**Discussion of Factor of Safety Against Buoyancy (Flotation) of Light Weight Cellular Concrete**

**Definitions and Background**

**Hydrostatic** – “It encompasses the study of the conditions under which fluids are at rest in [stable](https://en.wikipedia.org/wiki/Mechanical_equilibrium) [equilibrium](https://en.wikipedia.org/wiki/Hydrostatic_equilibrium) as opposed to [fluid dynamics](https://en.wikipedia.org/wiki/Fluid_dynamics), the study of fluids in motion. Hydrostatics are categorized as a part of the fluid statics, which is the study of all fluids, incompressible or not, at rest (<https://en.wikipedia.org/wiki/Hydrostatics>).”

**Hydrostatic Pressure** – “In a fluid at rest, all frictional and inertial stresses vanish and the state of stress of the system is called *hydrostatic*. When this condition of *V* = 0 is applied to the [Navier–Stokes equations](https://en.wikipedia.org/wiki/Navier%E2%80%93Stokes_equations), the gradient of pressure becomes a function of body forces only. For a [barotropic fluid](https://en.wikipedia.org/wiki/Barotropic_fluid) in a conservative force field like a gravitational force field, pressure exerted by a fluid at equilibrium becomes a function of force exerted by gravity.

The hydrostatic pressure can be determined from a control volume analysis of an infinitesimally small cube of fluid. Since [pressure](https://en.wikipedia.org/wiki/Pressure) is defined as the force exerted on a test area (*p* = *F*/*A*, with *p*: pressure, *F*: force normal to area *A*, *A*: area), and the only force acting on any such small cube of fluid is the weight of the fluid column above it, hydrostatic pressure can be calculated according to the following formula:

 {\displaystyle p(z)-p(z\_{0})={\frac {1}{A}}\int \_{z\_{0}}^{z}dz'\iint \_{A}dx'dy'\,\rho (z')g(z')=\int \_{z\_{0}}^{z}dz'\,\rho (z')g(z'),} (1)

where:

* *p* is the hydrostatic pressure (Pa),
* *ρ* is the fluid [density](https://en.wikipedia.org/wiki/Density) (kg/m3),
* *g* is [gravitational](https://en.wikipedia.org/wiki/Gravity) acceleration (m/s2),
* *A* is the test area (m2),
* *z* is the height (parallel to the direction of gravity) of the test area (m),
* *z*0 is the height of the [zero reference point of the pressure](https://en.wikipedia.org/wiki/Pressure_measurement#Absolute,_gauge_and_differential_pressures_-_zero_reference) (m).

For water and other liquids, this integral can be simplified significantly for many practical applications, based on the following two assumptions: Since many liquids can be considered [incompressible](https://en.wikipedia.org/wiki/Incompressible), a reasonable good estimation can be made from assuming a constant density throughout the liquid. (The same assumption cannot be made within a gaseous environment.) Also, since the height *h* of the fluid column between *z* and *z*0 is often reasonably small compared to the radius of the Earth, one can neglect the variation of [*g*](https://en.wikipedia.org/wiki/Gravity). Under these circumstances, the integral is simplified into the formula:

{\displaystyle p-p\_{0}=\rho gh,} (2)

where:

* *h* is the height *z* − *z*0 of the liquid column between the test volume and the zero-reference point of the pressure.

This formula is often called [Stevin's](https://en.wikipedia.org/wiki/Simon_Stevin) law[[3]](https://en.wikipedia.org/wiki/Hydrostatics#cite_note-3)[[4]](https://en.wikipedia.org/wiki/Hydrostatics#cite_note-4)  ([https://en.wikipedia.org/wiki/Hydrostatics)](https://en.wikipedia.org/wiki/Hydrostatics%29).”

For geotechnical applications, the zero-reference pressure is set to zero to represent zero gauge pressure; in other words, zero gauge pressure equals 1 atmosphere of absolute pressure. The hydrostatic pressure at depth h is simply rgh where g is the gravitational constant.

**“Buoyancy** or upthrust, is an upward [force](https://en.wikipedia.org/wiki/Force) exerted by a [fluid](https://en.wikipedia.org/wiki/Fluid) that opposes the [weight](https://en.wikipedia.org/wiki/Weight) of a partially or fully immersed object. In a column of fluid, pressure increases with depth as a result of the weight of the overlying fluid. Thus the pressure at the bottom of a column of fluid is greater than at the top of the column. Similarly, the pressure at the bottom of an object submerged in a fluid is greater than at the top of the object. The pressure difference results in a net upward force on the object ([https://en.wikipedia.org/wiki/Buoyancy)](https://en.wikipedia.org/wiki/Buoyancy%29).”

For 1-D geotechnical buoyancy calculations, the buoyant force per unit volume of soil or material is:

Fb = total – water (3)

where: total is the total unit weight of the soil or material per unit volume (i.e., weight of material solids + weight water), and water is the unit weight of water (i.e., 62.4 lb/ft3).

For completely saturated soils, there are two phases present: (1) solids and (2) water filled voids. In this case, the total is equal to (i.e., weight of material solids + weight of water) because there would be no volume associated with air voids in a completely saturated material.

For partially saturated soils or materials, ps, the unit weight is calculated from:

ps = dry (1+/100) (4)

where: dry is the dry unit weight (i.e., weight of the oven-dried soil or material) and is the moisture content of the soil (%).

For design purposes, values of ps are usually determined from long-duration laboratory saturation tests in controlled conditions. For materials such a permeable lightweight cellular concrete (P-LCC), the material does not obtain complete saturation due to isolated void/air pockets remaining with the P-LCC fabric that resists saturation because such are not hydraulically connected to the fluid in the fabric of the specimen.

A material specimen with a ps less than 62.4 lb/ft3 will have a net upward buoyant force per unit volume, Fb, that can be estimated for 1D conditions using:

 Fb = 62.4 - ps (5)

This is fundamentally a hydrostatic calculation. Hydrodynamic forces (i.e., the upward flow of water) are not considered in the evaluation of against buoyancy uplift. If such forces are present and significant, then a heave calculation is also performed that combines the buoyant and upward seepage forces.

Also, because this is a 1D calculation, any resisting forces due to upward shearing between the P-LCC block and surrounding soil are conservatively ignored. Furthermore, any basal bonding of the P-LCC with the underlying soil is also ignored. Thus, in reality the hydrostatic forces required to uplift a block of P-LCC are greater than those calculated from the 1D analysis.

**Allowable Stress Design**

Allowable stress design (ASD) addresses uncertainty in the capacity and demand equation by applying factor an overall factor of safety (FS).

FS = FC / FD (6)

where: FC is the forcing resisting potential failure of the system, and FD are the driving forces that cause failure.

In geotechnical calculations, the actual FS of a system is calculated using the best-estimate or most-likely values of FC and FD. The best-estimate values are represented as mean values for symmetrical distributions, median values for slightly skewed distributions, and geometrical mean for log-normal distributions (i.e., highly-skewed distributions). Generally, values of ps for P-LCC are assumed to be normally distributed with the mean representing the best-estimate for inputs for Equation (6).

**Load and Resistance Factor Design (LRFD)**

LRFD is widely used as an alternative to working stress design (WSD) and is popular in structural and geotechnical engineering codes. LRFD has been adopted by the following institutions and societies.

* + ACI (American Concrete Institute)
	+ AASHTO (American Association of Highway and Transportation Officials)
	+ ASCE (American Society of Civil Engineers)

LRFD considers uncertainty in both the loads and the resistance of the material(s). The overall equation is:

(LF)iQni ≤ (RF)iRni (7)

where LF = load factors, Qn = nominal loads, RF = resistance factors, Rn = nominal resistances.

The nominal loads and resistances are either code-specified or determined using best-estimate values similar to that of WSD. The load factors are generally greater than one and increase the design load to account for uncertainty, whereas the resistance factors are generally less than one and decrease the design resistance to account for uncertainty. The design is found to be acceptable as long as the sum of the factored resistance equals or exceeds the sum of the factored loads.

**Discussion of MRP Design Inputs**

The applicant for the Mission Rock Project (MRP) proposes to use ASD to calculate factors of safety against buoyancy uplift of the P-LCC based on guidance from U.S. Army Corp of Engineers (EM 1110-2-2100) and National Cooperative Highway Research Program (NCHRP) 529.

NCHRP 529 requires a FS of 1.2 for the design basis groundwater elevation or flood event. Consistent with ASD methodology, best-values of ps are recommended in calculating the factor of safety.

The guidance in EM 1110-2-2100 pertains to calculating the factor of safety against flotation of concrete structures:

 (8)



For the MRP evaluations, Equation (8) can be simplified to a 1D calculation:

FSf = S / U

where S = the sum of the total unit weights of all materials above the water table multiplied by their respective thickness and U = ps multiplied by the thickness of the submerged P-LCC. Note that FSf has units of F/L2 or pressure.



The value of FS varies according to the load conditions, as given in the table above. The return period of the loading condition category is given in the table below.



The factor of safety recommended in EM1110-2-2100 assumes that for critical and normal structures, the soil and material properties have been “conservatively” established through the explorations and testing. The MRP project proposes to use a design value for ps of 50 lb/ft3. Based on the laboratory testing performed for the Pilot project, most of the laboratory determined values of ps were in mid-range between 50 and 60 lb/ft3 with a minimum value of 52 lb/ft3. The laboratory saturated density testing of 14 samples of the 27 lb/ft3. The P-LCC of Lifts 1 through 4 of the Pilot Project averaged 54.8 lb/ft3 with a standard deviation of 2.1 lb/ft3.

Hence, the TAP agrees with the MRP design team that 50 lb/ft3 is an acceptable conservative design value for calculating buoyancy or flotation as required by EM1110-2-2100.

In its review, he San Francisco Department of Public Works (DPW) raised the issue that NCHRP 529 is not an appropriate design standard for P-LCC. The primary concern is that NCHRP 529 pertains to geofoam design, and geofoam due to is in-plant manufactured nature might be less variable in its unit weight properties when compared with P-LCC; hence the factor of safety of recommend 1.2 may not be adequately conservative for P-LCC design. Following this issue, there was considerable discussion about applying an “extra factor” of safety to the design value of ps of the P-LCC to account for additional variability suggested by the laboratory and field test values of ps. A lower bound estimate of mean minus two standard deviations was suggested for ps by City Engineers or Reviewers.

The TAP notes the following: (1) Both NCHRP 529 and EM1110-2-2100 have relatively consistent recommended factors of safety (1.2 vs. 1.1 to 1.3, respectively). (2) Using a mean minus two standard deviation estimate of ps is unprecedented in engineering design and is not required by current codes and documents. (3) The coefficient of variation (standard deviation divided by the mean) for both geofoam and P-LCC has not been determined. Therefore, it is premature to draw any conclusions regarding additional variability that might be present in ps when compared with the coefficient of variation for the density of geofoam. (4) EM1110-2-2100 requires that the designer consider the potential variation in soil properties, which are natural materials with considerable variability in ps values. Regarding this, as stated above, the TAP believes that the MRP designers have selected a conservative value for ps. (It should be noted that ps of 40 lb/ft3 discussed during the meeting was obtained from a “field” saturation and not a laboratory test; hence it is probably not representative of the true range of ps.) The MRP team has proposed a revised procedure for the “field” saturation test to ensure better consistency with laboratory-determined values. Stan Peters of the TAP has reviewed the revised procedure and concurs with the changes. Nonetheless, it is recommended that estimates of ps that support the basis of design should be obtained from laboratory-determined values to avoid confounding variability from the two test methods.

Discussion of LRFD Design Guidance

The load and resistant factors in LRFD allow for separate treatment of the uncertainty associated with loading conditions and soil properties. It is another design method that could be considered by the MRP design team, or the rationale and load factors of LRFD can be used to justify the ASD parameters proposed by the MRP design team.

The American Association of State Highway and Transportation Officials (AASHTO) in its LRFD specifications for bridge design (2017, 8th Edition) uses a load factor (WA) for extreme events of 1.00 (p. 3-15, Table 3.4.1-1). The extreme events include loading combinations relating to ice load, collision by vessels and vehicles, check floods, and certain hydrostatic events with a reduced live load other than that which is part of a vehicle collision (p. 3-10). Therefore, for LRFD, the TAP recommends a load factor of 1.00 for the extreme flood event, consistent with that of the WA load factor from AASHTO.

It should be noted that resistant factors, when unknown, are often selected so as to produce a factor of safety consistent with that obtained from allowable stress design. For example, this is the approach taken by BS 6349-3 for maritime structures (British Standard 6349-3 (1998) Maritime structures – Part 1: Code of Practice for general criteria). BS 6349-3 requires a factor of safety not less than 1.2 against hydrostatic uplift. For a favorable stability weight, a reduction factor applied to the mean value of ps of 1/1.2 = 0.83 is recommended by BS 6349-3. In addition, Simpson, Vogt and van Seters (2011) conclude “In uplift problems, it is necessary to vary either water pressure or the magnitudes of favorable, stabilizing weight, in order to ensure safety in view of possible secondary actions. In order to avoid factoring water pressure, the possibility of a reduced factor on favorable weight, perhaps between 0.8 and 0.9 should be considered (p. 517).”

Lastly, current AASTHO specifications do not suggest a resistance factor for buoyancy. Hence, until available, the TAP recommends an AASHTO resistance factor of 1/1.2 = 0.83333. . . to be consistent with the safety factors recommended by EM1110-2-2100 and NCHRP 529 if AASHTO LRFD is used.