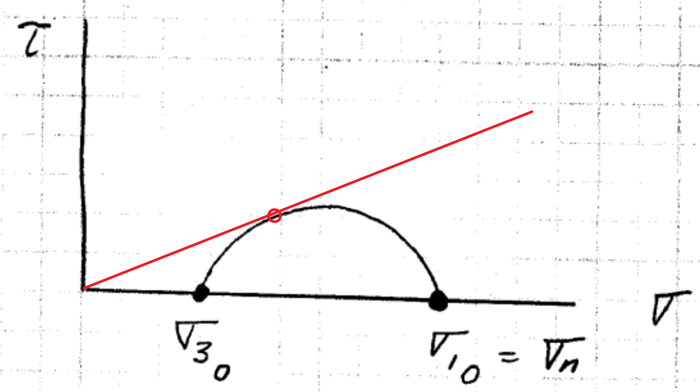
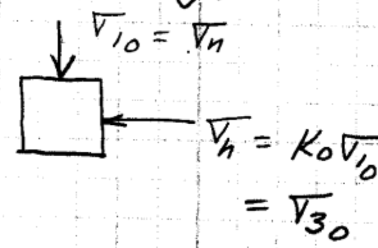
- Determination of Principal Axis for Direct Shear Test

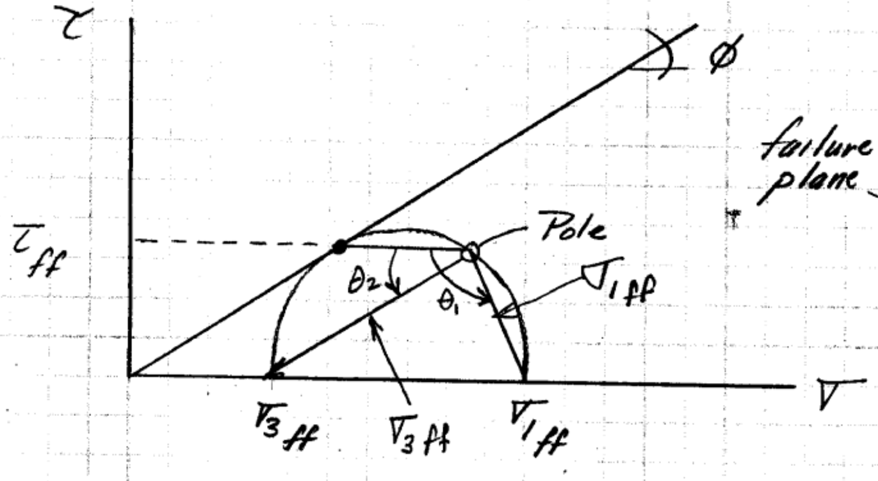


Initial conditions (before shearing)

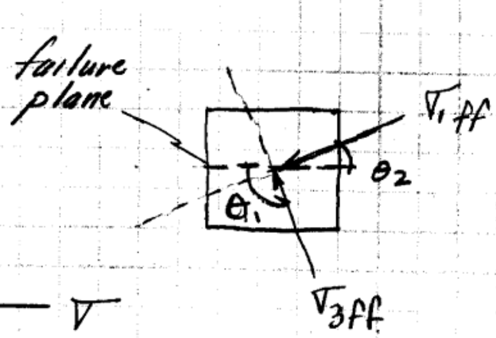


? What are σ_1 and σ_3 at failure?

? What is the orientation of the principal planes?



At failure



see p. 461 Holtz & Kovacs

see also p. 463 Holtz & Kovacs

Note: Slight rotation of σ_1 and σ_3 during shear

Triaxial Shear Test

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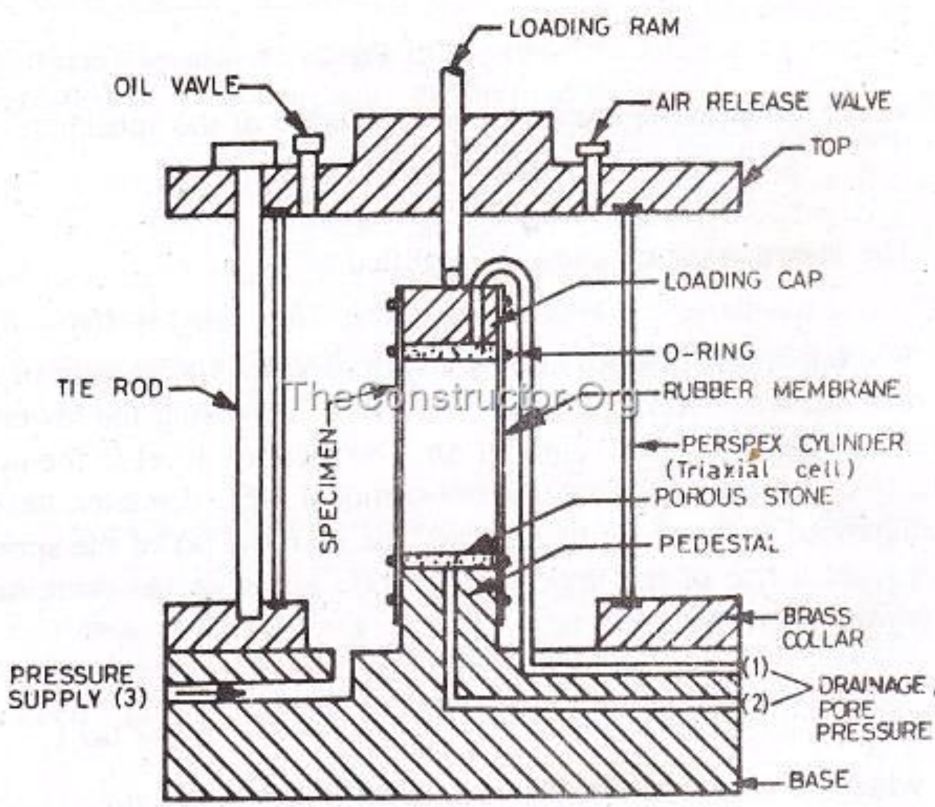
Triaxial Test Setup

https://www.youtube.com/watch?v=eiAzJAffbLo



Triaxial Test and Effects of Pore Water Pressure

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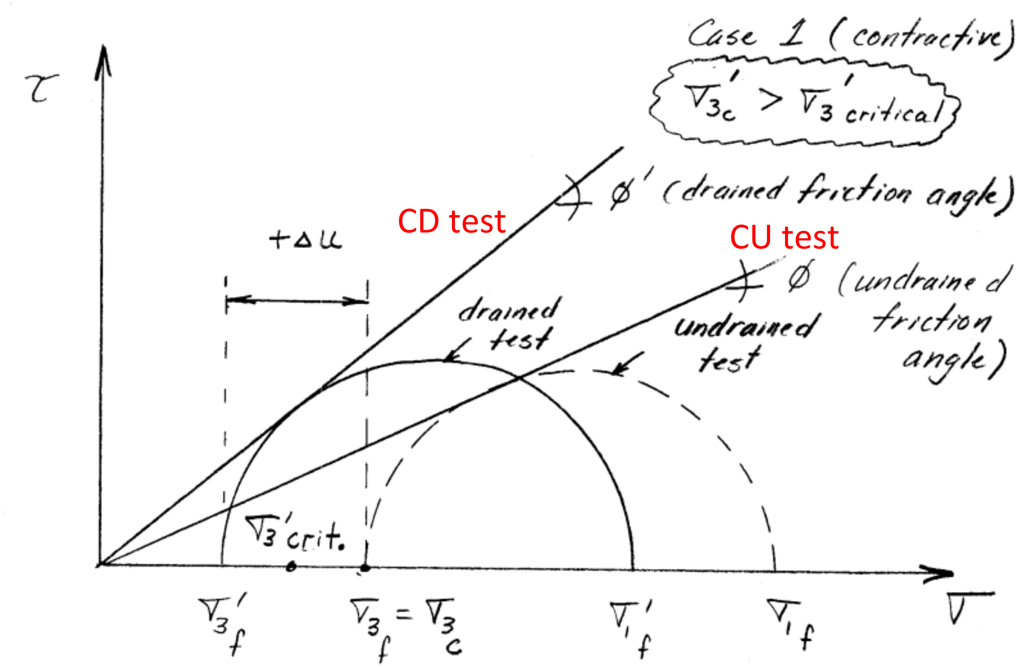
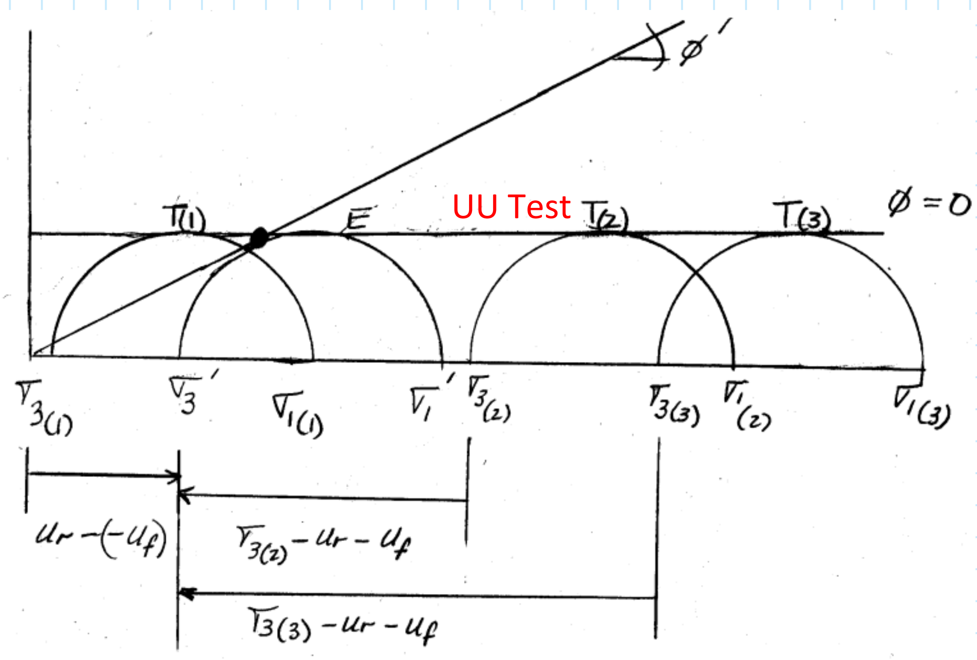
Table 4-1 Types of triaxial tests

Drainage during	TX tests CD	CU	UU
Consolidation	Drained	Drained	Undrained
Shearing	Drained	Undrained	Undrained





Axial Compression and Lateral Extension Triaxial Tests

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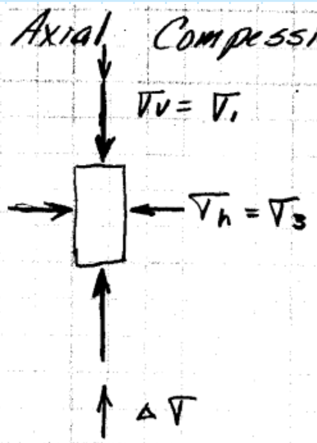
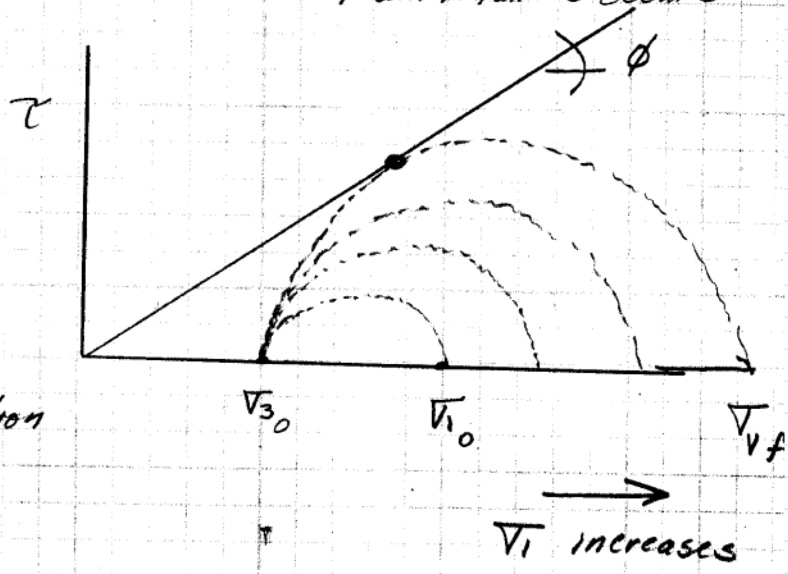
(1) Axial Compression

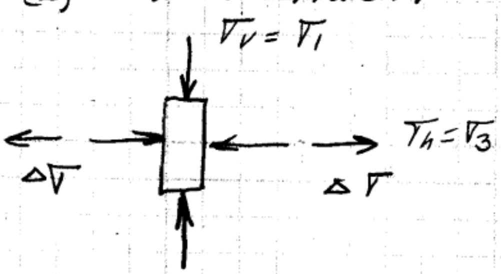


- Keep σ_3 constant
- Increase σ_1 until failure occurs

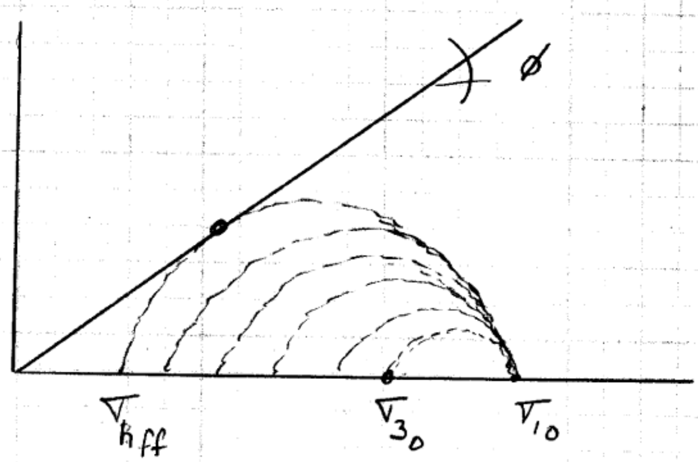


- Models foundation loading

(2) Lateral Extension

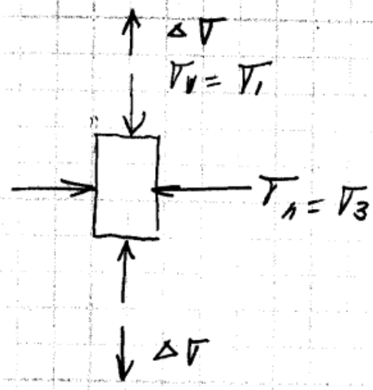


- Keep σ_1 constant
- Decrease σ_3 until failure occurs

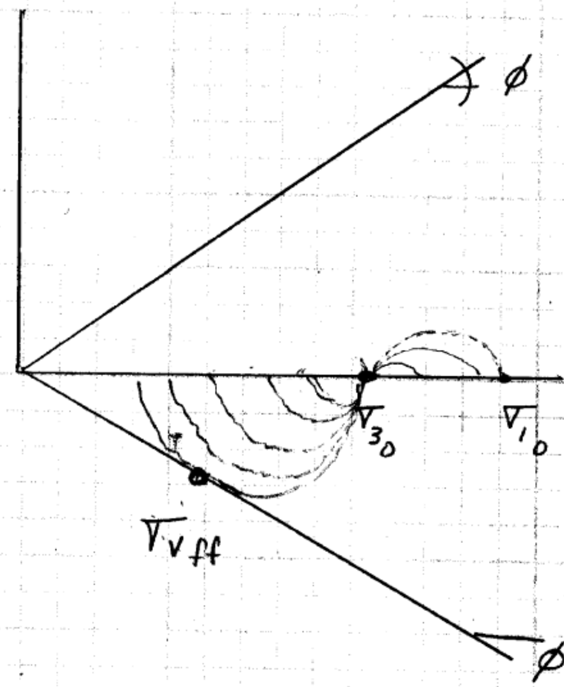


- models active earth pressure behind retaining wall

(3) Axial Extension

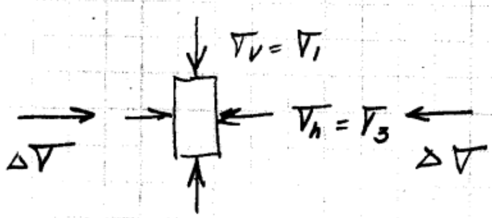


- Keep σ_3 constant
- Decrease σ_1 until failure occurs

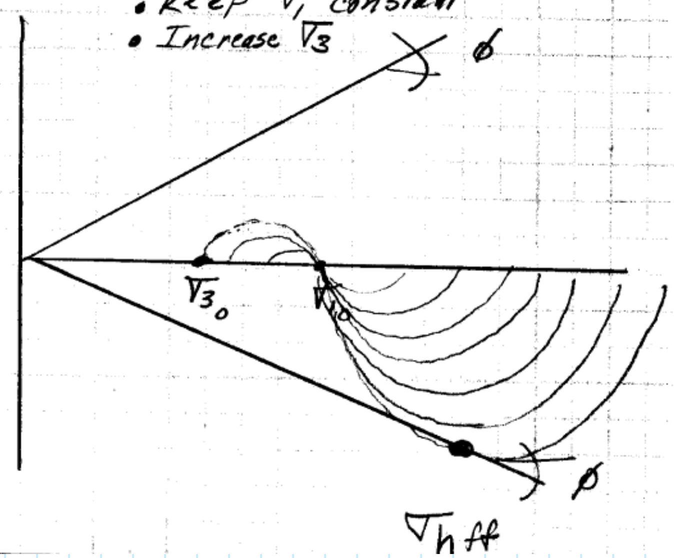


- models unloading due to an excavation

(4) Lateral Compression



- Keep σ_1 constant
- Increase σ_3



- models passive earth pressure

Triaxial Test Results shown with s-t plots

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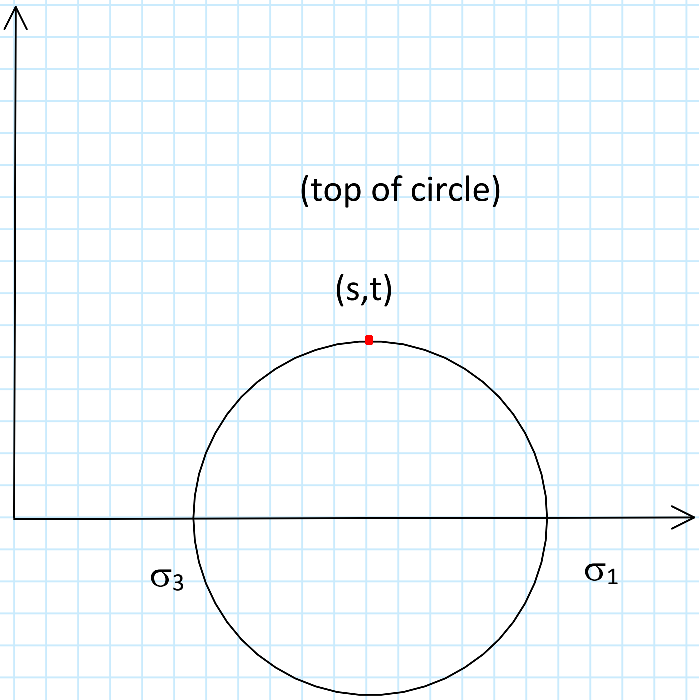
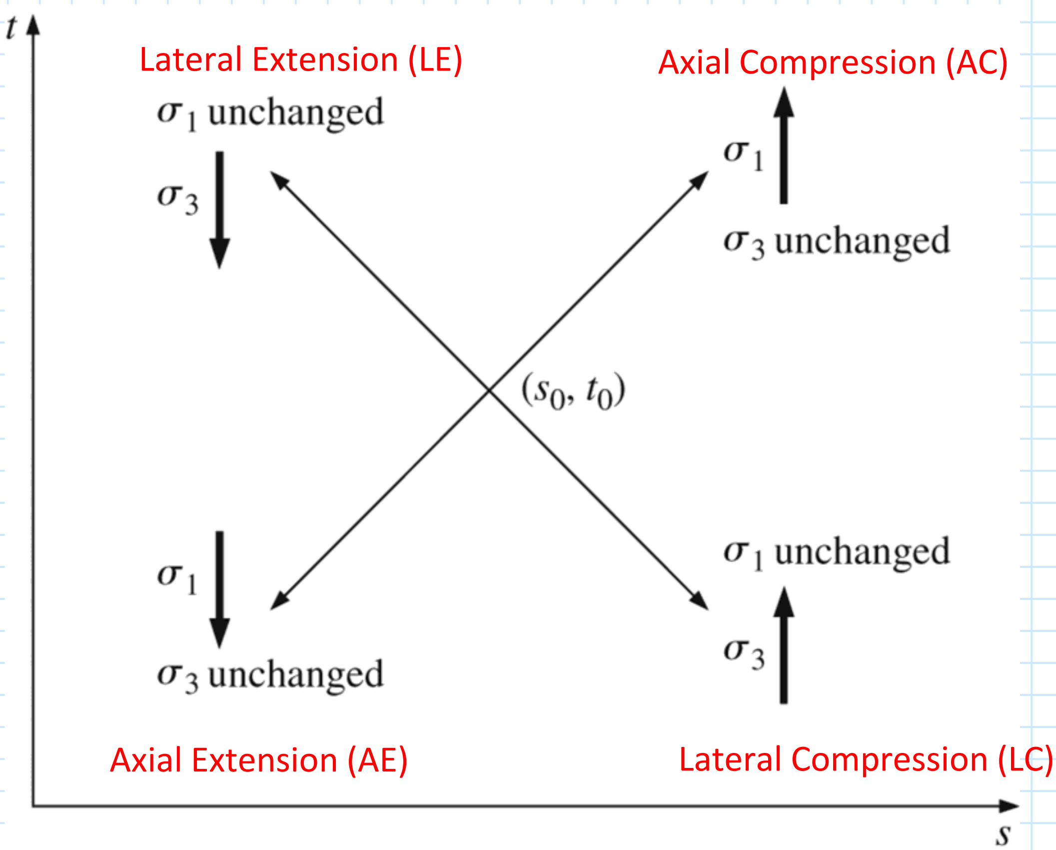
Stress paths in s-t space

$s = 1/2\ (\sigma_1 + \sigma_3)$ = ctr. circle
$t = 1/2\ (\sigma_1 - \sigma_3)$ = top circle

A stress path is the locus of points formed by the plot of s,t as the triaxial test progresses. It is useful in determining the potential path to failure and the state of stress at failure





s-t plots (cont.) in p-q space

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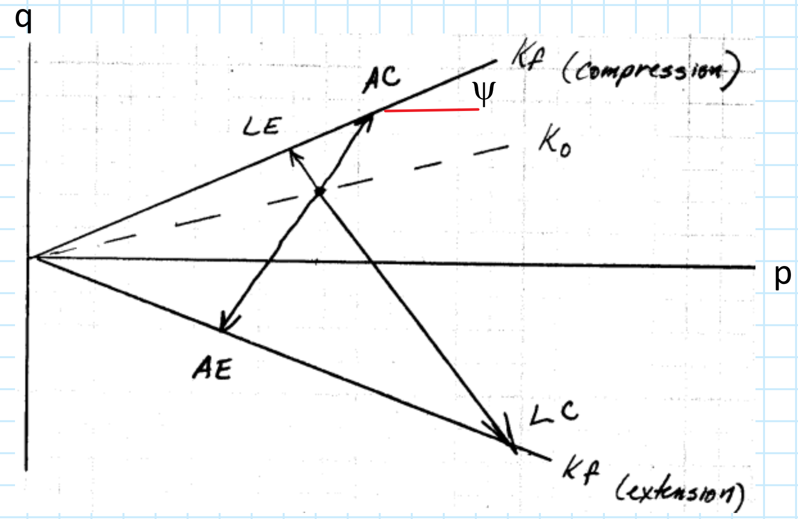
$p = 1/3\ (\sigma_1 + \sigma_2 + \sigma_3)$

$q = 1/2^{0.5} * [\ (\sigma_1 - \sigma_3)^2 + (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2]^{0.5}$



Potential stress paths for Ko Consolidated Specimen

Ko = anisotrophically consolidated specimens

Generally $\sigma_v = \sigma_h$ for Ko conditions

K_f line defines the ratio of the failure envelope.

$\sin \phi = \tan \psi$ where ϕ is the slope of the Mohr-Coulomb envelope and ψ is the slope of the K_f line

(Note that ψ is not the dilation angle. This is a bit confusing because we previously used ψ as the dilation angle.)

Blank

Thursday, September 30, 2021 12:45 PM

Ch 5 Strength and Stiffness of Granular Soils

Tuesday, September 19, 2023 12:45 PM

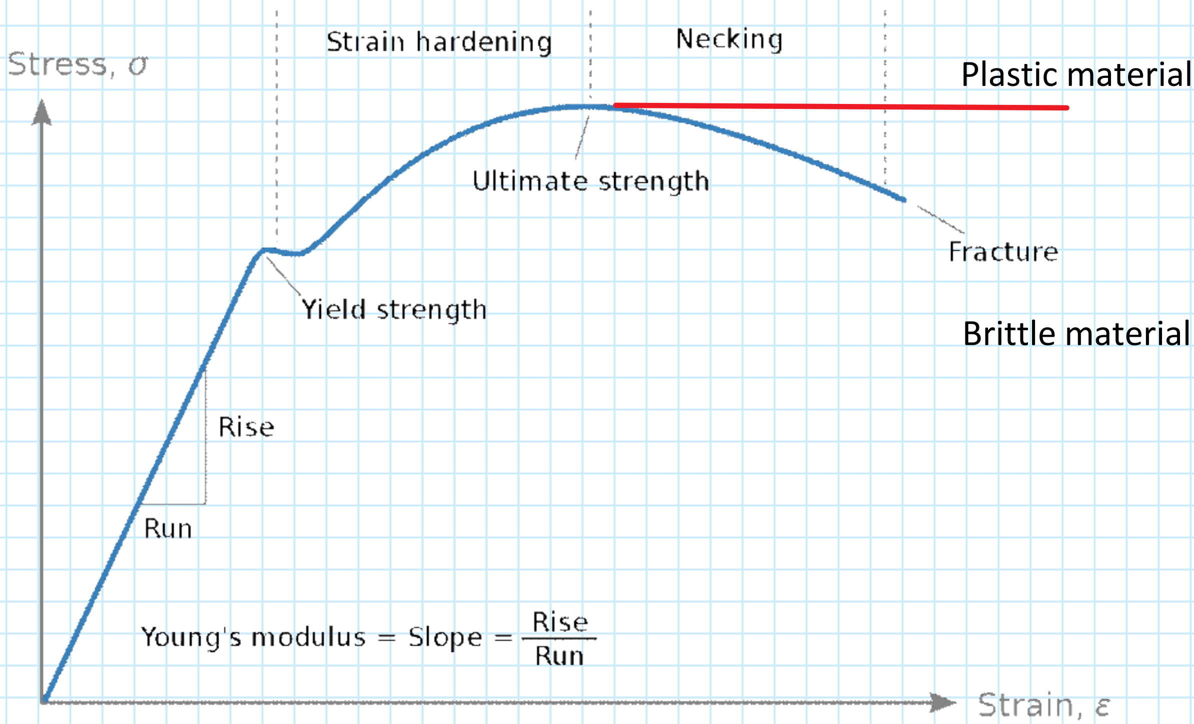
Reading Assignment

- Ch. 5 Lecture Notes
- Sections 5.1 - 5.7 (Salgado)

Other Materials

Homework Assignment 5

- Problems 5-9 (20 points), 5-12 (20 points), 5-23 (30 points)



Note that this plot is a generalized behavior plot. In the case of rock, fracture is possible. For soils, fracture does not occur, the material continues to deform along the internal failure plane. Also, for dense granular materials and overconsolidated clayey soils, there is not a necking phase. Instead, the soil continues to deform like a plastic material (i.e., at a constant shear stress).

Stress-Strain Behavior, Volume Change and Shear of Granular Soils

Tuesday, September 19, 2023 12:45 PM

What factors contribute to the strength of granular materials?

- Soil Fabric
- Density (void ratio)
- Confinement
- Cementation
- Aging

The effects of density and confinement can be accounted for using critical state theory for sands

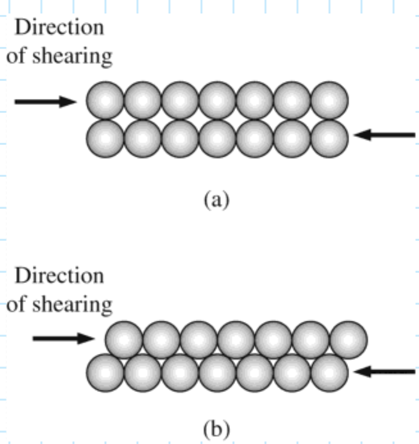
<u>SOIL FABRIC</u>

- Sources of shear resistance (at the particle (i.e., fabric level)
  - Friction between soil particles (influenced by angularity and roughness of particles)
  - Particle rearrangement and grain-size distribution (see diagrams below)
  - Interlocking between particles (influenced by angularity and particle arrangement

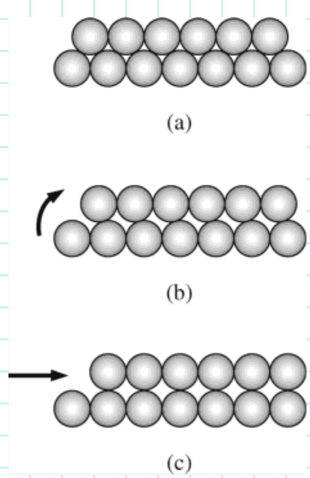
Particle arrangement

Sources of shearing strength

- Shearing strength of contractive sands is due primarily to friction between soil particles and particle rearrangement
- Shearing strength of dilative sands is due to interlocking between particles that has to be overcome by dilation (for dilation to occur, energy must be supplied to the soil for it to overcome the confining stress)



Contractive behavior



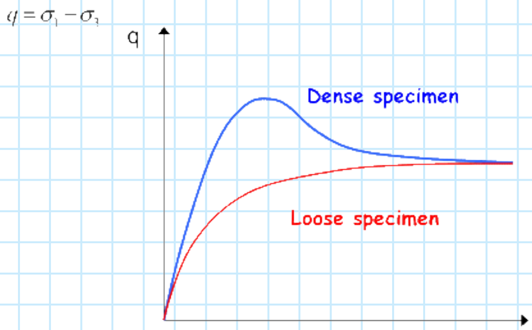
Dilative behavior

Stress-Strain Behavior, Volume Change and Shear of Granular Soils (cont.)

Tuesday, September 19, 2023 12:45 PM

Sand - Effect of D_R

- Drained ⇨ no excess pore pressure
- CD triaxial tests on dense and loose sand specimens
- Same initial confining stress but different initial D_R

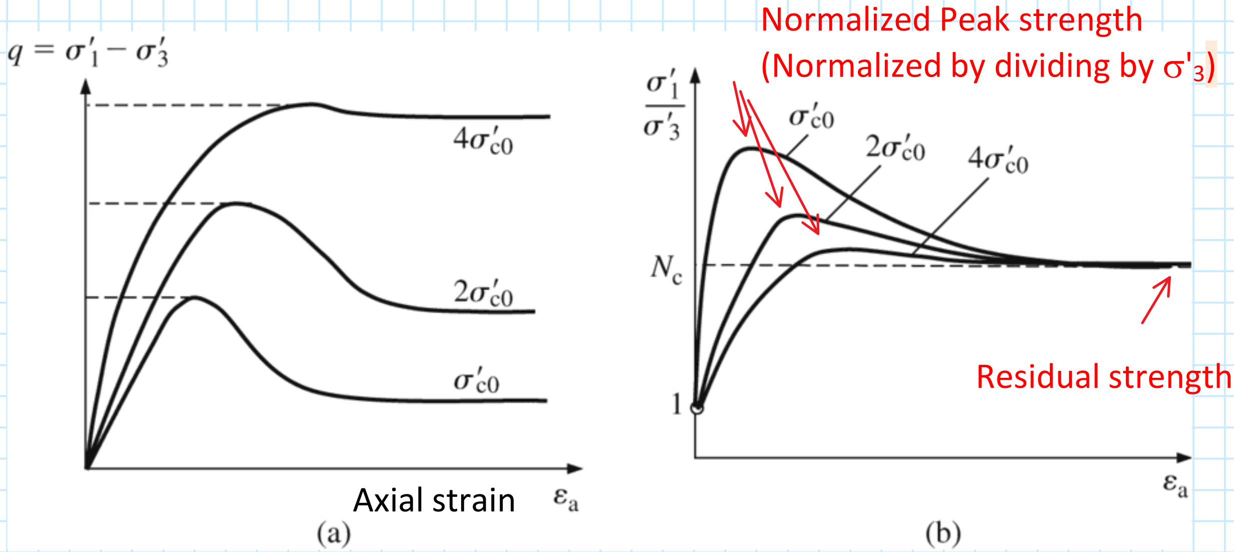


(D_R = relative density)

Relationship between relative density and void ratio
$(e_{max} - e)/(e_{max} - e_{min})$

e_{max}, e_{min} from
ASTM. 2006a. Standard test methods for maximum index density and unit weight of soils using a vibratory table. ASTM standard D4253.

Effects of confinement

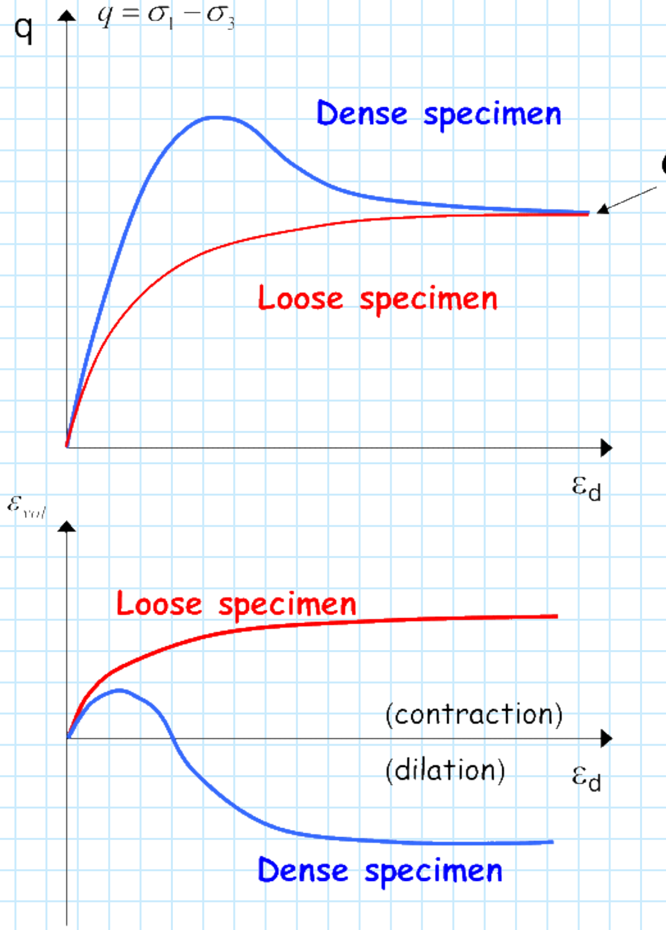


Note that the sand's **post-yield behavior changes as a function of the confining stress**. The sample with the lowest σ'_1/σ'_3 ratio has the lowest reduction in residual strength when compared with the peak strength. Samples with higher ratios have less reduction. At large strain, the residual strength is reached and the soil has reached the ***critical state***.

Stress-Strain Behavior, Volume Change and Shear of Granular Soils (cont.)

Tuesday, September 19, 2023 12:45 PM

Drained ⇨ no excess pore pressure



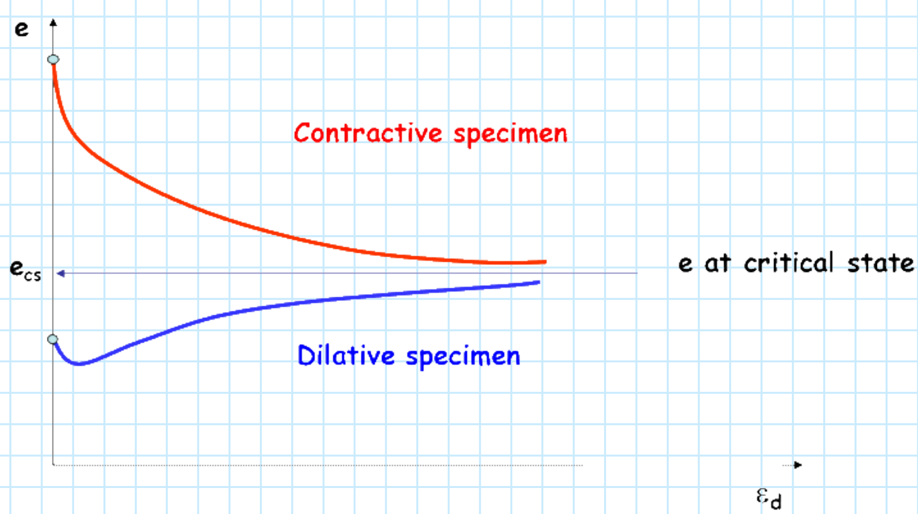
CRITICAL STATE

The critical state is a condition in which the soil is sheared at constant shear stress, constant effective confining stress, and constant volume (or void ratio). The critical state can be thought of as a state in which the soil is in equilibrium with the imposed stresses, no longer needing to either dilate or contract in order to shear.

Contractive

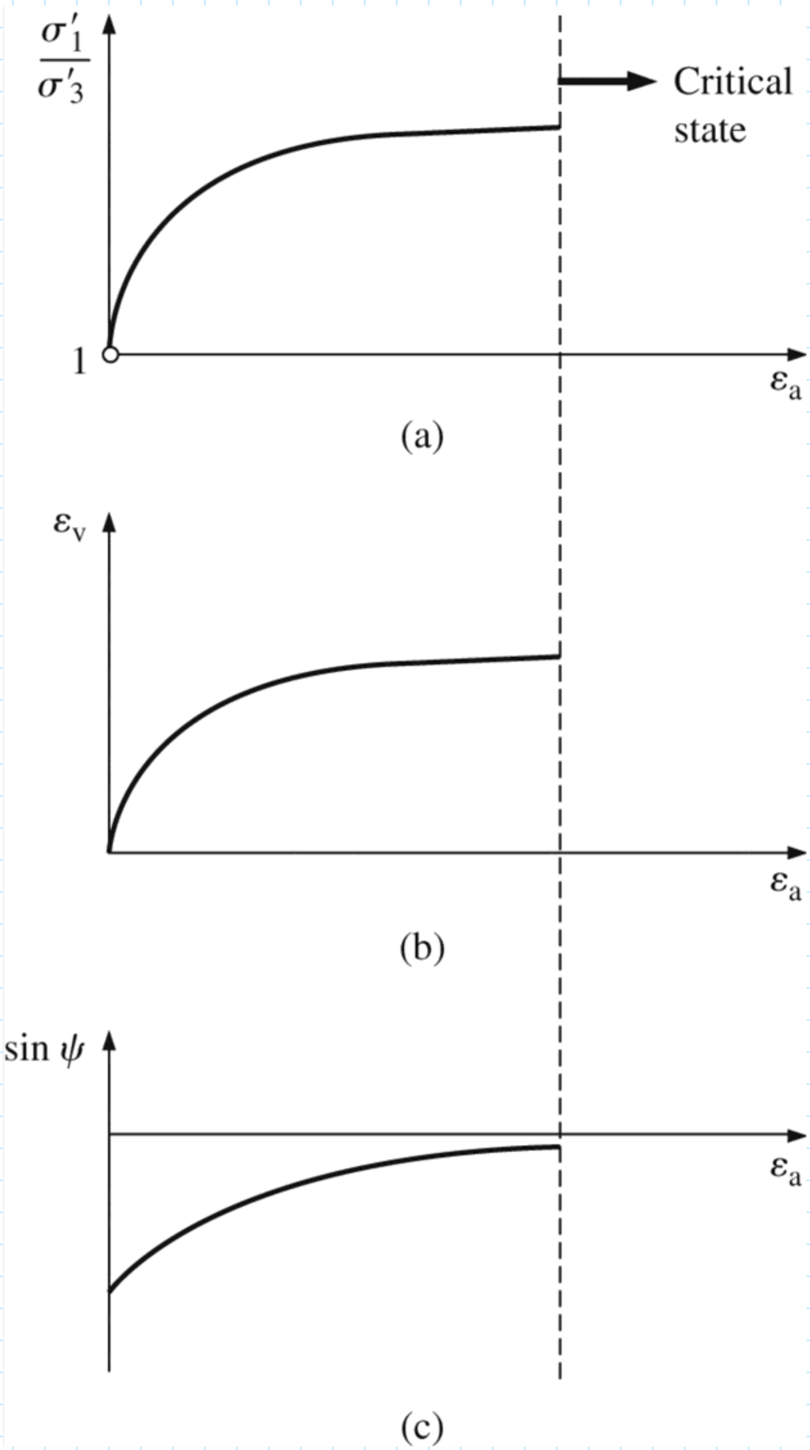
Dilative

Critical-state void ratio



Stress-Strain Behavior of Sands at the Critical State

Wednesday, August 17, 2011 12:45 PM



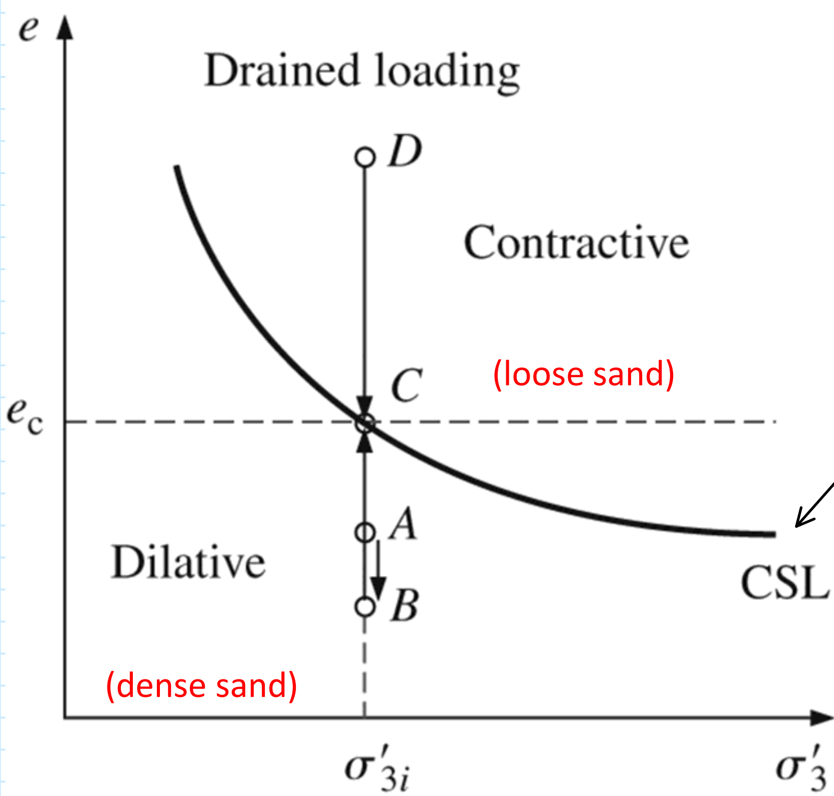
Note that at large axial strain the ratio of σ'_1/σ'_3 remains unchanged. Because σ'_3 remains constant during an axial compression test, this means that σ'_1 is not changing with axial strain. When this point is reached, this is called the critical state.

Note that the volumetric strain (y axis) remains unchanged with axial strain when the critical state is reached. This means that the sample is neither contracting or dilating, but straining at a constant void ratio.

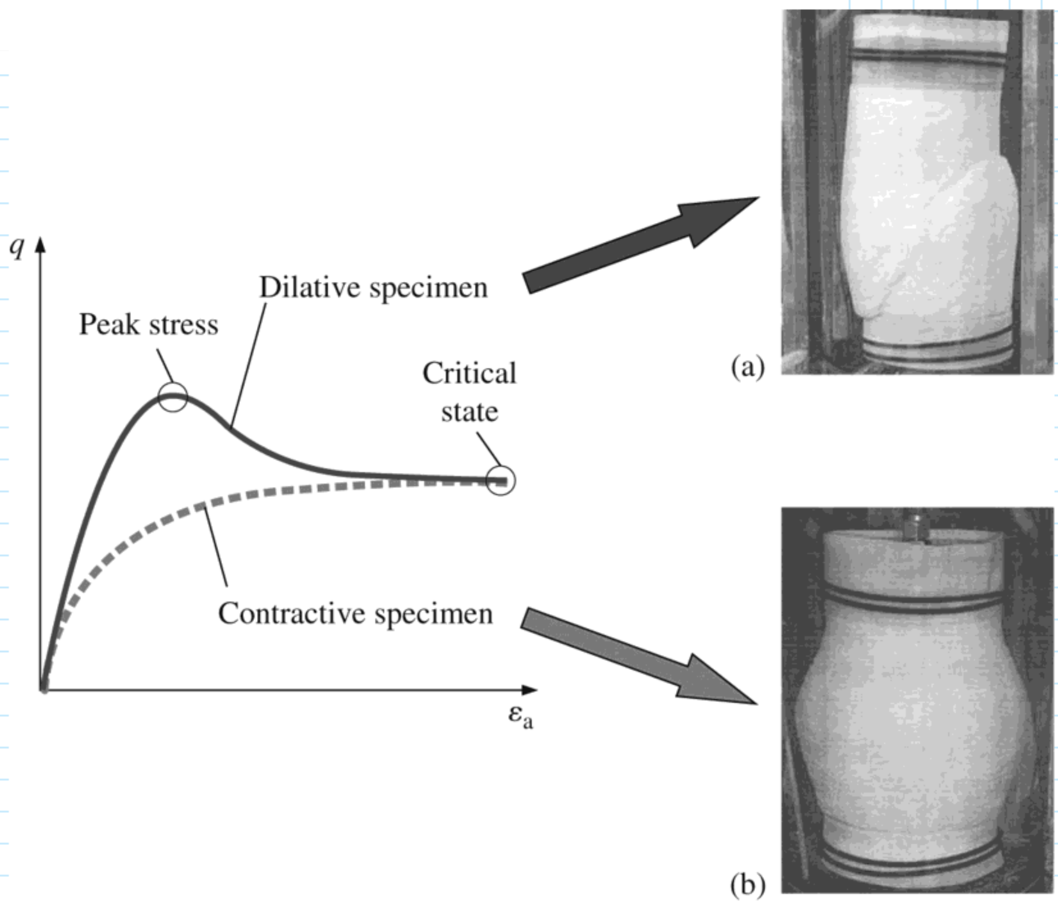
Note that the dilatancy angle is reducing with axial strain and is near zero at the critical state.

Stress-Strain Behavior of Sands (cont.)

Wednesday, September 29, 2021 12:45 PM



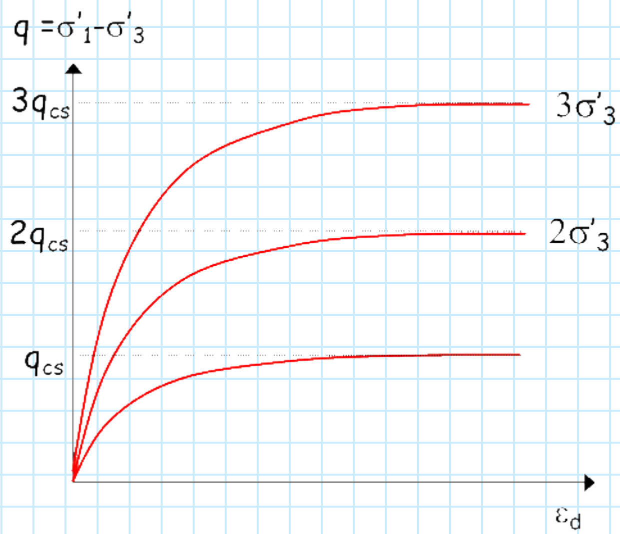
CSL = Critical State Line. This line shows the relationship between the critical void ratio, e_c and effective confining stress, σ'_3 that produces the critical state for a given soil

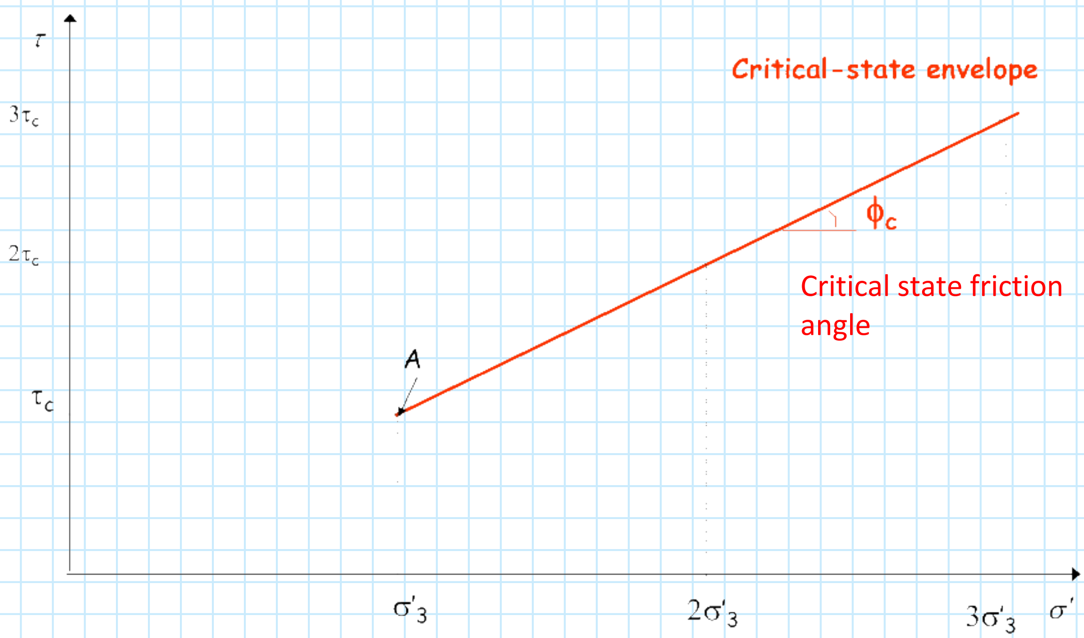


Critical State Friction Angle - Contractive Sand in 2D

Wednesday, August 17, 2011 12:45 PM

- Drained ⇨ no excess pore pressure
- CD triaxial tests on contractive sand specimens
- Same initial D_R but different initial confining stress





How does the **critical state phi angle** differ from the **peak friction angle** we have previously developed?

Critical State Friction Angle - Contractive Sand in 3D

Wednesday, September 29, 2021 12:45 PM

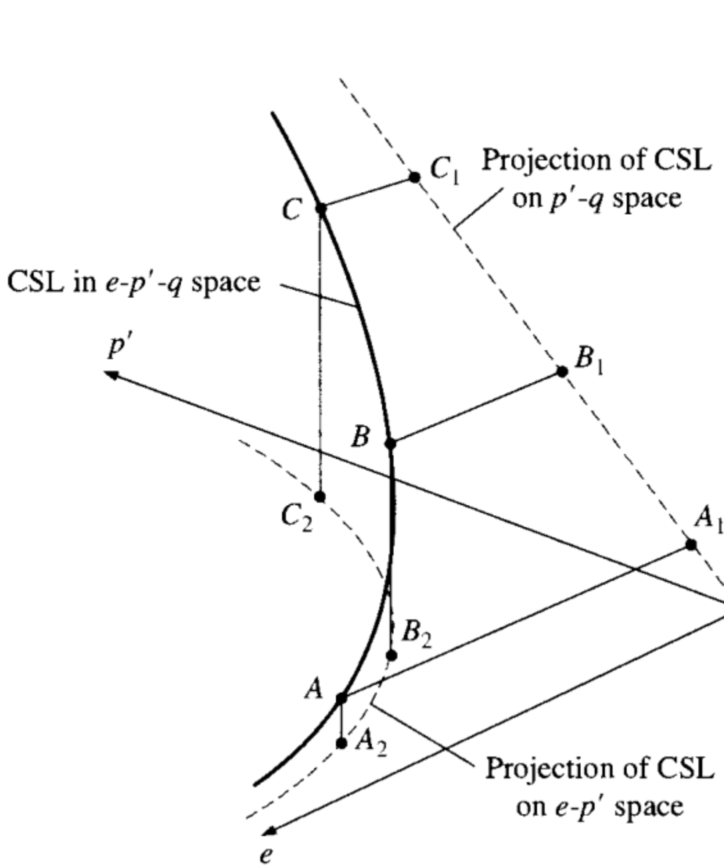


Figure 5-10
Critical-state line in e-p'-q space.

p = 1/3(σ1+σ2+σ3)

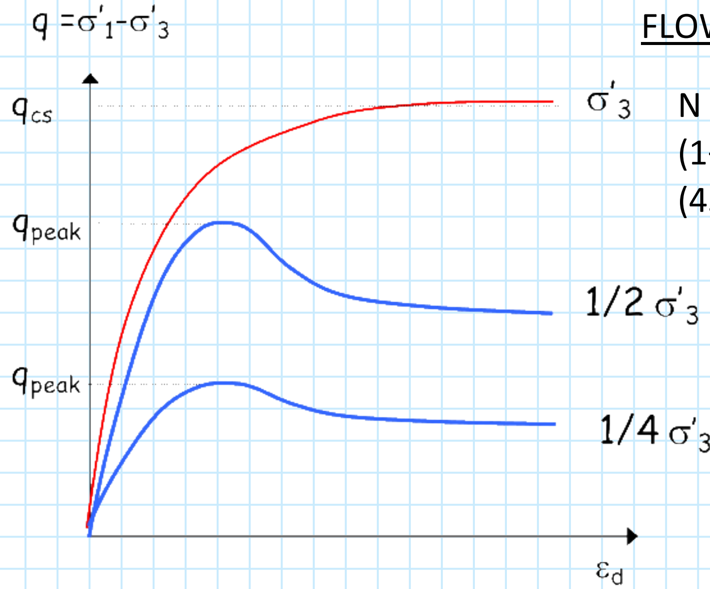
q = 1/2[(σ1–σ3)^2+σ1–σ2)^2+σ2–σ3)^2]

Figure 5-10 shows the critical-state line in e-p'-q space. This three-dimensional view of the CSL allows us to take a global view of the relationships between density, confining stress and shear stress at critical state. It also allows us to track what happens to void ratio and to the stresses as we shear a sample all the way to the critical state. We will return to this later in this chapter.

Critical State Friction Angle - Dilative Sand

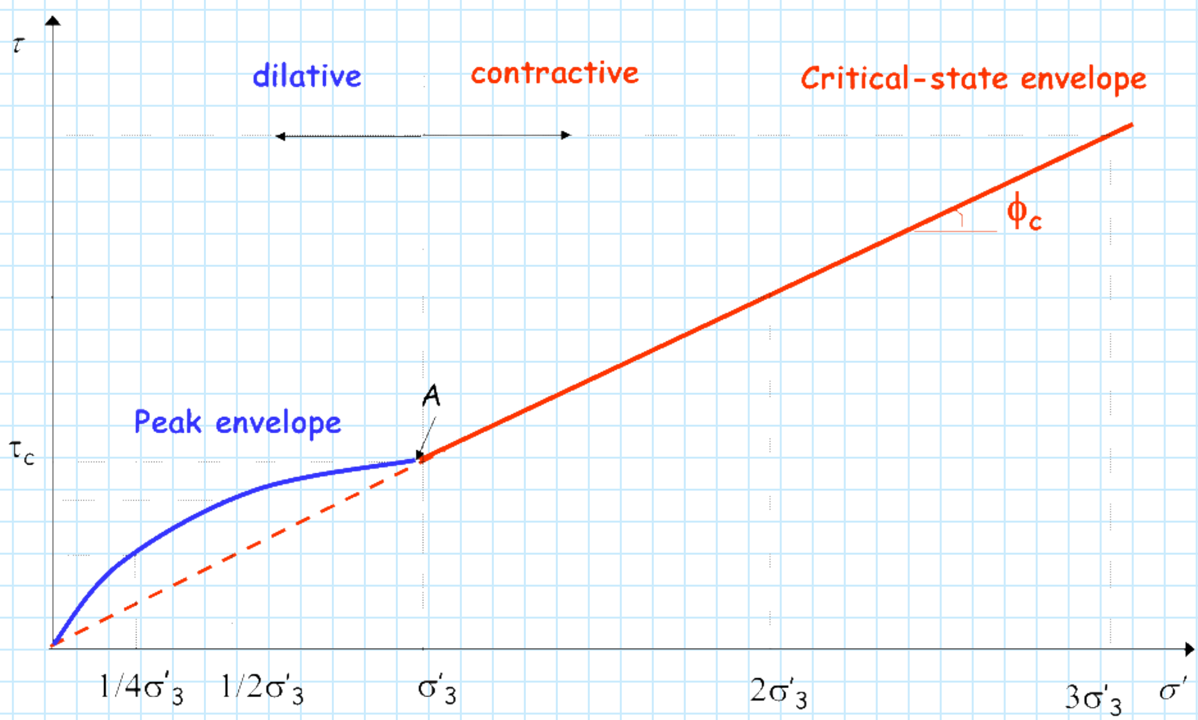
Wednesday, August 17, 2011 12:45 PM

- **Drained ⇨ no excess pore pressure**
- **CD triaxial tests on dilative sand specimens**
- **Same initial D_R but different initial confining stress**



<u>FLOW Number</u>

$N = \sigma'_1 / \sigma'_3 = (1+ \sin \phi) / (1 - \sin \phi) = \tan^2 (45 + \phi /2)$



Critical State Friction Angle - Dilative Sand (cont.)

Wednesday, August 17, 2011 12:45 PM

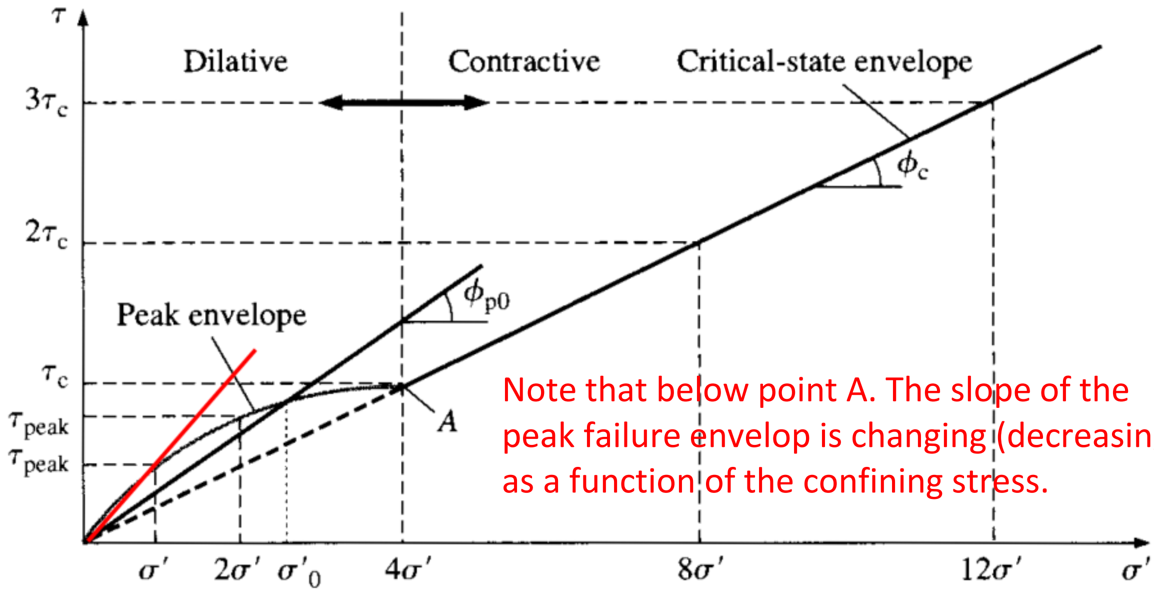


Figure 5-9
A shear envelope for a dilatant sand at a given value of relative density in which the friction angle decreases with increasing confining stress until the critical-state line is reached.

Initial Estimates of the Critical State Friction Angle

Wednesday, August 17, 2011 12:45 PM

Critical State Friction Angle

- Also called constant-volume ϕ

- The ϕ at very large strains

ϕ_c= 28° to 36° ⇨ silica sands

ϕ_c= 37° to 44° ⇨ carbonate sands

What friction angle do we use for design?

Do we use the peak friction angle ϕ_p or critical state friction angel ϕ_c?

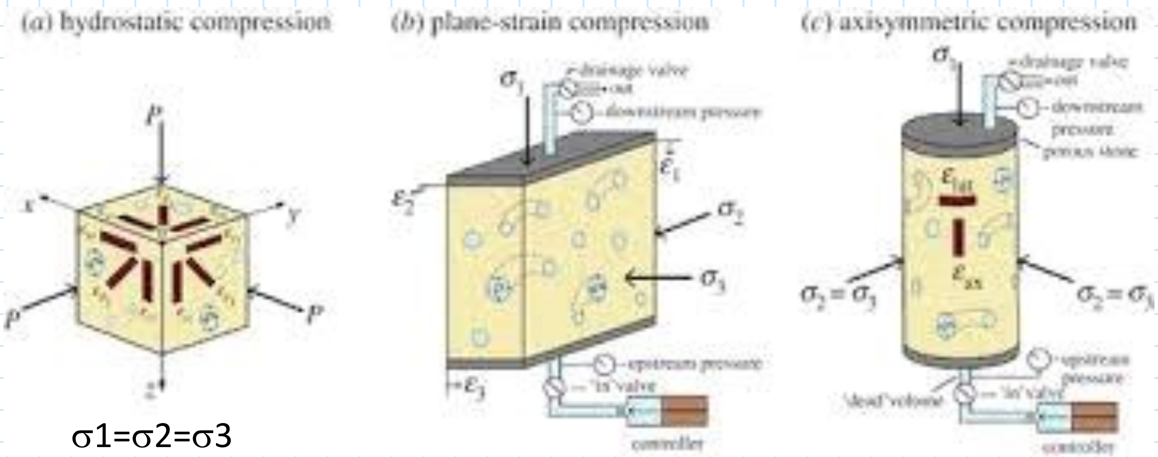
(see discussion on p. 203 of Salgado)

Which Friction Angle to Use in Design? We have seen that sand can have a friction angle ranging from the peak friction angle ϕ_p all the way down to the critical-state friction angle ϕ_c. The question of which ϕ to use in geotechnical design is an important one, which will be addressed specifically for each of the major classes of problems (shallow foundations, deep foundations, slopes and retaining structures) in the chapters in which they are discussed. However, there are some general statements that we can make.

The critical-state friction angle ϕ_c develops only after considerable deformation has already taken place. Therefore, it should only be used along slip surfaces where considerable sliding is expected or designed for. For all potential slip surfaces where sliding is to be prevented as part of the design of the geotechnical structure and where deformations are contained, the peak friction angle ϕ_p is used. There are cases where ϕ_p may be different along the same potential slip surface because of different levels of effective confining stress at different points of the slip surface; that needs to be accounted for. In cases where a slip surface is extensive and is subjected to different conditions, it may happen that different levels of deformation may develop along the slip surface, in which case each point may correspond to a different point of the stress-strain curve for the sand. This condition is called progressive failure. If we expect progressive failure to develop, we may indirectly account for it through the use of values within the range from ϕ_p to ϕ_c, depending on the strain levels, or even use of a single "representative" value of ϕ that would also be within that range. Advanced analyses relying on sophisticated numerical modeling and sophisticated soil modeling that have become possible in recent years may be used to shed light on this.

Plane Strain and Axisymmetric (Triaxial Compression) - Definitions

Wednesday, August 17, 2011 12:45 PM



σ1=σ2=σ3

σ1>σ2>σ3
Plane strain

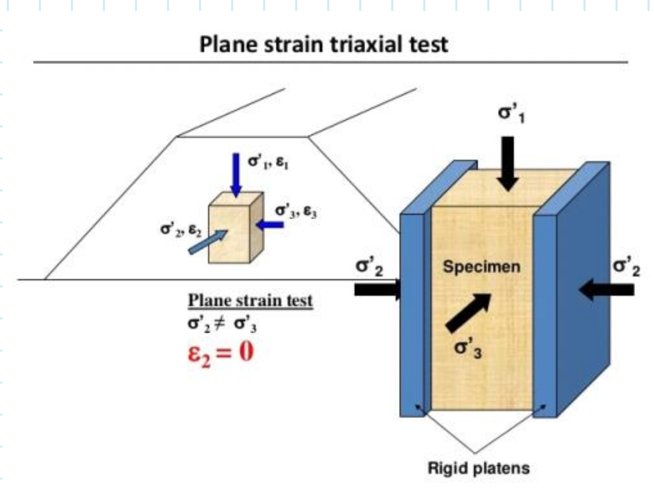
σ1>σ2=σ3
Triaxial (axisymmetric) compression

Plane strain conditions are applicable for what type of geotechnical structures?

- ?
- ?
- ?

Plane strain conditions are applicable for what type of geotechnical structures?

- ?
- ?
- ?



Estimating the Peak Drained Friction angle Shear Strength of Sands

Thursday, September 21, 2023 12:45 PM

The shear strength of sand has a component due to i**nterparticle friction and particle rearrangement** (i.e., critical-state shear strength) and a**nother due to dilatancy or contraction during shear**.

de Josselin de Jong (1976) showed that this can be expressed mathematically for plane-strain conditions as:

$N = MN_c$

where N is the flow number, M is the dilatancy number and N_c is the critical-state flow number. These are related by:

$N = (1 + \sin\phi) / (1 - \sin\phi) = \sigma_{1f} / \sigma_{3f}$ for c = 0
(from Lecture 4a, Inclination of Failure Plane)

$N_c = (1 + \sin\phi_c) / (1 - \sin\phi_c)$ (This is the flow number for the critical state)
$M = (1 + \sin\psi) / (1 - \sin\psi)$ (Similar to flow number but uses dilation angle)
where ϕ is the friction angle, **ψ is the dilatancy angle**, and **ϕ_c is the critical state friction angle.**

$\sin\psi = -(d\varepsilon_V/(d\varepsilon_1 - kd\varepsilon_3))$

see Eq. 4-19 in Salgado, where k = 1 for plane-strain conditions and 2 for triaxial conditions. (see next page)

For k =1 (plane-strain conditions), then
$\sin\psi = -d\varepsilon_V/(d\varepsilon_1 - d\varepsilon_3)$

Bolton (1986) examined a large number of **triaxial compression and plane-strain compression tests** and concluded that, for both types of loading, the following relationship held:

$$-(d\varepsilon_V / d\varepsilon_1)_p = 0.3I_R$$

where the p subscript indicates that quantity in parenthesis should be calculated at the peak strength and **I_R is the relative dilatancy index**

Estimating the Peak Drained Friction angle Shear Strength of Sands (cont.)

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Bolton defined the relative dilatancy index for the peak strength as:

$I_R = I_D\,[Q - \ln(100\,\sigma'_{mp}/p_A)] - R_Q$

where $I_D = D_R/100$ = relative density (%) divided by 100, Q and R_Q = fitted parameters that depend on the intrinsic characteristics of the sand (Table 5-1), p_A is the reference stress (1 atm ≈ 100 kPa = 0.1 Mpa ≈ 1 tsf = 2000 psf), and σ'_{mp} = mean effective stress at the peak shear strength.

$\sigma'_{mp} = (\sigma'_{1p} + \sigma'_{2p} + \sigma'_{3p})/3$

Table 5-1 Values of Q and R_Q for Ottawa sand with various percentages by weight of nonplastic silt and kaolinite clay

Fines (%)	Best fit Q	Best fit R	Best fit r^2
0%	9.9	0.86	0.95
5% silt	9.1	−0.33	0.99
10% silt	9.3	−0.30	0.98
2% clay	12.1	2.78	0.96
5% clay	11.7	3.17	0.95
10% clay	10.9	3.43	0.80

r^2 = coefficient of correlation

For triaxial compression test during shear phase of the test, $\sigma'_2 = \sigma'_3 = \sigma'_c$ where σ'_c is the confining or consolidation stress applied on the outer cell

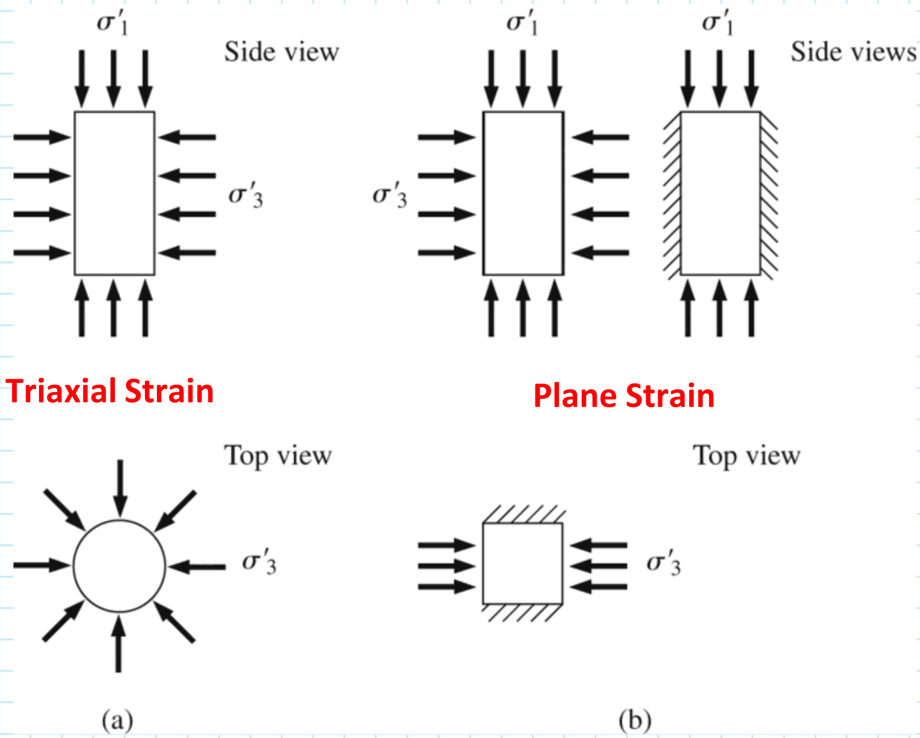
Bolton found that the following equation describes the peak friction angle very well for triaxial and plane-strain conditions.

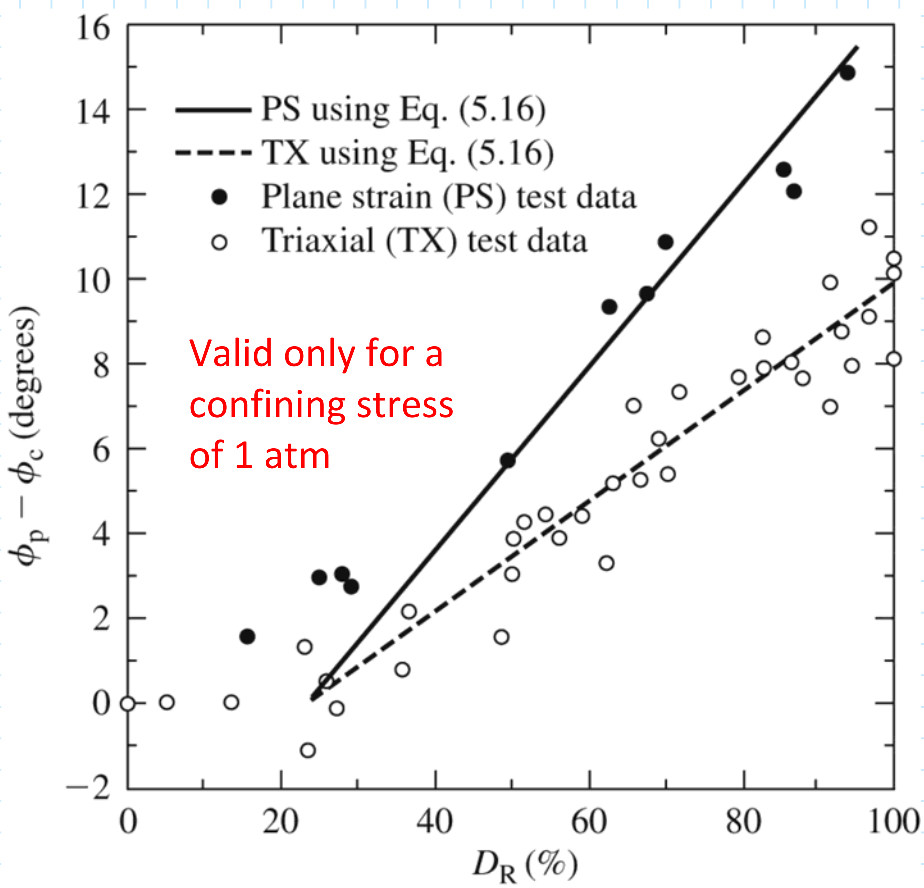
$\phi_p = \phi_c + A_\psi I_R$ (Eq. 5-16) Salgado

where **A_ψ = 3 for triaxial conditions and A_ψ = 5** for plane-strain conditions

Estimating the peak friction angle from the critical state friction angle for sand (cont.)

Wednesday, August 17, 2011 12:45 PM





(See Eq. 5-16 on previous page to relate ϕ_p and ϕ_c)

ϕ_p = peak friction angle

ϕ_C = critical state friction angle

Iteration to estimate peak friction angle from stress state and void ratio

- Practical application
  - If we know or can estimate the critical state friction angle of a soil, the horizontal earth pressure coefficient Ko, and the relative density of the deposit, we can estimate the peak friction angle. This is valuable for design because most often, the peak friction angle is used to define the strength (i.e., resistance) of the soil for shallow foundation calculations.

Problem 5-12 A deposit of clean sand has unit weight equal to 22 kN/m^3. The relative density increases approximately linearly from 60% at the surface of the deposit to 75% at a depth of 10 m. K_0 is 0.45 for this deposit. The sand can be assumed to have $\phi_c = 30^\circ$, Q = 10 and R_Q = 1. Calculate and plot the values of ϕ_p under triaxial and plane-strain compression conditions between 0 and 10 m depth assuming $\sigma'_c = \sigma'_m$. Consider the water table to be very deep (deeper than 10 m).

Let us consider the depth 1m from surface to show the sample calculation.

At 1m vertical stress $\sigma'_v = 1 \times 22 = 22\text{kPa}$

Given K_0 = 0.45 for this deposit. So horizontal stress at 1m

$\sigma'_h = 0.45 \times 22 = 9.9\text{kPa}$

Mean stress $\sigma'_m = \dfrac{\sigma'_v + 2\sigma'_h}{3} = \dfrac{22 + 2(9.9)}{3} = 13.9\text{kPa}$ ←

Now assuming mean stress σ'_m = consolidation stress σ'_c

Let's assume ϕ_p=40°, then:

$$N = \frac{\sigma'_{1p}}{\sigma'_{3p}} = \frac{1+\sin\phi_P}{1-\sin\phi_P} = \frac{1+\sin 40^\circ}{1-\sin 40^\circ} = 4.6 \rightarrow \sigma'_{1p} = 63.9\text{kPa}$$

$$\sigma'_{mp} = \frac{\sigma'_{1p} + 2\sigma'_{3p}}{3} = \frac{63.9 + 2\times 13.9}{3} = 30.6\text{kPa}$$

This is the peak mean effective principle stress for a peak friction angle of 40 degrees.

Estimation of the peak friction angle from critical state friction angle

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We can then use Bolton's equation to calculate the peak friction angle based on a peak mean effective principle stress of 30.6 kPa

$$D_R\big|_{x=1m} = 60 + \frac{75-60}{10} \times 1 = 61.5\%$$

$$I_D = \frac{D_R}{100} = \frac{61.5}{100} = 0.615$$

$A_\psi = 3$ (triaxial conditions) , $p_A = 100$kPa, $Q = 10$ and $R_Q = 1$

Therefore:

From our assumption for ϕ_p = 40 deg.

$$I_R = I_D\left[Q - \ln\left(\frac{100\sigma'_{mp}}{p_A}\right)\right] - R_Q = 0.615\left[10 - \ln\left(\frac{100 \times 30.6}{100}\right)\right] - 1 = 3.046$$

$$\phi_p = \phi_c + A_\psi I_R = 30° + 3 \times 3.046 = 39.1°$$

Note that in the above example, the peak friction angle calculated from Bolton's equation is not consistent with the assumed peak friction angle of 40 degrees. Therefore, our initial assumption of 40 degrees needs to be revised. Hence, another iteration is required.

This is done by adjusting the assumed peak friction angle to a new estimate of 39.1 degrees and recalculating the mean stress and resulting friction angle until convergence is reached. In practice, friction angles are usually reported to the rounded nearest whole number, so once the iteration converges to a stable whole number value, then iteration can cease.

For more information on the iterative process, see Example 5-2 in Salgado.

Estimation of the peak friction angle from critical state friction angle

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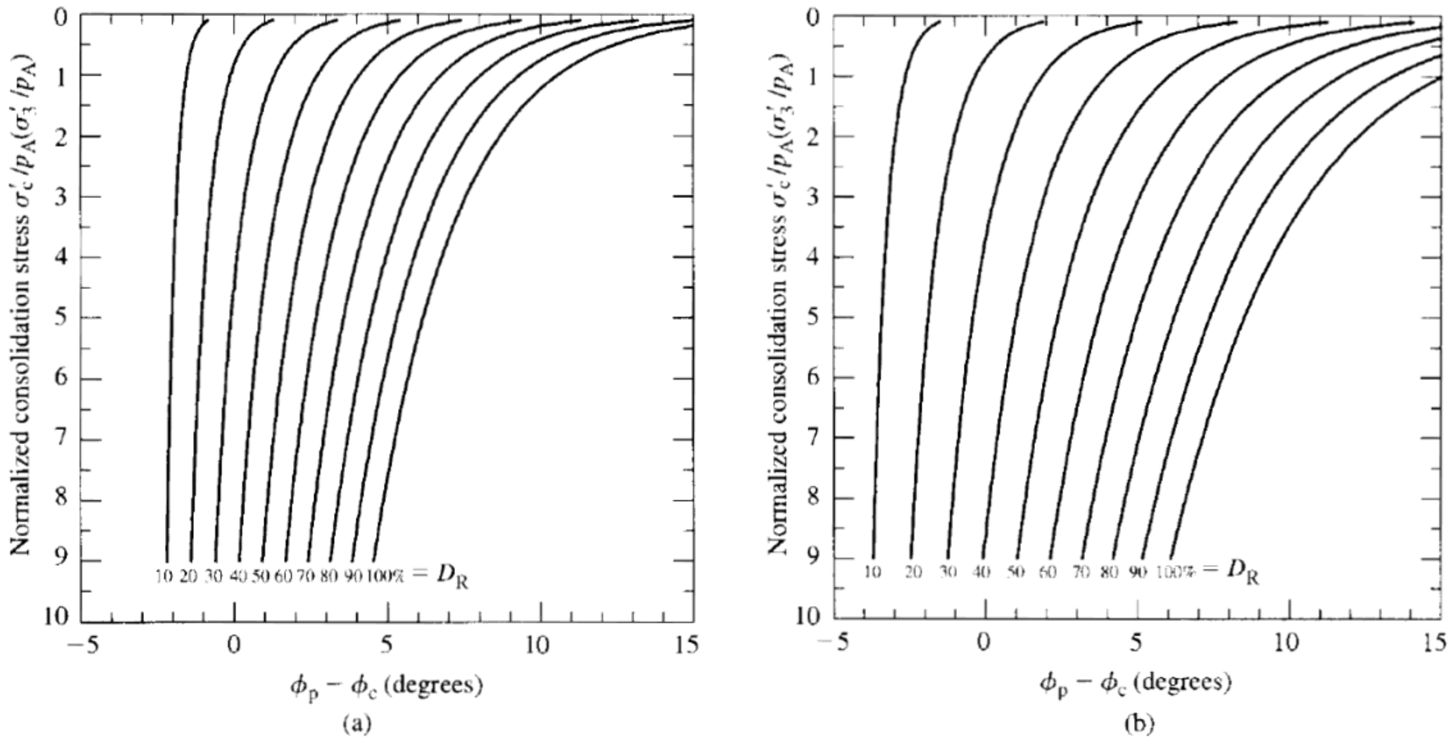
Note that these charts are useful for obtaining a first guess for the iterative procedure



Triaxial Compression

Plane Strain

Figure 5-13

Peak friction angle in (a) triaxial compression and (b) plane-strain compression as a function of stress state, relative density, and critical-state friction angle using $Q = 10$ and $R_Q = 1$.

A level, clean sand deposit has an average unit weight of 19 kN/m³ in the upper 8 m. Estimate the peak friction angle at a depth of 8 m if the water table is located at a depth of 10 m. The relative density is estimated to be equal to 65% at 8 m depth, and the critical-state friction angle of the sand is 32°.

■ Solution

The solution to this problem can be easily attained by using Fig. 5-13.

The vertical effective stress is found to be

$$8 \times 19 = 152 \text{ kPa}$$

Assuming K_0 to be 0.45, then the lateral stress follows directly:

$$\sigma'_h = 152 \times 0.45 = 68.4 \text{ kPa}$$

Using Fig. 5-13, we determine where $\sigma'_h = 68$ kPa intersects the $D_R = 65\%$ line. Now we project that down to the $\phi_p - \phi_c$ axis and read off a value of approximately 6.8°.

So

$$\phi_p - 32° = 6.8°$$

and

$$\phi_p = 38.8°$$

Undrained Shear Tests in Sands

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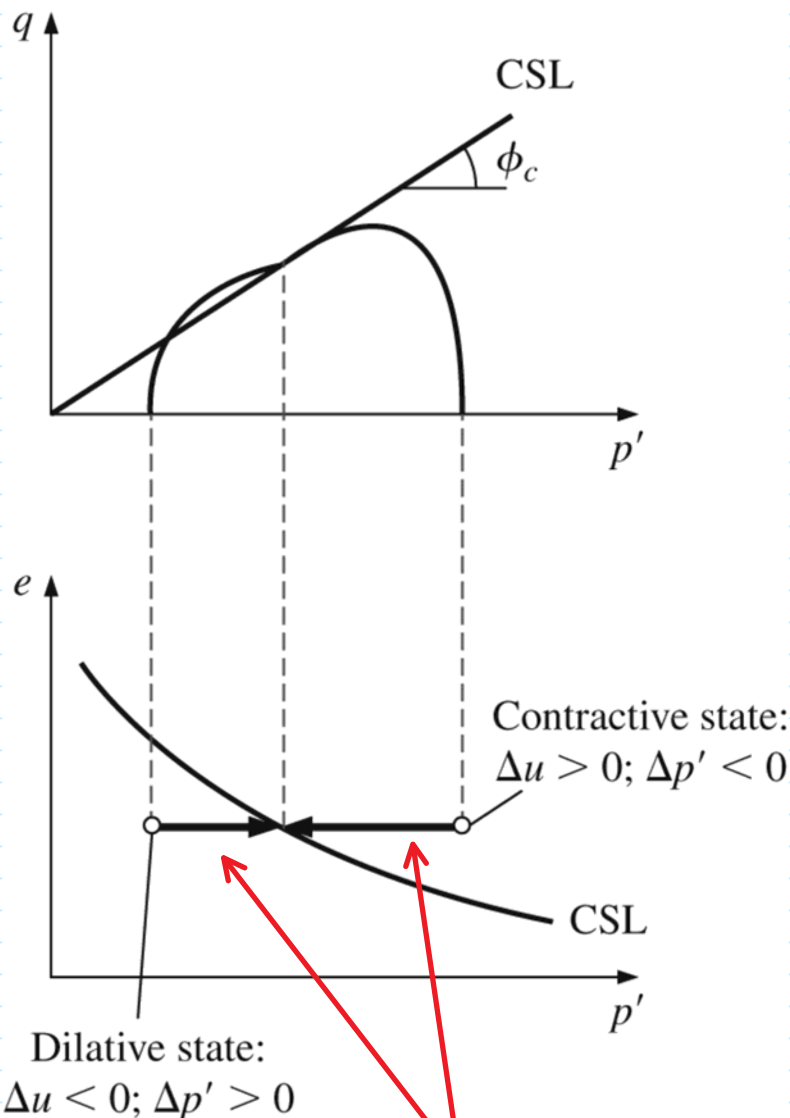

Note that there is no change in volume or void ratio during undrained shear because the drain value of the cell is closed.

Thus, the sample responds to shear by increasing or decreasing the pore pressures, u. The change in pore pressure during shear is called Δu, which in turn changes the effective stress during shear, as shown by changes in p'

The implications of pore pore pressures generated during shearing are further discussed in Ch. 6.

Δu = excess porewater change above hydrostatic

Note that no volume (i.e., void ratio) change is allowed because the drain value is close during shearing (undrained test)

Undrained tests will be more fully explained in Ch. 6.

What real world conditions could cause a saturated sand to behave undrained?

1.
2
3.

P1 of 5

Vertical Stress Profiles (example)

① $\gamma_m = 100 \text{ lb/ft}^3$ $D_{10} = 2 \text{ mm}$ 5'

② $\gamma_m = 105 \text{ lb/ft}^3$ $D_{10} = 0.075 \text{ mm}$ 5'
$\gamma_{sat} = 115 \text{ lb/ft}^3$

③ $\gamma_m = 110 \text{ lb/ft}^3$ $D_{10} = 0.001 \text{ mm}$ 5'
$\gamma_{sat} = 120 \text{ lb/ft}^3$

∇

④ $\gamma_{sat} = 120 \dfrac{\text{lb}}{\text{ft}^3}$ 10'

1) Calculate σ_v, u, σ_v' profiles

2) Consider capillary rise, as appropriate

TOPS 35502

Effective Stress Calculations (with capillary rise) (cont.)

Sunday, February 24, 2013 1:48 PM

① Calculate height capillary rise in layer ③

$$h_{c\,(m)} = \frac{-0.03m}{(0.2)(D_{10})mm}$$

$$h_c = \frac{-0.03}{(0.2)(0.001)} m$$

$$= 150\ (m)$$

∴ Layer ③ is saturated

Calculate height capillary rise in layer ②

$$h_{c\,(m)} = \frac{-0.03\ m}{(\cancel{0.02})(0.2)(0.075)}$$

$$= 2\ m$$

∴ Layer ② is saturated

Calculate height capillary rise in layer ①

$$h_{c\,(m)} = \frac{-0.03}{(0.2)(2)} m$$

$$= 0.075\ m$$

negligible (neglect)

TOPS 35502

Effective Stress Calculations((with capillary rise) cont.)

Sunday, February 24, 2013 1:48 PM

② Calculate σ_v

$\sigma_{v\,①\,bot} = (5)(100) = 500 \ \frac{lb}{ft^2}$

$\sigma_{v\,②\,bot} = (5)(115) + 500 = 1075 \ \frac{lb}{ft^2}$

$\sigma_{③\,bot} = (120)(5) + 1075 = 1675 \ \frac{lb}{ft^2}$

$\sigma_{④\,bot} = (120)(10) + 1675 = 2875 \ \frac{lb}{ft^2}$

Note that saturated units weights are use here because layers 2 & 3 are saturated with capillary rise

③ Calculate u

$u_{①\,top} = 0$

$* \begin{cases} u_{①\,bottom} = 0 \\ u_{①\,bottom \text{ in capillary zone}} = -h_c(\gamma_w) \end{cases}$

$= -(10\,ft)(62.4\,\frac{lb}{ft^3})$

$= -624\,\frac{lb}{ft^2}$

$u_{②\,bot} = (-5)(\gamma_w)$

$= -312\,\frac{lb}{ft^2}$

$u_{③\,bot} = 0$ (at water table)

$u = 0$

TOPS 35502

P4 of 5

③ (cont.)

$$u_{④ bot.} = z\gamma_w$$
$$= (10)(62.4)$$
$$= 624 \frac{lb}{ft^2}$$

④ Calculate σ_v'

$$\sigma_v' = \sigma_v - u$$

$$\sigma_v'{}_{① top} = 0$$

$$\sigma_v'{}_{① bottom} = 500 - 0$$
$$= 500 \text{ psf}$$

$$\sigma_v'{}_{① bottom \text{ in capillary zone}} = 500 - (-624)$$
$$= 500 + 624$$
$$= 1124 \text{ psf}$$

$$\sigma_v'{}_{② bottom} = 1075 - (-312)$$
$$= 1387 \text{ psf}.$$

$$\sigma_v'{}_{③ bottom} = 1675 - 0$$
$$= 1675 \text{ psf}.$$

$$\sigma_v'{}_{④ bottom} = 2875 - 624$$
$$= 2251 \text{ psf}.$$

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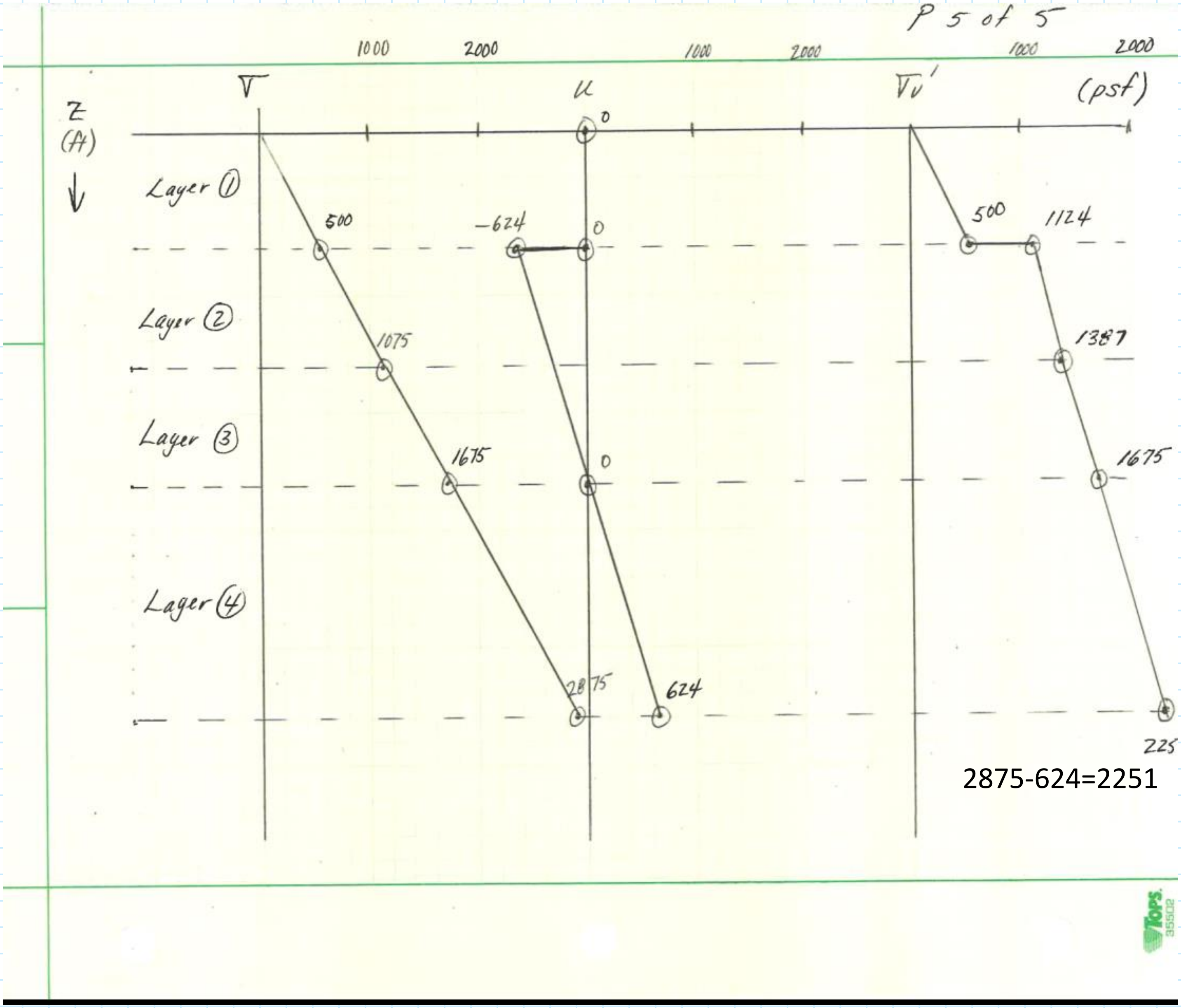
Effective Stress Calculations (with capillary rise) (cont.)

Sunday, February 24, 2013 1:48 PM



2875-624=2251

Effective Stress Calculations (without capillary rise)

Sunday, February 24, 2013 1:48 PM

Vertical Stress Profiles (example)

(1) $\gamma_m = 100$ lb/ft^3 $D_{10} = 2$ mm — 5'

(2) $\gamma_m = 105$ lb/ft^3 $D_{10} = 0.075$ mm — 5'
$\gamma_{sat} = 115$ lb/ft^3

(3) $\gamma_m = 110$ lb/ft^3 $D_{10} = 0.001$ mm — 5'
$\gamma_{sat} = 120$ lb/ft^3

▽

(4) $\gamma_{sat} = 120$ lb/ft^3 — 10'

1) Calculate σ_v, u, σ_v' profiles

2) Consider capillary rise, as appropriate

TOPS 35502

$\sigma_v' = \gamma_{m1}*t_1 + \gamma_{m2}*t_2 + \gamma_{m3}*t_3 + \gamma_{b4}*t_4$ where $\gamma_b = \gamma_{sat} - \gamma_w$

$\sigma_v' = (100*5)+(105*5)+(110*5)+(120-62.4)*10=2,151$ psf

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Ch 6 Strength and Stiffness of Clay

Wednesday, August 17, 2011 12:45 PM

Reading Assignment

- Ch. 6 Lecture Notes
- Sections 6.1 - 6.3, 6.5 (Salgado)

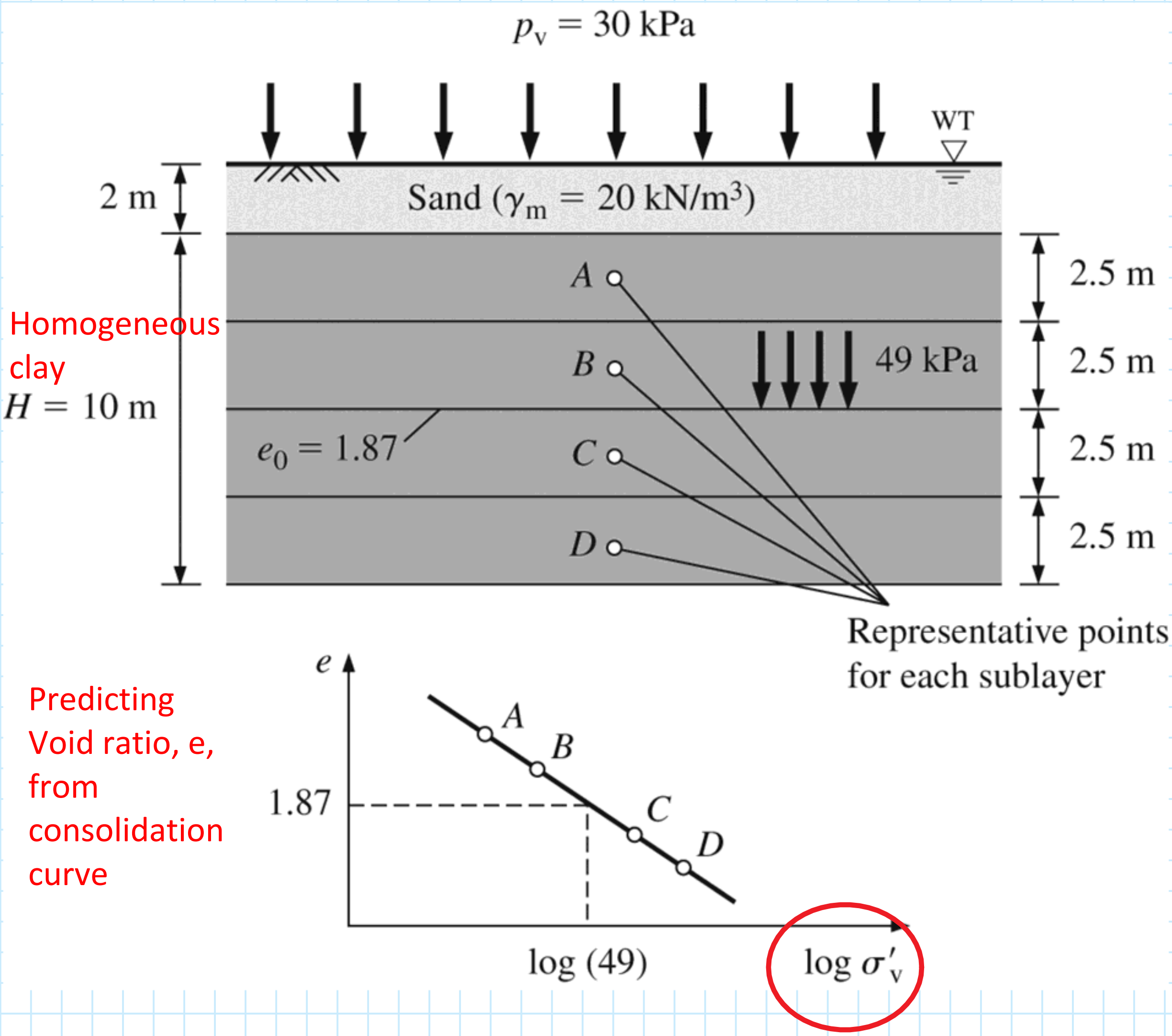
Other Materials

Homework Assignment 6

- Problems 6-11 (15 points), 6-12 a-b (15 points), 6-15 a-c (15 points), 6-17 (20 points)

Vertical Effective Stress and Void ratio changes versus depth

Monday, October 4, 2021 12:45 PM

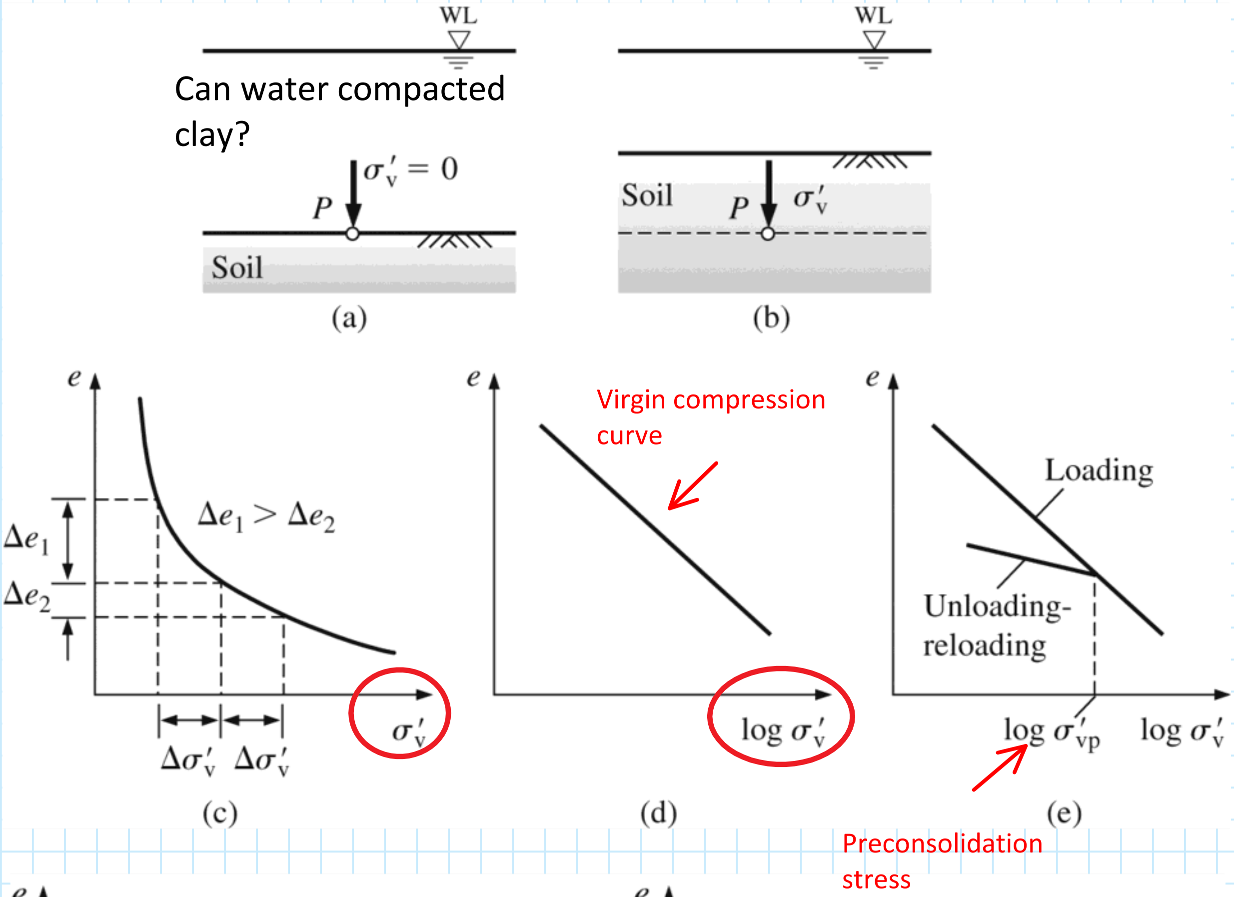


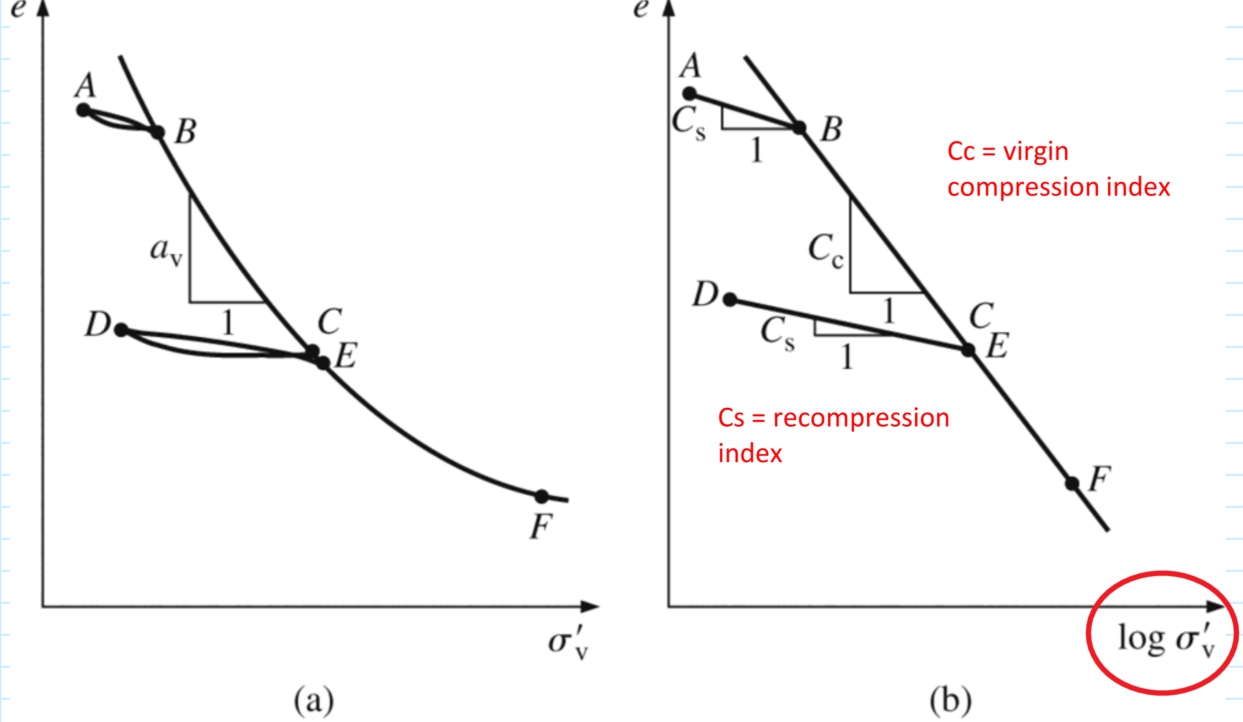
Questions:

1. How do we calculate the σ'v values in each of the layers?
    a. Do we consider the sand layer?
    b. Do we consider the applied semi-infinite load of P_v of 30 kPa?
    c. What about the load of 49 kPa at the depth of 7 m?

Compression and Void Ratio Change

Wednesday, August 17, 2011 12:45 PM





Time rate of consolidation

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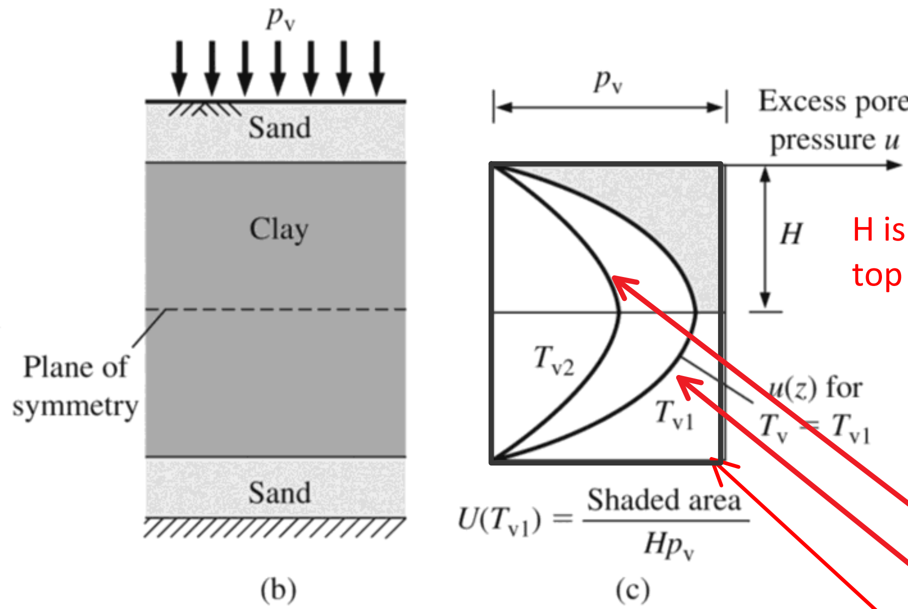


H is drained both top and bottom

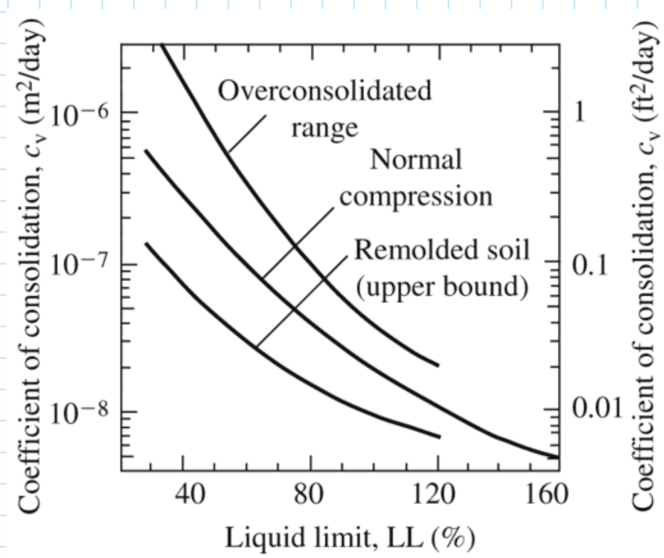
Isochrones of consolidation
Note the initial isochrone is a rectangle. Why?

Table 6-2 Values of the time factor T_v corresponding to various values of the degree of consolidation U expressed as a number

T_v	U	T_v	U
0.002	0.05	0.239	0.55
0.008	0.10	0.286	0.60
0.018	0.15	0.340	0.65
0.031	0.20	0.403	0.70
0.049	0.25	0.477	0.75
0.071	0.30	0.567	0.80
0.096	0.35	0.684	0.85
0.126	0.40	0.848	0.90
0.159	0.45	1.129	0.95
0.196	0.50	1.781	0.99

U = 0.99 means at 99 percent consolidation.

$$\boxed{T_v = \frac{c_v\, t}{H^2}}$$



Tv is the dimensionless time factor of consolidation,

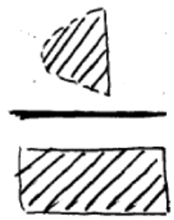
c_v is the coefficient of vertical consolidation

t is elapsed time

<u>Average Degree of Consolidation, U_{avg} (cont.)</u>

The value of U_{avg} can be conceptually thought of as:

$$U_{avg} = 1 - $$



$$= 1 - \frac{\int_{z=0}^{z=2H} u \, dz}{u_i \cdot 2H}$$

$$U_{avg} = \int_{z=0}^{z=2H} \frac{\left[(\sigma_f' - \sigma_i') - u\right]}{(\sigma_f' - \sigma_i')\, 2H} dz$$

- The boundary and initial conditions for the case of 1-D consolidation are:
  - Complete drainage at top and bottom of the compressible layer
  - $u_i = \Delta\sigma$ at the boundaries

    $z = 0,\ z = 2H \quad u = 0$

    $t = 0,\ u = u_i = \Delta\sigma = \sigma_f' - \sigma_i'$
- The general flow equation is:

$$\boxed{c_v \frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t}} \leftarrow$$

- For the above boundary conditions, the general solution to the above equation is:

$$u = \sum_{n=1}^{n=\infty} \left(\frac{1}{H} \int_{z=0}^{z=2H} u_i \sin \frac{n\pi z}{2H} dz \right) \sin \frac{n\pi z}{2H} \exp\left(\frac{-c_v t n^2 \pi^2}{4H^2} \right)$$

Consolidation of a Layer (cont.)

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Average Degree of Consolidation, U_{avg} (Cont.)

- When Δu_i is constant or varies linearly with depth

$$u = (\sigma_f' - \sigma_i') \sum_{n=0}^{\infty} \frac{4}{(2n+1)\pi} \cdot \sin\left(\frac{2n+1}{2} \pi \frac{z}{H}\right) \cdot$$

$$\exp - \left[\frac{(2n+1)^2}{4} \pi^2 \frac{k(1+e_o)}{a_v \rho_w g} \frac{t}{H^2} \right]$$

Note that: $c_v = \dfrac{k(1+e_o)}{a_v \rho_w g}$

See more explanation at the bottom of page

c_v - coef. of consolidation

k - coef. of permeability

a_v - coef. of compressibility

γ_ω - unit weight of water

$$T_v = \frac{c_v t}{H^2}$$

Recall that:

$$U_{avg} = \int_{z=0}^{z=2H} \frac{[(\sigma_f' - \sigma_i') - u]\, dz}{(\sigma_f' - \sigma_i')\, 2H}$$

$$U_{avg} = \frac{\int_0^{2H} (\sigma_f' - \sigma_i')\, dz - \int_0^{2H} u\, dz}{2H(\sigma_f' - \sigma_i')}$$

$$C_c = \frac{\Delta e}{\log_{10}\left(\dfrac{\sigma_2}{\sigma_1}\right)} \qquad (1)$$

Further, the coefficient of compressibility (a_v) and coefficient of volume compressibility (m_v) is evaluated using equations 2 and 3.

$$a_v = \frac{\Delta e}{\Delta \sigma} \qquad (2)$$

$$m_v = \frac{a_v}{1+e_o} \qquad (3)$$

Eq. 2 implies that the change in void ratio change and change in vertical stress are linearly related. This is only true over a narrow range of change in stress

Where Δe is change in void ratio, e_0 is the initial void ratio and σ2 & σ1 are being the final and initial effective stresses expressed in KN/m²

Average Degree of Consolidation, U_{avg} (cont.)

Using the results from the top of p. 6 and substituting it into the previous equation and intergrating yields.

$$U_{avg} = \frac{1}{2H(\sigma_f' - \sigma_i')} \Bigg[(\sigma_f' - \sigma_i') 2H - (\sigma_f' - \sigma_i') \cdot \sum_{n=0}^{\infty} \frac{4}{(2n+1)\pi} \cdot (-1) \cos \frac{(2n+1)\pi}{2H} z \cdot \left(\frac{2H}{(2n+1)\pi} \right) \cdot \exp - \left(\frac{(2n+1)^2 \pi^2}{4} T_v \right) \Bigg|_0^{2H} \Bigg]$$

Substituting in the limits yields:

$$U_{avg} = \left\{ 1 - \sum_{n=0}^{\infty} \frac{4}{(2n+1)^2 \pi^2} (-1)(-1-1) \cdot \exp - \left[\frac{(2n+1)^2 \pi^2}{4} T_v \right] \right\}$$

or

$$U_{avg} = \left\{ 1 - \sum_{n=0}^{\infty} \frac{8}{(2n+1)^2 \pi^2} \cdot \exp - \left[\frac{(2n+1)^2 \pi^2}{4} T_v \right] \right.$$

Consolidation of a Layer (cont.)

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Average Degree of Consolidation, U_{avg} (cont.)

The infinite series given above can be approximated by:

$$\boxed{U_{avg} = \frac{(4T_v/\pi)^{0.5}}{\left[1+(4T_v/\pi)^{2.8}\right]^{0.179}}}$$

This approximation (Sivaram, Swamee (1977)) gives less than 1% error for $U_{avg} < 0.9$ and less than 3% error for $0.9 < U_{avg} < 1.0$.

Estimating Settlement from Average Degree of Consolidation

- Recall that consolidation is essentially completed when all excess pore pressure has dissipated due to the new load

$$\boxed{U_{avg} = 1 - \frac{\int_0^{2H} u \, dz}{(\sigma_f' - \sigma_i')\,2H}}$$

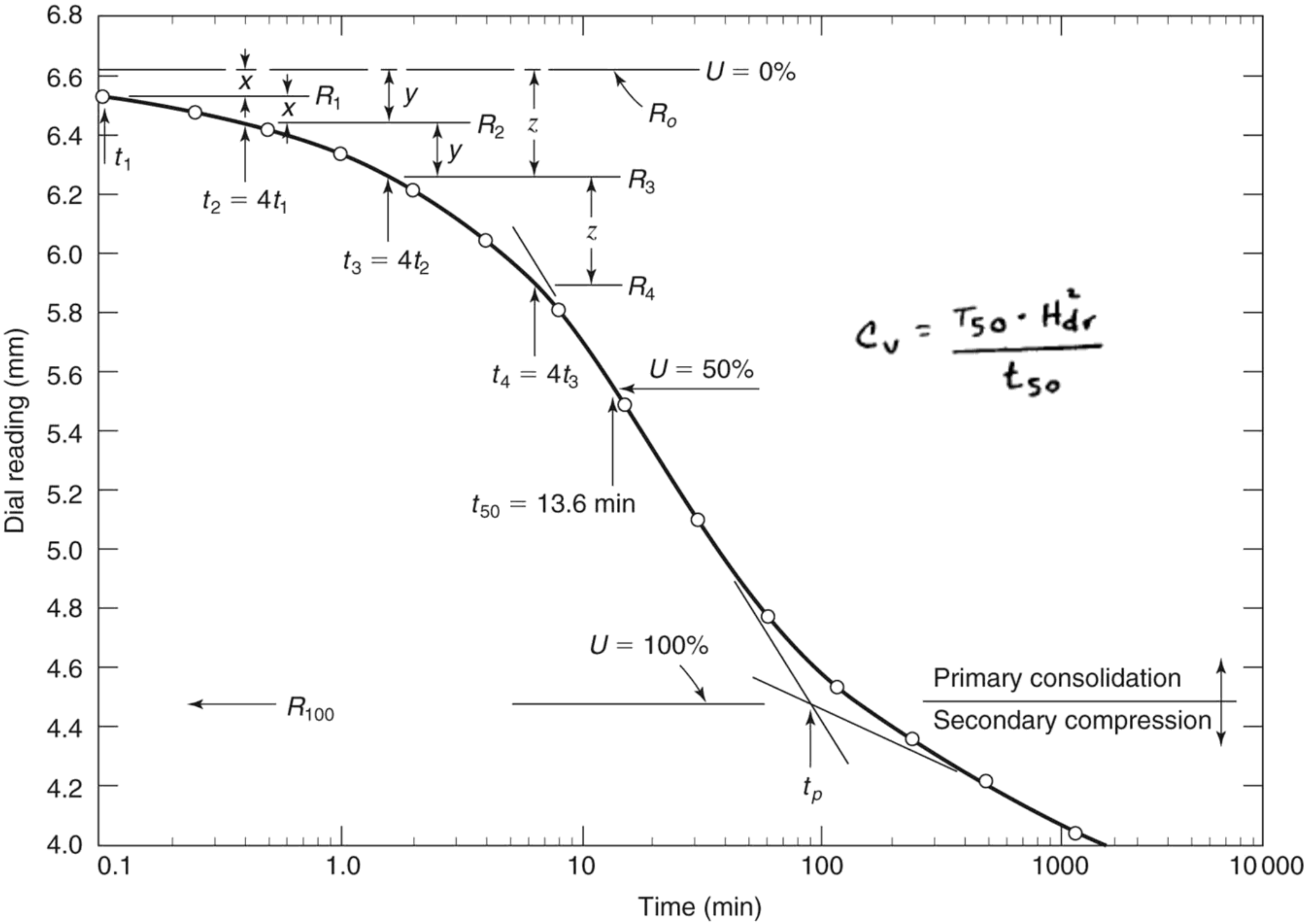
- However, the average percentage of pore pressure generation, U_{avg}, is not equal to the average degree of settlement in a layer

$$\boxed{U_{avg_t} \neq \frac{S_t}{S_{EOP}}}$$

U_{avg_t} = avg. degree of consolidation (pore pressure) at time t

Log of Time Method (Casagrande's Method)

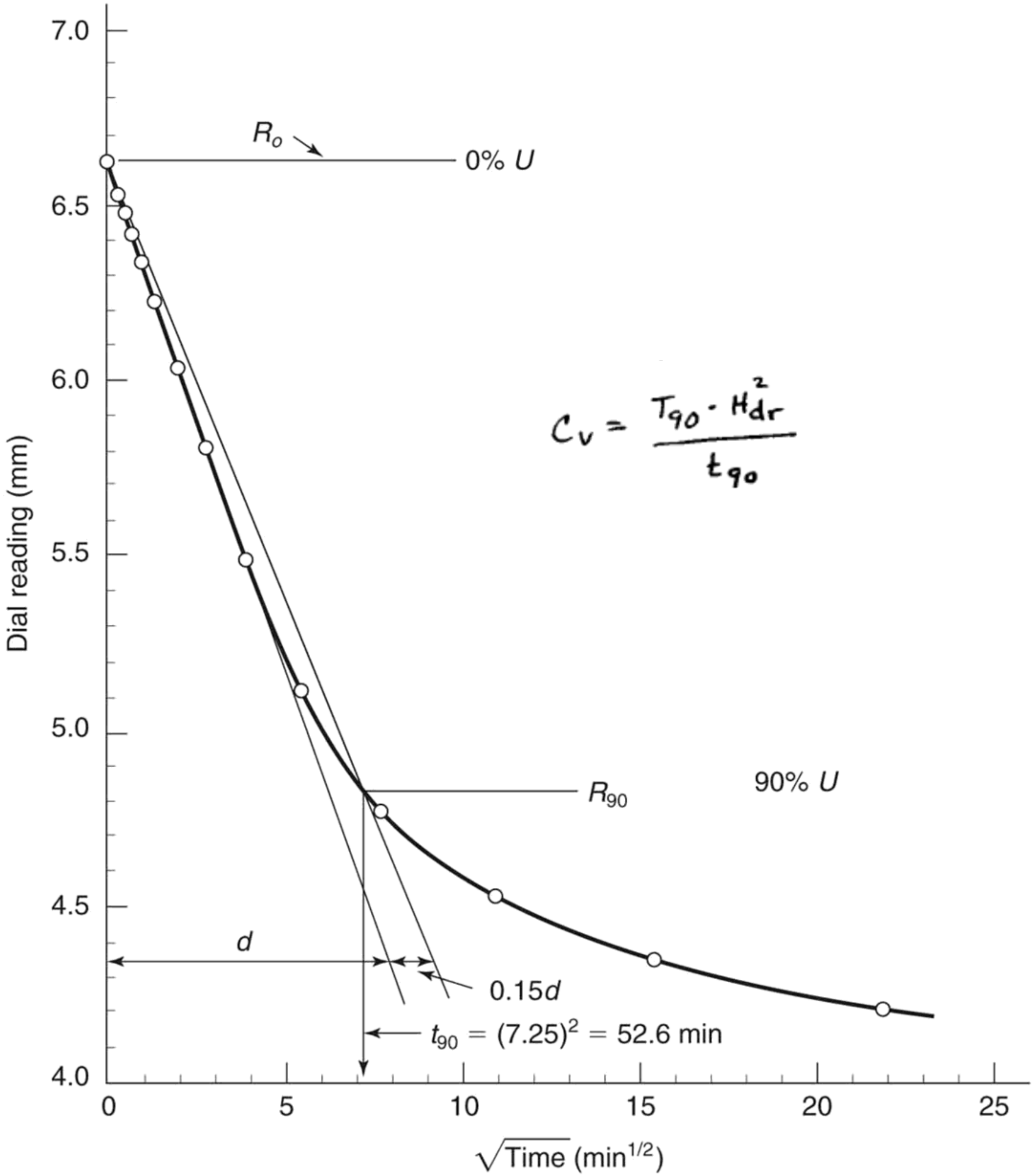
Thursday, March 11, 2010 11:43 AM



$$C_v = \frac{T_{50} \cdot H_{dr}^2}{t_{50}}$$

Square Root of Time Method

Thursday, March 11, 2010 11:43 AM



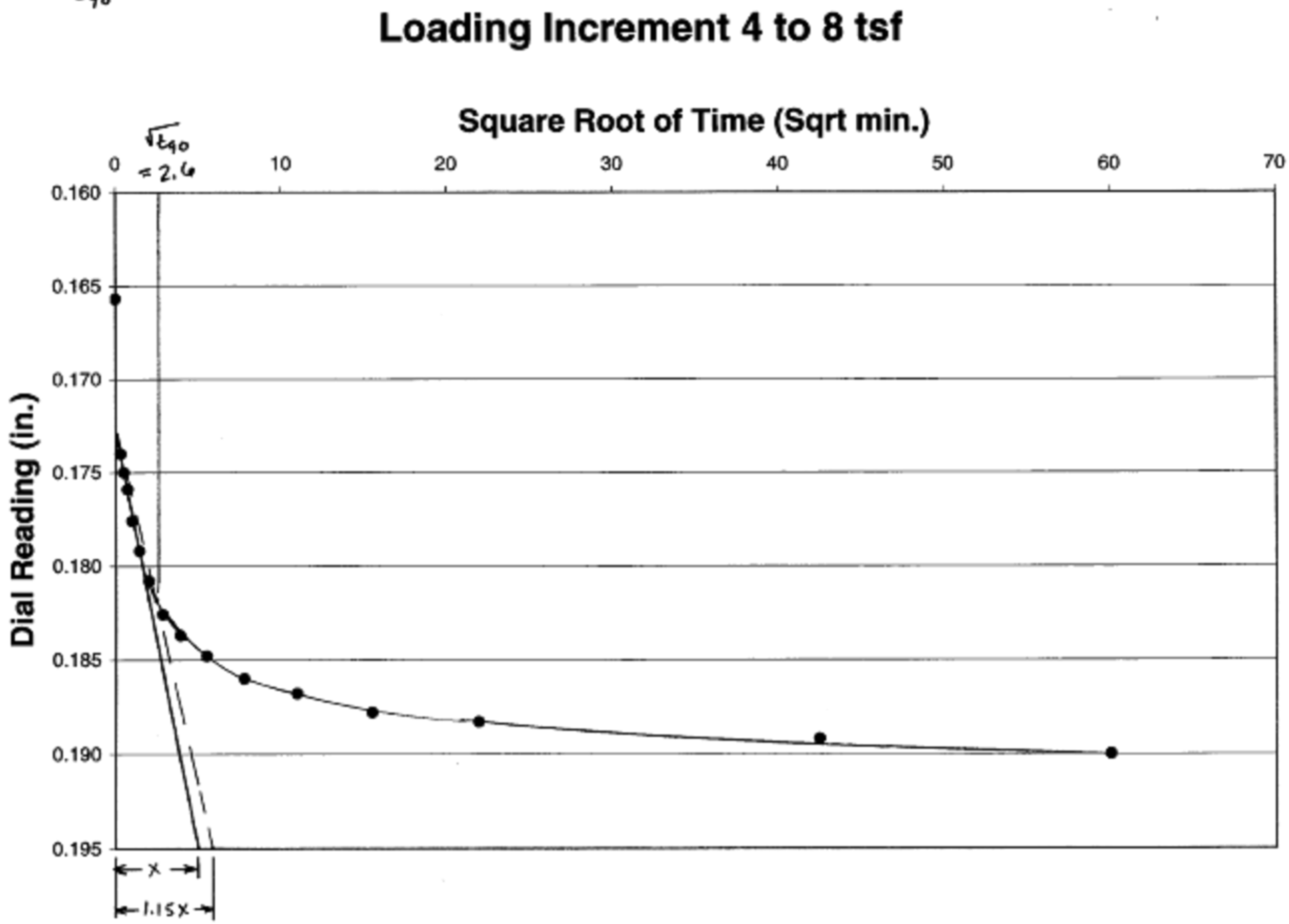
$$C_v = \frac{T_{90} \cdot H_{dr}^2}{t_{90}}$$

Square Root of Time Method (Example)

Thursday, March 11, 2010 11:43 AM

$t_{90} = (2.6)^2 = 6.8$ min.

$$C_v = \frac{T_{90} \cdot H_{dr}^2}{t_{90}} = \frac{0.848\,(0.4566)^2}{6.8} = 0.026\ \text{in}^2/\text{min} = 95\ \text{ft}^2/\text{yr} = 8.8\ \text{m}^2/\text{yr}$$



Example 1

Thursday, March 11, 2010 11:43 AM

Problem 1

Given: A settlement calculation estimates that the total amount of primary consolidation for a foundation is 7 inches.

S_c = 7 inches

Settlement measurements to-date show that the foundation has settled 4.9 inches and the foundation is still settling.

Find: The average degree of consolidation for the compressible layer?

$U_{avg} = ?$

Solution: $U_{avg} \approx \dfrac{S_t}{S_{EOP}}$

where: S_t = settlement at time t

S_{EOP} = settlement at the end of primary consolidation

for our case $S_{EOP} = S_c = 7$ inches

$U_{avg} = \dfrac{4.9\text{ in}}{7\text{ in}}$

$U_{avg}(\%) = \dfrac{4.9}{7} \times 100 = 70\%$

Note: Because S_c was estimated from a calculation and not measured, there is potential error in our estimate of U_{avg}.

It would be prudent to continue to monitor settlement even if we reach 7 inches of measured settlement because of possible additional settlement beyond 7 inches.

Example 2

Thursday, March 11, 2010 11:43 AM

Problem 2

Given: In the previous problem it was estimated that the consolidation process was about 71 % complete.

Find: If it took 180 days to reach 71% consolidation, estimate how long it will take to reach 90 and 95 percent consolidation respectively?

Solution: $$T_v = \frac{C_v\, t}{H^2}$$

where T_v = dimensionless time factor

C_v = coefficient of consolidation for vertical drainage

t = time elapsed

H = drainage path

- Values of U_{avg} versus T_v can be calculated from:

$$U_{avg} = \frac{(4T_v/\pi)^{0.5}}{\left[1+(4T_v/\pi)^{2.8}\right]^{0.179}}$$

or

obtained from Table 9-1 from Holtz and Kovacs.

$$\left.\begin{array}{l} \text{For } U = 90\%,\ T_v = 0.848 \\ \text{For } U = 95\%,\ T_v = 1.163 \end{array}\right\} \text{Table 9-1}$$

Example 2 (cont.)

Thursday, March 11, 2010 11:43 AM

Problem 2 (cont.)

<u>for</u> $t = 180$ days and $T_v = 0.403$ ($U = 70\%$) from Table 9-1

<u>then</u> $T_v = \frac{C_v t}{H^2}$

$$\frac{C_v}{H^2} = \frac{T_v}{t}$$

$$\frac{C_v}{H^2} = \frac{0.403}{180 \text{ days}}$$

$$\frac{C_v}{H^2} = 2.239 \times 10^{-3} / \text{day}$$

<u>assume</u> $\frac{C_v}{H^2}$ doesn't change significantly during consolidation (not <u>strictly</u> true)

<u>therefore</u>

$$t = \frac{T_v H^2}{C_v}$$

$$t_{90} = \frac{(0.848) \text{ days}}{2.239 \times 10^{-3}}$$

$$t_{90} = 379 \text{ days} \leftarrow$$

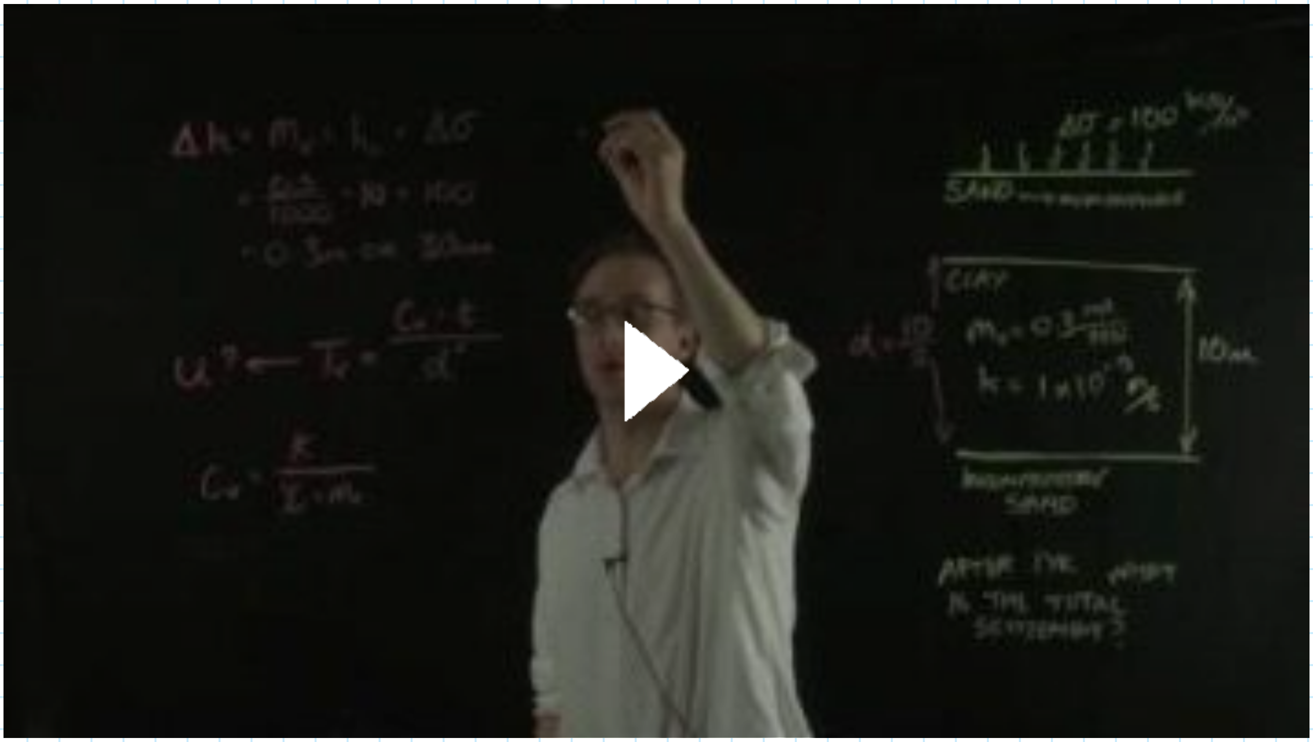
$$t_{95} = \frac{1.163 \text{ days}}{2.239 \times 10^{-3}}$$

$$t_{95} = 519 \text{ days} \leftarrow$$

Example 2 (cont.)

Thursday, March 11, 2010 11:43 AM

[Consolidation Time Example]



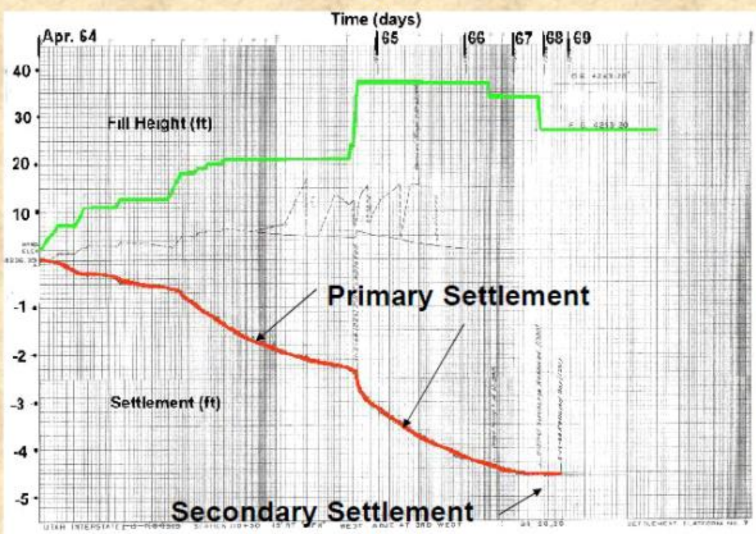
I-15 Reconstruction - Quick Facts

- Single Largest Highway Contract in U.S.
- 17 Miles of Urban Interstate
- $1.5 Billion Design-Build
- 4 Year Construction Duration (Summer 2001)
- 140 Bridges/Overpass Structures
- 160 Retaining Walls (mostly MSE Walls)
- 3.8 Million m³ of Embankment Fill
- 100,000 m³ Geofoam Embankment
- Approximate $4 to $6 M Research Program (4 years)

INTERSTATE 15 RECONSTRUCTION

UDOT GOING THE EXTRA MILE

Settlement of Soft Clays in Salt Lake Valley



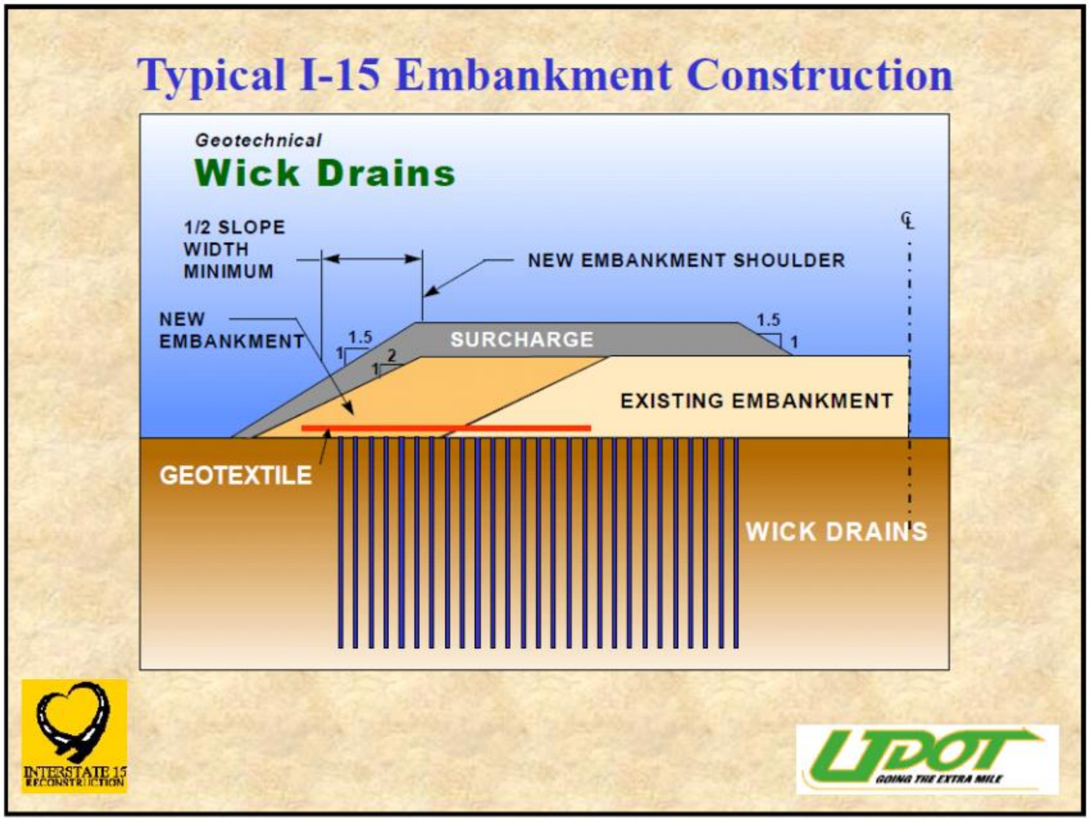
Approximate 2 years of primary settlement

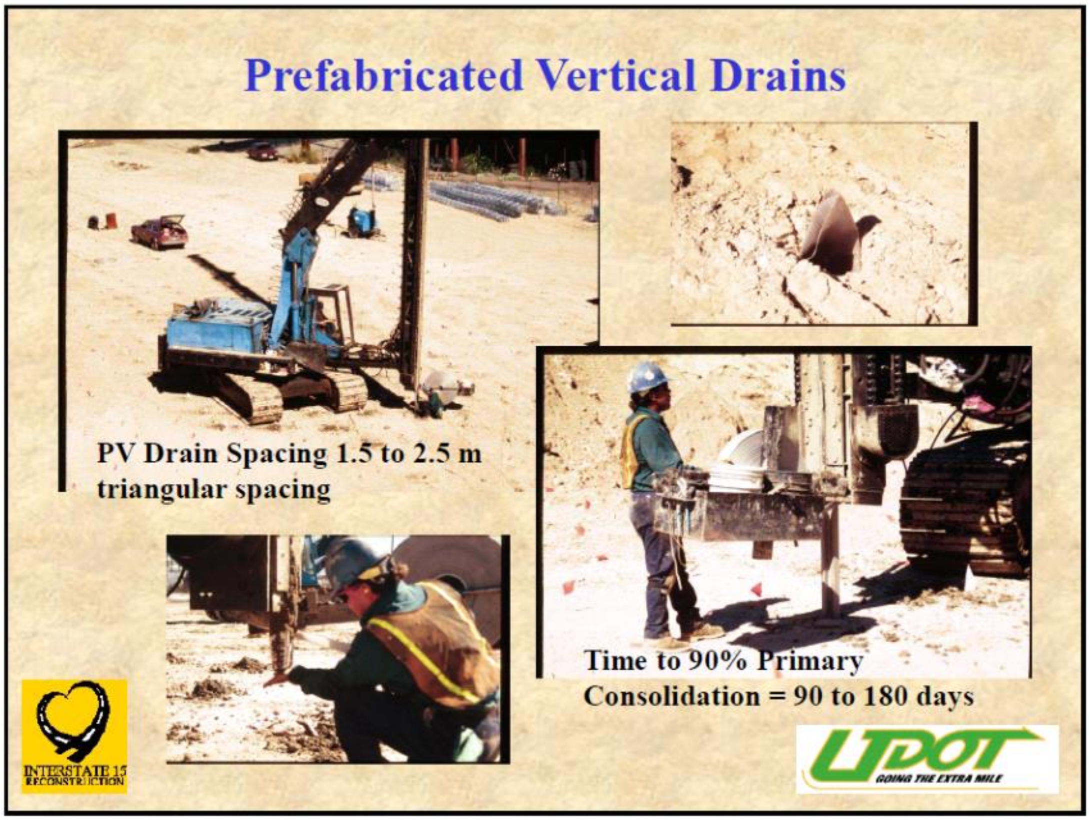
INTERSTATE 15 RECONSTRUCTION

UDOT GOING THE EXTRA MILE

Settlement Technologies (cont.)

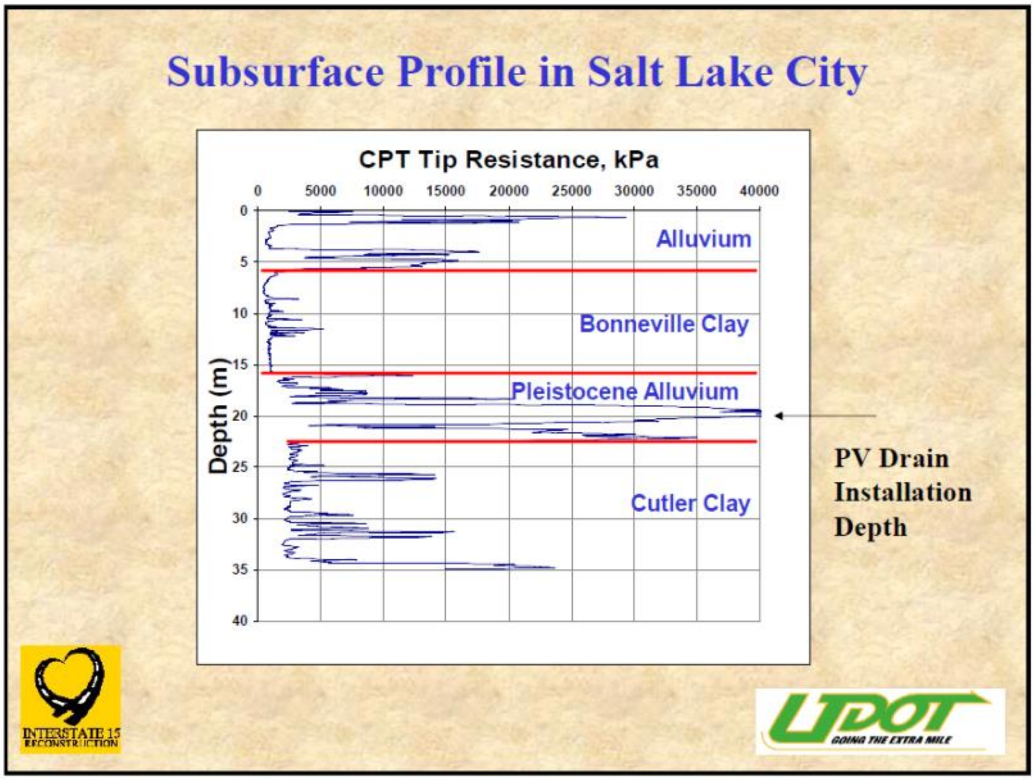
Thursday, October 19, 2023 12:45 PM

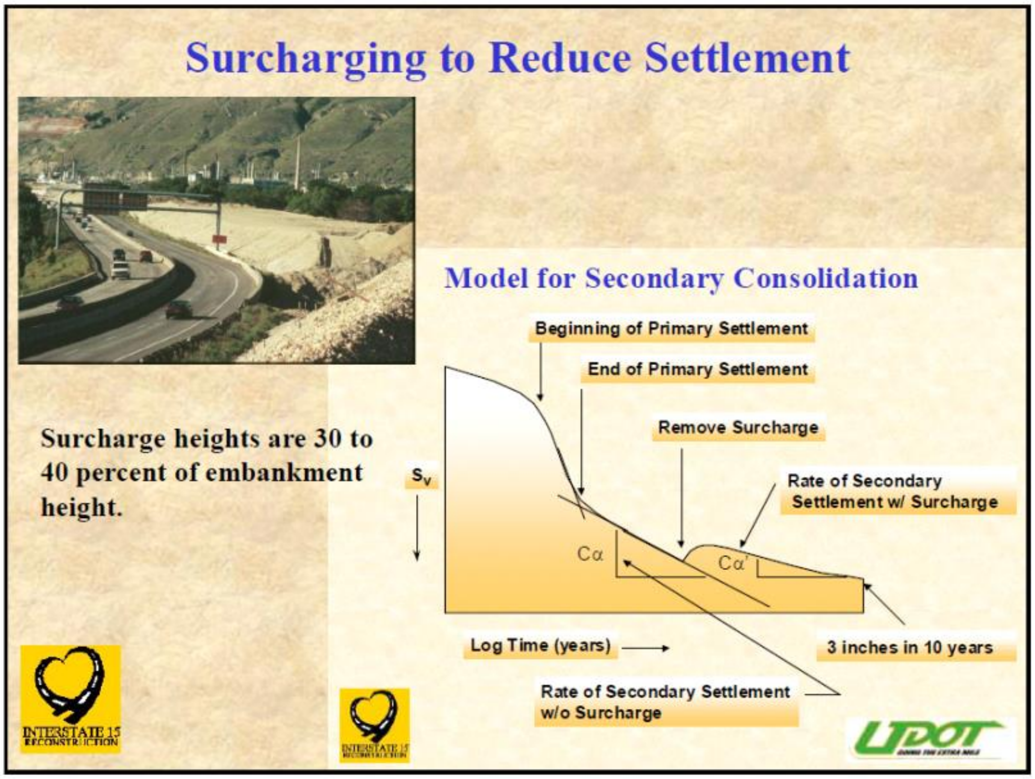




Settlement Technologies (cont.)

Thursday, October 19, 2023 12:45 PM





Settlement Technologies (cont.)

Thursday, October 19, 2023 12:45 PM

Geotextile Reinforced Slopes

Large embankment + surcharge heights (10-14 m) required geotextile reinforcement for short term global stability

Geotextile overlapped into MSE wall for global stability

INTERSTATE 15 RECONSTRUCTION

UDOT GOING THE EXTRA MILE

Mechanically Stabilized Earth Walls

Right-of-way constraints required many slopes to be built vertically.

Beginning of 2-stage MSE Wall

INTERSTATE 15 RECONSTRUCTION

UDOT GOING THE EXTRA MILE

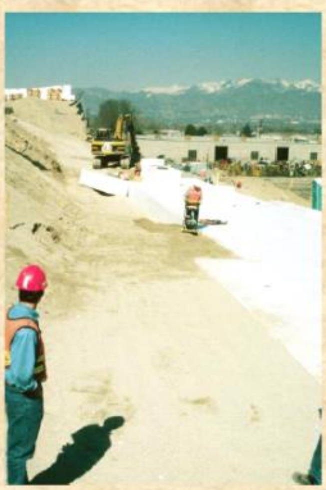
Settlement Technologies (cont.)

Thursday, October 19, 2023 12:45 PM

Geofoam Embankment

View of I-80 from State St. to 200 W. with Geofoam Embankment

Geofoam Embankment

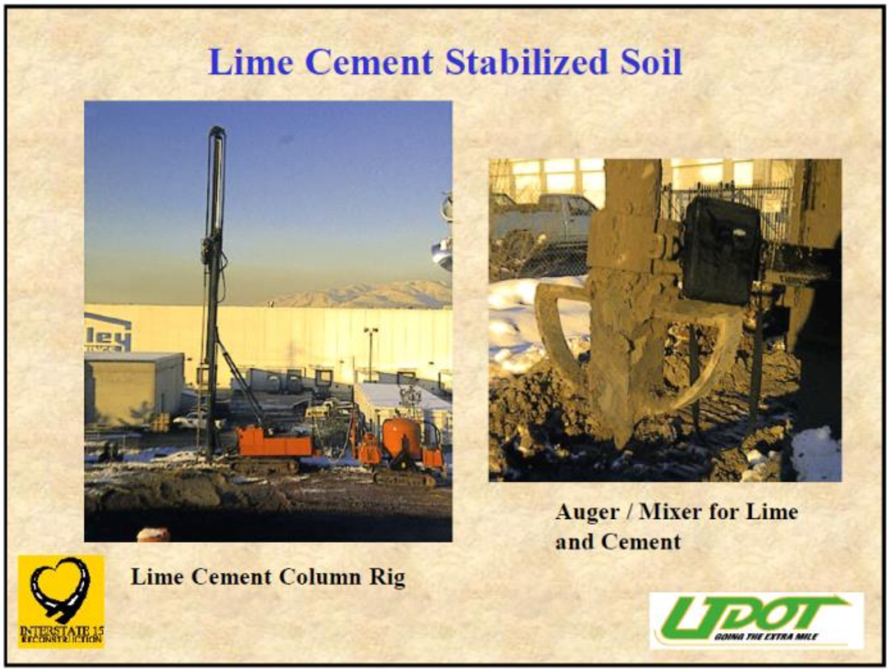


Leveling Course of Sand for Geofoam Embankment

Construction of Geofoam Embankment and Footing for Tilt-up Panel Wall

Settlement Technologies (cont.)

Thursday, October 19, 2023 12:45 PM

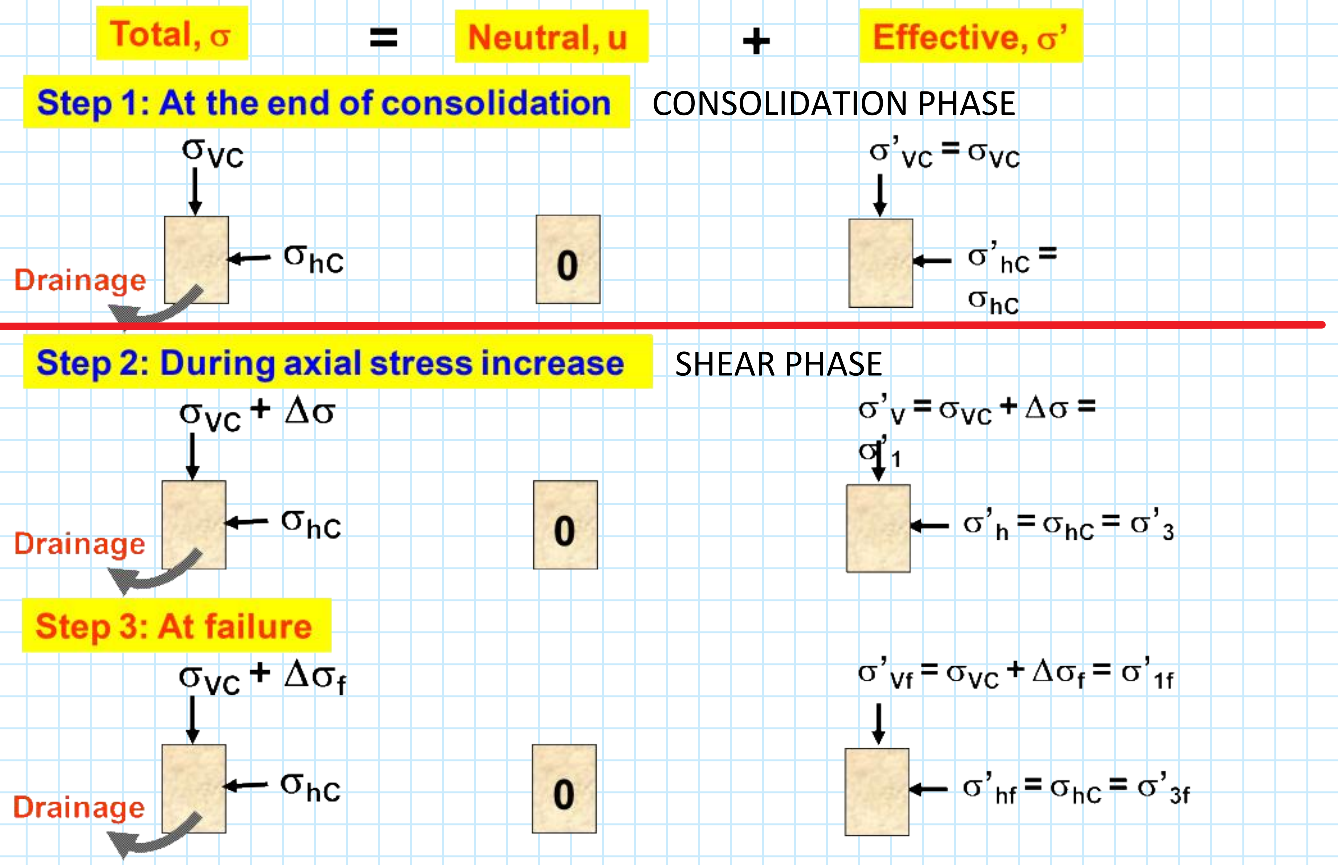


What factors contribute to the strength of granular materials?

- Excess Pore Pressure (Undrained Behavior under Loading
- Soil Compression (i.e., Preconsolidation Stress)
- Degree of Saturation
- Percentage of Clay and Clay Mineralogy
- Cementation
- Aging

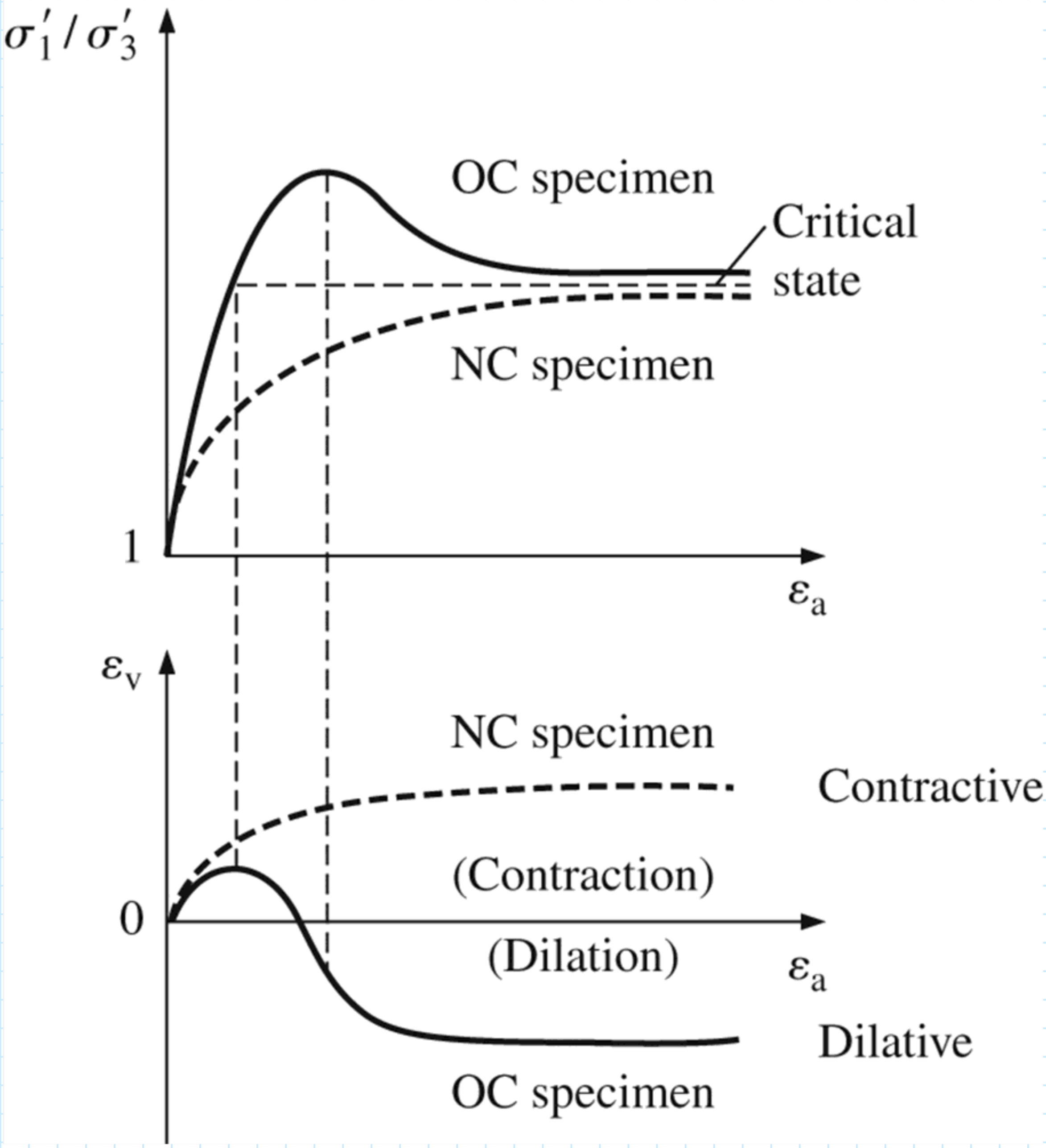
Consolidated Drained (CD) Triaxial Shear Test

Wednesday, August 17, 2011 12:45 PM



CD Strength of Saturated Clay

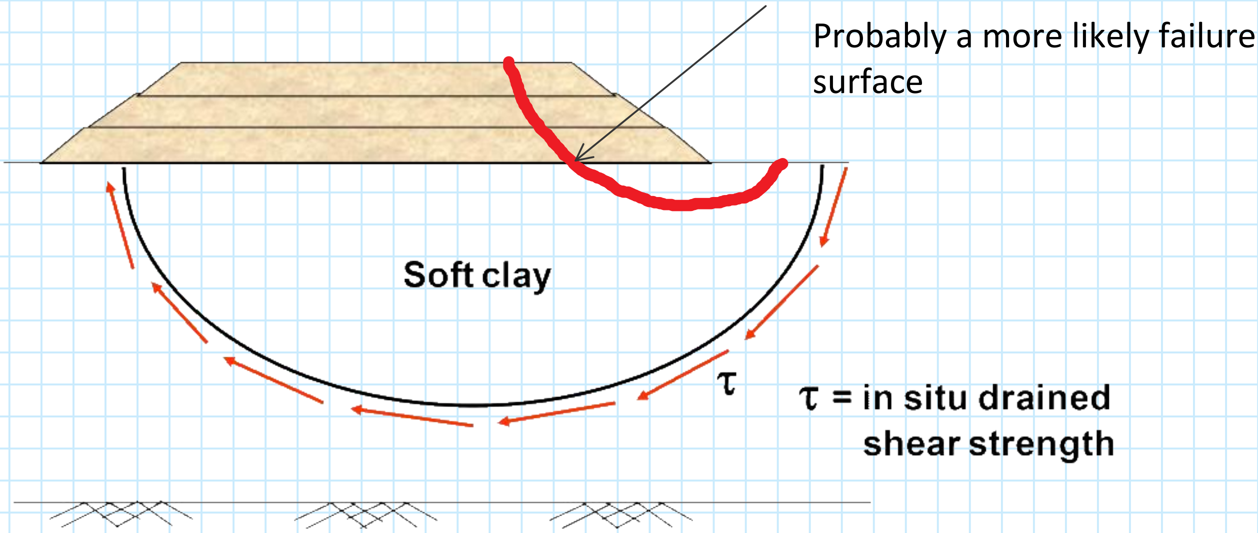
Wednesday, August 17, 2011 12:45 PM



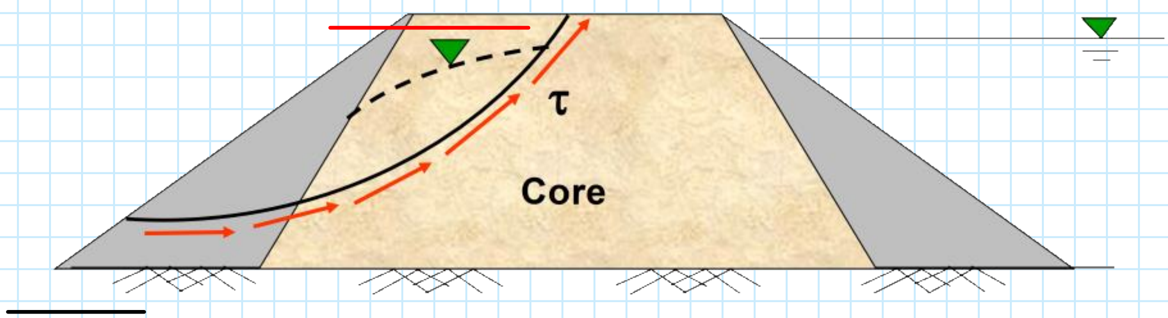
CD Test Application (from Nalin)

Monday, October 4, 2021 12:45 PM

1. Embankment constructed very slowly, in layers over a soft clay deposit



2. Earth dam with steady state seepage



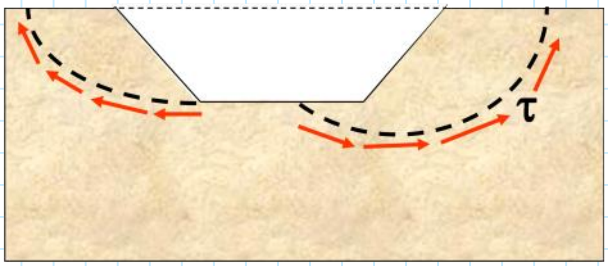
-

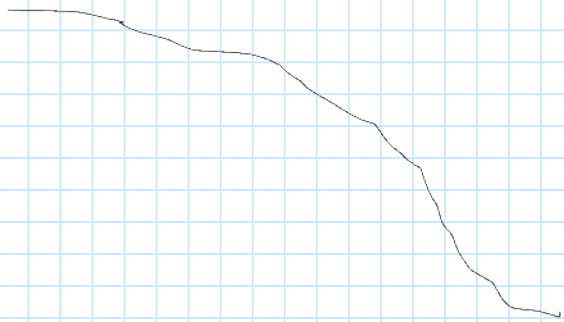
Would a CD test be appropriate if a steady-state seepage condition did not exist?

CD Test Application (from Nalin)

Monday, October 4, 2021 12:45 PM

3. Excavation or natural slope in clay



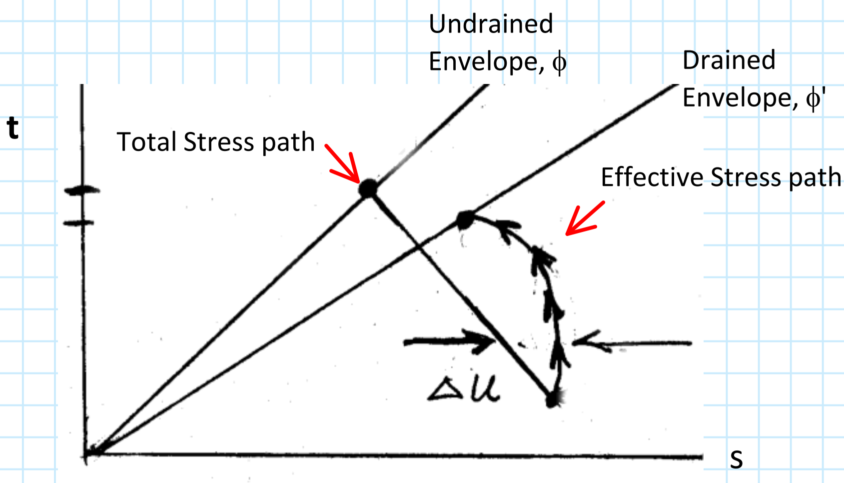


τ = In situ drained shear strength

Note: CD test simulates the <u>long term condition</u> in the field. Thus, c_d and ϕ_d should be used to evaluate the long term behavior of soils

In our textbook, we use the symbols c' and ϕ' to represent the drained cohesion intercept and drained friction angle.

Note that in the above case, the cut was made some time ago, so that any excess pore pressures in the slope has long dissipated. Also, there is no sudden change in the ground water conditions. Why is failure then reached after such a passage of time?



<u>Overconsolidated clay</u>

Initial excess pore water pressure, delta u, in slope was negative due to dilation as cut was made. This excess pore water pressure dissipates with time producing failure on the drained envelope, which has a lower strength.

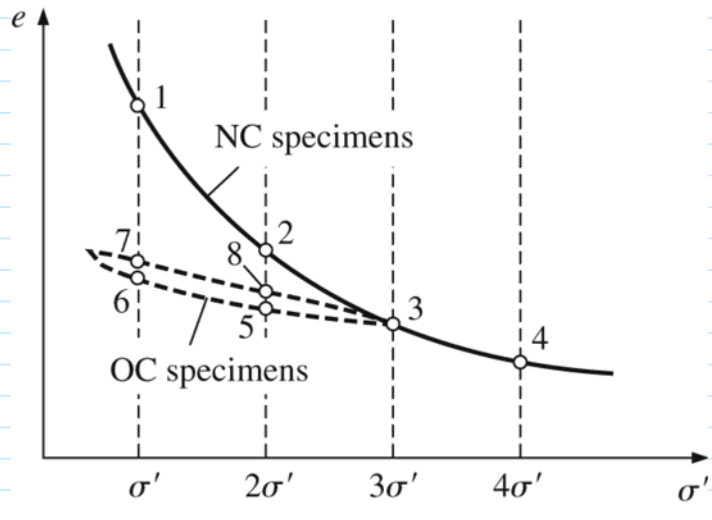
$$s = \tfrac{1}{2}(\sigma_1 + \sigma_3)$$

$$t = \tfrac{1}{2}(\sigma_1 - \sigma_3)$$

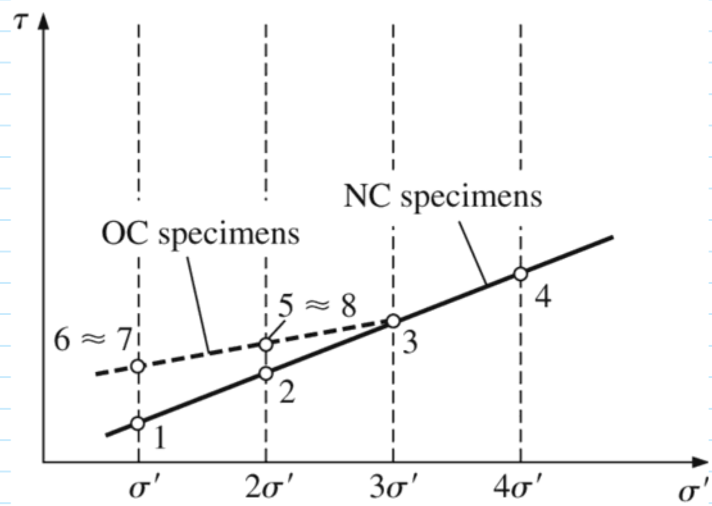
(See more on next page)

Effects of Overconsolidation (Drained Test)

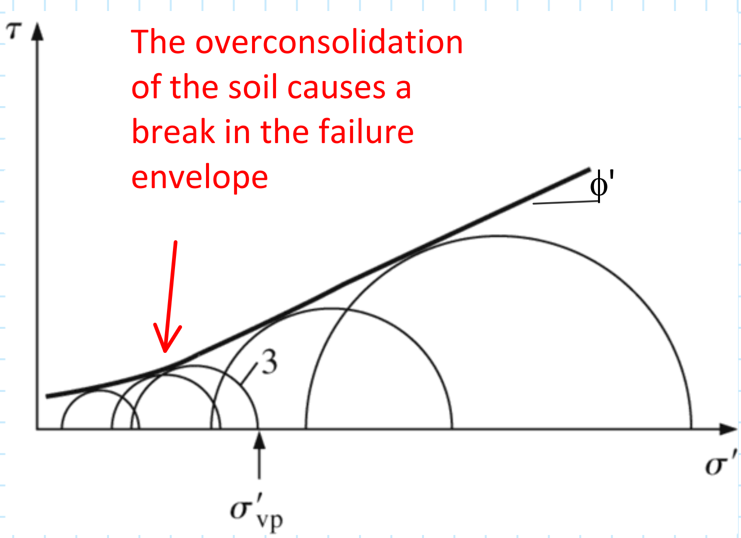
Wednesday, August 17, 2011 12:45 PM



(a)



(b)



CD Strength of Clay (Summary)

Monday, October 4, 2021 12:45 PM

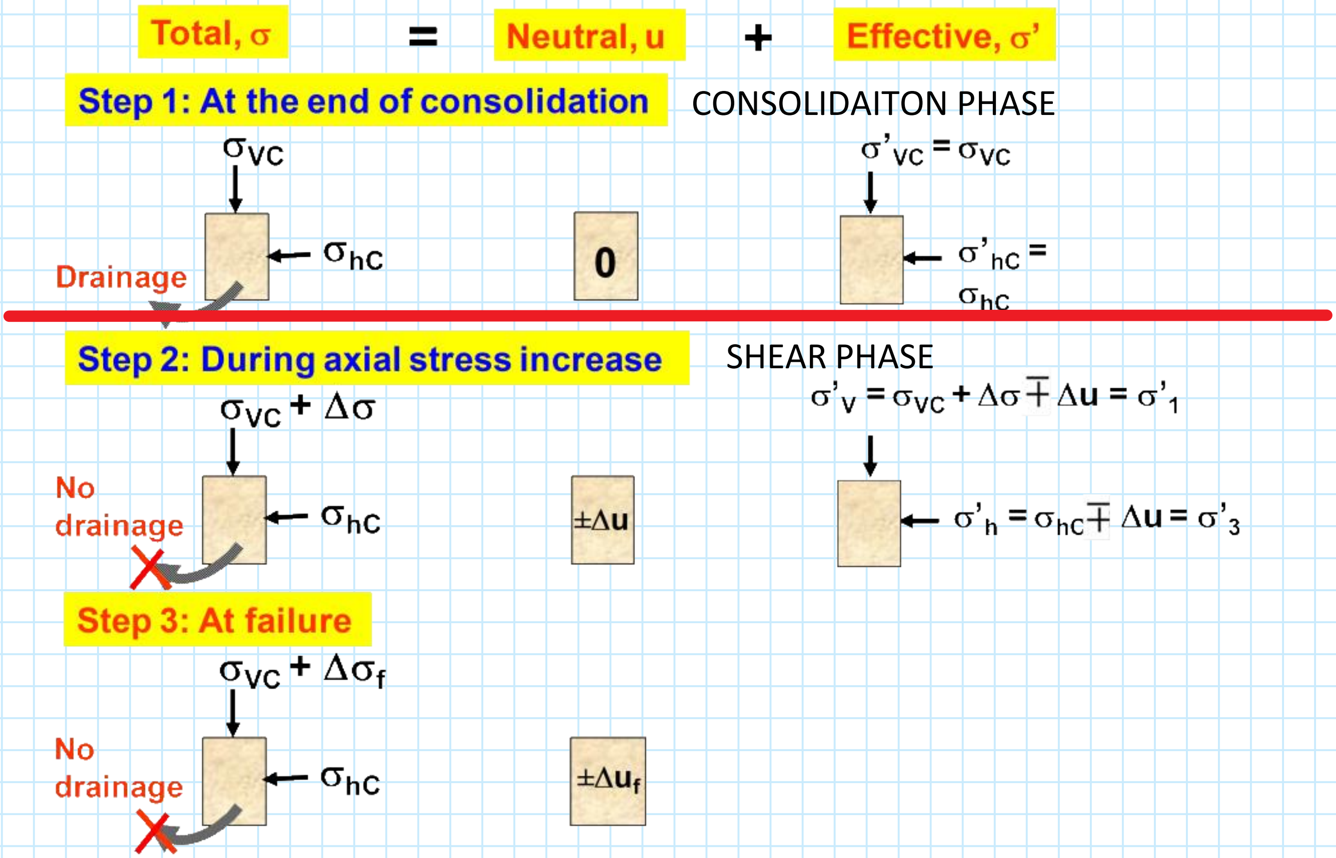
- Response of clay to drained loading is similar to sand
  - Normally consolidated (NC) clay is qualitatively similar to contractive sand
  - Overconsolidated (OC) clay is qualitatively similar to dilative sand

- The Mohr Coulomb envelope and critical state line of an NC clay passes through the origin.

- Differences between drain behavior in sands and clays
  - Clay, because of its cohesion is much easier to obtain an "undisturbed" sample. Sand, on the other hand is almost impossible to sample without causing significant disturbance, which is manifest by change in void ratio.
  - Clay critical state friction angles varies from 15 to 30 degrees
  - Sand critical state friction angle varies from about 28 to 36 degrees.
  - Clay are much less permeable and must be loaded very slowly to achieve a truly drained condition. In the laboratory, performing a drained test on a clay may take serval days or weeks to ensure that no excess pore pressure is built-up during shear.
  - <u>In many field situations, the rate of loading of clay is rapid enough to cause undrained behavior. Thus, drained test on clay have limited applicability for short-term loading construction situations.</u>
  - The effect of "stress history," such a preloading or preconsolidation is significant in clay and can affect is strength. OC clay can have an apparent cohesion which results from this preloading.
  - Clays that have been pre-sheared (e.g., landslide slip surface) may have a friction angle that is lower than the critical-state friction angle because of the alignment of clay particles in the direction of shearing. This friction angle is known as the residual friction angle, ϕ_r.

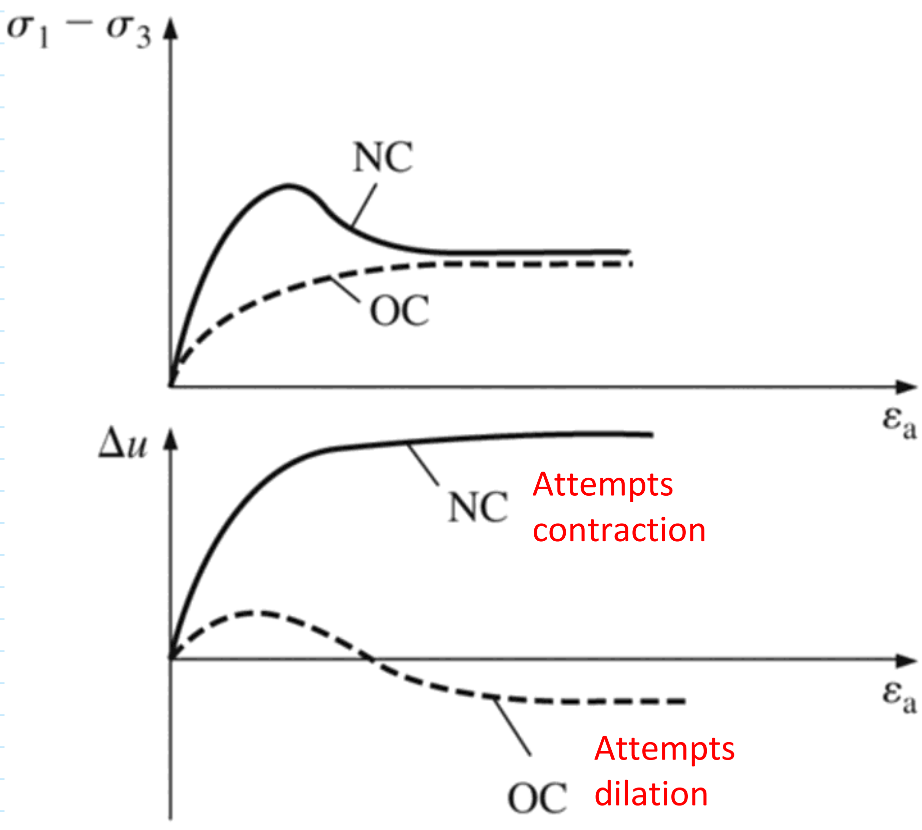
Consolidated Undrained (CU) Shear Test

Wednesday, August 17, 2011 12:45 PM



CU Shear

Wednesday, August 17, 2011 12:45 PM

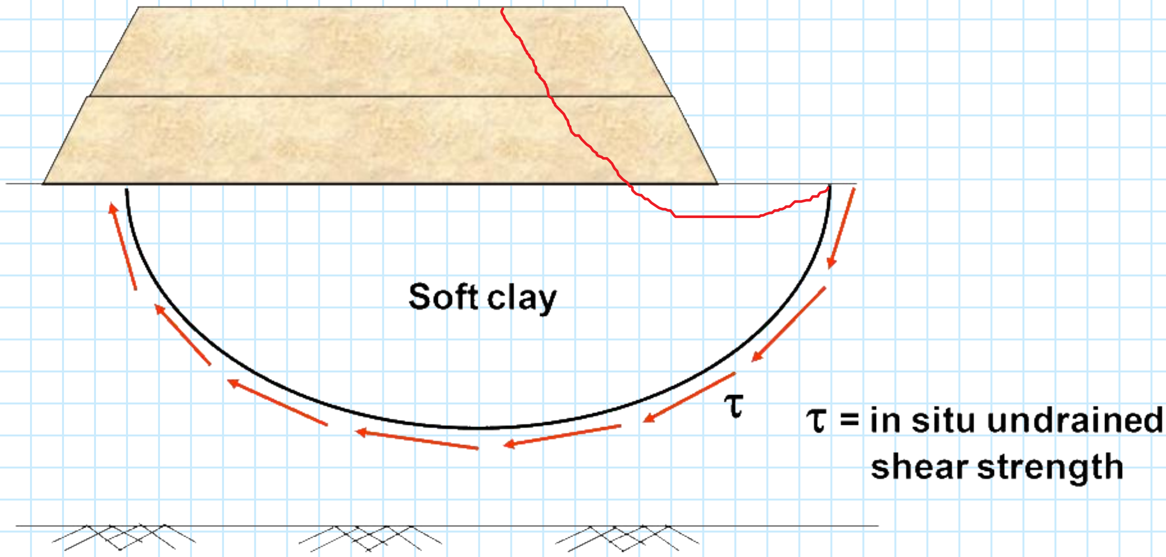


Note that because the drain value is closed during the shear phase of this test, excess pore water press, Δu is generated. NC clay generates positive pore water pressure at large shear; whereas, OC clay generates negative pore water pressure.

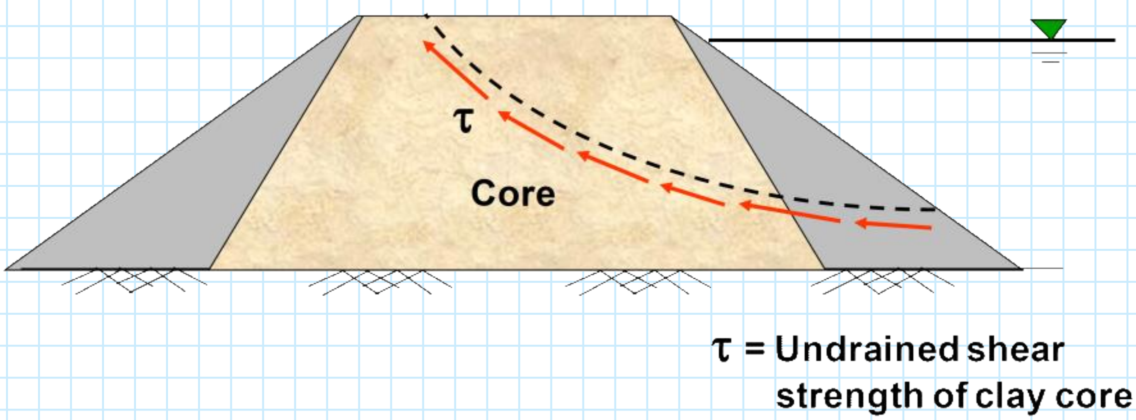
CU Test Application (from Nalin)

Wednesday, August 17, 2011 12:45 PM

1. Embankment constructed rapidly over a soft clay deposit



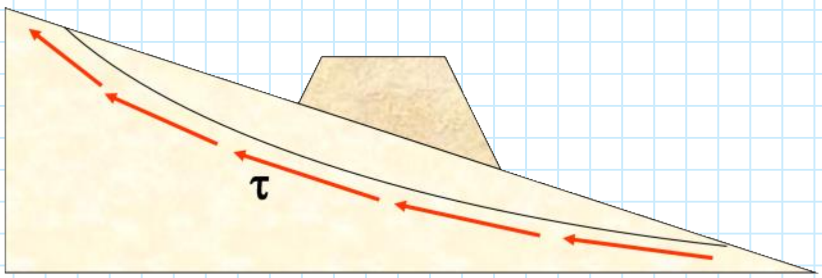
2. Rapid drawdown behind an earth dam



CU Test Application (from Nalin)

Wednesday, August 17, 2011 12:45 PM

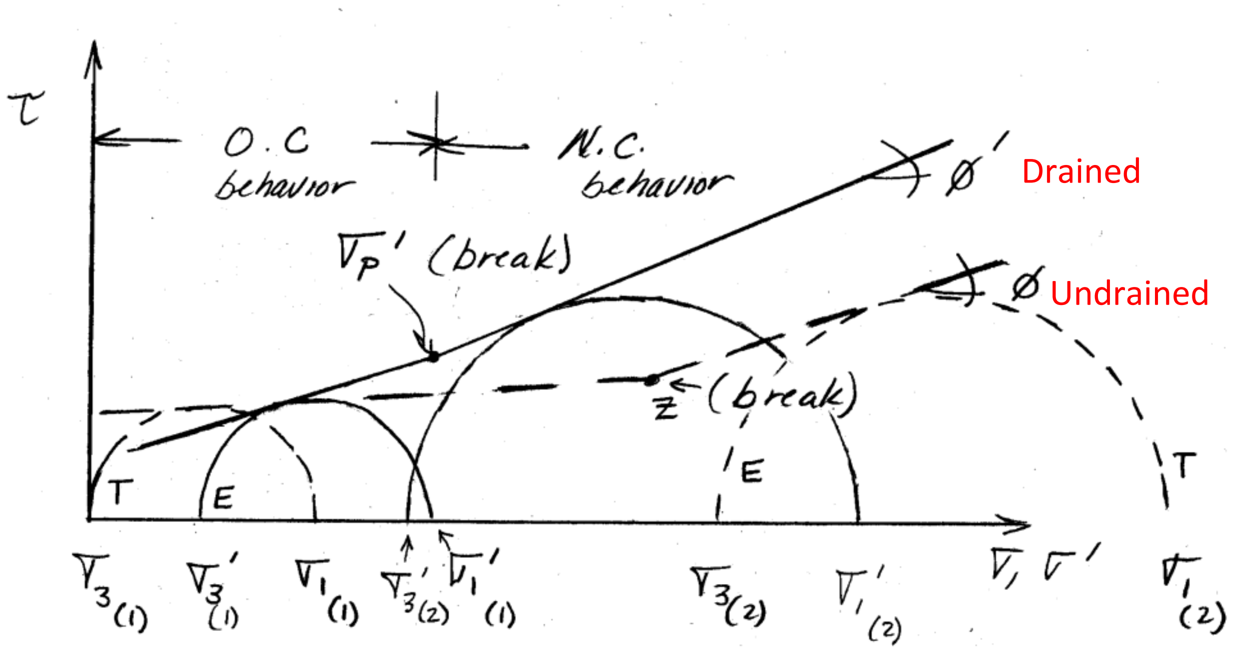
3. Rapid construction of an embankment on a natural slope



τ = In situ undrained shear strength

More on Effects of Overconsolidation (Undrained & Drained Tests)

Wednesday, August 17, 2011 12:45 PM



Skempton's Pore Water Pressure Equation and Parameters

Wednesday, August 17, 2011 12:45 PM

- During undrained shear, excess pore water pressure is generated because this pressure cannot escape from the triaxial cell.
- Skempton developed a method that attempts to predicted the magnitude of the generated excess pore water pressure in the sample.

$$\Delta u = B\Delta\sigma_3 + \bar{A}(\Delta\sigma_1 - \Delta\sigma_3)$$

Accounts for isotropic change in stress

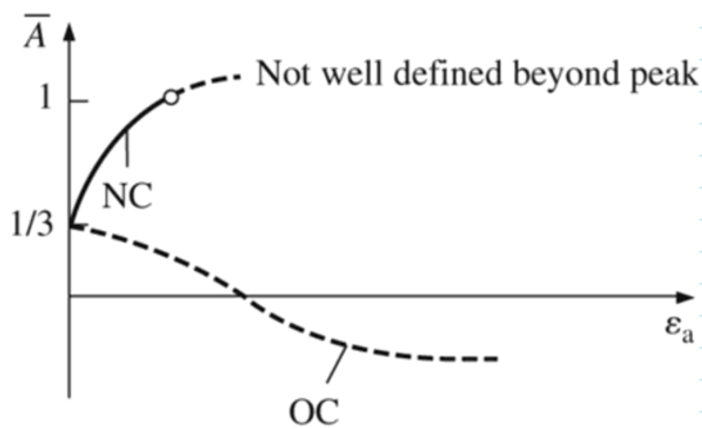
Accounts for deviatoric change in stress

- $B \approx 1$ (see table)

Table 6-3 Target *B* values for various materials

Material	*B*
Soft clay	1
Loose sand	>0.99
Dense sand	>0.95
Unsaturated soil	≪1
Concrete	≪1

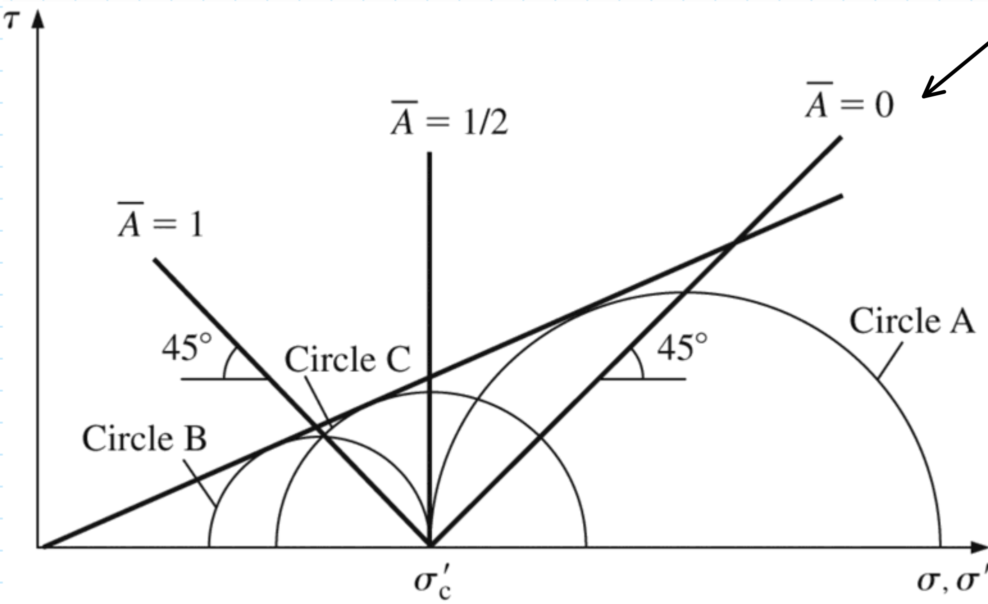
- $\bar{A}$ varies according to soil type and overconsolidation ratio (OCR)



- highly sensitive clays $0.75 < A < 1.5$
- NC clays $0.5 < A < 1$
- Lightly OC clays $0 < A < 0.5$
- Heavily OC clays $-0.5 < A < 0$

Skempton's Pore Water Pressure Equation and Parameters (cont.)

Wednesday, August 17, 2011 12:45 PM



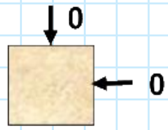
No pore pressure generation during shear

- Circle B is for $\bar{A}$ = 1 (Normally Consolidated)
- Circle C is for $\bar{A}$ = 1/2 (slightly Over Consolidated)
- Circle A is for $\bar{A}$ = 0 (heavily Over Consolidated)

Unconsolidated- Undrained test (UU Test)

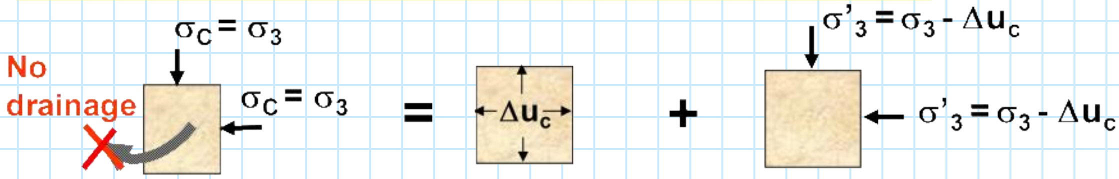
Step 1: Immediately after sampling

CONSOLIDATION PHASE (NOT DONE)



Step 2: After application of hydrostatic cell pressure

SHEAR PHASE



$$\Delta u_c = B \, \Delta\sigma_3$$

Increase of pwp due to increase of cell pressure

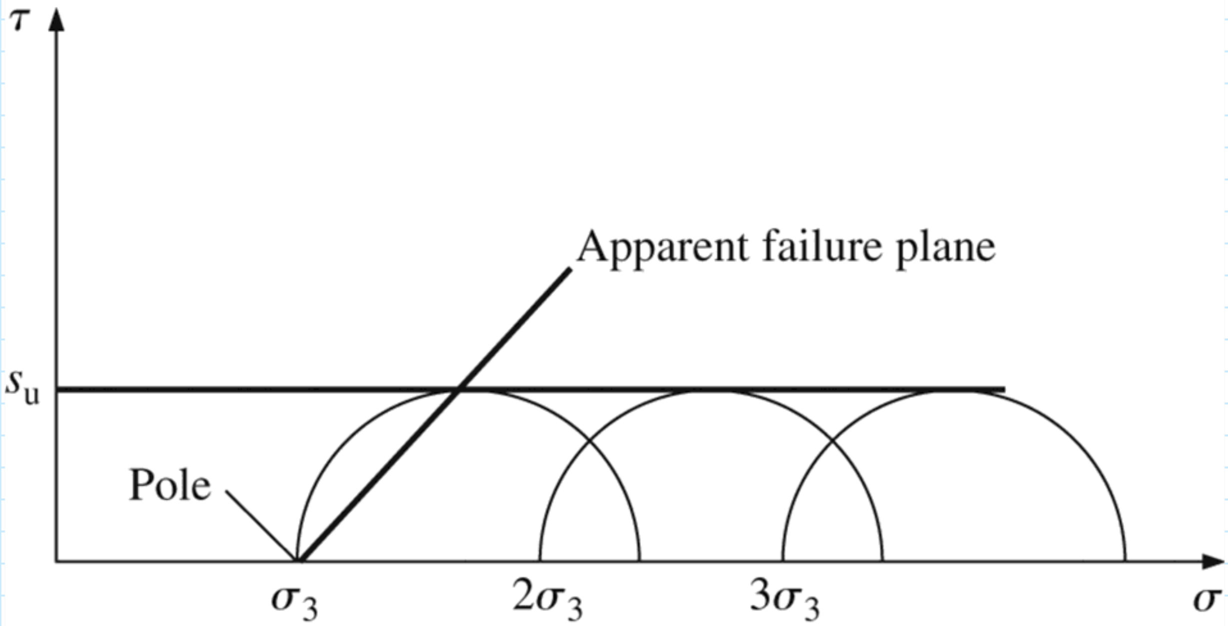
Skempton's pore water pressure parameter, B

Increase of cell pressure

Note: If soil is fully saturated, then B = 1 (hence, $\Delta u_c = \Delta\sigma_3$)

UU Shear

Wednesday, August 17, 2011 12:45 PM



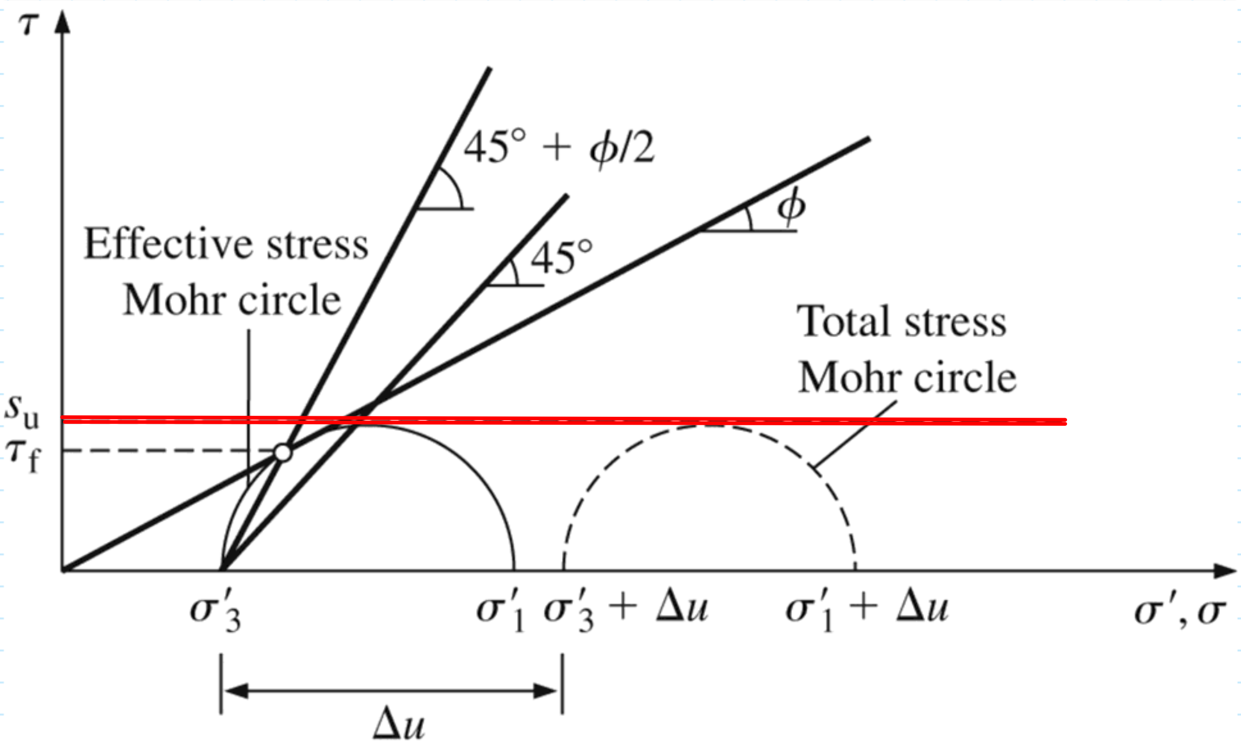
Explain why the failure envelope is horizontal for this test?

The above figure shows three Mohr's circles for differing confining stresses.

s_u is the undrained shear strength for the case where $\phi = 0$

UU Shear (cont.)

Wednesday, August 17, 2011 12:45 PM

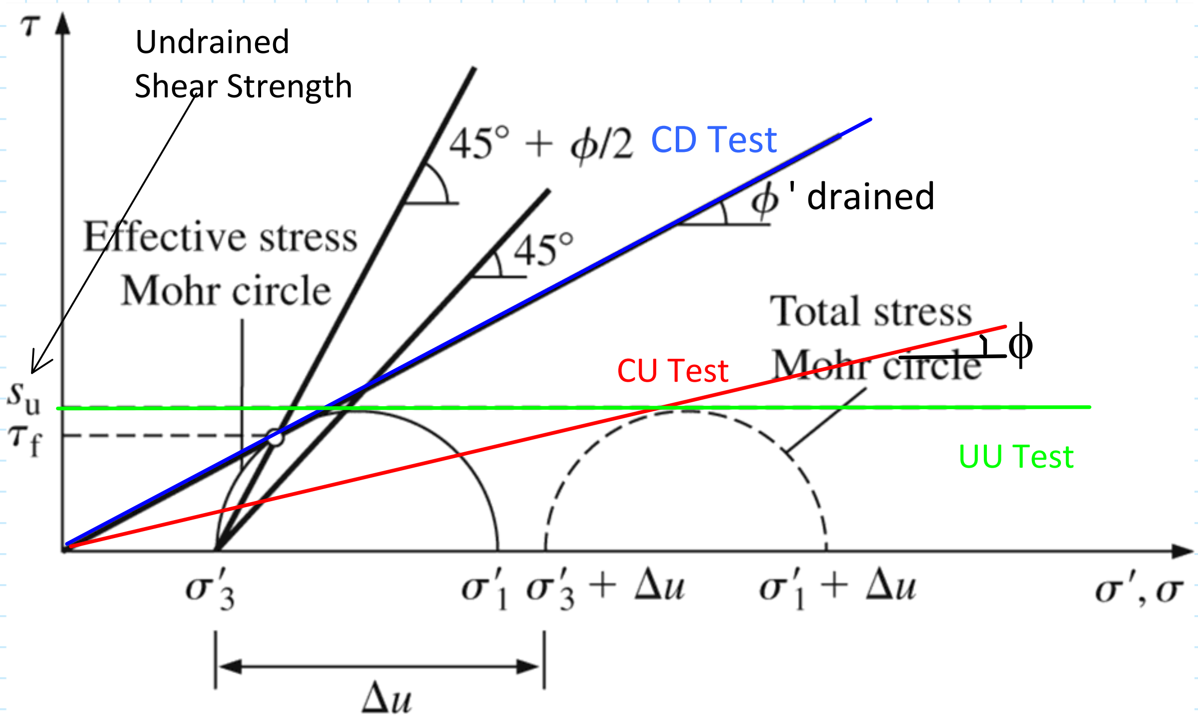


Why is the failure envelope for the UU test horizontal?

Hint, think about whether or not there is void ratio change during the consolidation and shear phase of this test and how the void ratio change or the lack thereof affects the shear strength during the shear phase of the test.

Comparison of Tests

Wednesday, August 17, 2011 12:45 PM



Correlations for Predicting Undrained Shear Strength, Su

Wednesday, August 17, 2011 12:45 PM

- NC Clay (Skempton 1957)

  $s_{u\,NC}/\sigma_v' = 0.11 + 0.0037(PI)$ where PI is the plasticity index

- NC Clay (Wroth 1984)

  $s_u/\sigma_v' = \phi_c/100$

- NC and OC Clay (Ladd 1977)

  $$\boxed{(s_{u\,OC}/\sigma_v') = (s_{u\,NC}/\sigma_v'_{NC})*OCR^{0.8}}$$

σ_v' is the current effective vertical stress, not the maximum past effective vertical stress.

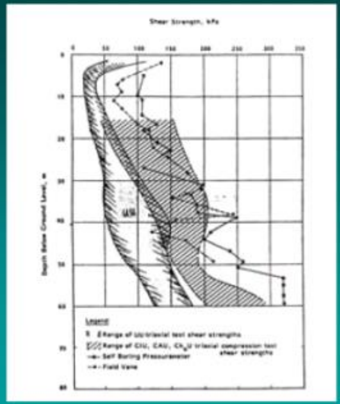
$s_{u\,NC}/\sigma_v'_{NC}$ varies from about 0.2 to 0.3 depending on the type clay and direction of shearing.

See following attachments

Evaluation of Undrained Shear Strength from In-Situ Tests

Paul W. Mayne
Georgia Institute of Technology

Mix & Match of Undrained Shear Strengths from different tests



Pentre, UK Site (Lambson, et al. 1996, Large Scale Pile Tests in Clay, Thomas Telford)

Undrained Shear Strength ($c = c_u = s_u$)

Undrained Strength Anisotropy and Effects of Strain Rate, Boundary Conditions, and Initial Stress State

COMPARISON OF UNDRAINED STRENGTHS FROM DIFFERENT LABORATORY AND IN-SITU TESTS ON NORMALLY-CONSOLIDATED BOSTON BLUE CLAY

(After Ladd, 1965 JSMFD ASCE; Kinner and Ladd, 1973 - 8th ICSMFE; Ladd, et al. 1977 - 9th ICSMFE; Ladd, et al. 1980 - FHWA Report on SBPMT).

TEST METHOD	s_u/σ_{vo}'
Self-boring pressuremeter (SBP)	0.42
Plane Strain Compression (PSC)	0.34
Triaxial Compression (CK_oUC)	0.33
Unconsolidated Undrained (UU)	0.275
Field Vane Shear Test (FV)	0.21
Direct Simple Shear (DSS)	0.20
Plane Strain Extension (PSE)	0.19
Triaxial Extension (CK_oUE)	0.16
Unconfined Compression (UC)	0.14

Classical Methods for s_u in Clays & Silts

IN-SITU TEST	REFERENCES	COMMENTS
VST: $s_{uv} = 6T/(7\pi D^3)$ for H/D = 2	Chandler (1988, ASTM 1014). Correction: $s_{u(field)} = \mu S_{uv}$	Static equilibrium analysis. Empirical: $\mu = 2.5(PI)^{-0.3} \leq 1.1$
PMT: $s_{upmt} = dp/d(\ln \varepsilon_v)$; $s_{upmt} = (p_L - p_o)/N_c$	Windle & Wroth (1977, ICSMFE). Baguelin et al. (1972, JSFMD)	Cavity expansion theory. Empirical bearing factor $N_c = 5.5$
SPT: $s_{u(spt)} = f_1 N_{60}p_a/100$	Stroud (1974, ESOPT-1). Stroud (1988, PTUK)	Empirical: $f_1 = 4.5$ for PI = 50. Empirical: $f_1 = 5.5$ for PI = 15
CPT: $s_{ucpt} = (q_c - \sigma_{vo})/N_c$	Meyerhof (1961). Vesic (1977, NCHRP 42)	Limit plasticity theory. Cavity Expansion Theory
$s_{ucpt} = (q_t - \sigma_{vo})/N_{kt}$	Aas, et al. (1986, ASCE GSP 6). Bearing factor is test-dependent	Corrected cone tip resistance, q_t. N_{kT} = 10 (TC) = 15 (DSS) = 20 (TE)
	Wroth (1988, ISOPT). Yu & Mitchell (1998, JGGE)	Finite Element Analysis. Strain Path Method
PCPT: $s_{upcpt} = \Delta u/N_\Delta$	Tavenas, et al. (1982, ESOPT). Campanella/Robertson (1983). Mayne & Holtz (1988, S&F)	$N_{\Delta u}$ = 7.9 (uncorrected vane). Charts: N_Δ = f(I_r, A_f, u_1 or u_2). Cavity expansion + critical state where $7 < N_{\Delta u} < 9$
DMT: $s_{udmt} = 0.22\,\sigma_{vo}'(\tfrac{1}{2}K_D)^{1.25}$; $s_{udmt} = (p_1 - u_o)/10$; $s_{udmt} = d_s\sigma_{vo}'(0.5K_D)^{1.25}$	Marchetti (1980, JGE). Schmertmann (1981). Lacasse & Lunne (1988, ISOPT)	Based on mix of UU, UC, VST. Cavity expansion theory. Empirical and test-dependent: TC: d_s = 0.20; VST: d_s = 0.19; DSS: d_s = 0.14
PLT: $s_{upl} = q_{pl}/9.18$	Meyerhof (1951, Geotechnique)	Limit plasticity theory
SPLT: $s_{uspl} = (q_{spl} - \sigma_{vo})/9.3$	Kay (1982, Ground Engrg.)	Limit plasticity theory

Undrained Shear Strength

- Classical interpretation from CPT in clays: undrained shear strength = $c_u = s_u$

$$s_u = \frac{q_t - \sigma_{vo}}{N_{kt}}$$

- Which s_u ? N_k = 15

Undrained Shear Strength of Clays

- Theoretical formulation from critical-state soil mechanics for intact clays (Cambridge Univ.):

$$s_u/\sigma_{vo}'{}_{DSS} = \tfrac{1}{2}\sin\phi' OCR^{\Lambda}$$

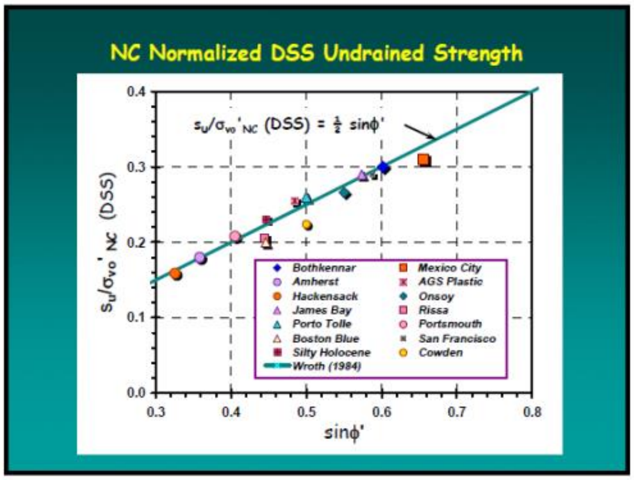
where $\Lambda \approx 1 - C_s/C_c$

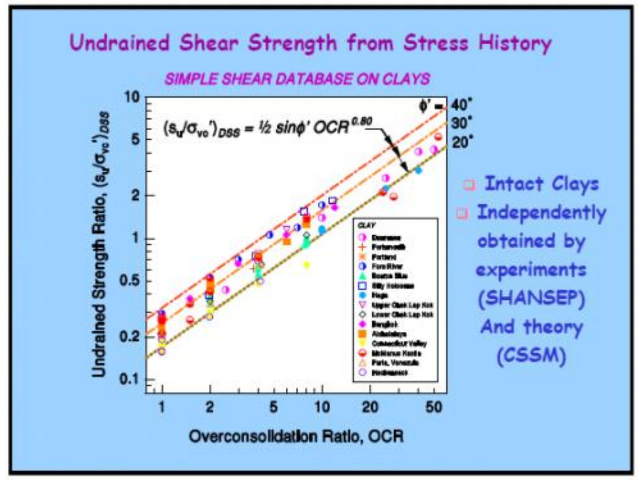
- Experimental work by MIT with simple shear device:

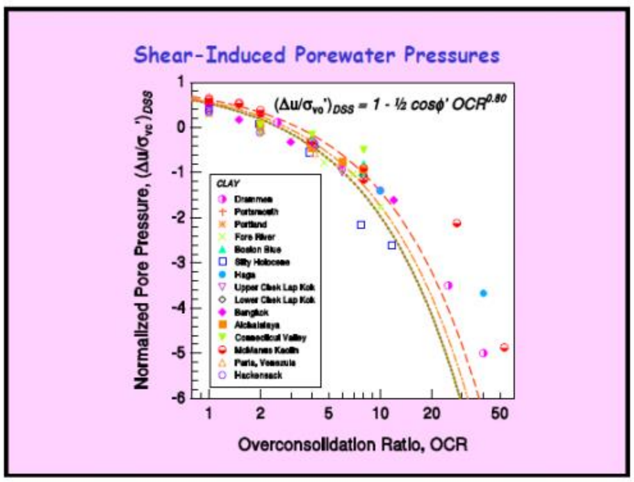
$$s_u/\sigma_{vo}'{}_{DSS} = 0.23\ OCR^{0.8}$$

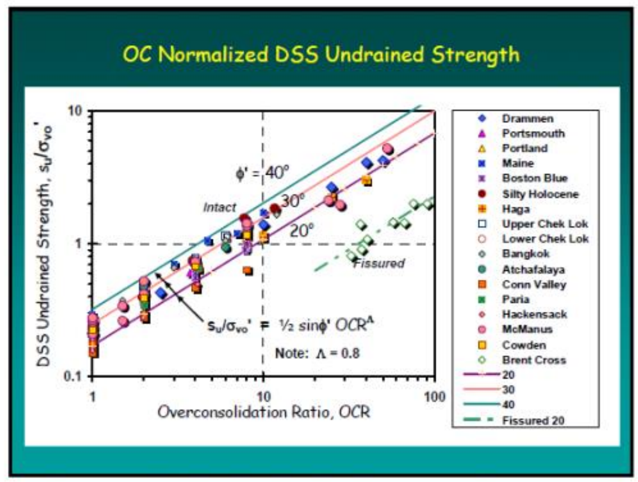
- Low OCRs (assume ϕ' = 28°): $s_{uDSS} \approx 0.2\ \sigma_p'$

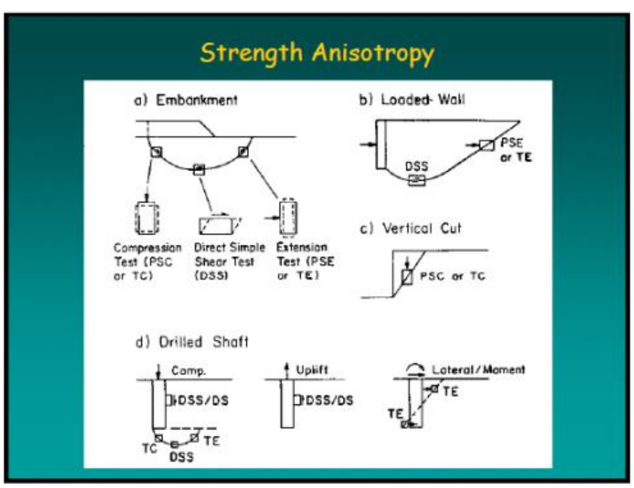
Evaluation of Undrained Shear Strength (cont.)

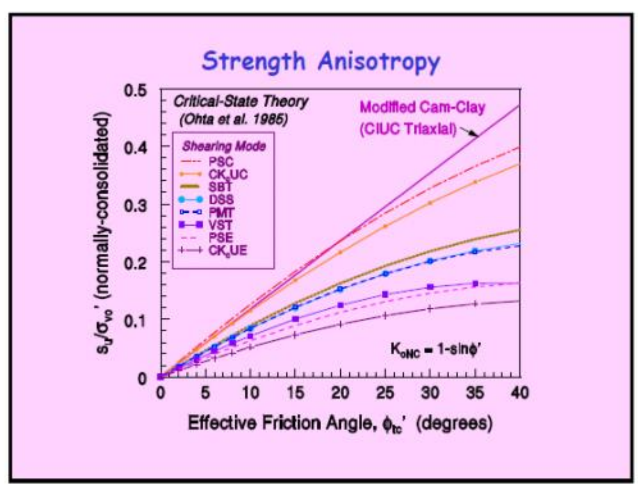




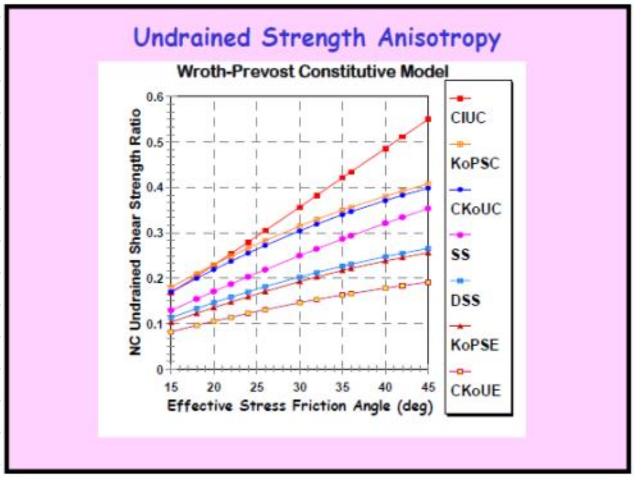


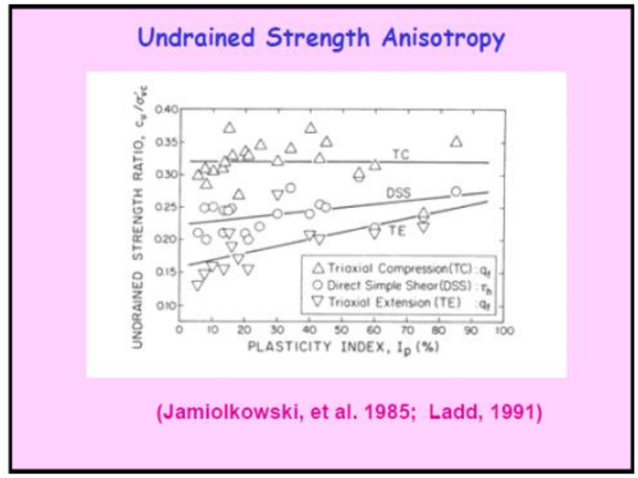


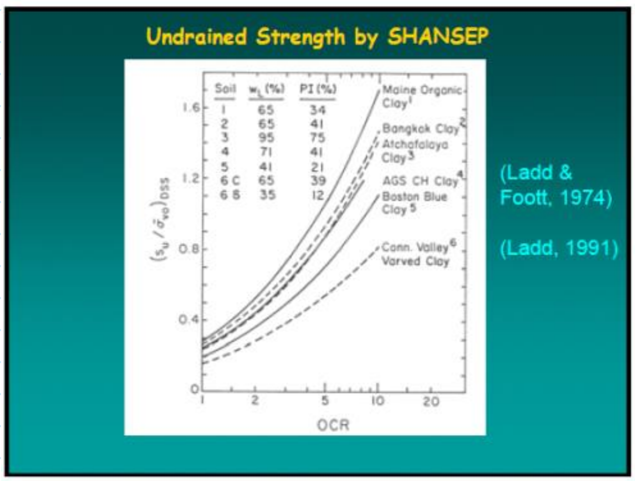


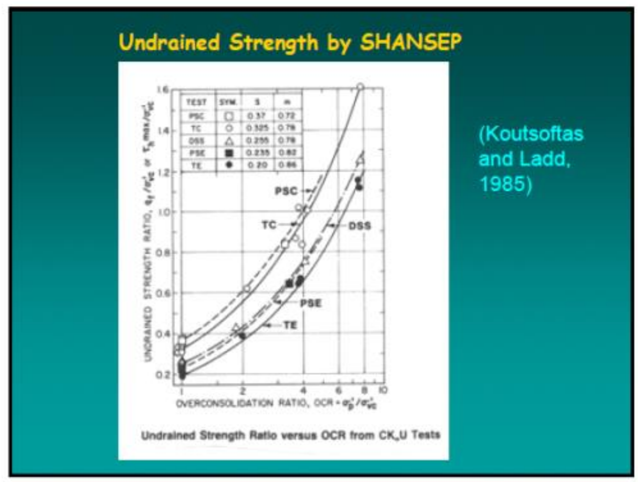


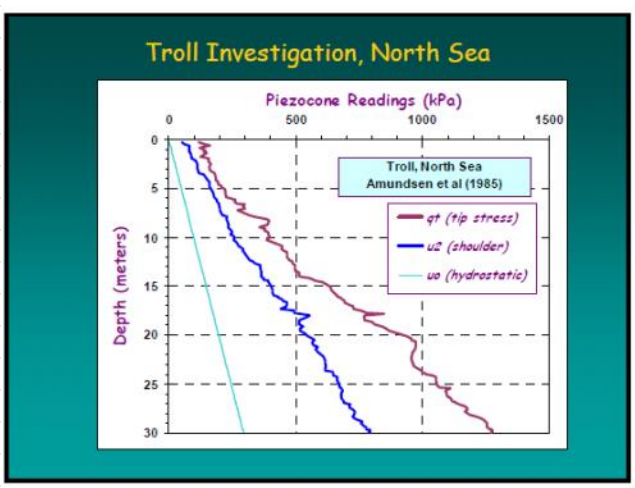
Evaluation of Undrained Shear Strength (cont.)

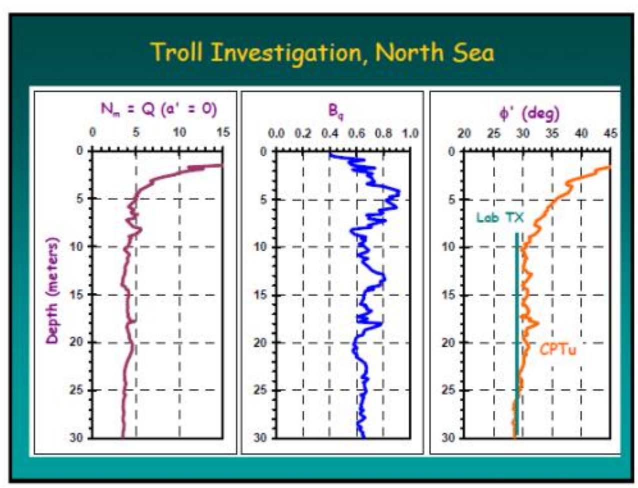




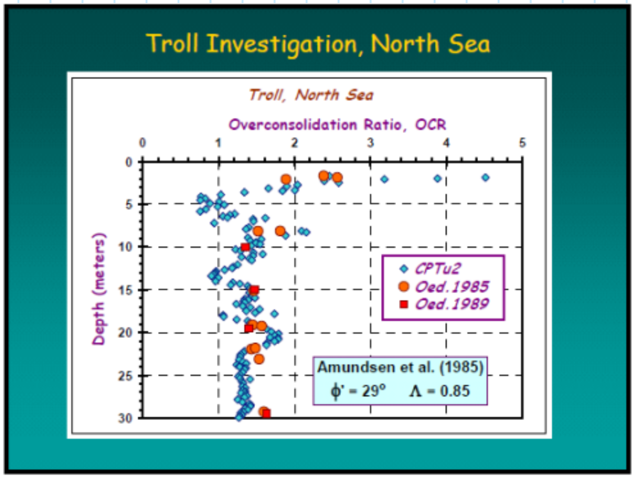


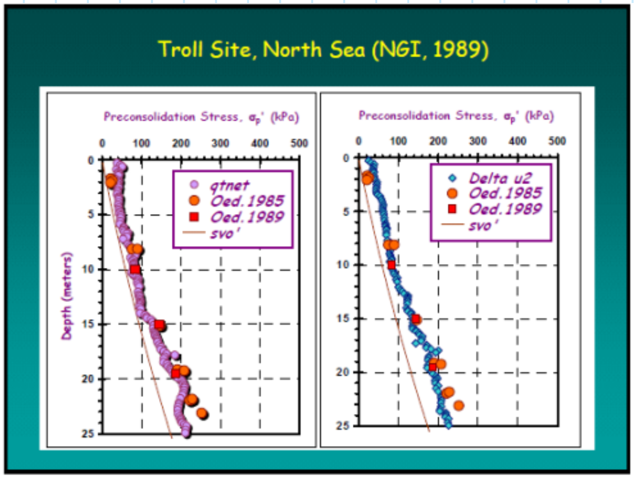


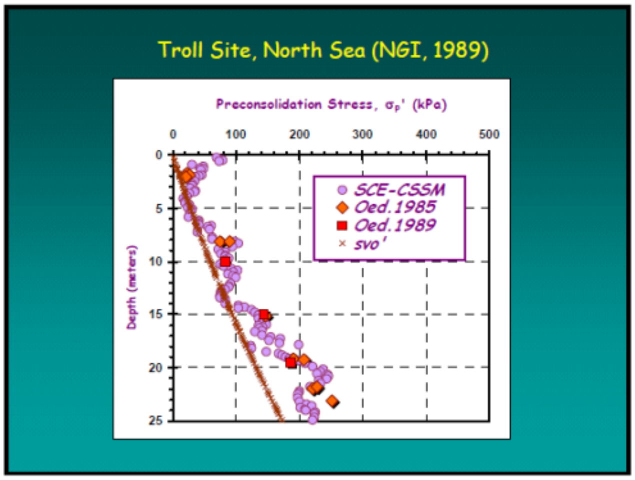


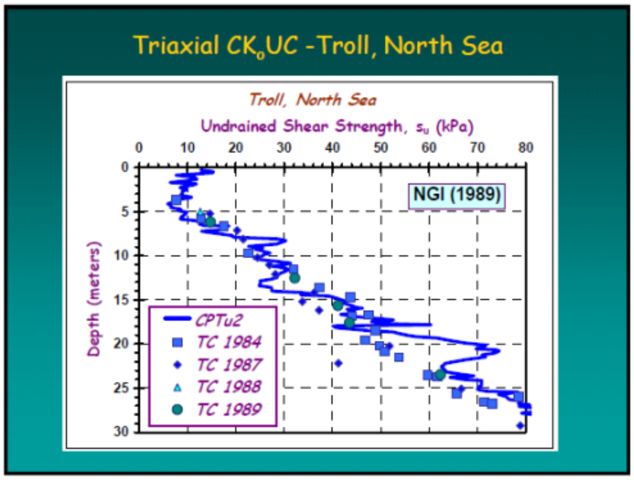


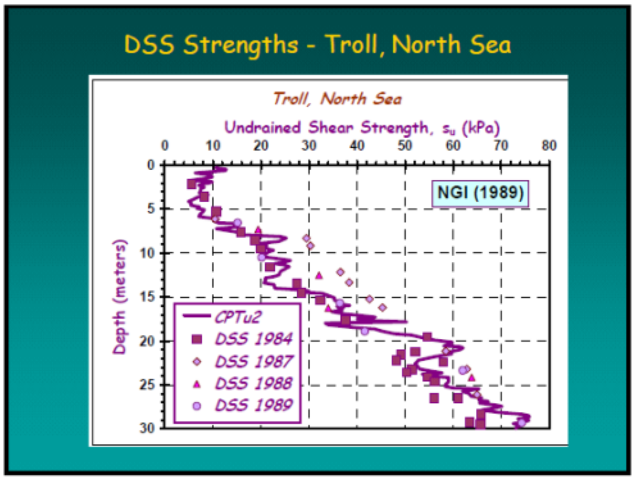
Evaluation of Undrained Shear Strength (cont.)

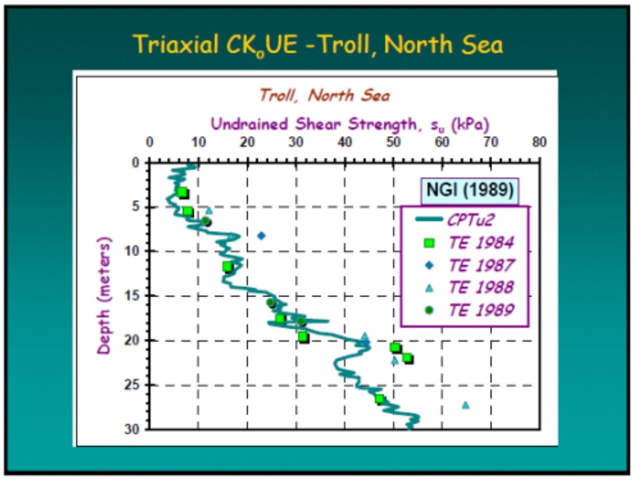




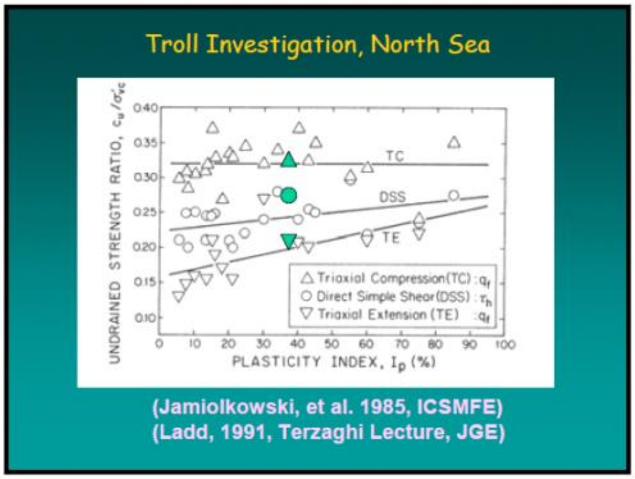


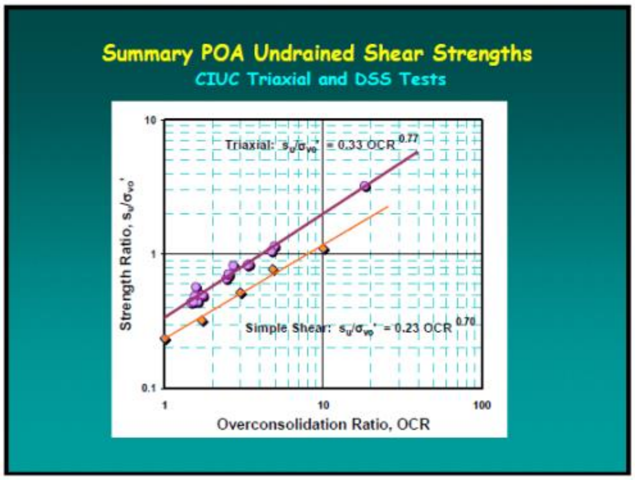


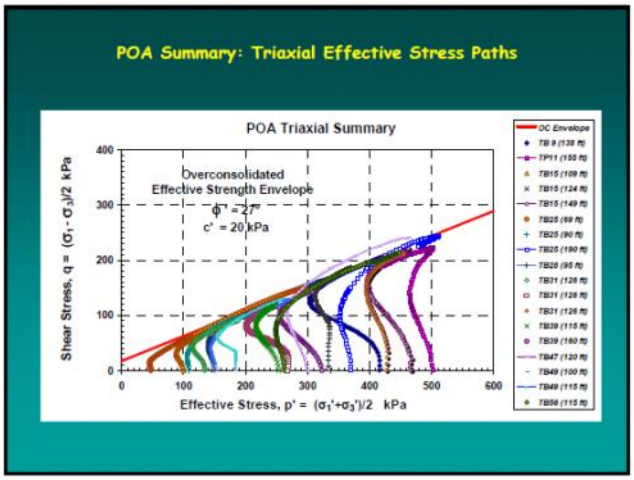


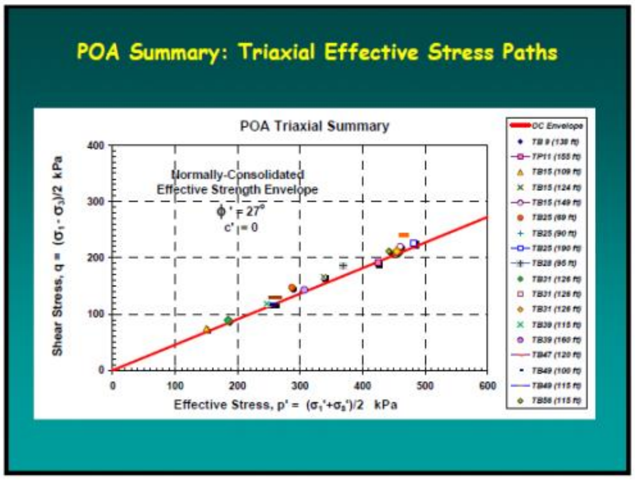


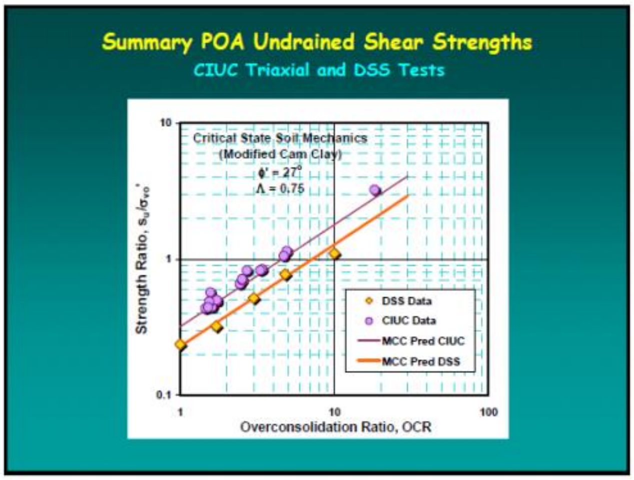
Evaluation of Undrained Shear Strength (cont.)



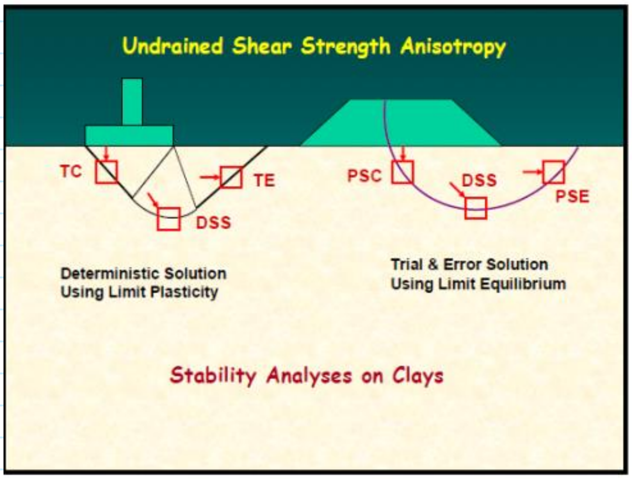


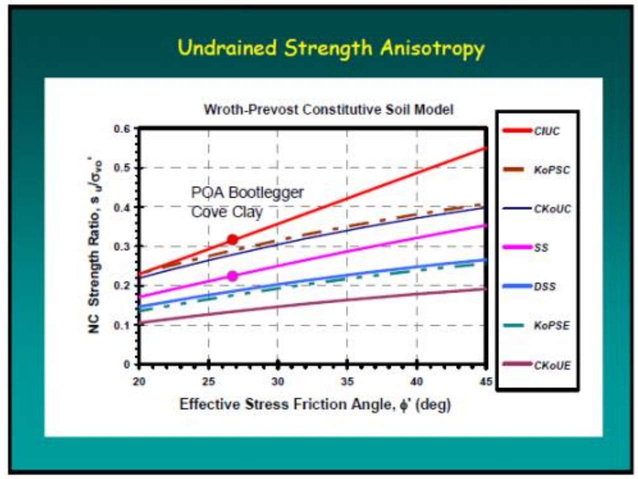


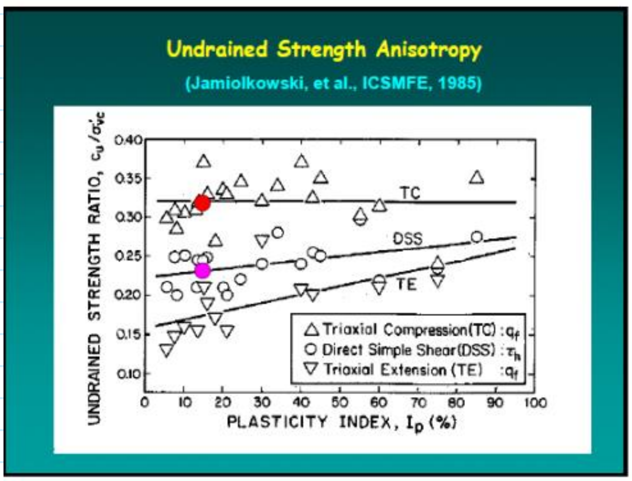


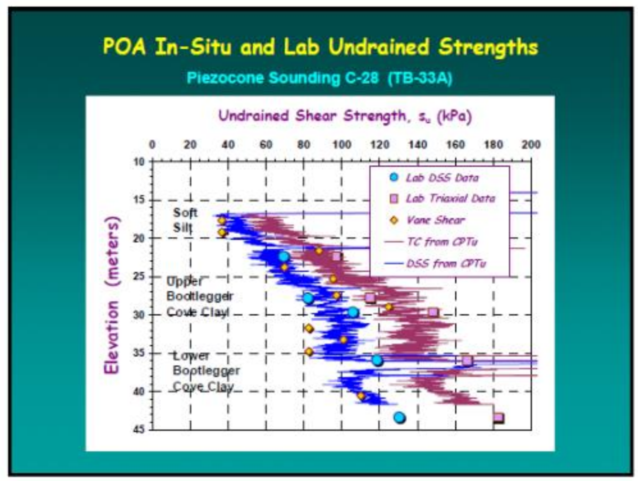


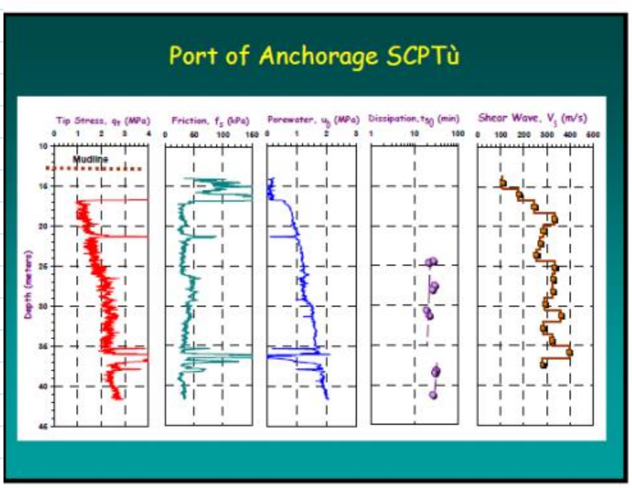
Evaluation of Undrained Shear Strength (cont.)

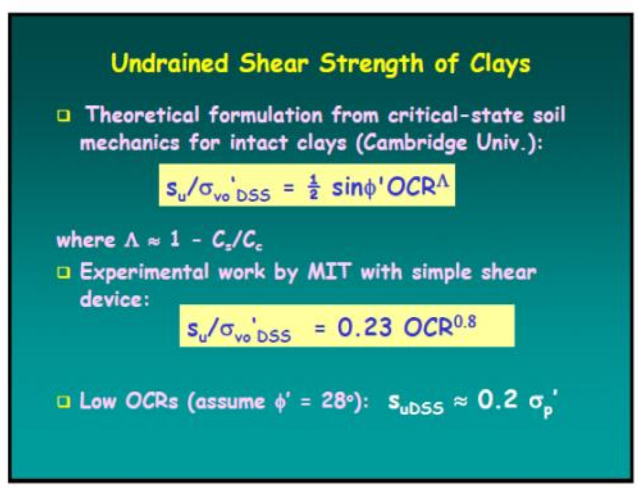












Settlement of Shallow Foundations

Wednesday, August 17, 2011 12:45 PM

Reading Assignment

- Ch. 9 Lecture Notes
- Ch. 8 and 9 (Salgado)

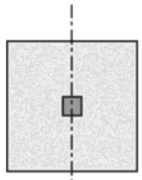
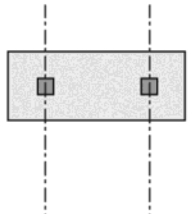
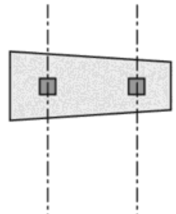
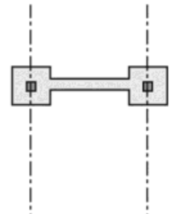
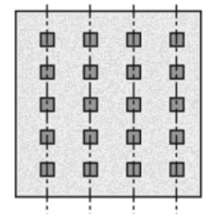
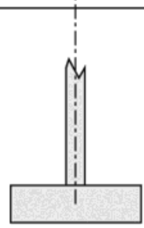
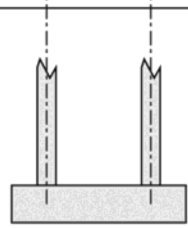
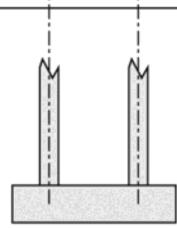
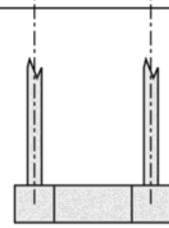
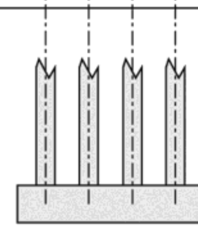
Other Materials

Homework Assignment 7

- Problems 9-5, 9-6, 9-8 (use Schmertmann's method), 9-16 (b) (use Myerhof's method)

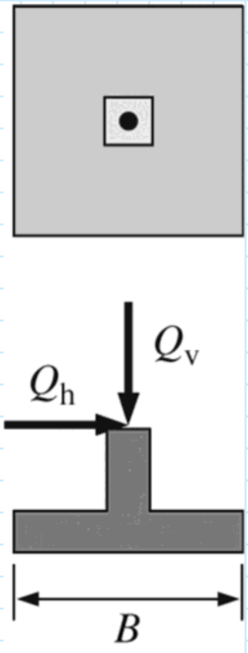
Introduction to Shallow Foundations

Wednesday, August 17, 2011 12:45 PM

Foundation type	Isolated footing	Combined footing		Strap footing	Mat foundation
		Rectangular	Trapezoidal		
Plan view					
Cross section					
Applicability	Relatively high ratio of soil resistance to structural loads	Columns too closely spaced Support of column too close to obstruction or property line (for column spacing $\leq \sim 7$ meters)	Same as rectangular but with large load difference	Support of column too close to obstruction or property line (for column spacing $> \sim 7$ m)	Relatively low ratio of soil resistance to structural loads

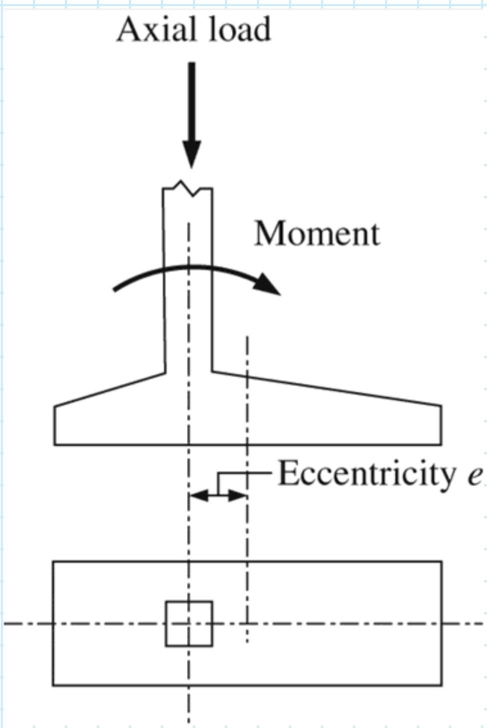
Loadings to Shallow Foundations

Wednesday, August 17, 2011 12:45 PM



Square footing

(Note that any horizontal loading to the footing that is not through the center of the footing causes an overturning moment.)

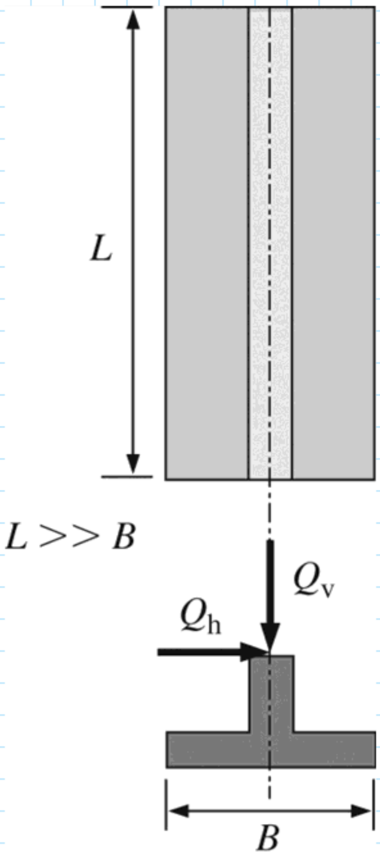


Rectangular footing

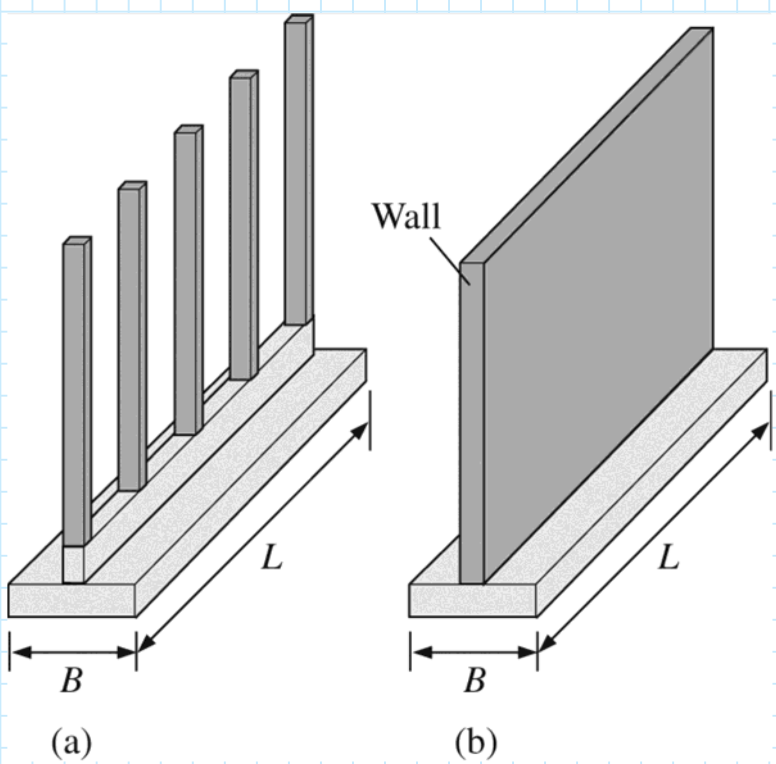
(Note that the moment is shown in this figure. The overturning moment causes eccentricity in the loading configuration that effects the resultant load and the pressure distribution that develops along the base of the footing.

Loadings to Shallow Foundations (cont.)

Wednesday, August 17, 2011 12:45 PM



Strip footing



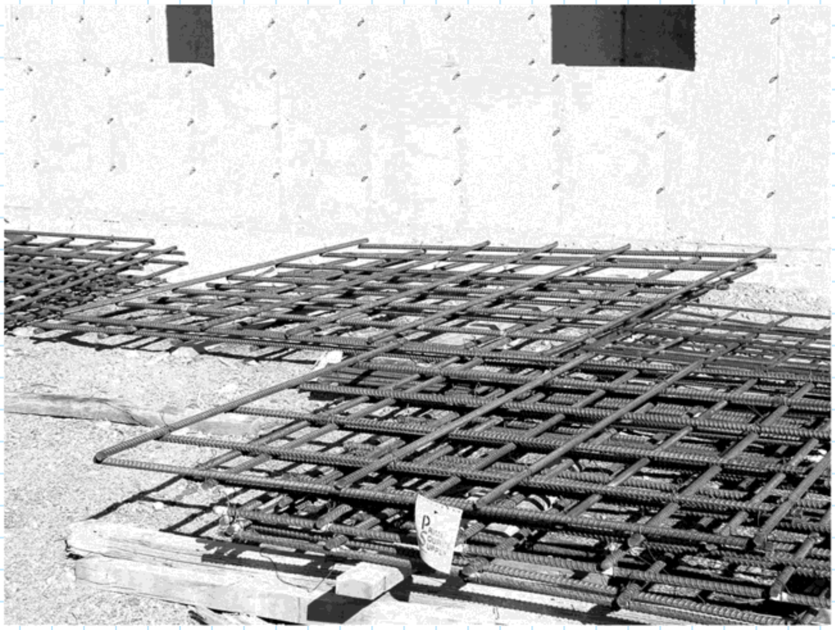
Strip footing with columns and strip footing with continuous wall panel

Construction of Footings

Wednesday, August 17, 2011 12:45 PM

Excavation and compaction of bearing level for foundation.

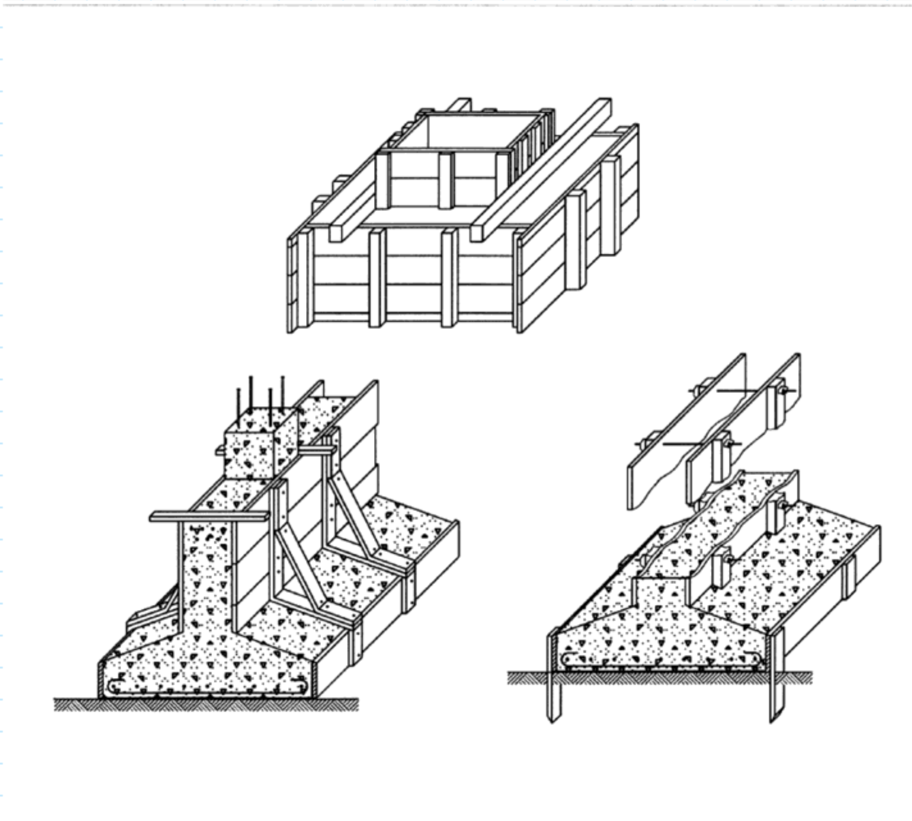
Note that the footing is placed below the surface at a depth below frost penetration.



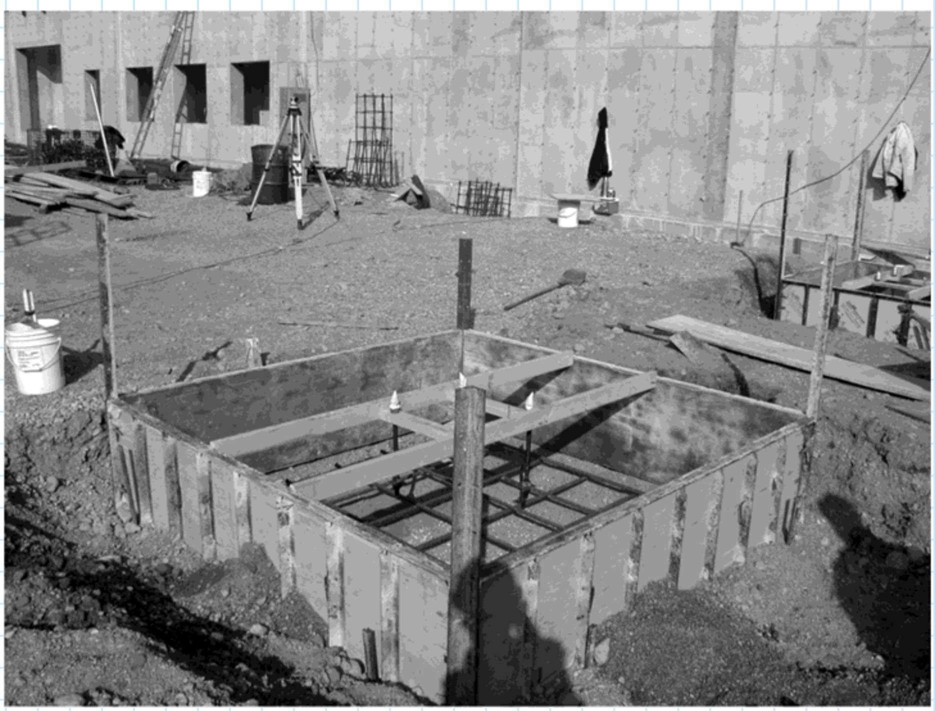
Reinforcement for footing.

Construction of Footings (cont.)

Wednesday, August 17, 2011 12:45 PM



Forming of footings



Forming of footings

Construction of Footings (cont.)

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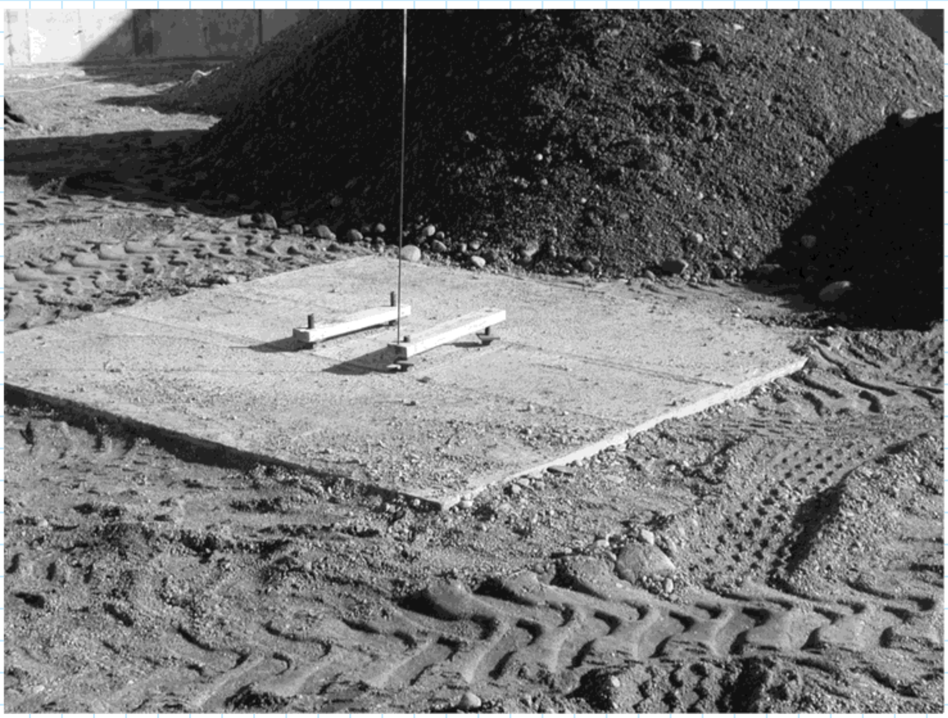
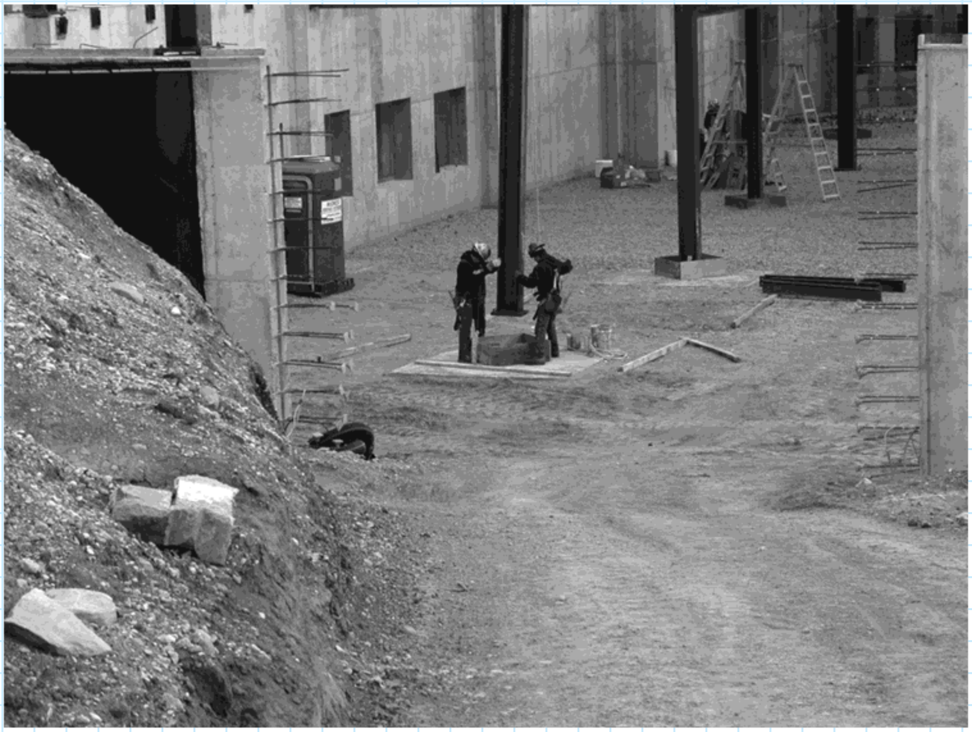
Pouring of concrete



Finished footing with anchor bolts

Construction of Footings (cont.)

Wednesday, August 17, 2011 12:45 PM

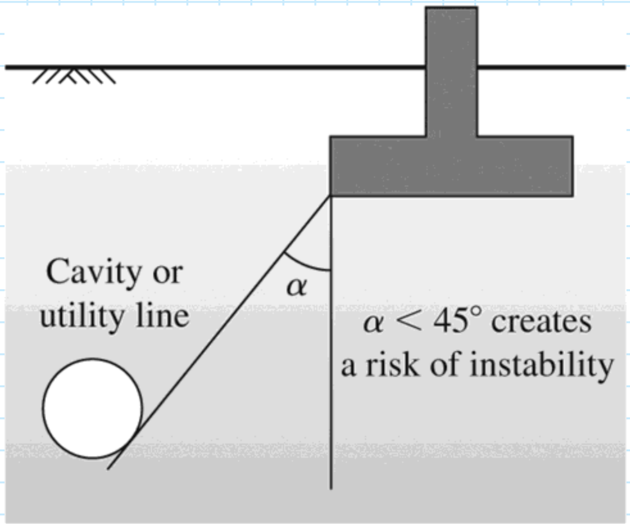


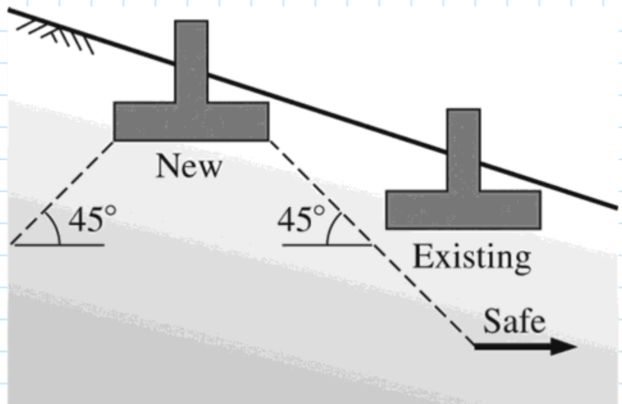
Attaching steel columns to anchor bolts

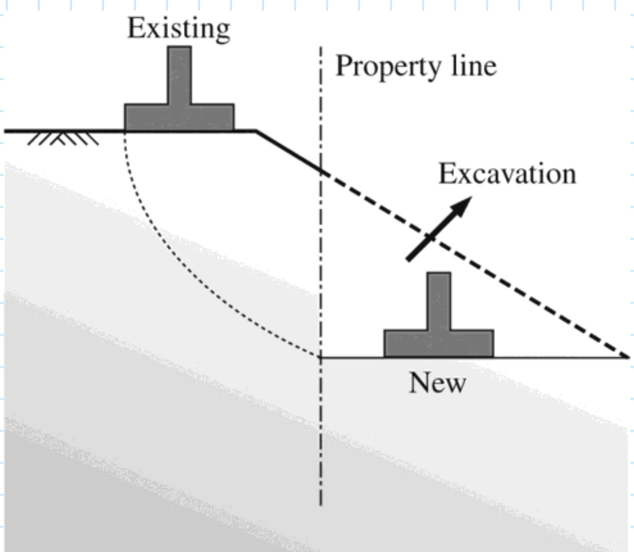
Tightening of nuts on anchor bolts

Proximity to Buried Utilities and Other Foundations

Wednesday, August 17, 2011 12:45 PM







Comparison of Settlement of Sands and Clays

Wednesday, August 17, 2011 12:45 PM

$\Delta u = 0$ negligible

Sands: $w_{total} = w_{immediate} + \cancel{w_{long-term}}$

Settlement due to both distortion and compression (volumetric) of the soil mass

- Large hydraulic conductivity
- Excess pore pressure Δu dissipates fast

$\Delta u > 0$

Clays: $w_{total} = w_{immediate} + w_{long-term}$

- Settlement due to distortion only (no volume change)

Settlement due to consolidation (volume change) and creep

- Small hydraulic conductivity
- Excess pore pressure generation
- Long-term settlement occurs as result of the dissipation of the excess pore pressures generated by the load application

Table 9-1 Sources of settlement of foundations

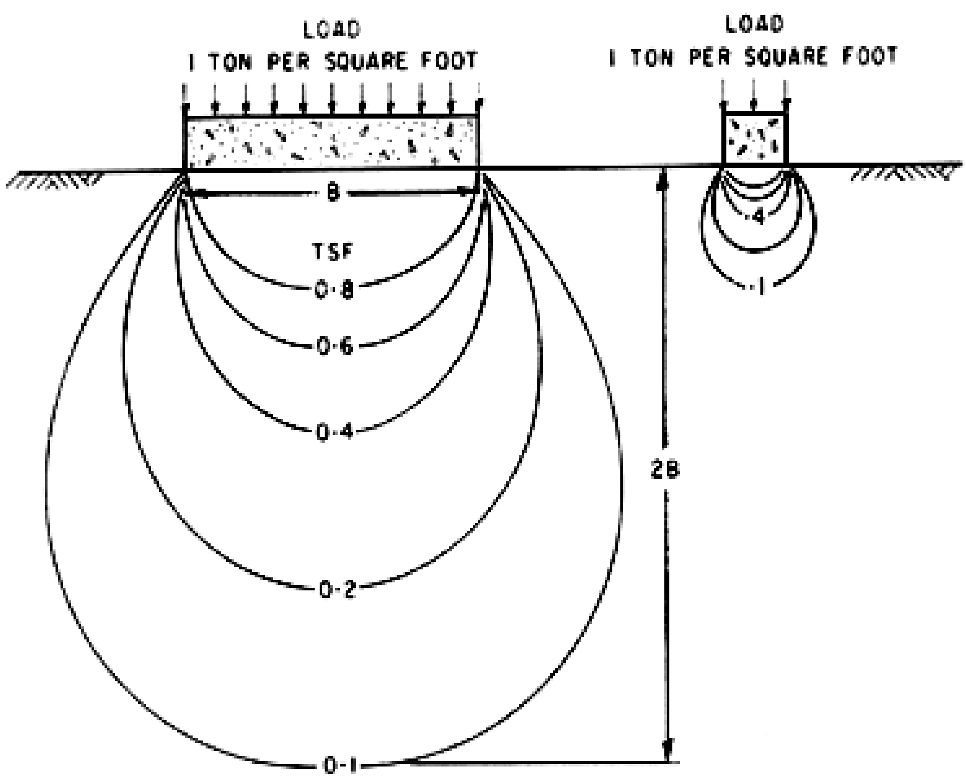
Settlement	Type of soil: Clay	Type of soil: Sand
Short-term or immediate	Distortion	Distortion Consolidation
Long-term or delayed	Consolidation Secondary	Secondary (small to negligible)

Note that this lecture deals with calculating the immediate settlement.

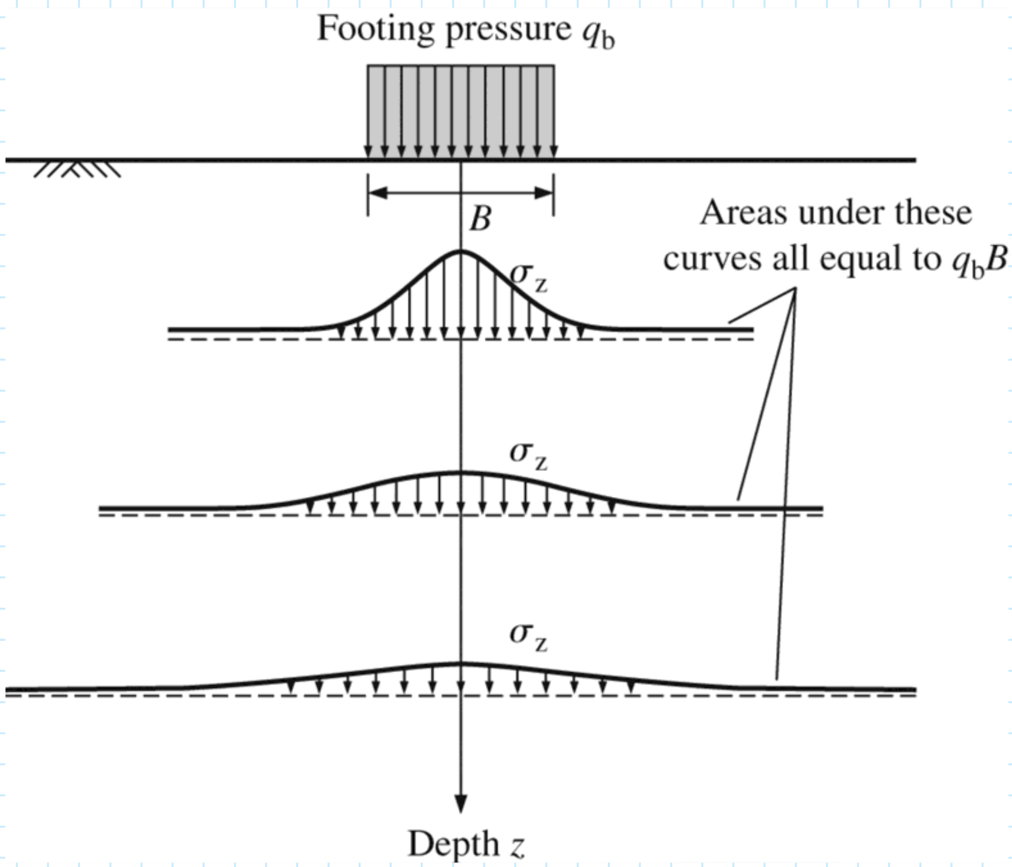
For calculating the consolidation settlement, see methods described in CVEEN 3310 or Salgado p. 389.

Pressure Distribution Under Footing Versus Depth

Monday, October 24, 2022 12:45 PM



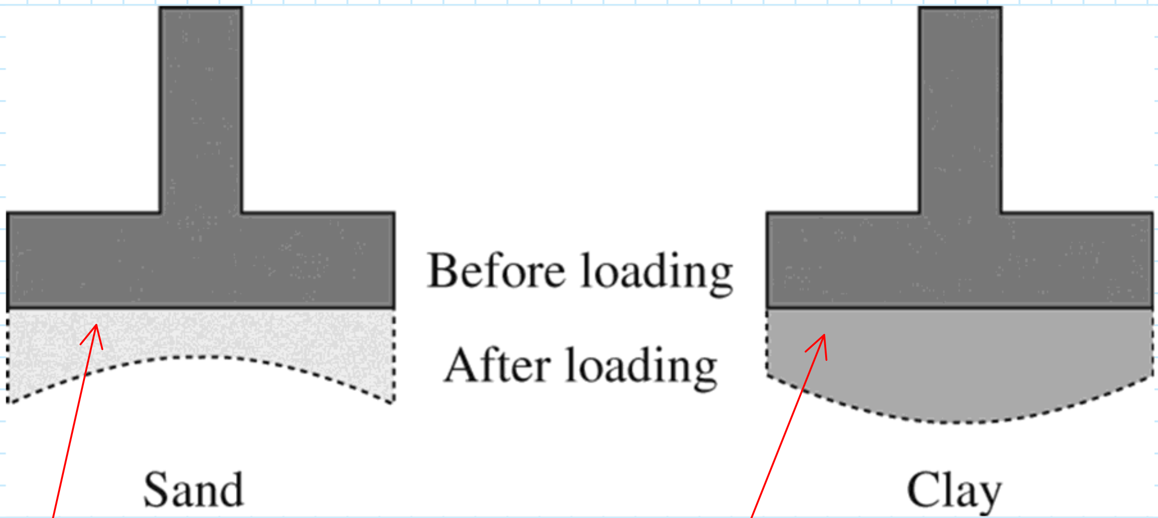
Rigid footing on elastic foundation



Contact Pressure at Base of Footing

Wednesday, August 17, 2011 12:45 PM

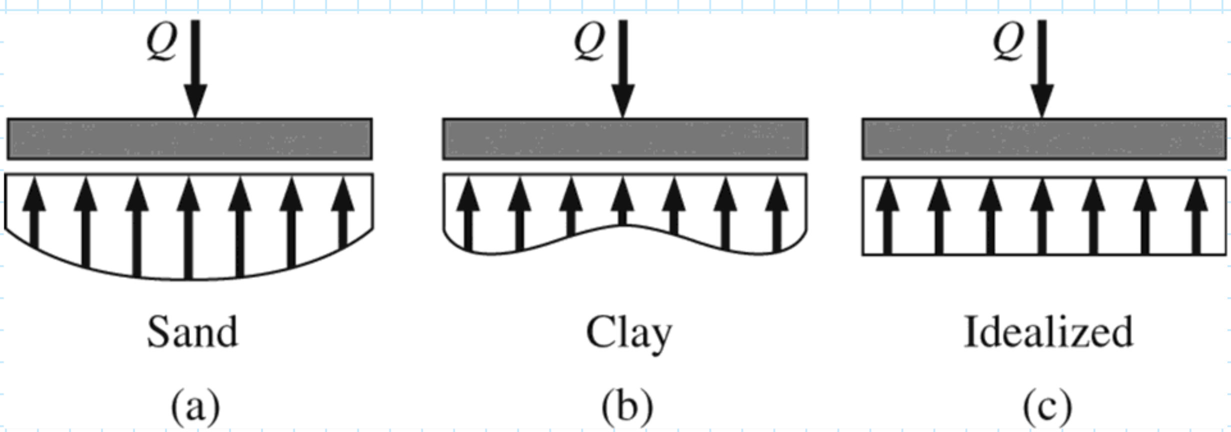
Influence of soil type on contact pressure (assumption of **rigid footing**)



Sand has inverted parabolic shape

Clay has parabolic shape

Influence of footing flexibility on contact pressure (**non-rigid footing**)



Contact Pressure (cont.)

Wednesday, August 17, 2011 12:45 PM

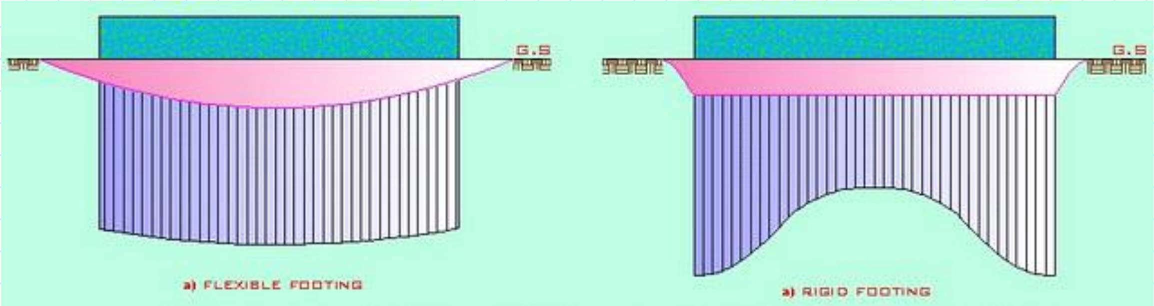


FIG. 1 CONTACT PRESSURE ON SATURATED CLAY

http://civil-engg-world.blogspot.com/2008/12/contact-pressure-distribution_24.html

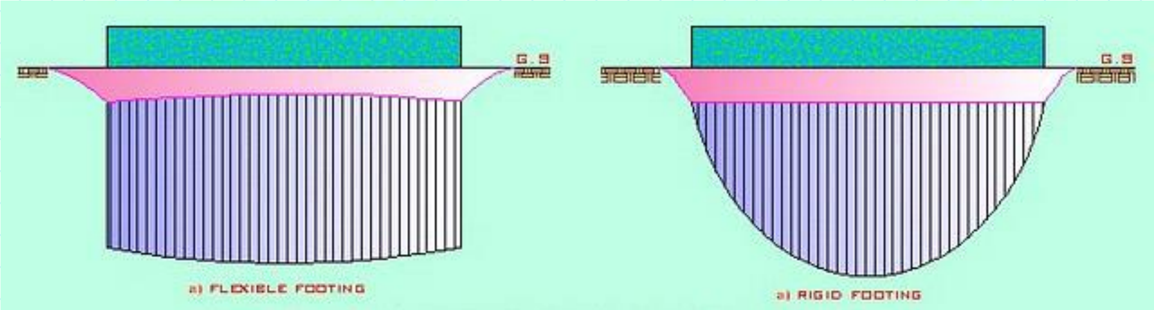


FIG. 2 CONTACT PRESSURE ON SAND.

http://civil-engg-world.blogspot.com/2008/12/contact-pressure-distribution_24.html

The fact that the contact pressure changes as a function of footing flexibility is called **soil-structure interaction**.

Contact Pressure - Uniform Distribution Incorporating Weight of Footing and Overlying Soil

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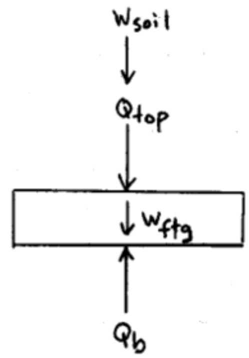
Calculation of q_b:



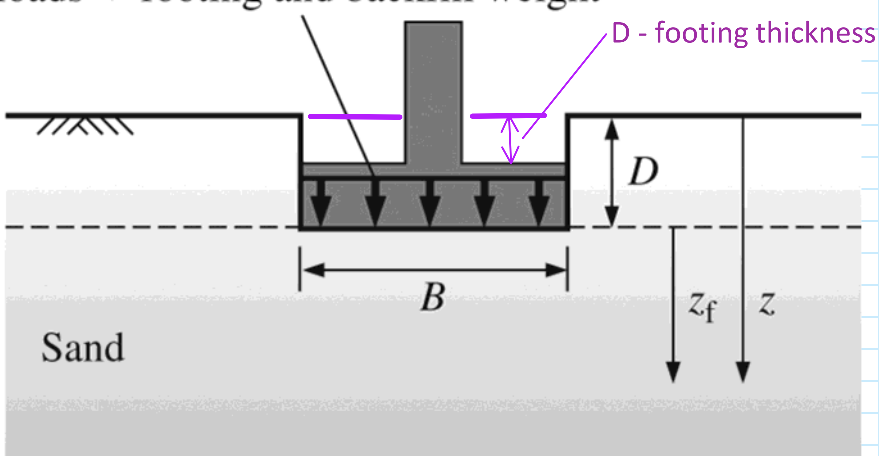
$$Q_b = Q_{top} + W_{ftg} + W_{soil}$$

$$q_b = \frac{Q_b}{A_{ftg}} \; ; \quad \text{For rectangular footing} \quad q_b = \frac{Q_b}{BL}$$

Need to know thickness (height) of footing, unit weight of footing, dimensions and shape of column, unit weight of soil.

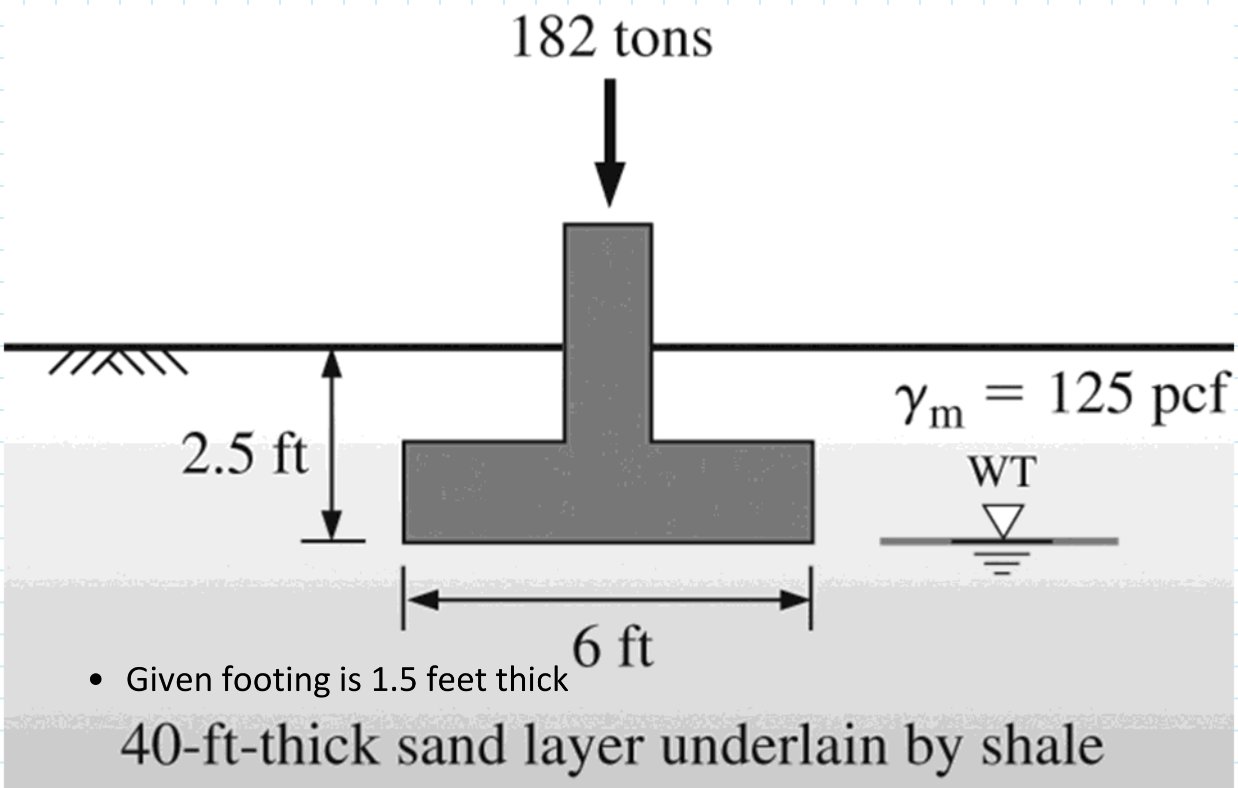
$Q = W_{soil} + Q_{top} + W_{ftg}$

q_b = unit load due to structural loads + footing and backfill weight



Contact Pressure - Uniform Distribution Incorporating Weight of Footing and Overlying Soil - Example

Wednesday, August 17, 2011 12:45 PM



- Given footing is 1.5 feet thick

Find qb

Show your calculations here:

(182*2000)/36=10111.1111

10111

Immediate Radial and Vertical Displacement at a Point from Elastic Theory (Point Load)

Wednesday, August 17, 2011 12:45 PM

$$w_r = \frac{Q}{4\pi G\sqrt{r^2+z^2}}\left[\frac{rz}{r^2+z^2} - \frac{(1-2v)r}{\sqrt{r^2+z^2}+z}\right] \qquad (9.4)$$

Radial displacement

$$w_z = \frac{Q}{4\pi G\sqrt{r^2+z^2}}\left[2(1-v) + \frac{z^2}{r^2+z^2}\right] \qquad (9.5)$$

Vertical Displacement

$$w_\theta = 0 \qquad (9.6)$$

Q = footing load (force)
G = Shear modulus
ν = Poisson's ratio
r = radius of footing

$$G = \frac{E}{2(1+\nu)}$$ (from Ch. 4)

Saint-Venant's Principle

The geometry of the loading has a negligible effect on the calculated stresses so long as the distance from the point where the stresses are desired to the loading is much larger than the scale of the load.

Example 9.1

A 500 kN load is applied at a point on the surface of a homogeneous soil deposit. Find the vertical displacement (a) at a depth of 3 m and a horizontal distance of 4 m from the load and (b) at the surface 4 m away from the load. This soil has elastic constants υ=0.15 and E=10 MPa.

Solution:

a. The distance R from the point where the vertical displacement is to be calculated to the point of application of the load is:

$$R = \sqrt{r^2+z^2} = \sqrt{4^2+3^2} = 5$$

Using Equation (9.5) with $G = E/[2(1+v)]$:

$$w = \frac{Q(1+v)}{2\pi ER}\left[2(1-v) + \frac{z^2}{R^2}\right]$$

$$= \frac{500(1+0.15)}{2\pi(10000)5}\left[2(1-0.15) + \frac{3^2}{5^2}\right] = 0.0038\,\text{m} = 3.8\,\text{mm}$$ answer

Immediate Settlement at Surface from Elastic Theory (Point Load)

Wednesday, August 17, 2011 12:45 PM

The settlement w of any point on the surface of the half-space is obtained by making $z = 0$ in Eq. (9.5), leading to

$$w = \frac{Q(1-\nu^2)}{\pi E R} = \frac{Q(1-\nu^2)}{\pi E r} \qquad (9.8)$$

The only point where we cannot use Eq. (9.8) is the point of application of the load, where the equation gives settlement equal to infinity, a result that is not useful in a practical world where ideal point loads do not really exist.

Q = footing load (force)
E = Young's modulus
ν = Poisson's ratio
r= radius of footing

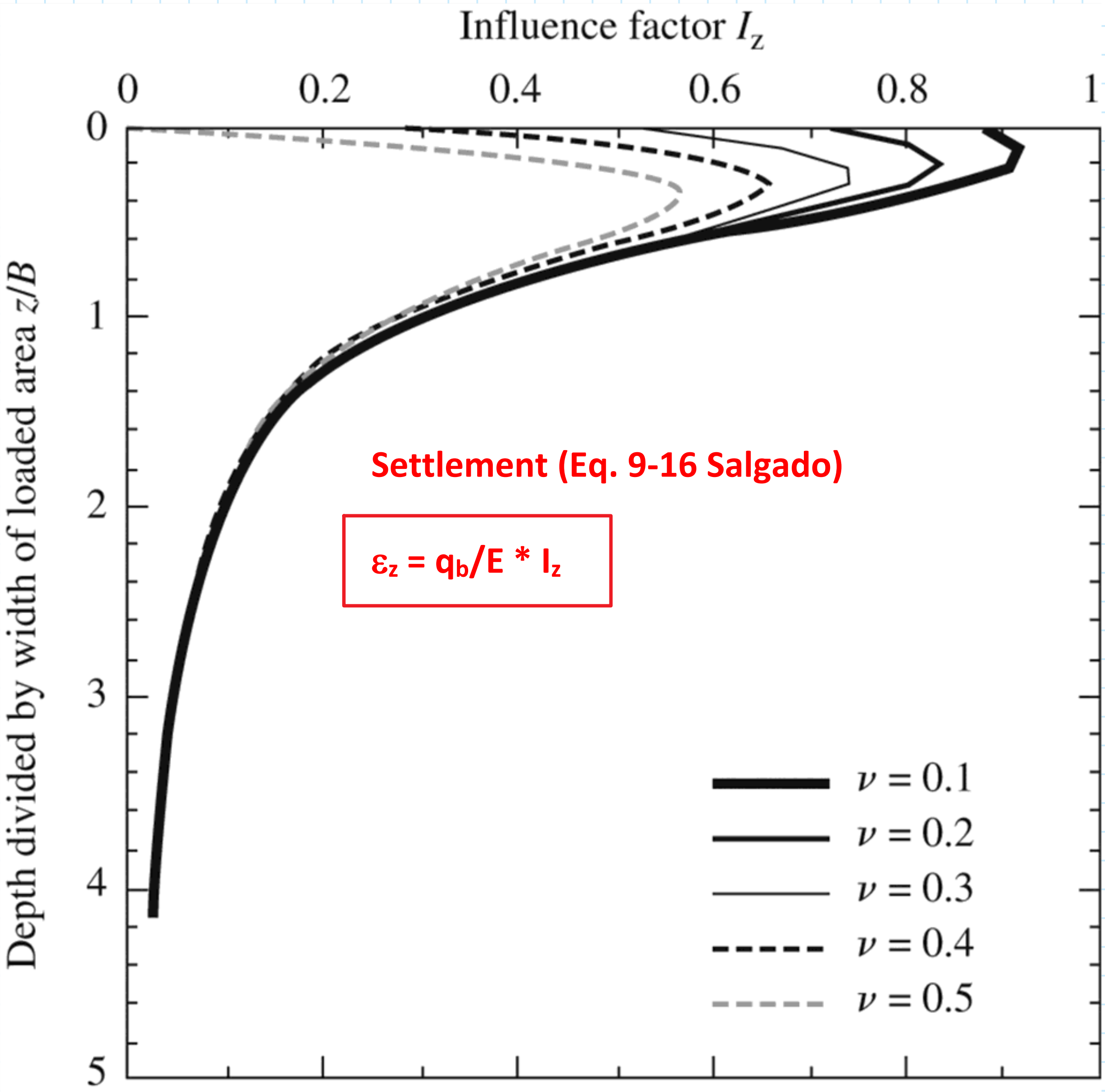
$$R^2 = r^2 + z^2$$

b. In this case, because the point of interest is on the surface, we can use Equation (9.8):

$$w = \frac{Q(1-\nu^2)}{\pi E R} = \frac{500(1-0.15^2)}{\pi(10000)4} = 3.9 \times 10^{-3}\ \text{m} = 3.9\,\text{mm}^{\text{answer}}$$

Immediate Settlement from Elastic Theory (Flexible Circular Load)

Wednesday, August 17, 2011 12:45 PM



Settlement (Eq. 9-16 Salgado)

$$\boxed{\varepsilon_z = q_b/E * I_z}$$

The above chart is for a uniform circular load:

$$\varepsilon_z\Big|_{r=0} = \frac{q_b}{E}\left\{1 - \nu(1+2\nu) + \frac{4\nu(1+\nu)\frac{z}{B}}{\sqrt{1+4\left(\frac{z}{B}\right)^2}} - \frac{8(1+\nu)\left(\frac{z}{B}\right)^3}{\left[1+4\left(\frac{z}{B}\right)^2\right]^{3/2}}\right\} \quad (9.15)$$

Or use chart above.

Immediate Settlement from Elastic Theory (Flexible Rectangular Load)

Wednesday, August 17, 2011 12:45 PM

To find the settlement under the corner of a rectangular foundation we can use equations (9.18) and (9.19). The settlement is given by

$$w = I\frac{q_b B(1-\nu^2)}{E}$$

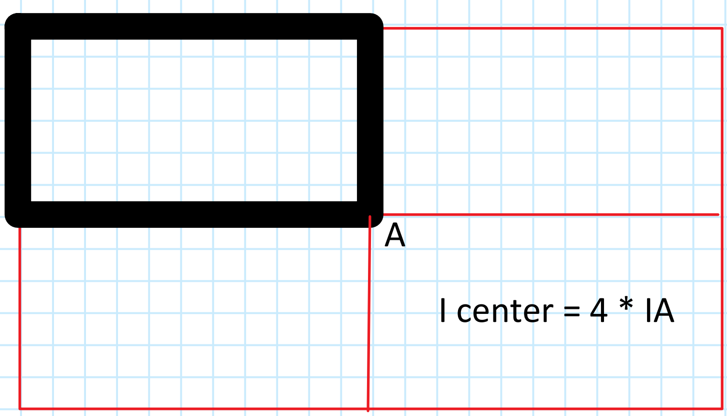
where the influence factor I is calculated as a function of m:

I is determined from Eq. 9.19

$$I = \frac{1}{2\pi}\left\{m\left[\ln\left(\frac{\sqrt{1+m^2}+1}{\sqrt{1+m^2}-1}\right)\right] + \ln\left(\frac{\sqrt{1+m^2}+m}{\sqrt{1+m^2}-m}\right)\right\} \qquad (9.19)$$

where $m = L/B$; and B, L = dimensions of rectangular load.

The settlement at the center of a flexible rectangular load can be calculated by superimposing the settlements at the corners of four rectangle, A, with the total area equal to the desired area.



Immediate Settlement from Elastic Theory (Flexible Rectangular Load)

Thursday, March 11, 2010 11:43 AM

- Rectangular shaped footing (cont.)
  - Influence factor, I, based on influence charts

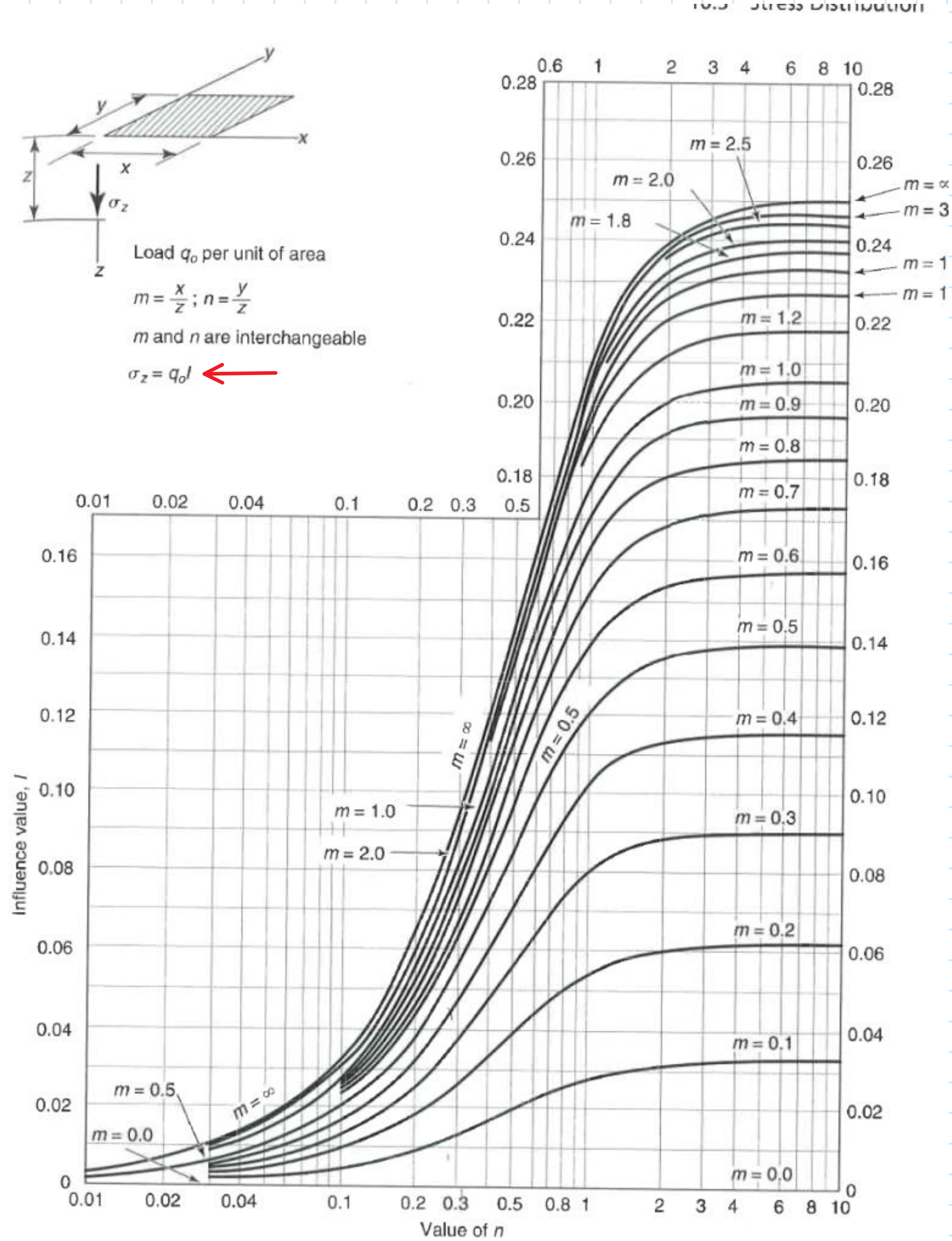


FIGURE 10.4 Influence value for vertical stress under corner of a uniformly loaded rectangular area (after Fadum, 1948).

- Principle of Superposition in Elastic Body
  - To find the stress at depth at a certain point under the foundation, we can simply find the $\Delta\sigma_V$ from each foundation element then sum the $\Delta\sigma_V$ values from each element to find the total stress.

Settlement of Rigid Footings

Wednesday, August 17, 2011 12:45 PM

Note that all previous methods from elastic theory are for perfectly flexible foundations. However, many concrete footings are closer to rigid than flexible, hence this should also be considered using the approximations given below.

The settlement of a rigid cylinder under a load Q is

$$w = w_z\Big|_{z=0} = \frac{Q(1-\nu^2)}{BE} \tag{9.20}$$

Equation (9.20) may also be written in terms of the average distributed load q_b:

$$w = \frac{\pi}{4}\frac{q_b B}{E}(1-\nu^2) \tag{9.21}$$

The settlement of other rigid areas can be estimated from the settlements calculated at the center and corner or edge of equivalent flexible areas as

$$w_{\text{rigid}} = \tfrac{1}{2}(w_{\text{center}} + w_{\text{edge}})_{\text{flexible}} \tag{9.22}$$

for circular or strip foundations, and

$$w_{\text{rigid}} = \tfrac{1}{3}(2w_{\text{center}} + w_{\text{corner}})_{\text{flexible}} \tag{9.23}$$

for square foundations.

The size B equals the diameter of the loaded circle in Equations 9.20 and 9.21

Methods of Estimating E (CPT)

Wednesday, August 17, 2011 12:45 PM

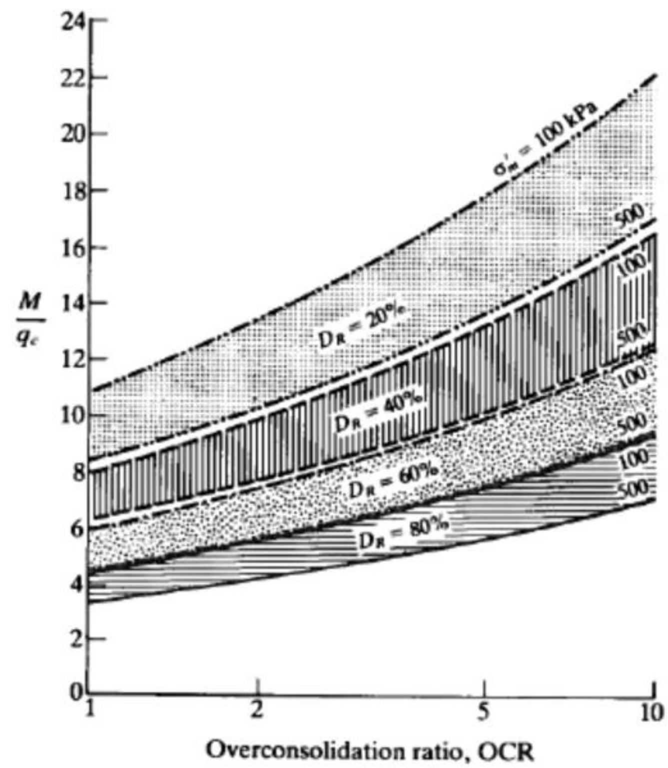
- Method for Estimating Young's Modulus E from CPT data
- Myerhof and Fellenius (1985) (Granular Materials)

$E = k\, q_c$

where: q_c = uncorrected CPT tip stress

k = 1.5 for silts and sand
= 2 for compacted sand
= 3 for dense sand
= 4 for sand and gravel

- Jamiolkowski (1988) (Cohesive Materials with



M = constrained modulus

$$M = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}$$

E=[M(v+1)(2v-1)]/(v-1)

qc is the uncorrected CPT tip stress

Penetration Testing 1988, ISOPT-1, De [illegible] (ed.)
© 1988 Balkema, Rotterdam, ISBN 90 [illegible]

New correlations of penetration tests for design practice

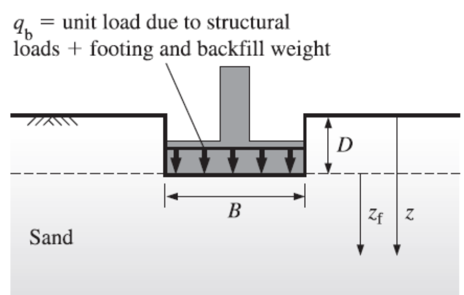
M.Jamiolkowski, V.N.Ghionna, R.Lancellotta & E.Pasqualini
Technological University Torino, Italy

Meyerhof (1965)

$$\frac{w}{L_R} = \frac{0.152}{\min\left(1+\dfrac{D}{3B},\ 1.33\right)N_{60}}\left(\frac{q_b - \sigma'_{vp}\big|_{z_f=0}}{p_A}\right)$$

for $B \le 1.2L_R$, and

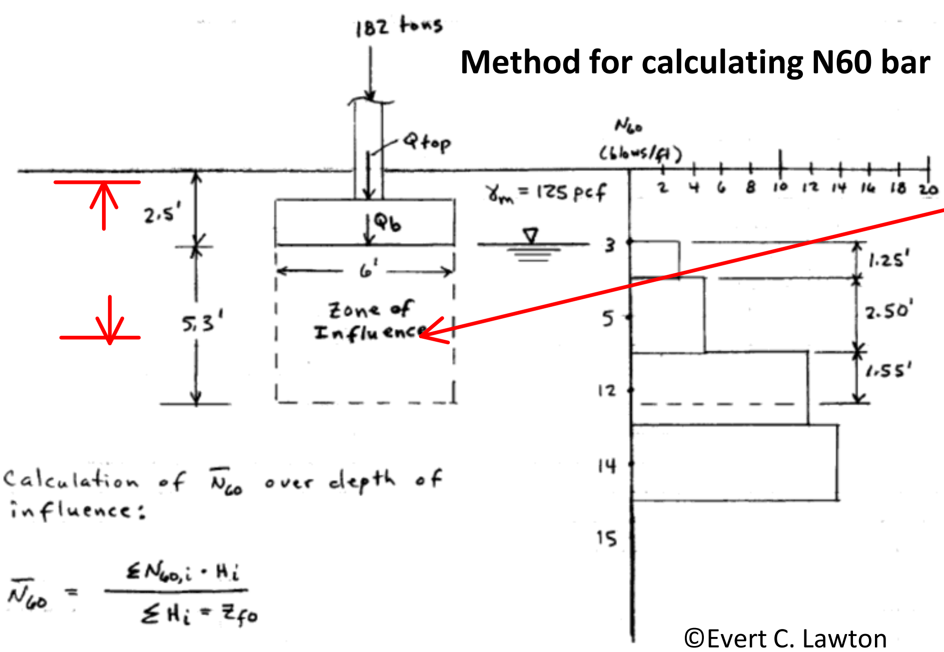
$$\frac{w}{L_R} = \frac{0.229}{\min\left(1+\dfrac{D}{3B},\ 1.33\right)N_{60}}\left(\frac{q_b - \sigma'_{vp}\big|_{z_f=0}}{p_A}\right)\left(\frac{B}{B+0.305L_R}\right)^2$$



Notes. Meyerhof (1965) suggested a relationships for settlements of spread footings on sand

for $B > 1.2L_R$, where w=footing settlement, z_f=depth measured from the level of the base of the footing (see Figure 9.6), q_b=gross unit load at the base of the footing (including both structural loads and the weight of the backfill and foundation element), $\sigma'_{vp}\big|_{z_f=0}$=maximum previous vertical effective stress experienced by the soil at the footing base level, N_{60}=average SPT blow count at 60% energy ratio over a depth of 1B below the footing base for square footings and 2B below the footing base for strip footings, B=footing width, L_R=reference length=1 m=3.281 ft=39.37 in., and p_A=reference stress=100 kPa ≈ 1 tsf. In Equations (9.24) and (9.25), the SPT N values are not corrected for the water table or overburden pressure and the min[1+D/(3B), 1.33] term is a depth factor that attempts to account for reduced settlement when the footing is embedded in the soil a depth equal to D, all else being the same. There is a small discontinuity at $B=1.2L_R$. Additionally, Equations (9.24) and (9.25) were intended for the calculation of settlements under working loads, so they should not be expected to be accurate for loads that are either much smaller than the limit bearing resistance (the load at which the footing will plunge into the ground, which we will discuss in Chapter 10), or close to it. The equations therefore are supposed to be used in design, when we are looking for the size of the footing that will give us a settlement that is close to the tolerable values we discussed in Chapter 2.

Method for calculating N60 bar



Zone of Influence=

1B for sq. ftg.

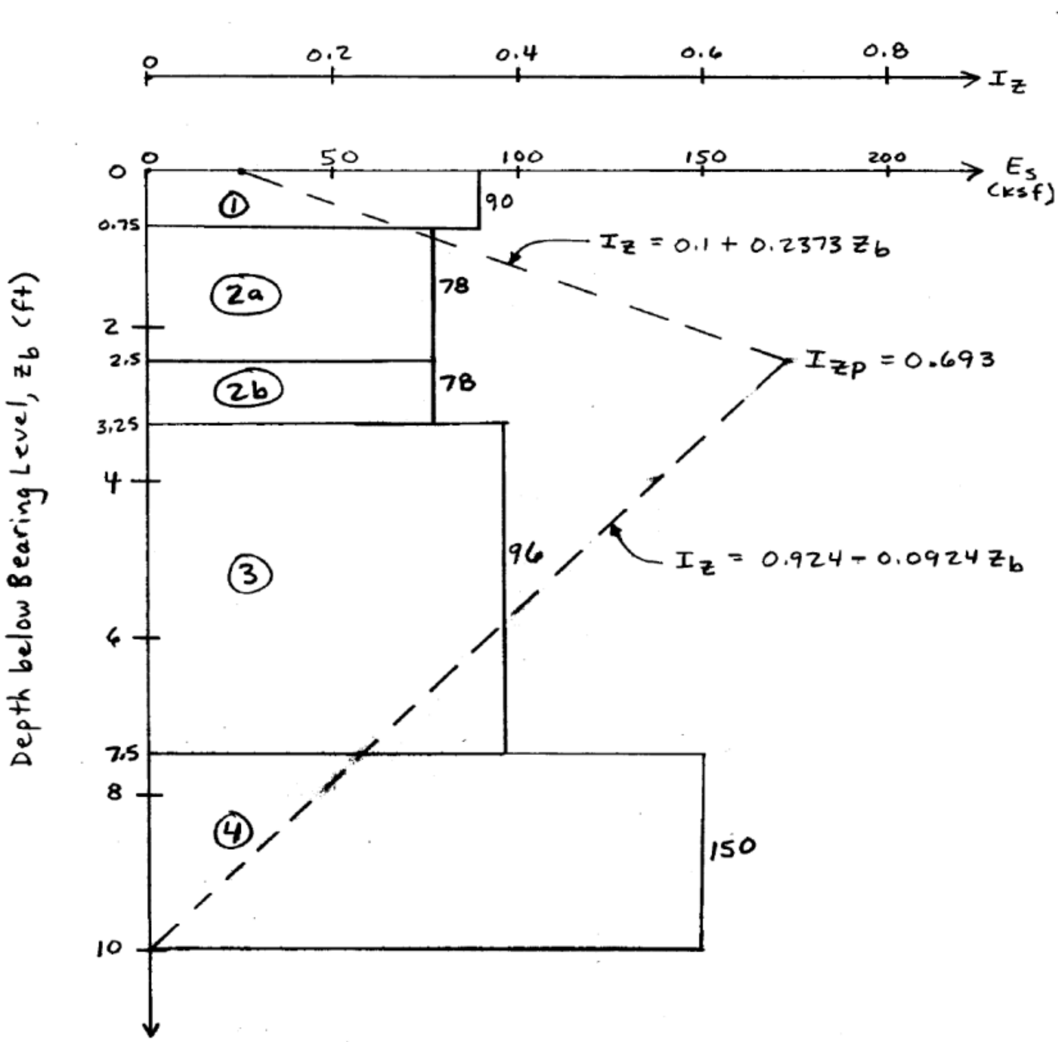
2B for strip ftg.

Calculation of $\bar{N}_{60}$ over depth of influence:

$$\bar{N}_{60} = \frac{\Sigma N_{60,i} \cdot H_i}{\Sigma H_i = z_{f0}}$$

$$\bar{N}_{60} = \frac{3(1.25) + 5(2.50) + 12(1.55)}{1.25 + 2.50 + 1.55} = \frac{34.85}{5.30} = 6.58$$

Example - Schmertmann's Method for S_i Using SPT Blowcounts to Estimate E_s



From the bearing level to z_p: $I_z = 0.1 + (I_{zp} - 0.1)/(B/2)*z_b$

$I_z =$ 0.1 + 0.2373 z_b

From z_p to z_0: $I_z = I_{zp} - I_{zp}/(1.5B)*(z_b - 0.5B) = 4/3*I_{zp} - I_{zp}/(1.5B)*z_b$

$I_z =$ 0.924 - 0.0924 z_b

$$\boxed{S_i = C_1*\Delta q* \Sigma I_z*\Delta z/E_s}$$

$C_1 = 1 - 0.5(\sigma'_{v0} / \Delta q)$ but not less than 0.5

Example - Schmertmann's Method for S_i Using SPT Blowcounts to Estimate E_s

Given:

A rigid, square footing for a building will be 5 ft wide and 2 ft thick and will bear at a depth of 5 ft below the existing ground surface. The load from the superstructure will be applied to the top of the footing by an 12 in. square reinforced concrete column that is centered over the footing. The best estimate of the load from the superstructure that will cause settlement is a vertical, concentric load of 50 kips. The soil conditions are as shown on the attached boring log. Note that the GWT is at a depth of 5 ft below the ground surface. The depths at which the blowcounts were taken are as follows: 2.0, 4.5, 7.0, 9.5, 14.5, 19.5, 24.5, and 29.5 ft below the ground surface. The drilling and sampling equipment used in the Standard Penetration Tests had the following characteristics: (1) The applied energy was 65% of the theoretical energy; (2) the length of rod was 5 ft plus the depth to the test; (3) a liner was not used in the sampler; and (4) the tests were conducted through the opening of a hollow stem auger. Calculate E_s for the upper silty sand layer using the equation for silts, sandy silt, or clayey silt in Table 5-5, p. 316; calculate E_s for the lower sand layer using the equation for saturated sand in Table 5-5. Assume that E_s is constant over a depth interval ranging from halfway between adjacent blowcounts.

The average unit weights were determined to be the following:

γ = 110 pcf for the silty sand above the GWT
γ_{sat} = 122.4 pcf for the silty sand below the GWT
γ_{sat} = 132.4 pcf for the dense sand below the GWT

Note: Assume that the footing excavation will be backfilled and that the excavation, backfilling, and loading will occur instantaneously. Calculate the load at the bearing level that will cause settlement as the load from the superstructure plus the weight of the footing and the weight of the backfill soil above the footing.

Required: Estimate the immediate settlement using Schmertmann's strain distribution method.

Solution:

$B =$	5	ft	$\gamma_{ss} =$	110	pcf			
$L =$	5	ft	$\gamma_{sat\text{-}ss} =$	122.4	pcf	$\gamma'_{ss} =$	60	pcf
$T =$	2	ft	$\gamma_{sat\text{-}ds} =$	132.4	pcf	$\gamma'_{ds} =$	70	pcf
$D =$	5	ft	$\gamma_w =$	62.4	pcf			
$z_{gwt} =$	5	ft	$\gamma_{conc} =$	150	pcf			
$V_{super} =$	50	kips	$b_{col} =$	1.0	ft			

Calculate the vertical load that will cause settlement at the bearing level (V_{brg}):

$$W_{ftg} = B \times L \times T \times \gamma_{conc} = 7.50 \text{ kips}$$
$$W_{soil} = (BL - b_{col}^2)(D - T)\gamma_{ss} = 7.92 \text{ kips}$$
$$V_{brg} = V_{super} + W_{soil} + W_{ftg} = 65.42 \text{ kips}$$

Example - Schmertmann's Method for S_i Using SPT Blowcounts to Estimate E_s

RECORD OF SOIL EXPLORATION
(Boring Log)

CLIENT: American Architects
BORING NO.: 13
PROJECT NAME: New Warehouse
JOB NO.: 76-1
DATE: 3/76
PROJECT LOCATION: Naples, Florida
FOREMAN: Bob Jones
INSPECTOR: R. Smith
BORING METHOD: HSA
RC DIA. ___ in.
SHELBY TUBE O.D. ___ in.

⊗ UNCONFINED COMPRESSIVE STRENGTH, TSF: 1 2 3 4 5
○ NATURAL DRY DENSITY, PCF: 90 100 110 120 130
□ WATER CONTENT, % ◪ LIQ. LIM., % ◪ PLAST. LIM., %: 10 20 30 40 50
● STANDARD PENETRATION, BLOWS/FT: 10 20 30 40 50

SOIL CLASSIFICATION
SURFACE ELEVATION — 40.0

SOIL CLASSIFICATION	STRATUM DEPTH, FT.	DEPTH SCALE, FT.	SAMPLE NO.	SPT* BLOWS/6in. 3-6 in. INCREMENTS	RECOVERY, %	SHELBY TUBE NO.
Topsoil	0.5					
Tan moist loose silty fine sand (SM) with trace shells			1	3/4/4	70	
		5	2	4/4/6	80	
			3	4/4/4	70	
		10	4	4/5/5	50	
	12.5					
Light gray wet medium dense Sand (SP) with some shells		15	5	6/7/8	40	
		20	6	8/10/10	40	
		25	7	8/8/10	50	
	30.0	30	8	10/12/11	40	
Bottom of test boring @ 30'						

BORING METHOD
HSA - HOLLOW STEM AUGER
CFA - CONTINUOUS FLIGHT AUGER
DC - DRIVEN CASING
MD - MUD DRILLING
RC - ROCK CORING

GROUND WATER
▽ NOTED ON RODS 1.0 FT.
▼ AT COMPLETION ___ FT.
▼ AFTER 24 HRS. 5.0 FT.

SHELBY TUBE SAMPLES OBTAINED IN BORING ___ DRILLED A FEW FEET FROM BORING ___
*STANDARD PENETRATION TEST

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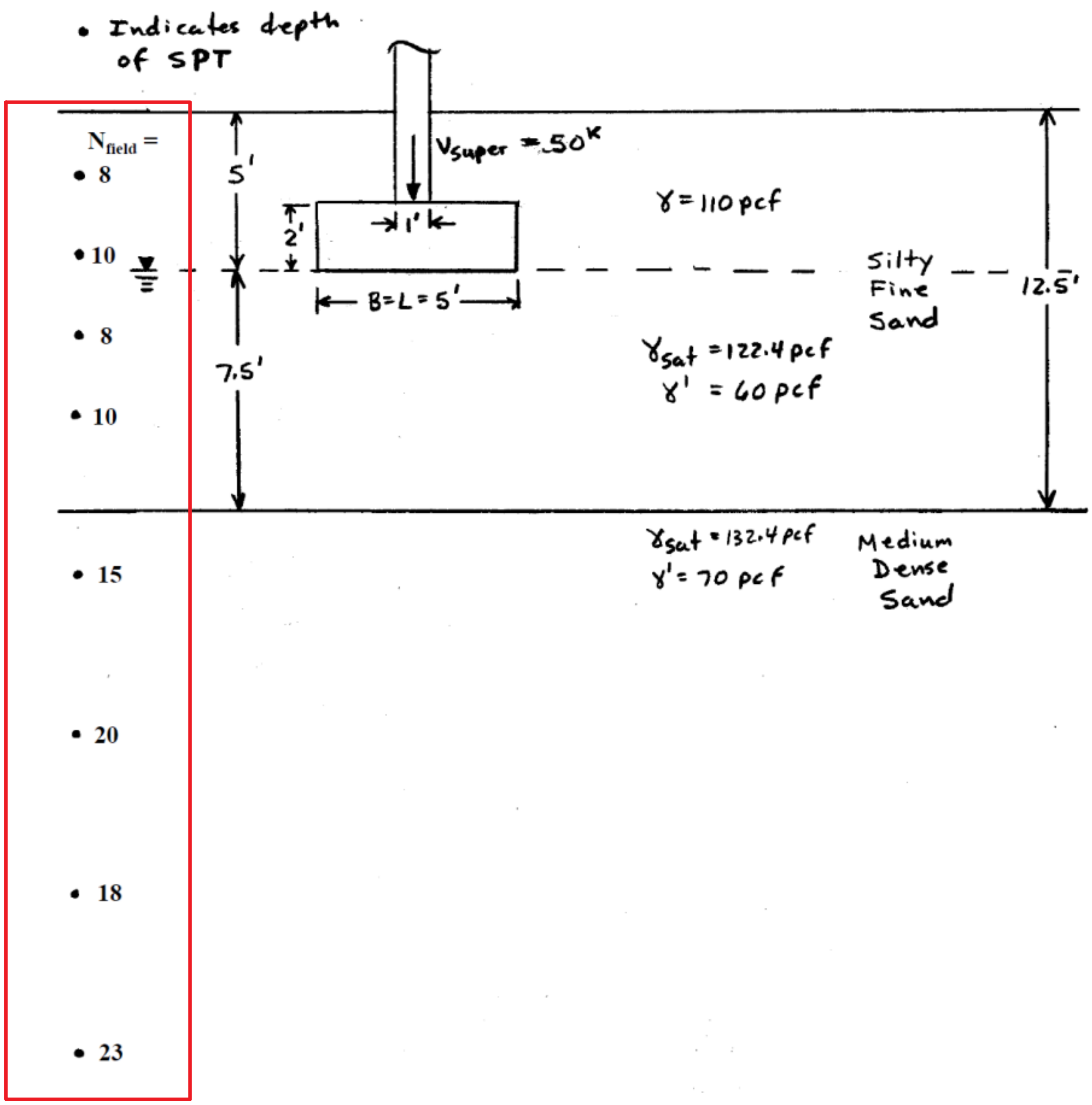
Example - Schmertmann's Method for S_i Using SPT Blowcounts to Estimate E_s



Example - Schmertmann's Method for S_i Using SPT Blowcounts to Estimate E_s

Correct blowcounts for η_1 - η_4 but not C_N and then calculate E_s from corrected N:

Notes: Blowcounts will be rounded to nearest integer before calculating E_s.
Only soil from bearing level and deeper contributes to settlement.
For silty sand (z ≤ 12.5 ft): E_s (ksf) = 6(N′$_{55}$+6)
For sand (z > 12.5 ft): E_s (ksf) = 5(N′$_{55}$+15)
z = depth below ground surface
z_b = depth below bearing level = z - D
L_r (m) = length of the drill rod = (z + 5)(0.3048)
η_3 = 1.0 for all samples because a liner was not used
η_4 = 1.0 for all samples because SPT taken through opening of hollow-stem auger

Blow Count Corrections

$$N_{60} = C_h \cdot C_r \cdot C_s \cdot C_d \cdot N$$

- η1 C_h = hammer type correction factor
- η2 C_r = rod length correction factor
- η3 C_s = sampler correction factor
- η4 C_d = borehole diameter correction factor

Soil Type	Sample No.	z (ft)	z_{brg} (ft)	$N_{field} = N_{65}$ (blows/ft)	η_1	L_r (m)	η_2	N'_{55} (blows/ft)	E_s (ksf)
Silty sand	2	4.5	-0.5	10	1.18	2.90	0.75	9	90
Silty sand	3	7.0	2.0	8	1.18	3.66	0.75	7	78
Silty sand	4	9.5	4.5	10	1.18	4.42	0.85	10	96
Dense sand	5	14.5	9.5	15	1.18	5.94	0.85	15	150
Dense sand	6	19.5	14.5	20	1.18	7.47	0.95	22	185
Dense sand	7	24.5	19.5	18	1.18	8.99	0.95	20	175
Dense sand	8	29.5	24.5	23	1.18	10.52	1.00	27	210

Layers of constant E_s are assumed to be halfway between adjacent blowcounts except at the interface of the silty sand and dense sand at z = 12.5 ft (z_{brg} = 7.5 ft).

Footing is square so use solution for axisymmetric conditions (closely approximated by square footing).

$\sigma'_{v0} = \sigma'_v$ at bearing level prior to excavation and placement of footing = $D\gamma_{ss}$ = 0.550 ksf

$\sigma'_{vp} = \sigma'_v$ at depth of I_{zp} prior to excavation and placement of footing = $\sigma'_{v0} + 0.5B\gamma'_{ss}$ = 0.700 ksf

$\Delta q = V_{brg}/(BL)$ = 2.617 ksf

$I_{zp} = 0.5 + 0.1(\Delta q / \sigma'_{vp})^{1/2}$ = 0.693

$C_1 = 1 - 0.5(\sigma'_{v0} / \Delta q)$ but not less than 0.5 = 0.895

Let z_b = depth below the bearing level.
The breakpoint in the I_z vs. z relationship occurs at $z_b = z_p = B/2$ for axisymmetric conditions.
The depth of influence is $z_b = z_0 = 2B$ for axisummetric conditions.

x

Example - Schmertmann's Method for S_i Using SPT Blowcounts to Estimate E_s

$z_p =$ 2.5 ft

$z_0 =$ 10 ft

Write equations for lines of I_z vs. z_b. There are two lines - one from bearing level to z_p, and one from z_p to z_0.

From the bearing level to z_p: $I_z = 0.1 + (I_{zp} - 0.1)/(B/2)*z_b$

$I_z =$ 0.1 + 0.2373 z_b

From z_p to z_0: $I_z = I_{zp} - I_{zp}/(1.5B)*(z_b - 0.5B) = 4/3*I_{zp} - I_{zp}/(1.5B)*z_b$

$I_z =$ 0.924 - 0.0924 z_b

Layer i	Soil Type	z_b(top) (ft)	z_b(bot) (ft)	Δz (ft)	z_b (ft)	Avg. I_z	E_s (ksf)	$I_z*\Delta z/E_s$ (ft/ksf)
1	Silty sand	0.00	0.75	0.75	0.375	0.189	90	0.00158
2a	Silty sand	0.75	2.50	1.75	1.625	0.486	78	0.01090
2b	Silty sand	2.50	3.25	0.75	2.875	0.659	78	0.00633
3	Silty sand	3.25	7.50	4.25	5.375	0.428	96	0.01893
4	Dense sand	7.50	10.00	2.50	8.75	0.116	150	0.00193
			$\Sigma\Delta z = 2B =$	10			$\Sigma I_z*\Delta z/E_s =$	0.03966

$S_i = C_1*\Delta q*\Sigma I_z*\Delta z/E_s =$ 0.0929 ft = 1.11 in.

Could also define layers strictly halfway between adjacent blowcounts, which would modify layers 3 and 4 only.

Layer i	Soil Type	z_b(top) (ft)	z_b(bot) (ft)	Δz (ft)	z_b (ft)	Avg. I_z	E_s (ksf)	$I_z*\Delta z/E_s$ (ft/ksf)
1	Silty sand	0.00	0.75	0.75	0.375	0.189	90	0.00158
2a	Silty sand	0.75	2.50	1.75	1.625	0.486	78	0.01090
2b	Silty sand	2.50	3.25	0.75	2.875	0.659	78	0.00633
3	Silty sand	3.25	7.00	3.75	5.125	0.451	96	0.01760
4	Dense sand	7.00	10.00	3.00	8.5	0.139	150	0.00277
			$\Sigma\Delta z = 2B =$	10			$\Sigma I_z*\Delta z/E_s =$	0.03918

$S_i = C_1*\Delta q*\Sigma I_z*\Delta z/E_s =$ 0.0918 ft = 1.10 in.

SUMMARY OF ANSWERS: From Schertmann's method, $S_i =$ 1.1 in.

CPT Based Methods

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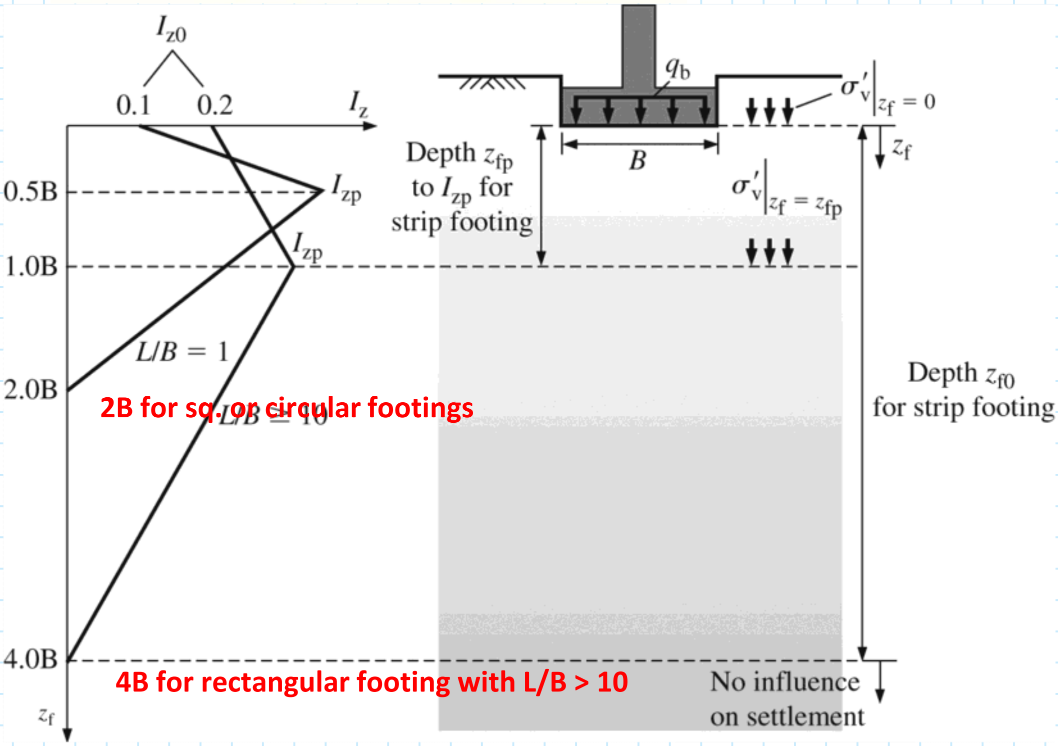
$$S_i = C_1 C_2 \Delta q \sum \left(\frac{I_z}{E}\right)_i \Delta z_i$$



$$\frac{z_{f0}}{B} = 2 + 0.222\left(\frac{L}{B} - 1\right) \le 4$$

For lowest y-coordinate for $I_{z0} = 0$ (y-axis intercept)

$$\frac{z_{fp}}{B} = 0.5 + 0.0555\left(\frac{L}{B} - 1\right)$$

For y-coordinate of break point (i.e., y-coordinate for $I_{zp\ max}$)

$$I_{z0} = 0.1 + 0.0111\left(\frac{L}{B} - 1\right) \le 0.2$$

For x-coordinate for $z_f = 0$

$$I_{zp} = 0.5 + 0.1\sqrt{\frac{q_b - \sigma'_v|_{z_f=0}}{\sigma'_v|_{z_f=z_{fp}}}}$$

For x-coordinate of break point

$$I_z = I_{z0} + \frac{z_f}{z_{fp}}\left(I_{zp} - I_{z0}\right)$$

For $z_f < z_{fp}$

For I_z between z_f and z_{fp} (i.e., bottom part)

$$I_z = \frac{z_{f0} - z_f}{z_{f0} - z_{fp}} I_{zp}$$

For $z_{fp} \le z_f \le z_{f0}$

For I_z between z_0 and z_{fp} (i.e., top part)

CPT Based Methods

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Layer i	Δz_i	$q_{c,i}$	E_i	$I_{z,i}$	$\frac{I_{z,i}}{E_i}\Delta z_i$
1					
2					
3					
⋮					
					$\sum_i \left(\frac{I_z \cdot \Delta z_i}{E_i}\right)$

$$w = C_1 \cdot C_2 \cdot \left(q_b - \sigma'_v\big|_{z_f=0}\right) \cdot \sum_i \left(\frac{I_{z,i} \cdot \Delta z_i}{E_i}\right)$$

Ei = 2.5qc for young NC sand
Ei = 3.5qc for aged NC sand
Ei = 6.0qc for overconsolidated silica sand

C1 from Eq. 9.43 Salgado = $C1 = 1-0.5(\sigma'_v|z_{f=0} / \Delta q) \geq 0.5$
C2 = 1 (see below)
$\Delta q = q_b - \sigma'_v|z_{f=0}$

C2 from Eq. 9.44 or usually set to 1 i.e., 1σ'v|zf=0 (see note below)

Schmertmann (1970) also included a second correction factor, C_2, to account for some time-independent increase in settlement that was observed even for foundations on presumably cohesionless soils. In the cases studied by Schmertmann, time-dependent settlements probably occurred as a result of the consolidation of thin strata of silts and clays within the sands. Consequently, because the elastic distribution is inappropriate for cohesive soils and the method uses the Dutch cone penetration test (CPT) to estimate modulus, which is questionable for cohesive soils, the use of the correction factor C_2 is not recommended; therefore, use C_2 equal to 1.0 in

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Example

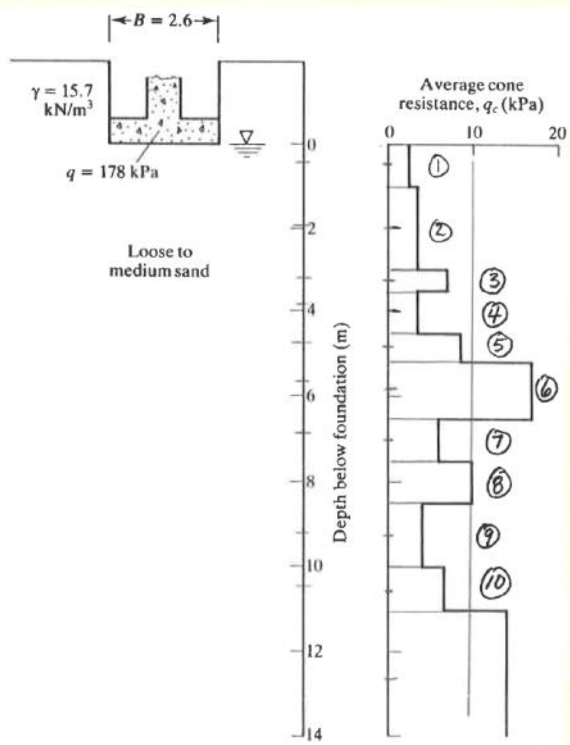
Given: $L = 22.8\ m$ GW table = 2m

$B = 2.6\ m$ m. loose to dense sand

$\gamma_{soil} = 15.7\ kN/m^3$ $D = 2\ m$

$q = 178\ kPa$

Find: immediate settlement of pier



Calculations:

1. $L/B = 22.8/2.6 = 8.8 \approx 10$

$\therefore$ use solution for plane strain

CPT Based Methods (Example cont.)

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Calculations (cont.)

2. Determine strain distribution influence factor

$$\Delta q = q - q_{vo}'$$

$$\Delta q = 178 - (15.7)(2) = 147 \text{ kPa}$$

depth to base of ftg.

$$\sigma_{vp}' = (15.7)(2) + (15.7 - 9.81)(2.6) = 47.6 \text{ kPa}$$

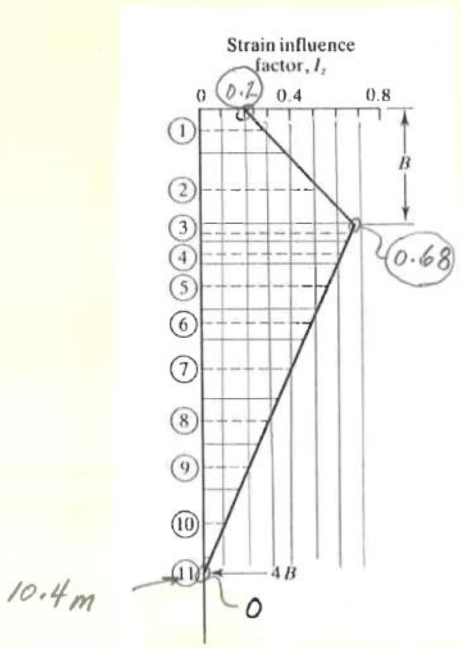
depth B below ftg. for plane strain

$$I_{zp} = 0.5 + 0.1\sqrt{\frac{\Delta q}{\sigma_{vp}'}}$$

$$I_{zp} = 0.5 + 0.1\sqrt{\frac{147}{47.6}}$$

$$I_{zp} = 0.68$$

3. Draw strain influence factor diagram



Equation of line ($z = 0$ to 2.6 m)

$$I_z = \frac{0.68 - 0.20}{2.6} z + 0.20$$

$$I_z = 0.1846 z + 0.20$$

Equation of line ($z = 2.6$ to 10.4 m)

$$I_z = \frac{-0.68}{(10.4 - 2.6)}(z - 2.6) + 0.68$$

$$-0.08718 (z - 2.6) + 0.68$$

4. Divide the profile in layers using q_c (see pg. 8)

	Thickness (Δz)	Dist. to center
Layer (1)	1	0.5
Layer (2)	1.6	1.8
Layer (3)	0.4	2.8
Layer (4)	0.5	3.25
Layer (5)	1.0	4.0
Layer (6)	0.7	4.85
Layer (7)	1.3	5.85
Layer (8)	1.0	7.0
Layer (9)	1.0	8.0
Layer (10)	1.5	9.25
Layer (11)	0.4	10.2

5. Determine I_z for each layer

$$I_{z(1)} = 0.1846(0.5) + 0.2 = 0.29$$

$$I_{z(2)} = 0.1846(1.8) + 0.2 = 0.53$$

$$I_{z(3)} = -0.08718(2.8 - 2.6) + 0.68 = 0.66$$

$$I_{z(4)} = -0.08718(3.25 - 2.6) + 0.68 = 0.62$$

$I_{z(5)} = 0.55$

$I_{z(6)} = 0.48$

$I_{z(7)} = 0.39$

$I_{z(8)} = 0.29$

$I_{z(9)} = 0.20$

$I_{z(10)} = 0.09$

$I_{z(11)} = 0.02$

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6. Calculate E_s for each layer

$$E_s = 3.5\, q_c \quad \text{(plane strain conditions)}$$

layer	q_c (MPa)	E_s (MPa)	$I_z\,\Delta z / E_s$ (m/MPa)
1	2.5	8.75	0.033
2	3.5	12.25	0.069
3	3.5	12.25	0.022
4	7.0	24.5	0.013
5	3.0	10.5	0.052
6	8.5	29.75	0.011
7	17.0	59.5	0.009
8	6.0	21.0	0.014
9	10.0	35.0	0.006
10	4.0	14.0	0.010
11	6.5	22.75	0.0003
			$\Sigma = 0.239$

7. Calculate $I_z\,\Delta z / E_s$ for each layer

(see above)

8. Sum $I_z\,\Delta z / E_s$ for each layer

(see above)

9. Determine C_1 & C_2

$$C_1 = 1 - 0.5\left(\frac{\sigma_{v_0}'}{\Delta q}\right) \qquad C_2 = 1 \leftarrow$$

$$C_1 = 1 - 0.5\left(\frac{(2)(15.7)}{147}\right) = 0.89 \leftarrow$$

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10. Calculate settlement

$$S_i = C_1 C_2 \Delta q \sum \left(\frac{I_z}{E}\right)_i \Delta z_i$$

$$S_i = (0.89)(1.0)\left(147 \frac{kN}{m^2}\right)\left(0.239 \frac{m \cdot m^2}{MN} \cdot \frac{1M}{1000k}\right)$$

$$S_i = 0.031\, m$$

$$S_i = 31\, mm \leftarrow$$

Notes

1) Method should be used for normally loaded sands
2) Bearing capacity should be checked
3) Method will over predict preloaded sands Schmertmann recommends dividing the solution by 2, if preloading has occurred (this is still probably conservative)
4) May be additional settlement from dynamic, cyclic or vibratory loading (especially in loose sands below the water table)

Stresses Under Embankment and Slopes

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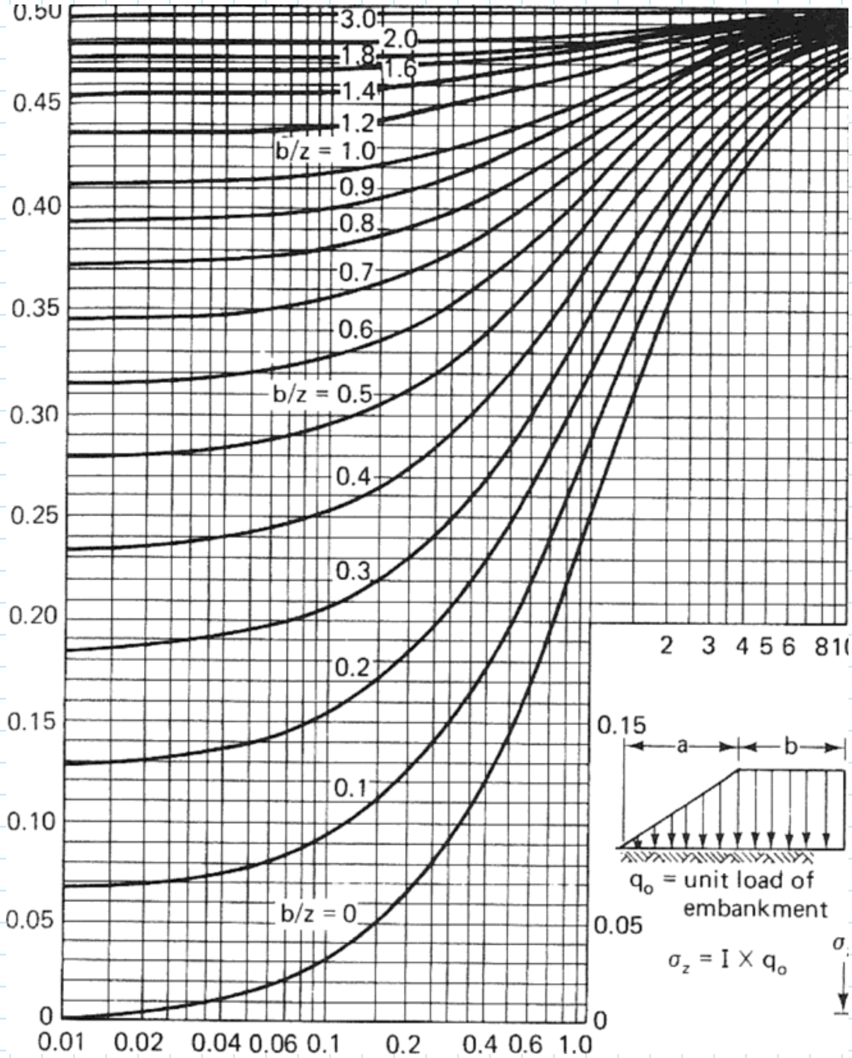
Elastic Theory

I = (influence factor)

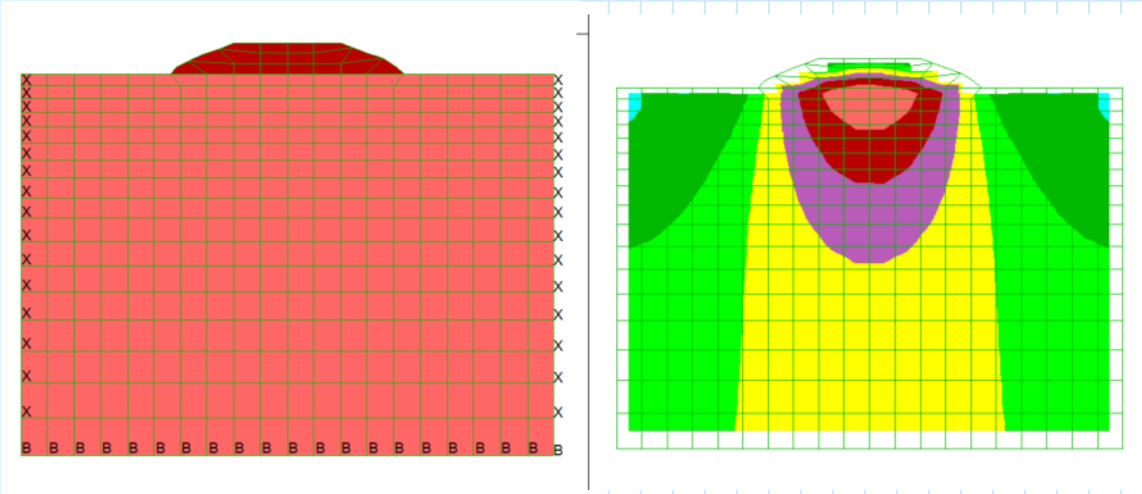
Note: Influence factor values from this chart must be double to account for the right side of the embankment.



(z = depth below ground surface (i.e., depth below base of embankment)

a/z

Numerical Approach



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Bearing Capacity

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Reading Assignment

- Salgado 10.1 -10.6
- Ch. 10 Lecture Notes

Other Materials

- None

Homework Assignment 9

1. Develop a spreadsheet program or use and AI engine to calculate q_{ult} for a shallow foundation using Meyerhof's method as discussed in the lecture notes with the following additions and clarifications:
   a. To help you in this task, an example printout is shown in the course notes. This output is given for your information only. You may use the format shown or any other format you wish so long as the input section is first and distinctly separate from the output section.
   b. Include r_γ at the end of the N_γ term of the Myerhof Bearing Capacity equations for all footings with B' > 2.0 m. Note: κ = 2.0 m and B' should have the same units as κ. For example, the general equation for a vertical load will be as follows:

$$q_{ult} = cN_c s_c d_c + \overline{q}N_q s_q d_q + 0.5\gamma B' N_\gamma s_\gamma d_\gamma r_\gamma$$

   (see correction for B > 2m in course notes and Bowles, Ch. 4 p. 230 for further discussion)
   c. Include a conditional statement to determine if the load is vertical or inclined and then use the appropriate main equation.
   d. When appropriate, include a correction to the triaxial friction angle of the soil for plane-strain conditions. Use the following rules and have the plane-strain fiction angle rounded off to the nearest whole degree:

For $\phi_{tr} \le 34°$: Use $\phi = \phi_{tr}$ regardless of the value for L/B.

For $\phi_{tr} > 34°$: If $L/B \le 2.0$, use $\phi = \phi_{tr}$.
If $L/B > 2.0$, use $\phi = \phi_{ps} = 1.5\phi_{tr}$ - 17°

Bearing Capacity (cont.)

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e. Account for eccentricities in either direction (in the direction of the width of the footing and/or the length of the footing) using Meyerhof's effective area method illustrated in Fig. 4-4 on p. 237 and described on p. 236. (Note: Use B' in place of B and L' in place of L in all calculations.

f. You will need to include conditional statements on some of the factors. For example, the equations for s_q and s_γ are different for $\phi = 0$ and $\phi > 10°$. Note also that the equation for N_c will "blow up" when $\phi = 0$. The correct value of N_c for $\phi = 0$ is 5.14 which is $\pi + 2$ limit as ϕ approaches zero. Therefore, when N_c is calculated in your spreadsheet, you will need to include a conditional statement for the special case when $\phi = 0$. You do not need to worry about the case where ϕ is between 0 and 10 degrees. This case will not occur for almost all soils.

g. The value of γ input for the soil data is the value to be used in the $N\gamma$ term. This value needs to be adjusted to γ_e if the watertable falls within a zone of $0.5B \tan(45 + \phi/2)$ (see effects of water table). Then q is calculated as: $q = \gamma_e D$.

h. Note: The answer for the spreadsheet are somewhat different than in Bowles, Ch. 4 owing to the use of B and L in Bowles' solution given in the textbook compared to B' and L' in the spreadsheet program. Also, Bowles rounded the N factors to integer values.

2. Each of the following footings below bears in a relatively homogeneous soil deposit and will support a centric, vertical load at the bearing level. Calculate the ultimate bearing capacity (q_{ult}) using Meyerhof's method and the program you developed in Problem 1. Also calculate the allowable bearing capacity (q_a), and the allowable vertical load for each footing (V_a) for the case below. Use a factor of safety (SF) of 3.0.
   a. B = 6.0 ft, L = 6.0 ft, D = 3.0 ft, γ = 110 pcf, ϕ = 35°, and c = 250 psf. The water table is far below the bearing zone. The strength parameters were determined from triaxial tests. (Answers: q_{ult} = 58.8 ksf, V_a = 705 kips).
   b. Same as footing in Part a except that L = 15 ft. (Partial Answer: q_{ult} = 50.5 ksf).
   c. Same as footing in Part a except that the footing is a continuous (strip) footing (L = ∞). (Partial Answer: qult = 42.0 kips/ft).

Bearing Capacity (cont.)

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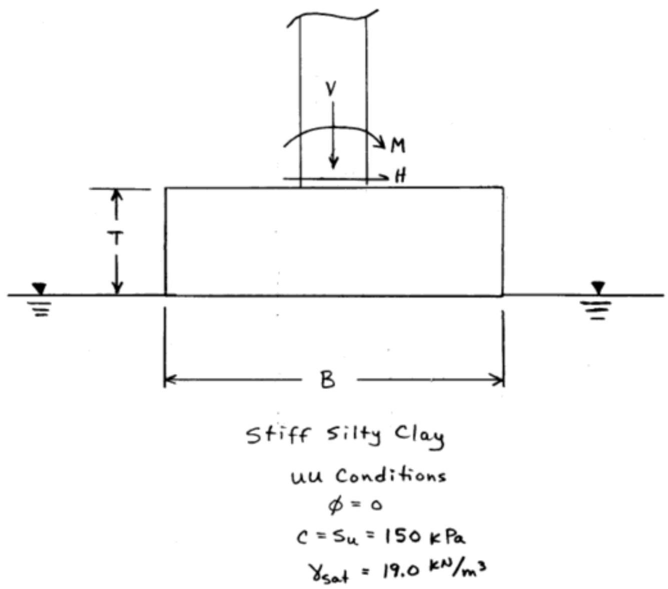
3. The square footing of width B shown in the figure below will bear on the ground surface of a deep silty clay deposit and will be subjected to loads from the superstructure V , H, and M as shown in the figure. The footing is composed of reinforced concrete with a unit weight, γ_{conc} = 23.6 kN/m3 and will be 1.00 m thick (T). Values of UU strength parameters and unit weight for the silty clay are given in the figure.



a. Write an equation for the weight of the footing (W_{ftg}) in terms of B, T, and γ_{conc}. (5 points)
b. Draw a free body diagram of the footing. Use the following terminology for the resisting moment, vertical force, and horizontal force from the soil at the bearing level: M_b, V_b, and H_b. (10 points)
c. Derive equations for M_b, V_b, and H_b that satisfy static equilibrium. (10 points)
d. A structural engineer has determined that the design loads from the superstructure are as follows: M = 750 kN-m, V = 2,000 kN, H = 250 kN Using the program you developed in Problem 1, determine the required width of the footing (B) to the nearest 0.05 m that will provide a minimum factor of safety (SF) of 3.0 against ultimate bearing capacity failure. The input loads are Mbrg, Vbrg, and Hbrg. (5 points)
e. For the size of footing you determined in Part d, determine the maximum value of H_b that can be developed along the interface of the bottom of the footing and the silty clay. Assume that the interface is perfectly rough so that the maximum shearing resistance is equal to the shearing strength of the soil. Assume the shearing resistance will act only on the effective area of the footing not the total area. (5 points)

Types of Bearing Capacity Failures

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Punching failure

Local shear failure

General shear failure

Bearing Capacity Failure Examples

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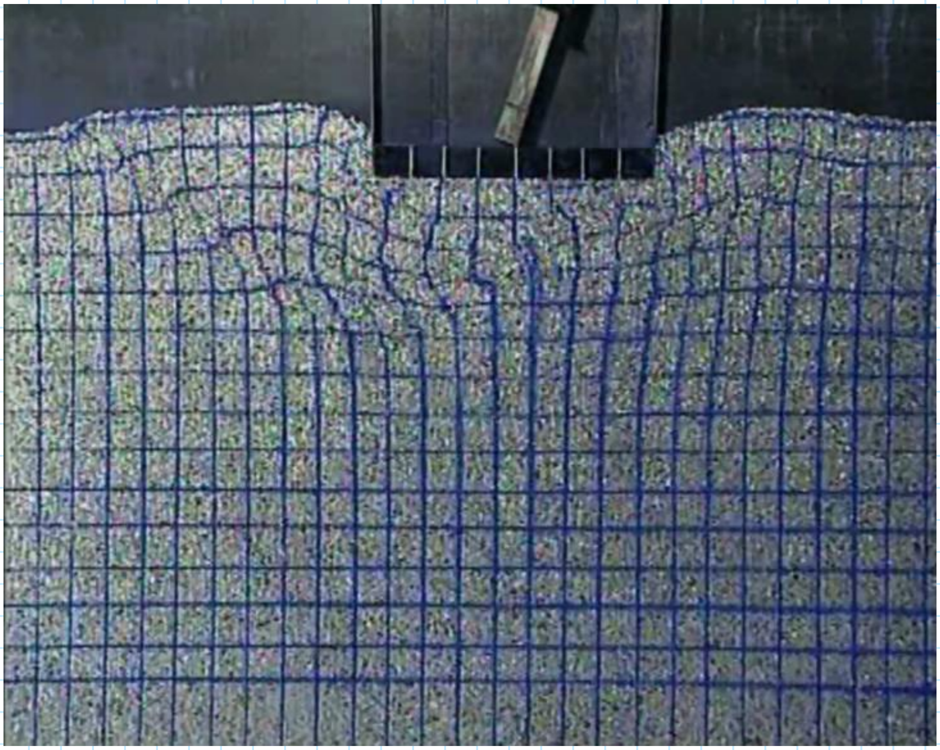
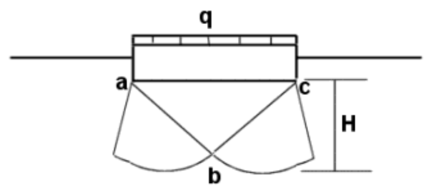
Development of shear planes under footing



General Shear Failure



Punching Shear Failure

Bearing Capacity Failure Examples (cont.)

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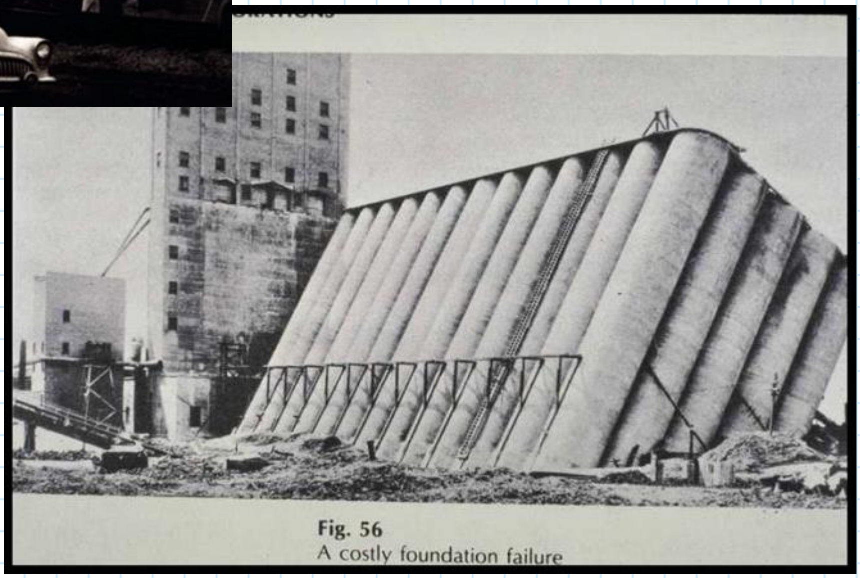
General Bearing Capacity Failure (note relatively large failure surface)

General bearing capacity failure of grain silos



Estimating Type of Failure

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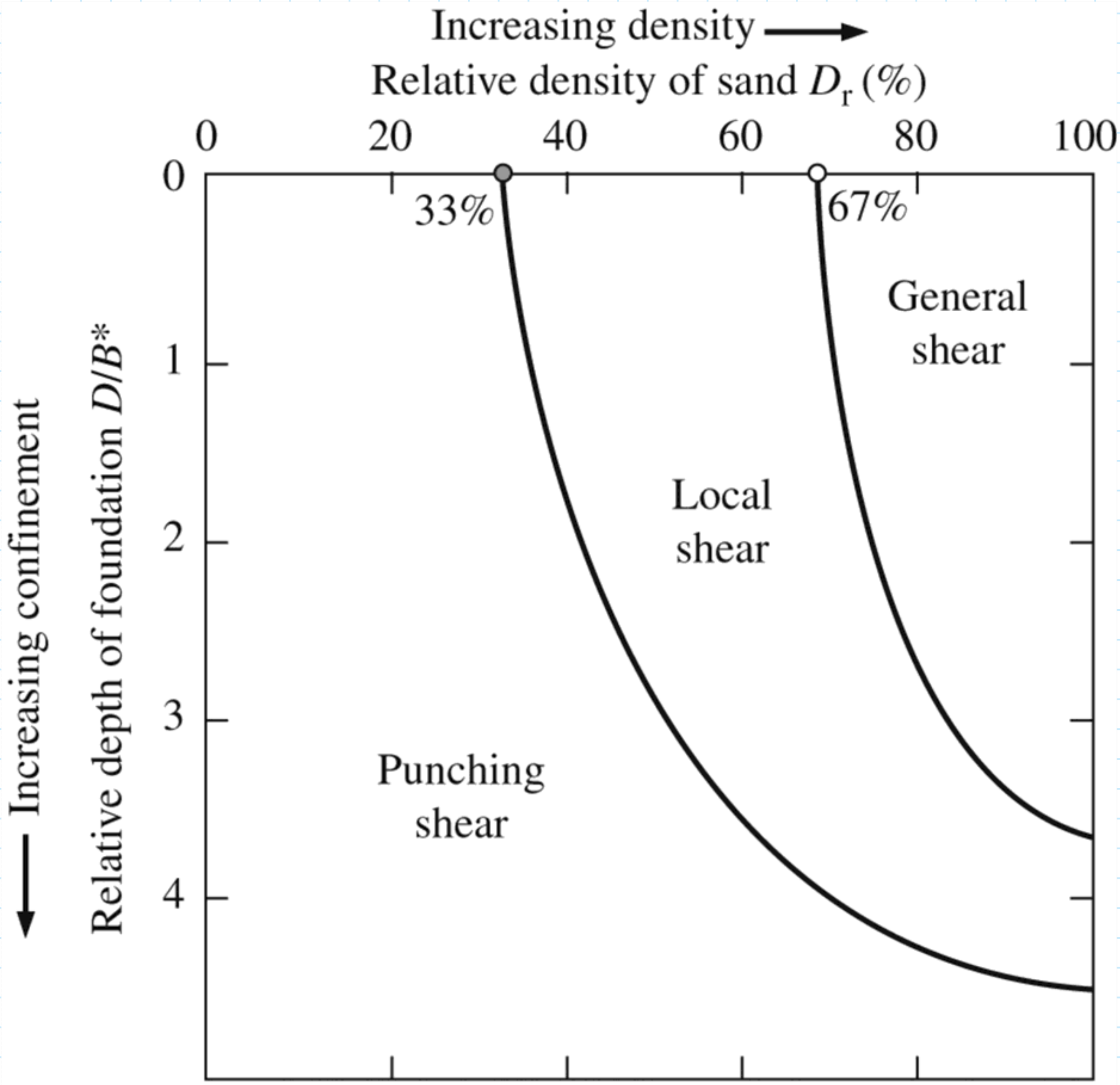

$B^* = $ B for a square or circular footing

$B^* = 2BL/(B + L)$ for a rectangular footing

Failure zones

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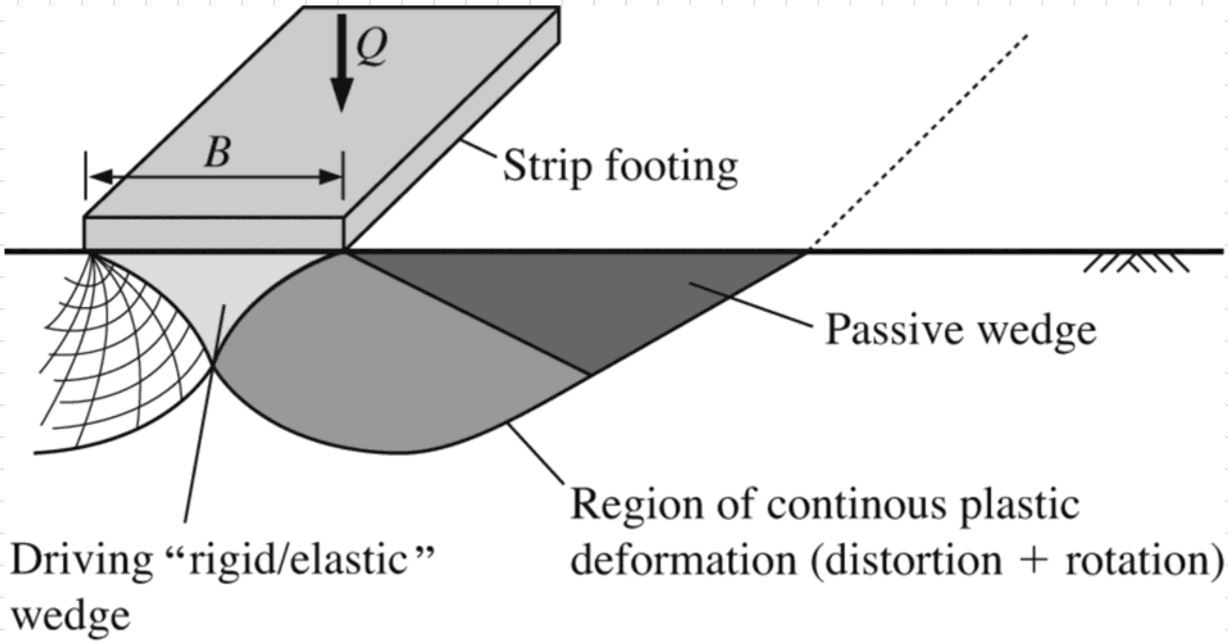

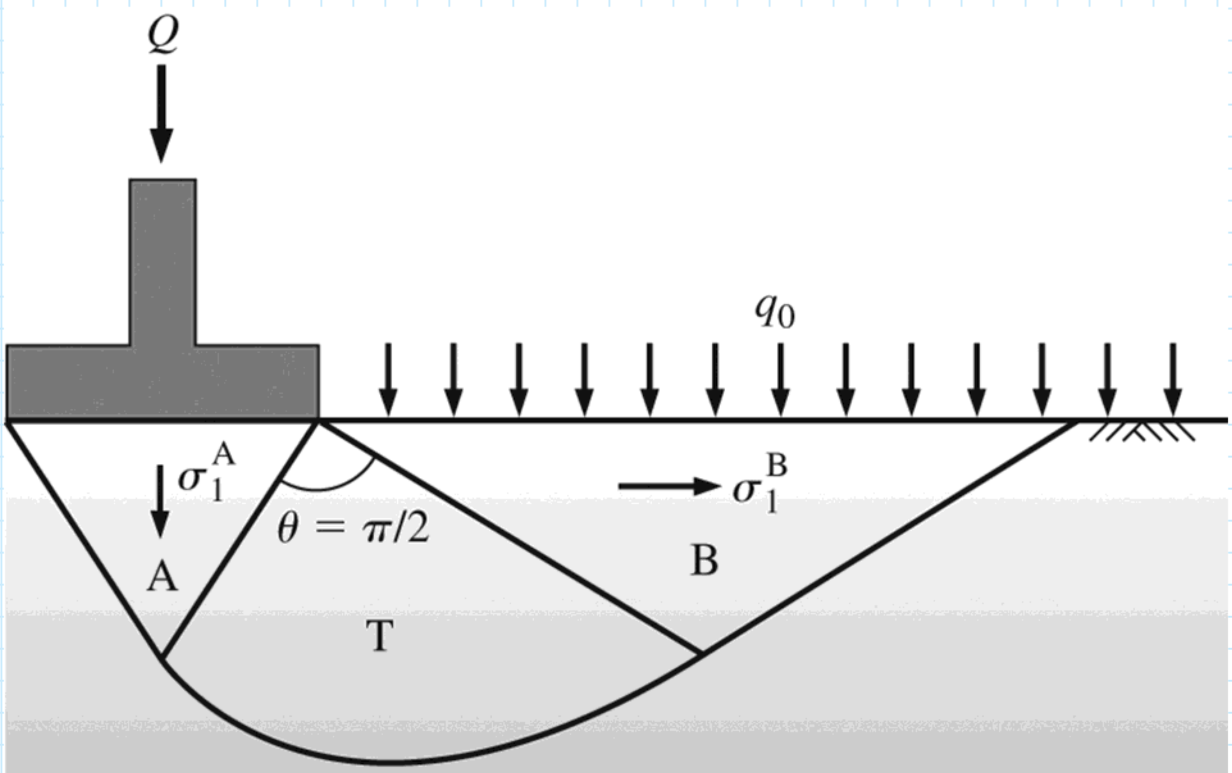

Terzaghi's Bearing Capacity Theory

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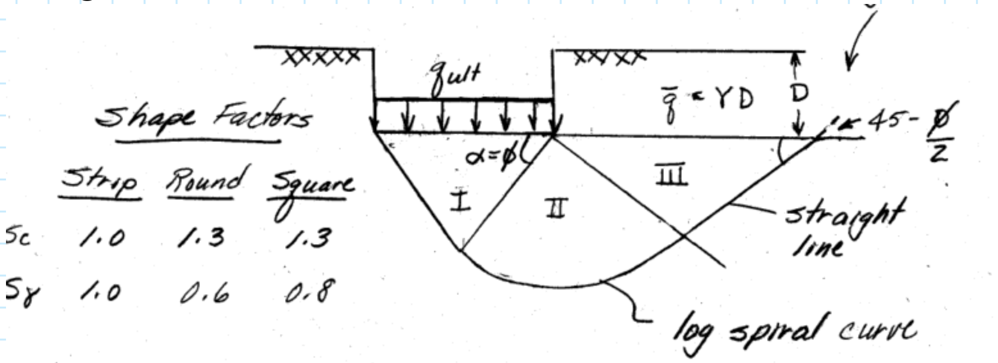
Karl von Terzaghi was the first to present a comprehensive theory for the evaluation of the ultimate bearing capacity of rough shallow foundations. This theory states that a foundation is shallow if its depth is less than or equal to its width.[3] Later investigations, however, have suggested that foundations with a depth, measured from the ground surface, equal to 3 to 4 times their width may be defined as shallow foundations(Das, 2007). Terzaghi developed a method for determining bearing capacity for the general shear failure case in 1943. The equations are given below.



For square foundations:

$$q_{ult} = 1.3c' N_c + \sigma'_{zD} N_q + 0.4\gamma' B N_\gamma$$

For continuous foundations:

$$q_{ult} = c' N_c + \sigma'_{zD} N_q + 0.5\gamma' B N_\gamma$$

For circular foundations:

$$q_{ult} = 1.3c' N_c + \sigma'_{zD} N_q + 0.3\gamma' B N_\gamma$$

where

$$N_q = \frac{e^{2\pi(0.75-\phi'/360)\tan\phi'}}{2\cos^2(45+\phi'/2)}$$

$$N_c = 5.7$$

for ϕ' = 0

$$N_c = \frac{N_q - 1}{\tan\phi'}$$

for ϕ' > 0

$$N_\gamma = \frac{\tan\phi'}{2}\left(\frac{K_{p\gamma}}{\cos^2\phi'} - 1\right)$$

Terzaghi's Bearing Capacity Theory (cont.)

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c' is the effective cohesion.
σ_{zD}' is the vertical effective stress at the base of the foundation
γ' *is the effective unit weight when saturated or the total unit weight when not fully saturated.*
B is the width or the diameter of the foundation.
φ' is the effective internal angle of friction.

$K_{p\gamma}$ is obtained graphically. Simplifications have been made to eliminate the need for $K_{p\gamma}$. One such was done by Coduto, given below, and it is accurate to within 10%.[2]

$$N_\gamma = \frac{2\,(N_q+1)\tan\phi'}{1+0.4\sin 4\phi'}$$

For foundations that exhibit the local shear failure mode in soils, Terzaghi suggested the following modifications to the previous equations. The equations are given below.

For square foundations:
$$q_{ult} = 0.867c'N_c' + \sigma_{zD}'N_q' + 0.4\gamma'BN_\gamma'$$
For continuous foundations:
$$q_{ult} = \frac{2}{3}c'N_c' + \sigma_{zD}'N_q' + 0.5\gamma'BN_\gamma'$$
For circular foundations:
$$q_{ult} = 0.867c'N_c' + \sigma_{zD}'N_q' + 0.3\gamma'BN_\gamma'$$

N'_c, N'_q *and* N'_γ, the modified bearing capacity factors, can be calculated by using the bearing capacity factors equations(for N_c, N_q, *and* N_γ, respectively) by replacing the effective internal angle of friction(ϕ') by a value equal to
: $tan^{-1}\left(\frac{2}{3}tan\phi'\right)$

Pasted from <http://en.wikipedia.org/wiki/Bearing_capacity>

Meyerhof's Bearing Capacity Equation

- Similar to Terzaghi's eq., but has a shape factor, s_q, included with depth terms.
- Also includes depth factor (depth of embedment), d_i and inclination factor, i_i, for loads inclined from vertical.
- developed for strip loadings, but can be applied to square and retangular footings using proper shape factors.
- uses $\alpha = 45 + \phi/2$ (Remember Terzaghi used $\alpha = \phi$ for the inclination of the failure plane with the horizontal.
- Can be used for $D > B$ (Terzaghi's eq. is limited to $D \le B$).
- shear resistance of soil above the base of the footing is neglected.

Meyerhof Equation Vertical Loads

$$q_{ult} = c N_c s_c d_c + \bar{q} N_q s_q d_q + 0.5 \gamma B N_\gamma s_\gamma d_\gamma$$

Meyerhort Equation Inclined Loads

All shape factors, s, are equal to 1.0 when inclined load is present.

$$q_{ult} = c N_c d_c i_c + \bar{q} N_q d_q i_q + 0.5 \gamma B N_\gamma d_\gamma i_\gamma$$

N Factors (Meyerhof)

$$N_q = e^{\pi \tan\phi} \tan^2(45 + \phi/2)$$

$$N_c = (N_q - 1) \cot \phi$$

Nc = 5.14 for $\phi = 0$

$$N_\gamma = (N_q - 1) \tan(1.4\phi)$$

If L'/B' <= 2.0, the phi = phi from triaxial device, else use phi plain strain = 1.5 phi triaxial - 17.

Shape Factors

$$s_c = 1 + 0.2 K_p B/L$$

$$s_q = s_\gamma = 1 + 0.1 K_p B/L \quad (\text{for } \phi > 10°)$$

$$s_q = s_\gamma = 1 \quad (\text{for } \phi = 0)$$

$$K_p = \tan^2(45 + \phi/2)$$

Meyerhof's Bearing Capacity (cont)

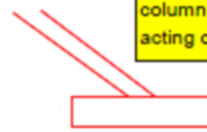
Depth Factors

$d_c = 1 + 0.2\sqrt{K_p}\,\frac{D}{B}$ where $K_p = \tan^2(45 + \phi/2)$

$d_q = d_\gamma = 1 + 0.1\sqrt{K_p}\,\frac{D}{B}$ for $\phi > 10$

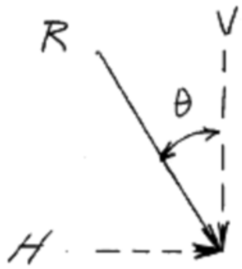
$d_q = d_\gamma = 1$ for $\phi = 0$

Note: An inclined load acting through the center of the footing does not cause a moment about the footing



Inclined load from column or strut acting on footing.

Inclination Factors



$\theta^* < \phi$

θ^* = Angle of resultant measured w/o sign (i.e., θ = positive number regardless of direction from vertical)

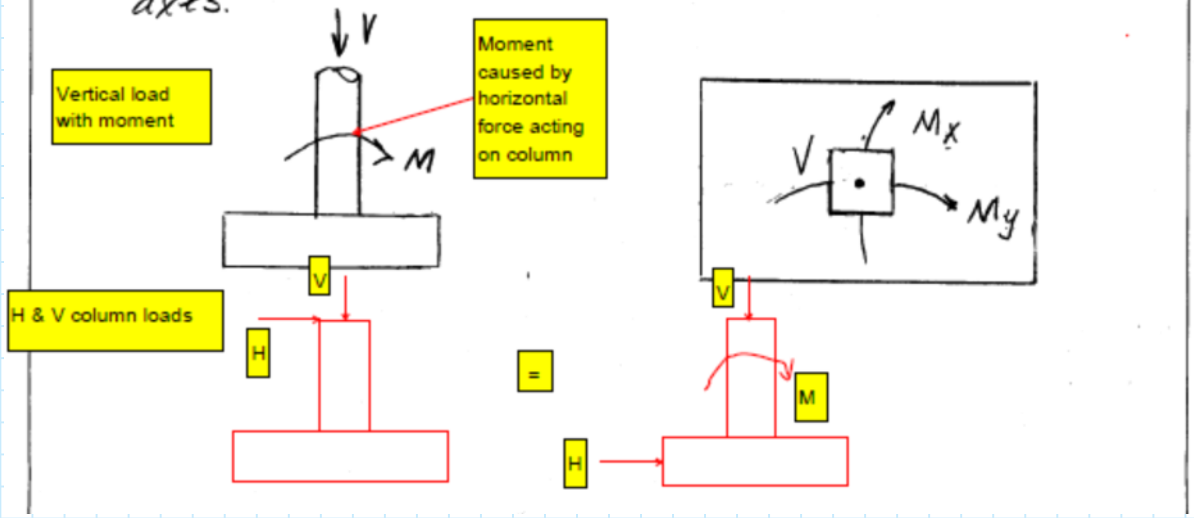
$i_c = i_q = \left(1 - \frac{\theta^\circ}{90}\right)^2$ for any ϕ

$i_\gamma = \left(1 - \frac{\theta^\circ}{\phi^\circ}\right)^2$ for $\phi > 0$

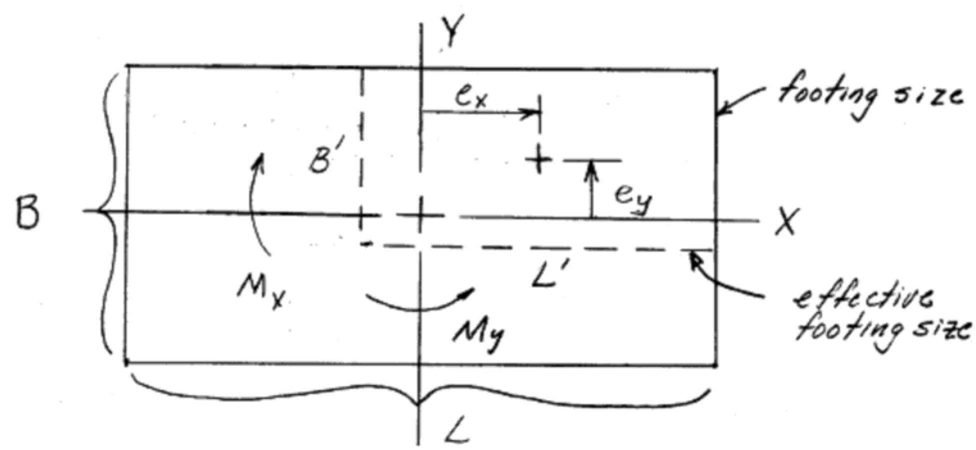
$i_\gamma = 0$ for $\phi = 0$

Modification of Meyerhof's Equation for Eccentric Loads

- footing may be eccentrically loaded from a concentric column with an axial load and moments about one or both axes.



Eccentric Loads (cont.) Definitions



$L' = L - 2e_x$ effective footing length

$B' = B - 2e_y$ effective footing width

$A_f = B'L'$ effective footing area

$e_x = M_y/V$ My = ML eccentricity in x-direction

$e_y = M_x/V$ Mx = MB eccentricity in y-direction

$P_{ult} = q_{ult}(B'L')$

Eccentric Loads (cont.)

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Notes on eccentricity

- Eccentricity should be avoided, when possible
- If req'd, it should be applied in the direction of the largest dimension of footing (i.e., M_y)
- $e/B < 0.2$

B' in eqs. below is the minimum effective footing dimension (see Step 2 on next page.

Final Equations (Eccentric Loads)

Vertical Load with moment

$$q_{ult} = cN_c s_c d_c + \bar{q} N_q s_q d_q + 0.5 \gamma B' N_\gamma s_\gamma d_\gamma \leftarrow$$

Inclined Load with moment

$$q_{ult} = cN_c d_c i_c + \bar{q} N_q d_q i_q + 0.5 \gamma B' N_\gamma d_\gamma i_\gamma \leftarrow$$

Application of Myerhoff's equations for <u>one-way</u> eccentricity

- Must calculate the effective area

<u>Steps</u>

1) For eccentricity in x direction (M_y)

$$L' = L - 2e_x$$
$$B' = B$$

However if eccentricity in y direction (M_x)

$$B' = B - 2e_y$$
$$L' = L$$

2) Compare B' with L', <u>if</u> $B' < L'$

<u>then</u> $B' = B'$ and $L' = L'$

<u>else</u> $B' > L'$

<u>then</u> $B' = L'$ and $L' = B'$

3) Apply Meyerhof's equations using

B' for B in shape and depth factor eq's

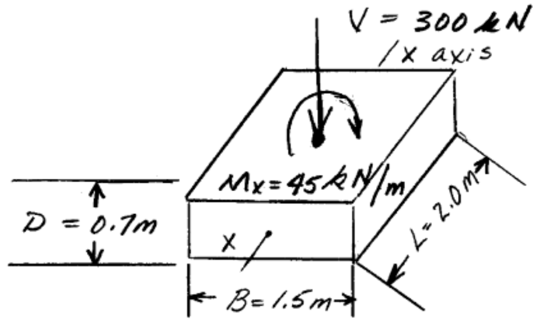
L' for L in shape factor eq's

- Problem Statement:

Calculate the ultimate bearing capacity (q_{ult}), the allowable bearing pressure ($q_{allowable}$) and the factor of safety against bearing capacity for the footing, soil and loading conditions shown below.

- $q_{ult} = ?$
- $q_{allowable} = ?$
- $F.S. = ?$

- Given:



D = depth of embedment

B = footing width

L = footing length

V = vertical load

M_x = moment about x axis (horizontal axis)

Sand (med. dense)

$\gamma = 18\ kN/m^3$

$\phi' = \phi = 30$ deg (Use peak friction angle, ϕ_p for phi)

$c = 0$

- Solution

- $q_{ult} = \overset{0}{\cancel{c}} N_c \overset{0}{\cancel{s_c}} d_c + \bar{q} N_q s_q d_q + 0.5 \gamma B' N_\gamma s_\gamma d_\gamma$

- $q_{allowable} = \dfrac{q_{ult}}{F.S.}$

- $F.S. = \dfrac{q_{ult}}{q_{actual}}$

Note: B' is used instead of B because of eccentric load

Example (cont.)

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- Solution (cont.)

$$\bar{q} = (D)(\gamma) = (0.7m)(18 \frac{kN}{m^3}) = 12.6\ kPa$$

$$N_q = e^{\pi \tan\phi} \tan^2(45 + \phi/2)$$

$$N_q = (e^{\pi \tan 30^\circ}) \tan^2(45 + \frac{30}{2}) = 18.40$$

$$s_q = 1 + 0.1 K_p\ B'/L'$$

(Note: In the above equation, we have used B' and L' instead of B and L because an eccentric load exists)

$$B' = B - 2e_y$$
$$e_y = M_x/V$$
$$M_x = 45\ kN/m$$
$$e_y = 45 \frac{kN}{m} / 300\ kN$$
$$e_y = 0.15\ m$$
$$B' = 1.5m - 2(0.15m)$$
$$B' = 1.2\ m$$
$$L' = \cancel{B - 2e_x}\ \ L - 2e_x$$
$$e_x = M_y/V$$
$$M_y = 0$$
$$e_x = 0$$
$$L' = L$$
$$L' = 2.0m$$

(Check to see that $B' < L'$ otherwise $B' = L'$ and $L' = B'$. See page 8b of 8 in notes)

$$s_q = 1 + 0.1\left(\tan^2(45 + \frac{\phi}{2})\right)\frac{B'}{L'}$$
$$= 1 + 0.1\left(\tan^2(45 + \frac{30}{2})\right)(1.2m)/(2.0m)$$
$$s_q = 1.18$$
$$d_q = 1 + 0.1\sqrt{K_p}\ \frac{D}{B'}$$
$$d_q = 1 + 0.1\sqrt{\tan^2(45 + \frac{30}{2})}\ \left(\frac{0.7m}{1.2m}\right)$$
$$d_q = 1.10$$

Example (cont.)

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$N_\gamma = (N_q - 1)\tan(1.4\phi)$

$N_q = 18.4$ (previous page)

$N_\gamma = (18.4 - 1)(\tan(1.4(30°))$

$N_\gamma = 15.67$

$s_\gamma = s_q$

$s_q = 1.18$ (from previous page)

$d_\gamma = d_q$

$d_q = 1.10$ (from previous page)

$q_{ult} = \bar{q}\, N_q\, s_q\, d_q + 0.5\,\gamma\, B'\, N_\gamma\, s_\gamma\, d_\gamma$

$q_{ult} = (12.6\ kPa)(18.40)(1.18)(1.10) + (0.5)(18\ \frac{kN}{m^3})(1.2\ m)$
$(15.67)(1.18)(1.10) =$

$q_{ult} = (301 + 220)$

$q_{ult} = 521\ kPa \leftarrow$

$q_{allowable} = \frac{q_{ult}}{F.S.}$

$F.S. = ?$ Use Fig. 10-20 of Salgado

- $F.S. = 2$ (local shear failure)
- $F.S. = 3$ (general shear failure)

$D_f = D = 0.7$

$B^o = \frac{2B'L'}{B' + L'}$

$B^o = \frac{(2)(1.2m)(2.0m)}{(1.2 + 2.0)m} = 1.5$

$D_f/B' = 0.7/1.5 = 0.47$

$D_r \approx 35\% = 0.35$ from Table 3-4 (Bowles)

$\therefore$ Local shear failure controls from Fig. 3.3 (Das)

F.S. = 2 (design factor of safety)

$$q_{allowable} = \frac{521 \text{ kPa}}{2}$$

$q_{allowable} = 260$ kPa ← Report this to structural engineer w/ a F.S. = 2.0

$$F.S._{actual} = \frac{V_{ult}}{V_{actual}}$$

$$F.S._{actual} = \frac{(q_{ult})(B')(L')}{V}$$

$$F.S._{actual} = \frac{(521)(1.2)(2.0)}{300}$$

$F.S._{actual} = 4.2$ ←

Footing could be resized because F.S. 4.2 > 2.0

Try B = 1.2, L = 1.5 ⇒

$q_{ult} = 480$ kPa

$V_{ult} = 649$ kPa

$$F.S. = \frac{649}{300}$$

F.S. = 2.16 ←

Spreadsheet Calculation

Wednesday, August 17, 2011 12:45 PM

Blue = inputs, red = equations

Myerhoff Bearing Capacity Equation 4.168624

Footing Data:			Loading Data:			Soil Data:		
B =	1.50	m	V =	300	kN	γ' =	18.00	kN/m^3
L =	2.00	m	H =	0	kN	ϕ_{tr} =	30	deg.
L'/B' =	1.667		M_B =	45	kN-m	c =	0	kPa
D =	0.70	m	M_L =	0	kN-m	ϕ_{ps} =	30	deg.
D/B' =	0.583		θ =	0	deg.	If L'/B' >= 2.0, ϕ_{ps} is used in place of ϕ_{tr}.		
$\overline{q} = \gamma' D$ =	12.6	kPa						

$$\phi_{ps} = 1.5\phi_{tr} - 17^\circ \; for \; \phi_{tr} > 34^\circ$$

Use ϕ = 30 deg.

$$B' = B - 2e_B = B - \frac{2M_B}{V} = \quad 1.20 \text{ m}$$

$$L' = L - 2e_L = L - \frac{2M_L}{V} = \quad 2.00 \text{ m}$$

Shape Factors:

$$s_c = 1 + 0.2\tan^2\left(45^\circ + \phi/2\right)\frac{B'}{L'} = \quad 1.360$$

$$s_q = 1 + 0.1\tan^2\left(45^\circ + \phi/2\right)\frac{B'}{L'} = \quad 1.180$$

$$s_\gamma = 1 + 0.1\tan^2\left(45^\circ + \phi/2\right)\frac{B'}{L'} = \quad 1.180$$

Depth Factors:

$$d_c = 1 + 0.2\tan\left(45^\circ + \phi/2\right)\frac{D}{B'} = \quad 1.202$$

$$d_q = 1 + 0.1\tan\left(45^\circ + \phi/2\right)\frac{D}{B'} = \quad 1.101$$

$$d_\gamma = 1 + 0.1\tan\left(45^\circ + \phi/2\right)\frac{D}{B'} = \quad 1.101$$

Inclination Factors:

$$i_c = \left(1 - \theta/90^\circ\right)^2 = \quad 1.000$$

$$i_q = \left(1 - \theta/90^\circ\right)^2 = \quad 1.000$$

$$i_\gamma = \left(1 - \theta^\circ/\phi^\circ\right)^2 = \quad 1.000 \quad \text{for } \phi > 0$$

$$i_\gamma = \quad 0 \quad \text{for } \phi = 0$$

Use i_γ = 1.000

Bearing Capacity Factors:

$$N_q = e^{\pi\tan\phi} \cdot \tan^2\left(45^\circ + \phi/2\right) = \quad 18.40$$

$$N_c = \frac{(N_q - 1)}{\tan\phi} = \quad 30.14$$

$$N_\gamma = (N_q - 1) \cdot \tan(1.4\phi) = \quad 15.67$$

Vertical Load: $q_{ult} = cN_c s_c d_c + \overline{q} N_q s_q d_q + 0.5\gamma B' N_\gamma s_\gamma d_\gamma$

Inclined Load: $q_{ult} = cN_c d_c i_c + \overline{q} N_q d_q i_q + 0.5\gamma B' N_\gamma d_\gamma i_\gamma$

LOAD IS VERTICAL

N_c term (kPa)	N_q term (kPa)	N_γ term (kPa)	q_{ult} (kPa)
0.0	301.2	219.8	521.1

$$V_{ult} = q_{ult} \cdot B' \cdot L' = \quad 1,251 \text{ kN}$$

SF = 4.2

Meyerhof.xlsx, Page 1 of 1

Correction of phi for plane strain

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If L'/B' > 2.0, ϕ_{Ps} is used in place of ϕ_{tr}.

$\phi_{ps} = 1.5\phi_{tr} - 17° \textit{ for}, \phi_{tr} > 34°$

ϕ_{ps} = phi adjusted for plane strain conditions (L'/B' > 2) for Myerhof Method

ϕ_{tr} = phi from peak friction angle ϕ_p obtained from triaxial conditions

Correction for B > 2 m

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There is some evidence, from using small footings up to about 1 m for B, that the BN_γ term does not increase the bearing capacity without bound, and for very large values of B both Vesić (1969) and De Beer (1965) suggest that the limiting value of q_{ult} approaches that of a deep foundation. The author suggests the following reduction factor:

$$r_\gamma = 1 - 0.25 \log\left(\frac{B}{\kappa}\right) \qquad B \geq 2 \text{ m (6 ft)}$$

where $\kappa = 2.0$ for SI and 6.0 for fps. This equation gives the following results:

$B = 2$	2.5	3	3.5	4	5	10	20	100 m
$r_\gamma = 1.0$	0.97	0.95	0.93	0.92	0.90	0.82	0.75	0.57

One can use this reduction factor with any of the bearing-capacity methods to give

$$\boxed{0.5\gamma B N_\gamma s_\gamma d_\gamma r_\gamma}$$

Effects of Water Table

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The *effective* unit weight of the soil is used in the bearing-capacity equations for computing the ultimate capacity. This has already been defined for $\overline{q}$ in the $\overline{q}N_q$ term. A careful inspection of Fig. 4-3 indicates that the wedge term $0.5\gamma BN_\gamma$ also uses the effective unit weight for the soil.

The water table is seldom above the base of the footing, as this would, at the very least, cause construction problems. If it is, however, the $\overline{q}$ term requires adjusting so that the surcharge pressure is an effective value. This computation is a simple one involving computing the pressure at the GWT using that depth and the wet unit weight + pressure from the GWT to the footing base using that depth × effective unit weight γ'. If the water table is at the ground surface, the effective pressure is approximately one-half that with the water table at or below the footing level, since the effective unit weight γ' is approximately one-half the saturated unit weight.

When the water table is below the wedge zone [depth approximately $0.5B\tan(45 + \phi/2)$], the water table effects can be ignored for computing the bearing capacity. When the water table lies within the wedge zone, some small difficulty may be obtained in computing the effective unit weight to use in the $0.5\gamma BN_\gamma$ term. In many cases this term can be ignored for a conservative solution since we saw in Example 4-1 that its contribution is not substantial (see also following Example). In any case, if B is known, one can compute the average effective weight γ_e of the soil in the wedge zone as

$$\gamma_e = (2H - d_w)\frac{d_w}{H^2}\gamma_{\text{wet}} + \frac{\gamma'}{H^2}(H - d_w)^2 \qquad (4\text{-}4)$$

where $H = 0.5B\tan(45° + \phi/2)$

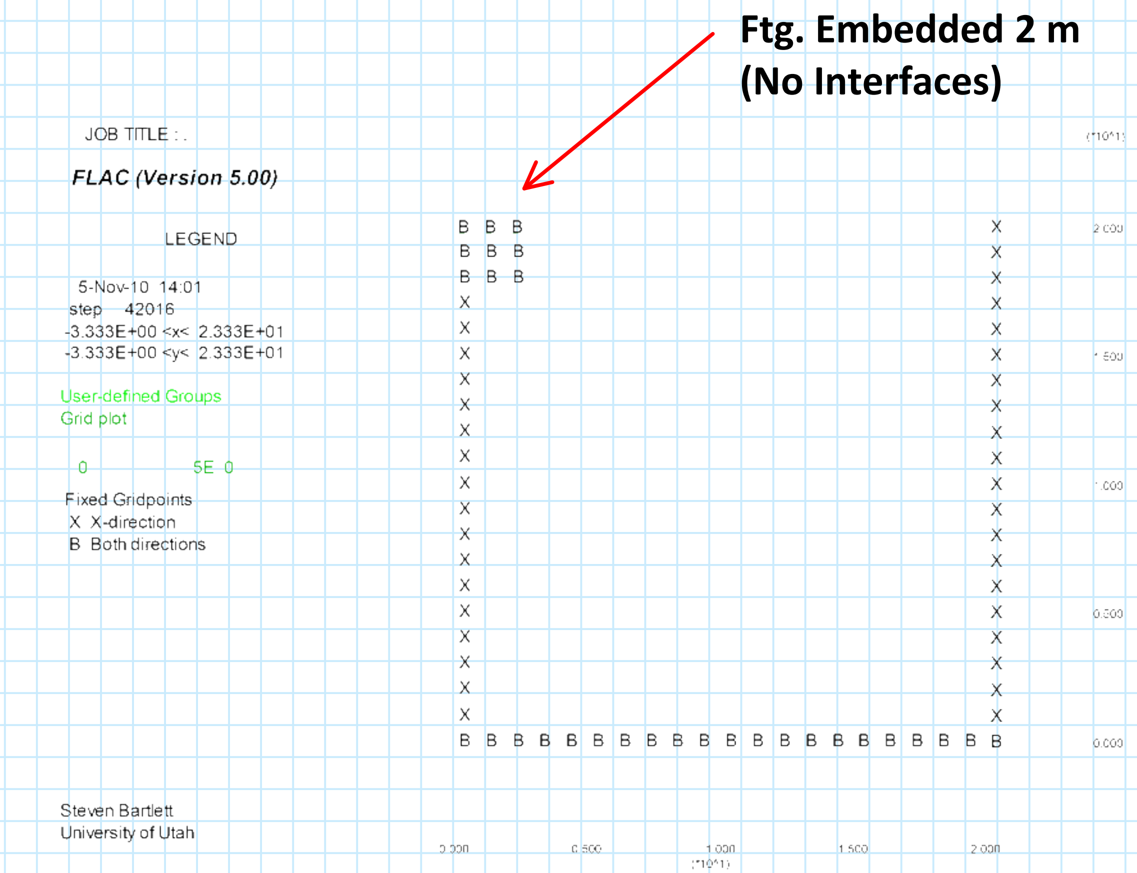
d_w = depth to water table below base of footing

γ_{wet} = wet unit weight of soil in depth d_w

γ' = submerged unit weight below water table = $\gamma_{\text{sat}} - \gamma_w$

Numerical Model - Embedment of Footing

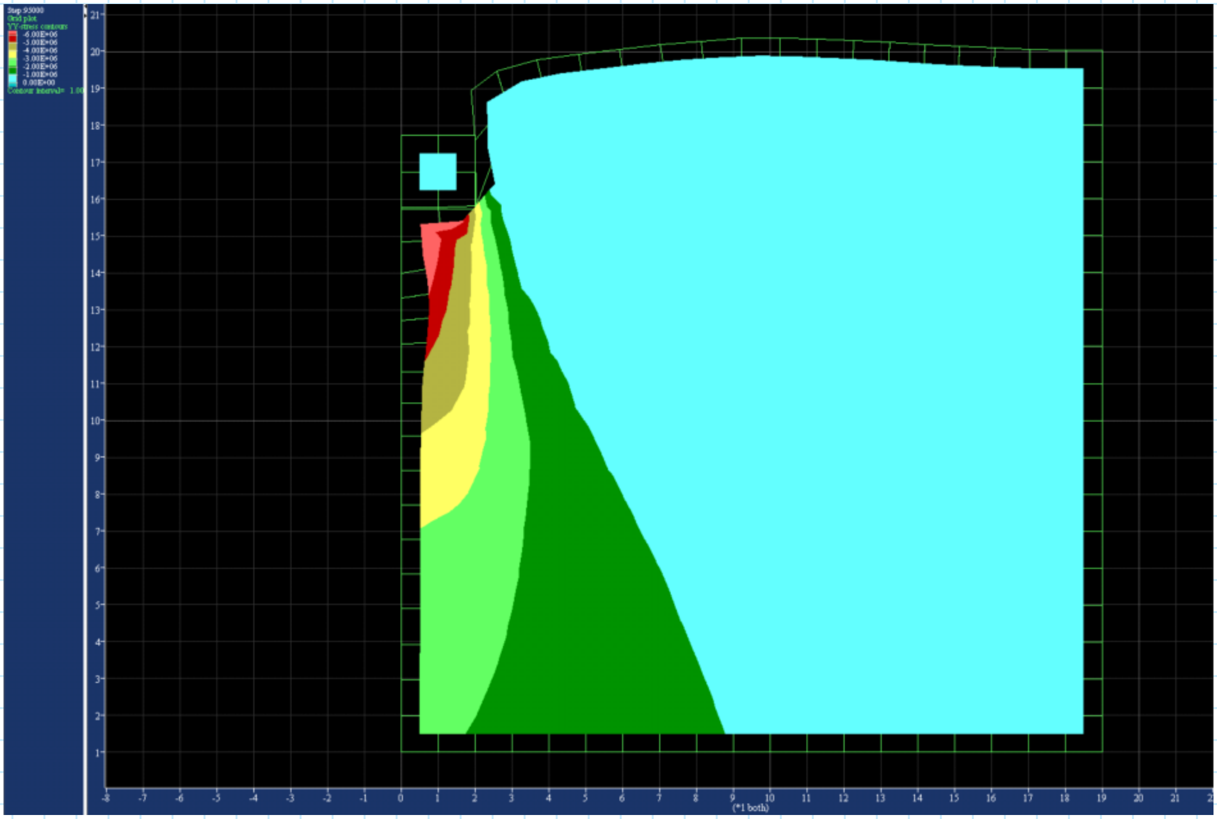
Thursday, March 11, 2010 11:43 AM



properties s=40e6 bul=80e6 d 2000 coh 0 fric 35.0 dilation 0 ten 0

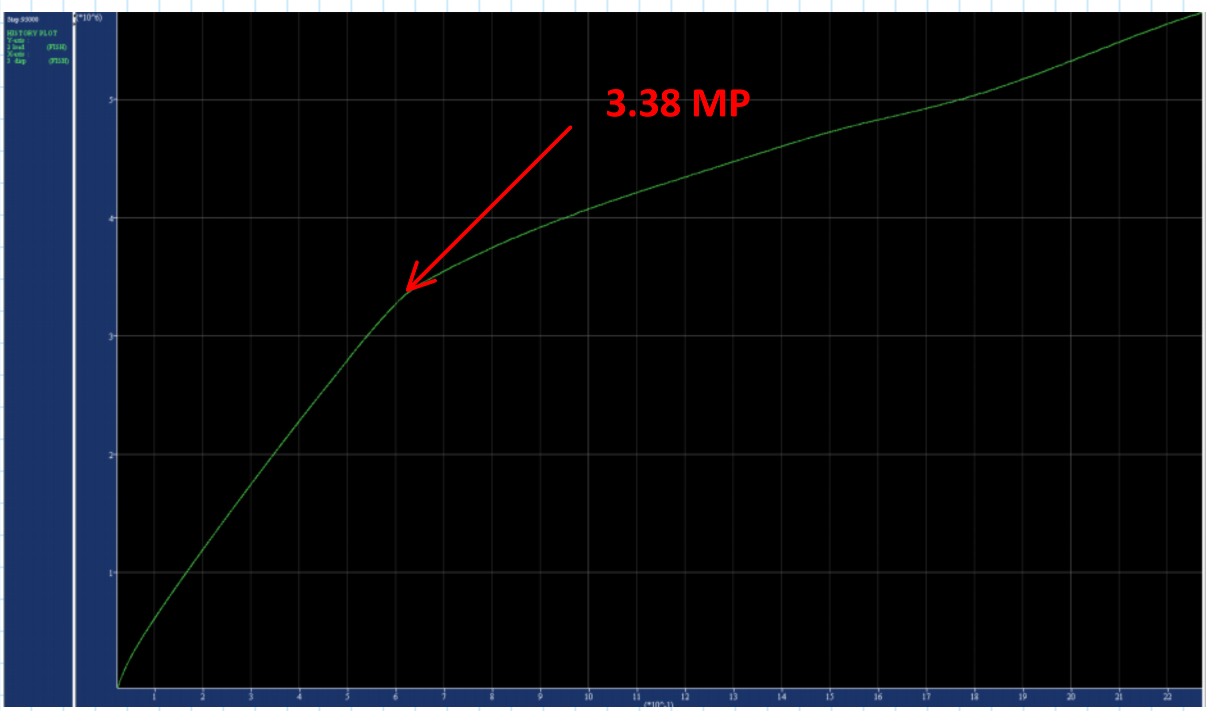
Embedment of Footing - Numerical Model

Thursday, March 11, 2010 11:43 AM



Vertical = Horizontal scale = 1 m

(Mpa)



(cm)

Embedment of Footing - Numerical Model

Thursday, March 11, 2010 11:43 AM

Bearing Capacity, Vesic Equation (*Valid only for ϕ>0)
Reference: American Society of Civil Engineers, US Army Corps of Engineers Technical Engineering and Design Guide No. 7, Bearing Capacity of Soils, 1993, pp. 26-32.
Friction Angle estimate: Peck, Hansen, and Thornburn, Foundation Engineering, 1974, p. 310.

Effective Friction Angle	$\phi = 35 \cdot deg$	Footing Width	$B_1 = 16.4 \cdot ft$
Cohesion	$C = 0 \cdot psf$	Footing Depth	$D_1 = 6.56 \cdot ft$
Effective unit weight of soil above footing	$\gamma_H = 124.8 \cdot pcf$	Footing Length	$L_1 = 1.64 \times 10^3 \cdot ft$
		Factor of Safety	$FS = 1$ ←
Effective unit weight of soil beneath footings	$\gamma_D = 124.8 \cdot pcf$	N-value	$N = 26.205$

Bearing Capacity Factors

$N_q := e^{\pi \cdot \tan(\phi)} \cdot \left(\frac{1 + \sin(\phi)}{1 - \sin(\phi)} \right)$ $N_q = 33.3$

$N_\gamma := 2 \cdot (N_q + 1) \cdot \tan(\phi)$ $N_\gamma = 48.03$

$N_c := (N_q - 1) \cdot \cot(\phi)$ $N_c = 46.12$

Shape Factors

$s_c := 1 + \frac{N_q}{N_c} \cdot \frac{B_1}{L_1}$ $s_q := 1 + \frac{B_1}{L_1} \cdot \tan(\phi)$ $s_\gamma := 1 - 0.4 \cdot \frac{B_1}{L_1}$

$s_c = 1.01$ $s_q = 1.01$ $s_\gamma = 1.00$

Depth Factors $k := \text{if}\left(\frac{D_1}{B_1} \le 1, \frac{D_1}{B_1}, \text{atan}\left(\frac{D_1}{B_1} \right) \right)$ $k = 0.4$

$d_c := 1 + 0.4 \cdot k$ $d_q := 1 + 2 \cdot \tan(\phi) \cdot (1 - \sin(\phi))^2 \cdot k$ $d_\gamma := 1.0$

$d_c = 1.16$ $d_q = 1.10$ $d_\gamma = 1.00$

Allowable Bearing Capacity, q_a

$q_{u2} := C \cdot N_c \cdot s_c \cdot d_c + D_1 \cdot \gamma_D \cdot N_q \cdot s_q \cdot d_q + 0.5 \cdot \gamma_H \cdot B_1 \cdot N_\gamma \cdot s_\gamma \cdot d_\gamma$

$q_{a2} := \frac{q_{u2}}{FS}$ $q_{a2} = 79.20 \cdot ksf$ ← 3.8 MPa

Embedment of Footing - Numerical Model

Thursday, March 11, 2010 11:43 AM

Bearing Capacity, Meyerhof Equation (*Valid only for ϕ>0)
Reference: American Society of Civil Engineers, US Army Corps of Engineers Technical Engineering and Design Guide No. 7, Bearing Capacity of Soils, 1993, pp. 26-32.
Friction Angle estimate: Peck, Hansen, and Thornburn, Foundation Engineering, 1974, p. 310.

Effective Friction Angle	$\phi = 35 \cdot deg$	Footing Width	$B_1 = 16.4 \cdot ft$
Cohesion	$C = 0 \cdot psf$	Footing Depth	$D_1 = 6.56 \cdot ft$
Effective unit weight of soil above footing	$\gamma_H = 124.8 \cdot pcf$	Footing Length	$L_1 = 1.64 \times 10^3 \cdot ft$
Effective unit weight of soil beneath footings	$\gamma_D = 124.8 \cdot pcf$	Factor of Safety	$FS = 1$
		N-value	$N = 26.205$

Bearing Capacity Factors

$$N_\phi := \left(\frac{1+\sin(\phi)}{1-\sin(\phi)}\right) \qquad N_\phi = 3.69$$

$$N_q := e^{\pi \cdot \tan(\phi)} \cdot N_\phi \qquad N_q = 33.3$$

$$N_\gamma := (N_q - 1) \cdot \tan(1.4 \cdot \phi) \qquad N_\gamma = 37.15$$

$$N_c := (N_q - 1) \cdot \cot(\phi) \qquad N_c = 46.12$$

Shape Factors

$$s_c := 1 + 0.2 \cdot N_\phi \cdot \frac{B_1}{L_1} \qquad s_q := 1 + 0.1 \cdot N_\phi \cdot \frac{B_1}{L_1} \qquad s_\gamma := 1 + 0.1 \cdot N_\phi \cdot \frac{B_1}{L_1}$$

$$s_c = 1.01 \qquad s_q = 1.00 \qquad s_\gamma = 1.00$$

Depth Factors

$$d_c := 1 + 0.2 \cdot N_\phi^{0.5} \cdot \frac{D_1}{B_1} \qquad d_q := 1 + 0.1 \cdot N_\phi^{0.5} \cdot \frac{D_1}{B_1} \qquad d_\gamma := 1 + 0.1 \cdot N_\phi^{0.5} \cdot \frac{D_1}{B_1}$$

$$d_c = 1.15 \qquad d_q = 1.08 \qquad d_\gamma = 1.08$$

Allowable Bearing Capacity, q_a

$$q_{u3} := C \cdot N_c \cdot s_c \cdot d_c + D_1 \cdot \gamma_D \cdot N_q \cdot s_q \cdot d_q + 0.5 \cdot \gamma_H \cdot B_1 \cdot N_\gamma \cdot s_\gamma \cdot d_\gamma$$

$$q_{a3} := \frac{q_{u3}}{FS} \qquad q_{a3} = 70.55 \cdot ksf \longleftarrow \text{3.4 Mpa}$$

Numerical Model - Embedment of Footing

Thursday, March 11, 2010 11:43 AM

Bearing Capacity, Hansen Equation (*Valid only for ϕ>0)
Reference: American Society of Civil Engineers, US Army Corps of Engineers Technical Engineering and Design Guide No. 7, Bearing Capacity of Soils, 1993, pp. 26-32.
Friction Angle estimate: Peck, Hansen, and Thornburn, Foundation Engineering, 1974, p. 310.

$pcf := 1 \cdot \frac{lb}{ft^3}$ $psf := 1 \cdot \frac{lb}{ft^2}$ $ksf := 1000 \cdot psf$

Effective Friction Angle	$\phi := 35deg$	Footing Width	$B_1 := 16.4 \cdot ft$
Cohesion	$C := 0 \cdot psf$	Footing Depth	$D_1 := 6.56 \cdot ft$
Effective unit weight of soil above footing	$\gamma_H := 124.8 \cdot pcf$	Footing Length	$L_1 := 1640 \cdot ft$
		Factor of Safety	$FS := 1$
Effective unit weight of soil beneath footings	$\gamma_D := 124.8 \cdot pcf$	N-value	$N = 26.205$

Bearing Capacity Factors

$N_q := e^{\pi \cdot \tan(\phi)} \cdot \left(\frac{1+\sin(\phi)}{1-\sin(\phi)}\right)$ $N_q = 33.3$

$N_\gamma := 1.5 \cdot (N_q - 1) \cdot \tan(\phi)$ $N_\gamma = 33.92$

$N_c := (N_q - 1) \cdot \cot(\phi)$ $N_c = 46.12$

Shape Factors

$s_c := 1 + \frac{N_q}{N_c} \cdot \frac{B_1}{L_1}$ $s_q := 1 + \frac{B_1}{L_1} \cdot \tan(\phi)$ $s_\gamma := 1 - 0.4 \cdot \frac{B_1}{L_1}$

$s_c = 1.01$ $s_q = 1.01$ $s_\gamma = 1.00$

Depth Factors $k := \mathrm{if}\left(\frac{D_1}{B_1} \le 1, \frac{D_1}{B_1}, \mathrm{atan}\left(\frac{D_1}{B_1}\right)\right)$ $k = 0.4$

$d_c := 1 + 0.4 \cdot k$ $d_q := 1 + 2 \cdot \tan(\phi) \cdot (1 - \sin(\phi))^2 \cdot k$ $d_\gamma := 1.0$

$d_c = 1.16$ $d_q = 1.10$ $d_\gamma = 1.00$

Allowable Bearing Capacity, q_a

$q_{u2} := C \cdot N_c \cdot s_c \cdot d_c + D_1 \cdot \gamma_D \cdot N_q \cdot s_q \cdot d_q + 0.5 \cdot \gamma_H \cdot B_1 \cdot N_\gamma \cdot s_\gamma \cdot d_\gamma$

$q_{a1} := \frac{q_{u2}}{FS}$ $q_{a1} = 64.82 \cdot ksf$ ← 3.1 Mpa

Embedment of Footing - Model Comparison

Thursday, March 11, 2010 11:43 AM

Summary

Method	Allowable Bearing Capacity
1. Hansen	$q_{a1} = 64.8 \cdot ksf$
2. Vesic	$q_{a2} = 79.2 \cdot ksf$
3. Meyerhof	$q_{a3} = 70.6 \cdot ksf$
4. Meyerhof (settlement = 1 in.)	$q_{a4} = 8.3 \cdot ksf$

$$Average := g \cdot \frac{q_{a1} + q_{a2} + q_{a3}}{3}$$

$$Average = 3.425 \times 10^{6} m^{-1} \cdot kg \cdot s^{-2} \quad \text{or Pa}$$

More Spreadsheet Examples - Example 1

Wednesday, August 17, 2011 12:45 PM

Myerhoff Bearing Capacity Equation 2.049238

Footing Data:

B =	1.00	m
L =	4.20	m
L'/B' =	4.100	
D =	0.70	m
D/B' =	0.700	
$\bar{q} = \gamma' D =$	12.6	kPa

Loading Data:

V =	3,000	kN
H =	1,000	kN
M_B =	0	kN-m
M_L =	150	kN-m
θ =	18.43495	deg.

Soil Data:

γ' =	18.00	kN/m³
ϕ_{tr} =	40	deg.
c =	0	kPa
ϕ_{ps} =	43	deg.

If L'/B' >= 2.0, ϕ_{ps} is used in place of ϕ_{tr}.

$\phi_{ps} = 1.5\phi_{tr} - 17^\circ$ for $\phi_{tr} > 34^\circ$

Use ϕ = 43 deg.

$$B' = B - 2e_B = B - \frac{2M_B}{V} = \quad 1.00 \text{ m}$$

$$L' = L - 2e_L = L - \frac{2M_L}{V} = \quad 4.10 \text{ m}$$

Shape Factors:

$$s_c = 1 + 0.2\tan^2\left(45^\circ + \phi/2\right)\frac{B'}{L'} = \quad 1.000$$

$$s_q = 1 + 0.1\tan^2\left(45^\circ + \phi/2\right)\frac{B'}{L'} = \quad 1.000$$

$$s_\gamma = 1 + 0.1\tan^2\left(45^\circ + \phi/2\right)\frac{B'}{L'} = \quad 1.000$$

Depth Factors:

$$d_c = 1 + 0.2\tan\left(45^\circ + \phi/2\right)\frac{D}{B'} = \quad 1.322$$

$$d_q = 1 + 0.1\tan\left(45^\circ + \phi/2\right)\frac{D}{B'} = \quad 1.161$$

$$d_\gamma = 1 + 0.1\tan\left(45^\circ + \phi/2\right)\frac{D}{B'} = \quad 1.161$$

Inclination Factors:

$$i_c = \left(1 - \theta/90^\circ\right)^2 = \quad 0.632$$

$$i_q = \left(1 - \theta/90^\circ\right)^2 = \quad 0.632$$

$$i_\gamma = \left(1 - \theta^\circ/\phi^\circ\right)^2 = \quad 0.326 \quad \text{for } \phi > 0$$

$$i_\gamma = \quad 0 \quad \text{for } \phi = 0$$

Use i_γ = 0.326

Bearing Capacity Factors:

$$N_q = e^{\pi \tan\phi} \cdot \tan^2\left(45^\circ + \phi/2\right) = \quad 99.01$$

$$N_c = \frac{(N_q - 1)}{\tan\phi} = \quad 105.11$$

$$N_\gamma = (N_q - 1)\cdot \tan(1.4\phi) = \quad 71.14$$

Vertical Load: $q_{ult} = cN_c s_c d_c + \bar{q} N_q s_q d_q + 0.5\gamma B' N_\gamma s_\gamma d_\gamma$

Inclined Load: $q_{ult} = cN_c d_c i_c + \bar{q} N_q d_q i_q + 0.5\gamma B' N_\gamma d_\gamma i_\gamma$

LOAD IS INCLINED

N_c term (kPa)	N_q term (kPa)	N_γ term (kPa)	q_{ult} (kPa)
0.0	915.8	583.6	1,499.4

$$V_{ult} = q_{ult} \cdot B' \cdot L' = \quad 6,148 \text{ kN}$$

SF = 2.0

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More Spreadsheet Examples - Example 2

Myerhoff Bearing Capacity Equation Wednesday, August 17, 2011 2:00 PM

Footing Data:			Loading Data:			Soil Data:		
B =	1.00	m	V =	320	kN	γ' =	18.00	kN/m^3
L =	1.00	m	H =	90	kN	ϕ_{tr} =	40	deg.
L'/B' =	1.455		M_B =	50	kN-m	c =	0	kPa
D =	0.70	m	M_L =	0	kN-m	ϕ_{ps} =	43	deg.
D/B' =	1.018		θ =	15.70864	deg.			
$\bar{q} = \gamma' D =$	12.6	kPa						

If L'/B' >= 2.0, ϕ_{ps} is used in place of ϕ_{tr}.

$\phi_{ps} = 1.5\phi_{tr} - 17^\circ \; for \; \phi_{tr} > 34^\circ$

Use ϕ = 40 deg.

$$B' = B - 2e_B = B - \frac{2M_B}{V} = \quad 0.69 \text{ m}$$

$$L' = L - 2e_L = L - \frac{2M_L}{V} = \quad 1.00 \text{ m}$$

Shape Factors:

$$s_c = 1 + 0.2\tan^2\left(45^\circ + \frac{\phi}{2}\right)\frac{B'}{L'} = \quad 1.000$$

$$s_q = 1 + 0.1\tan^2\left(45^\circ + \frac{\phi}{2}\right)\frac{B'}{L'} = \quad 1.000$$

$$s_\gamma = 1 + 0.1\tan^2\left(45^\circ + \frac{\phi}{2}\right)\frac{B'}{L'} = \quad 1.000$$

Depth Factors:

$$d_c = 1 + 0.2\tan\left(45^\circ + \frac{\phi}{2}\right)\frac{D}{B'} = \quad 1.437$$

$$d_q = 1 + 0.1\tan\left(45^\circ + \frac{\phi}{2}\right)\frac{D}{B'} = \quad 1.218$$

$$d_\gamma = 1 + 0.1\tan\left(45^\circ + \frac{\phi}{2}\right)\frac{D}{B'} = \quad 1.218$$

Inclination Factors:

$$i_c = \left(1 - \frac{\theta}{90^\circ}\right)^2 = \quad 0.681$$

$$i_q = \left(1 - \frac{\theta}{90^\circ}\right)^2 = \quad 0.681$$

$$i_\gamma = \left(1 - \frac{\theta^\circ}{\phi^\circ}\right)^2 = \quad 0.369 \quad \text{for } \phi > 0$$

$$i_\gamma = \quad 0 \quad \text{for } \phi = 0$$

Use i_γ = 0.369

Bearing Capacity Factors:

$$N_q = e^{\pi\tan\phi}\cdot\tan^2\left(45^\circ + \frac{\phi}{2}\right) = \quad 64.20$$

$$N_c = \frac{(N_q - 1)}{\tan\phi} = \quad 75.31$$

$$N_\gamma = (N_q - 1)\cdot\tan(1.4\phi) = \quad 93.69$$

Vertical Load: $q_{ult} = cN_c s_c d_c + \bar{q}N_q s_q d_q + 0.5\gamma B' N_\gamma s_\gamma d_\gamma$

Inclined Load: $q_{ult} = cN_c d_c i_c + \bar{q}N_q d_q i_q + 0.5\gamma B' N_\gamma d_\gamma i_\gamma$

LOAD IS INCLINED

N_c term (kPa)	N_q term (kPa)	N_γ term (kPa)	q_{ult} (kPa)
0.0	671.5	260.5	932.0

$$V_{ult} = q_{ult}\cdot B' L' = \quad 641 \text{ kN}$$

SF = 2.0

Meyerhof.xlsx, Page 1 of 1

More Spreadsheet Examples - Example 3

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Myerhoff Bearing Capacity Equation 2.079963

Footing Data:

B = 0.80 m
L = 1.50 m
L'/B' = 3.077
D = 0.70 m
D/B' = 1.436
$\bar{q} = \gamma' D =$ 12.6 kPa

Loading Data:

V = 320 kN
H = 90 kN
M_B = 50 kN-m
M_L = 0 kN-m
θ = 15.70864 deg.

Soil Data:

γ' = 18.00 kN/m³
ϕ_{tr} = 0 deg.
c = 200 kPa
ϕ_{ps} = 0 deg.
If L'/B' >= 2.0, ϕ_{ps} is used in place of ϕ_{tr}.

$\phi_{ps} = 1.5\phi_{tr} - 17^\circ \; for \; \phi_{tr} > 34^\circ$

Use ϕ = 0 deg.

$$B' = B - 2e_B = B - \frac{2M_B}{V} = \quad 0.49 \text{ m}$$

$$L' = L - 2e_L = L - \frac{2M_L}{V} = \quad 1.50 \text{ m}$$

Shape Factors:

$$s_c = 1 + 0.2\tan^2\left(45^\circ + \frac{\phi}{2}\right)\frac{B'}{L'} = \quad 1.000$$

$$s_q = 1 + 0.1\tan^2\left(45^\circ + \frac{\phi}{2}\right)\frac{B'}{L'} = \quad 1.000$$

$$s_\gamma = 1 + 0.1\tan^2\left(45^\circ + \frac{\phi}{2}\right)\frac{B'}{L'} = \quad 1.000$$

Depth Factors:

$$d_c = 1 + 0.2\tan\left(45^\circ + \frac{\phi}{2}\right)\frac{D}{B'} = \quad 1.287$$

$$d_q = 1 + 0.1\tan\left(45^\circ + \frac{\phi}{2}\right)\frac{D}{B'} = \quad 1.000$$

$$d_\gamma = 1 + 0.1\tan\left(45^\circ + \frac{\phi}{2}\right)\frac{D}{B'} = \quad 1.000$$

Inclination Factors:

$$i_c = \left(1 - \frac{\theta}{90^\circ}\right)^2 = \quad 0.681$$

$$i_q = \left(1 - \frac{\theta}{90^\circ}\right)^2 = \quad 0.681$$

$$i_\gamma = \left(1 - \frac{\theta^\circ}{\phi^\circ}\right)^2 = \quad \#DIV/0! \quad \text{for } \phi > 0$$

$$i_\gamma = \quad 0 \quad \text{for } \phi = 0$$

Use i_γ = 0.000

Bearing Capacity Factors:

$$N_q = e^{\pi\tan\phi}\cdot\tan^2\left(45^\circ + \frac{\phi}{2}\right) = \quad 1.00$$

$$N_c = \frac{(N_q - 1)}{\tan\phi} = \quad 5.14$$

$$N_\gamma = (N_q - 1)\cdot\tan(1.4\phi) = \quad 0.00$$

Vertical Load: $q_{ult} = cN_c s_c d_c + \bar{q}N_q s_q d_q + 0.5\gamma B' N_\gamma s_\gamma d_\gamma$

Inclined Load: $q_{ult} = cN_c d_c i_c + \bar{q}N_q d_q i_q + 0.5\gamma B' N_\gamma d_\gamma i_\gamma$

LOAD IS INCLINED

N_c term (kPa)	N_q term (kPa)	N_γ term (kPa)	q_{ult} (kPa)
901.6	8.6	0.0	910.2

$$V_{ult} = q_{ult}\cdot B' \cdot L' = \quad 666 \text{ kN}$$

SF = 2.1

Meyerhof.xlsx, Page 1 of 1

Myerhoff Bearing Capacity Equation 1.592144

Footing Data:			Loading Data:			Soil Data:		
B =	0.80	m	V =	500	kN	γ' =	18.00	kN/m^3
L =	1.50	m	H =	50	kN	ϕ_{tr} =	20	deg.
L'/B' =	1.375		M_B =	0	kN-m	c =	50	kPa
D =	0.70	m	M_L =	100	kN-m	ϕ_{ps} =	20	deg.
D/B' =	0.875		θ =	5.710593	deg.			
$\bar{q} = \gamma' D$ =	12.6	kPa						

If L'/B' >= 2.0, ϕ_{ps} is used in place of ϕ_{tr}.

$\phi_{ps} = 1.5\phi_{tr} - 17^\circ$ for $\phi_{tr} > 34^\circ$

Use ϕ = 20 deg.

$$B' = B - 2e_B = B - \frac{2M_B}{V} = \quad 0.80 \text{ m}$$

$$L' = L - 2e_L = L - \frac{2M_L}{V} = \quad 1.10 \text{ m}$$

Shape Factors:

$$s_c = 1 + 0.2\tan^2\left(45^\circ + \frac{\phi}{2}\right)\frac{B'}{L'} = \quad 1.000$$

$$s_q = 1 + 0.1\tan^2\left(45^\circ + \frac{\phi}{2}\right)\frac{B'}{L'} = \quad 1.000$$

$$s_\gamma = 1 + 0.1\tan^2\left(45^\circ + \frac{\phi}{2}\right)\frac{B'}{L'} = \quad 1.000$$

Depth Factors:

$$d_c = 1 + 0.2\tan\left(45^\circ + \frac{\phi}{2}\right)\frac{D}{B'} = \quad 1.250$$

$$d_q = 1 + 0.1\tan\left(45^\circ + \frac{\phi}{2}\right)\frac{D}{B'} = \quad 1.125$$

$$d_\gamma = 1 + 0.1\tan\left(45^\circ + \frac{\phi}{2}\right)\frac{D}{B'} = \quad 1.125$$

Inclination Factors:

$$i_c = \left(1 - \frac{\theta}{90^\circ}\right)^2 = \quad 0.877$$

$$i_q = \left(1 - \frac{\theta}{90^\circ}\right)^2 = \quad 0.877$$

$$i_\gamma = \left(1 - \frac{\theta^\circ}{\phi^\circ}\right)^2 = \quad 0.510 \quad \text{for } \phi > 0$$

$$i_\gamma = \quad 0 \quad \text{for } \phi = 0$$

Use i_γ = 0.510

Bearing Capacity Factors:

$$N_q = e^{\pi\tan\phi}\cdot\tan^2\left(45^\circ + \frac{\phi}{2}\right) = \quad 6.40$$

$$N_c = \frac{(N_q - 1)}{\tan\phi} = \quad 14.83$$

$$N_\gamma = (N_q - 1)\cdot\tan(1.4\phi) = \quad 2.87$$

Vertical Load: $q_{ult} = cN_c s_c d_c + \bar{q}N_q s_q d_q + 0.5\gamma B' N_\gamma s_\gamma d_\gamma$

Inclined Load: $q_{ult} = cN_c d_c i_c + \bar{q}N_q d_q i_q + 0.5\gamma B' N_\gamma d_\gamma i_\gamma$

LOAD IS INCLINED

N_c term (kPa)	N_q term (kPa)	N_γ term (kPa)	q_{ult} (kPa)
813.2	79.6	11.9	904.6

$$V_{ult} = q_{ult}\cdot B'\cdot L' = \quad 796 \text{ kN}$$

SF = 1.6

Approximate Bearing Capacity by Soil Type - BS 8004

Tuesday, November 1, 2022 12:45 PM

Allowable bearing capacity: The maximum pressure that can be applied to the soil from the foundation so that the two requirements are satisfied:

1. Acceptable safety factor against shear failure below the foundation
2. Acceptable total and differential settelement

Ultimate bearing capacity: The minimum pressure that would cause the shear failure of the supporting soil immediately below and adjacent to the foundation.

Typical values of soil bearing capacity

https://www.thenbs.com/PublicationIndex/documents/details?Pub=BSI&DocID=310971

For preliminary design purposes, BS 8004 [1] gives typical values of allowable bearing capacity which should result in an adequate factor of safety against shaer failure without accounting for the setllemenet criteria [2].

Soil type	Bearing value (kPa)	Remarks
Dense gravel or dense sand and gravel	> 600	Width of foundation not less than 1 m. Water table at least at the depth equal to the width of foundation, below base of foundation.
Dense dense gravel or medium dense sand and gravel	200-600	-
Loose gravel or loose sand and gravel	< 200	-
Compact sand	> 300	-
Medium dense sand	100 - 300	-
Very stiff boulder clays and hard clays	300 - 600	Susceptible to long term consolidation settlement
Stiff clays	150 - 300	-
Firm clays	75 -150	-
Soft clays and silts	< 75	-
Very soft clays and silts	-	-

Ultimate bearing capacity for shallow foundations according to Terzaghi

The utimate bearing capacity for shallow foundations can be calculated using the relation proposed by Terzaghi [3]:

http://www.geotechdata.info/parameter/bearing-capacity

Approximate Bearing Capacity by Soil Type - IBC 2018

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1806.2 Presumptive load-bearing values.

The load-bearing values used in design for supporting soils near the surface shall not exceed the values specified in Table 1806.2 unless data to substantiate the use of higher values are submitted and *approved*. Where the *building official* has reason to doubt the classification, strength or compressibility of the soil, the requirements of Section 1803.5.2 shall be satisfied.

Presumptive load-bearing values shall apply to materials with similar physical characteristics and dispositions. Mud, organic silt, organic clays, peat or unprepared fill shall not be assumed to have a presumptive load-bearing capacity unless data to substantiate the use of such a value are submitted.

Exception: A presumptive load-bearing capacity shall be permitted to be used where the *building official* deems the load-bearing capacity of mud, organic silt or unprepared fill is adequate for the support of lightweight or temporary structures.

TABLE 1806.2 PRESUMPTIVE LOAD-BEARING VALUES

CLASS OF MATERIALS	VERTICAL FOUNDATION PRESSURE (psf)	LATERAL BEARING PRESSURE (psf/ft below natural grade)	LATERAL SLIDING RESISTANCE	
			Coefficient of friction[a]	Cohesion (psf)[b]
1. Crystalline bedrock	12,000	1,200	0.70	—
2. Sedimentary and foliated rock	4,000	400	0.35	—
3. Sandy gravel and gravel (GW and GP)	3,000	200	0.35	—
4. Sand, silty sand, clayey sand, silty gravel and clayey gravel (SW, SP, SM, SC, GM and GC)	2,000	150	0.25	—
5. Clay, sandy clay, silty clay, clayey silt, silt and sandy silt (CL, ML, MH and CH)	1,500	100	—	130

For SI: 1 pound per square foot = 0.0479kPa, 1 pound per square foot per foot = 0.157 kPa/m.

a. Coefficient to be multiplied by the dead load.

b. Cohesion value to be multiplied by the contact area, as limited by Section 1806.3.2.

Ch. 16 Retaining Structures

Monday, November 9, 2015 11:43 AM

Reading Assignment

- Salgado 16.1 -16.3
- Ch. 16 Lecture Notes

Homework Assignment

1. Develop a spreadsheet for the design of an embedded gravity wall (see example in class notes.) Turn in your solution for the case given in the lecture notes
2. Determine the factor of safety against overturning and sliding for the following case:

Wall Dimensions				**Fill Properties**			
Top	4	ft		$\alpha_{backfill}$ deg	20	*0.349*	*radians*
Bottom	4	ft		α_{toe} deg	0	*0.000*	*radians*
$\gamma_{concrete}$	150	pcf		φ deg	40	*0.698*	*radians*
H	15	ft		δ deg	20	*0.349*	*radians*
D	3	ft		$\gamma_{backfill}$	120		*pcf*

Partial Answer

Factors of Safety	
$FS_{sliding}$	**2.813**
FS_{oturn}	**1.462**

3. Determine the factor of safety against overturning and sliding for the following case:

Wall Dimensions				**Fill Properties**			
Top	3	ft		$\alpha_{backfill}$ deg	0	*0.000*	*radians*
Bottom	3	ft		α_{toe} deg	0	*0.000*	*radians*
$\gamma_{concrete}$	150	pcf		φ deg	40	*0.698*	*radians*
H	10	ft		δ deg	20	*0.349*	*radians*
D	0	ft		$\gamma_{backfill}$	120		*pcf*

Foundation Soil Properties				
φ deg	40	*0.698*	*radians*	
δ deg note 2	40	*0.698*	*radians*	

Introduction to Retaining Structures

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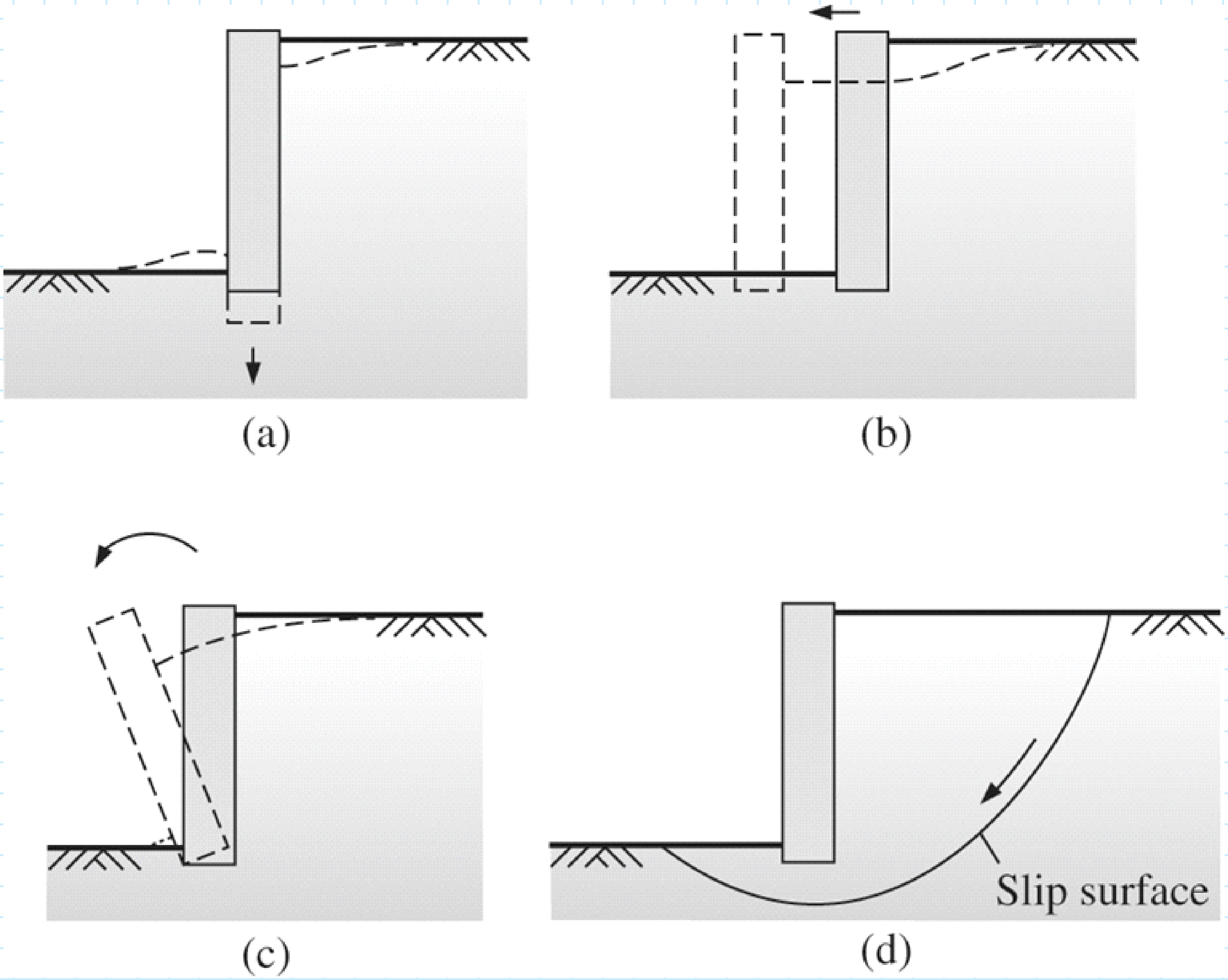

Failure modes

(a) bearing capacity failure of base or footing

(b) sliding

(c) overturning

(d) global instability

Retaining Walls - General

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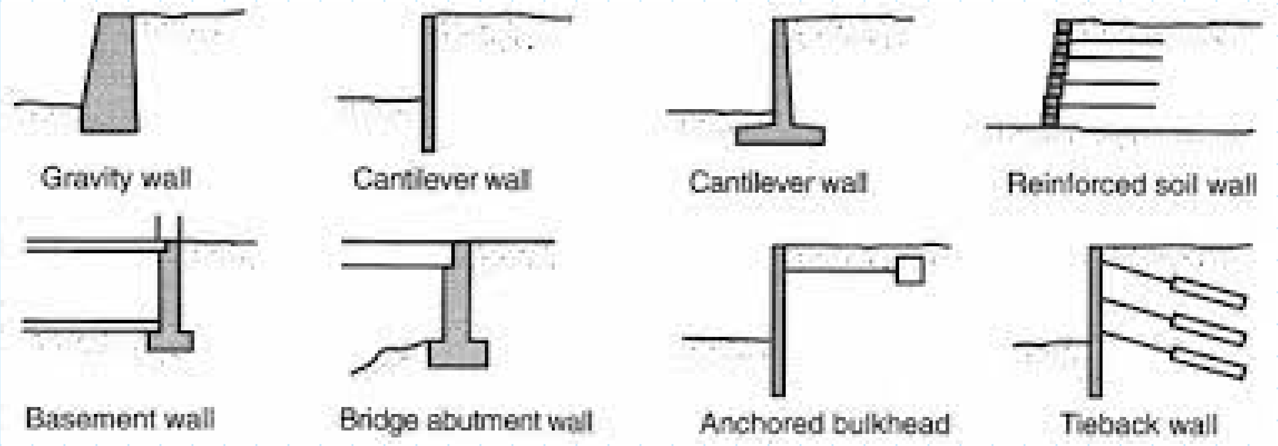

[Retaining wall || Types of retaining walls || Retaining wall construction || Gabion retaining wall]

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Gravity Wall

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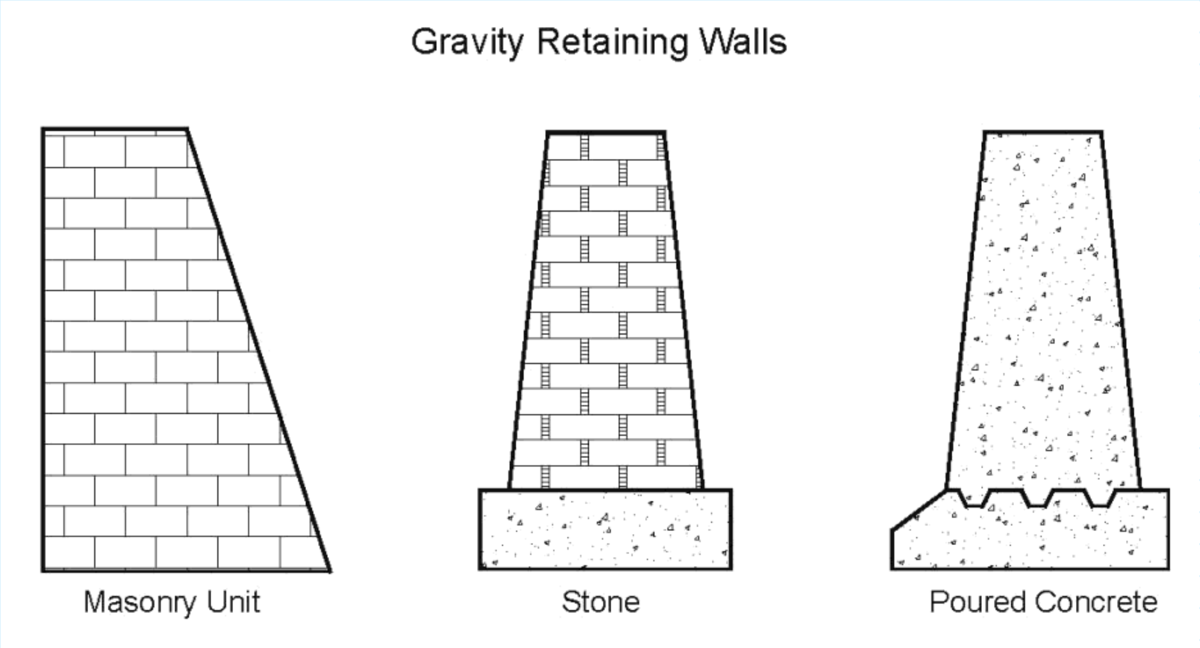

Cantilever Wall

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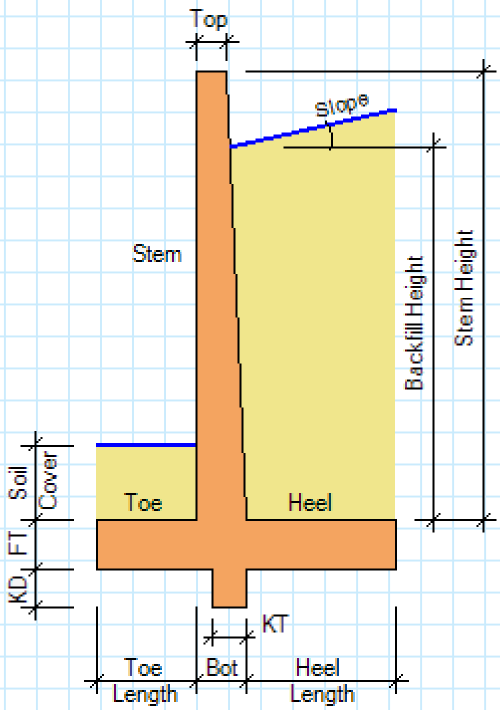

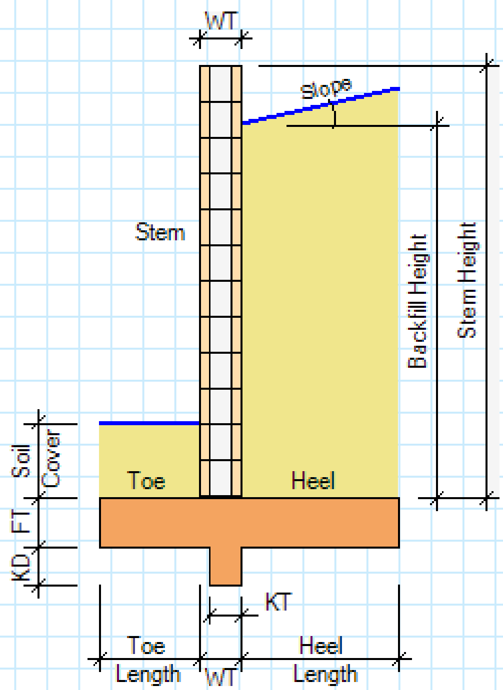

Reinforced Soil Wall or Mechanically Stabilized Earth Wall

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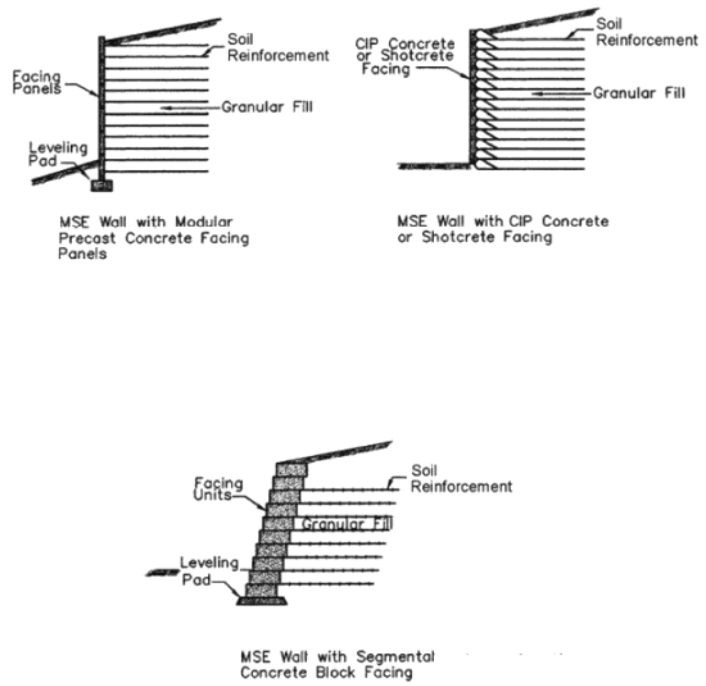

Figure C11.10.1-1—Typical Mechanically Stabilized Earth Walls

1-stage MSE placed over columns

Finished MSE wall

Tieback Walls

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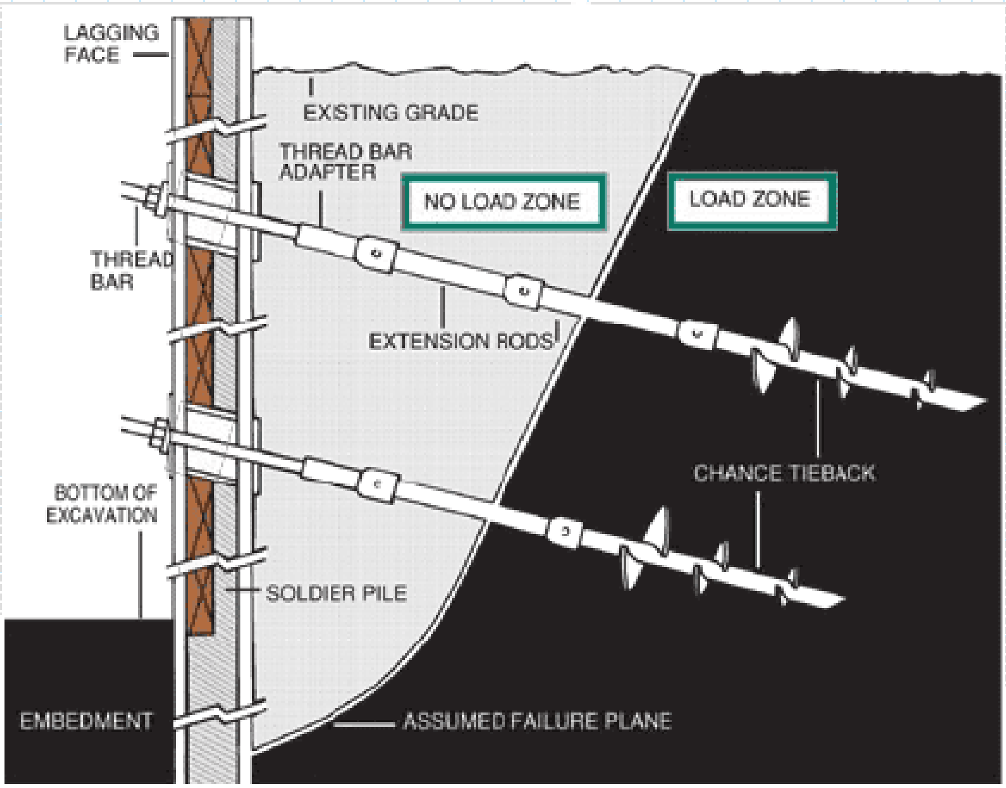

Soil Nailing

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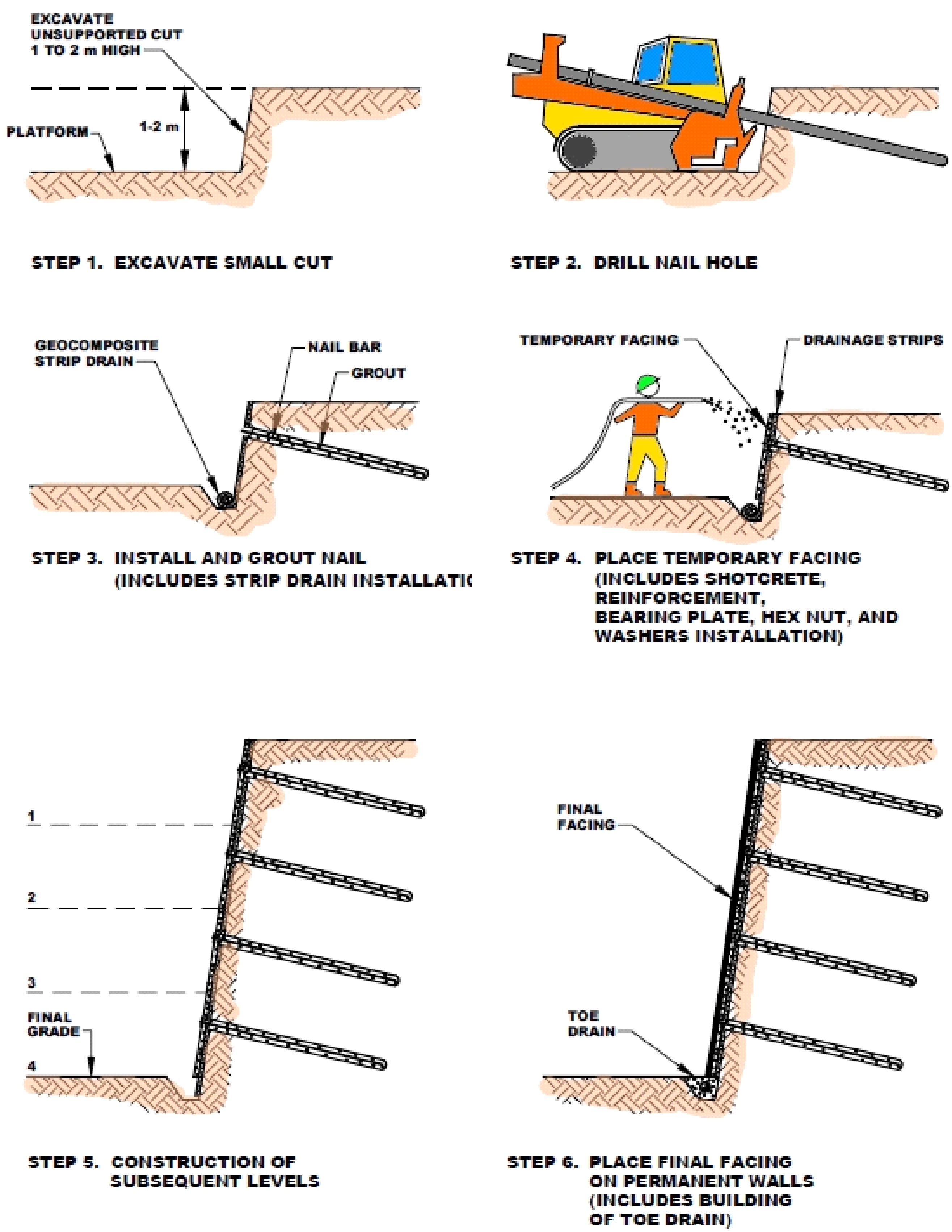
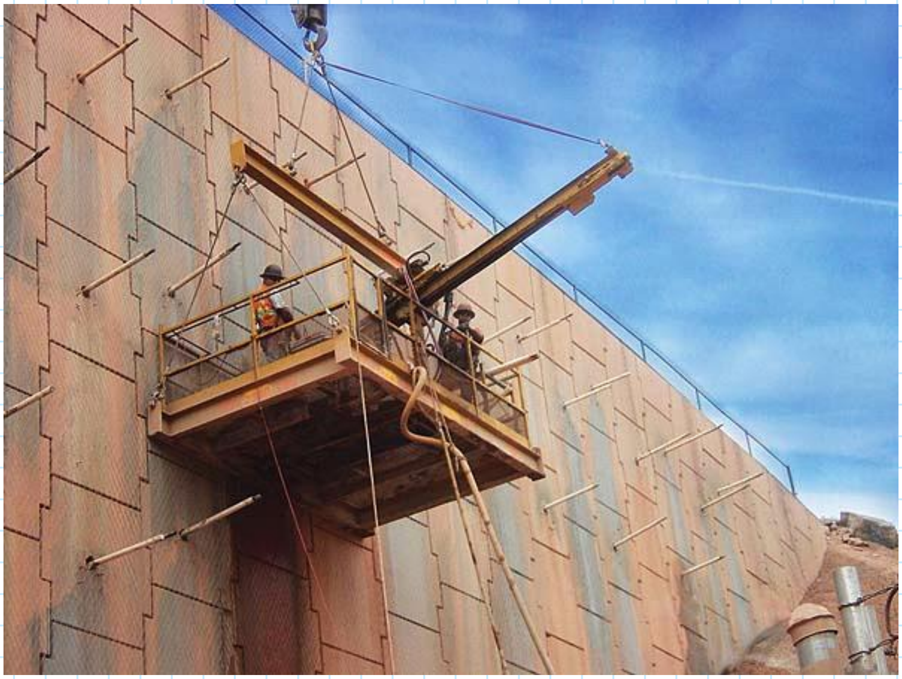


Soil Nailing

Monday, November 9, 2015 11:43 AM



Remediation of a MSE wall using soil nailing

Soil nailing - Provo Canyon

To stabilize a potential landslide, which could be triggered by a cut for the new roadway alignment, Schnabel Foundation Company installed 208 encapsulated 14-strand anchors in the sidehill cut. The anchors have a design load of 400 kips and an average installed length of 160 feet in order to make capacity behind the failure plain

From <http://www.schnabel.com/projects/view/75>

Introduction to Retaining Structures (cont.)

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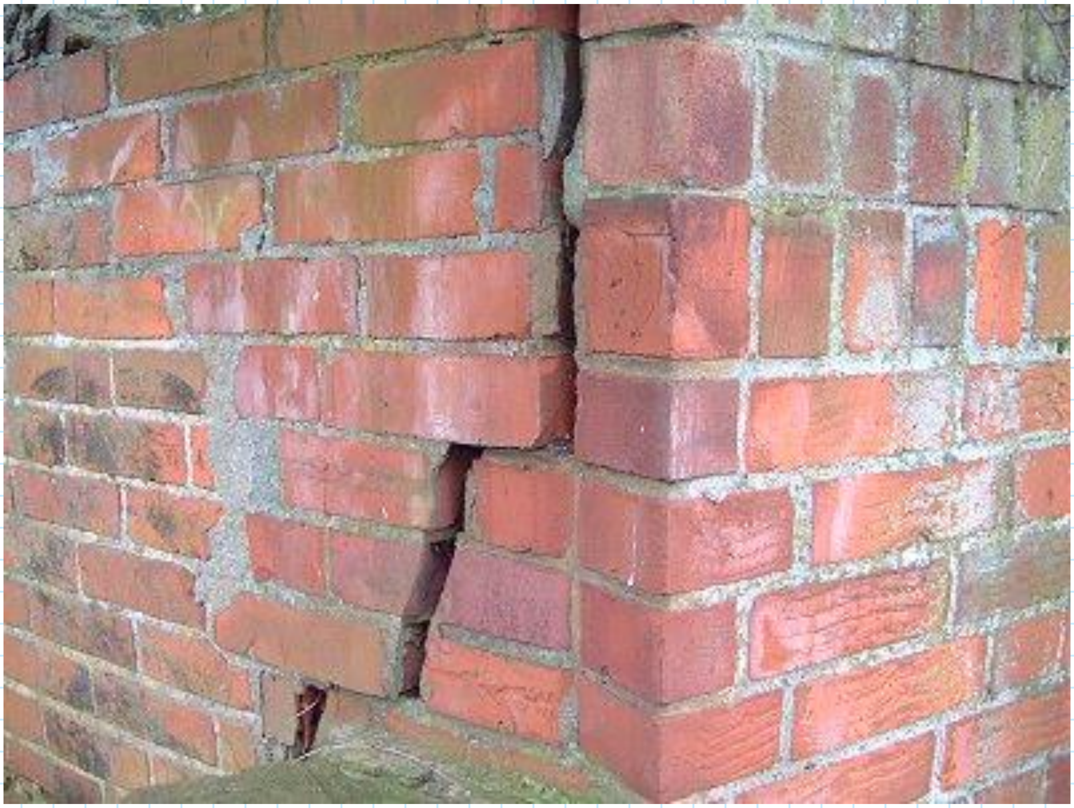
(a) bearing capacity failure

Figure 262. Photo. Wall movement near Chepe railroad bridge due to lateral spreading and settlement.

http://www.fhwa.dot.gov/publications/research/infrastructure/structures/11030/005.cfm

This failure was due to liquefaction of the foundation soils - 2010 Chilean earthquake

(b) sliding



http://www.michael-roberts.co.uk/photo/pmisc0.jpg

Introduction to Retaining Structures (cont.)

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(c) overturning

http://designandbuildllc.com/design-and-build-gallery/

Introduction to Retaining Structures (cont.)

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(d) global instability

Introduction to Retaining Structures (cont.)

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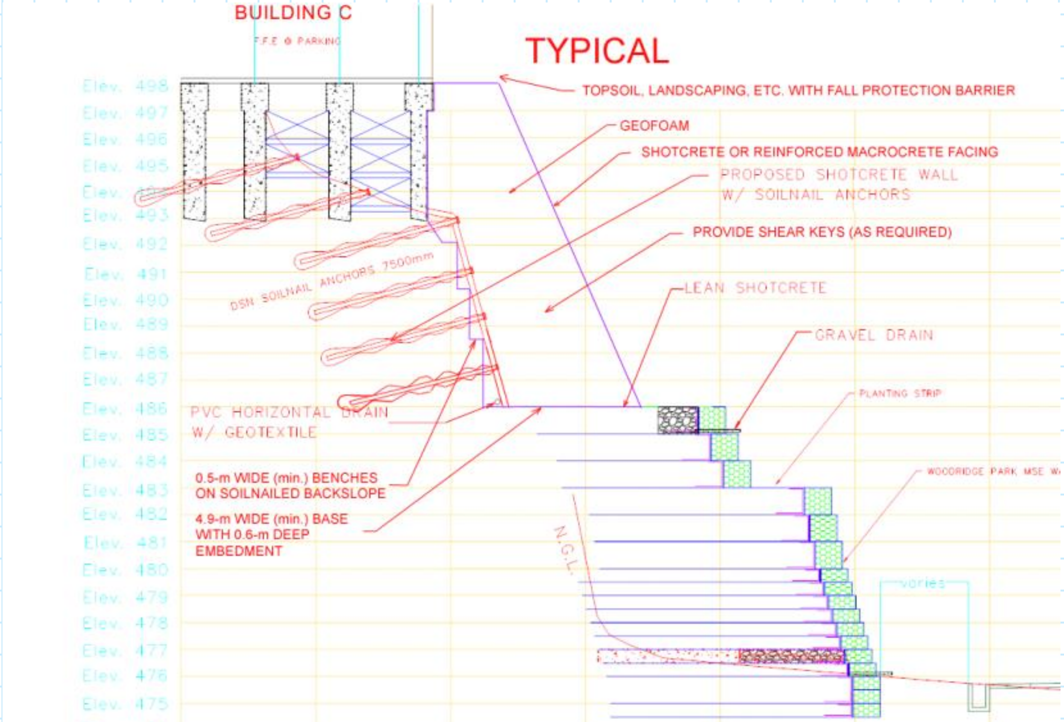
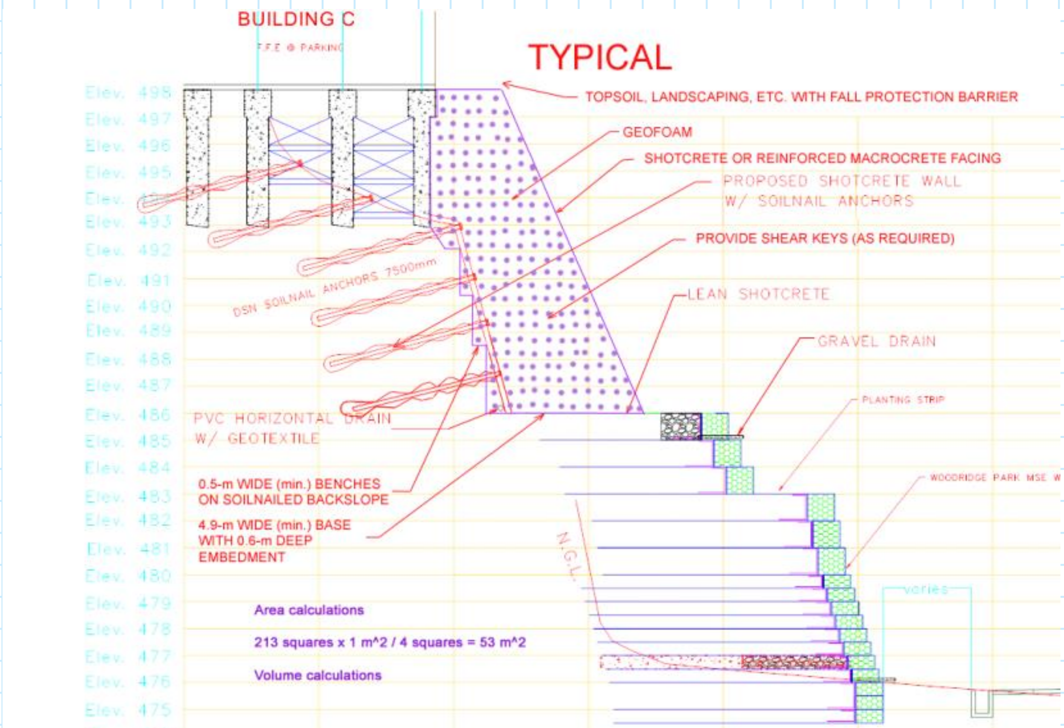
Remediation of MSE wall failure - Woodlands Building C, Philippines





Introduction to Retaining Structures (cont.)

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Remediation of MSE wall failure - Woodlands Building C, Philippines

Final configuration without EPS Geofoam

Earth Pressure Theory - Which Theory?

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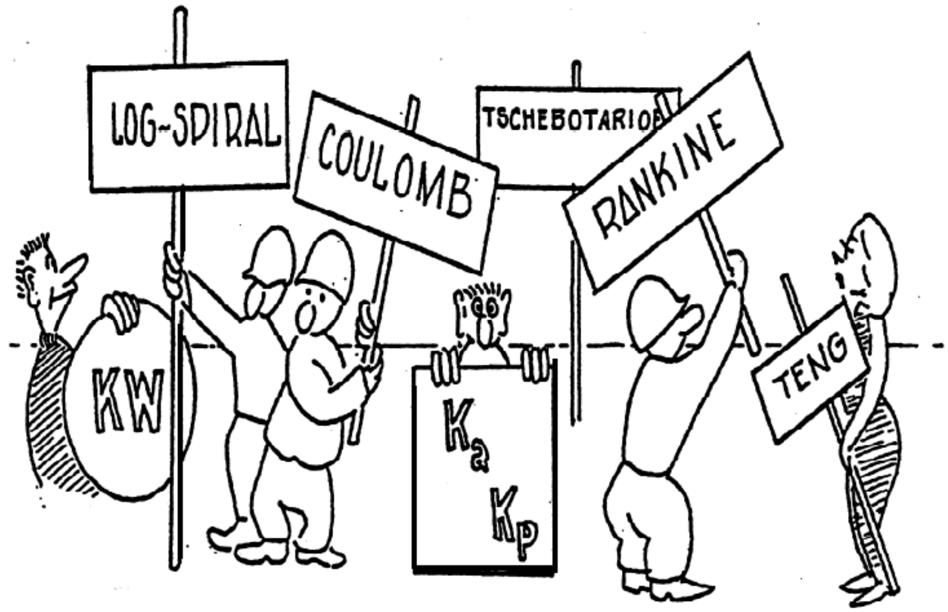

EARTH PRESSURE THEORY AND APPLICATION

Earth pressure is the lateral force exerted by the soil on a shoring system. It is dependent on the soil structure and the interaction or movement with the retaining system. Due to many variables, shoring problems can be highly indeterminate. Therefore, it is essential that good engineering judgment be used.

COEFFICIENT OF EARTH PRESSURE

The coefficient of earth pressure (K) is the term used to express the ratio of the lateral earth pressure to the vertical earth pressure

$$\boxed{K = \sigma'_h / \sigma'_v}$$

Earth Pressure Theory, Ko Conditions

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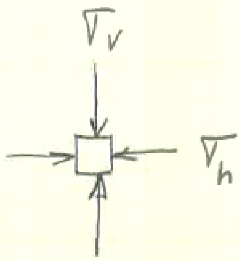
Earth Pressure at Rest, K_o



$$K_o = \frac{\sigma_h'}{\sigma_v'}$$

K_o = earth pressure coefficient at rest

- State of elastic equilibrium (i.e., horizontal strain is zero).
- σ_h and σ_v are both effective and total stresses
- Note that there are no shear stresses on the vertical and horizontal planes (i.e., σ_h & σ_v are principal stresses).
- $\sigma_v = \gamma z$
  where: γ = unit weight, z = depth from ground surface

K_o for loose coarse-grained soils, (Jaky 1944 equation)

- $K_o = 1 - \sin \phi'$

  where ϕ' is the drained friction angle

- Note: Jaky's equation gives good results when the backfill behind a retaining wall is loose. However for a dense sand, Jaky's equation may grossly underestimate the earth pressure coefficient, hence underestimate the horizontal stress against the wall.

K_o for various densities of sand (Sherif, Fang, Sherif (1984))

- $$K_o = (1 - \sin \phi') + \left[\frac{\gamma_d}{\gamma_{d(min)}} - 1\right] 5.5$$

  where γ_d = actual compacted dry unit weight behind wall

  $\gamma_{d(min)}$ = dry unit weight of sand in loosest state

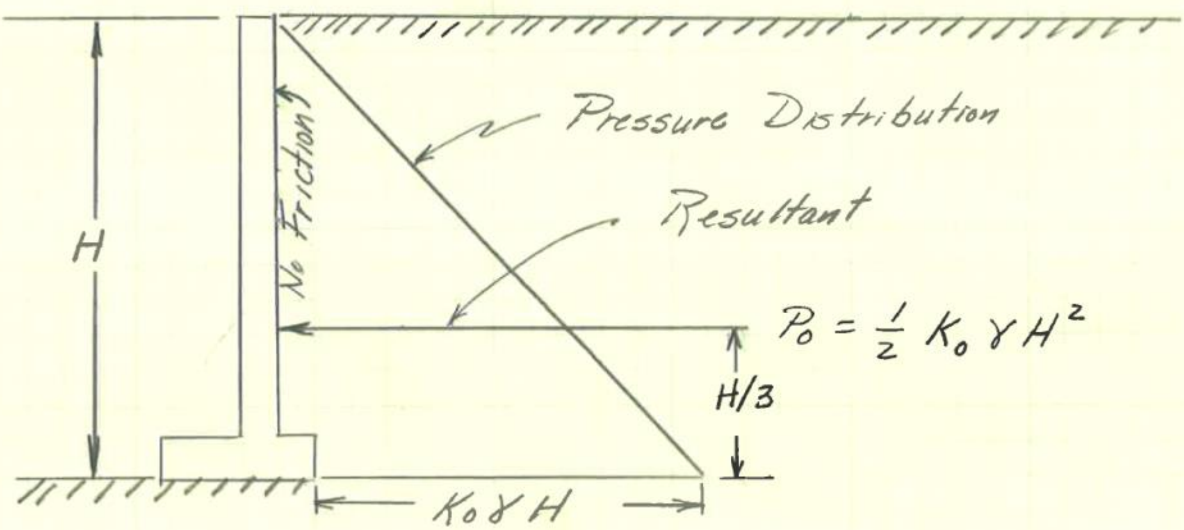
K_o for fine-grained, normally consolidated soils (Massarsch (1979))

- $K_o = 0.44 + 0.42\left[\frac{PI(\%)}{100}\right]$

K_o for fine-grained, overconsolidated soils (Massarsch (1979))

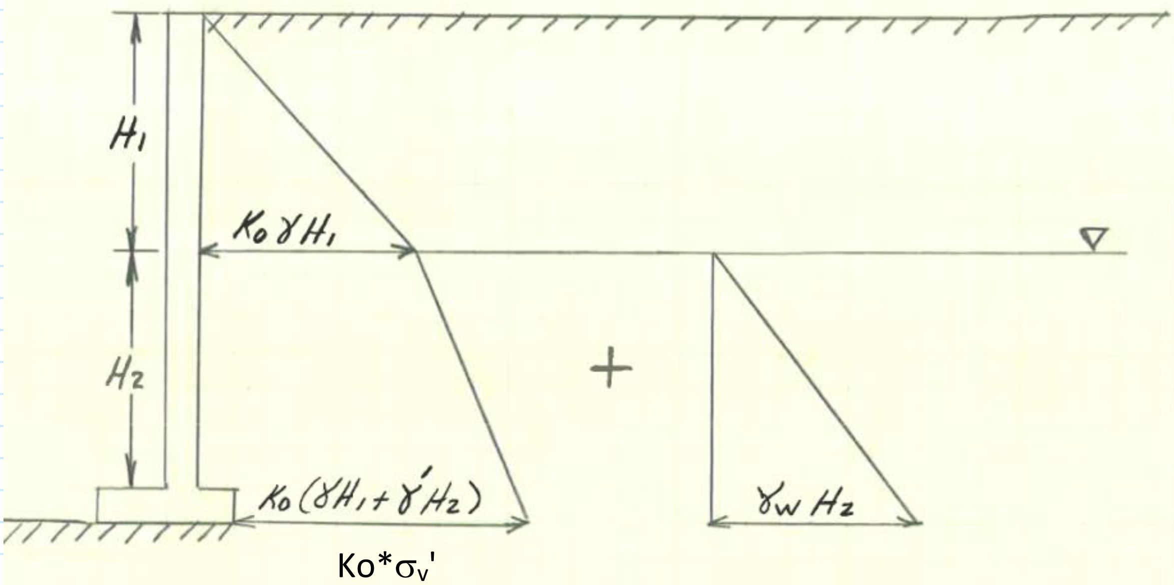
- $K_o(\text{overconsolidated}) = K_o(nc)\sqrt{OCR}$

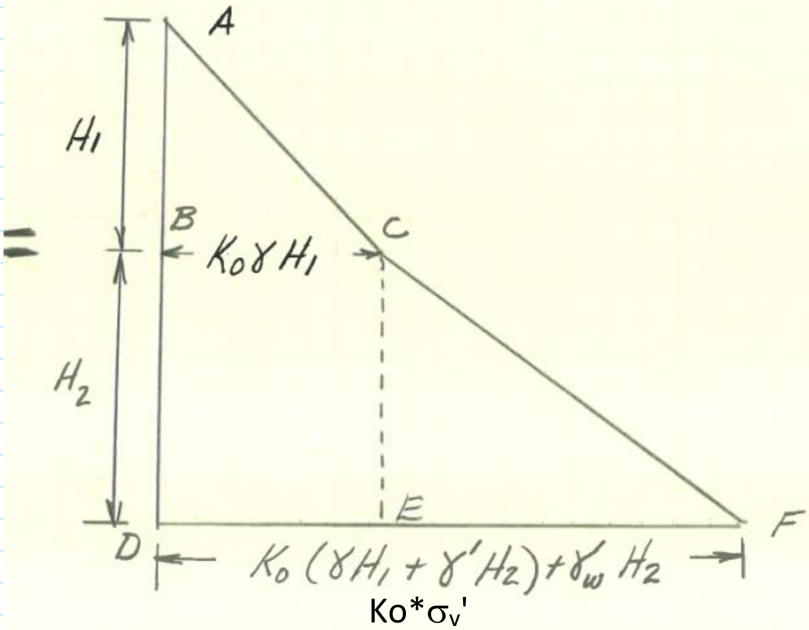
Earth Pressure at Rest Behind Wall (No water table)



- K_o = earth pressure coefficient at rest
- γ = unit weight of soil
- H = height of wall above base
- P_o acts 1/3 of height from base (force per unit width)
- No embedment or compensating soil at face
- No friction along side of wall (i.e., no shear stresses along stem)

Earth Pressure at Rest Behind Wall (Partially Submerged)





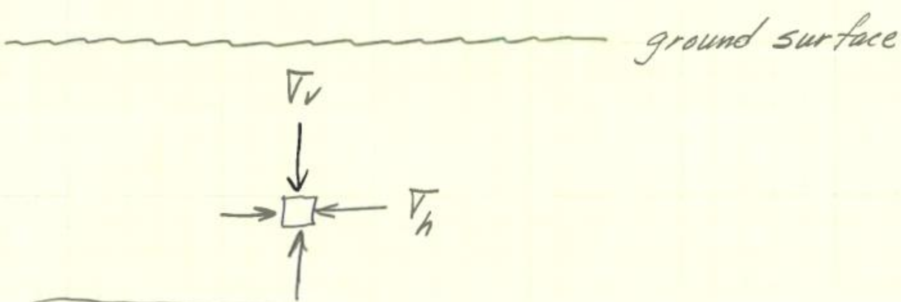
- P_0 = resultant force (per unit width of wall)

$$P_0 = \underbrace{\tfrac{1}{2} K_0 \gamma H_1^2}_{\text{Area } ABC} + \underbrace{K_0 \gamma H_1 H_2}_{\text{Area } BCDE} + \underbrace{\tfrac{1}{2}(K_0 \gamma' + \gamma_w) H_2^2}_{\text{Area } CEF}$$

Application of Lateral Earth Pressure At Rest

- Sometimes refered to as K_o conditions
- Soil mass in state of elastic equilibrium (i.e., no horizontal strain)

Case 1 - No Excavation or Cut

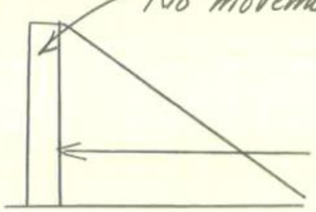


$$\boxed{\sigma_h = K_o \, \sigma_v}$$

Case 2 - Infinitely Rigid Wall (No defection allowed in horizontal direction)

No movement of wall (i.e, wall is very stiff and short)

$\varepsilon_h = 0$



$$P_o = \frac{1}{2} K_o \gamma H^2$$

Earth Pressure Theory, Ko Conditions (cont.)

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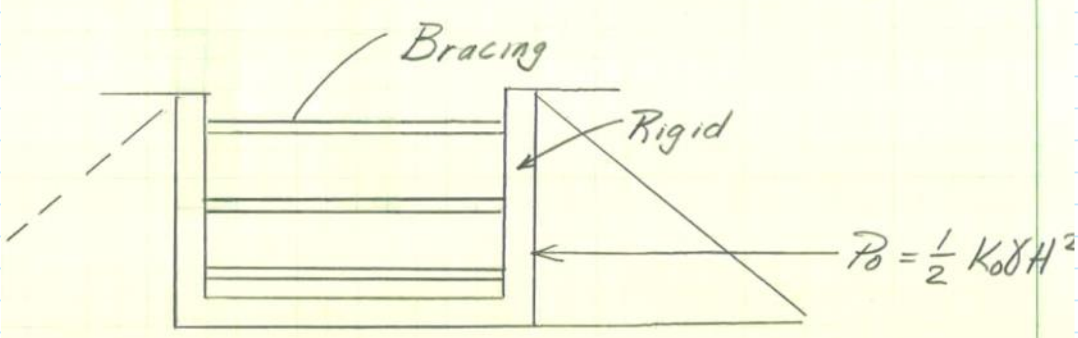
<u>Case 3</u> - Braced, Very Rigid Wall

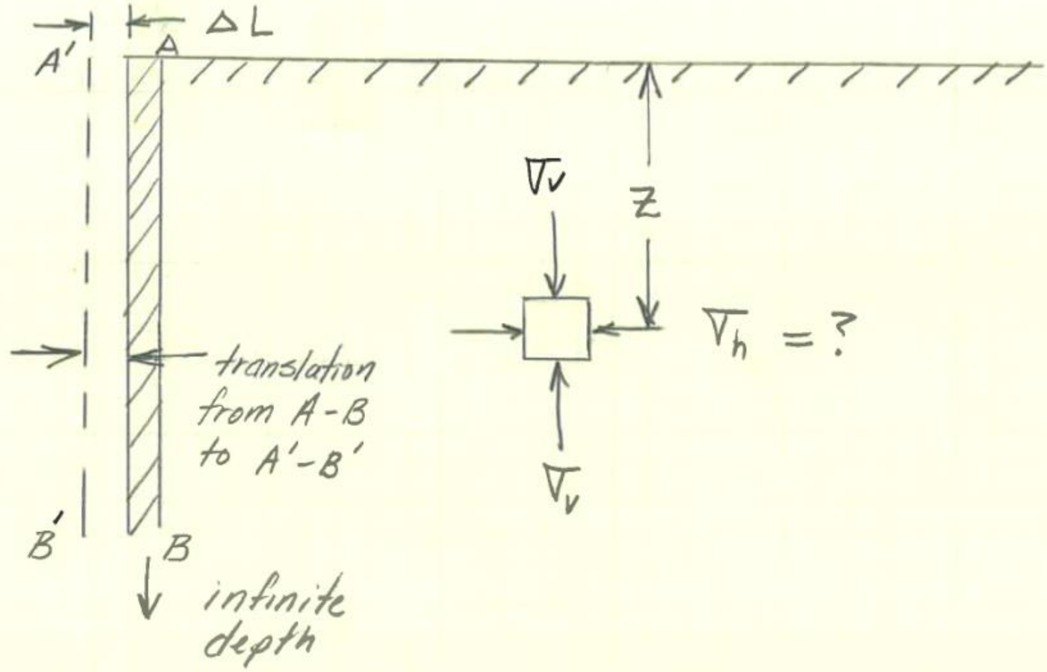


<u>Other Cases?</u>

-
-

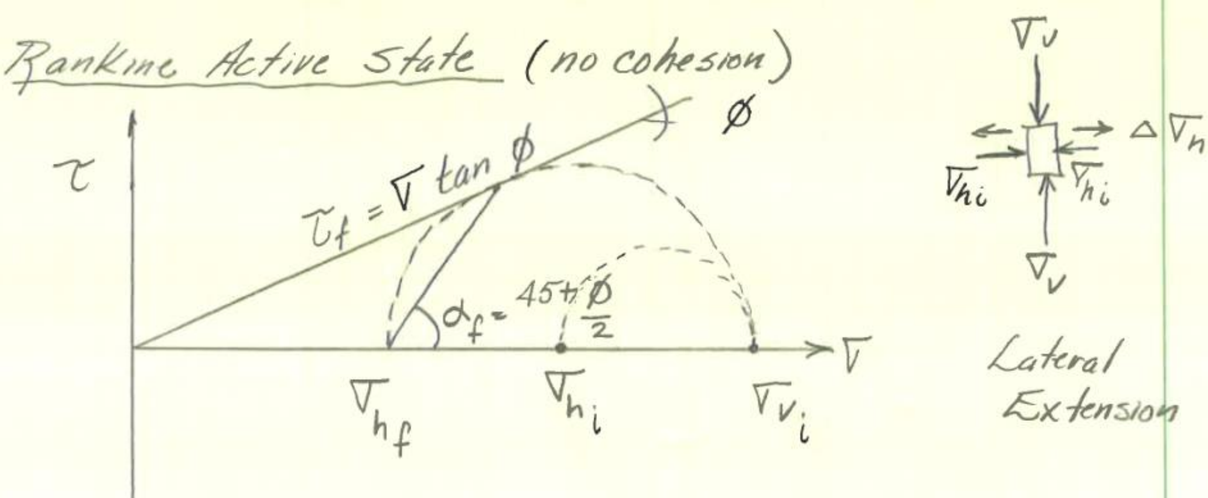
Active Earth Pressure, K_a

- State of plastic equilibrium (condition where every point in soil mass in on verge of failure)



- Note that outward translation is necessary to mobilize active earth pressure conditions.

Rankine Active State (no cohesion)



From Holtz & Kovac's Eq. 10-17

$$\left(\frac{\sigma_h}{\sigma_v}\right)_f = \left(\frac{\sigma_3}{\sigma_1}\right)_f = \tan^2\left(45 - \frac{\phi}{2}\right) = K_{a\,(RANKINE)}$$

Earth Pressure Theory, Active State (cont.)

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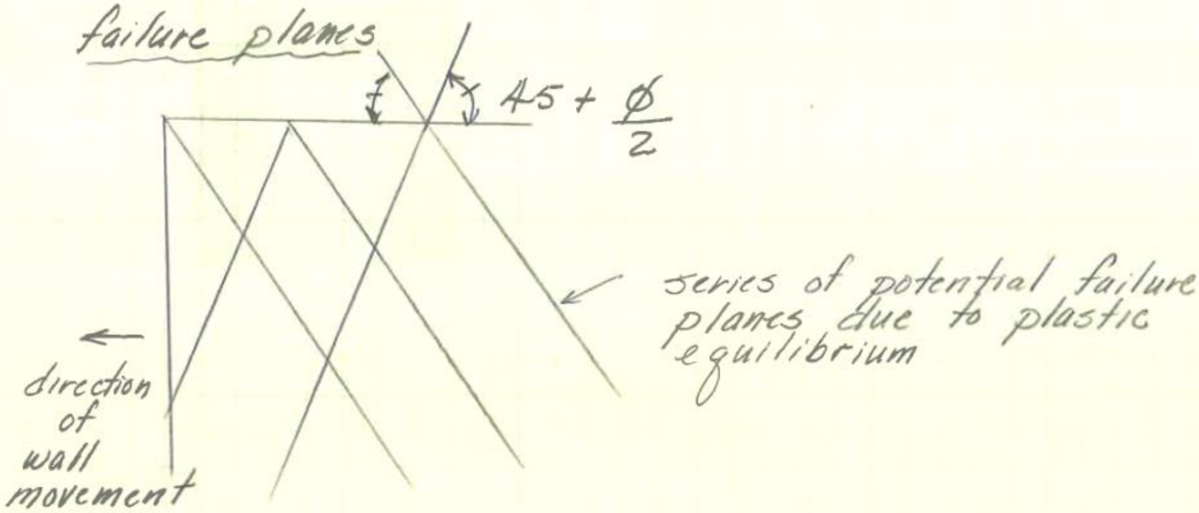
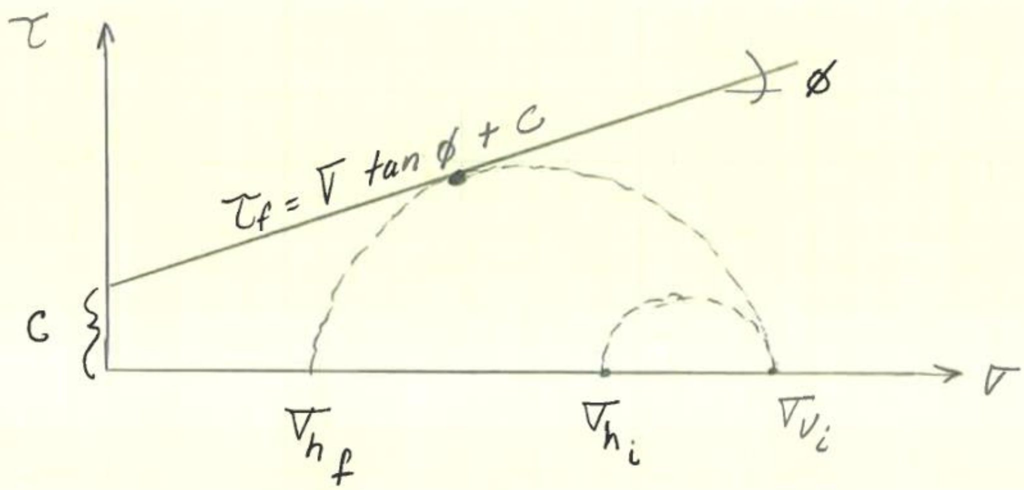
Rankine Active State (cont.) (no cohesion)



Rankine Active State (with cohesion)



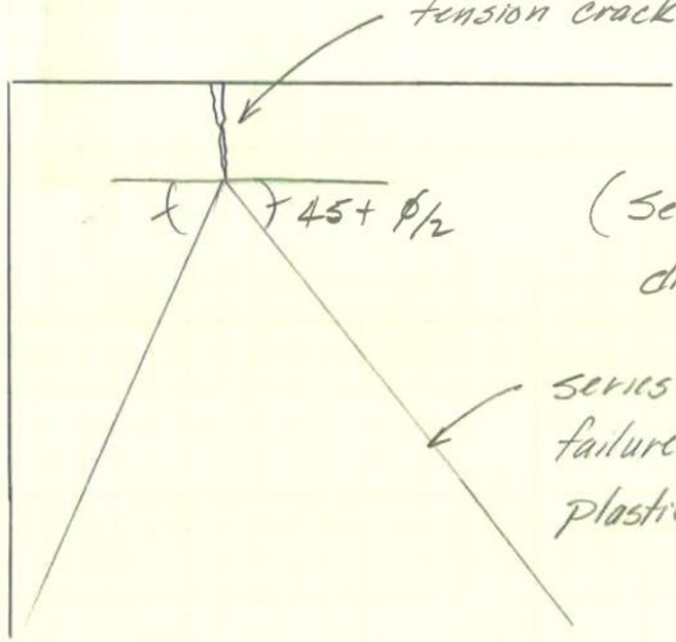
$$\left(\frac{\sigma_h}{\sigma_v}\right)_f = \left(\frac{\sigma_3}{\sigma_1}\right)_f = \frac{\gamma z \tan^2\left(45 - \frac{\phi}{2}\right) - 2c\tan\left(45 - \frac{\phi}{2}\right)}{\gamma z}$$

$$= K_{a\ (\text{RANKINE})}$$

Rankine Active State (cont.) (with cohesion)

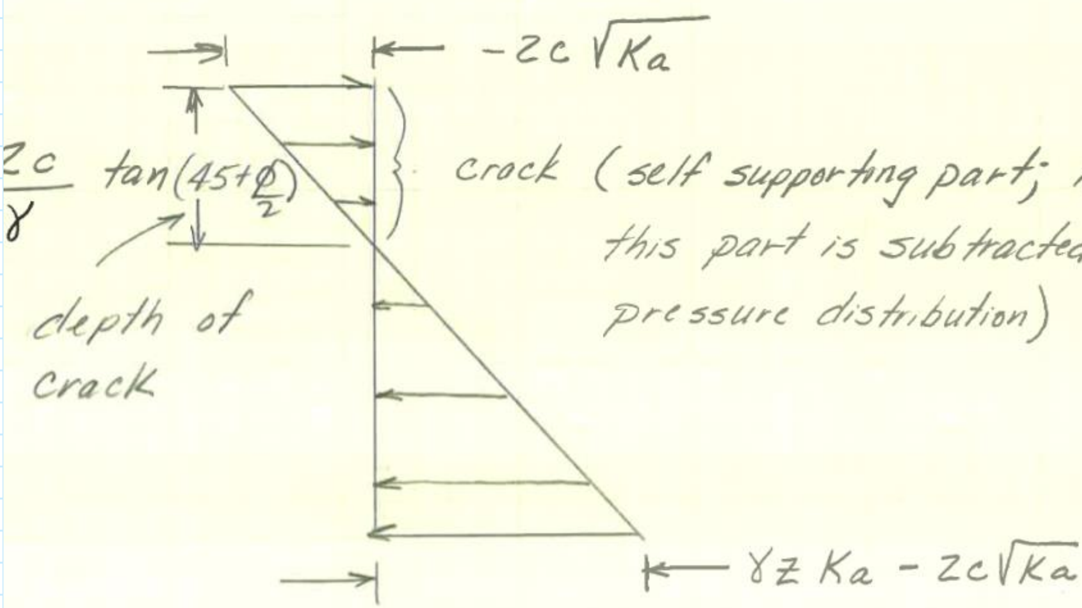
failure planes

tension crack (self supporting)



(See pressure diagram below)

series of potential failure planes due to plastic equilibrium



depth of crack

crack (self supporting part; hence this part is subtracted from pressure distribution)

Pressure Diagram

Earth Pressure Theory, Active State (cont.)

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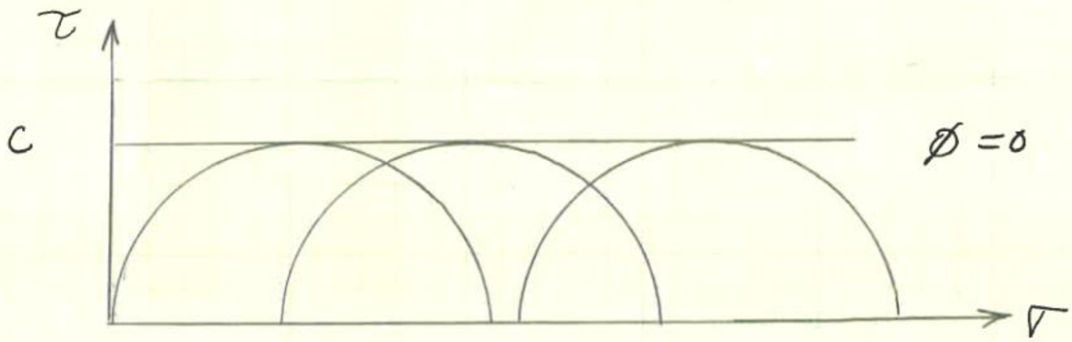
Rankine Active State (cohesion only)



$$\tau_f = \sigma_v \tan\phi + c$$

$$\tau_f = c$$

$$\left(\frac{\sigma_3}{\sigma_1}\right)_f = \frac{\gamma z \tan^2\left(45 - \frac{\phi}{2}\right) - 2c\tan\left(45 - \frac{\phi}{2}\right)}{\gamma z}$$

$$= \frac{\gamma z \tan^2(45) - 2c\tan(45)}{\gamma z}$$

$$= \frac{\gamma z - 2c}{\gamma z}$$

Note: $-2c$ indicates tension, neglect this part

$$= \frac{\gamma z}{\gamma z}$$

$$= 1$$

$K_{A,\text{RANKINE}} \simeq 1$ (cohesive soil)

Active Failure State Pressure Diagrams

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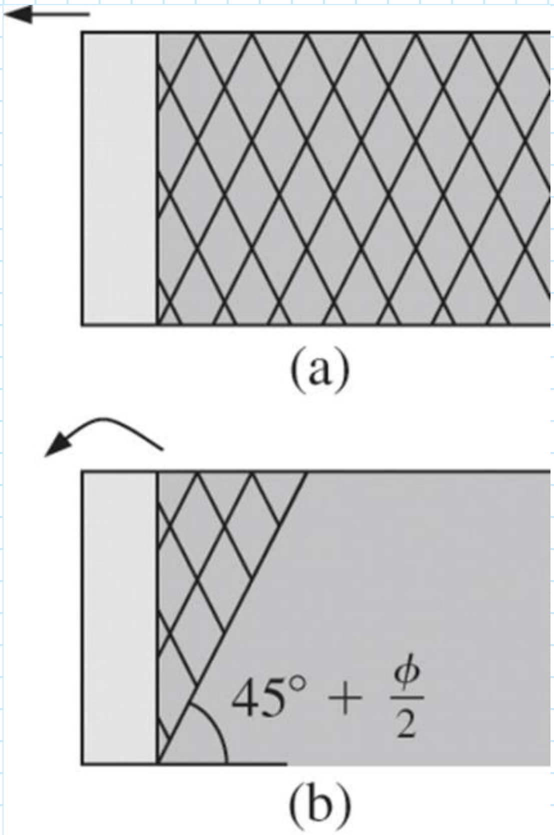
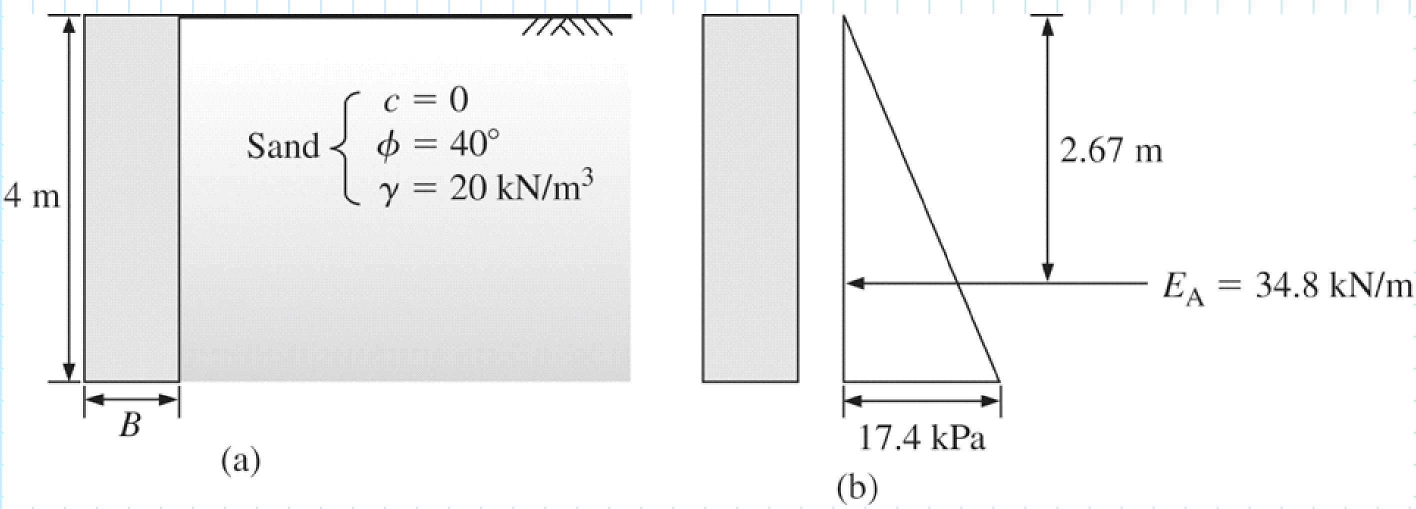


For sliding

For toppling



Earth pressure diagram

Earth pressure diagram for the active case (for case without water in backfill).

Active Failure State Pressure Diagrams (cont.)

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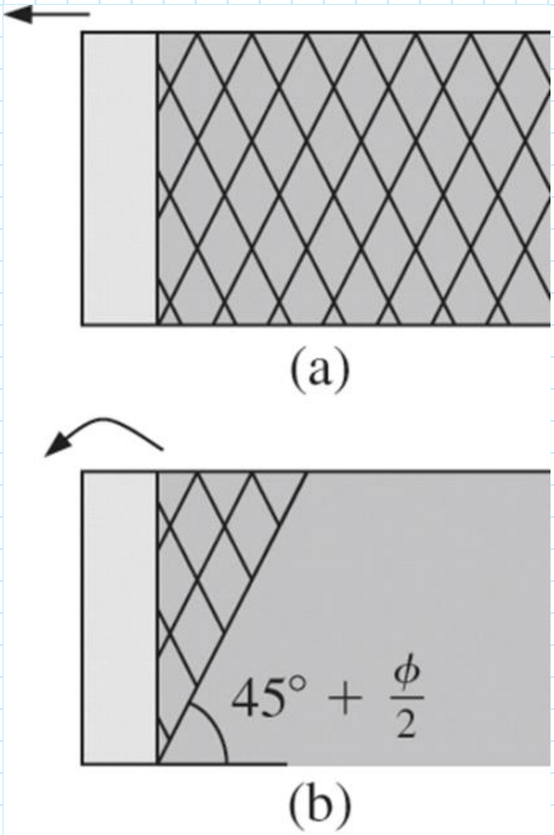
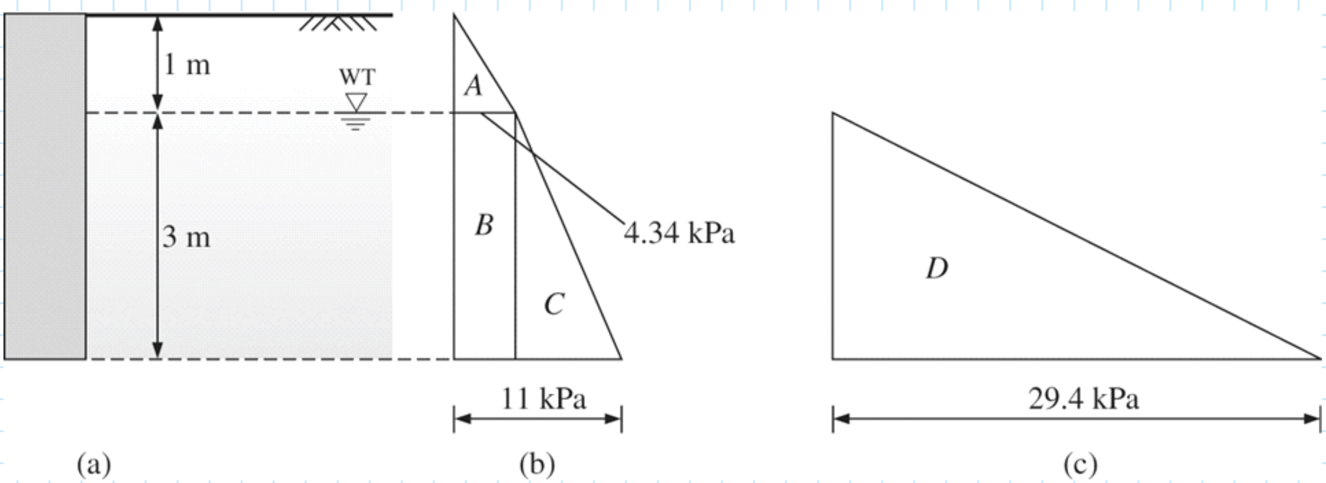


For sliding

For toppling



Earth pressure diagram

Hydrostatic water pressure diagram

Earth pressure diagram for the active case (for case with water)

Active Failure State - Sloped Backfill

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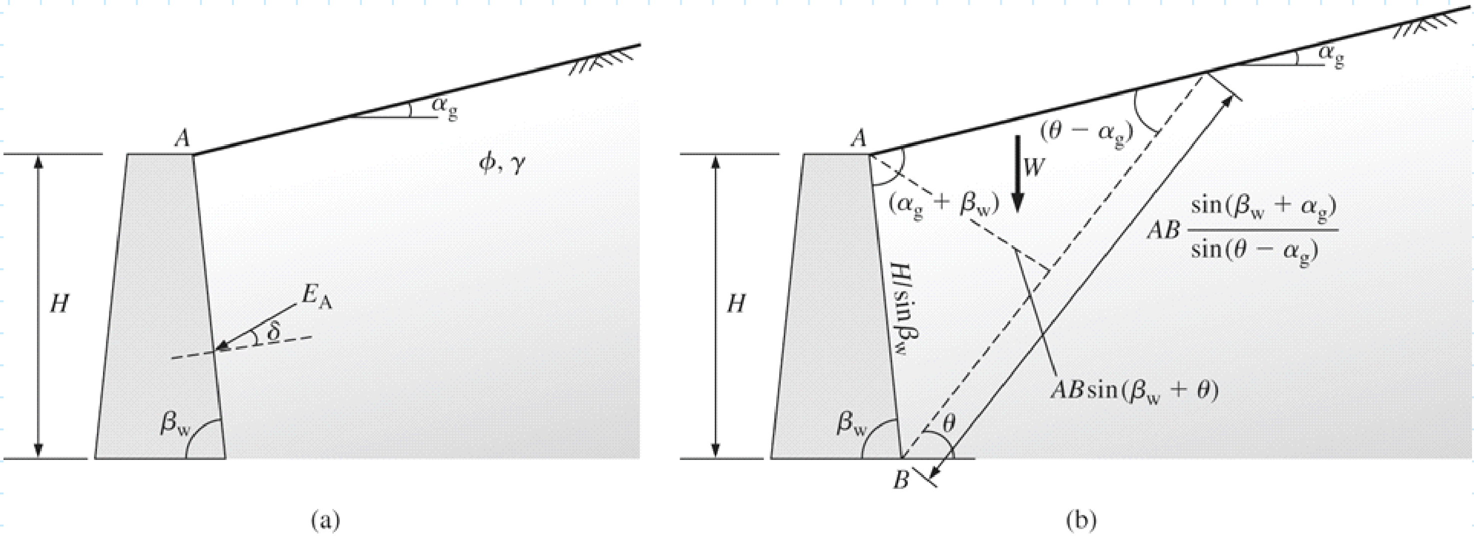
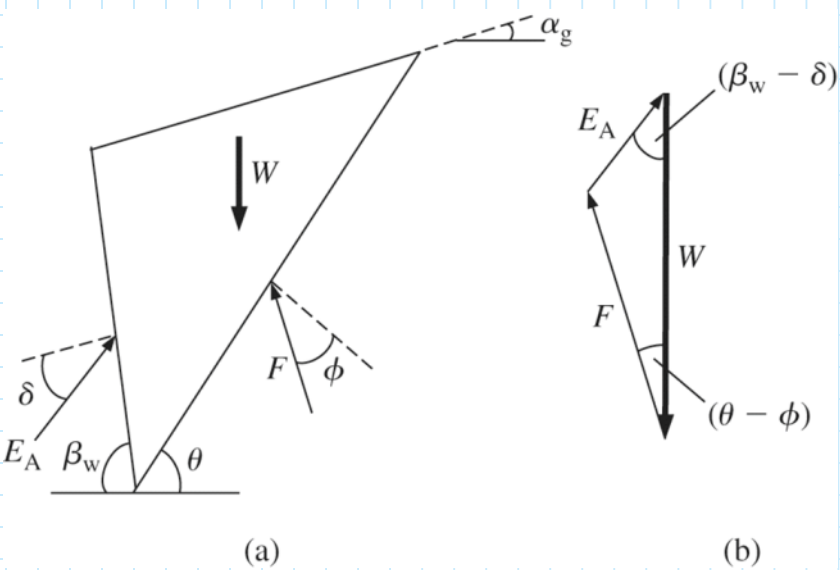
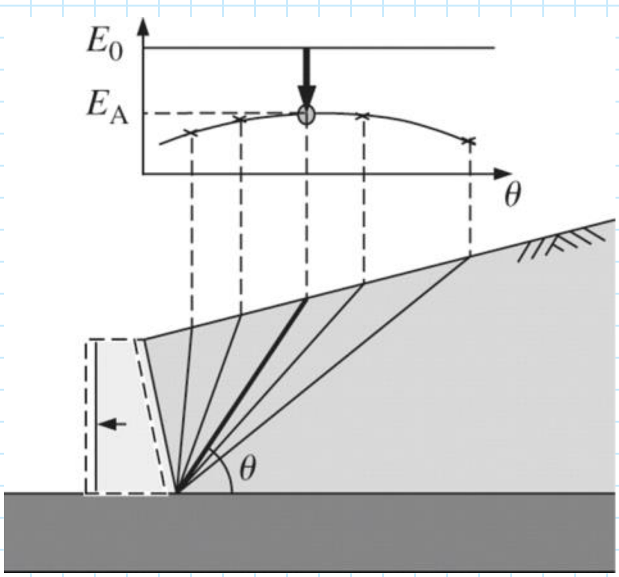




Force diagram



E_A is unknown. Maximize E_A by solving for $dE_A / d\theta = 0$

For more information, see Coulomb Theory

Earth Pressure Theory, Passive State

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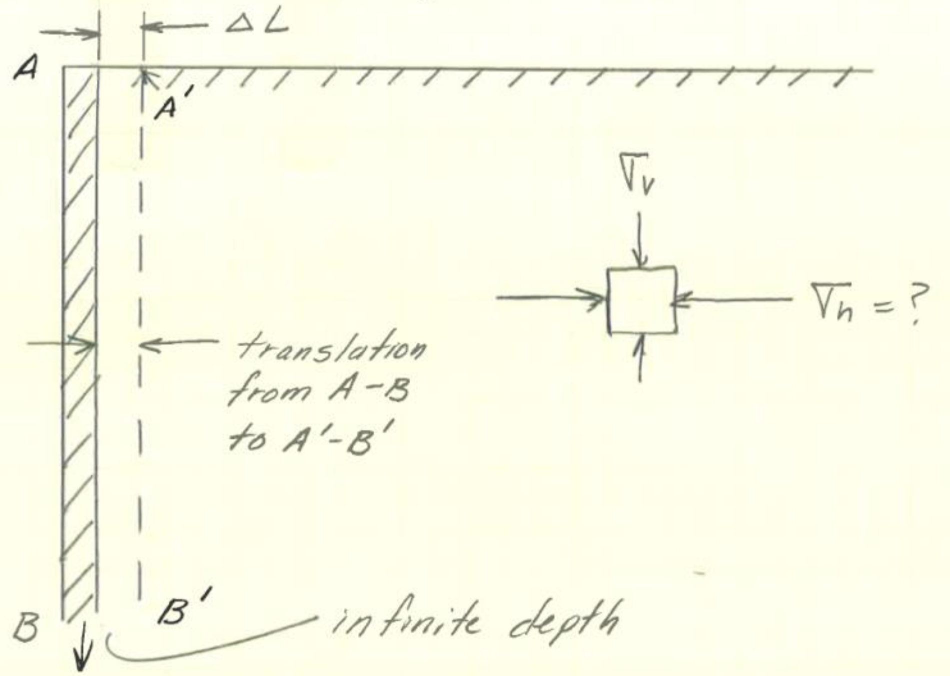
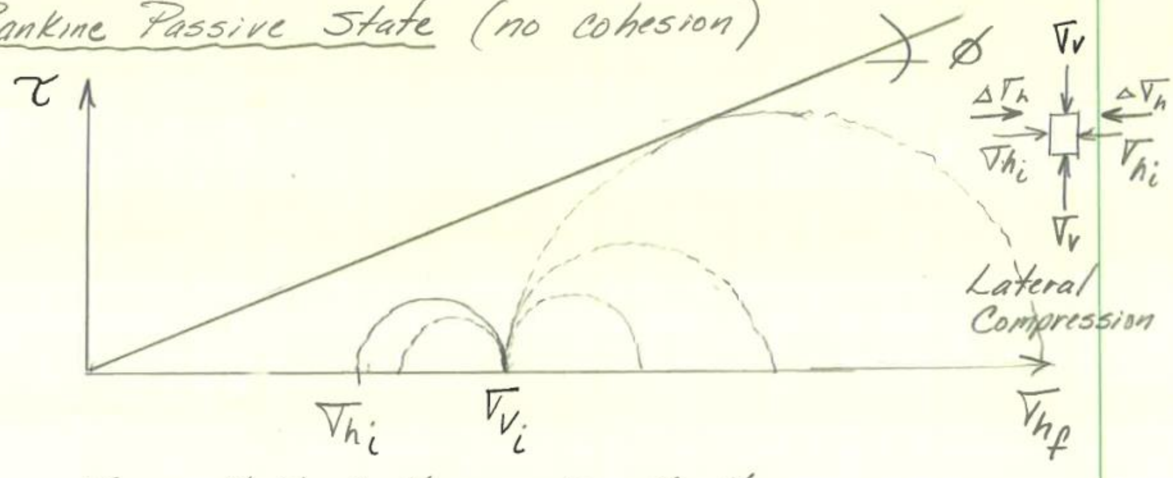
Passive Earth Pressure, Kp

- State of Plastic equilibrium



- Note that inward translation, pushing the wall into the soil, is necessary to mobilize passive earth pressure conditions.

Rankine Passive State (no cohesion)



From Holtz & Kovacs Eq. 10-16

$$\left(\frac{\sigma_h}{\sigma_v}\right)_f = \left(\frac{\sigma_1}{\sigma_3}\right)_f = \tan^2\left(45 + \frac{\phi}{2}\right) = K_{P\,(\text{RANKINE})}$$

Earth Pressure Theory, Passive State (cont.)

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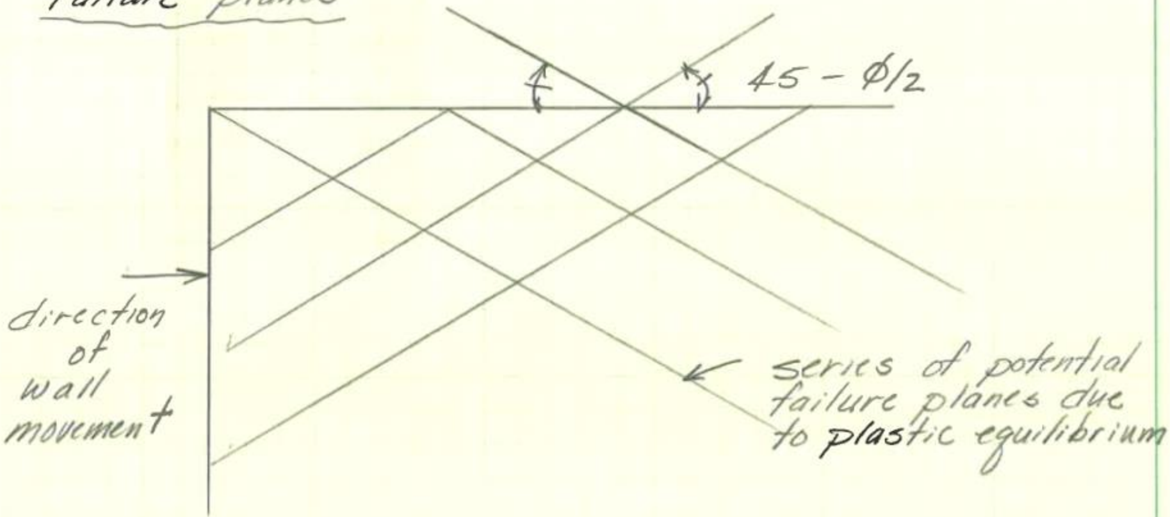
Rankine Passive State (cont.) (no cohesion)

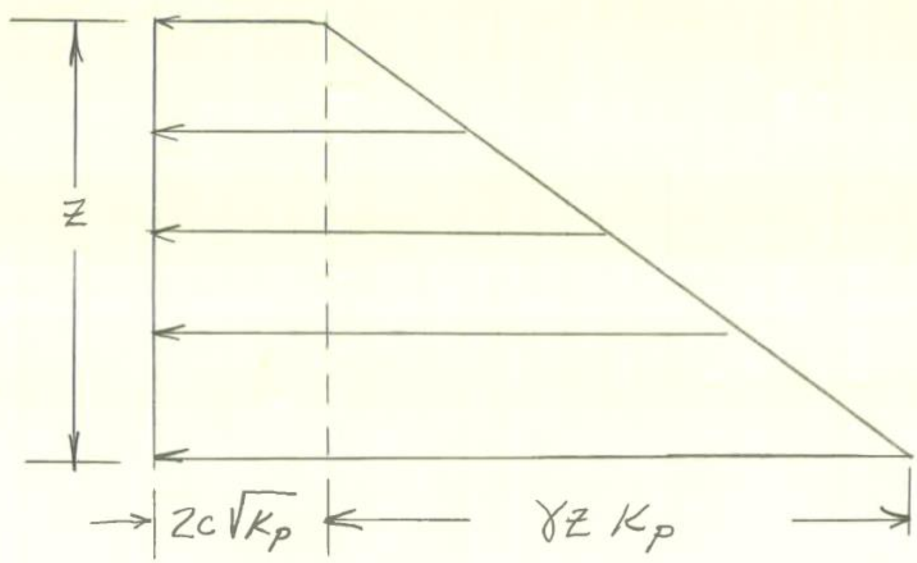
failure planes



Rankine Passive State (cont.) (with cohesion)

$$\left(\frac{\sigma_h}{\sigma_v}\right)_f = \left(\frac{\sigma_1}{\sigma_3}\right)_f = \frac{\gamma z \tan^2\left(45+\frac{\phi}{2}\right) + 2c\tan\left(45+\frac{\phi}{2}\right)}{\gamma z}$$

$$= K_P \text{ (RANKINE)}$$



Pressure Diagram

Earth Pressure Theory, Passive State (cont.)

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Rankine Passive state (cohesion only)

$$\left(\frac{\sigma_h}{\sigma_v}\right)_f = \left(\frac{\sigma_1}{\sigma_3}\right)_f = \frac{\gamma z + 2c}{\gamma z}$$

$$= K_P \text{ (RANKINE)}$$

Earth Pressure Theory - More Pressure Diagrams

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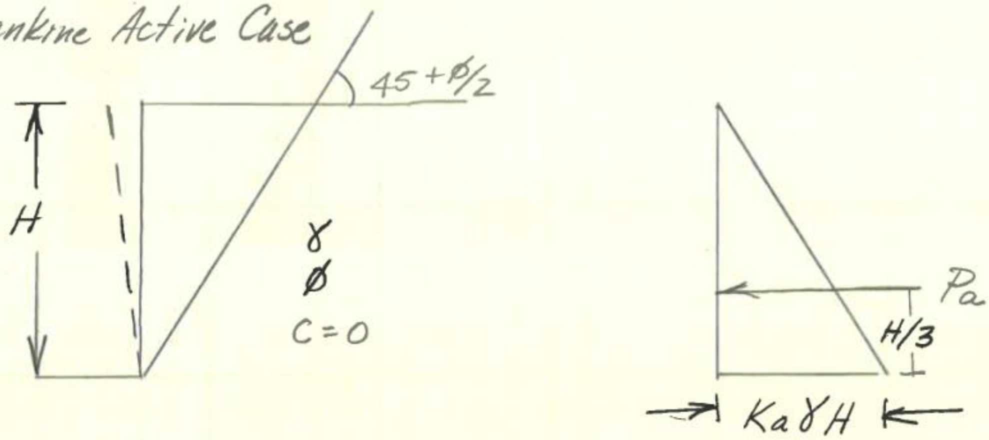
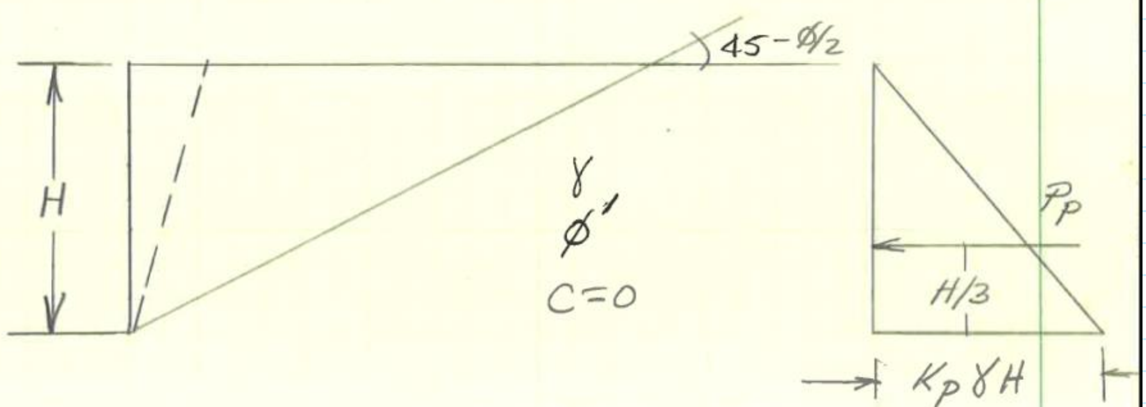
CASE 1 - Cohesionless Soil - Horizontal Ground Surface

Rankine Active Case



$P_a = \frac{1}{2} K_a \gamma H^2$

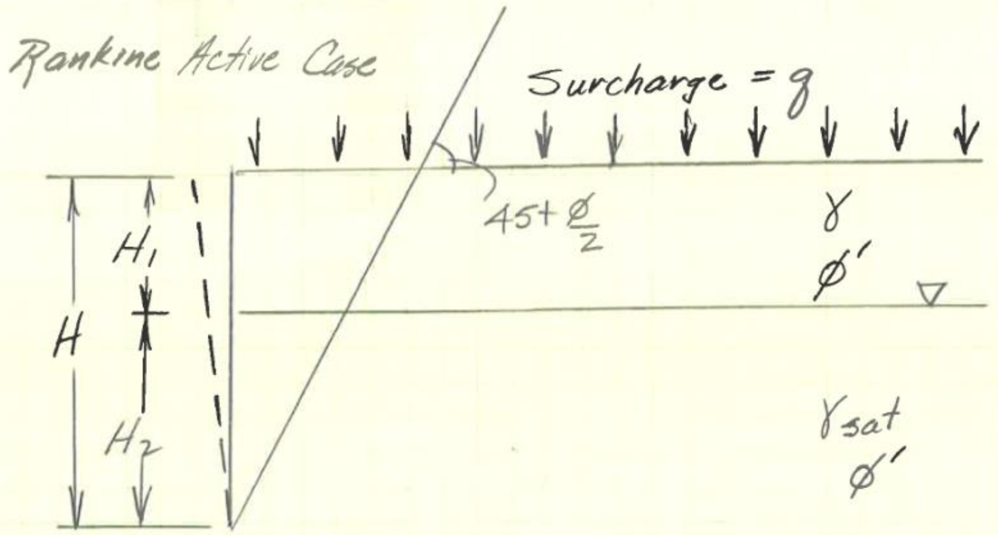
Rankine Passive Case

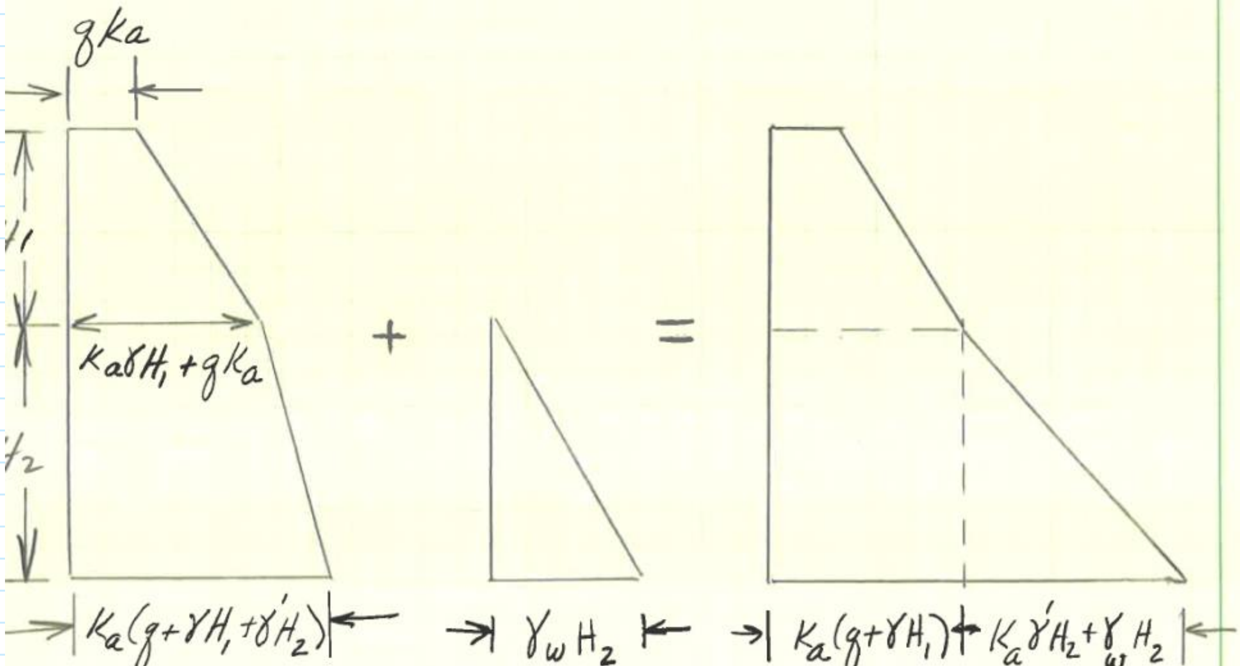


$P_p = \frac{1}{2} K_p \gamma H^2$

CASE 2 – Cohesionless Soil – Horizontal Ground Surface – Partially Submerged with Surcharge

Rankine Active Case





Earth Pressure Theory - More Pressure Diagrams (cont.)

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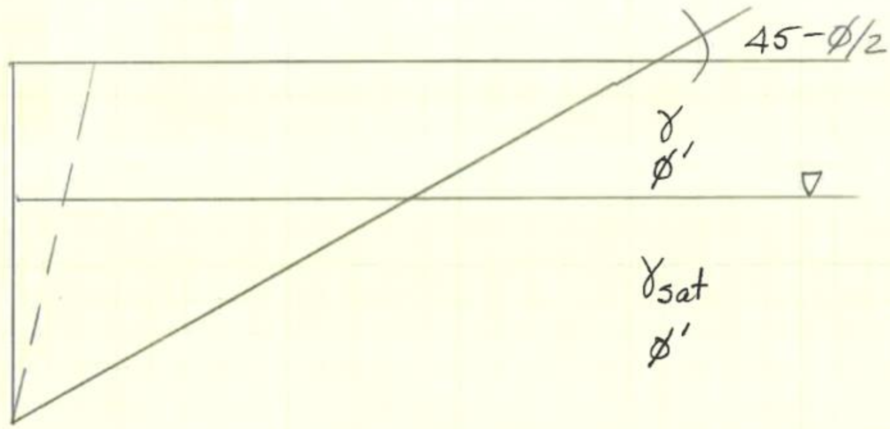
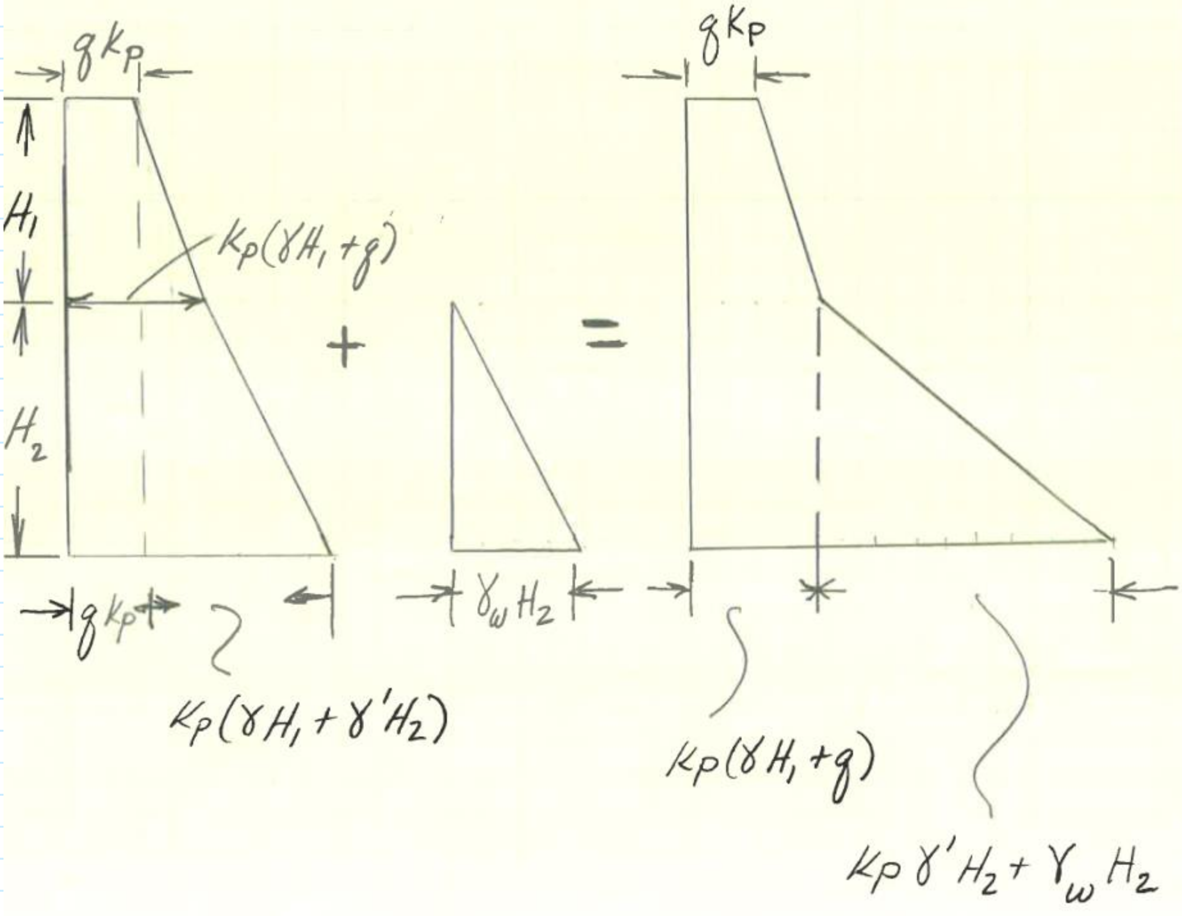
CASE 2 (cont) - Cohesionless Soil - Horizontal Ground Surface
Partially Submerged with Surcharge

Rankine Passive Case





Earth Pressure Theory - More Pressure Diagrams (cont.)

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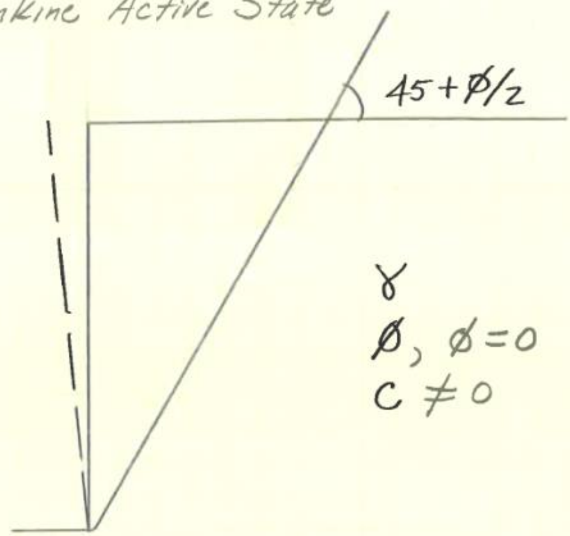
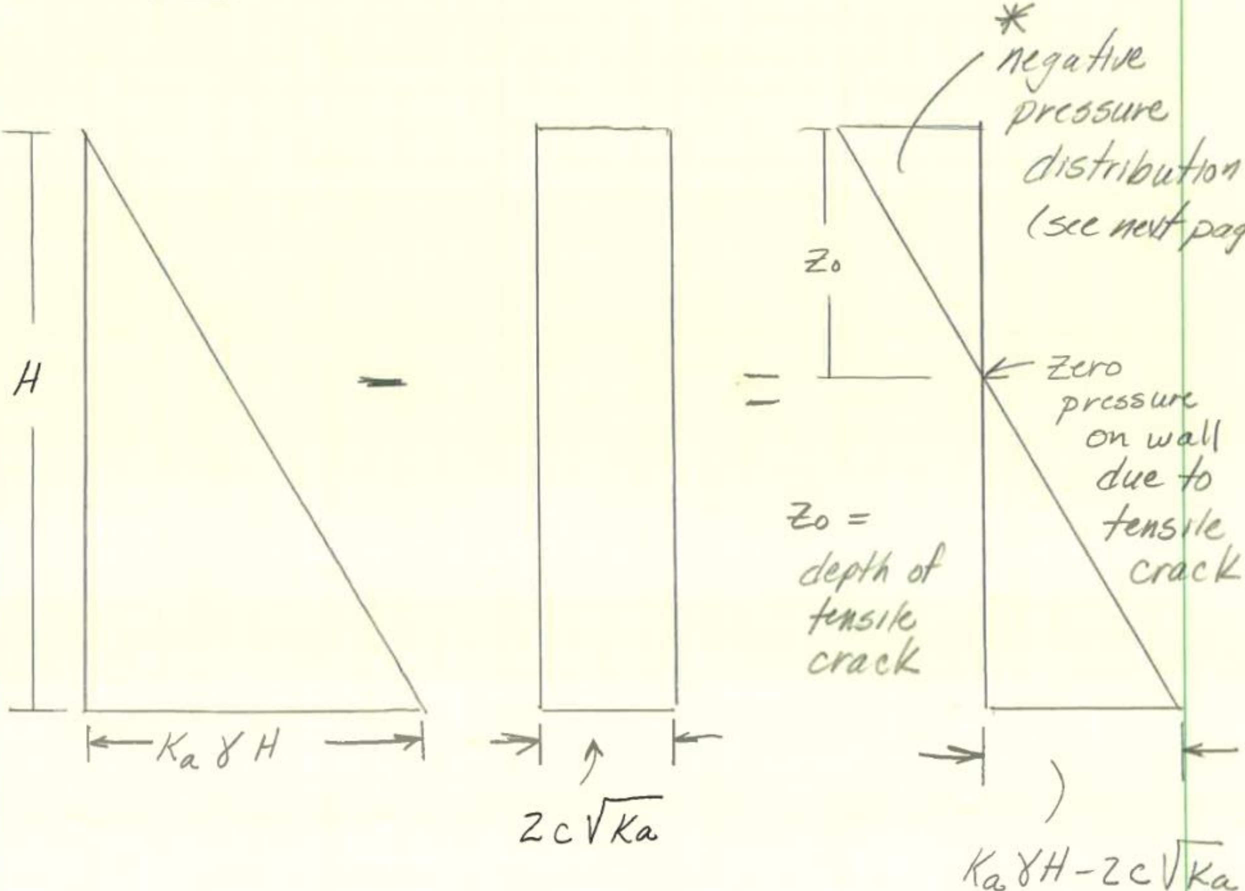
Case 3 - Cohesive Soil, Horizontal Ground Surface

Rankine Active State





$z_o = \dfrac{2c}{\gamma\sqrt{K_a}}$ (for $\phi \neq 0$) | $P_a = \frac{1}{2}K_a \gamma H^2 - 2c\sqrt{K_a}\,H$

$z_o = \dfrac{2c}{\gamma}$ (for $\phi = 0$) | $P_a = \frac{1}{2}\gamma H^2 - 2cH$

Case 3 - (cont)

* In practice it is common to ignore the negative pressure distribution due to the cohesion of the soil, in this case:

$$P_a = \frac{1}{2} K_a \gamma H^2 - 2c\sqrt{K_a}\, H \quad \text{is modified to}$$

$$P_a = \frac{1}{2}\left(K_a \gamma H - 2c\sqrt{K_a}\right)\left(H - \frac{2c}{\gamma\sqrt{K_a}}\right) \longleftarrow$$

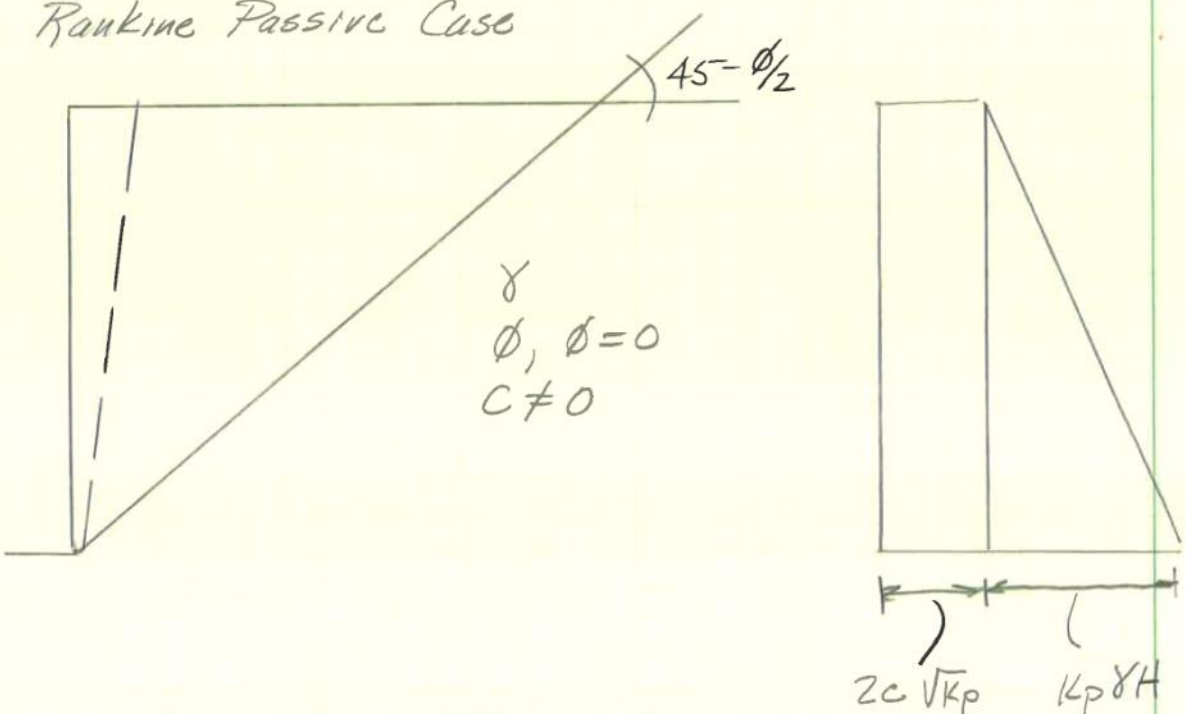
For $\phi \neq 0$

For the case of $\phi = 0$, then

$$P_a = \frac{1}{2}\gamma H^2 - 2cH + \frac{2c^2}{\gamma} \longleftarrow$$

Case 3 - (cont.)

Rankine Passive Case

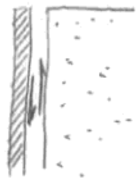


$$P_p = \frac{1}{2} K_p \gamma H^2 + 2c\sqrt{K_p}\, H \quad \text{for } \phi \neq 0$$

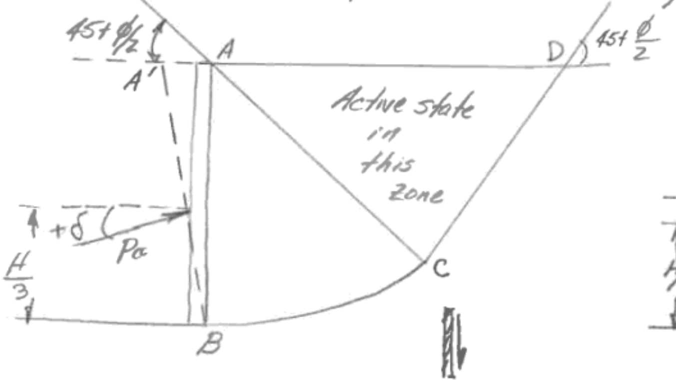
$$P_p = \frac{1}{2}\gamma H^2 + 2cH \quad \text{for } \phi = 0$$

Retaining Walls With Friction (Coulomb Theory) Page 1 of 3

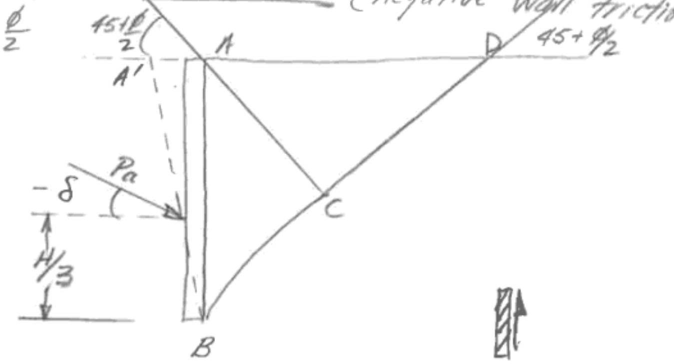
- In reality, retaining walls are rough and shear forces develop between the face of the wall and the backfill.



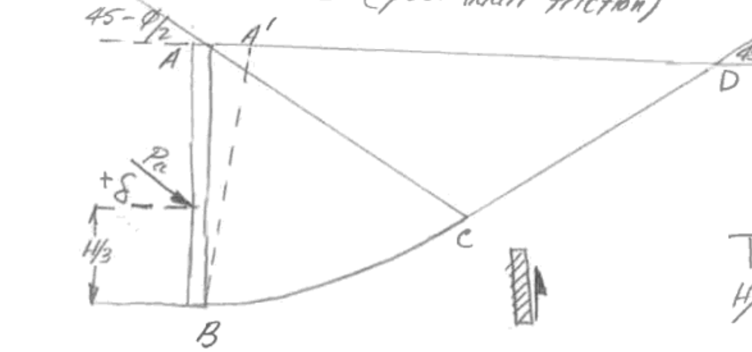
Active Case (positive wall friction)



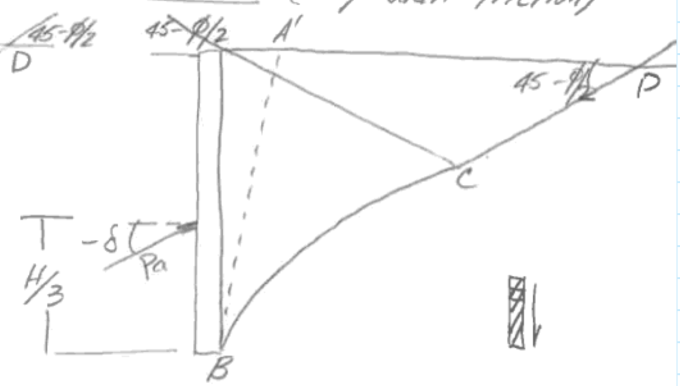
Active Case (negative wall friction)



Passive Case (pos. wall friction)



Passive Case (neg. wall friction)



Earth Pressure Theory, Coulomb Theory - Active State (cont.)

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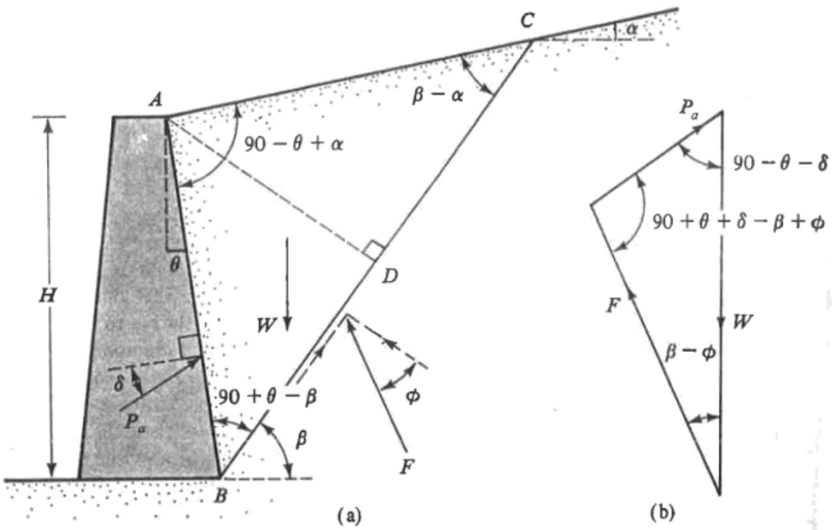


Figure 9.22 Coulomb's active pressure: (a) trial failure wedge; (b) force polygon

From force triangle

$$\frac{W}{\sin(90+\theta+\delta-\beta+\phi)} = \frac{P_a}{\sin(\beta-\phi)} \quad \text{(law of sines)}$$

$$P_a = \frac{\sin(\beta-\phi)}{\sin(90+\theta+\delta-\beta+\phi)} \cdot W \quad \text{(solving above for } P_a\text{)}$$

$$W = \tfrac{1}{2}(\overline{AD})(\overline{BC}) \cdot \gamma \quad (\tfrac{1}{2}\ \text{h} \cdot \text{b} \cdot \text{unit weight})$$

but $\overline{AD} = \overline{AB}\ \sin(90+\theta-\beta)$ (side opposite = hypot $\cdot \sin\alpha$)

$$= \frac{H}{\cos\theta} \cdot \sin(90+\theta-\beta)$$

$$= H \cdot \frac{\cos(\theta-\beta)}{\cos\theta} \quad (\sin(90+\theta) = \cos\theta)$$

to get $\overline{BC}$

$$\frac{\overline{AB}}{\sin(\beta-\alpha)} = \frac{\overline{BC}}{\sin(90-\theta+\alpha)} \quad \text{(law of sines)}$$

$$\overline{BC} = \frac{\cos(\theta-\alpha)}{\sin(\beta-\alpha)} \cdot \overline{AB} = \frac{\cos(\theta-\alpha)}{\sin(\beta-\alpha)\cdot\cos\theta} \cdot H$$

$$W = \frac{1}{2}(AD)(BC)\cdot\gamma$$

$$= \frac{1}{2}\gamma H^2 \frac{\cos(\theta-\beta)\cdot\cos(\theta-\alpha)}{\cos^2\theta\cdot\sin(\beta-\alpha)}$$

$$P_a = \frac{\sin(\beta-\phi)}{\sin(90+\theta+\delta+\beta+\phi)}\cdot W$$

$$P_a = \frac{1}{2}\gamma H^2\left[\frac{\cos(\theta-\beta)\cdot\cos(\theta-\alpha)\cdot\sin(\beta-\phi)}{\cos^2\theta\cdot\sin(\beta-\alpha)\cdot\sin(90+\theta+\delta-\beta+\phi)}\right]$$

$\gamma, H, \theta, \alpha, \phi$ and δ are known

β is unknown

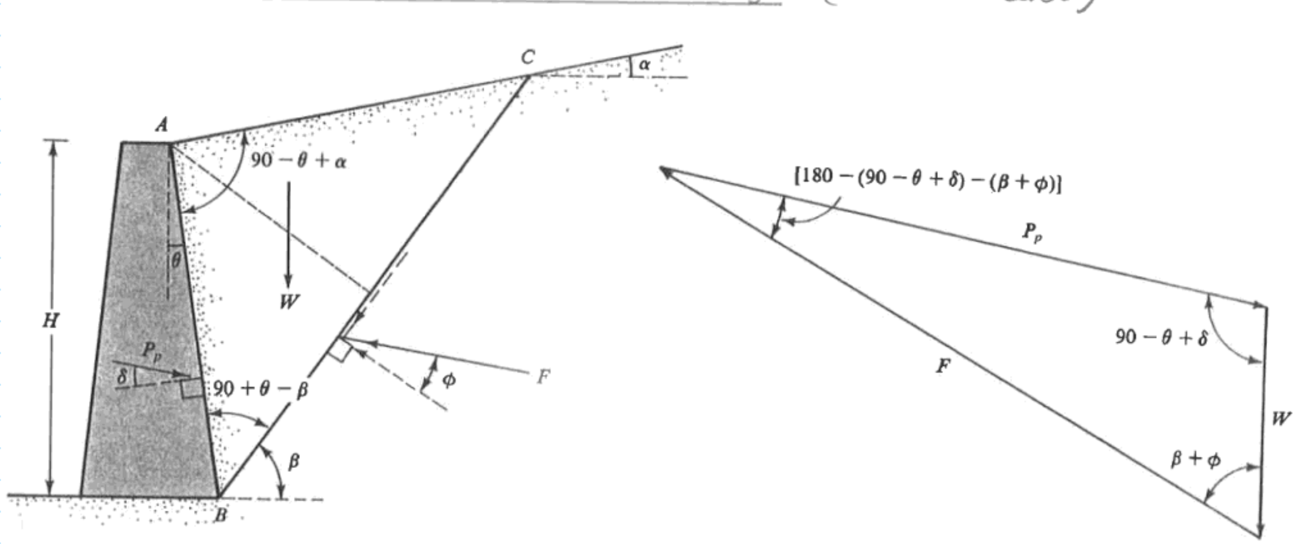
To determine critical value of β for maximum P_a

$$\frac{dP_a}{d\beta} = 0$$

$$\rightarrow \boxed{P_a = \frac{1}{2}K_a\gamma H^2} \quad \text{where } K_a = \frac{\cos^2(\phi-\theta)}{\cos^2\theta\cdot\cos(\delta+\theta)\left[1+\sqrt{\frac{\sin(\delta+\phi)\cdot\sin(\phi-\alpha)}{\cos(\delta+\theta)\cdot\cos(\theta-\alpha)}}\right]^2}$$

Note: when $\alpha = 0°$, $\theta = 0°$ and $\delta = 0°$ Coulomb's K_a = Rankine K_a

Coulomb's Earth Pressure Theory (Passive Case)



$$\boxed{P_p = \frac{1}{2}K_p\gamma H^2} \text{ where } \frac{\cos^2(\phi+\theta)}{}$$

$$K_p = \frac{\cos^2(\phi+\theta)}{\cos^2\theta\cos(\delta-\theta)\left[1-\sqrt{\frac{\sin(\phi+\delta)\sin(\phi+\alpha)}{\cos(\delta-\theta)\cos(\alpha-\theta)}}\right]^2}$$

Note: $K_{p,Coul} = K_{p,Rankin}$

Kp equation from Das, Principles of Geotechnical Engineering 4th Edition. I have corrected the error in this equation.

Earth Pressure Theory, Coulomb Theory - Active and Passive State

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TABLE 11-1

Coulomb active earth pressure coefficients K_a using Eq. (11-3)

ALPHA = 90 BETA = -10

δ	φ = 26	28	30	32	34	36	38	40	42
0	0.354	0.328	0.304	0.281	0.259	0.239	0.220	0.201	0.184
16	0.311	0.290	0.270	0.252	0.234	0.216	0.200	0.184	0.170
17	0.309	0.289	0.269	0.251	0.233	0.216	0.200	0.184	0.169
20	0.306	0.286	0.267	0.249	0.231	0.214	0.198	0.183	0.169
22	0.304	0.285	0.266	0.248	0.230	0.214	0.198	0.183	0.168

ALPHA = 90 BETA = -5

δ	φ = 26	28	30	32	34	36	38	40	42
0	0.371	0.343	0.318	0.293	0.270	0.249	0.228	0.209	0.191
16	0.328	0.306	0.284	0.264	0.245	0.226	0.209	0.192	0.176
17	0.327	0.305	0.283	0.263	0.244	0.226	0.208	0.192	0.176
20	0.324	0.302	0.281	0.261	0.242	0.224	0.207	0.191	0.175
22	0.322	0.301	0.280	0.260	0.242	0.224	0.207	0.191	0.175

ALPHA = 90 BETA = 0

δ	φ = 26	28	30	32	34	36	38	40	42
0	0.390	0.361	0.333	0.307	0.283	0.260	0.238	0.217	0.198
16	0.349	0.324	0.300	0.278	0.257	0.237	0.218	0.201	0.184
17	0.348	0.323	0.299	0.277	0.256	0.237	0.218	0.200	0.183
20	0.345	0.320	0.297	0.276	0.255	0.235	0.217	0.199	0.183
22	0.343	0.319	0.296	0.275	0.254	0.235	0.217	0.199	0.183

ALPHA = 90 BETA = 5

δ	φ = 26	28	30	32	34	36	38	40	42
0	0.414	0.382	0.352	0.323	0.297	0.272	0.249	0.227	0.206
16	0.373	0.345	0.319	0.295	0.272	0.250	0.229	0.210	0.192
17	0.372	0.344	0.318	0.294	0.271	0.249	0.229	0.210	0.192
20	0.370	0.342	0.316	0.292	0.270	0.248	0.228	0.209	0.191
22	0.369	0.341	0.316	0.292	0.269	0.248	0.228	0.209	0.191

ALPHA = 90 BETA = 10

δ	φ = 26	28	30	32	34	36	38	40	42
0	0.443	0.407	0.374	0.343	0.314	0.286	0.261	0.238	0.216
16	0.404	0.372	0.342	0.315	0.289	0.265	0.242	0.221	0.201
17	0.404	0.371	0.342	0.314	0.288	0.264	0.242	0.221	0.201
20	0.402	0.370	0.340	0.313	0.287	0.263	0.241	0.220	0.201
22	0.401	0.369	0.340	0.312	0.287	0.263	0.241	0.220	0.201

ALPHA = 90 BETA = 15

δ	φ = 26	28	30	32	34	36	38	40	42
0	0.482	0.440	0.402	0.367	0.334	0.304	0.276	0.251	0.227
16	0.447	0.408	0.372	0.340	0.310	0.283	0.258	0.234	0.213
17	0.447	0.407	0.372	0.339	0.310	0.282	0.257	0.234	0.212
20	0.446	0.406	0.371	0.338	0.309	0.282	0.257	0.234	0.212
22	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

TABLE 11-2

Coulomb passive earth pressure coefficients K_p using Eq. (11-6)

ALPHA = 90 BETA = -10

δ	φ = 26	28	30	32	34	36	38	40	42
0	1.914	2.053	2.204	2.369	2.547	2.743	2.957	3.193	3.452
16	2.693	2.956	3.247	3.571	3.934	4.344	4.807	5.335	5.940
17	2.760	3.034	3.339	3.679	4.062	4.493	4.983	5.543	6.187
20	2.980	3.294	3.645	4.041	4.488	4.997	5.581	6.255	7.039
22	3.145	3.490	3.878	4.317	4.816	5.389	6.050	6.819	7.720

ALPHA = 90 BETA = -5

δ	φ = 26	28	30	32	34	36	38	40	42
0	2.223	2.392	2.577	2.781	3.004	3.250	3.523	3.826	4.163
16	3.367	3.709	4.094	4.529	5.024	5.591	6.243	7.000	7.883
17	3.469	3.828	4.234	4.694	5.218	5.820	6.516	7.326	8.277
20	3.806	4.226	4.704	5.250	5.879	6.609	7.462	8.468	9.665
22	4.064	4.532	5.067	5.684	6.399	7.236	8.222	9.397	10.809

ALPHA = 90 BETA = 0

δ	φ = 26	28	30	32	34	36	38	40	42
0	2.561	2.770	3.000	3.255	3.537	3.852	4.204	4.599	5.045
16	4.195	4.652	5.174	5.775	6.469	7.279	8.229	9.356	10.704
17	4.346	4.830	5.385	6.025	6.767	7.636	8.661	9.882	11.351
20	4.857	5.436	6.105	6.886	7.804	8.892	10.194	11.771	13.705
22	5.253	5.910	6.675	7.574	8.641	9.919	11.466	13.364	15.726

ALPHA = 90 BETA = 5

δ	φ = 26	28	30	32	34	36	38	40	42
0	2.943	3.203	3.492	3.815	4.177	4.585	5.046	5.572	6.173
16	5.250	5.878	6.609	7.464	8.474	9.678	11.128	12.894	15.076
17	5.475	6.146	6.929	7.850	8.942	10.251	11.836	13.781	16.201
20	6.249	7.074	8.049	9.212	10.613	12.321	14.433	17.083	20.468
22	6.864	7.820	8.960	10.334	12.011	14.083	16.685	20.011	24.352

ALPHA = 90 BETA = 10

δ	φ = 26	28	30	32	34	36	38	40	42
0	3.385	3.712	4.080	4.496	4.968	5.507	6.125	6.840	7.673
16	6.652	7.545	8.605	9.876	11.417	13.309	15.665	18.647	22.497
17	6.992	7.956	9.105	10.492	12.183	14.274	16.899	20.254	24.633
20	8.186	9.414	10.903	12.733	15.014	17.903	21.636	26.569	33.270
22	9.164	10.625	12.421	14.659	17.497	21.164	26.012	32.601	41.863

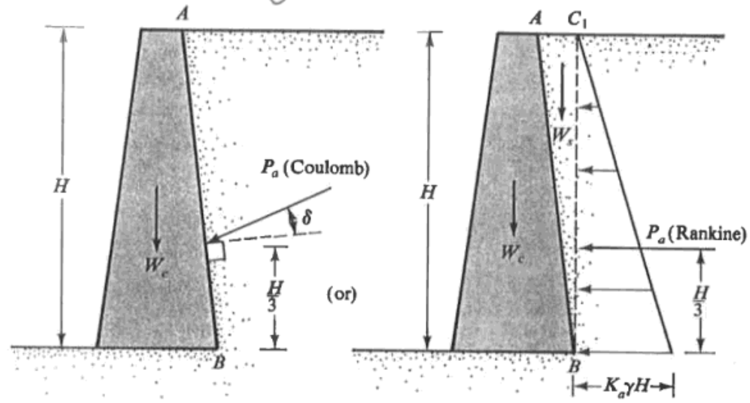
ALPHA = 90 BETA = 15

δ	φ = 26	28	30	32	34	36	38	40	42
0	3.913	4.331	4.807	5.362	5.980	6.710	7.563	8.570	9.768
16	8.611	9.936	11.555	13.557	16.073	19.291	23.494	29.123	36.894
17	9.139	10.590	12.373	14.595	17.413	21.054	25.867	32.409	41.603
20	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
22	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

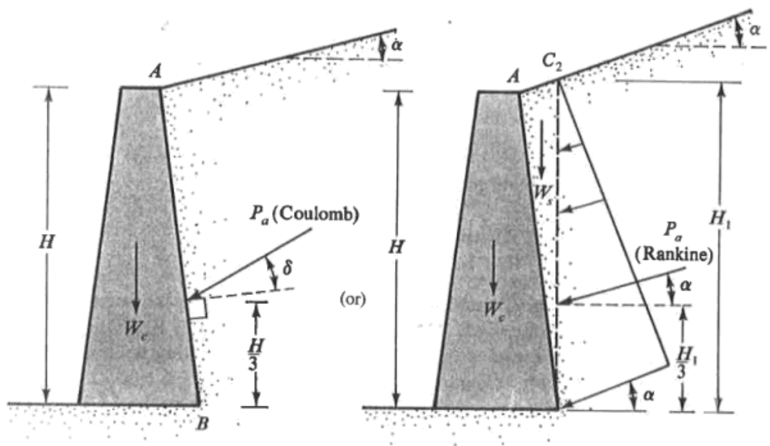
The notation in this table is different from that used in the free body diagrams in the previous pages. For this table, α (ALPHA) is the inclination of the backwall measured from the <u>horizontal</u> (i.e., α is 90 for a vertical backwall); β (BETA) is the inclination of the slope behind the wall, ϕ (phi) is the internal angle of friction of the backfill soil, and δ (delta) is the interface friction between the backfill soil and the wall (i.e., concrete).

The values in this table can be used to check your functions entered in the homework spreadsheet.

Approximate Analysis of Active Force on Retaining Walls Using Rankine Theory for non-vertical backwall & back slope



(a)



1) Gravity Retaining Walls with granular backfill
2) horizontal or sloped backfill
3) For Rankine analysis, Ws, is added for stability analysis

also $P_a = \frac{1}{2} K_a \gamma H_1^2$

$$K_a = \cos\alpha \cdot \frac{\cos\alpha - \sqrt{\cos^2\alpha - \cos^2\phi}}{\cos\alpha + \sqrt{\cos^2\alpha - \cos^2\phi}} \Bigg\} \text{ for sloped backfill}$$

Active and Passive Pressure - Movement of Wall

Monday, November 9, 2015 11:43 AM

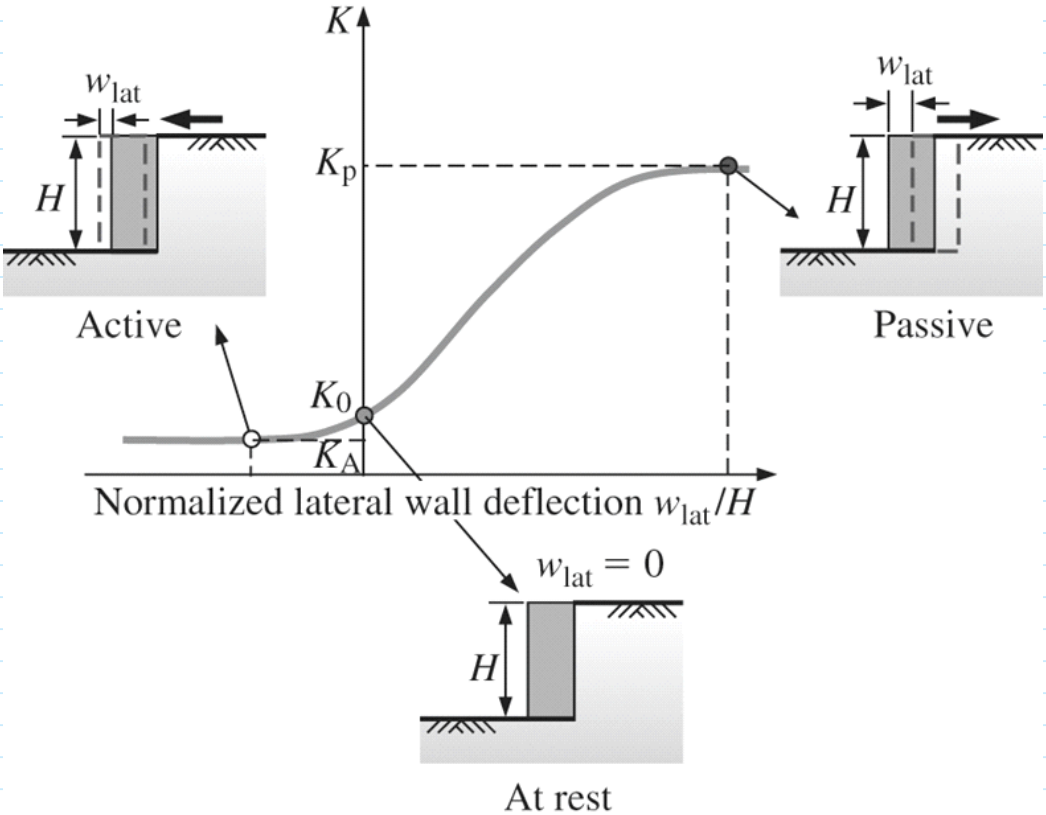


Table 16-1 Movement required to mobilize active and passive pressures

Soil	Horizontal displacement/ wall height	
	Active state	**Passive state**
Dense sand	0.001	0.020
Loose sand	0.004	0.060
Stiff clay	0.010	0.020
Soft clay	0.020	0.040

Based on values proposed in CGS (1992).

Note that the amount of lateral displacement wlat is much larger for the passive state e when compare with that of active state. This means that full active earth pressure is mobilized after only small displacements; whereas full passive earth pressure requires more displacement. Hence, especially for passive earth pressure, one must consider the amount of displacement that is expected in order to estimate the passive earth pressure.

No. 5505 Engineer's Computation Pad

Application of Earth Pressure Theory in Practice

- Coulomb theory is more general than Rankine Theory because the following can be considered
  - irregular backfill (sloped)
  - concentrated loads on back fill slope
  - seepage forces
  - wall soil interaction (i.e., no frictionless wall)
- For active earth pressure, Rankine K_a is a special case of Coulomb theory where:

  $\alpha = 0°$, $\theta = 0°$ and $\delta = 0°$
- For passive earth pressure, Rankine theory underpredicts (unconservative) the maximum earth pressure.
- For passive earth pressure, Coulomb theory over predicts (conservative) the maximum earth pressure.
- Rankine theory is rarely used for passive earth pressure.
- Coulomb theory over predicts actual passive earth pressures by about 11% for $\delta = \phi/2$ and by 100% for $\delta = \phi$. For this reason, Coulomb theory is rarely used to evaluate passive earth pressure for $\delta > \phi/2$.
- Log-spiral Method is slightly more accurate than Rankine and Coulomb theory for active earth pressures and is considerably more accurate for passive earth pressure.
- Typical values of δ for soils against concrete range from $\delta = \phi/2$ to $\delta = \frac{2}{3}\phi$, thus Coulomb theory will give a conservative estimate for concrete retaining walls with soil back fill.

Retaining Wall Design (Free Body Diagrams - Gravity Wall)

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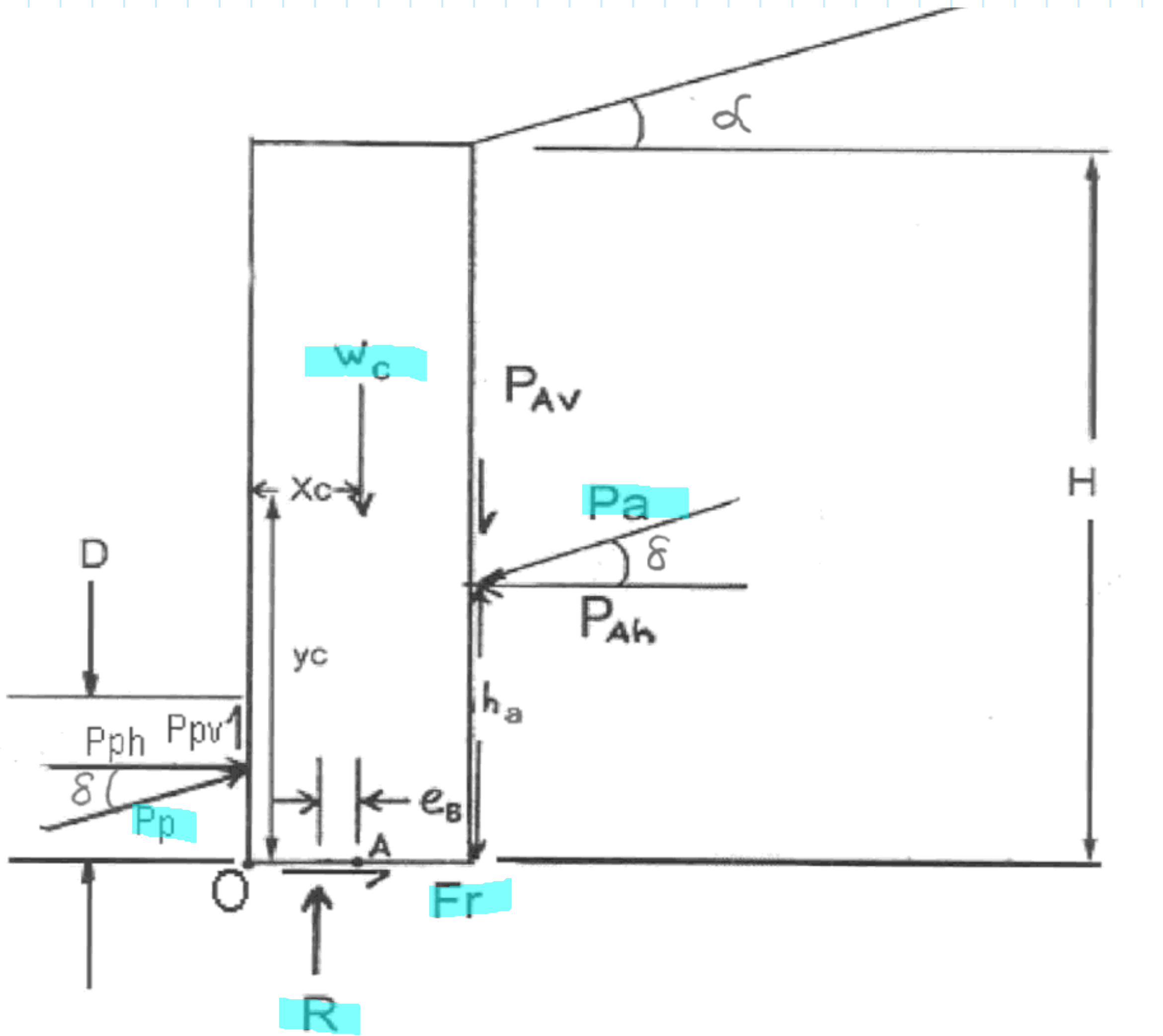

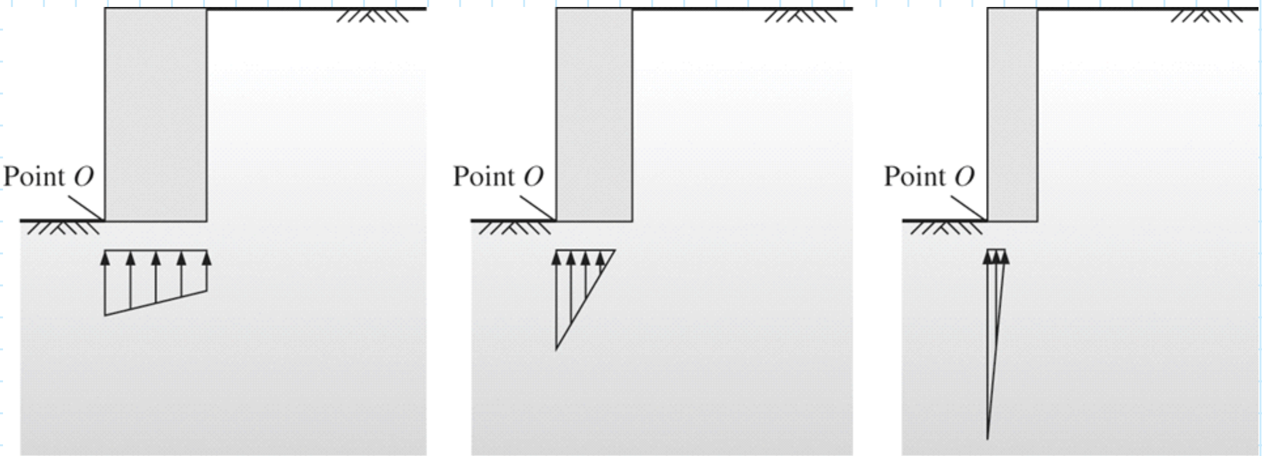

(a) (b) (c)

Retaining Wall Design (Gravity Wall - Spreadsheet)

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Gravity Wall Design

Wall Dimensions

Top	2.75	ft
Bottom	2.75	ft
$\gamma_{concrete}$	150	pcf
H	10	ft
D	3	ft

Centroid Location

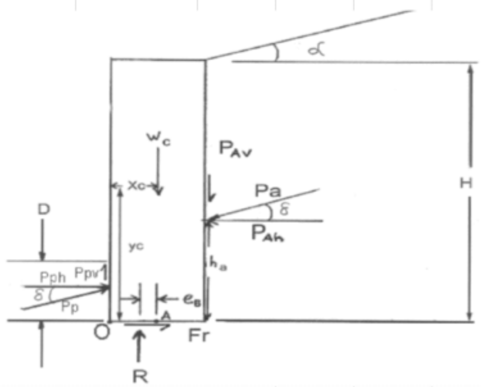
x_c	1.375	ft
y_c	5.000	ft

Fill Properties

$\alpha_{backfill}$ deg	10	0.175	radians
α_{front} deg	0	0.000	radians
φ deg	40	0.698	radians
δ deg [note 1]	20.0	0.349	radians
$\gamma_{backfill}$	120		pcf

Foundation Soil Properties

φ deg	40	0.698	radians
δ deg [note 2]	40	0.698	radians



Earth Pressures

K_A	0.220	unitless	
K_P	11.77	unitless	
factor for K_P [note 3]	0.1000		
K_0	0.419	unitless	$(1-\sin\phi)^*(1+\sin\beta)$

Resisting Moments on Wall

$P_{av} * B$	1242.7
$P_{ph} * D/3$	597.3
$W_c * x_c$	5671.9
ΣM_r	7511.9

Forces

P_a	1321.2	lb/ft	
P_{ah}	1241.5	lb/ft	$P_a \cos\delta$
P_{av}	451.9	lb/ft	$P_{ah}\tan\delta$ or $P_a\cos\delta$
W_c	4125	lb/ft	
R	4359.5	lb/ft	$W_c + P_{av}$ - P_{pv}
F_r	3658.0	lb/ft	R tan (δ or ϕ) [note 2]
$P_{p\ factored}$ x 0.5	635.7	lb/ft	
P_{ph}	597.33	lb/ft	$P_{p\ factored}\cos\delta$
P_{pv}	217.4	lb/ft	$P_{p\ factored}\sin\delta$

Overturning Moments on Wall

$P_{ah} * h_a$	4138.4
ΣMo	4138.4

Factors of Safety

$FS_{sliding}$	3.428
FS_{oturn}	1.815

Acceptable FS

$FS_{sliding}$= 1.5

FS_{oturn} = 1.5

Values for bearing capacity calculation

xbar	0.774	$(\Sigma M_r - \Sigma Mo)/R$	(from left edge to location of R, must be positive)
e	0.601	x_c-xbar	(distance from R to center of mass)
e/B	0.219	OK	(e recommended to be less than B/6)
V	4576.9	Wc + Pav (used for bearing capacity; P_{pv} neglected)	
H	1067.249	Fr / FS sliding	
M	2751.5	V*e	

Remember to check bearing capacity and global stability of wall

note 1 δ = 0.5 to 0.67 of ϕ for concrete-soil interface.

note 2 δ = 0.5 to 0.67 of ϕ for concrete-soil interface where concrete footing is pre-cast; otherwise use $\delta = \phi$ concrete cast directly on foundation soil.

note 3 (see below)

11.6.3.5—Passive Resistance

Passive resistance shall be neglected in stability computations, unless the base of the wall extends below the depth of maximum scour, freeze-thaw or other disturbances. In the latter case, only the embedme below the greater of these depths shall be consider effective.

Where passive resistance is utilized to ensu adequate wall stability, the calculated passive resistan of soil in front of abutments and conventional wal shall be sufficient to prevent unacceptable forwa movement of the wall.

The passive resistance shall be neglected if the sc providing passive resistance is, or is likely to becon soft, loose, or disturbed, or if the contact between tl soil and wall is not tight.

Unacceptable deformations may occur before passive resistance is mobilized. Approximate deformations required to mobilize passive resistance are discussed in Article C3.11.1, where H in Table C3.11.1-1 is the effective depth of passive restraint.

Table C3.11.1-1—Approximate Values of Relative Movements Required to Reach Active or Passive Earth Pressure Conditions (Clough and Duncan, 1991)

Type of Backfill	Values of Δ/H Active	Passive
Dense sand	0.001	0.01
Medium dense sand	0.002	0.02
Loose sand	0.004	0.04
Compacted silt	0.002	0.02
Compacted lean clay	0.010	0.05
Compacted fat clay	0.010	0.05

4

Notes

The table is not factors, but shows that the amount of movement to mobilize full passive pressure is 10 times that of active. Hence, when 100 percent of active is mobilized, only 10 percent of passive pressure is mobilized, Thus, the reduction factor for KP is 10/100 or 0.1.

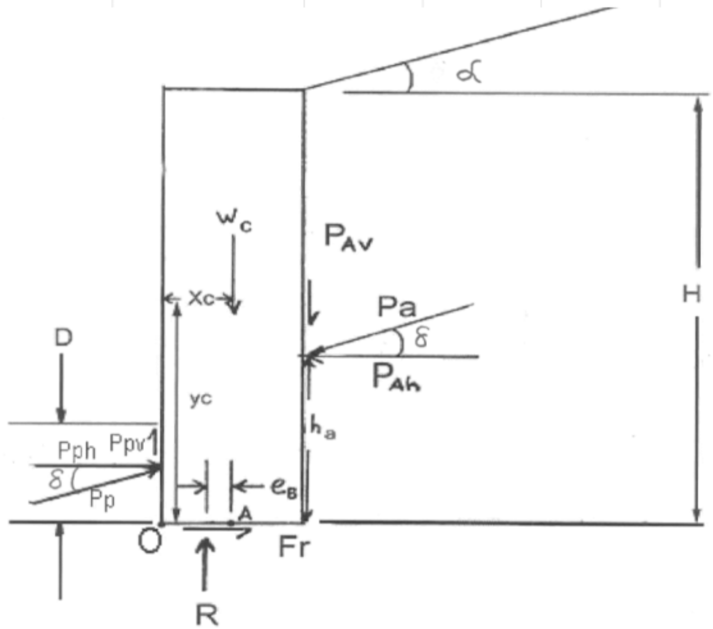
The Kp factor recommended is using 0.1. This reduces Kp to 10 percent. This is done because the displacement to produce the peak passive pressure is 10 times larger than that to produce peak active pressure (see Table C3.11.1-1 above)

Also, Pp factored has been additionally factored by 0.5 as recommended by geotechnical guidance documents. (see cell labeled "**P_p factored by 0.5**")

Gravity Wall Design

Wall Dimensions				Fill Properties			
Top	4	ft		$\alpha_{backfill}$ deg	20	0.349	*radians*
Bottom	4	ft		α_{front} deg	0	0.000	*radians*
$\gamma_{concrete}$	150	pcf		φ deg	40	0.698	*radians*
H	15	ft		δ deg note 1	20.0	0.349	*radians*
D	3	ft		$\gamma_{backfill}$	120		*pcf*
Centroid Location							
x_c	2.000	ft		**Foundation Soil Properties**			
y_c	7.500	ft		φ deg	40	0.698	*radians*
				δ deg note 2	40	0.698	*radians*

Earth Pressures				Resisting Moments on Wall	
K_A	0.250	unitless		P_{av} * B	4625.0
K_P	11.77	unitless		P_{ph} * D/3	597.3
factor for K_P note 3	0.1000			W_c * x_c	18000.0
K_0	0.479	unitless	(1-sin ϕ)*(1+ sin β)	ΣM_r	23222.3



Forces				Overturning Moments on Wall	
P_a	3380.6	lb/ft			
P_{ah}	3176.8	lb/ft	P_a cos δ	P_{ah} * h_a	15883.8
P_{av}	1156.2	lb/ft	P_{ah} tan δ or P_a cos (	ΣMo	15883.8
W_c	9000	lb/ft			
R	9938.8	lb/ft	W_c + $P_{av'}$ - $P_{pv'}$	**Factors of Safety**	
F_r	8339.7	lb/ft	R tan (δ or ϕ) note 2	**$FS_{sliding}$**	**2.813**
$P_{p\ factored}$ x 0.5	635.7	lb/ft		**FS_{oturn}**	**1.462**
P_{ph}	597.33	lb/ft	$P_{p\ factored}$ cos δ	Acceptable FS	
P_{pv}	217.4	lb/ft	$P_{p\ factored}$ sin δ	$FS_{sliding}$ = 1.5	
				FS_{oturn} = 1.5	

Values for bearing capacity calculation			
xbar	0.738	(ΣM_r-ΣMo)/R	(from left edge to location of R, must be positive)
e	1.262	x_c-xbar	(distance from R to center of mass)
e/B	0.315	OK	(e recommended to be less than B/6)
V	10156.2	Wc + Pav (used for bearing capacity; P_{pv} neglected)	
H	2964.434	Fr / FS sliding	
M	12813.5	V*e	

Remember to check bearing capacity and global stability of wall

HW Solutions (cont.)

Monday, December 5, 2022 3:52 PM

Gravity Wall Design

Wall Dimensions			
Top	3	ft	
Bottom	3	ft	
$\gamma_{concrete}$	150	pcf	
H	10	ft	
D	0	ft	
Centroid Location			
x_c	1.500	ft	
y_c	5.000	ft	
Earth Pressures			
K_A	0.199	unitless	
K_P	11.77	unitless	
factor for K_P note 3	0.1000		
K_0	0.357	unitless	$(1-\sin\phi)*(1+\sin\beta)$
Forces			
P_a	1196.4	lb/ft	
P_{ah}	1124.3	lb/ft	$P_a \cos\delta$
P_{av}	409.2	lb/ft	$P_{ah}\tan\delta$ or $P_a\cos\delta$
W_c	4500	lb/ft	
R	4909.2	lb/ft	$W_c + P_{av'} - P_{pv'}$
F_r	4119.3	lb/ft	R tan (δ or ϕ) note 2
$P_{p\ factored}$ x 0.5	0.0	lb/ft	
P_{ph}	0.00	lb/ft	$P_{p\ factored}\cos\delta$
P_{pv}	0.0	lb/ft	$P_{p\ factored}\sin\delta$

Fill Properties			
$\alpha_{backfill}$ deg	0	0.000	*radians*
α_{front} deg	0	0.000	*radians*
φ deg	40	0.698	*radians*
δ deg note 1	20	0.349	*radians*
$\gamma_{backfill}$	120		*pcf*
Foundation Soil Properties			
φ deg	40	0.698	*radians*
δ deg note 2	40	0.698	*radians*

Resisting Moments on Wall	
P_{av} * B	1227.6
P_{ph} * D/3	0.0
W_c * x_c	6750.0
ΣM_r	7977.6
Overturning Moments on Wall	
P_{ah} * h_a	3747.6
ΣMo	3747.6

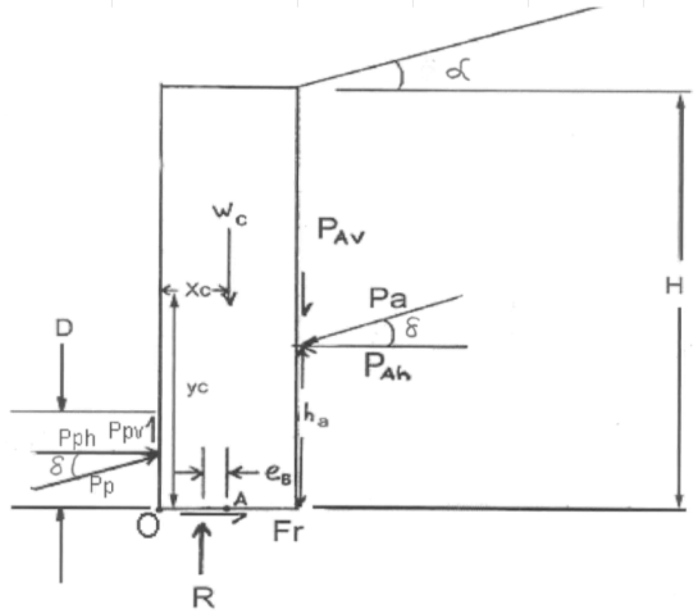
Factors of Safety

$FS_{sliding}$	**3.664**
FS_{oturn}	**2.129**

Acceptable FS

$FS_{sliding}$= 1.5

FS_{oturn} = 1.5



Values for bearing capacity calculation			
xbar	0.862	$(\Sigma M_r - \Sigma Mo)/R$	(from left edge to location of R, must be positive)
e	0.638	x_c-xbar	(distance from R to center of mass)
e/B	0.213	OK	(e recommended to be less than B/6)
V	4909.2	Wc + Pav (used for bearing capacity; P_{pv} neglected)	
H	1124.277	Fr / FS sliding	
M	3133.8	V*e	

Remember to check bearing capacity and global stability of wall

Additional Examples

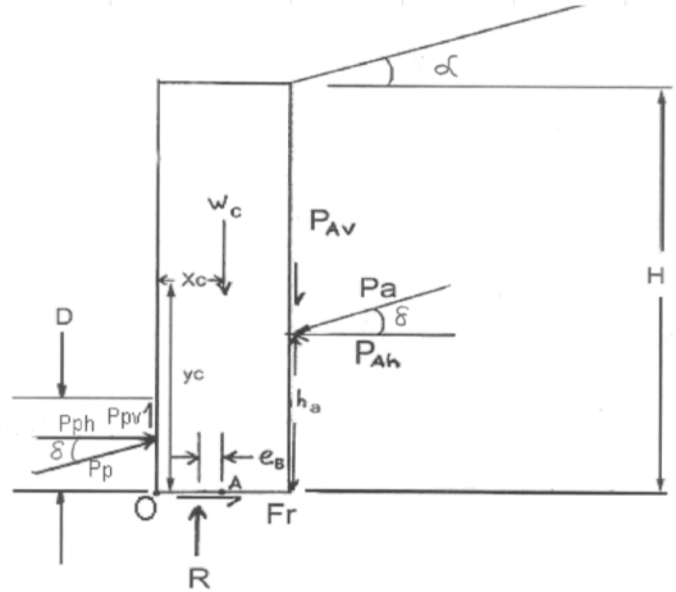
Monday, December 5, 2022 3:52 PM

Gravity Wall Design

Wall Dimensions		
Top	3	ft
Bottom	3	ft
$\gamma_{concrete}$	150	pcf
H	11	ft
D	2	ft
Centroid Location		
x_c	1.500	ft
y_c	5.500	ft

Fill Properties			
$\alpha_{backfill}$ deg	20	*0.349*	*radians*
α_{front} deg	0	*0.000*	*radians*
φ deg	40	*0.698*	*radians*
δ deg note 1	20	*0.349*	*radians*
$\gamma_{backfill}$	120		*pcf*

Foundation Soil Properties			
φ deg	40	*0.698*	*radians*
δ deg note 2	40	*0.698*	*radians*



Earth Pressures			
K_A	0.250	unitless	
K_P	11.77	unitless	
factor for K_P note 3	0.1000		
K_0	0.479	unitless	(1-sin ϕ)*(1+ sin β)

Resisting Moments on Wall	
P_{av} * B	1865.4
P_{ph} * D/3	177.0
W_c * x_c	7425.0
ΣM_r	9467.4

Forces			
P_a	1818.0	lb/ft	
P_{ah}	1708.4	lb/ft	P_a cos δ
P_{av}	621.8	lb/ft	P_{ah} tan δ or P_a cos δ
W_c	4950	lb/ft	
R	5475.2	lb/ft	W_c + $P_{av'}$ - $P_{pv'}$
F_r	4594.2	lb/ft	R tan (δ or ϕ) note 2
$P_{p\ factored}$ x 0.5	282.5	lb/ft	
P_{ph}	265.48	lb/ft	$P_{p\ factored}$ cos δ
P_{pv}	96.6	lb/ft	$P_{p\ factored}$ sin δ

Overturning Moments on Wall	
P_{ah} * h_a	6264.1
ΣMo	6264.1

Factors of Safety	
$FS_{sliding}$	**2.845**
FS_{oturn}	**1.511**
Acceptable FS	
$FS_{sliding}$= 1.5	
FS_{oturn} = 1.5	

Values for bearing capacity calculation			
xbar	0.585	$(\Sigma M_r - \Sigma Mo)/R$	(from left edge to location of R, must be positive)
e	0.915	x_c-xbar	(distance from R to center of mass)
e/B	0.305	OK	(e recommended to be less than B/6)
V	5571.8	Wc + Pav (used for bearing capacity; P_{pv} neglected)	
H	1615.064	Fr / FS sliding	
M	5097.9	V*e	

Remember to check bearing capacity and global stability of wall

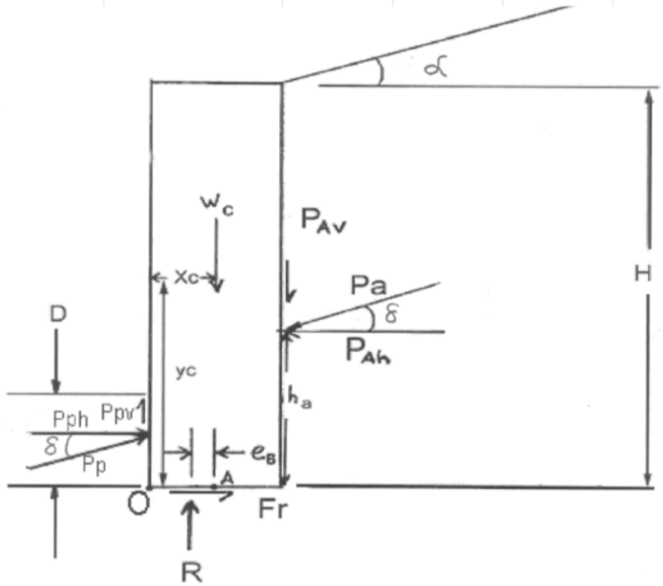
Additional Examples (cont.)

Monday, December 5, 2022 3:52 PM

Gravity Wall Design

Wall Dimensions			Fill Properties			
Top	4	ft	$\alpha_{backfill}$ deg	0	0.000	radians
Bottom	4	ft	α_{front} deg	0	0.000	radians
$\gamma_{concrete}$	150	pcf	φ deg	40	0.698	radians
H	16	ft	δ deg note 1	20.0	0.349	radians
D	4	ft	$\gamma_{backfill}$	135		pcf

Centroid Location			Foundation Soil Properties			
x_c	2.000	ft	φ deg	40	0.698	radians
y_c	8.000	ft	δ deg note 2	40	0.698	radians



Earth Pressures				Resisting Moments on Wall	
K_A	0.199	unitless		P_{av} * B	4714.0
K_P	11.77	unitless		P_{ph} * D/3	1592.9
factor for K_P note 3	0.1000			W_c * x_c	19200.0
K_0	0.357	unitless	$(1-\sin\phi)*(1+\sin\beta)$	ΣM_r	25506.9

Forces			
P_a	3445.7	lb/ft	
P_{ah}	3237.9	lb/ft	$P_a \cos\delta$
P_{av}	1178.5	lb/ft	$P_{ah} \tan\delta$ or $P_a \cos\delta$
W_c	9600	lb/ft	
R	10343.7	lb/ft	$W_c + P_{av'} - P_{pv'}$
F_r	8679.4	lb/ft	R tan (δ or ϕ) note 2
$P_{p\ factored}$ x 0.5	1271.3	lb/ft	
P_{ph}	1194.65	lb/ft	$P_{p\ factored} \cos\delta$
P_{pv}	434.8	lb/ft	$P_{p\ factored} \sin\delta$

Overturning Moments on Wall	
P_{ah} * h_a	17268.9
ΣMo	17268.9

Factors of Safety	
$FS_{sliding}$	3.050
FS_{oturn}	1.477

Acceptable FS

$FS_{sliding}$ = 1.5

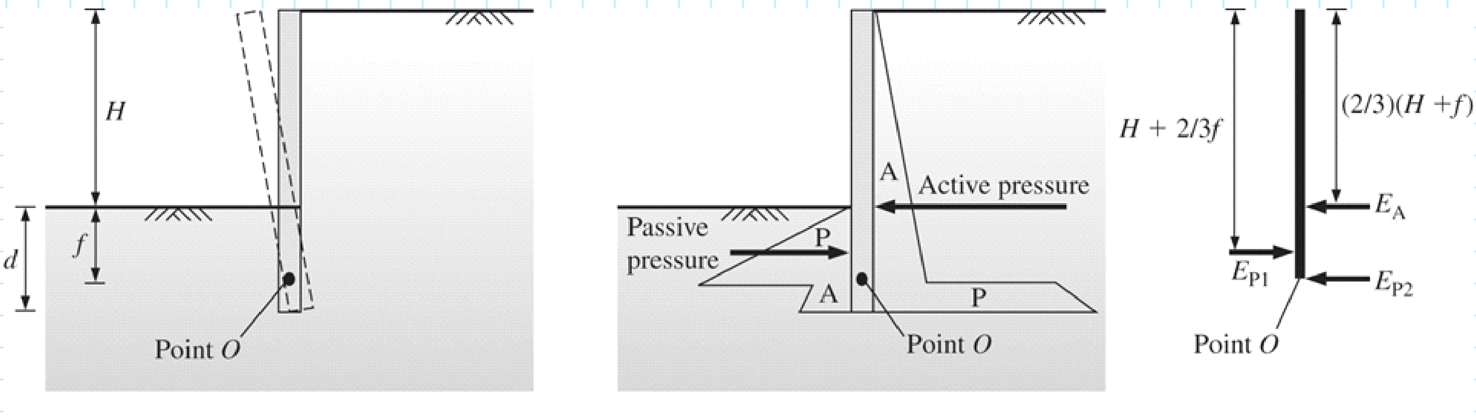
FS_{oturn} = 1.5

Values for bearing capacity calculation			
xbar	0.796	$(\Sigma M_r - \Sigma Mo)/R$	(from left edge to location of R, must be positive)
e	1.204	x_c-xbar	(distance from R to center of mass)
e/B	0.301	OK	(e recommended to be less than B/6)
V	10778.5	Wc + Pav (used for bearing capacity; P_{pv} neglected)	
H	2846.164	Fr / FS sliding	
M	12972.7	V*e	

Remember to check bearing capacity and global stability of wall

Retaining Wall Design (FBD - Cantilevered Wall)

Monday, November 9, 2015 11:43 AM



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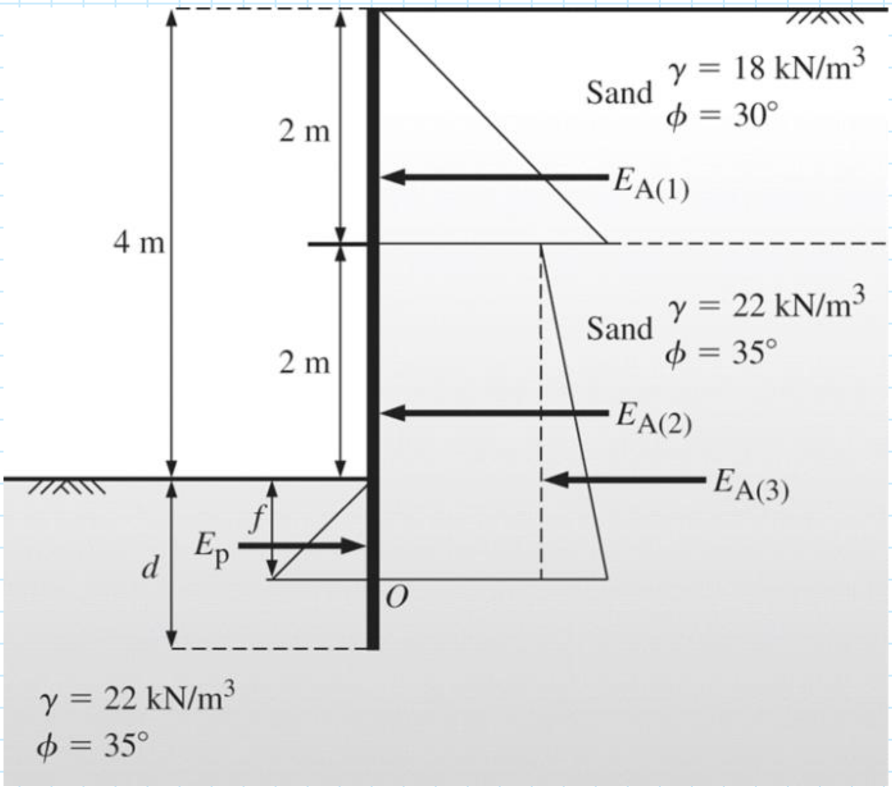

For more information on the design of cantilevered wall systems, see handout on course website

Retaining Wall Design (FBD - Cantilevered Wall with Tieback)

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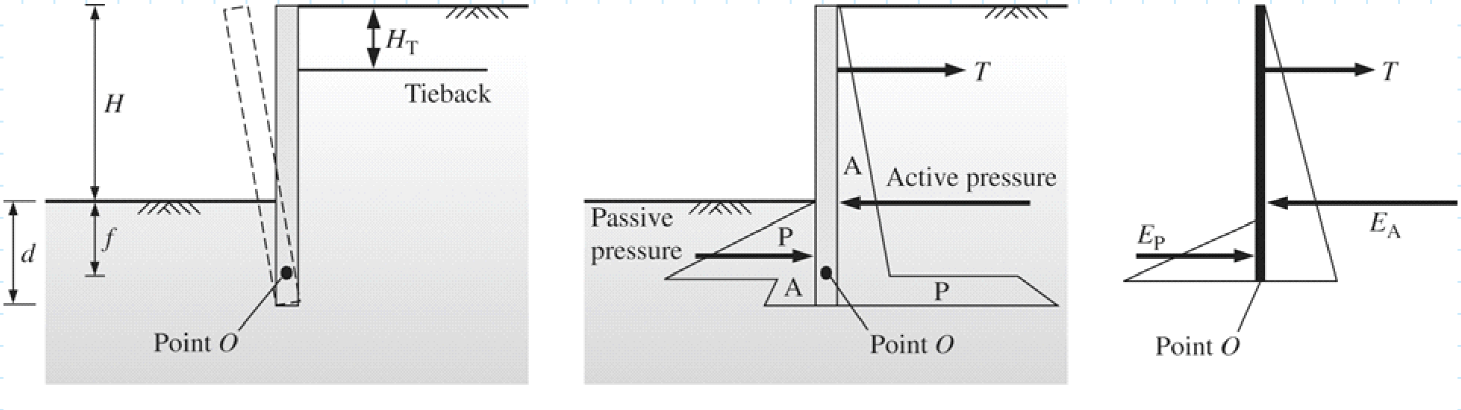
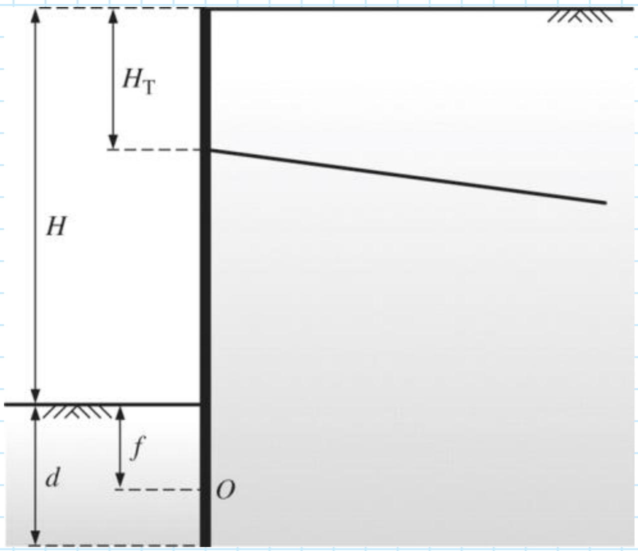
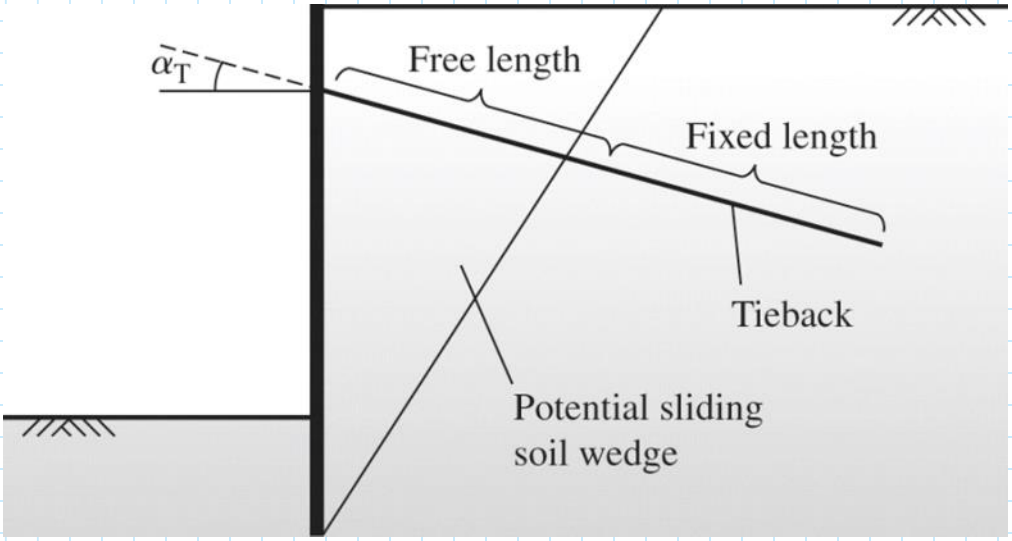


(a) (b) (c)

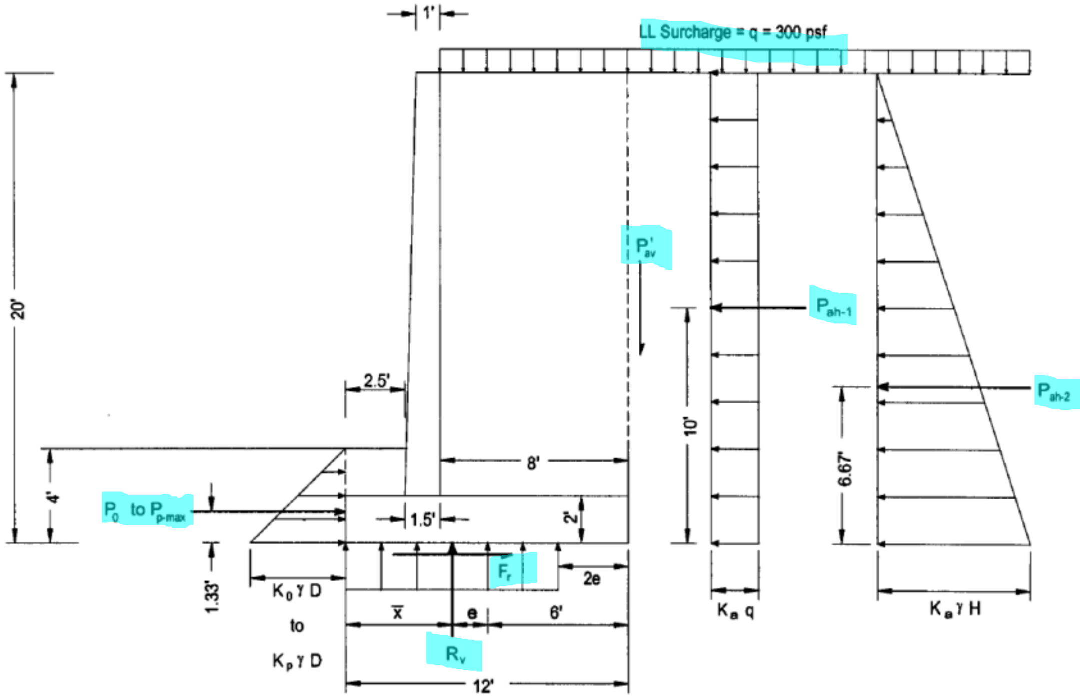




REINFORCED CONCRETE CANTILEVERED RETAINING WALL DESIGN

EXAMPLE PROBLEM

All soil is fine sand with $\gamma = 110$ pcf, $\phi' = 34°$, and $c' = 0$.



Cantilevered Retaining Wall Calculations (cont.)

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R.C. Cantilevered Retaining Wall Design Example

γ =	110	pcf	γ_{conc} =	150	pcf	
ϕ' =	34	degrees	T =	2	ft	(Thickness of footing)
c' =	0	psf	B =	12	ft	(Width of footing)
q =	300	psf	SB =	1	ft	(Width at top of stem)
H =	20	ft	BA =	2.5	ft	(Toe width along top of footing)
D =	4	ft	BB =	1.5	ft	(Width at bottom of stem)
			BC =	8	ft	(Heel width along top of footing)

Solution:

1. Calculate earth pressures using Rankine's method.

$$K_a = \tan^2\left(45^\circ - \phi'/2\right) = \tan^2\left(45^\circ - 34^\circ/2\right) = \quad 0.2827$$

$\sigma'_v(\text{top}) = q =$	300	psf	**(Note: If c' > 0 must check for**
$\sigma'_v(\text{bot}) = q + \gamma H =$	2500	psf	**tensile crack at top)**
$\sigma'_a(\text{top}) = \sigma'_v(\text{top})K_a - 2c'(K_a)^{1/2} =$	84.81	psf	
$\sigma'_a(\text{bot}) = \sigma'_v(\text{bot})K_a - 2c'(K_a)^{1/2} =$	706.79	psf	
$P_{ah-1} = \sigma'_a(\text{top}) * H =$	1.696	k/ft	
$P_{ah-2} = 0.5\,[\sigma'_a(\text{bot}) - \sigma'_a(\text{top})] * H =$	6.220	k/ft	

Lateral earth pressure on toe side generally varies from P_0 to $P_{p\text{-max}}$.
However, full passive resistance ($P_{p\text{-max}}$) requires significant movement to mobilize,
so generally design for somewhere between P_0 and $P_{p\text{-max}}$.
Calculate both values to give range of expected resistance.
Assume sand is normally consolidated (NC) although in reality, the compaction of the backfill sand will produce some overconsolidation.
In addition, some of the toe overburden may be lost over time owing to a variety of possible phenomena. Thus, sometimes the lateral resisting pressure at the toe is ignored.

$$K_0 = 1 - \sin\phi' = \quad 0.4408$$

$$P_0 = \frac{1}{2}K_0\gamma D^2 = \quad 0.388 \text{ k/ft}$$

$$K_p = \tan^2\left(45^\circ + \phi'/2\right) = \quad 3.5371$$

$$P_{p-\max} = \frac{1}{2}K_p\gamma D^2 + 2c'\sqrt{K_p}\,D = \quad 3.113 \text{ k/ft}$$

One could also use an intermediate value for the maximum passive thrust to be used in the design. For example, one could use a value halfway between at-rest and limiting passive:

$P_p(\text{max,design}) = 0.5(P_0 + P_{p\text{-max}}) =$ 1.750 k/ft

This value of P_p(max,design) will be used in subsequent calculations.

Cantilevered Retaining Wall Calculations (cont.)

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R.C. Cantilevered Retaining Wall Design Example

2. Calculate lateral forces and moments about toe at bottom of footing.

Part	Force (k/ft)	Moment Arm (ft)	Moment (ft-k/ft)
Toe soil - left	2.5*2*0.110 = 0.550	1.25	0.688
Toe soil - right	0.5*0.0556*2*0.110 = 0.006	2.52	0.015
Stem - left	0.5*18*0.5*0.150 = 0.675	2.83	1.913
Stem - right	18*1*0.150 = 2.700	3.50	9.450
Heel soil	8*18*0.110 = 15.840	8.00	126.720
Footing	12*2*0.150 = 3.600	6.00	21.600
Surcharge	0.3*8 = 2.400	8.00	19.200
	V_{wq} = 25.771	M_{wq} =	179.585
	$e_{wq} = M_{wq} / V_{wq}$ =	6.9685	ft
P_{ah-1}	= 1.696	10.00	16.963
P_{ah-2}	= 6.220	6.67	41.465
		M_o =	58.428
P'_{av}(max)	= 5.339	12.00	64.073
P_p(max,design)	= 1.750	1.33	2.334
		M_{p-av} =	66.407

Note: $P'_{av}(\max) = (P_{ah-1} + P_{ah-2}) \tan \phi' + c' H$

3. Check sliding stability

For concrete footing poured directly against the ground that has been excavated by a backhoe or similar equipment, use $\delta = \phi$ and $c_a = c$.

δ_b = 34 degrees c_a = 0 psf

The factor of safety against sliding failure will be calculated three ways: Using P_0, using $P_{p\text{-max}}$, and using P_p(max,design).

$$F_r(\max) = V_{wq} \tan \delta_b + c_a B' = \quad 17.38 \text{ k/ft}$$

$$FS_{sliding} = \frac{F_r(\max) + P_0}{P_{ah-1} + P_{ah-2}} = \quad 2.24$$

$$FS_{sliding} = \frac{F_r(\max) + P_{p-\max}}{P_{ah-1} + P_{ah-2}} = \quad 2.59$$

$$FS_{sliding} = \frac{F_r(\max) + P_p(\max, design)}{P_{ah-1} + P_{ah-2}} = \quad 2.42$$

Generally want a factor of safety against sliding greater than 1.25 to 2.0. This is acceptable even using a conservative value of P_0 on the toe side.

4. Check overturning stability

$$FS_{overturning} = \frac{\sum \text{Resisting Moments}}{\sum \text{Overturning Moments}} + \frac{M_{wq} + M_{p-av}}{M_o} = \quad 4.21$$

Some engineers will ignore the resisting moment developing from P_{av}.
For this case, the factor of safety is as follows:

$$FS_{overturning} = \frac{M_{wq} + M_p}{M_o} = \quad 3.11$$

Generally, want a factor of safety against overturning of greater than 2.0 to 3.0, so this wall is safe against overturning for both design methods.

5. Determine bearing pressure and effective width of footing for actual design loads.

Must make assumptions about how the resisting thrusts will develop in response to the active thrusts. Will assume that the $FS_{sliding}$ calculated using P_p(max,design) applies to all the resisting thrusts.

P_p(design) = P_p(max,design) / $FS_{sliding}$ = 0.724 k/ft

F_r(design) = F_r(max) / $FS_{sliding}$ = 7.192 k/ft

Now sum forces in the horizontal direction to make sure that static equilibrium is satisfied. Use a sign convention that a positive horizontal force acts to the left.

$\Sigma F_h = P_{ah-1} + P_{ah-2}$ - F_r(design) - P_p(design) = 0.00000 OK

Now must make an assumption about what value of P'_{av} develops. Will assume that P'_{av}(design) = 0. With this assumption and summing moments in the vertical direction to solve for R_v:

P'_{av}(design) = 0.000

$\Sigma F_v = V_{wq} + P_{av}$(design) - R_v(design) = 0 ===> R_v(design) = $V_{wq} + P_{av}$(design) = 25.771 k/ft

Find the location of R_v from the toe of the footing by summing moments and solving for xbar:

xbar = [M_{wq} + P_p(design) * Arm + P_{av}(design) * B - M_o] / R_v(design) = 4.7388 ft

e = 0.5B - xbar = 1.2612 ft

Note: Some engineers prefer to keep the eccentricity (e) less than B/6 based on an assumed trapezoidal distribution of pressure (so the entire width of the footing has bearing pressure acting on it).

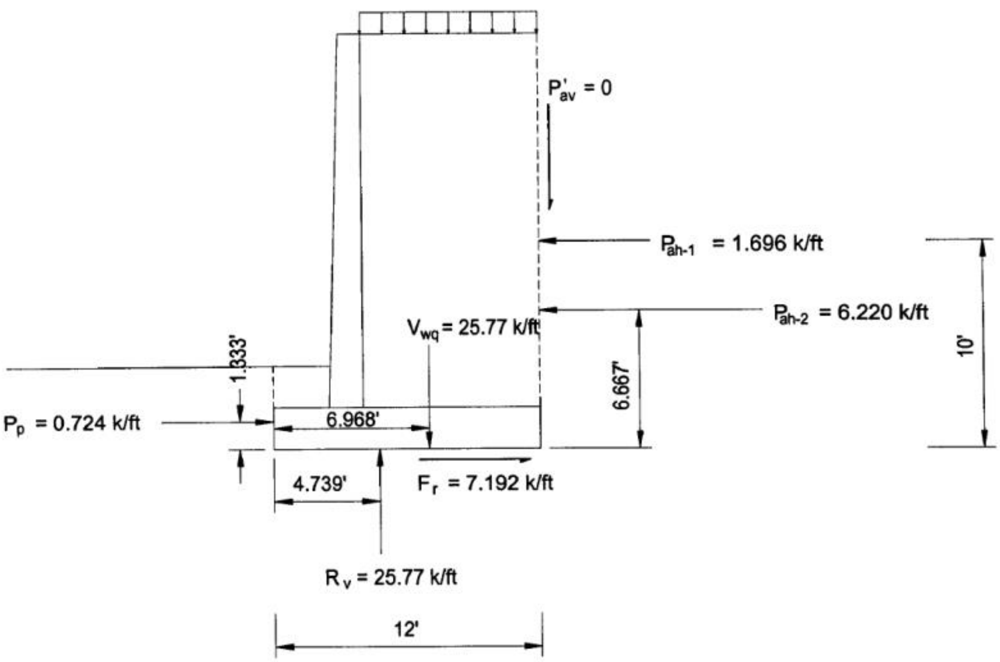
In this case, B/6 = 2.000 ft ===> e <= B/6 OK

B′ = B - 2e = 9.478 ft

$q_0 = R_v$(design) / B′ = 2.719 ksf

The free body diagram for the stable (design) condition is shown in the diagram below.

R.C. Cantilevered Retaining Wall Design Example



6. Estimate settlement of retaining wall

Estimate settlement using q_0 and B'. Settlement is generally not of major concern in a retaining wall unless the settlement is very large. However, this is not true for all situations.

7. Calculate factor of safety against ultimate bearing capacity failure

The design loads at the bearing level have now been established:

$V = R_v(\text{design}) =$ 25.771 k/ft

$H = F_r(\text{design}) =$ 7.192 k/ft

$M = R_v(\text{design}) * e =$ 32.503 ft-k/ft

Calculate q_{ult} using a spreadsheet developed specifically for Meyerhof's method. From this spreadsheet (see attached sheet), the calculated value of q_{ult} is

$q_{ult} =$ 14.69 ksf

$FS_{BC} = q_{ult} / q_0 =$ 5.4

For a retaining wall in sand, typically the factor of safety should be greater than about 2 to 4, but other values might be used depending on a number of factors. So this value of FS_{BC} is acceptable.

R.C. Cantilevered Retaining Wall Design Example

8. Perform structural design of retaining wall and footing

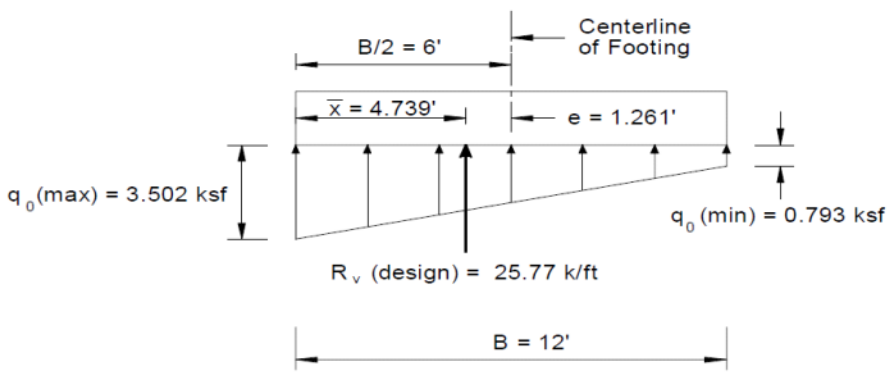
a. Design footing for shear and bending (toe and heel sides)
b. Design stem for shear and moment

Note: The structural design of the footing is usually conducted using an assumed trapezoidal distribution of bearing pressure based on an elastic analysis for a beam-column:

Beam-Column: $\sigma(\max, \min) = \dfrac{P}{A} \pm \dfrac{Mc}{I}$

Footing: $q_0(\max, \min) = \dfrac{R_v(design)}{B \cdot 1} \pm \dfrac{R_v(design) \cdot e \cdot B/2}{1 \cdot B^3/12} = \dfrac{R_v(design)}{B}\left(1 \pm \dfrac{6e}{B}\right)$

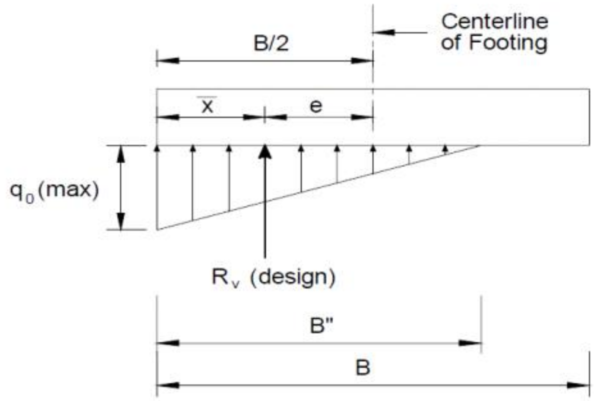
So for this retaining wall:	$q_0(\max)$ =	3.502	ksf (left side)
	$q_0(\min)$ =	0.793	ksf (right side)



The values of pressure shown in all the previous figures could be used if the structural design were performed using working stress design. However, in a factored design, the pressures would need to be re-calculated based on factored loads (for example, 1.4*DL, 1.7*LL, etc.).

R.C. Cantilevered Retaining Wall Design Example

Note: If e > B/6, the distribution is triangular and the value of q_0(max) and the width of the triangle (B") can be found by setting the resultant force, R_v(design), at the centroid of the triangle, and knowing the width of the triangle, solving for the magnitude of q_0(max) that gives the area of the pressure diagram equal to R_v(design).



$$\bar{x} = \frac{B''}{3} = \frac{B}{2} - e \quad \Rightarrow \quad B'' = 3\left(\frac{B}{2} - e\right)$$

$$R_v(design) = \frac{1}{2} q_0(\max) B'' \quad \Rightarrow \quad q_0(\max) = \frac{2R_v(design)}{B''}$$

9. Check global (slope) stability using an appropriate slope stability program (UTEXAS3, PCSTABL5, etc.)

See Bowles' Figure 12-13 on p. 687.

10. Repeat analysis and design until a retaining wall design is achieved that meets all requirements (structural, geotechnical, others) and is the most economical design overall.
This wall is too conservative from a geotechnical standpoint and probably would need to be re-designed to achieve the most economical design.

R.C. Cantilevered Retaining Wall Design Example

Spreadsheet for Calculating q_{ult} Using Meyerhof's Method Including r_γ

RED INDICATES REQUIRED INPUT

UNITS: ENTER "EN" FOR ENGLISH UNITS, "SI" FOR SI UNITS ===> EN

Footing Data:

B = 12 ft

L = 999999.00 ft

L'/B' = 105509.778

D = 4.00 ft

D/B' = 0.422

$\overline{q} = \gamma D =$ 0.440 ksf

Loading Data:

V = 25.771 kips

H_B = 7.192 kips

M_B = 32.50 kip-ft

M_L = 0 kip-ft

θ = 15.593 degrees

<== (Override if eqn. is not correct)

Soil Data:

γ = 0.110 kcf

ϕ_{tr} = 34 degrees

c = 0 ksf

ϕ_{ps} = 34 degrees

If L'/B' >= 2.0, ϕ_{ps} is used in place of ϕ_{tr}.

$\phi_{ps} = 1.5\phi_{tr} - 17^\circ$ for $\phi_{tr} > 34^\circ$

Use ϕ = 34 deg.

$$B' = B - 2e_B = B - \frac{2M_B}{V} = \quad 9.478 \text{ ft}$$

$$L' = L - 2e_L = L - \frac{2M_L}{V} = \quad 999999 \text{ ft}$$

S

$$s_c = 1 + 0.2\tan^2\left(45^\circ + \phi/2\right)\frac{B'}{L'} = \quad 1.000$$

$$s_q = 1 + 0.1\tan^2\left(45^\circ + \phi/2\right)\frac{B'}{L'} = \quad 1.000$$

$$s_\gamma = 1 + 0.1\tan^2\left(45^\circ + \phi/2\right)\frac{B'}{L'} = \quad 1.000$$

Depth Factors:

$$d_c = 1 + 0.2\tan\left(45^\circ + \phi/2\right)\frac{D}{B'} = \quad 1.159$$

$$d_q = 1 + 0.1\tan\left(45^\circ + \phi/2\right)\frac{D}{B'} = \quad 1.079$$

$$d_\gamma = 1 + 0.1\tan\left(45^\circ + \phi/2\right)\frac{D}{B'} = \quad 1.079$$

Inclination Factors:

$$i_c = \left(1 - \theta/90^\circ\right)^2 = \quad 0.684$$

$$i_q = \left(1 - \theta/90^\circ\right)^2 = \quad 0.684$$

$$i_\gamma = \left(1 - \theta^\circ/\phi^\circ\right)^2 = \quad 0.293 \quad \text{for } \phi > 0$$

$$i_\gamma = \quad 0 \quad \text{for } \phi = 0$$

Use i_γ = 0.293

Reduction Factor for Wide Footings:

$r_\gamma = 1 - 0.25 \log(B'/\kappa)$ for $B' > \kappa$, where κ = 6 ft or 2 m: r_γ = 0.950

Cantilevered Retaining Wall Calculations (cont.)

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Bearing Capacity Factors:

$$N_q = e^{\pi \tan\phi} \cdot \tan^2\left(45^\circ + \phi/2\right) = \quad 29.44$$

$$N_c = \frac{(N_q - 1)}{\tan\phi} = \quad 42.16$$

$$N_\gamma = (N_q - 1) \cdot \tan(1.4\phi) = \quad 31.15$$

Vertical Load: $q_{ult} = cN_c s_c d_c + \bar{q} N_q s_q d_q + 0.5\gamma B' N_\gamma s_\gamma d_\gamma r_\gamma$

Inclined Load: $q_{ult} = cN_c d_c i_c + \bar{q} N_q d_q i_q + 0.5\gamma B' N_\gamma d_\gamma i_\gamma r_\gamma$

LOAD IS INCLINED

N_c term (ksf)	N_q term (ksf)	N_γ term (ksf)	q_{ult} (ksf)
0.00	9.56	4.88	14.44

$q_0 = V / B' =$ 2.719 ksf

$SF = q_{ult} / q_0 =$ 5.3

Retaining Wall Design (Mechanically Stabilized Earth Walls (MSE))

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