Numerical Methods in Geotechnical Engineering

Tuesday, August 21, 2012
12:43 PM

Course Notes

Prepared by:

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CIVIL & ENVIRONMENTAL ENGINEERING
THE UNIVERSITY OF UTAH

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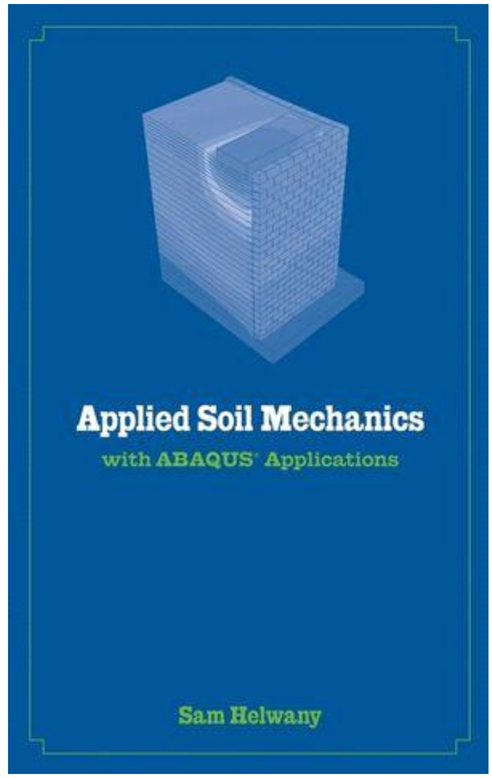
Course Information

Friday, August 20, 2010
12:43 PM

Textbooks (Required)

Applied Soil Mechanics with ABAQUS Applications
Sam Helwany
ISBN: 978-0-471-79107-2
Hardcover
400 pages



Software (provided)
FLAC V. 5.0 (Student License)
FLAC V. 5.0 (Network License)

Note: The text uses the finite element method, we will be using finite difference method but applying it to the same types of problems in the text

Reading Assignments

To facilitate the learning, each student will be required to read the assignment and be prepared to discuss in class the material that was read. Because it is nearly impossible to cover the material exactly according to the schedule, it is each student's responsibility to follow the lectures to determine what the appropriate reading assignment is for the next class period. PLEASE BRING THE TEXTBOOK, LECTURE NOTES, AND/OR OTHER APPROPRIATE REFERENCES TO EACH CLASS!

You should bring laptops to class. They will be used at various times to develop numerical models in class.

Course Information (continued)

Tuesday, August 21, 2012
12:43 PM

Participation

At various times during each lecture, students will be asked questions or be given the opportunity to answer questions posed by the instructor. Each student is expected to participate in these discussions during the lectures throughout the semester. Relevant information from students with practical working experience on a particular topic is encouraged.

Homework

Homework will generally be due at the beginning of class as assigned. The due date of the homework will be shown on the course website by the homework link. Late homework assignments will be assessed a penalty of 20% per class period. For example, if homework is due on Tuesday at 2:00 p.m. and it is turned in on Wednesday morning, then a 20% late penalty will be assessed. Homework that is more than 2 class periods late will receive a maximum of 50 percent reduction and <u>will not be checked</u>. A grade of zero will be given on any homework that is copied from someone else. Unauthorized copying of or help from others on homework will result in an E for the course.

Attendance

Attendance is necessary to learn the material. Non-attendance increases the amount of time you spend on the course and reduces the quality of your educational experience. Also, examination questions will come from items covered in lecture that may not be present on the course notes or textbook. Y<u>our grade will be reduced by 3 percent for each unexcused</u>. If you are sick, please inform the instructor via e mail.

Course Information (continued)

Tuesday, August 21, 2012
12:43 PM

Course Grading

Weight		Grade	Score	Grade	Score
Homework	60%	A	94-100	A-	90-93
Quizzes	20%	B+	87-89	B	83-86
Final Project	20%	B-	80-82	C+	77-79
		C	73-76	C-	70-72
		D+	67-69	D	63-66
		D	60-62	E	<60

Grading Guidelines

- **Simple error (mathematic, coding error, etc.) = 10 percent deduction**
- **Conceptual error (wrong approach, formula, etc.) = 50 percent deduction**

Topics

- **ELASTICITY AND PLASTICITY**
- **STRESSES IN SOIL**
- **SHEAR STRENGTH OF SOIL**
- **SHALLOW FOUNDATIONS**
- **LATERAL EARTH PRESSURE AND RETAINING WALLS**
- **SLOPE STABILITY**
- **CONSOLIDATION**
- **PERMEABILITY AND SEEPAGE**
- **OTHER ADVANCED TOPICS**

Academic Calendar

Tuesday, August 21, 2012
12:43 PM

Events	Dates
Deadline to apply for graduation	Friday, June 1
Class Schedule & Registration appointments available	Monday, March 5
Admission/readmission deadline	Friday, April 1
Registration by appointment begins	Monday, April 9
Open enrollment	Monday, July 30
House Bill 60 registration - opens new window	Tuesday, August 14
Labor Day holiday	Monday, September 3
Tuition payment due - opens new window	Tuesday, September 4
Census deadline	Monday, September 10
Fall break	Sun.-Sun., October 7-14
Thanksgiving break	Thurs.-Fri., Nov. 22-23
Holiday recess	Sat. Dec. 15-Sun. Jan. 6
Grades available	Thursday, December 27

General Calendar Dates

Pasted from <http://registrar.utah.edu/academic-calendars/fall2012.php>

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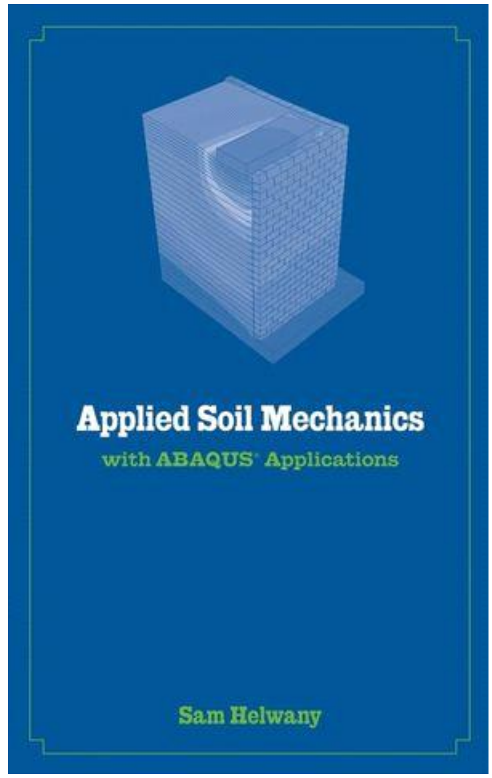

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Introduction to Modeling

Tuesday, August 21, 2012
12:43 PM

Numerical Techniques Covered in this Course

- Finite Difference Method (FDM)
- Finite Element Method (FEM) (Introduction)

Commercially Available Software Packages

- FLAC (Fast Lagrangian Analysis of Continua) (General FDM)
  ABAQUS (FEM) (General FEM with some geotechnical relations)
- ANSYS (FEM) (Mechanical/Structural)
- PLAXIS (FEM) (Geotechnical)
- SIGMA/W (FEM) (Geotechnical)
- SEEP/W (FEM) (Seepage Analysis)
- MODFLOW (FEM) (Groundwater Modeling)

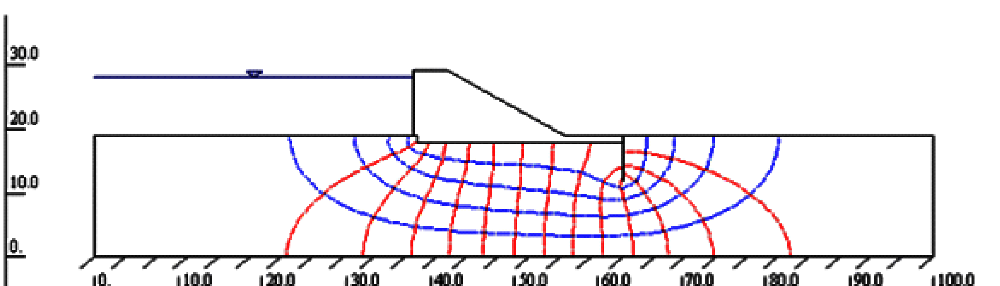
FLAC and PLAXIS are the most commonly used by advanced geotechnical consultants

Common Applications of Modeling in Geotechnical Engineering

- Numerical approximation for various types of differential equations commonly encountered in geotechnical engineering

- LaPlace's Equation (governing equation for 3D steady-state flow)

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

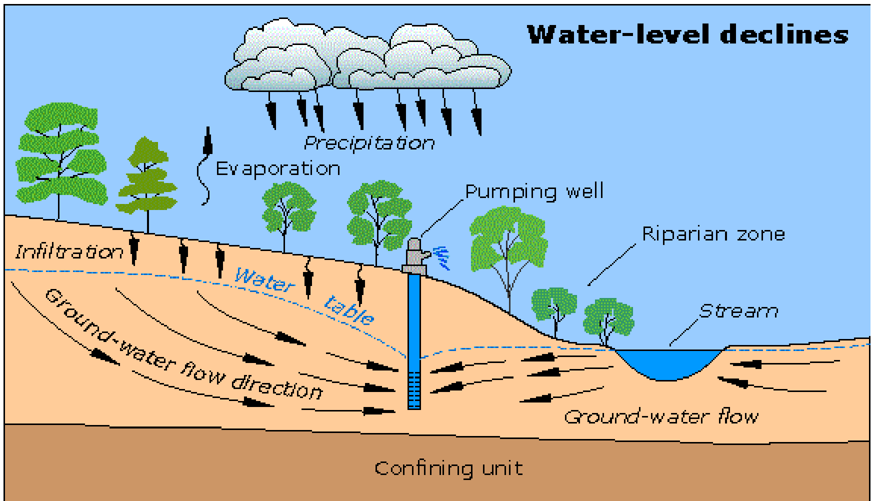


Flow under a concrete dam with cutoff

Common Applications (continued)

Tuesday, August 21, 2012
8:41 AM

- Groundwater Flow Equation (3D transient flow)



$$\frac{\partial h}{\partial t} = \alpha \left[\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} \right] - G.$$

α = k/Ss = hydraulic conductivity / Specific Storage
G = source/sink term

- Equation of motion for forced damped vibration system

The behavior of the spring mass damper model when we add a harmonic force takes the form below. A force of this type could, for example, be generated by a rotating imbalance.

$$F = F_0 \cos(2\pi f t).$$

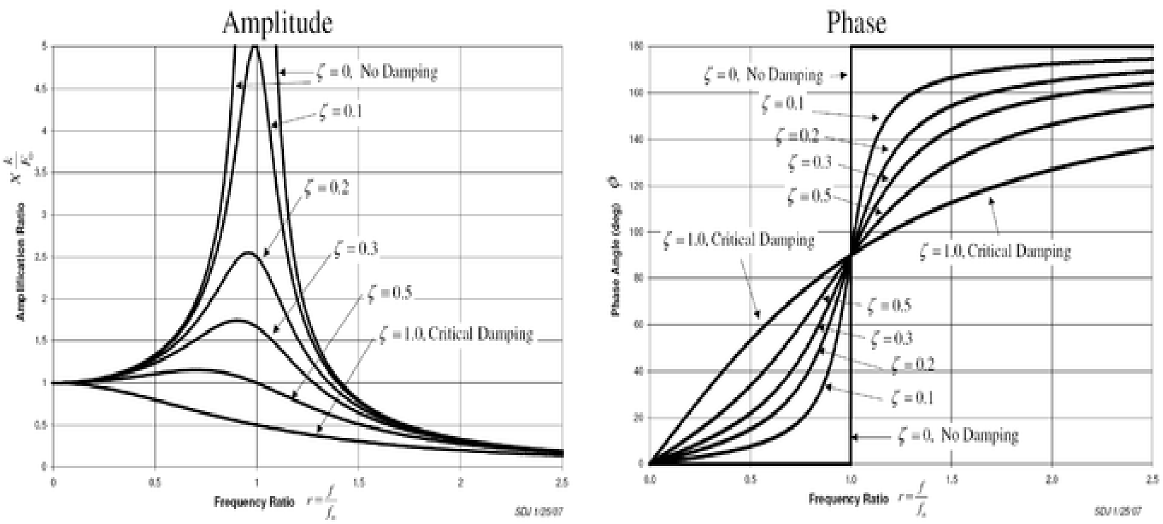
If we again sum the forces on the mass we get the following ordinary differential equation:

$$m\ddot{x} + c\dot{x} + kx = F_0 \cos(2\pi f t).$$

See next page for solution for homogeneous material; however heterogeneous materials require numerical methods.

Common Applications (continued)

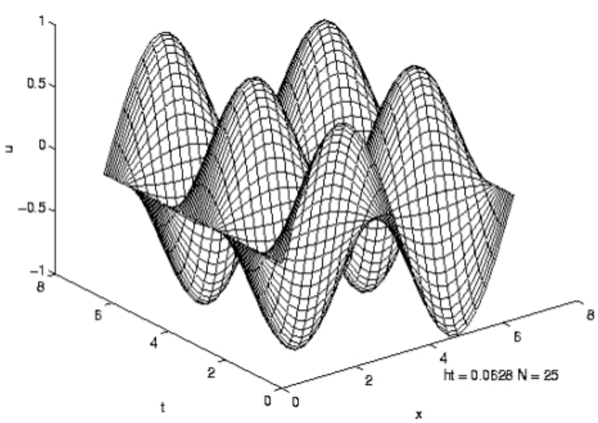
Thursday, March 11, 2010
11:43 AM



- Wave equation for solid materials

The wave equation is an important second-order linear partial differential equation of waves, such as sound waves, light waves and water waves. It arises in fields such as acoustics, electromagnetics, and fluid dynamics (from Wikipedia).

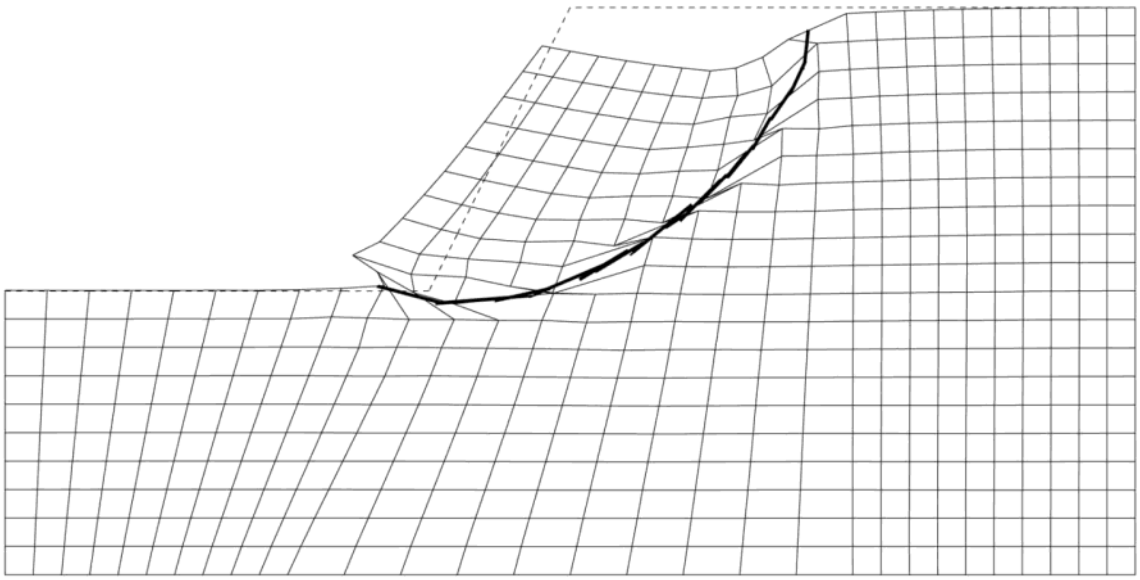
$$\frac{\partial u^2}{\partial t^2} = c^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$



Common Applications (continued)

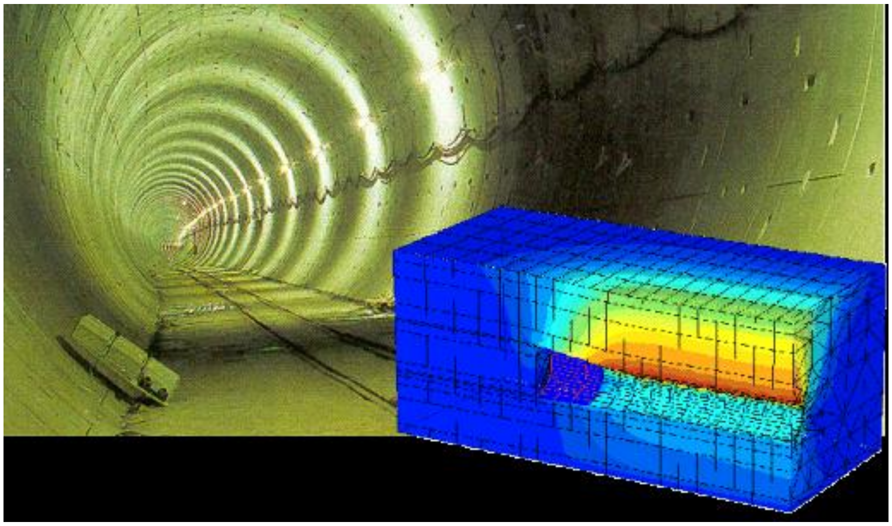
Thursday, March 11, 2010
11:43 AM

- Deformation Analysis of Slopes



In deformation analysis we seek to estimate how much the slope will move or deform. This is much more of an involved process than simply calculating the factor of safety against failure from pseudo-static techniques.

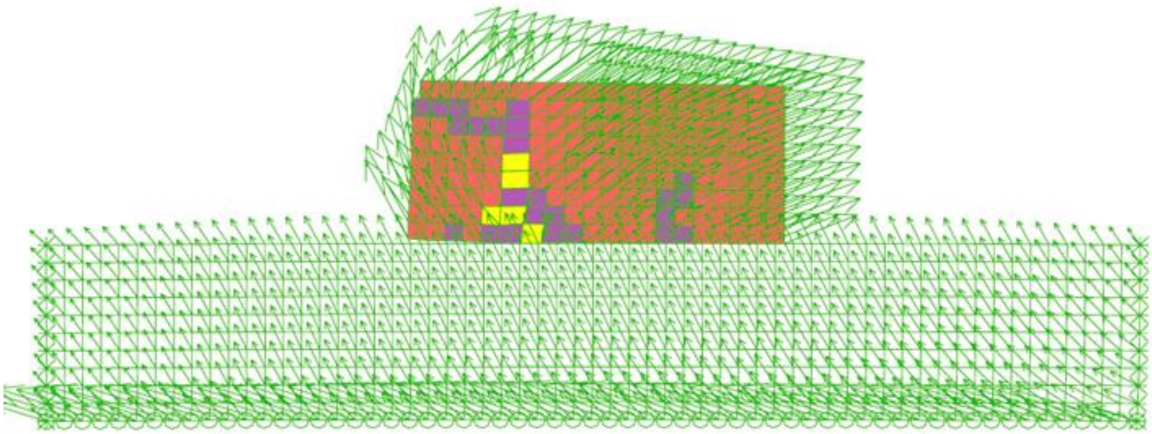
- Deformation Analysis of Tunnels



Common Applications (continued)

Thursday, March 11, 2010
11:43 AM

- **Dynamic Analyses**



Rocking analysis of a geofoam embankment undergoing earthquake excitation.

Reading

Tuesday, August 21, 2012
12:43 PM

- FLAC v. 5.0 User's Guide, Section 1: Introduction
- FLAC v. 5.0 User's Guide, Section 2 (p. 2-1 to 2-12)
- Applied Soil Mechanics, Ch. 1 Properties of Soils

Assignment 1

Tuesday, August 21, 2012
12:43 PM

- Assigned Reading
- Install FLAC v 5.0 software on your computer
- Run the following code to check the model (see FLAC manual Example 4.8 Slip in a bin-flow problem)

```
config
grid 7 10
model mohr i=1,5
model elastic i=7
gen 0,0 0,5 5,5 3,0 i=1,6 j=1,6
gen 3,0 5,5 6,5 6,0 i=7,8 j=1,6
gen 5,5 5,10 6,10 6,5 i=7,8 j=6,11
fix x y i=7,8
fix x i=1
prop dens=2000 shear=1e8 bulk=2e8 fric=30 i=1,5
prop dens=2000 shear=1e8 bulk=2e8 i=7
int 1 Aside from 6,1 to 6,11 Bside from 7,1 to 7,11
int 1 ks=2e9 kn=2e9 fric=15
set large, grav=10
step 3000
ret
```

Blank

Thursday, March 11, 2010
11:43 AM

Steps to Modeling

Thursday, August 23, 2012
12:43 PM

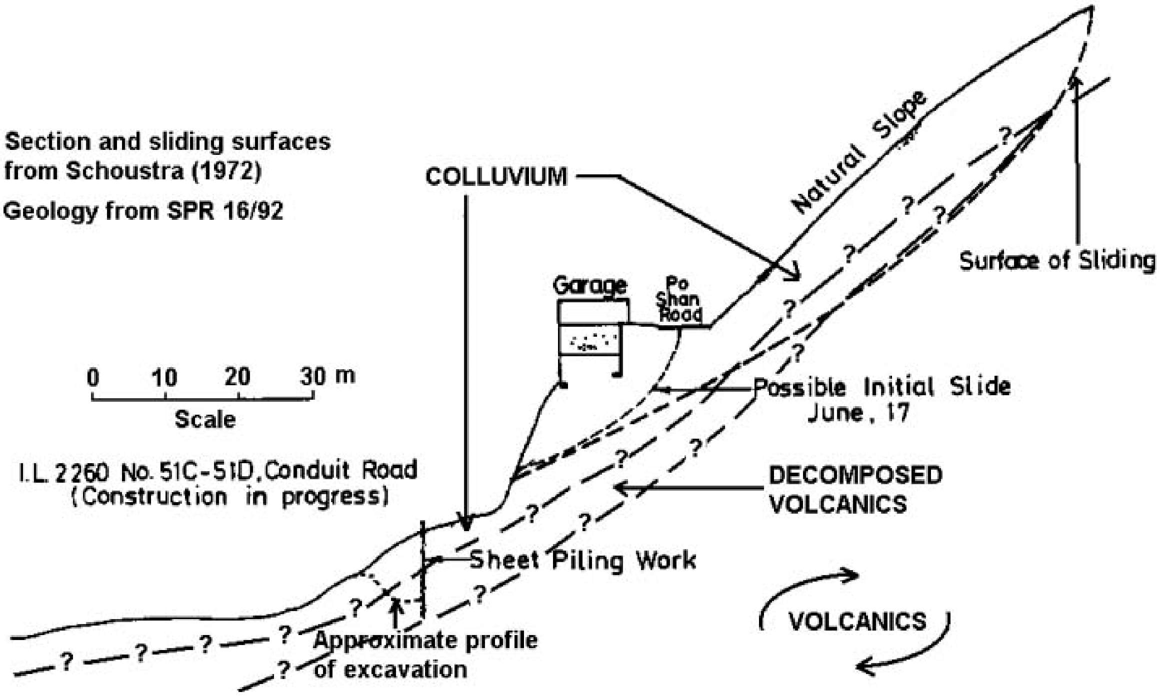
Modeling of real systems takes a fundamental understanding of how the system will function or perform. There is a need to simplify the real situation so that one can reasonably deal with the geometries and properties in the numerical scheme.

Steps to Modeling

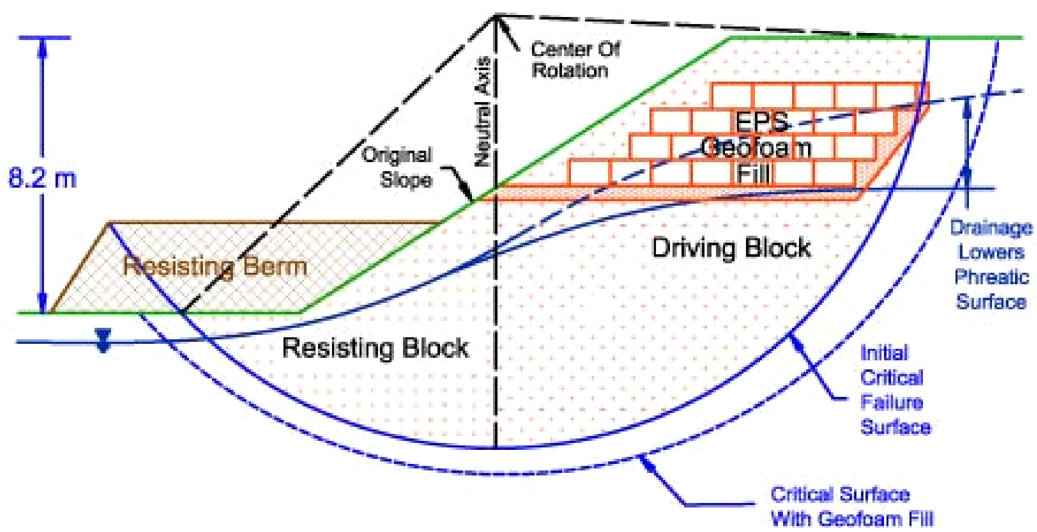
- Selection of representative cross-section
  - Idealize the field conditions into a design X-section
  - Plane strain vs. axisymmetrical models
- Choice of numerical scheme and constitutive relationship
  - FEM vs FDM
  - Elastic vs Mohr-Coulomb vs. Elastoplastic models
- Characterization of material properties for use in model
  - Strength
  - Stiffness
  - Stress - Strain Relationships
- Grid generation
  - Discretize the Design X-section into nodes or elements
- Assign of materials properties to grid
- Assigning boundary conditions
- Calculate initial conditions
- Determine loading or modeling sequence
- Run the model
- Obtain results
- Interpret of results

Idealize Field Conditions to Design X-Section

Thursday, March 11, 2010
11:43 AM



The above X-section has a significant amount of complexity. This must be somewhat simplified for modeling, or, several cases must be modeled.



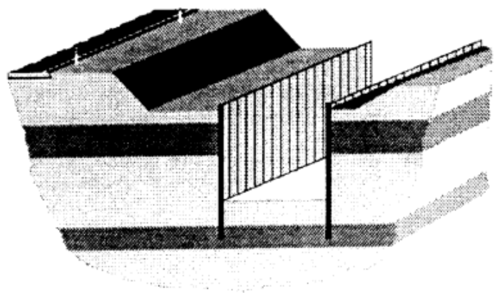
Design X-section for a landslide stabilization using EPS Geofoam

Selection of X-Section

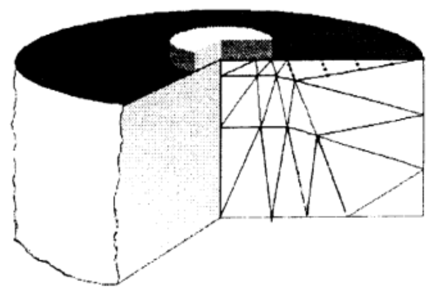
Tuesday, August 21, 2012
12:43 PM

- Many 3D problems can be reduced to 2D problems by selection of the appropriate X-sections. This make the modeling much easier when this can be done.

• Plane Strain



• Axi-symmetry



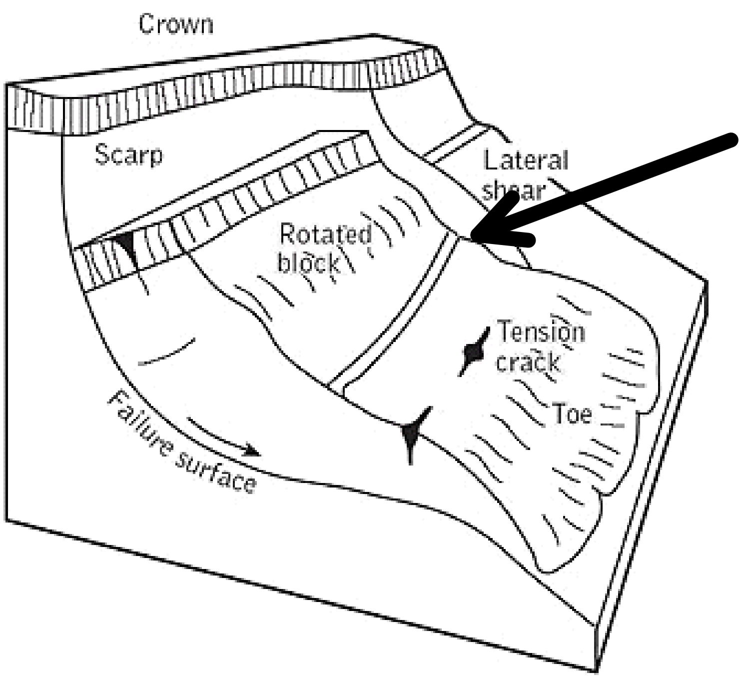
- **Plane strain conditions**
  - Dams
    - Relatively long dams with 2D seepage
  - Roadway Embankments and Pavements
  - Landslides and slope stability
  - Strip Footings
  - Retaining Walls

Note for plane strain conditions to exist all strains are in the x-y coordinate system (i.e., x-y plane). There is no strain in the z direction (i.e., out of the paper direction). This usually implies that the structure or feature is relatively long, so that the z direction and the balanced stresses in this direction have little influence on the behavior within the selected cross section.

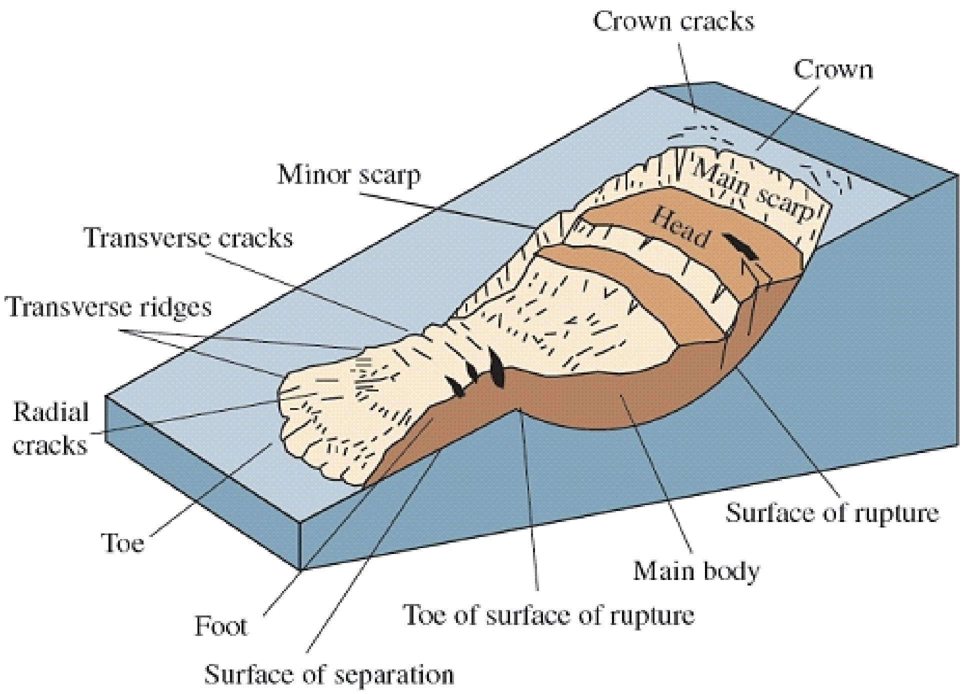
Selection of X-Section (continued)

Thursday, March 11, 2010
11:43 AM

Landslides and slope stability (plane strain conditions)



Note that in the above drawing , the 2D plain strain condition would assume that the shear resistance on the back margin of the slide has little influence on the behavior of the landslide. If this is not true, then a 3D model would be required to capture this effect.

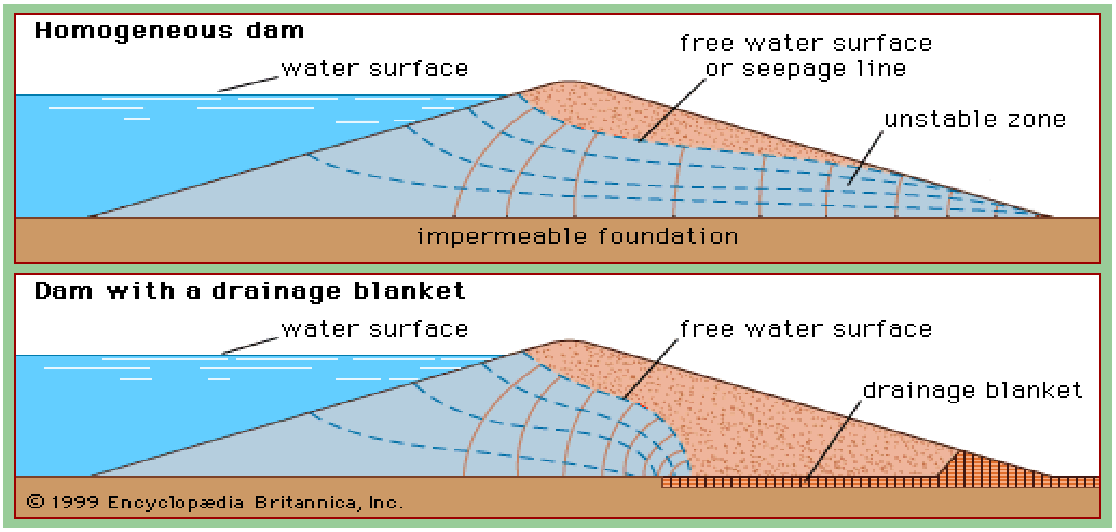


In the case of a rotational slump (above) the sides of the landslide have significant impact on the sliding resistance and this requires a 3D model.

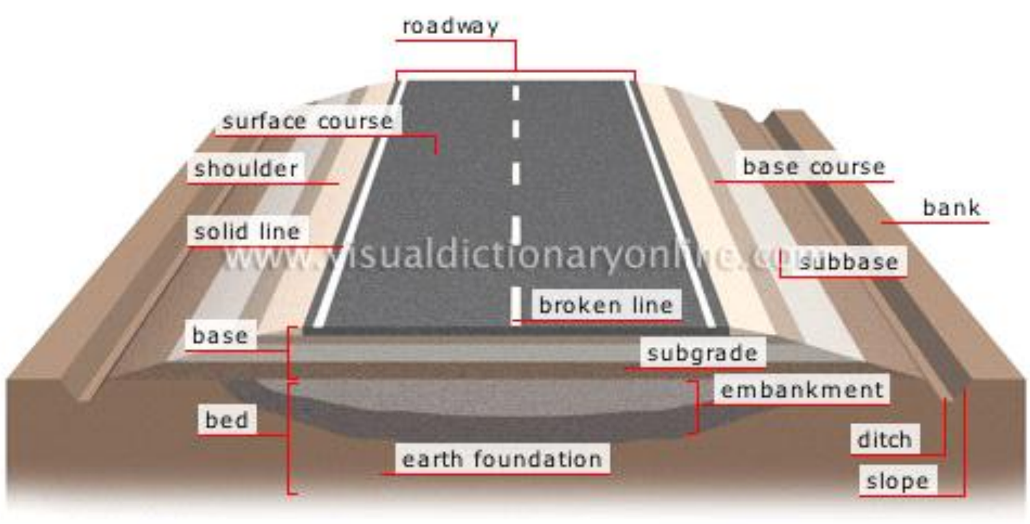
Selection of X-Section (continued)

Thursday, March 11, 2010
11:43 AM

Dam with 2D seepage (2D flow and plane strain conditions)



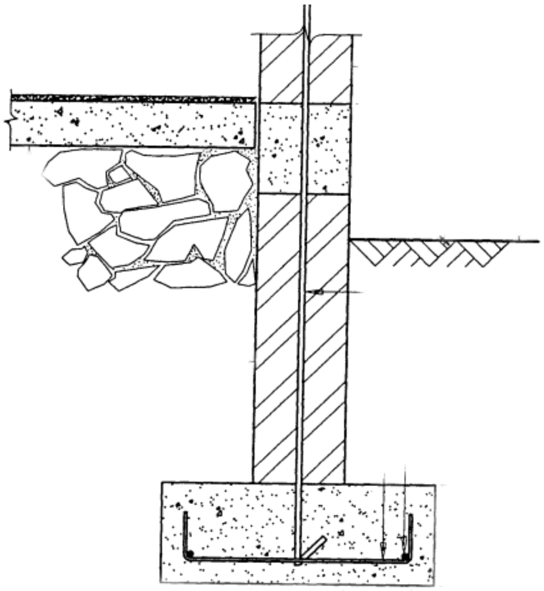
Typical Roadway Embankment (plane strain conditions)



Selection of X-Section (continued)

Thursday, March 11, 2010
11:43 AM

Strip footings (plane strain conditions)



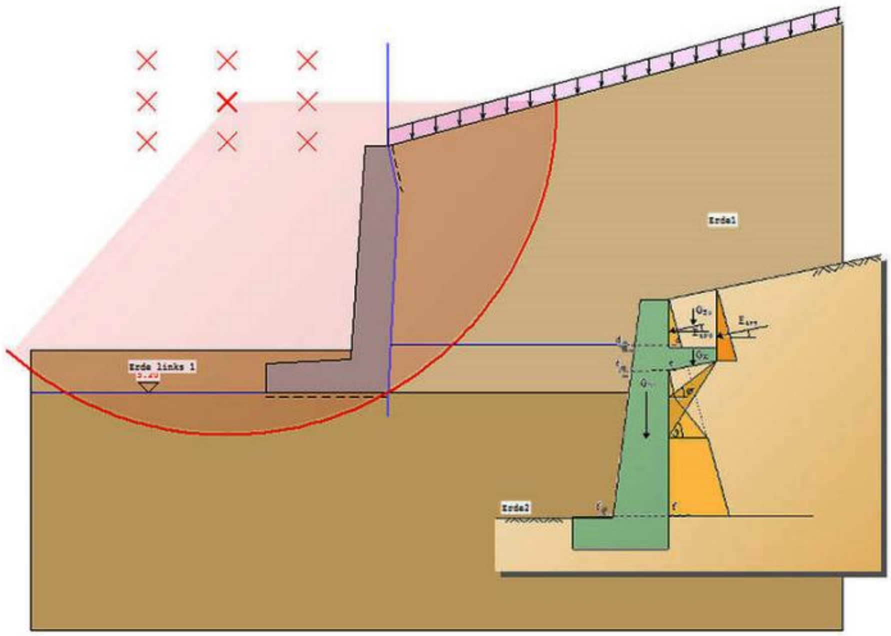
Note: To be a plane strain condition, the loading to the footing must be uniform along its length and the footing must be relatively long.

Tunnel (plane strain conditions)

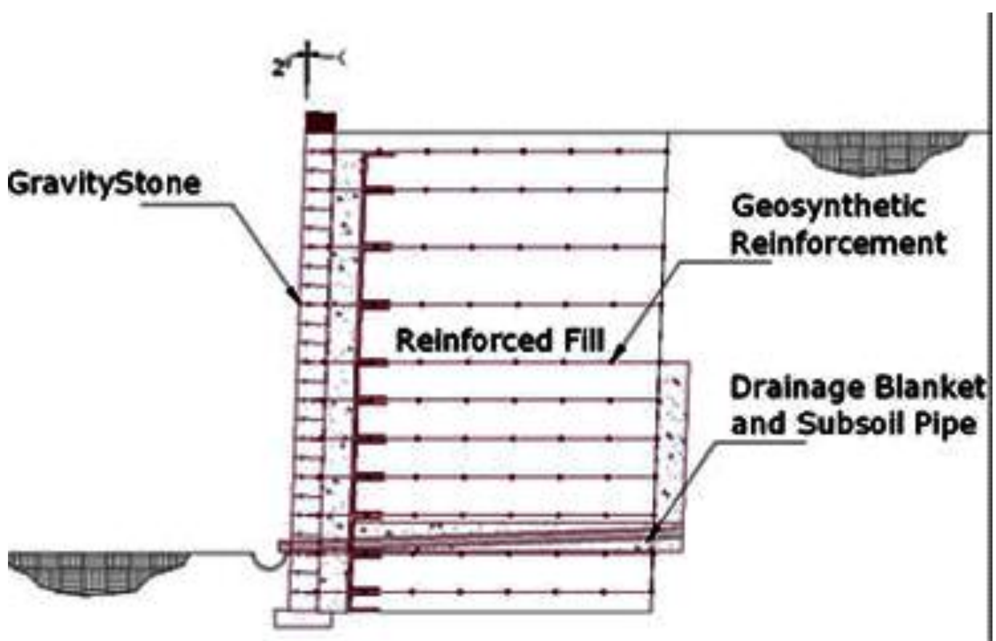
Selection of X-Section (continued)

Thursday, March 11, 2010
11:43 AM

Retaining Wall (plane strain conditions)



MSE Wall (plane strain conditions)

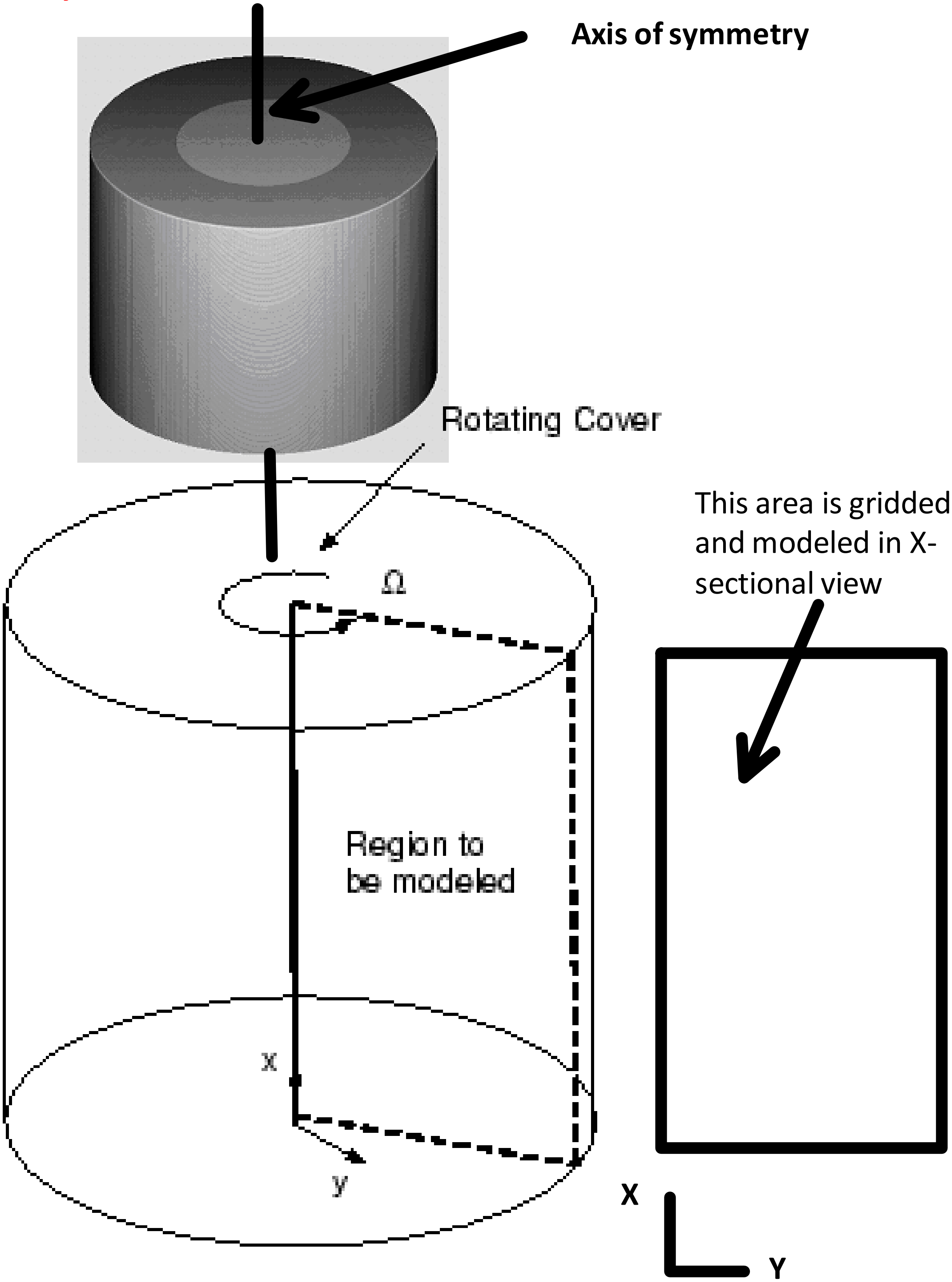


Note that MSE walls have a complex behavior due to their flexibility

Selection of X-Section (continued)

Thursday, March 11, 2010
11:43 AM

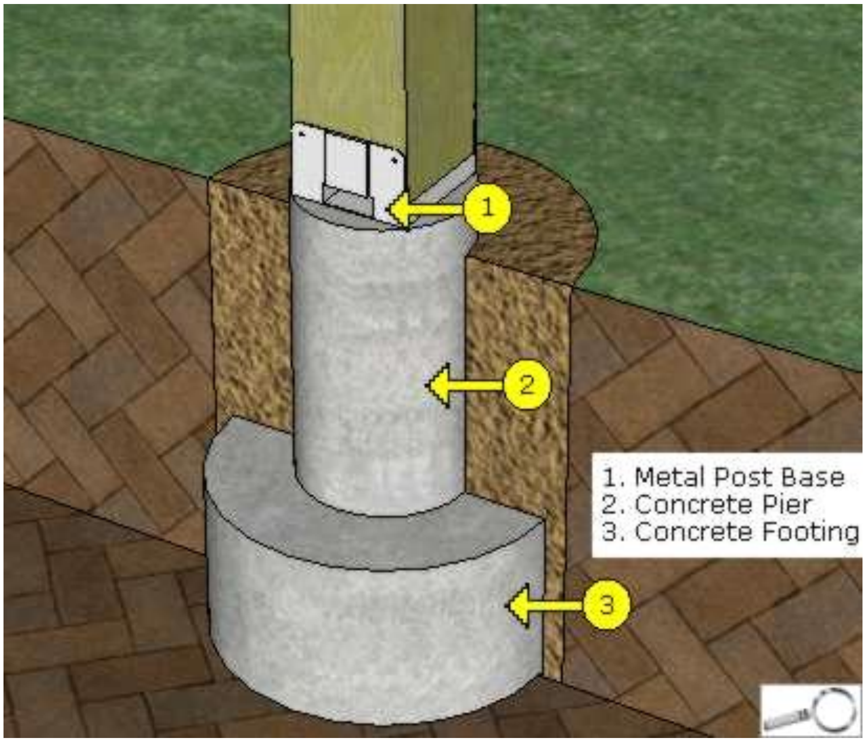
Axisymmetrical conditions



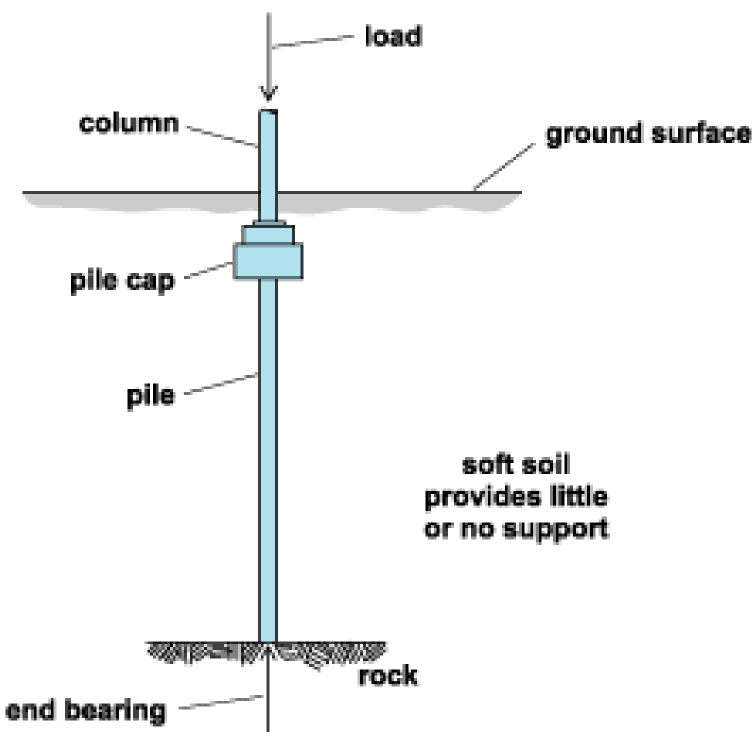
Selection of X-Section (continued)

Thursday, March 11, 2010
11:43 AM

Circular footing (Axisymmetrical Conditions)



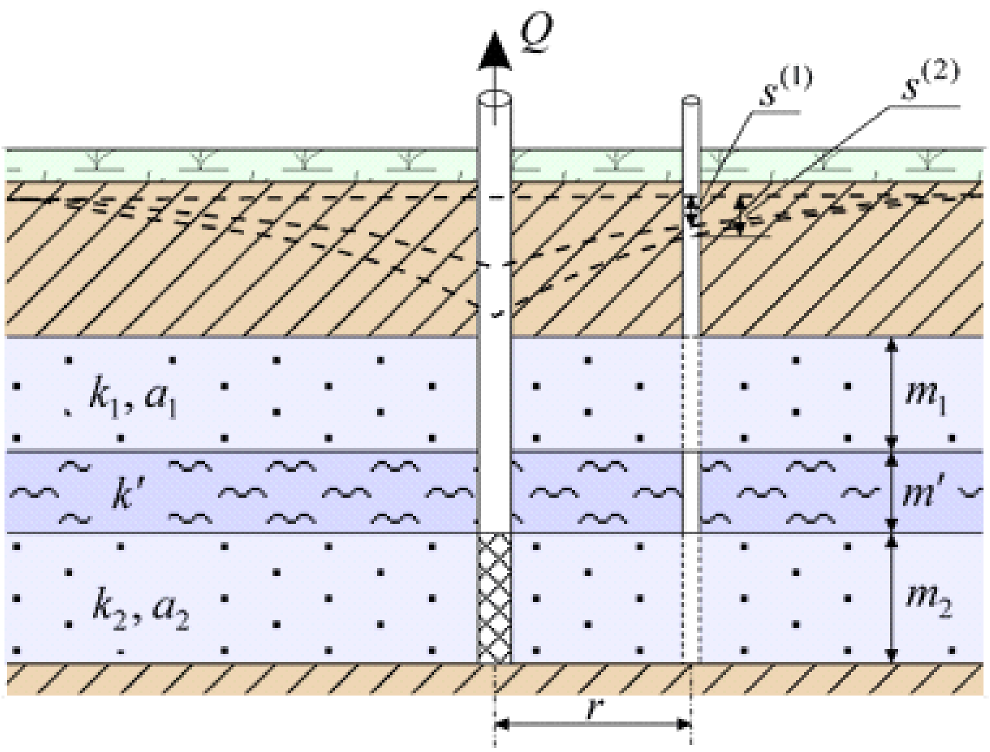
Single Pile (Axisymmetrical Conditions)



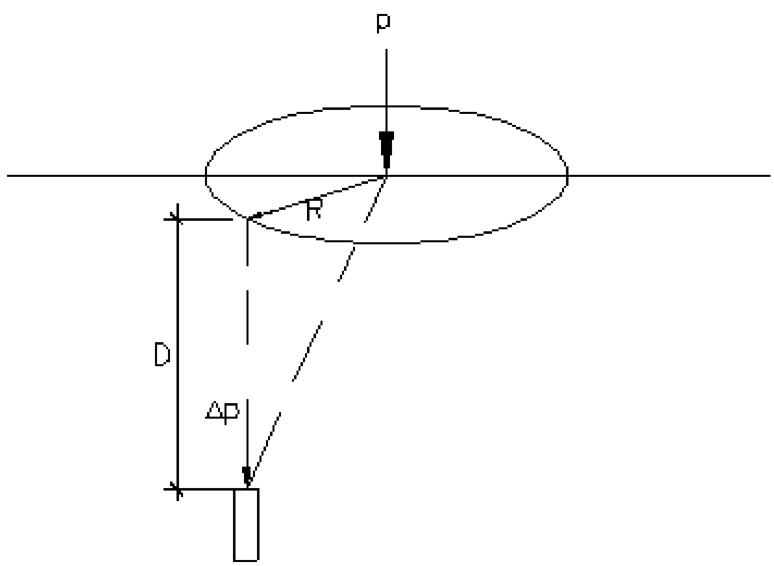
Selection of X-Section (continued)

Thursday, March 11, 2010
11:43 AM

Flow to an injection and/or pumping well (Axisymmetrical Conditions)



Point Load on Soil (Axisymmetrical Conditions)

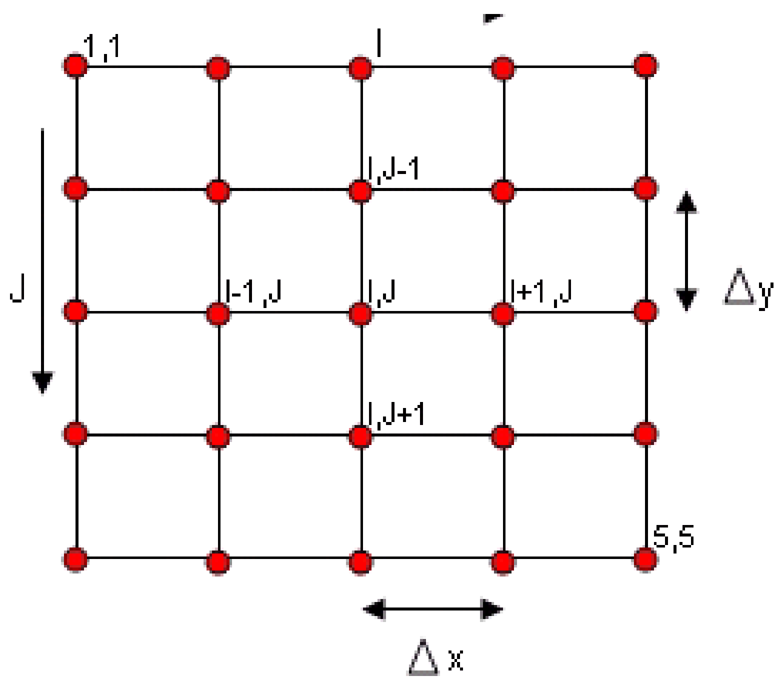


FDM vs FEM

Thursday, March 11, 2010
11:43 AM

Finite Difference Method (in brief)

- Oldest technique and simplest technique
- Requires knowledge of initial values and/boundary values
- Derivatives in the governing equation replaced by algebraic expression in terms of field variables
  - Field variables
    - Stress or pressure
    - Displacement
    - Velocity
- Field variables described at discrete points in space (i.e., nodes)
- Field variables are not defined between the nodes (are not defined by elements)
- No matrix operations are required
- Explicit method generally used
  - Solution is done by time stepping using small intervals of time
  - Grid values generated at each time step
  - Good method for dynamics and large deformations



LaPlace's Eq.

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0$$

LaPlace's Eq. using Central difference formula for 2nd order derivative

$$\frac{\partial^2 h}{\partial x^2} \approx \frac{h_{i-1,j} - 2h_{i,j} + h_{i+1,j}}{(\Delta x)^2}$$

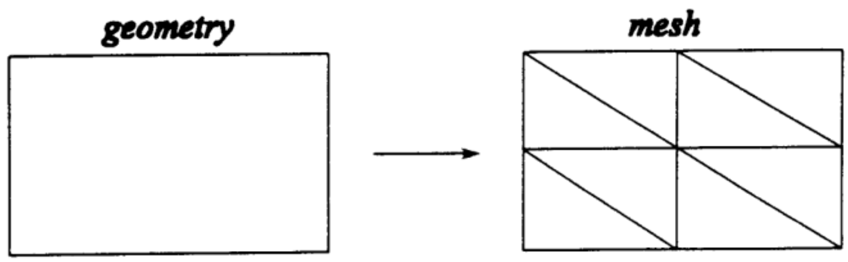
$$\frac{\partial^2 h}{\partial y^2} \approx \frac{h_{i,j-1} - 2h_{i,j} + h_{i,j+1}}{(\Delta y)^2}$$

FDM vs FEM

Tuesday, August 21, 2012
12:43 PM

Finite Element Method (in brief)

- Evolved from mechanical and structural analysis of beams, columns, frames, etc. and has been generalized to continuous media such as soils
- General method to solve boundary value problems in an approximate and discretized manner
- Division of domain geometry into finite element mesh
- Field variables are defined by elements



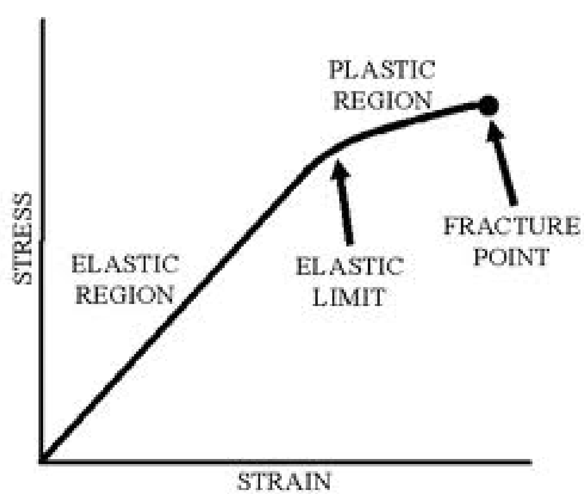
- FEM requires that field variables vary in prescribed fashion using specified functions (interpolation functions) throughout the domain. Pre-assumed interpolation functions are used for the field variables over elements based on values in points (nodes).
- Implicit FEM more common
  - Matrix operations required for solution
  - Stiffness matrix formed. Formulation of stiffness matrix, K, and force vector, r
- Adjustments of field variables is made until error term is minimized in terms of energy

Constitutive (i.e., Stress - Strain) Relationships

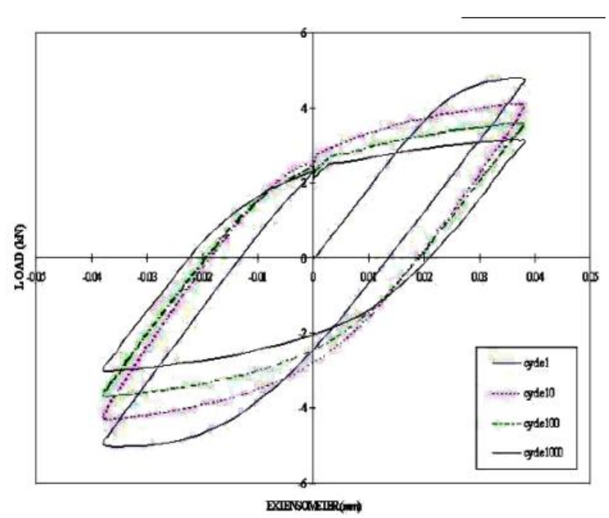
Thursday, March 11, 2010
11:43 AM

FDM and FEM required constitutive relations (i.e., stress-strain laws). There are three general classes of behavior that describe how a solid responds to an applied stress: (from Wikipedia)

- Elastic– When an applied stress is removed, the material returns to its undeformed state. Linearly elastic materials, those that deform proportionally to the applied load, can be described by the linear elasticity equations such as Hooke's law.
- Viscoelastic– These are materials that behave elastically, but also have damping: when the stress is applied and removed, work has to be done against the damping effects and is converted in heat within the material resulting in a hysteresis loop in the stress–strain curve. This implies that the material response has time-dependence.
- Plastic – Materials that behave elastically generally do so when the applied stress is less than a yield value. When the stress is greater than the yield stress, the material behaves plastically and does not return to its previous state. That is, deformation that occurs after yield is permanent.



Elastic - Plastic Behavior



Viscoelastic Behavior

Characterization of Material Properties

Thursday, March 11, 2010
11:43 AM

The type of constitutive relation selected with dictate the type of testing required. More advance models need more parameters and testing, especially if nonlinear or plastic analyses are required.

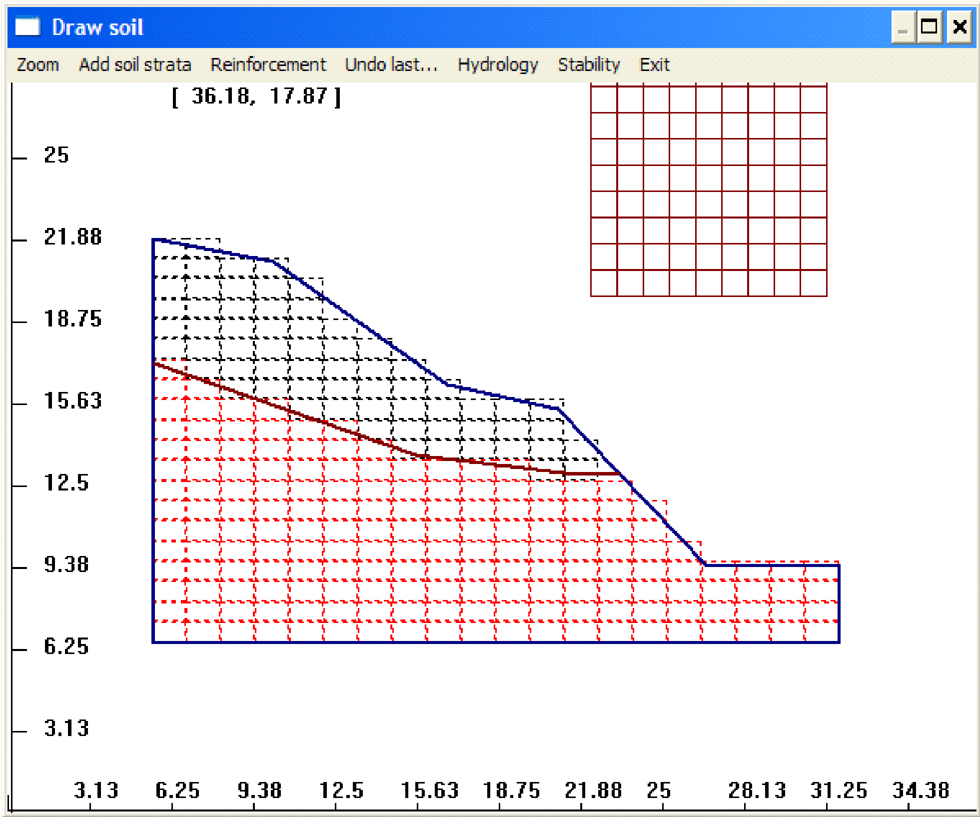
Methods

- Laboratory Testing
    - Index tests
    - Strength Testing
        - Direct Shear Tests
        - Direct Simple Shear Test
        - Triaxial Testing
            - UU (Unconsolidated Undrained)
            - CU (Consolidated Undrained)
            - CD (Consolidated Drained
        - Ring Shear
    - Consolidation Testing
        - Incremental load
        - Constant Rate of Strain
        - Row Cell
    - Permeability Testing
        - Constant Head
        - Falling Head
- In situ Testing
        - SPT
        - CPT
        - DMT
        - Vane Shear
        - Borehole Shear
        - Pressuremeter
        - Packer Testing
- Back analysis of case histories or performance data
    - Back analysis of cases of failure

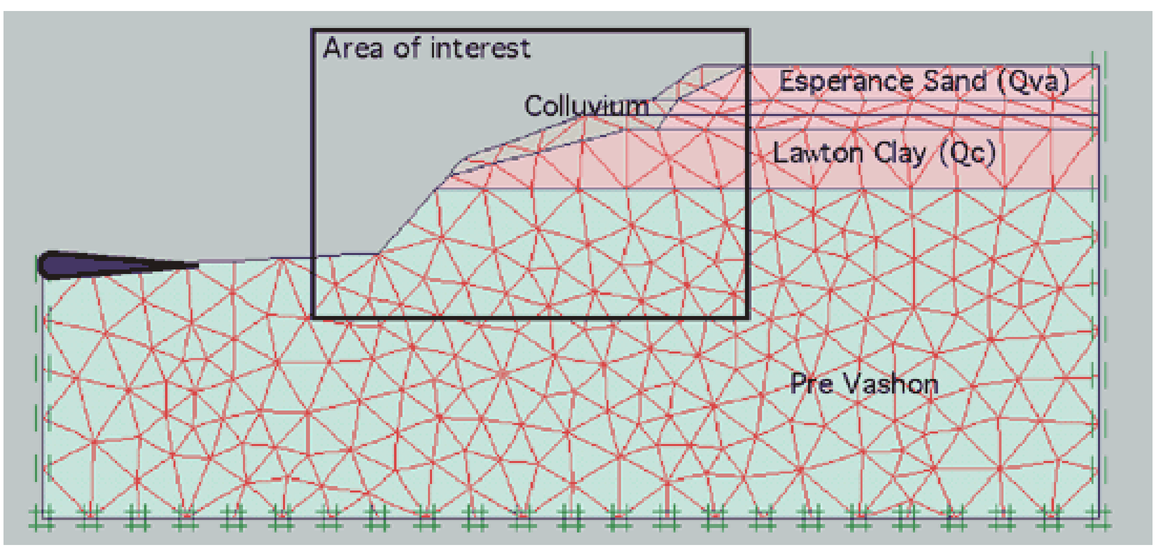
Grid Generation

Thursday, March 11, 2010
11:43 AM

Typical Finite difference grid

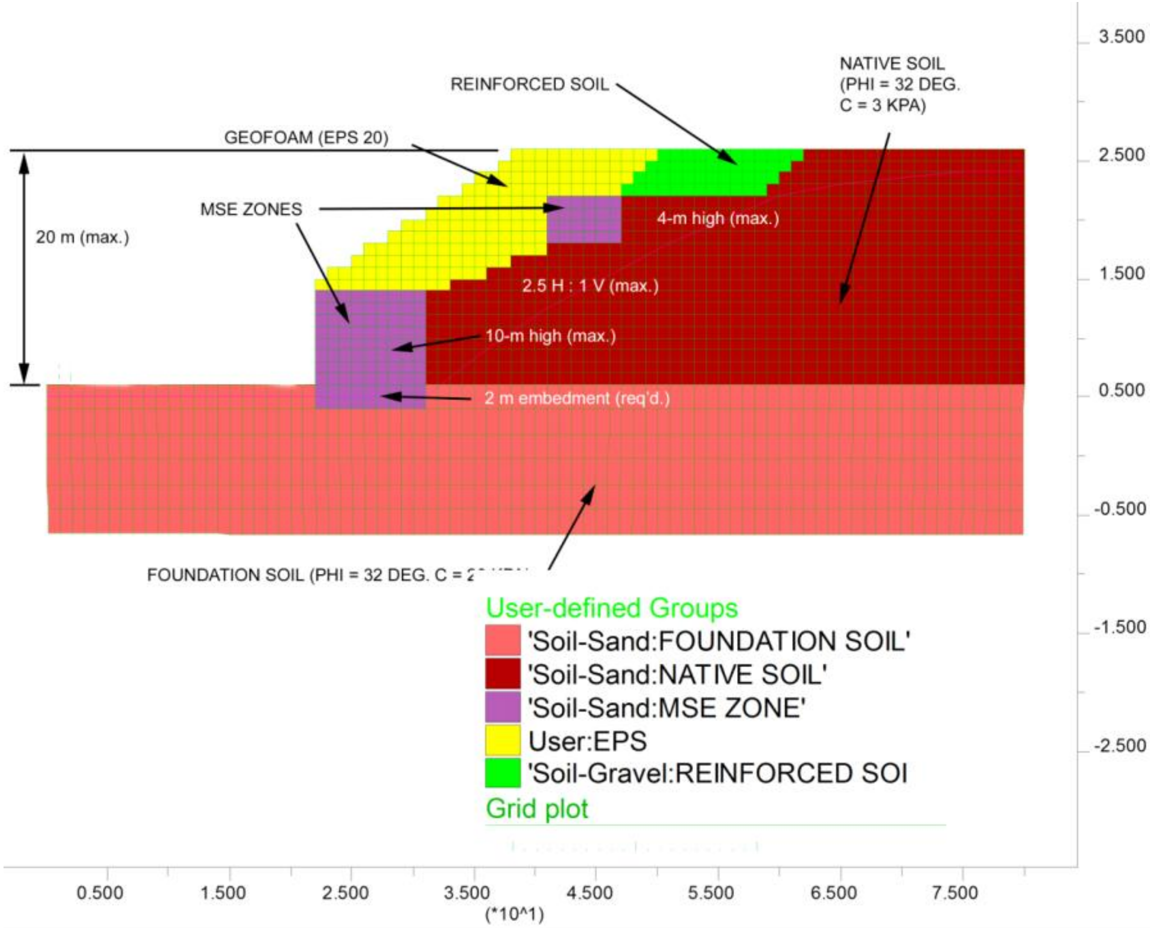


Typical Finite Element Grid



Assign Material Properties

Thursday, March 11, 2010
11:43 AM



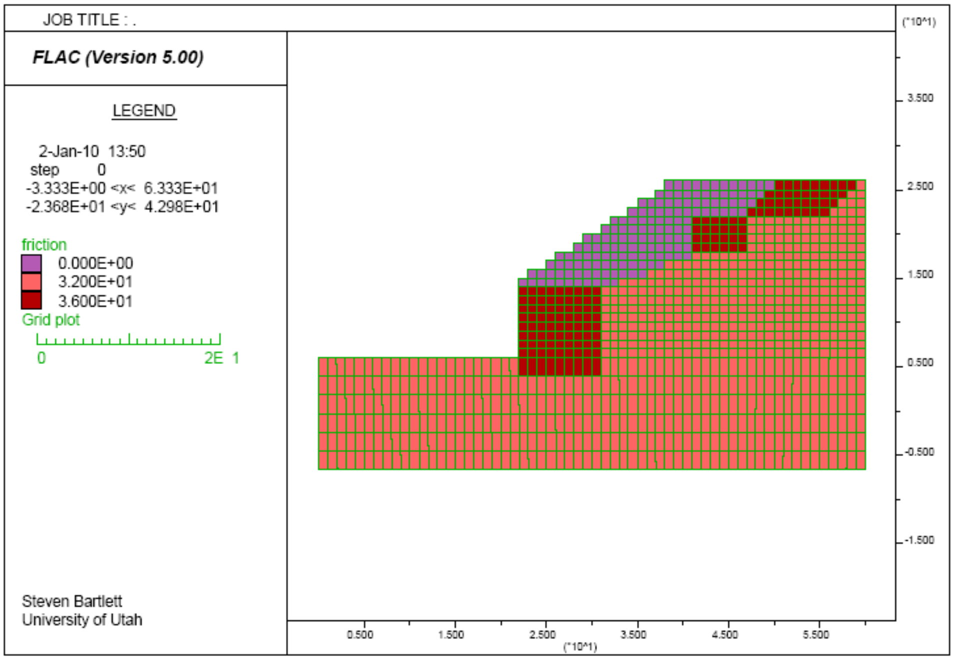
Determine major soil units

Assign properties to soil units:

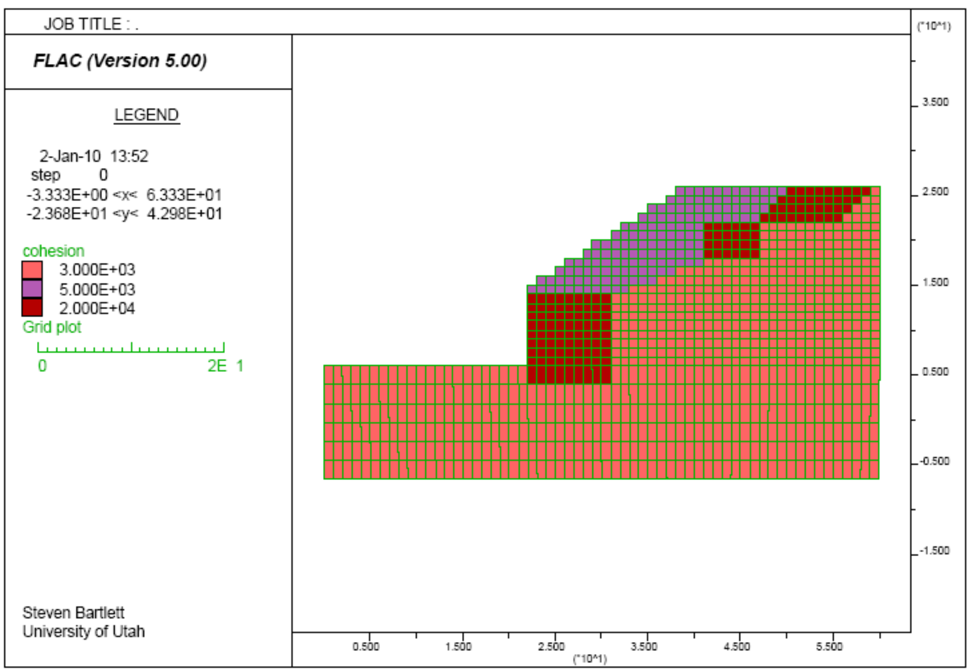
- Unit weight
- Young's modulus
- Bulk modulus
- Constitutive model
  - Pre-failure model (usually elastic model)
  - Failure criterion (failure envelope)
  - Post-failure model (plastic model)

Assign Material Properties

Thursday, March 11, 2010
11:43 AM



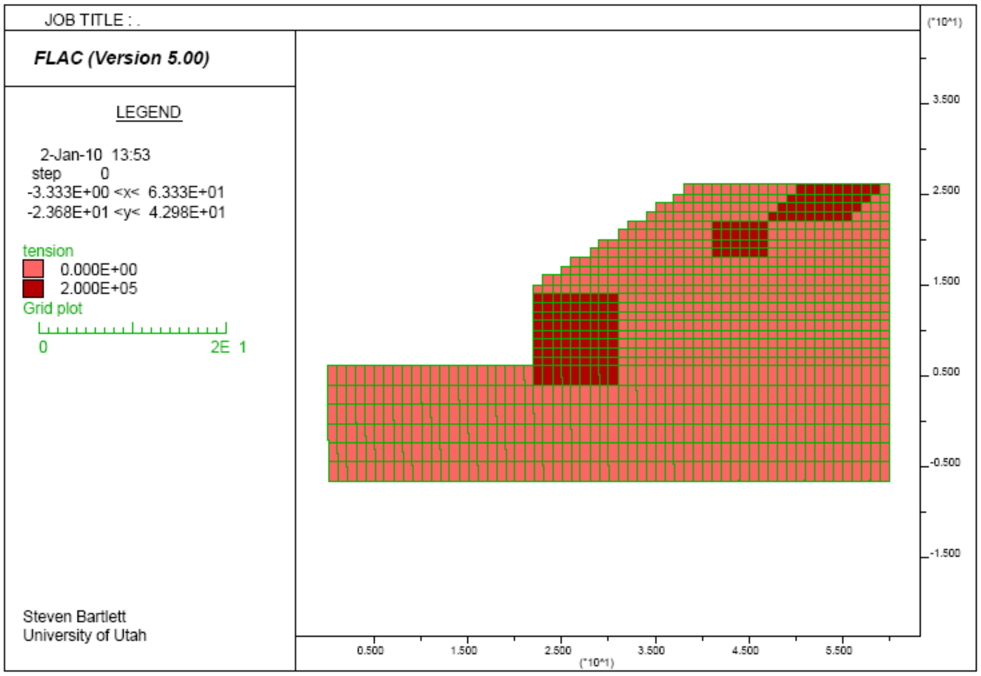
Friction Angle



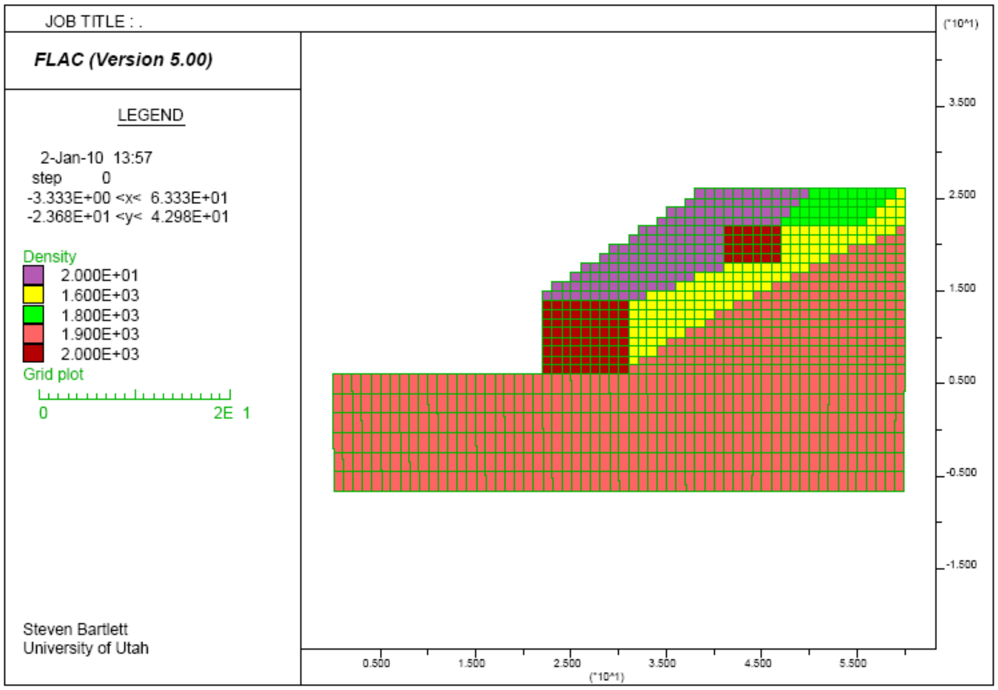
Cohesion

Assign Material Properties

Thursday, March 11, 2010
11:43 AM



Tensile Strength (for reinforced zones)

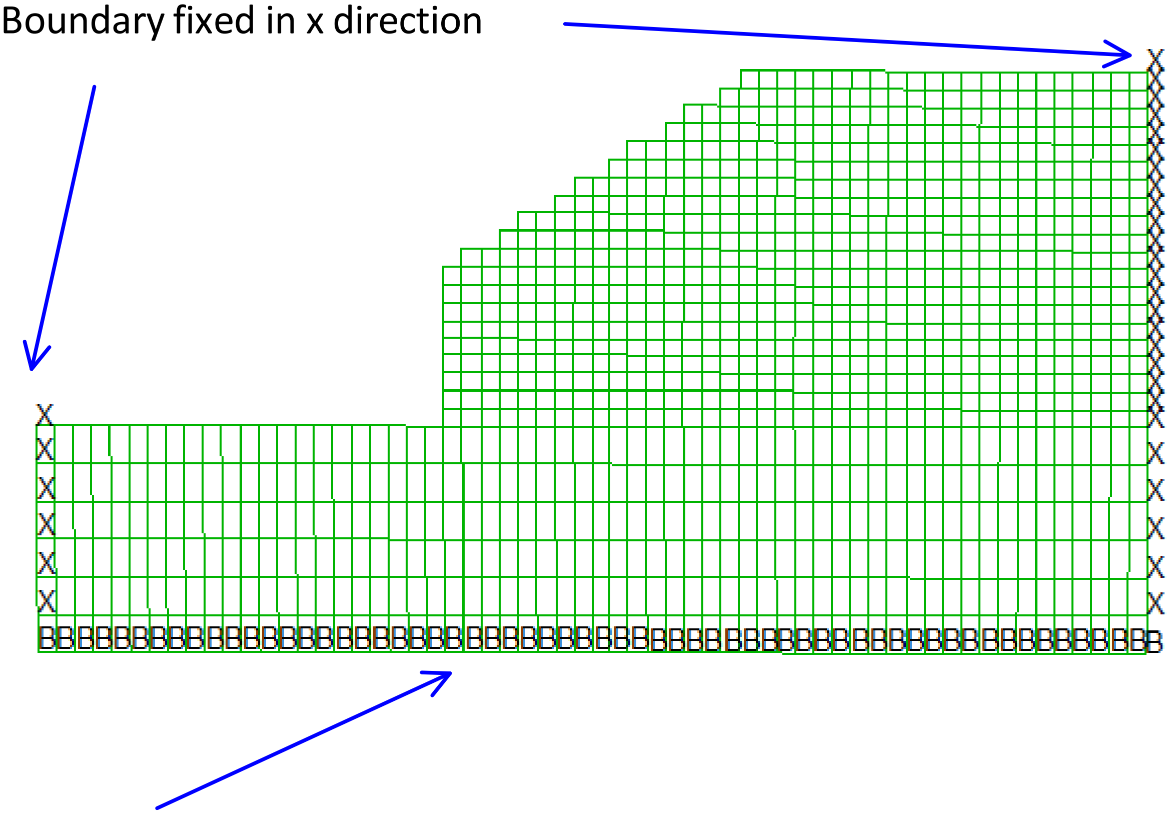


Soil Density

Assign Boundary Conditions

Thursday, March 11, 2010
11:43 AM

Boundary fixed in x direction



Boundary fixed in x and y direction (i.e., B is used to indicate boundary is fixed in both directions).

Typical boundary conditions

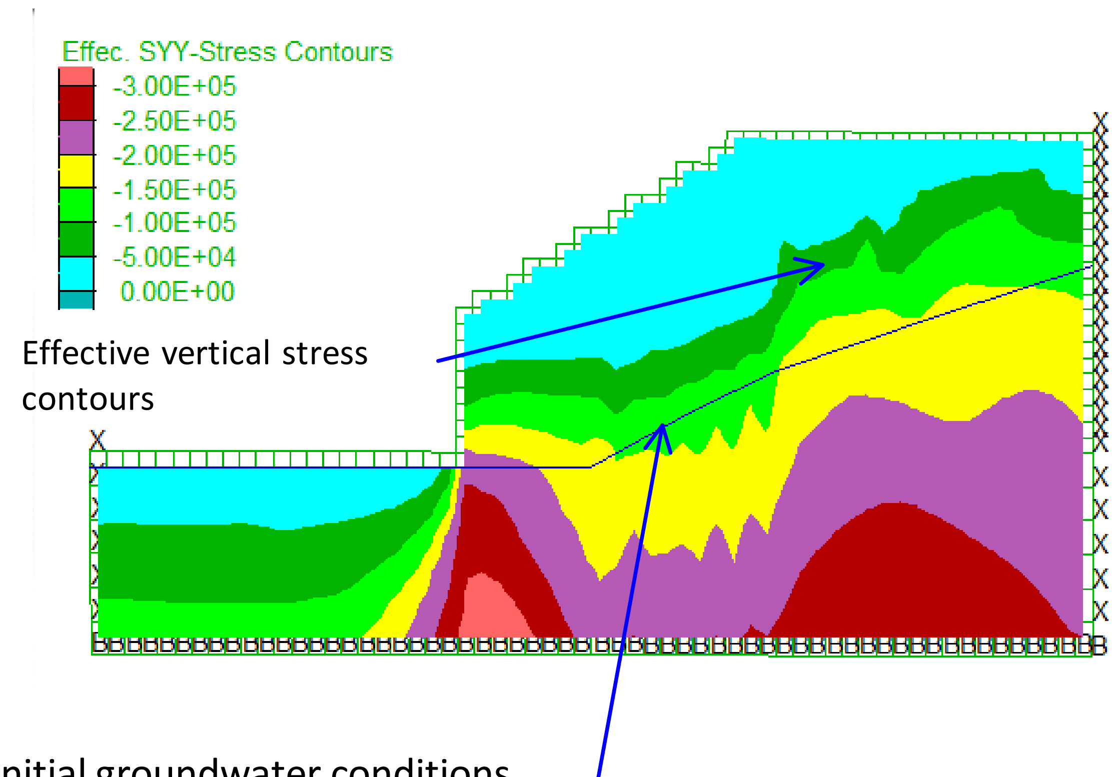
- Fixed in x direction
- Fixed in y direction
- Fixed in both directions
- Free in x and y directions (no boundary assigned)

Calculate Initial Conditions

Tuesday, August 21, 2012
12:43 PM

Initial Conditions that are generally considered:

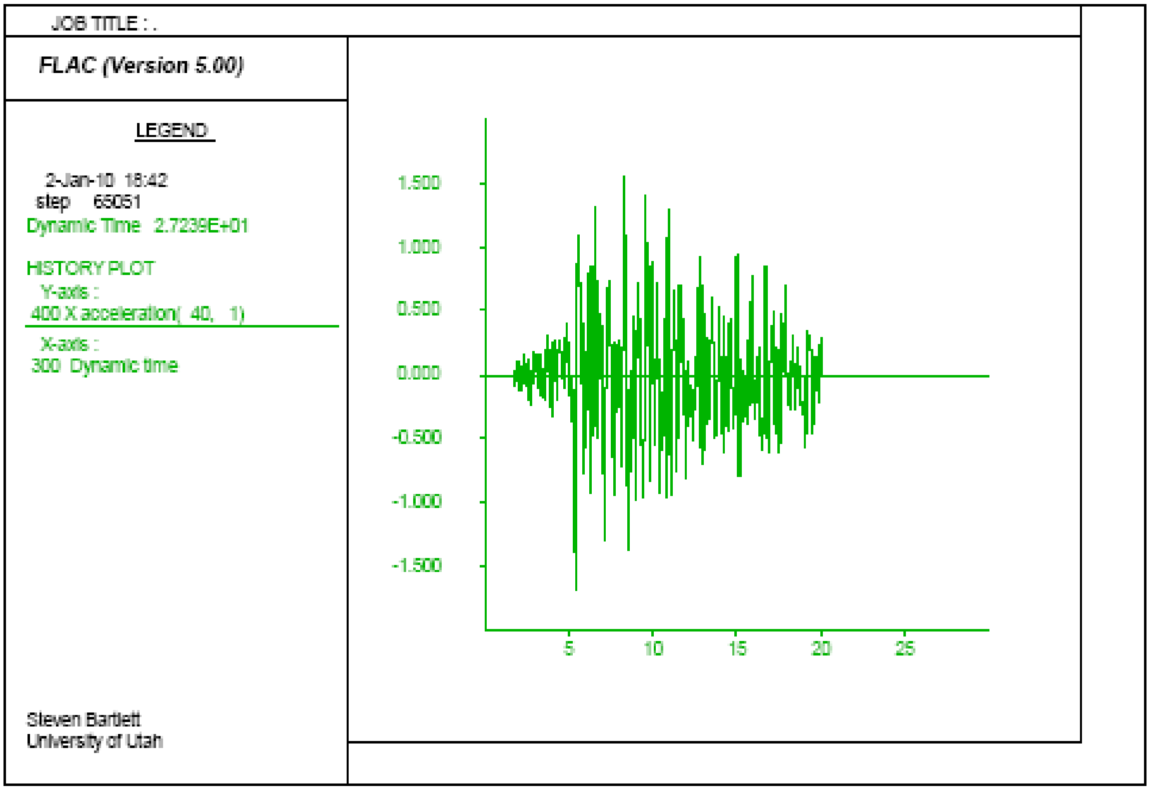
- Initial shear stresses
- Groundwater conditions
  - Hydrostatic water table
  - Flow gradient (non-steady state)
- For dynamic problems
  - Acceleration, velocity or stress time history



Note that for this case, the initial effective vertical stresses were calculated by the computer model for the given boundary conditions, water table elevations and material properties.

Determine modeling or load sequence

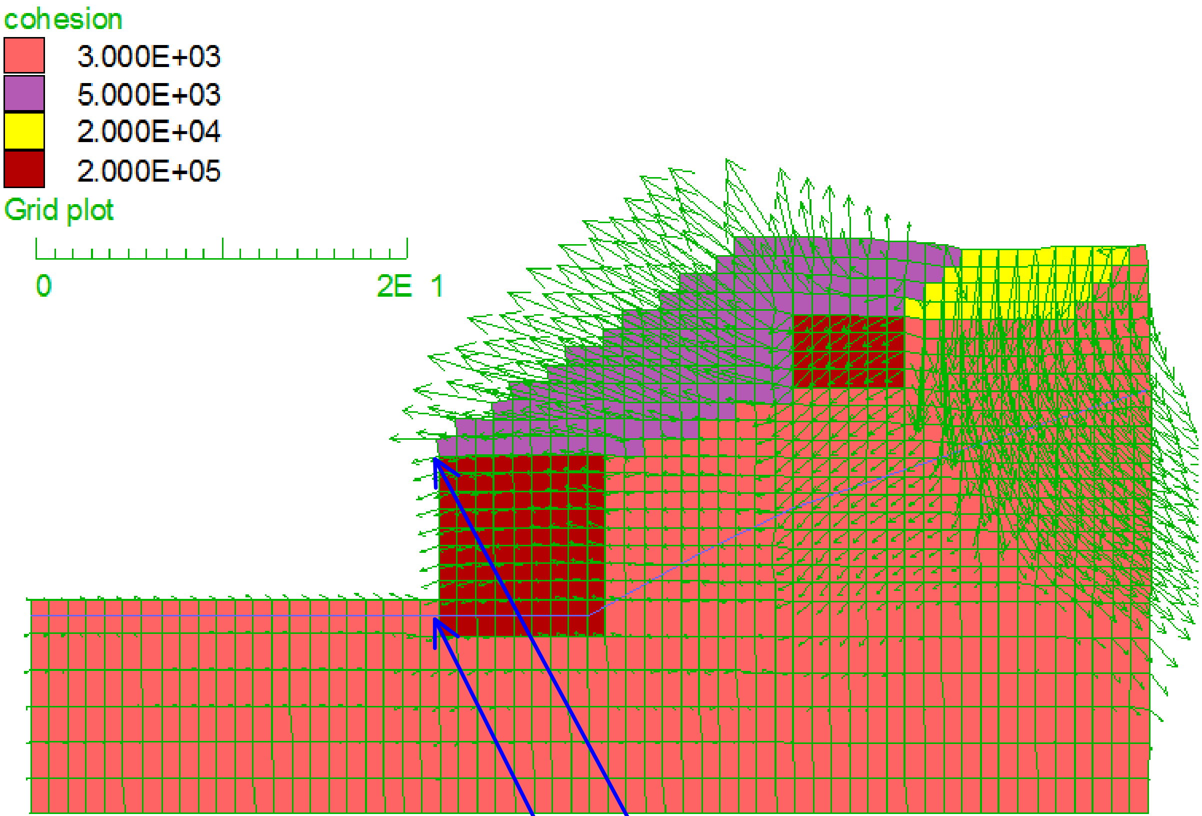
Thursday, March 11, 2010
11:43 AM



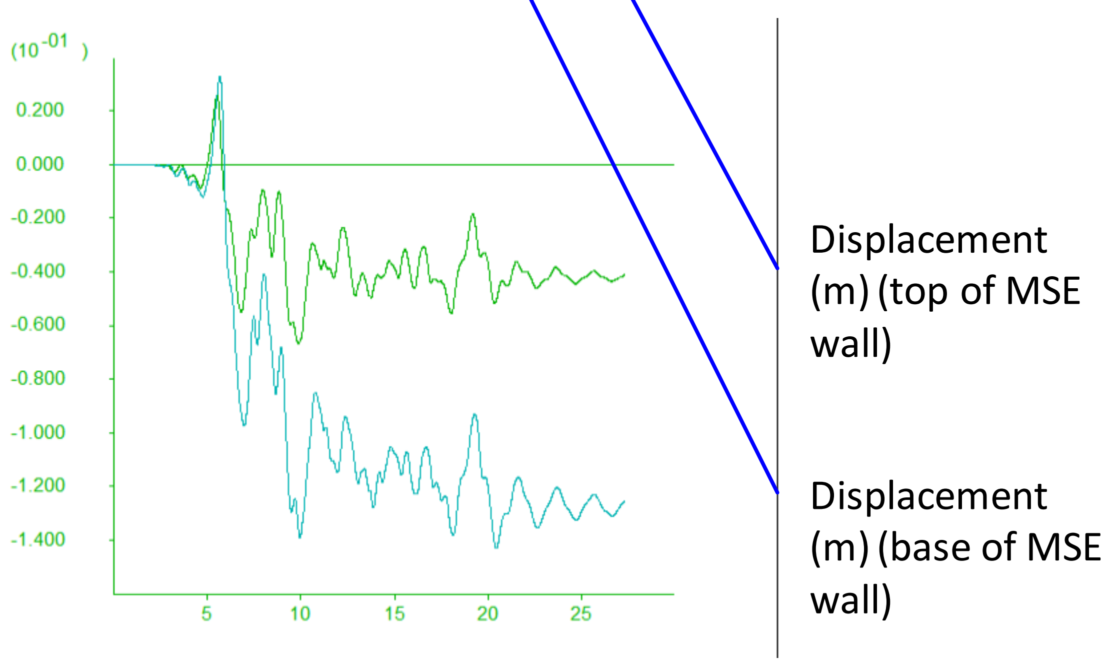
Input acceleration time history that is input into base of the model for dynamic modeling

Obtain Results

Thursday, March 11, 2010
11:43 AM



Final vector displacement pattern for input acceleration time history



Interpret Results

Monday, August 23, 2010
6:03 PM

The figures on the previous page show that the horizontal displacement of the top and base of the MSE wall during the earthquake event. The top and base of the MSE wall have moved outward about 4 and 12 cm, respectively during the seismic event.

This amount of displacement is potentially damaging to the overlying roadway and the design must be modified or optimized to reduce these displacements.

The figures on the previous page show that the horizontal displacement of the top and base of the MSE wall during the earthquake event. The top and base of the MSE wall have moved outward about 40 and 120 cm, respectively during the seismic event.

This amount of displacement is potentially damaging to the overlying roadway and the design must be modified or optimized to reduce these displacements.

Reading

Thursday, March 11, 2010
11:43 AM

- FLAC v. 5.0 User's Guide, Section 3.0 PROBLEM SOLVING WITH FLAC
- FLAC v. 5.0 User's Guide, Section 3.1 General Approach

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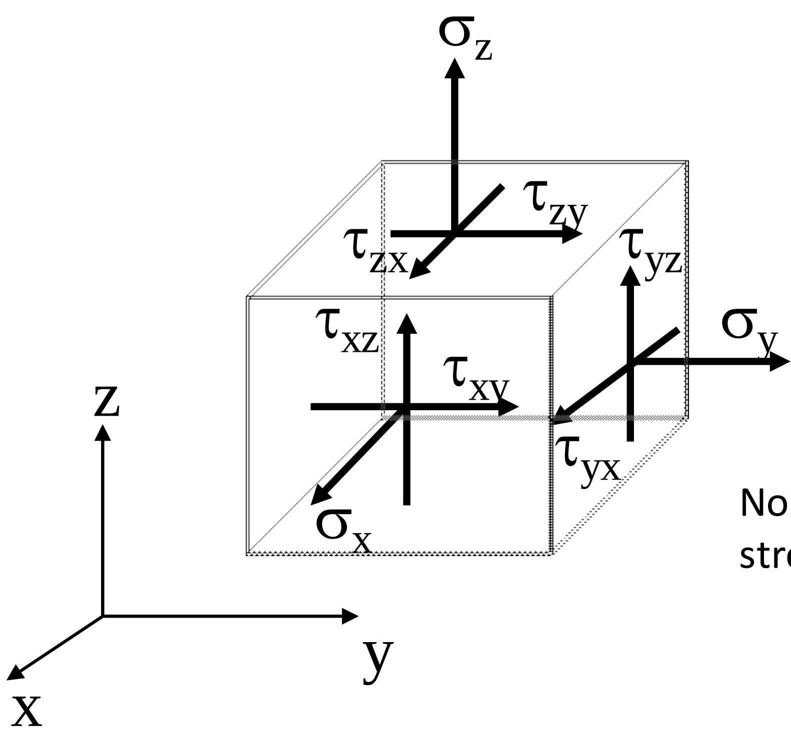
Thursday, March 11, 2010
11:43 AM

Elastic Theory

Thursday, March 11, 2010
11:43 AM



Normal and shear stresses

$$\underline{\sigma} = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix}$$

Normal stress in the x direction

Normal stress in the y direction

Normal stress in the z direction

Shear stress on the xy plane

Shear stress on the yz plane

Shear stress on the zx plane

Recall that:

$$\tau_{xy} = \tau_{yx}$$

$$\tau_{yz} = \tau_{zy}$$

$$\tau_{zx} = \tau_{xz}$$

There are 6 independent unknown stresses

3D State of Stress (continued)

Thursday, March 11, 2010
11:43 AM

Strain and displacement relations for 3D

$$\underline{\varepsilon} = \underline{\partial}\ \underline{u}$$

Definitions of axial and shear strain

$$\varepsilon_x = \frac{\partial u}{\partial x}$$

Axial strain in the x-direction

$$\varepsilon_y = \frac{\partial v}{\partial y}$$

Axial strain in the y-direction

$$\varepsilon_z = \frac{\partial w}{\partial z}$$

Axial strain in the z-direction

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

Shear strain in the x-y plane

$$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}$$

Shear strain in the y-z plane

$$\gamma_{zx} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}$$

Shear strain in the z-x plane

Hooke's Law in frequently written in terms of the *engineering shear strain,*

$$\gamma$$

Recall, that <u>the engineering shear strain is defined to be twice that of the tensor shear strain</u>; for example,

$$\gamma_{xy} = 2\varepsilon_{xy}$$

3D State of Stress (continued)

Thursday, March 11, 2010
11:43 AM

6 independent and unknown strains

$$\underline{\varepsilon} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix}$$

Axial strain in the x direction

Axial strain in the y direction

Axial strain in the z direction

Shear strain in the xy plane

Shear strain in the yz plane

Shear strain in the zx plane

3 independent and unknown displacement

$$\underline{\mathbf{u}} = \begin{Bmatrix} \mathbf{u} \\ \mathbf{v} \\ \mathbf{w} \end{Bmatrix}$$

Displacement in the x direction

Displacement in the y direction

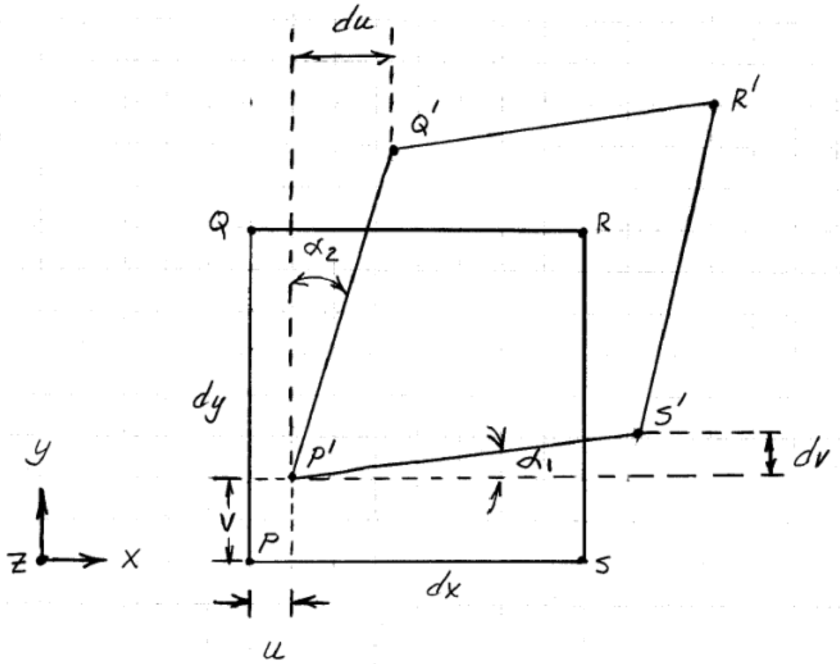
Displacement the z direction

2D State of Stress (continued)

Wednesday, August 29, 2012
12:43 PM

- Review of Strain Notation

Z-D Plane strain (w/ displacement, distortion & rotation)



Relationships

- $\tan \alpha_1 = dv/dx$ v = displacement in y direction
- $\tan \alpha_2 = du/dy$ u = displacement in x direction

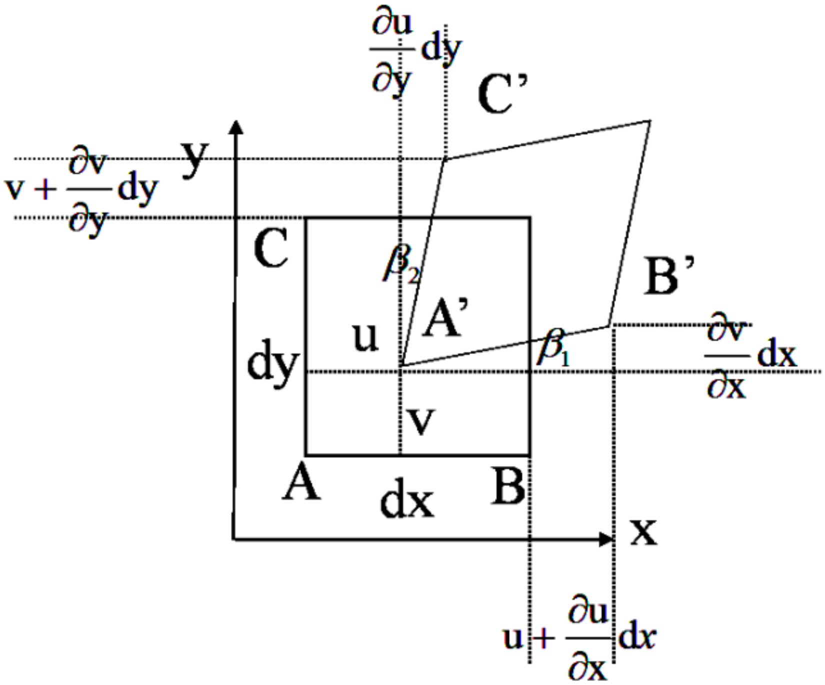
- shear strain in x-y plane
  $\gamma_{xy} = \alpha_1 + \alpha_2$
- For small deformation

  $\alpha_1 \approx \tan \alpha_1$
  $\alpha_2 \approx \tan \alpha_2$

  $\gamma_{xy} = dv/dx + du/dy$
- Rotation $\Omega_z = (\alpha_1 - \alpha_2)/2$

2D Strain - displacement relations

Thursday, March 11, 2010
11:43 AM



Note that the square shown below has undergone translation, deformation and distortion

$$\varepsilon_x = \frac{A'B' - AB}{AB} = \frac{\left(dx + \left(u + \frac{\partial u}{\partial x} dx\right) - u\right) - dx}{dx} = \frac{\partial u}{\partial x}$$

$$\varepsilon_y = \frac{A'C' - AC}{AC} = \frac{\left(dy + \left(v + \frac{\partial v}{\partial y} dy\right) - v\right) - dy}{dy} = \frac{\partial v}{\partial y}$$

$$\gamma_{xy} = \frac{\pi}{2} - \text{angle}\ (C'A'B') = \beta_1 + \beta_2 \approx \tan\beta_1 + \tan\beta_2$$

$$\approx \frac{\partial v}{\partial x} + \frac{\partial u}{\partial x}$$

Note that shear strain is an angular distortion measured in radians. For small distortions, the angles above may be taken equal to their tangents or dv/dx + du/dx.

3D Hooke's Law

Thursday, March 11, 2010
11:43 AM

To solve for these **15 unknowns**, we have:

- 3 equations of force equilibrium (from the stresses)
- 6 equations of compatibility (from the strains)

Hence, the system is statically indeterminate and to overcome this deficiency, we need 6 more equations. These equations can be obtained from relating stress and strain and assuming an isotropic medium.

For the **linear elastic, isotropic case** (i.e., stiffness the same in all directions), the stresses and strains can be related through Hooke's law and the system of equations is solvable.

$$\underline{\sigma} = \underline{D}\,\underline{\varepsilon}$$

Stresses from Hooke's Law

$$\underline{D} = \frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}$$

E = Young's Modulus or the Elastic Modulus
υ = Poisson's ratio

Hooke's Law (Strains and Stresses)

Thursday, March 11, 2010
11:43 AM

Stains from Hooke's Law

$$\varepsilon_x = \frac{\sigma_x}{E} - \upsilon\frac{\sigma_y}{E} - \upsilon\frac{\sigma_z}{E}$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \upsilon\frac{\sigma_x}{E} - \upsilon\frac{\sigma_z}{E}$$

$$\varepsilon_z = \frac{\sigma_z}{E} - \upsilon\frac{\sigma_x}{E} - \upsilon\frac{\sigma_y}{E}$$

and

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

$$\gamma_{yz} = \frac{\tau_{yz}}{G}$$

$$\gamma_{zx} = \frac{\tau_{zx}}{G}$$

Stresses from Hooke's Law

$$\sigma_x = \frac{E}{(1+\upsilon)(1-2\upsilon)}\left[(1-\upsilon)\varepsilon_x + \upsilon\left(\varepsilon_y + \varepsilon_z\right)\right]$$

$$\sigma_y = \frac{E}{(1+\upsilon)(1-2\upsilon)}\left[(1-\upsilon)\varepsilon_y + \upsilon\left(\varepsilon_x + \varepsilon_z\right)\right]$$

$$\sigma_z = \frac{E}{(1+\upsilon)(1-2\upsilon)}\left[(1-\upsilon)\varepsilon_z + \upsilon\left(\varepsilon_x + \varepsilon_y\right)\right]$$

E= elastic modulus

ν = poisson's ratio

G= shear modulus

$$G = \frac{E}{2(1+\upsilon)}$$

Elastic Moduli - Young's Modulus

Thursday, March 11, 2010
11:43 AM

In solid mechanics, Young's modulus, also known as the tensile modulus, is a measure of the stiffness of an isotropic elastic material. It is also commonly, but incorrectly, called the *elastic modulus* or *modulus of elasticity*, because Young's modulus is the most common elastic modulus used, but there are other elastic moduli measured, too, such as the bulk modulus and the shear modulus.
Young's modulus is the ratio of stress, which has units of pressure, to strain, which is dimensionless; therefore, Young's modulus has units of pressure.

For many materials, Young's modulus is essentially constant over a range of strains. Such materials are called linear, and are said to obey Hooke's law. Examples of linear materials are steel, carbon fiber and glass. Non-linear materials include rubber and soils, except under very small strains.

Definition:

$$E \equiv \frac{\text{tensile stress}}{\text{tensile strain}} = \frac{\sigma}{\varepsilon} = \frac{F/A_0}{\Delta L/L_0} = \frac{FL_0}{A_0 \Delta L}$$

where

E is the Young's modulus (modulus of elasticity)
F is the force applied to the object;
A_0 is the original cross-sectional area through which the force is applied;
ΔL is the amount by which the length of the object changes;
L_0 is the original length of the object.

Pasted from <http://en.wikipedia.org/wiki/Young%27s_Modulus>

Elastic Moduli - Shear Modulus

Thursday, March 11, 2010
11:43 AM

In materials science, shear modulus or modulus of rigidity, denoted by G, or sometimes S or μ, is defined as the ratio of shear stress to the shear strain:[1]

$$G \stackrel{\text{def}}{=} \frac{\tau_{xy}}{\gamma_{xy}} = \frac{F/A}{\Delta x/I} = \frac{FI}{A\Delta x}$$

where

$$\tau_{xy} = F/A$$

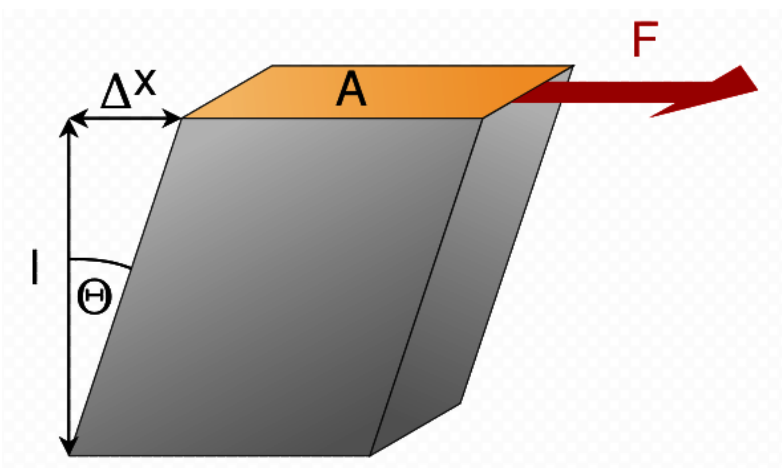
shear stress

F is the force which acts
A is the area on which the force acts

$$\gamma_{xy} = \Delta x/I = \tan\theta$$

shear strain;

Δx is the transverse displacement and
I is the initial length



Pasted from <http://en.wikipedia.org/wiki/Shear_modulus>

Elastic Moduli - Bulk Modulus

Thursday, March 11, 2010

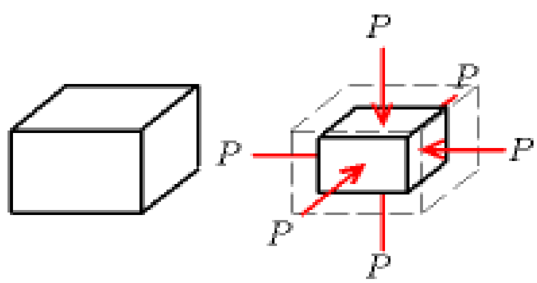
The bulk modulus (*K*) of a substance measures the substance's resistance to uniform compression. It is defined as the pressure increase needed to cause a given relative decrease in volume. Its base unit is that of pressure.

As an example, suppose an iron cannon ball with bulk modulus 160 GPa is to be reduced in volume by 0.5%. This requires a pressure increase of 0.005×160 GPa = 0.8 GPa (116,000 psi).

Definition

The bulk modulus *K* can be formally defined by the equation:

$$K = -V\frac{\partial P}{\partial V}$$



where *P* is pressure, *V* is volume, and ∂*P*/∂*V* denotes the partial derivative of pressure with respect to volume. The inverse of the bulk modulus gives a substance's compressibility.

Other moduli describe the material's response (strain) to other kinds of stress: the shear modulus describes the response to shear, and Young's modulus describes the response to linear strain. For a fluid, only the bulk modulus is meaningful. For an anisotropic solid such as wood or paper, these three moduli do not contain enough information to describe its behavior, and one must use the full generalized Hooke's law

Elastic Constants - Relationships

Thursday, March 11, 2010
11:43 AM

	(λ, G)	(E, G)	(K, λ)	(K, G)	(λ, ν)
$K =$	$\lambda + \frac{2G}{3}$	$\frac{EG}{3(3G-E)}$			$\frac{\lambda(1+\nu)}{3\nu}$
$E =$	$\frac{G(3\lambda+2G)}{\lambda+G}$		$\frac{9K(K-\lambda)}{3K-\lambda}$	$\frac{9KG}{3K+G}$	$\frac{\lambda(1+\nu)(1-2\nu)}{\nu}$
$\lambda =$		$\frac{G(E-2G)}{3G-E}$		$K - \frac{2G}{3}$	
$G =$			$\frac{3(K-\lambda)}{2}$		$\frac{\lambda(1-2\nu)}{2\nu}$
$\nu =$	$\frac{\lambda}{2(\lambda+G)}$	$\frac{E}{2G} - 1$	$\frac{\lambda}{3K-\lambda}$	$\frac{3K-2G}{2(3K+G)}$	
$M =$	$\lambda + 2G$	$\frac{G(4G-E)}{3G-E}$	$3K - 2\lambda$	$K + \frac{4G}{3}$	$\frac{\lambda(1-\nu)}{\nu}$

	(G, ν)	(E, ν)	(K, ν)	(K, E)	(M, G)
$K =$	$\frac{2G(1+\nu)}{3(1-2\nu)}$	$\frac{E}{3(1-2\nu)}$			$M - \frac{4G}{3}$
$E =$	$2G(1+\nu)$		$3K(1-2\nu)$		$\frac{G(3M-4G)}{M-G}$
$\lambda =$	$\frac{2G\nu}{1-2\nu}$	$\frac{E\nu}{(1+\nu)(1-2\nu)}$	$\frac{3K\nu}{1+\nu}$	$\frac{3K(3K-E)}{9K-E}$	$M - 2G$
$G =$		$\frac{E}{2(1+\nu)}$	$\frac{3K(1-2\nu)}{2(1+\nu)}$	$\frac{3KE}{9K-E}$	
$\nu =$				$\frac{3K-E}{6K}$	$\frac{M-2G}{2M-2G}$
$M =$	$\frac{2G(1-\nu)}{1-2\nu}$	$\frac{E(1-\nu)}{(1+\nu)(1-2\nu)}$	$\frac{3K(1-\nu)}{1+\nu}$	$\frac{3K(3K+E)}{9K-E}$	

http://en.wikipedia.org/wiki/Bulk_modulus

Bulk modulus (K) • Young's modulus (E) • Lamé's first parameter (λ) •

Shear modulus (G) • Poisson's ratio (ν) • P-wave modulus (M)

Plane Strain

Thursday, March 11, 2010
11:43 AM

Nonzero **stress**: $\sigma_x, \sigma_y, \sigma_z, \tau_{xy}$

Nonzero **strain** components: $\varepsilon_x, \varepsilon_y, \gamma_{xy}$

Isotropic linear elastic stress-strain law $\boxed{\underline{\sigma} = \underline{D}\,\underline{\varepsilon}}$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

$$\sigma_z = \nu\left(\sigma_x + \sigma_y\right)$$

Note that for the plane strain case the normal stress in the z direction is not zero. However, since this stress is balanced, it produces no strain in this direction.

Hence, the D matrix for the **plane strain case** is

$$\underline{D} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}$$

Plane Strain

Thursday, March 11, 2010
11:43 AM

Strains for Plane Strain Case

$$\varepsilon_x = \frac{\sigma_x}{E} - \upsilon\frac{\sigma_y}{E} - \upsilon\frac{\sigma_z}{E}$$
$$\varepsilon_y = \frac{\sigma_y}{E} - \upsilon\frac{\sigma_x}{E} - \upsilon\frac{\sigma_z}{E}$$

$$0 = \frac{\sigma_z}{E} - \upsilon\frac{\sigma_x}{E} - \upsilon\frac{\sigma_y}{E}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

Stresses for Plane Strain Case

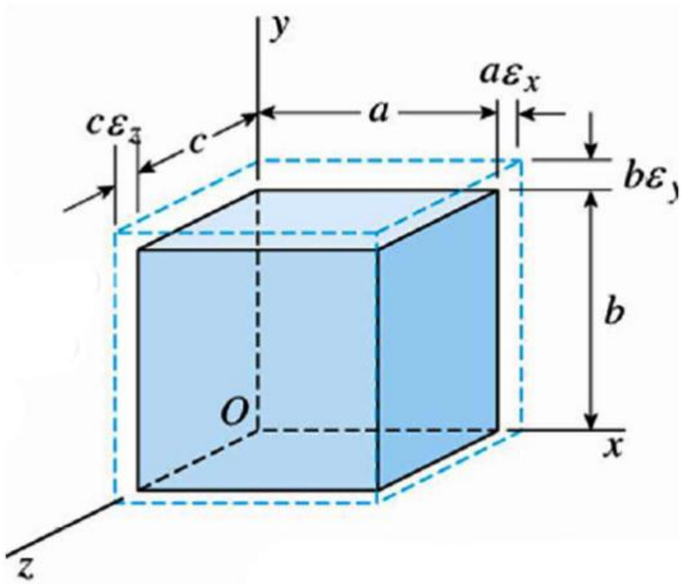
$$\sigma_x = \frac{E}{(1+\upsilon)(1-2\upsilon)}\left[(1-\upsilon)\varepsilon_x + \upsilon\varepsilon_y\right]$$
$$\sigma_y = \frac{E}{(1+\upsilon)(1-2\upsilon)}\left[(1-\upsilon)\varepsilon_y + \upsilon\varepsilon_x\right]$$

$$\sigma_z = \frac{E}{(1+\upsilon)(1-2\upsilon)}\left[\upsilon(\varepsilon_x + \varepsilon_y)\right]$$

$$\tau_{xy} = G\gamma_{xy}$$

Volumetric Change

Thursday, March 11, 2010
11:43 AM



Original Volume

$$V_o = abc$$

Final Volume

$$V_1 = (a + a\varepsilon_x)(b + b\varepsilon_y)(c + c\varepsilon_z)$$
$$= abc(1+\varepsilon_x)(1+\varepsilon_y)(1+\varepsilon_z)$$
$$\approx V_o(1+\varepsilon_x+\varepsilon_y+\varepsilon_z)$$

Ignoring square terms of strain, since they are small

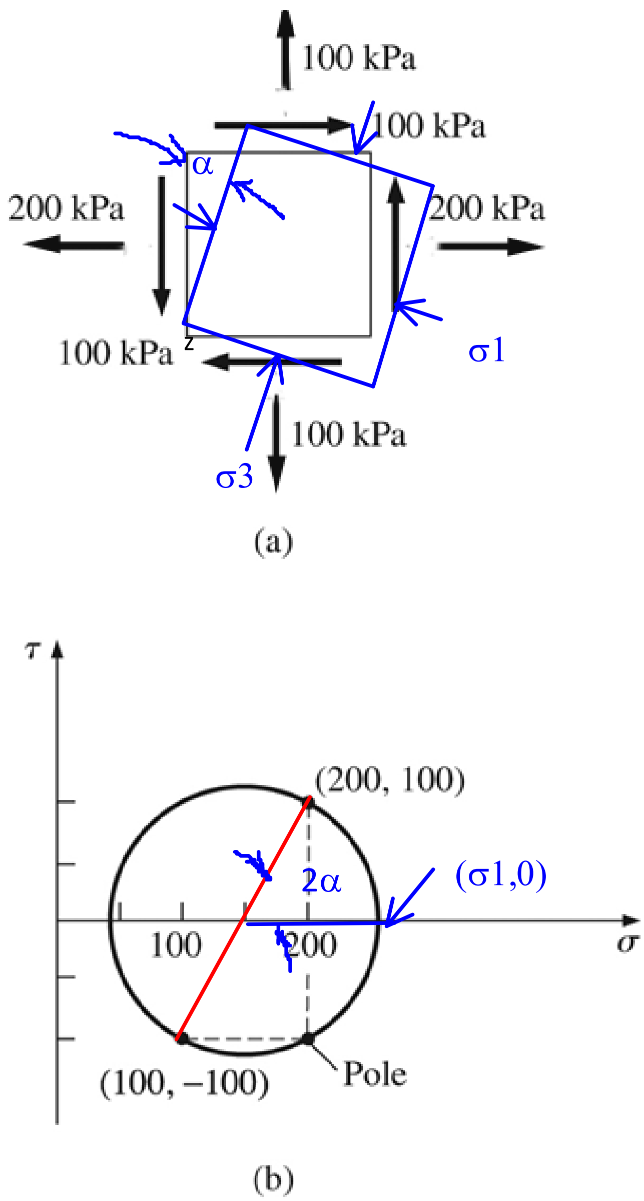
Volume change: $\Delta V = V_1 - V_o = V_o(\varepsilon_x + \varepsilon_y + \varepsilon_z)$

Dilation = change in unit volume

$$e = \frac{\Delta V}{V_o} = \varepsilon_x + \varepsilon_y + \varepsilon_z$$
$$= \frac{1-2\upsilon}{E}\left(\sigma_x + \sigma_y + \sigma_z\right)$$

Major and Minor Principal Stresses

Tuesday, August 28, 2012
12:45 PM



Note:

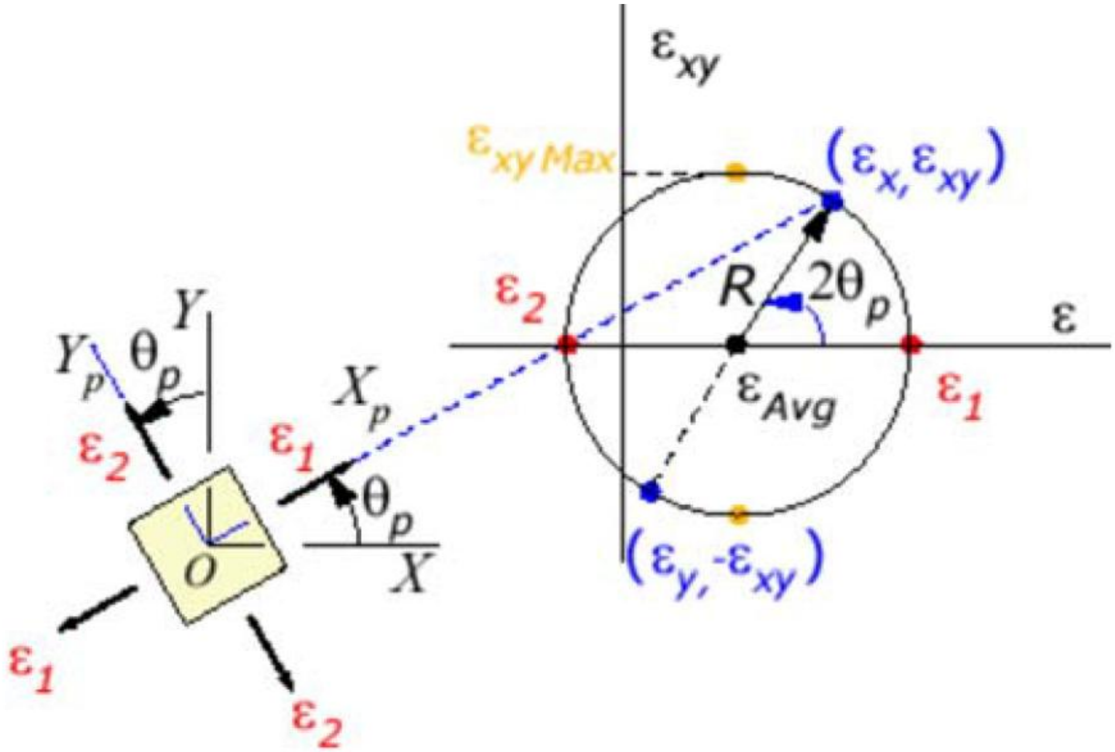
Normal stress in tension has been shown as positive

Note that the major principal stress, σ1, has the largest value of normal stress and the shear stress is zero. The plane upon which this stress acts is called the major principal plane.

The minor principal stress, σ3, has the smallest value of the normal stress and the shear stress is also zero. The plane upon which this stress acts is called the minor principal plane

Mohr's Circle of Strain

Thursday, March 11, 2010
11:43 AM



ε_1, ε_2 are major and minor principal strains, respectively

θ_p is the angle to the major principal strain

ε_{xy} is the strain tensor which is equal to $\gamma_{xy}/2$ when using the Mohr's circle of strain.

Principal Directions, Principal Strain

Tuesday, August 28, 2012
12:43 PM

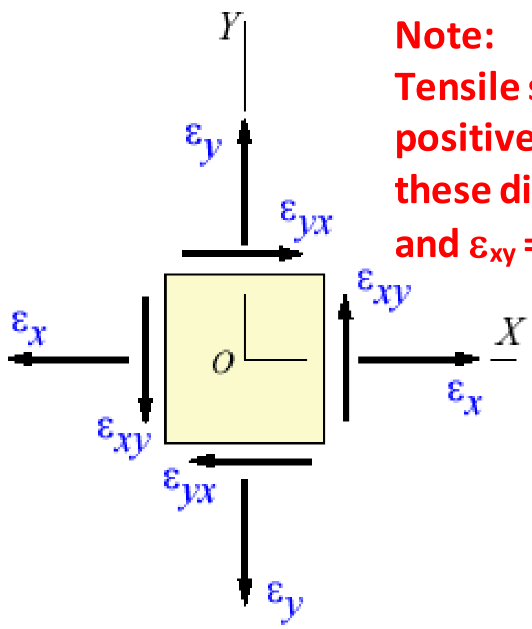
The normal strains ($\varepsilon_{x'}$ and $\varepsilon_{y'}$) and the shear strain ($\varepsilon_{x'y'}$) vary smoothly with respect to the rotation angle θ, in accordance with the transformation equations given above. There exist a couple of particular angles where the strains take on special values.
First, there exists an angle θ_p where the shear strain $\varepsilon_{x'y'}$ vanishes. That angle is given by:

$$\tan 2\theta_P = \gamma_{xy} / (\varepsilon_x - \varepsilon_y)$$

This angle defines the *principal directions*. The associated *principal strains* are given by,

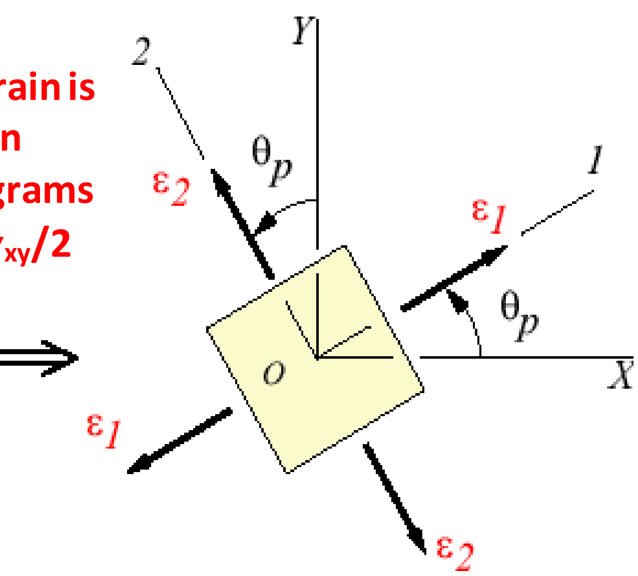
$$\varepsilon_{1,2} = ((\varepsilon_x + \varepsilon_y)/2) \pm [((\varepsilon_x - \varepsilon_y)/2)^2 + (\gamma_{xy}/2)^2]^{0.5}$$

The transformation to the principal directions with their principal strains can be illustrated as:



Strains in given coordinate system

Note:
Tensile strain is positive on these diagrams and $\varepsilon_{xy} = \gamma_{xy}/2$



Strains transformed to principal directions

Maximum Shear Stress Direction

Tuesday, August 28, 2012
12:43 PM

An important angle, θ_s, is where the maximum shear strain occurs and is given by:

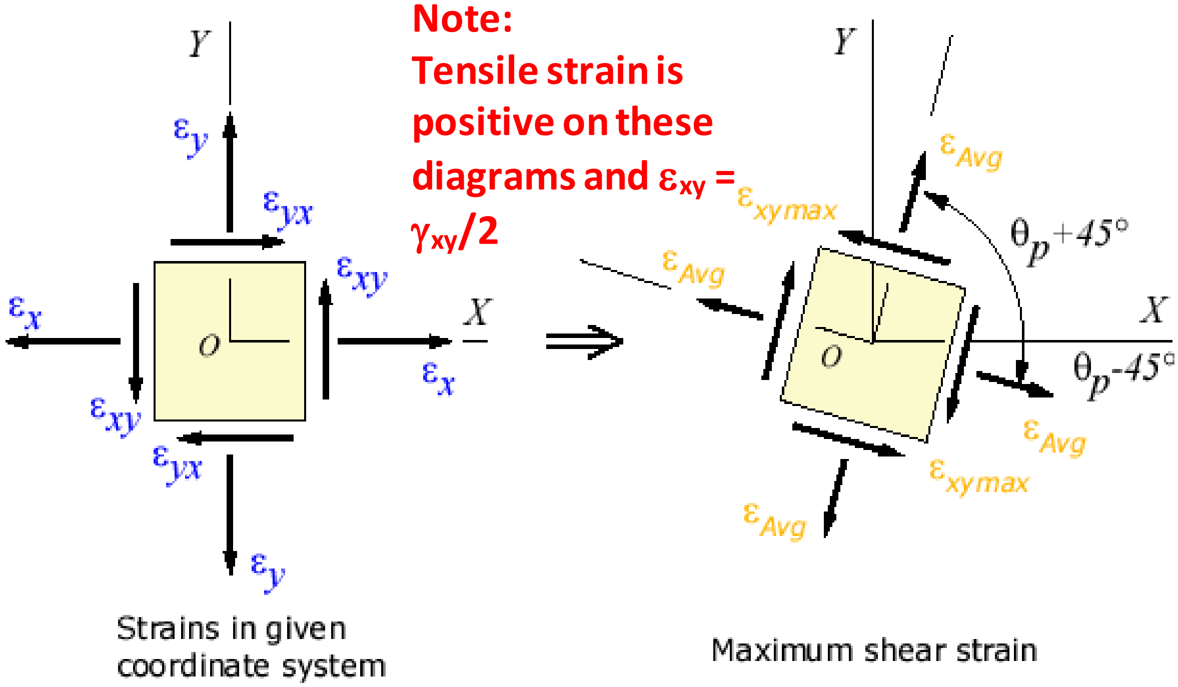
$\tan 2\theta_s = - (\varepsilon_x - \varepsilon_y)/\gamma_{xy}$

The maximum shear strain is found to be one-half the difference between the two principal strains:

$\gamma_{max}/2 = [((\varepsilon_x - \varepsilon_y)/2)^2 + (\gamma_{xy}/2)^2]^{0.5} = (\varepsilon_1 - \varepsilon_2)/2$

The transformation to the maximum shear strain direction can be illustrated as:

Note: Tensile strain is positive on these diagrams and $\varepsilon_{xy} = \gamma_{xy}/2$



Pasted from <http://www.efunda.com/formulae/solid_mechanics/mat_mechanics/plane_strain_principal.cfm>

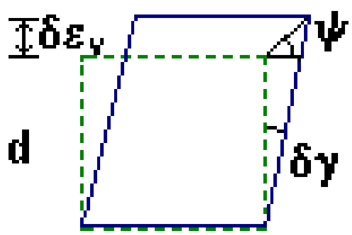
Dilation Angle - Plastic Strain

Tuesday, August 28, 2012
12:43 PM

The Mohr–Coulomb yield surface is often used to model the plastic flow of geomaterials (and other cohesive-frictional materials). Many such materials show dilatational behavior under triaxial states of stress which the Mohr–Coulomb model does not include. Also, since the yield surface has corners, it may be inconvenient to use the original Mohr–Coulomb model to determine the direction of plastic flow (in the flow theory of plasticity).

Pasted from <http://en.wikipedia.org/wiki/Mohr%E2%80%93Coulomb_theory>

Definition of Dilation Angle for a Unit Cube



$$\tan\psi = \frac{\delta\varepsilon_v}{\delta\gamma}$$

Obtaining the Dilation Angle from a Triaxial Test

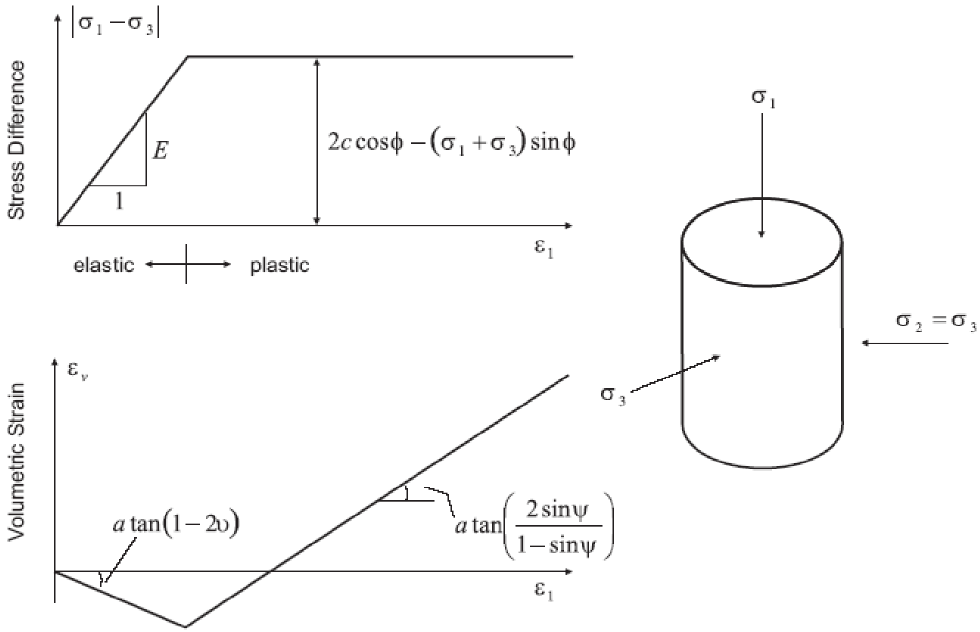


Figure 3.58 Idealized relation for dilation angle, ψ, from triaxial test results [Vermeer and de Borst (1984)]

More Reading

Thursday, March 11, 2010
11:43 AM

Reading

- Applied Soil Mechanics, Ch. 2.0 to 2.3
- The Engineering of Foundations Ch. 4.1 to 4.2
- Geotechnical Earthquake Engineering Ch. 5.2.2 to 5.2.2.3

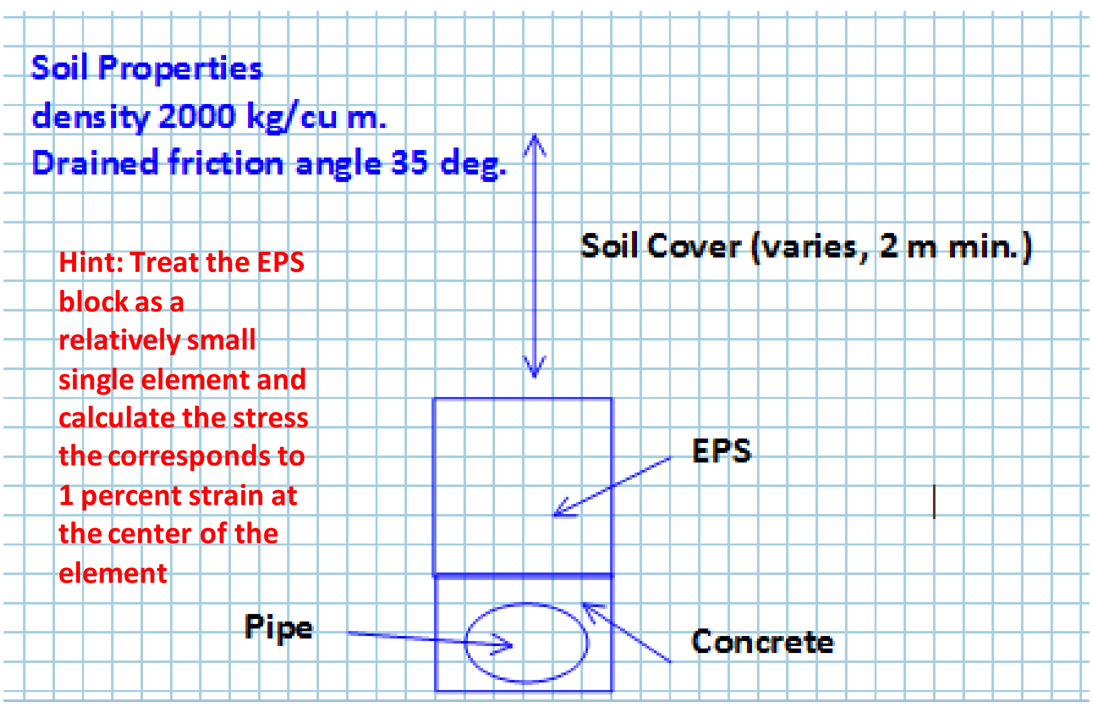
Assignment 2

Thursday, March 11, 2010
11:43 AM

1. A 0.3-m cube of Expanded Polystyrene (EPS) geofoam is subjected to the state of stress given below. Calculate the axial strains in the x, y and z directions that corresponding to this state of stress for the properties below. (10 points)
   - E = 5 Mpa
   - Density = 20 kg/m^3
   - Poison's ratio = 0.1
   - σ_{xx} = 60 Kpa (compression)
   - σ_{yy} = 30 Kpa (compression)
   - σ_{zz} = 30 Kpa (compression)

2. For the information given in problem 1, calculate the volumetric strain of the EPS cube. (5 points)

3. A roadway embankment is planned where EPS will be used to protect a buried culvert. Using elastic theory, approximate the maximum cover for plane strain conditions that will limit the EPS vertical strain to 1 percent axial strain. (30 points)



Assignment 2 (cont)

Thursday, March 11, 2010
11:43 AM

4. Develop a simple 6 x 6 FDM grid for a unit cube of EPS using the properties given in the previous problems. Use the FLAC model to estimate the axial strains in the y and x-directions for the loading conditions used in problem 3. Compare the axial strain in the y-direction with that calculated in problem 3. (30 points)

5. The normal strain in the x-direction is 1.0 percent, the normal strain in the y-direction is 0.5 percent and the shear strain, γ, in the x-y plane is 0.5 percent. From this information, calculate the following: (10 points)
   a. Maximum normal strain, ε_1
   b. Minimum normal strain, ε_2
   c. Principal angle, θ_p
   d. Maximum shear strain, γ_{max}
   e. Maximum shear strain angle, θ_s

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Thursday, March 11, 2010
11:43 AM

Finite Difference Method

Thursday, March 11, 2010
11:43 AM

Steps

1. Generate a grid for the domain where we want an approximate solution.
2. Assign material properties
3. Assign boundary/loading conditions
4. Use the finite difference equations as a substitute for the ODE/PDE system of equations. The ODE/PDE, thus substituted, becomes a linear or non-linear system of algebraic equations.
5. Solve for the system of algebraic equations using the initial conditions and the boundary conditions. This usually done by time stepping in an explicit formulation.
6. Implement the solution in computer code to perform the calculations.

Grid Generation

Thursday, March 11, 2010
11:43 AM

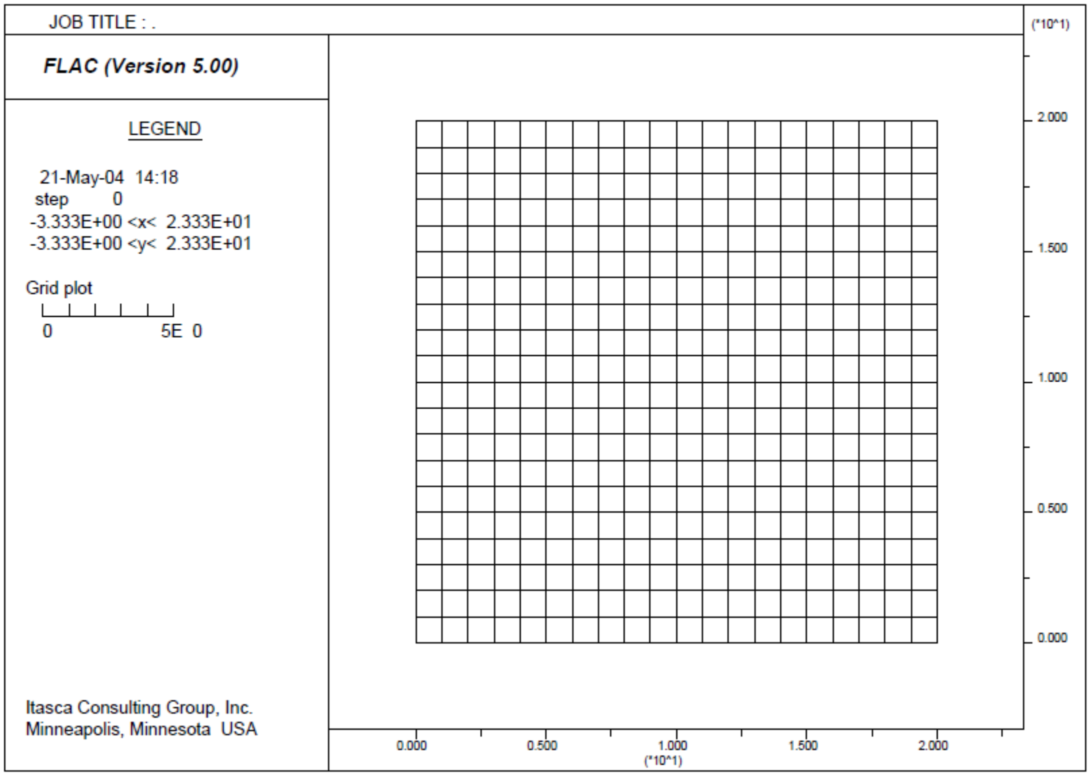
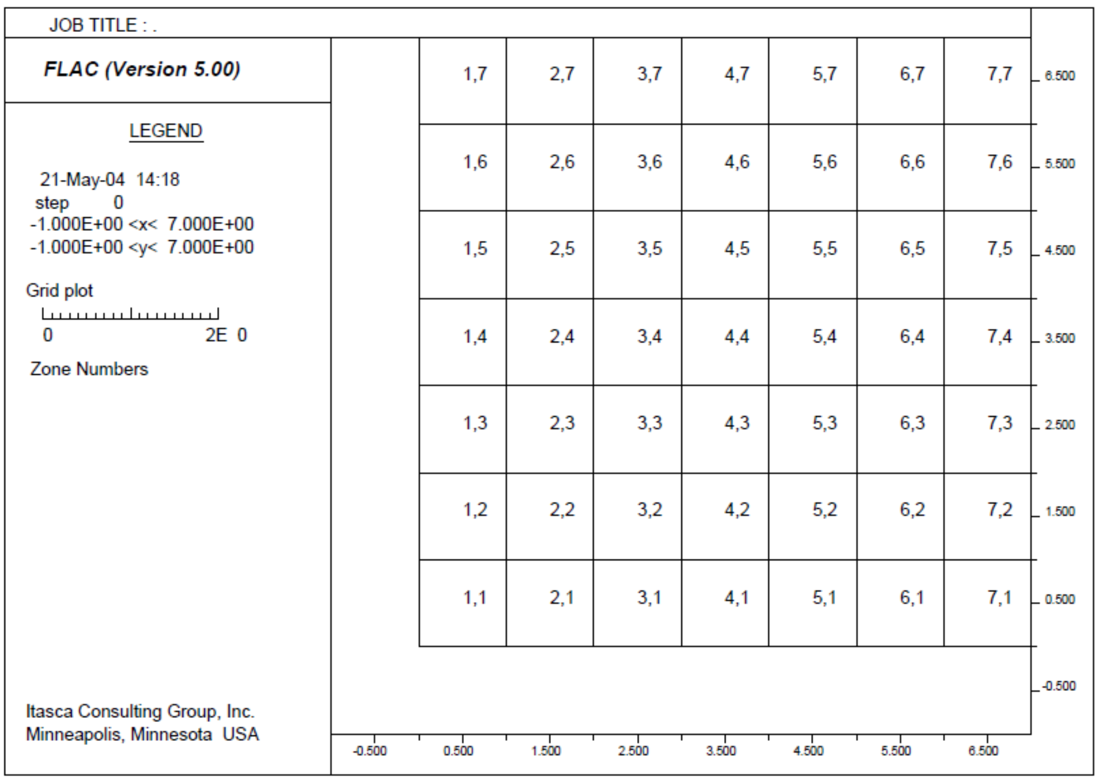


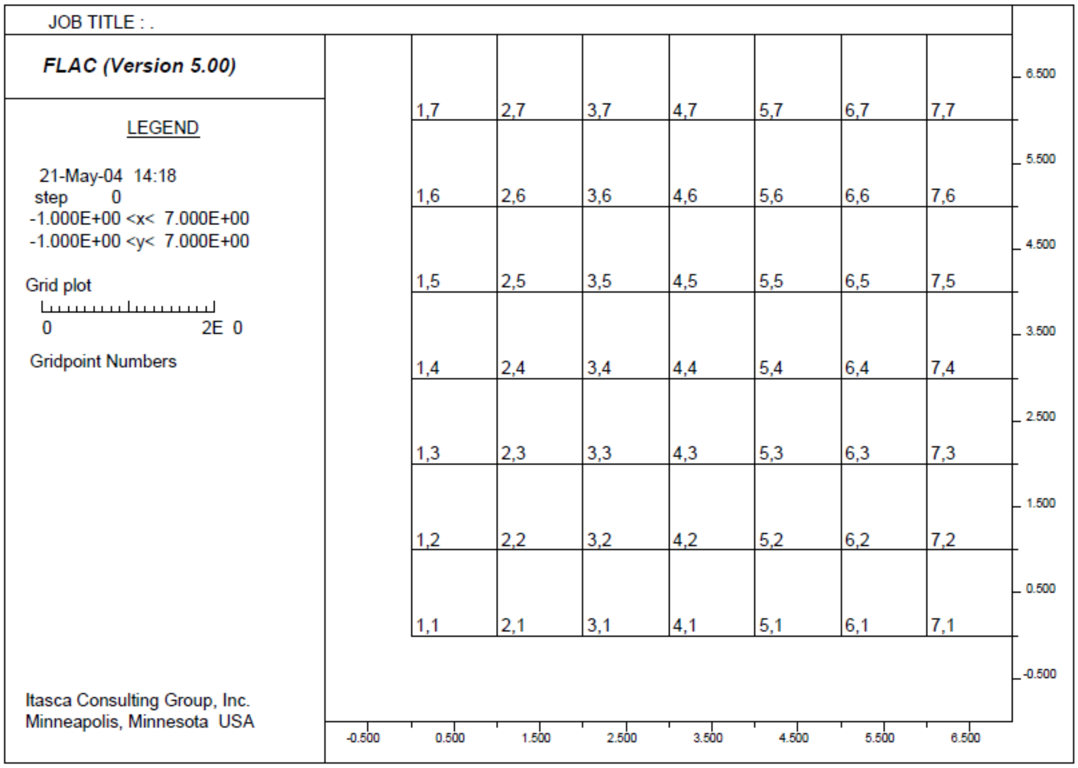
Figure 2.37 Finite difference grid with 400 zones



(a) zone numbers

Grid Generation (continued)

Thursday, March 11, 2010
11:43 AM

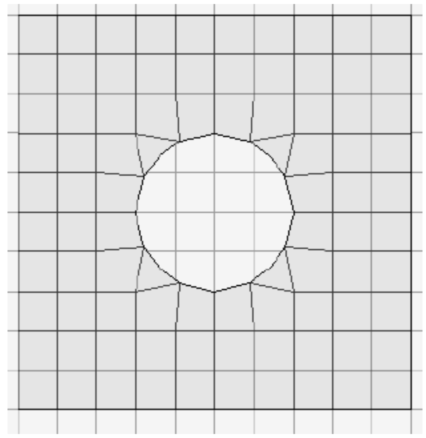


(b) gridpoint numbers

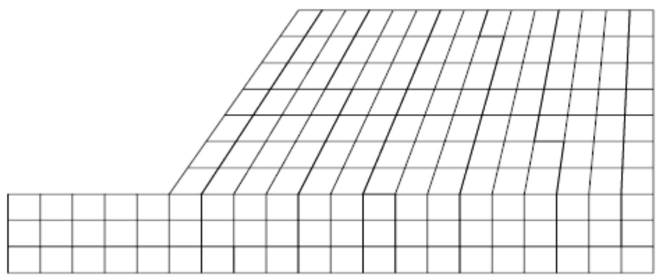
The finite difference grid also identifies the storage location of all state variables in the model. The procedure followed by *FLAC* is that all vector quantities (e.g.. forces. velocities. displacements. flow rates) are stored at gridpoint locations. while all scalar and tensor quantities (e.g.. stresses. pressure. material properties) are stored at zone centroid locations. There are three exceptions: saturation and temperature are considered gridpoint variables: and pore pressure is stored at both gridpoint and zone centroid locations.

Irregular Grids

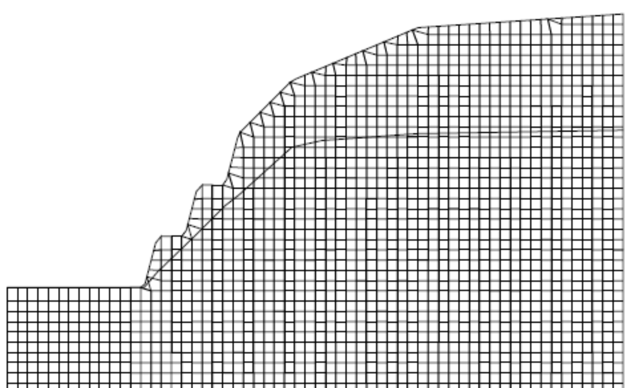
Thursday, March 11, 2010
11:43 AM



Tunnel



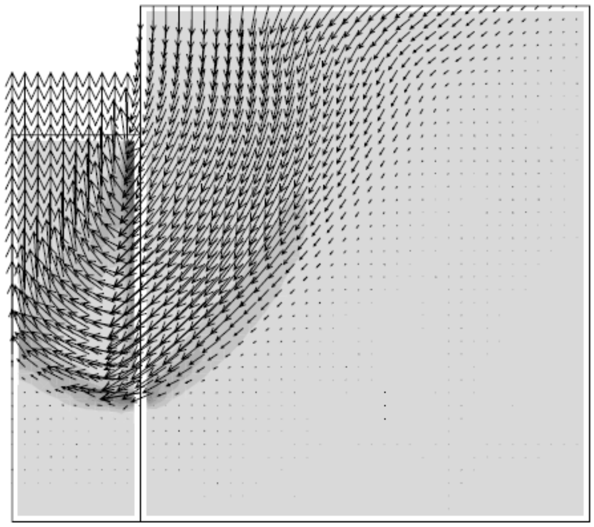
Slope or Embankment



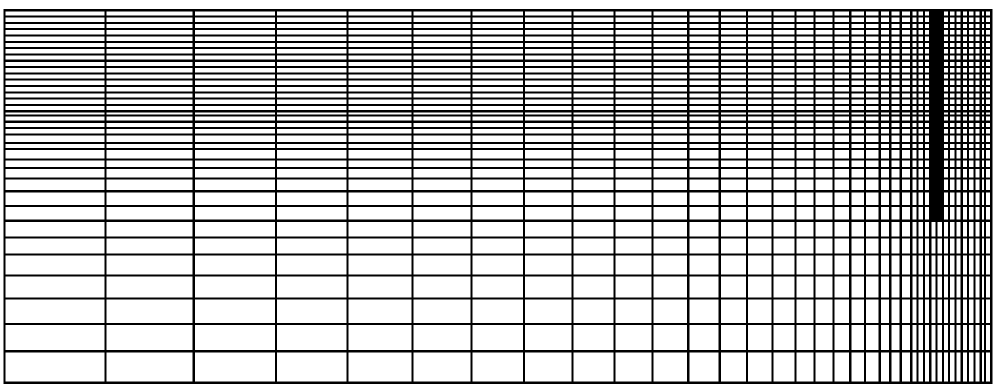
Rock Slope with groundwater

Irregular grids (cont.)

Thursday, March 11, 2010
11:43 AM



Braced Excavation



Concrete Diaphragm Wall

Material Properties

Tuesday, August 28, 2012
12:43 PM

Elastic and Mohr Coulomb Models

- Density
- Bulk Modulus
- Shear Modulus
- Cohesion (MC only)
- Tension (MC only)
- Drained Friction Angle (MC only)
- Dilation Angle (MC only)

Hyperbolic Model

$$\sigma_d = \frac{\epsilon_1}{\frac{1}{E_i} + \frac{\epsilon_1}{Y}}$$

Functional Form of Hyperbolic Model

where: $\sigma_d = |\sigma_1 - \sigma_3|$;
ϵ_1 = axial strain;
Y = maximum value of $|\sigma_1 - \sigma_3|$; and
E_i = initial Young's modulus (at $\sigma_d = 0$)

The equation can be differentiated to obtain the slope of the stress/strain curve:

$$\frac{d\sigma_d}{d\epsilon_1} = E = \frac{E_i\,(Y - \sigma_d)^2}{Y^2}$$

Required Input for Hyperbolic Model

`b_mod`	K = bulk modulus
`y_initial`	E_i = initial Young's modulus
`yield`	$Y = (\sigma_1 - \sigma_3)_{max}$

Units for FLAC

Thursday, March 11, 2010
11:43 AM

FLAC accepts any consistent set of engineering units. Examples of consistent sets of units for basic parameters are shown in Tables 2.5. 2.6 and 2.7. The user should apply great care when converting from one system of units to another. No conversions are performed in *FLAC* except for friction and dilation angles. which are entered in degrees.

Table 2.5 Systems of units — mechanical parameters

	SI				Imperial	
Length	m	m	m	cm	ft	in
Density	kg / m^3	10^3 kg / m^3	10^6 kg / m^3	10^6 g / cm^3	slugs / ft^3	snails / in^3
Force	N	kN	MN	Mdynes	lb$_f$	lb$_f$
Stress	Pa	kPa	MPa	bar	lb$_f$ / ft^2	psi
Gravity	m / sec^2	m / sec^2	m / sec^2	cm / s^2	ft / sec^2	in / sec^2
Stiffness*	Pa / m	kPa / m	MPa / m	bar / cm	lb$_f$ / ft^3	lb / in^3

Table 2.6 Systems of units — groundwater flow parameters

	SI		Imperial	
Water Bulk Modulus	Pa	bar	lbf/ft^2	psi
Water Density	kg / m^3	10^6 g / cm^3	slugs / ft^3	snails / in^3
Permeability	m^{3}sec / kg	10^{-6} cm sec / g	ft^3 sec / slug	in^3 sec / snail
Intrinsic Permeability	m^2	cm^2	ft^2	in^2
Hydraulic Conductivity	m / sec	cm / sec	ft / sec	in / sec

Table 2.7 Systems of units — structural elements

Property	Unit	SI				Imperial	
area	length^2	m^2	m^2	m^2	cm^2	ft^2	in^2
axial or shear stiffness	force/disp	N/m	kN/m	MN/m	Mdynes/cm	lb$_f$/ft	lb$_f$/in
bond stiffness	force/length/disp	N/m/m	kN/m/m	MN/m/m	Mdynes/cm/cm	lb$_f$/ft/ft	lb$_f$/in/in
bond strength	force/length	N/m	kN/m	MN/m	Mdynes/cm	lb$_f$/ft	lb$_f$/in
exposed perimeter	length	m	m	m	cm	ft	in
moment of inertia	length^4	m^4	m^4	m^4	cm^4	ft^4	in^4
plastic moment	force-length	N-m	kN-m	MN-m	Mdynes-cm	ft-lb$_f$	in-lb$_f$
yield strength	force	N	kN	MN	Mdynes	lb$_f$	lb$_f$
Young's modulus	stress	Pa	kPa	MPa	bar	lb$_f$/ft^2	psi

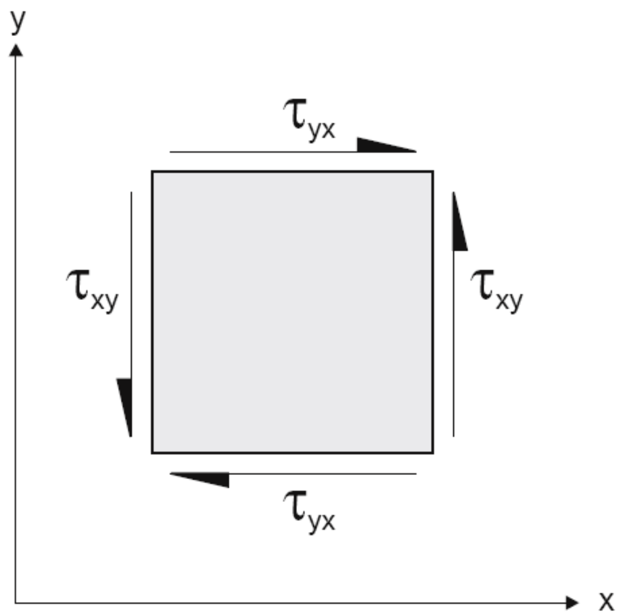
Sign Conventions for FLAC

Thursday, March 11, 2010
11:43 AM

Normal or direct stress

- **Positive = tension**
- **Negative = compression**

Shear stress



With reference to the above figure, a positive shear stress points in the positive direction of the coordinate axis of the second subscript if it acts on a surface with an outward normal in the positive direction. Conversely, if the outward normal of the surface is in the negative direction, then the positive shear stress points in the negative direction of the coordinate axis of the second subscript. The shear stresses shown in the above figure are all positive (from FLAC manual).

In other words, τ_{xy} is positive in the counter-clockwise direction; likewise τ_{yx} is positive in the clockwise direction.

Sign Conventions (cont.)

Thursday, March 11, 2010
11:43 AM

DIRECT OR NORMAL STRAIN

- Positive strain indicates extension: negative strain indicates compression.

SHEAR STRAIN

- Shear strain follows the convention of shear stress (see figure above). The distortion associated with positive and negative shear strain is illustrated in Figure 2.44.

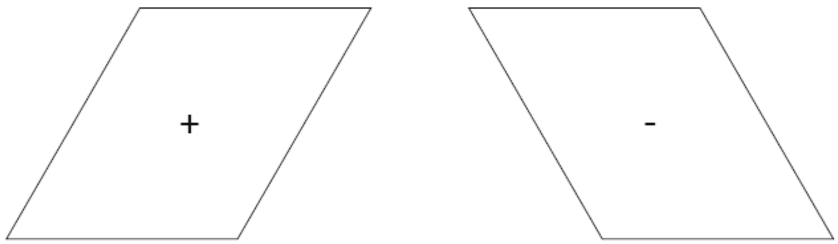


Figure 2.44 Distortion associated with positive and negative shear strain

PRESSURE

- *A* positive pressure will act normal to. and in a direction toward. the surface of a body (i.e.. push), A negative pressure will act normal to. and in a direction away from. the surface of a body (i.e.. pull). Figure 2.45 illustrates this convention.

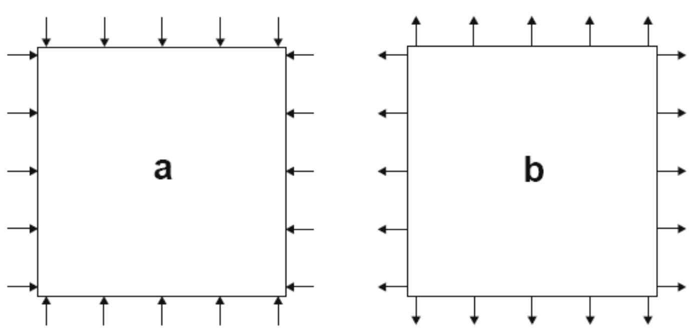


Figure 2.45 Mechanical pressure: (a) positive; (b) negative

Sign Conventions (cont.)

Thursday, March 11, 2010
11:43 AM

PORE PRESSURE

- Fluid pore pressure is positive in compression. Negative pore pressure indicates *fluid* tension.

GRAVITY

- Positive gravity will pull the mass of a body downward (in the negative y-direction). Negative gravity will pull the mass of a body upward.

GFLOW

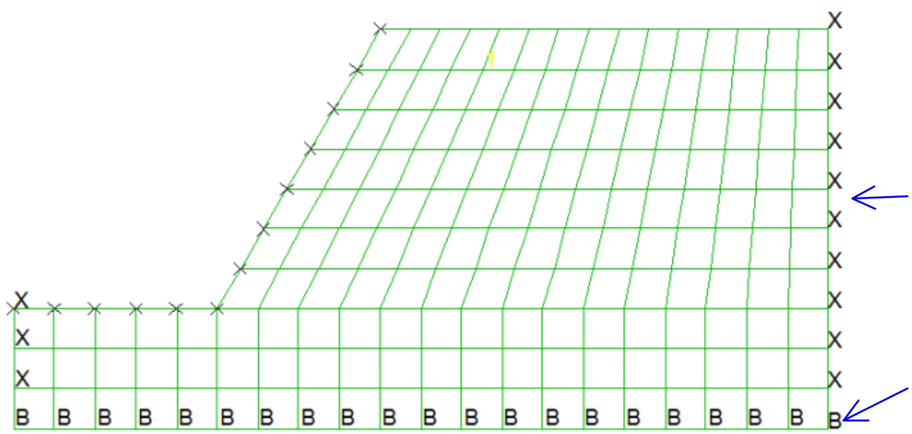
- This is a *FISH* parameter (see Section 2 in the *FISH* volume which denotes the net fluid flow associated with a gridpoint. A positive gflow corresponds to flow into a gridpoint. Conversely, a negative gflow corresponds to flow out of a gridpoint.

Boundary Conditions

Thursday, March 11, 2010
11:43 AM

Boundary Conditions

- Fixed (X or Y) or both (B)
- Free

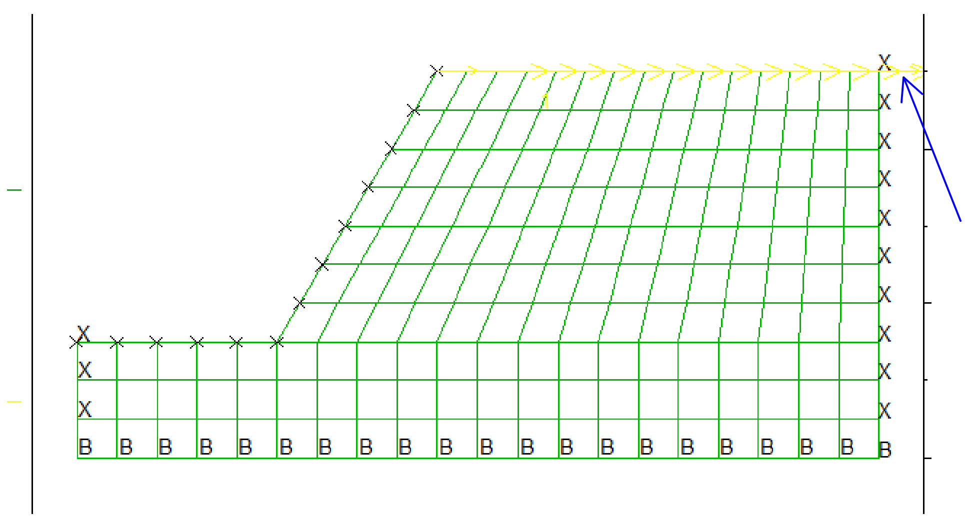


X means fixed in x direction

B means fixed in both directions

Applied Conditions at Boundary

- Velocity or displacement
- Stress or force



Yellow line with circle means force, velocity or stress has been applied to this surface.

Fundamentals of FDM

Thursday, March 11, 2010
11:43 AM

Finite-difference methods approximate the solutions to differential equations by replacing derivative expressions with approximately equivalent difference quotients. That is, because the first derivative of a function f is, by definition,

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h},$$

then a reasonable approximation for that derivative would be to take

$$f'(a) \approx \frac{f(a+h) - f(a)}{h}$$

for some small value of h. In fact, this is the forward difference equation for the first derivative. Using this and similar formulae to replace derivative expressions in differential equations, one can approximate their solutions without the need for calculus

Pasted from <http://en.wikipedia.org/wiki/Finite_difference_method>

Only three forms are commonly considered: forward, backward, and central differences.

A forward difference is an expression of the form

$$\Delta_h[f](x) = f(x+h) - f(x).$$

Depending on the application, the spacing *h* may be variable or held constant.

A backward difference uses the function values at *x* and *x* − *h*, instead of the values at *x* + *h* and *x*:

$$\nabla_h[f](x) = f(x) - f(x-h).$$

Finally, the central difference is given by

$$\delta_h[f](x) = f(x + \tfrac{1}{2}h) - f(x - \tfrac{1}{2}h).$$

Pasted from <http://en.wikipedia.org/wiki/Forward_difference>

Fundamentals of FDM (cont.)

Thursday, March 11, 2010
11:43 AM

Higher-order differences

2nd Order Derivative

In an analogous way one can obtain finite difference approximations to higher order derivatives and differential operators. For example, by using the above central difference formula for $f'(x + h / 2)$ and $f'(x - h / 2)$ and applying a central difference formula for the derivative of f' at x, we obtain the central difference approximation of the second derivative of f:

$$f''(x) \approx \frac{\delta_h^2[f](x)}{h^2} = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}.$$

Pasted from <http://en.wikipedia.org/wiki/Finite_difference>

Examples of 2nd Order Differential Equations

$$\frac{\partial h}{\partial t} = \alpha \left[\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} \right] - G.$$

Groundwater flow equation where h is this equation is head.

Pasted from <http://en.wikipedia.org/wiki/Groundwater_flow_equation>

$$\frac{\partial^2 v}{\partial t^2} = c^2 \left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right]$$

2D wave equation where ν In this equation is position.

Pasted from <https://ccrma.stanford.edu/~jos/pasp/D_Mesh_Wave.html>

Fundamentals of FDM - Explicit vs Implicit Methods

Thursday, March 11, 2010
11:43 AM

Explicit and implicit methods are approaches used in numerical analysis for obtaining numerical solutions of time-dependent ordinary and partial differential equations, as is required in computer simulations of physical processes such as groundwater flow and the wave equation.

Explicit methods calculate the state of a system at a later time from the state of the system at the current time, while implicit methods find a solution by solving an equation involving both the current state of the system and the later one. Mathematically, if $Y(t)$ is the current system state and $Y(t + \Delta t)$ is the state at the later time (Δt is a small time step), then, for an explicit method

$$Y(t + \Delta t) = F(Y(t))$$

while for an implicit method one solves an equation

$$G(Y(t), Y(t + \Delta t)) = 0 \qquad (1)$$

to find $Y(t + \Delta t)$.

It is clear that implicit methods require an extra computation (solving the above equation), and they can be much harder to implement. Implicit methods are used because many problems arising in real life are stiff, for which the use of an explicit method requires impractically small time steps Δt to keep the error in the result bounded (see numerical stability). For such problems, to achieve given accuracy, it takes much less computational time to use an implicit method with larger time steps, even taking into account that one needs to solve an equation of the form (1) at each time step. That said, whether one should use an explicit or implicit method depends upon the problem to be solved.

Pasted from <http://en.wikipedia.org/wiki/Explicit_method>

Explicit versus Implicit Formulation

Thursday, March 11, 2010
11:43 AM

The previous page contains explains the explicit method which is implemented in FLAC. The central concept of an explicit method is that the calculational "wave speed" always keeps ahead of the physical wave speed, so that the equations always operate on known values that are fixed for the duration of the calculation. There are several distinct advantages to this (and at least one big disadvantage!): most importantly, no iteration process is necessary. Computing stresses from strains in an element, even if the constitutive law is wildly nonlinear.

In an implicit method (which is commonly used in finite element programs), every element communicates with every other element during one solution step: several cycles of iteration are necessary before compatibility and equilibrium are obtained.

Table 1.1 (next page) compares the explicit amid implicit methods. The disadvantage of the explicit method is seen to be the small timestep, which means that large numbers of steps must be taken.

Overall, explicit methods are best for ill-behaved systems e.g., nonlinear, large—strain, physical instability; they are not efficient for modeling linear, small—strain problems.

Explicit versus Implicit Formulation (cont.)

Thursday, March 11, 2010
11:43 AM

Table 1.1 Comparison of Explicit versus Implicit Formulations

Explicit Method

- Timestep must be smaller than a critical value for stability
- Small amount of computational effort per timestep.
- No significant numerical damping introduced for dynamic solution
- No iterations necessary to follow nonlinear constitutive law.
- Provided that the timestep criterion is always satisfied, nonlinear laws are always followed in a valid physical way.
- Matrices are never formed.
- Memory requirements are always at a minimum. No bandwidth limitations. Since matrices are never formed large displacements and strains are accommodated without additional computing effort.

Implicit Method

- Timestep can be arbitrarily large with unconditionally stable schemes
- Large amount of computational effort per timestep.
- Numerical damping dependent on timestep present with unconditionally stable schemes.
- Iterative procedure necessary to follow nonlinear constitutive law.
- Always necessary to demonstrate that the above-mentioned procedure is: (a) stable: and (b) follows the physically correct path (for path-sensitive problems).
- Stiffness matrices must be stored. Ways must be found to overcome associated problems such as bandwidth.
- Memory requirements tend to be large.
- Additional computing effort needed to follow large displacements and strains.

Explicit Method for FLAC (Fast Lagrangian Analysis of Continua)

Thursday, March 11, 2010
11:43 AM

Explicit, Time-Marching Scheme

Even though we want FLAC to find a static solution to a problem, the dynamic equations of motion are included in the formulation. One reason for doing this is to ensure that the numerical scheme is stable when the physical system being modeled is unstable. With nonlinear materials, there is always the possibility of physical instability—e.g., the sudden collapse of a pillar. In real life, some of the strain energy in the system is converted into kinetic energy, which then radiates away from the source and dissipates. FLAC models this process directly, because inertial terms are included — kinetic energy is generated and dissipated. One penalty for including the full law of motion is that the user must have some physical feel for what is going on; FLAC is not a black box that will give "the solution." The behavior of the numerical system must be interpreted.

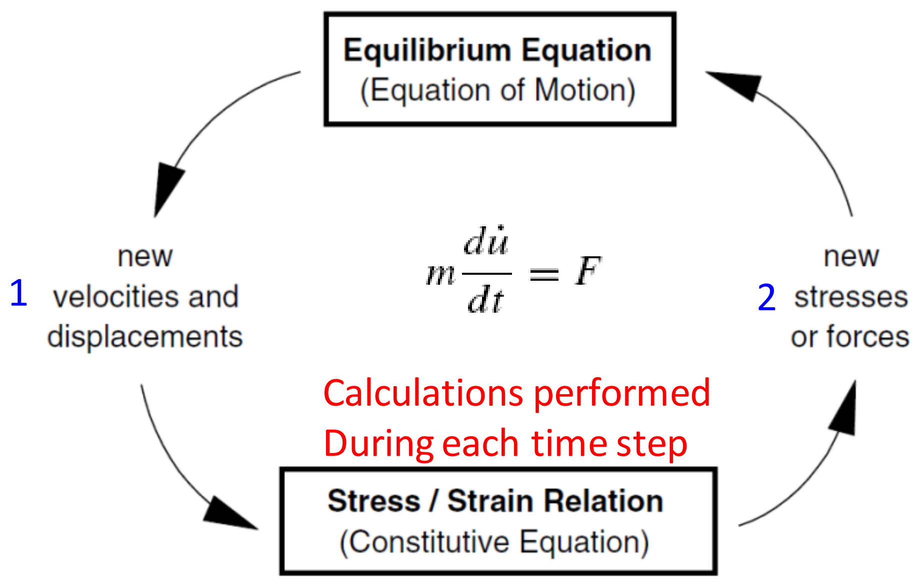


Figure 1.1 Basic explicit calculation cycle

Calculation cycle can begin with 1 or 2. These conditions are imposed on the model and the model timesteps until equlibrium is reached throughout the domain.

Lagrangian Analysis

Thursday, March 11, 2010
11:43 AM

Lagrangian analysis is the use of Lagrangian coordinates to analyze various problems in continuum mechanics. Such analysis may be used to analyze currents and flows of various materials by analyzing data collected from gauges/sensors embedded in the material which freely move with the motion of the material.[1] A common application is study of ocean currents in oceanography, where the movable gauges in question called Lagrangian drifters.

Pasted from <http://en.wikipedia.org/wiki/Lagrangian_analysis>

Example of Lagrangian analysis of golf club head striking ball. Note that the tracking and movement of the sand with the striking of the ball requires a Lagrangian analysis. (from ANSYS)

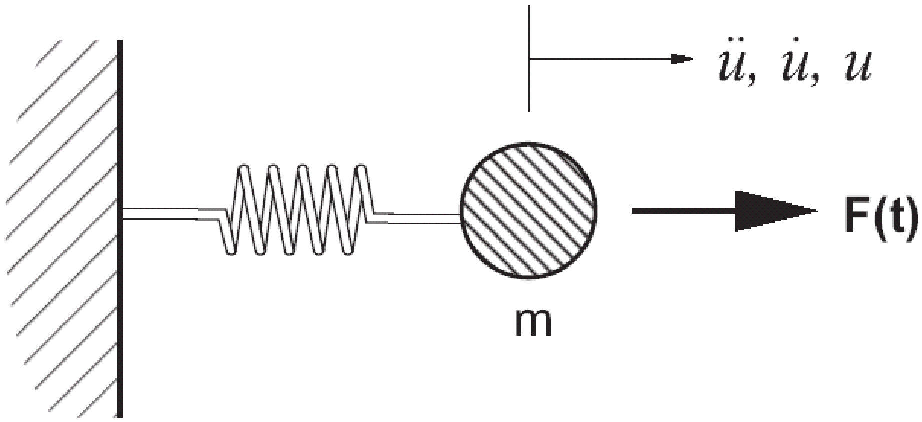
Pasted from <http://www.ansys.com/products/images/new-features-1.jpg>

Since FLAC using a Lagrangian method, it does not need to form a global stiffness matrix, thus it is a trivial matter to update coordinates at each timestep in large-strain mode. The incremental displacements are added to the coordinates so that the grid moves and deforms with the material it represents. This is termed a “Lagrangian” formulation. in contrast to an “Eulerian” formulation. in which the material moves and deforms relative to a fixed grid. The constitutive formulation at each step is a small—strain one. but is equivalent to a large-strain formulation over many steps.

See example (bin.prj) in the FLAC manual to see large deformation scheme used in the Lagrangian analysis.

Equation of Motion

Thursday, March 11, 2010
11:43 AM



$$m\frac{d\dot{u}}{dt} = F$$ **Eq. (1.1)**

In a continuous solid body, Eq. (1.1) is generalized as follows:

$$\rho\frac{\partial \dot{u}_i}{\partial t} = \frac{\partial \sigma_{ij}}{\partial x_j} + \rho g_i$$

where ρ = mass density;

t = time;

x_i = components of coordinate vector;

g_i = components of gravitational acceleration (body forces);

σ_{ij} = components of stress tensor.

Note that the above partial differential equation is a 2nd order partial differential equation because u dot is a derivative of u (displacement). This equation expresses dynamic force equilibrium which relates the inertial and gravitational forces to changes in stress. The above equation is also called the wave equation.

Constitutive Relations

Thursday, March 11, 2010
11:43 AM

The constitutive relation that is required in the PDE given before relates changes in stress with strain.

However, since FLAC's formulation is essentially a dynamic formulation, where changes in velocities are easily calculated, then strain rate is used and is related to velocity as shown below.

$$\dot{e}_{ij} = \frac{1}{2}\left[\frac{\partial \dot{u}_i}{\partial x_j} + \frac{\partial \dot{u}_j}{\partial x_i}\right]$$

where $\dot{e}_{ij}$ = strain-rate components; and

$\dot{u}_i$ = velocity components.

The mechanical constitutive law has the form:

$$\sigma_{ij} := M(\sigma_{ij},\ \dot{e}_{ij},\ \kappa)$$

where $M(\ )$ is the functional form of the constitutive law;

κ is a history parameter(s) which may or may not be present the particular law; and

$:=$ means "replaced by."

FDM - Elastic Example from FLAC manual

Thursday, March 11, 2010
11:43 AM

$$\sigma_{xx} = E\,\frac{\partial u_x}{\partial x}$$

Stress Strain Constitutive Law (Hooke's Law)

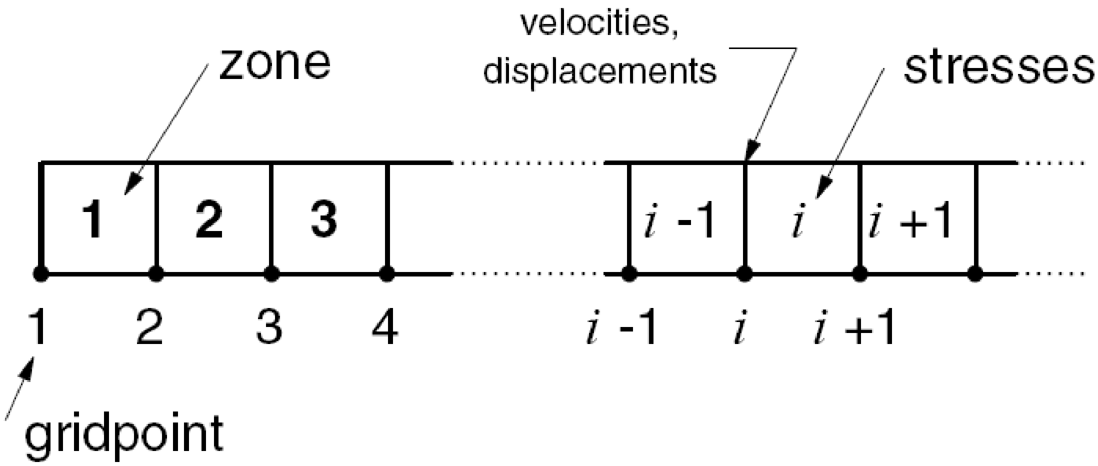
$$\rho\,\frac{\partial^2 u_x}{\partial t^2} = \frac{\partial \sigma_{xx}}{\partial x}$$

Equation of Motion for Dynamic Equilibrium (wave equation)

$$\sigma_{xx}^{i}(t) = E\,\frac{u_x^{i+1}(t) - u_x^{i}(t)}{\Delta x}$$

Eq. (1.2)
FDM formulation using central finite difference equation.

The central finite difference equation corresponding is for a typical zone i is given by the above equation. Here the quantities in parentheses — e.g.. (i) — denote the time, t, at which quantities are evaluated: the superscripts. *i,* denote the zone number, not that something is raised to a power.



Numbering scheme for a 1-D body using FDM.

FDM - Elastic Example (cont.)

Thursday, March 11, 2010
11:43 AM

$$\frac{\rho}{\Delta t}\left\{\dot{u}_x^i(t+\tfrac{\Delta t}{2}) - \dot{u}_x^i\,(t-\tfrac{\Delta t}{2})\right\} = \frac{1}{\Delta x}\left\{\sigma_{xx}^i(t) - \sigma_{xx}^{i-1}(t)\right\}$$

Finite difference equation for equation of motion using central finite difference equation. Note that on the left side of the equation a change in velocity (i.e., acceleration) is represented; on the right side of the equation a change in stress with respect to position is represented for the time step. In other words, an acceleration (unbalanced force) causes a change is the stress, or stress wave.

Rearrange the above equation, produces Eq. 1.3

$$\dot{u}_x^i\,(t+\tfrac{\Delta t}{2}) = \dot{u}_x^i\,(t-\tfrac{\Delta t}{2}) + \frac{\Delta t}{\rho\,\Delta x}\left\{\sigma_{xx}^i(t) - \sigma_{xx}^{i-1}(t)\right\}$$

Integrating this equation, produces displacements as shown in Eq. 1.4

$$u_x^i(t+\Delta t) = u_x^i(t) + \dot{u}_x^i\,(t+\tfrac{\Delta t}{2})\;\Delta t$$

This equation says that the position and time t + delta t is equal to the position and time t + (velocity at time t + 1/2 delta t) * delta t.

FDM - Elastic Example (cont.)

Thursday, March 11, 2010
11:43 AM

In the explicit method. the quantities on the right-hand sides of all difference equations are "known"; therefore. we must evaluate Eq. 1.2) for all zones before moving on to Eqs. (1.3) and (1.4). which are evaluated for all grid points. Conceptually. this process is equivalent to a *simultaneous* update of variables.

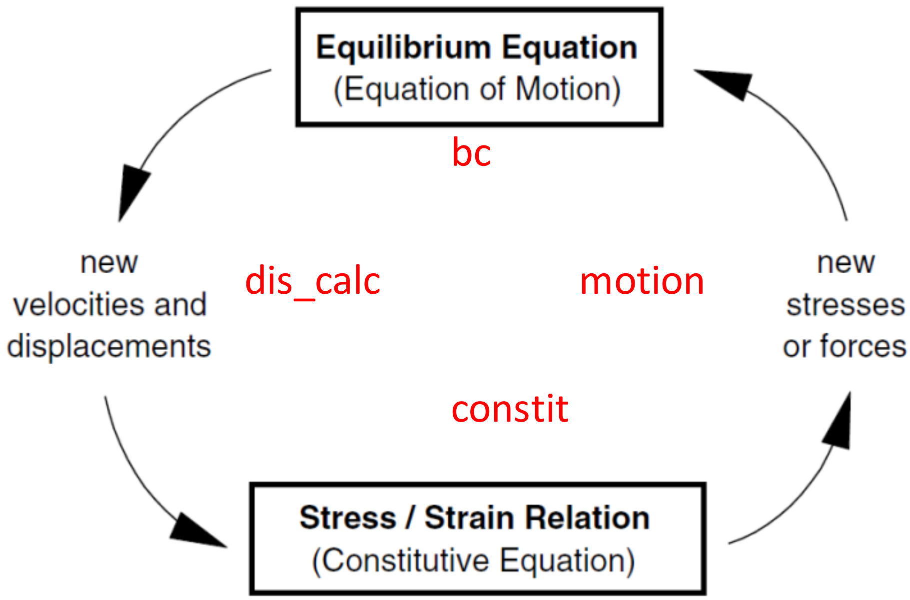


Figure 1.1 Basic explicit calculation cycle

bc	velocity pulse applied to boundary condition
dis_calc	displacements from velocity
constit	stresses are derived from strain
motion	velocity calculated stress

FDM - Elastic Example (cont.)

Thursday, March 11, 2010
11:43 AM

The following is an example of implementing the FDM for to calculate the behavior of an elastic bar. To do this, we must write FISH code. The primary subroutine, scan_all, and the other routines described in the following pages can be obtained from bar.dat in the Itasca folder.

```
def scan_all
 while_stepping
 time = time + dt
 bc                    ; pulse applied to boundary condition
 dis_calc              ; displacements calculated from velocity
 constit               ; stresses are derived from strain
 motion                ; velocity calculated stress
end
```

The subroutine, bc, applies a one-sided cosine velocity pulse to the left end of the rod.

```
def bc ; boundary conditions - cosine pulse applied to left end
 if time >= twave then
  xvel(1,1) = 0.0
 else
  xvel(1,1) = vmax * 0.5 * (1.0 - cos(w * time))
 end_if
End
```

The subroutine, dis_calc, calculates the displacements from the velocities.

```
def dis_calc
 loop i (1,nel)
    xdisp(i,1) = xdisp(i,1) + xvel(i,1) * dt
 end_loop
end
```

FDM - Elastic Example (cont.)

Thursday, March 11, 2010
11:43 AM

The subroutine, called constit, calculates the stress as derived from strain using Hooke's law. The value of e is Young's modulus.

```
def constit
  loop i (1,nel)
    sxx(i,1) = e * (xdisp(i+1,1) - xdisp(i,1)) / dx
  end_loop
end
```

This subroutine, called motion, calculates the new velocity from stress. Recall that an unbalanced stress causes an unbalanced force, which in turn produces an acceleration which is a change in velocity.

```
def motion
  loop i (2,nel)
    xvel(i,1) = xvel(i,1) + (sxx(i,1) - sxx(i-1,1)) * tdx
  end_loop
end
```

FDM - Elastic Example (cont.)

Thursday, March 11, 2010
11:43 AM

Table 1.2 Variables defined in start-up

FISH name	Name within equations	Meaning
nel		number of elements
e	E	Young's modulus
ro	ρ	density
dx	Δx	element size
p		number of wavelengths per element
vmax		amplitude of velocity pulse
frac		fraction of critical timestep
c	c	wave speed
dt	Δt	timestep
twave		duration of input pulse
freq	f	frequency of input pulse
tdx	$\Delta t/(\rho \Delta x)$	
w	$\omega = 2\pi f$	
ncyc		number of timesteps for 50 "seconds"

As described previously, the explicit-solution procedure is not unconditionally stable, the speed of the "calculation front" must be faster than the maximum speed at which information propagates (i.e., wave speed).

A timestep must be chosen that is smaller than some critical timestep. The stability condition for an elastic solid discretized into elements of size x is

$$\Delta t < \frac{\Delta x}{C}$$

dt = frac * dx / c

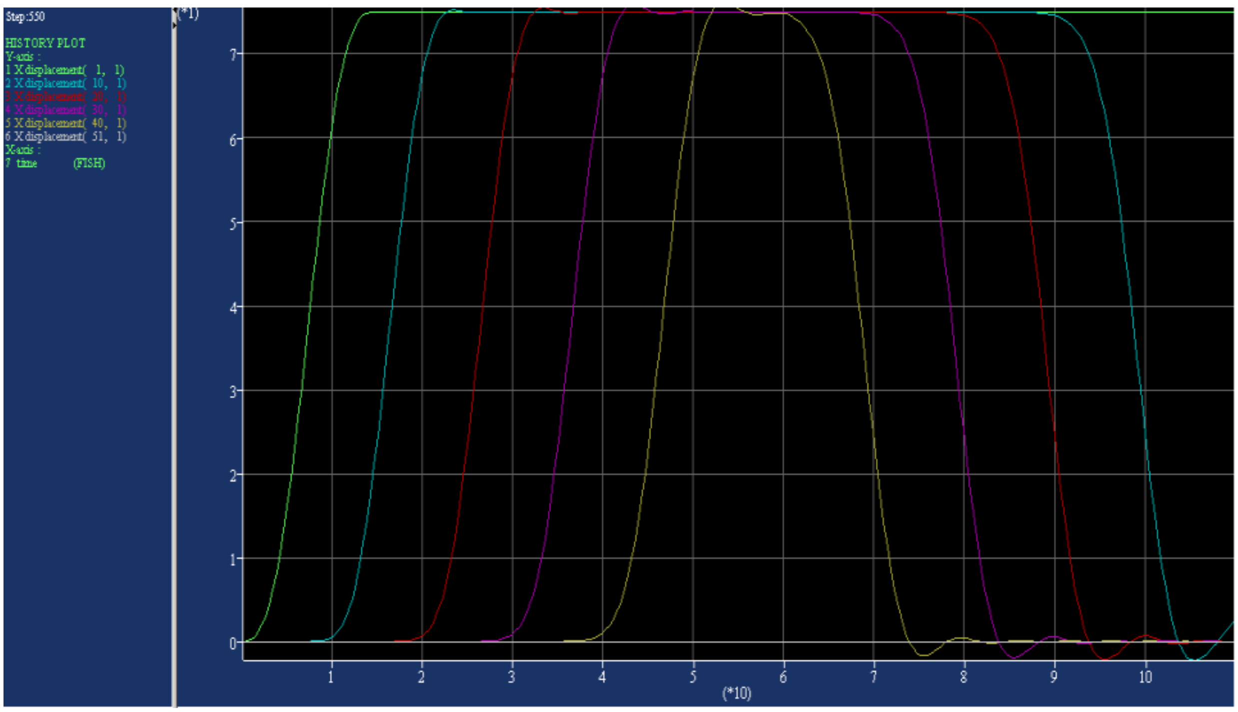
where *C* is the maximum speed at which information can propagate — typically, the p-wave speed. *C* where

$$C_p = \sqrt{\frac{K + 4G/3}{\rho}}$$

FDM - Elastic Example (cont.) - Solution

Thursday, March 11, 2010
11:43 AM

nel = 50 ; no. of elements
e = 1.0 ; Young's modulus
ro = 1.0 ; density
dx = 1.0 ; element size
p = 15.0 ; number of wavelengths per elements
vmax = 1.0 ; amplitude of velocity pulse
frac = 0.2 ; fraction of critical timestep



More Reading

Thursday, March 11, 2010
11:43 AM

- Watch FDM videos on course website
- FLAC manual: Theory and Background, Section 1 - Background - The Explicit Finite Difference Method

Assignment 3

Thursday, March 11, 2010
11:43 AM

1. Use MS Excel or a similar computer program to develop a solution for the program bar.dat.
   a. The grid should consist of 11 nodes and 10 zones.
   b. Use 0.2 for the fraction of the critical time step.
   c. Select the other input properties that are consistent with the properties use in bar.dat
2. Show by plots that the developed program matches the solution from bar.dat by plotting the x displacement histories at nodes 1, 3, 6, 9 and 11. Compare the maximum displacements at these nodes with those calculated by FLAC. To find the maximum displacement at these nodes, make sure that the model has run for at least 20 cycles. (50 points)
3. Provide the Excel spreadsheet or computer code used to solve the problem. Also, provide the modified bar.dat routine in FLAC. These should be e mailed to bartlett@civil.utah.edu

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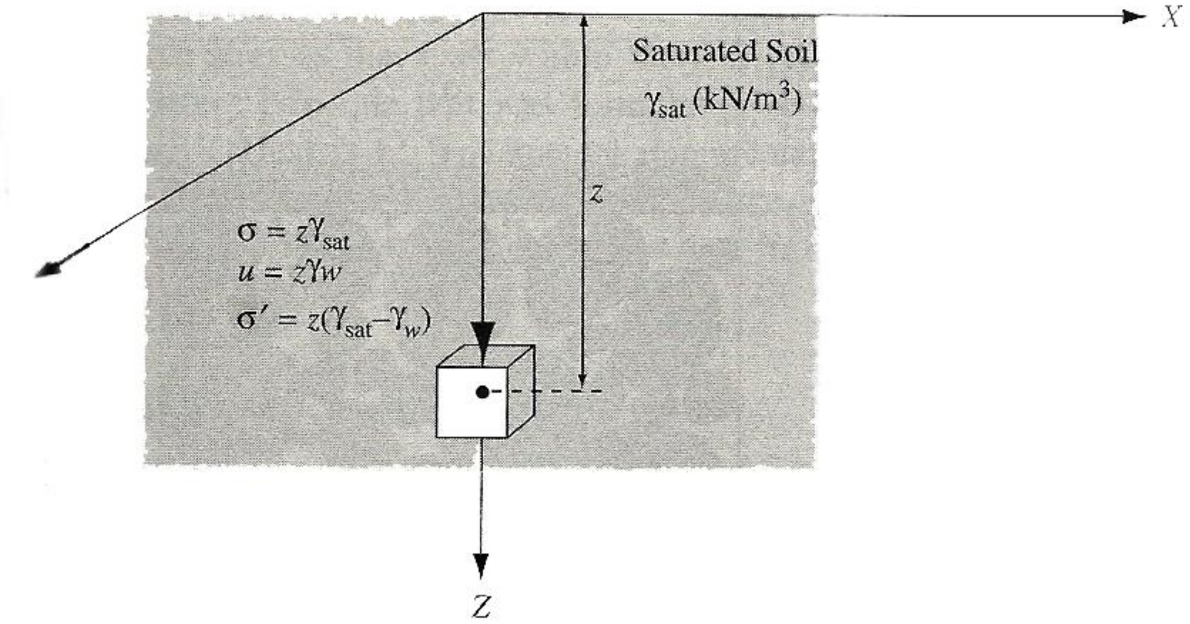
Thursday, March 11, 2010
11:43 AM

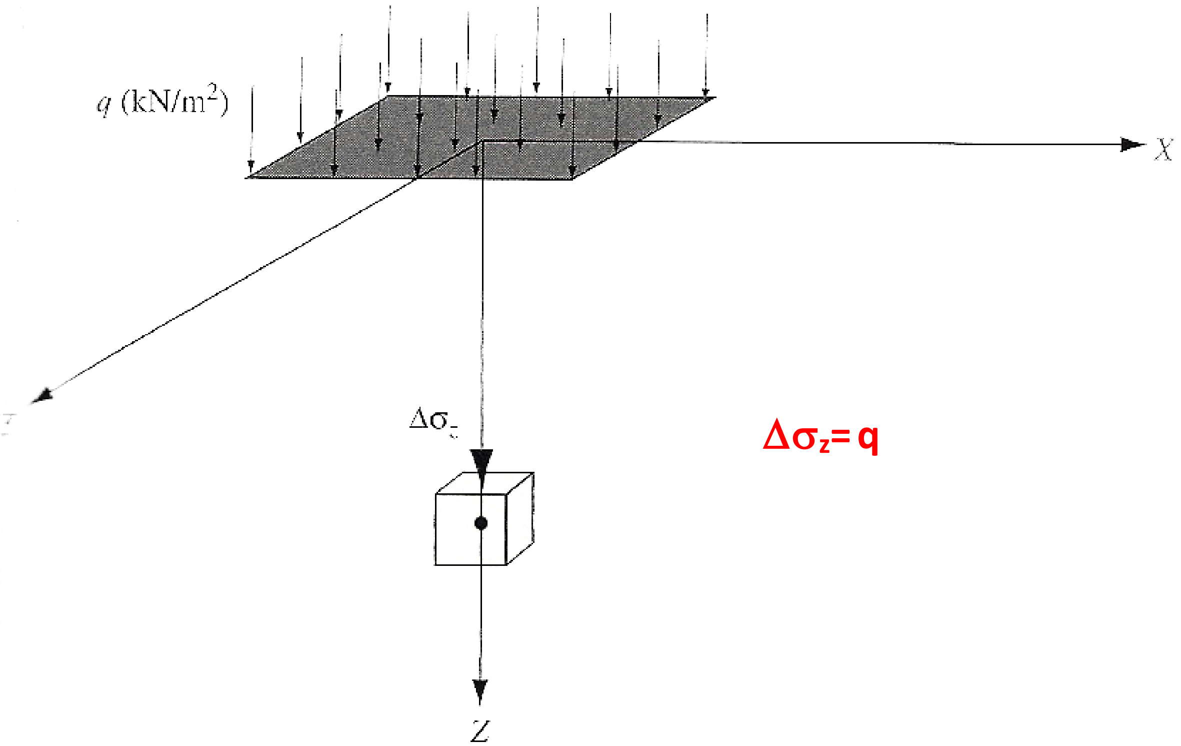
Vertical Stress

Thursday, March 11, 2010
11:43 AM

Vertical Stress in a Semi-infinite Half Space from Self-weight (i.e., gravity)



Increase in Vertical Stress from a Large (i.e., infinite) Uniform Load

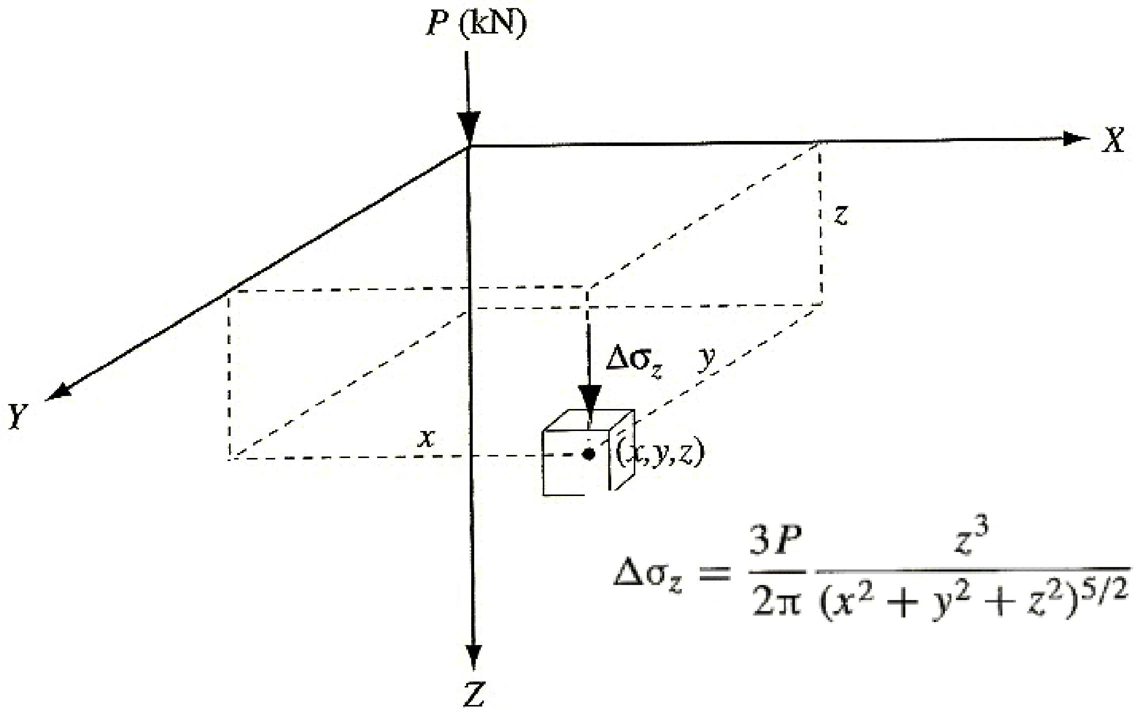


$\Delta\sigma_z = q$

Vertical Stress from Point Load

Thursday, March 11, 2010
11:43 AM

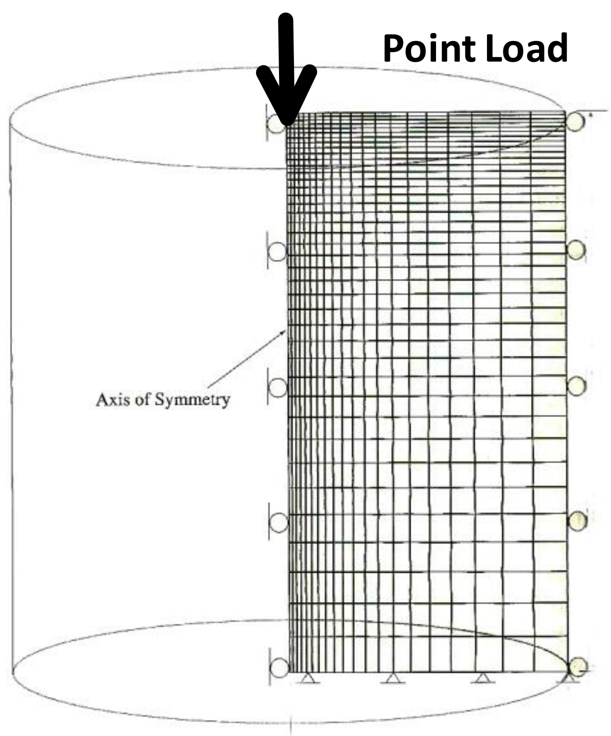
Elastic Theory



$$\Delta\sigma_z = \frac{3P}{2\pi}\frac{z^3}{(x^2+y^2+z^2)^{5/2}}$$

Numerical Approach

Axisymmetrical model

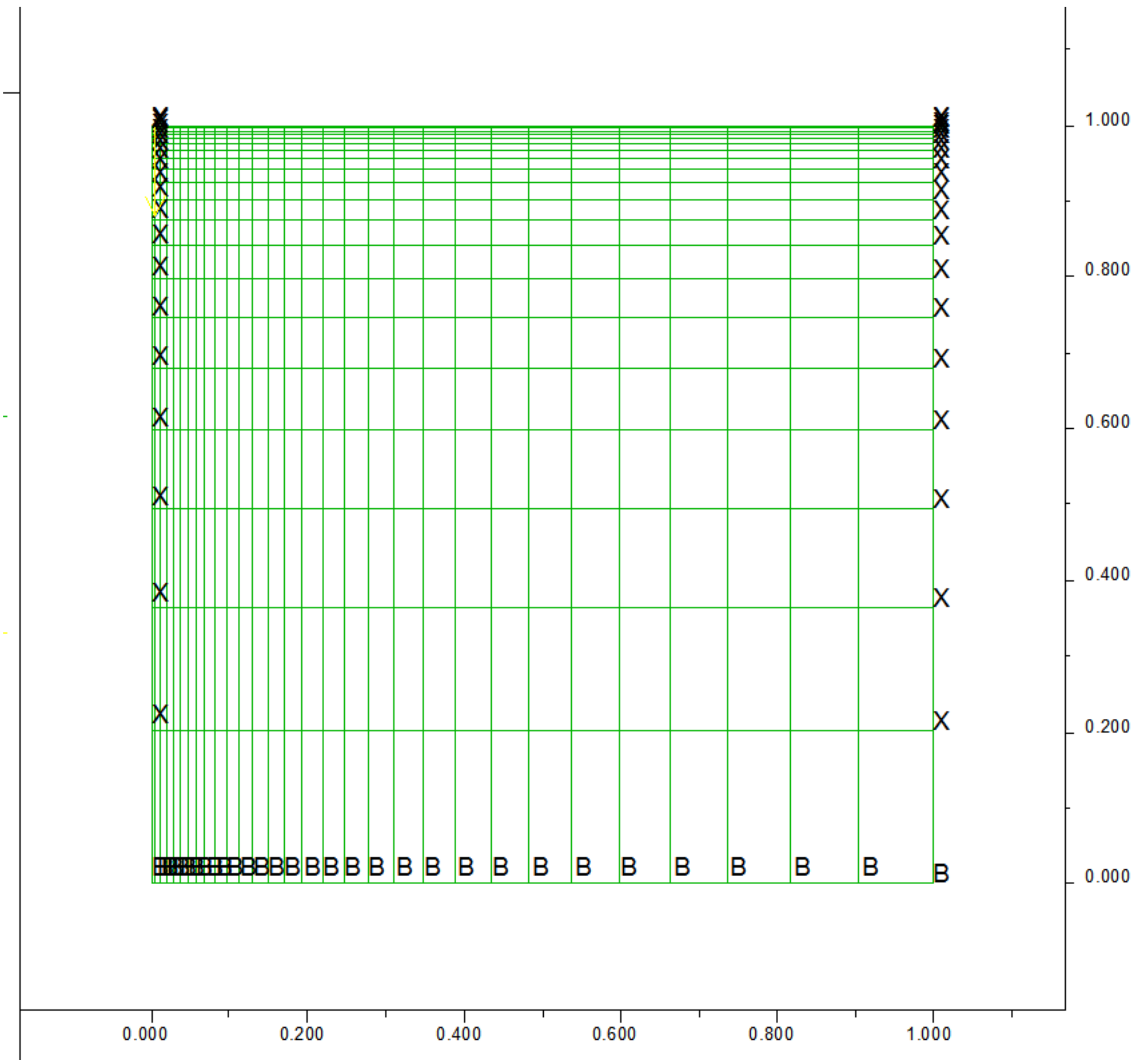


Note: Solution is sensitive to grid spacing. To minimize this try using an equivalent circular stress instead of force.

Vertical Stress from Point Load (cont.)

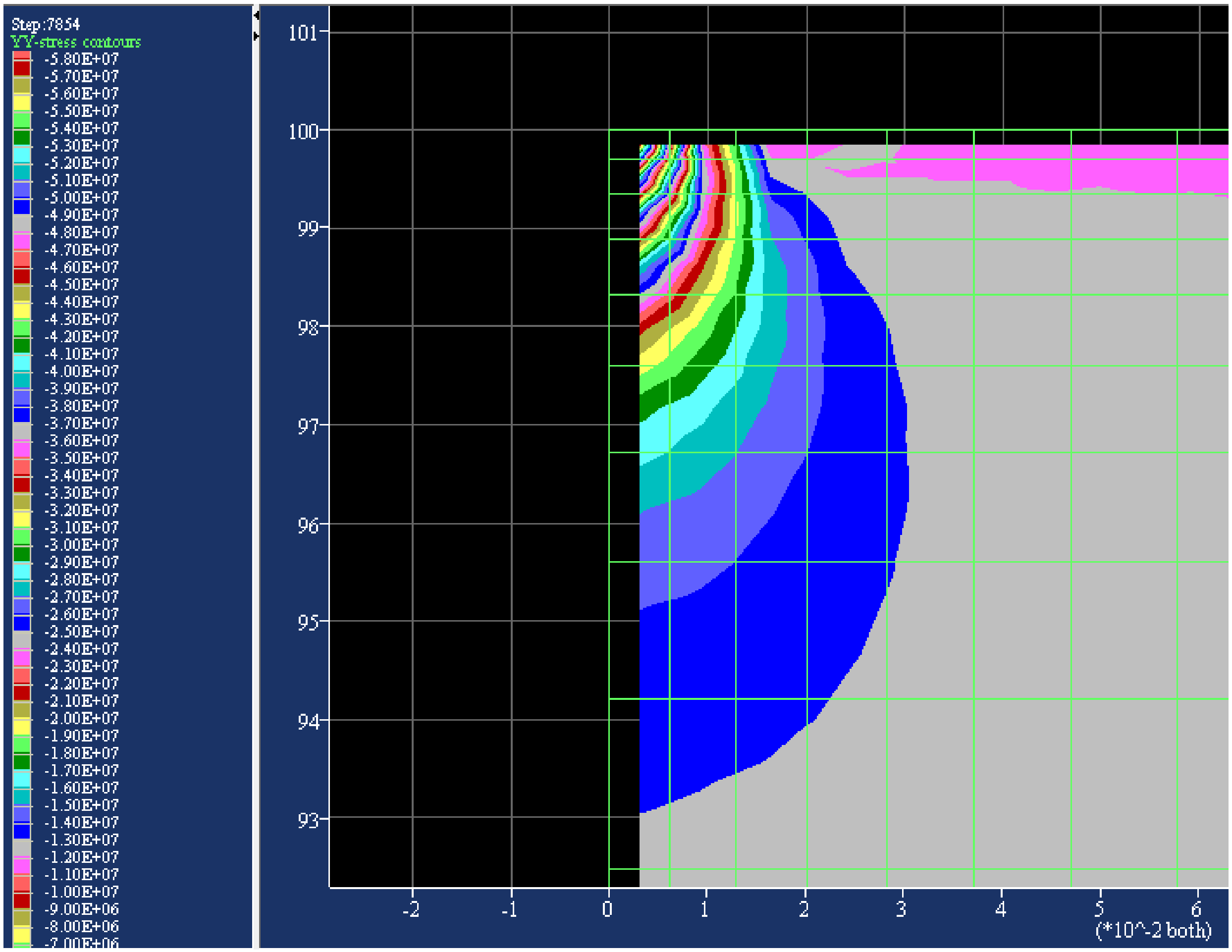
Tuesday, September 11, 2012
12:43 PM

```
config axisymmetry
grid 30 20
gen 0,0 0,1 1,1 1,0 ratio 1.1 .8
model elastic
prop density=1800  bulk=8333E6  shear=3846E6;  E=10000e6  v = 0.35
fix  x y j 1
fix  x i 31
apply syy -86391844  from 1,21  to 2,21;  -10000/(0.00607^2*pi)
;apply yforce -10000  from 1,21  to 1,21
solve
save point_load.sav  'last project state'
```



Vertical Stress from Point Load (cont.)

Thursday, March 11, 2010
11:43 AM

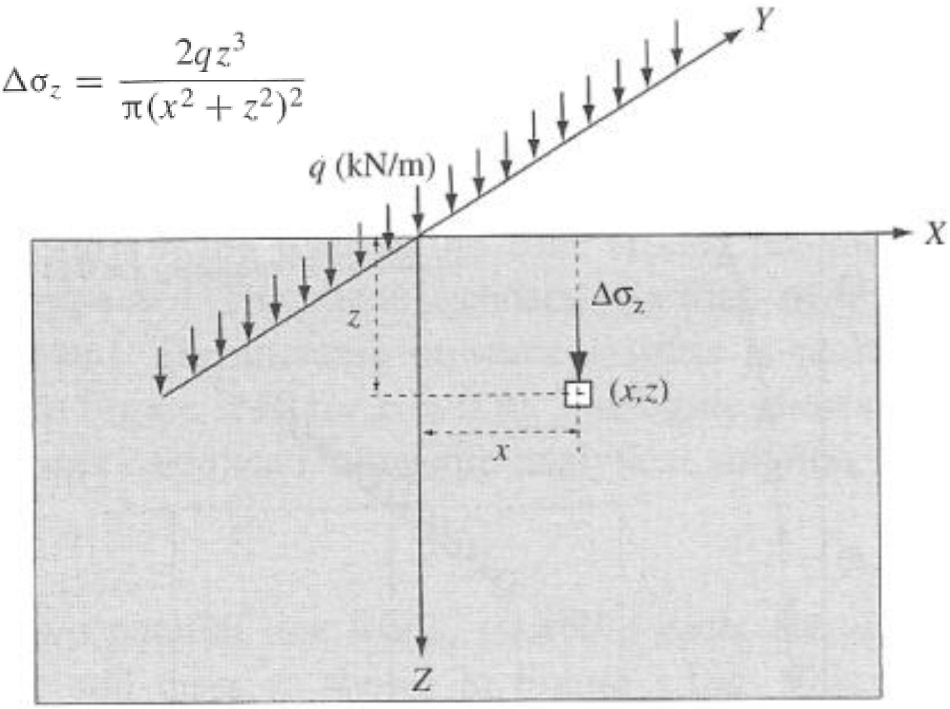


Vertical Stress from Line Load

Thursday, March 11, 2010
11:43 AM

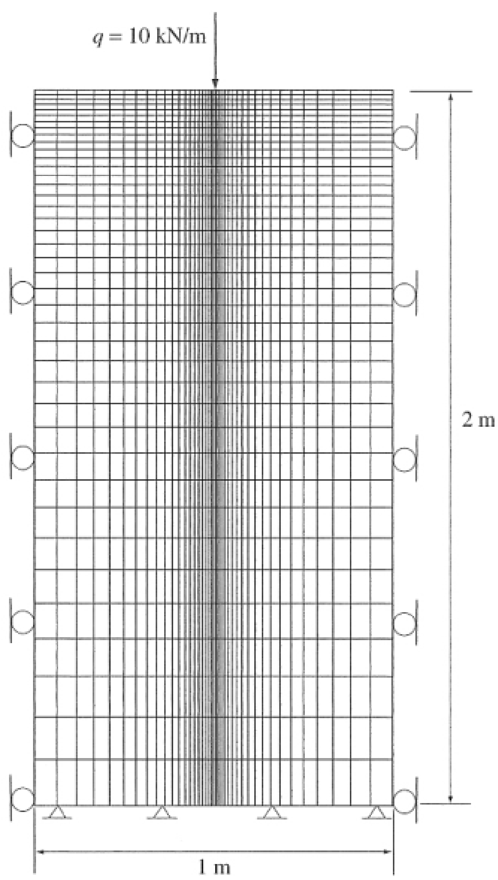
Elastic Theory

$$\Delta\sigma_z = \frac{2qz^3}{\pi(x^2 + z^2)^2}$$



Numerical Approach

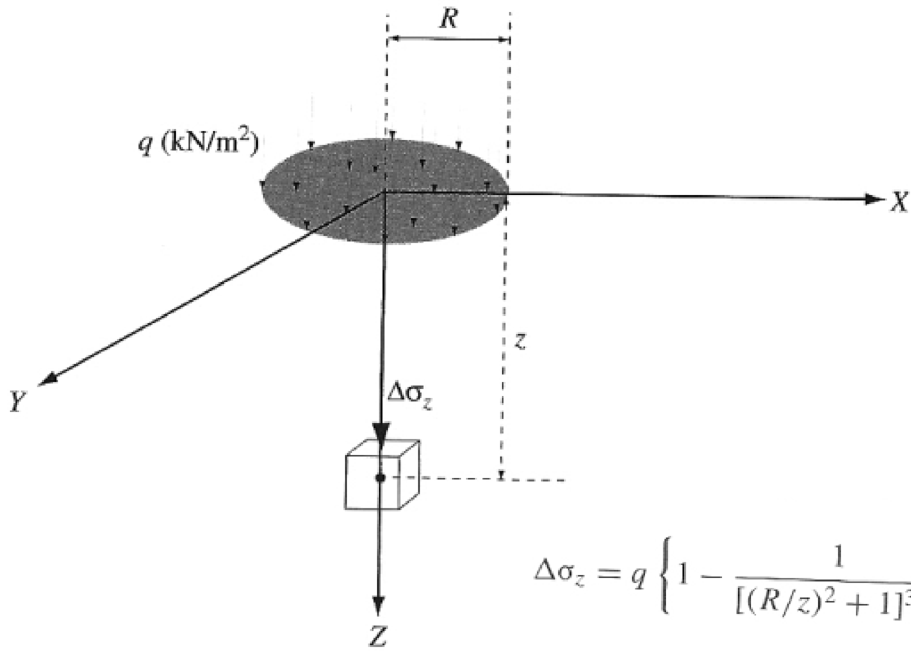
Plane Strain Model



Vertical Stress Under a Uniformly Loaded Circle

Thursday, March 11, 2010
11:43 AM

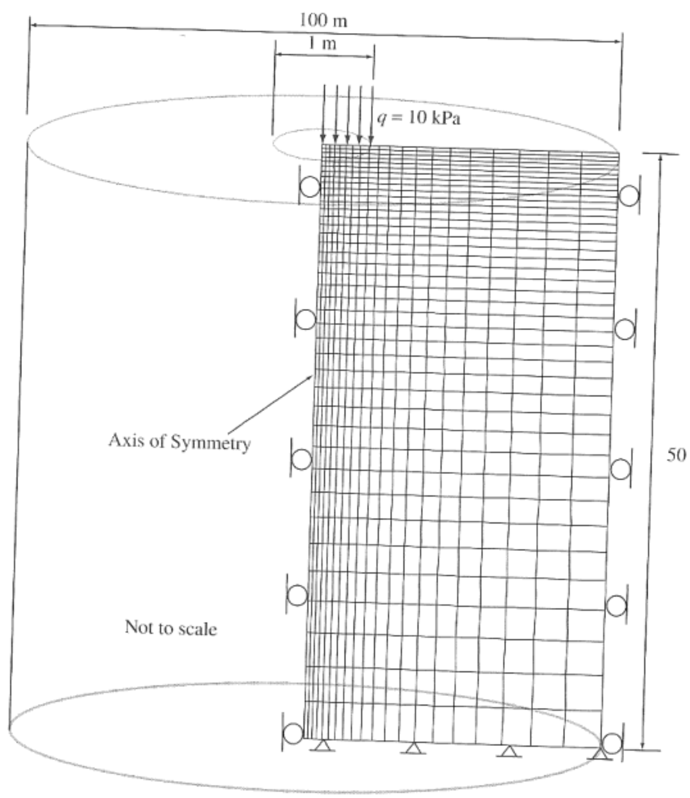
Elastic Theory



$$\Delta\sigma_z = q\left\{1 - \frac{1}{[(R/z)^2 + 1]^{3/2}}\right\}$$

Numerical Approach

Axisymmetrical Model

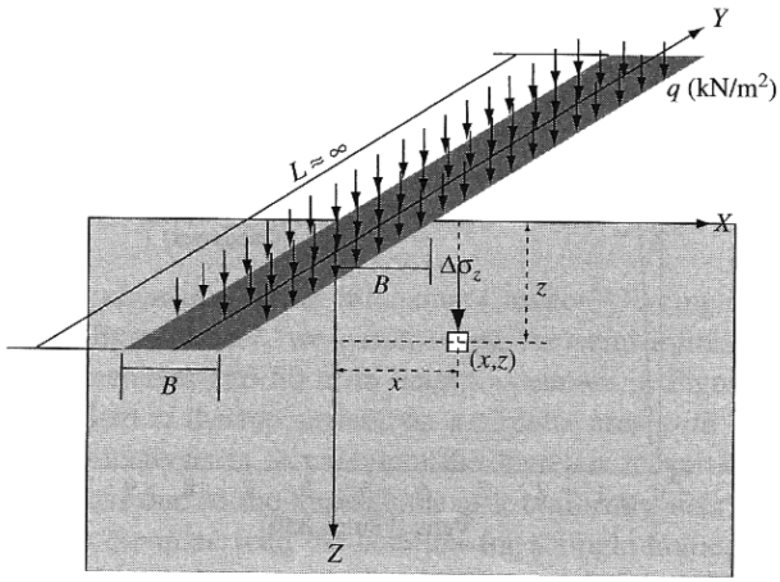


Vertical Stress Under a Strip Load

Thursday, March 11, 2010
11:43 AM

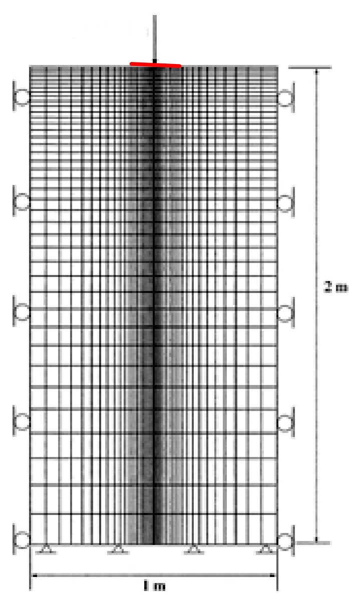
Elastic Model

$$\Delta\sigma_z = \frac{q}{\pi}\left\{\tan^{-1}\left(\frac{x}{z}\right) - \tan^{-1}\left(\frac{x-B}{z}\right) + \sin\left[\tan^{-1}\left(\frac{x}{z}\right) - \tan^{-1}\left(\frac{x-B}{z}\right)\right]\cos\left[\tan^{-1}\left(\frac{x}{z}\right) + \tan^{-1}\left(\frac{x-B}{z}\right)\right]\right\}$$



q= 10 kPa/m

Numerical Approach



Plane Strain Model

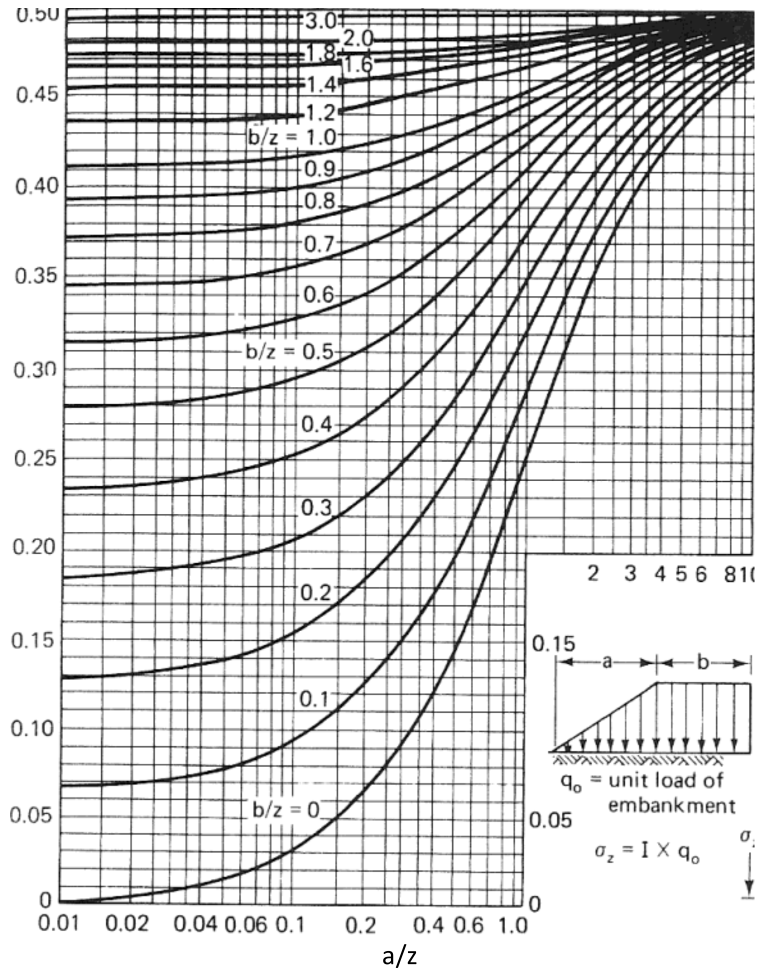
Embankment and Slopes

Thursday, March 11, 2010
11:43 AM

Elastic Theory

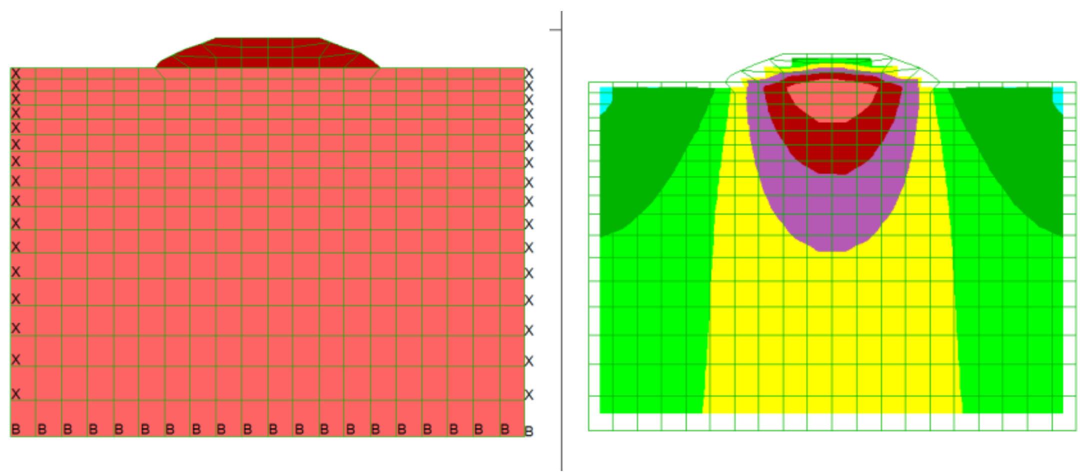
I = (influence factor)

Note: Influence factor values from this chart must be double to account for the right side of the embankment.



(z = depth below ground surface (i.e., depth below base of embankment)

Numerical Approach



Ⓒ Steven F. Bartlett, 2010

Grading a Mesh in FLAC

Thursday, March 11, 2010
11:43 AM

Example 3 — The GENERATE command can be used to grade a mesh to represent far boundaries. For example, in many cases, an excavation is to be created at a great depth in a rock mass. Detailed information on the stresses and displacements is to be determined around the excavation, where the disturbance is large, but little detail is necessary at greater distances. In the following example, the
lower left-hand portion of the grid is left finely discretized, and the boundaries are graded outward in the x- and y-directions. Try issuing the commands in Example 2.3.

Example 2.3 Grading the mesh (2 way)

```
new
grid 20,20
m e
gen 0,0 0,100 100,100 100,0 rat 1.25 1.25
plot hold grid
```

The GENERATE command forces the grid lines to expand to 100.0 units at a rate 1.25 times the previous grid spacing in the x- and y-directions. (Example 2.3 also illustrates that command words can be truncated: MODEL elas becomes m e.) Note that if the ratio entered on the GEN command is between 0 and 1, the grid dimensions will decrease with increasing coordinate value. For example, issue the commands in Example 2.4.

Example 2.4 Applying different gradients to a mesh

```
new
gr 10,10
m e
gen -100,0 -100,100 0,100 0,0 rat .80,1.25
plot hold grid
```

You will see a grid graded in the negative x- and positive y-directions.

Layered Systems

Thursday, March 11, 2010
11:43 AM

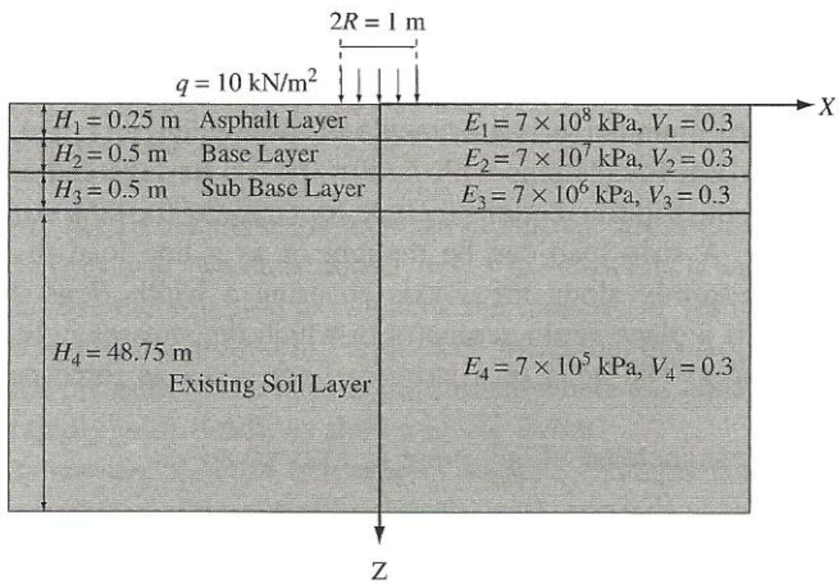


FIGURE 3.23 Stress increase in a layered soil system with a uniformly loaded circular area.

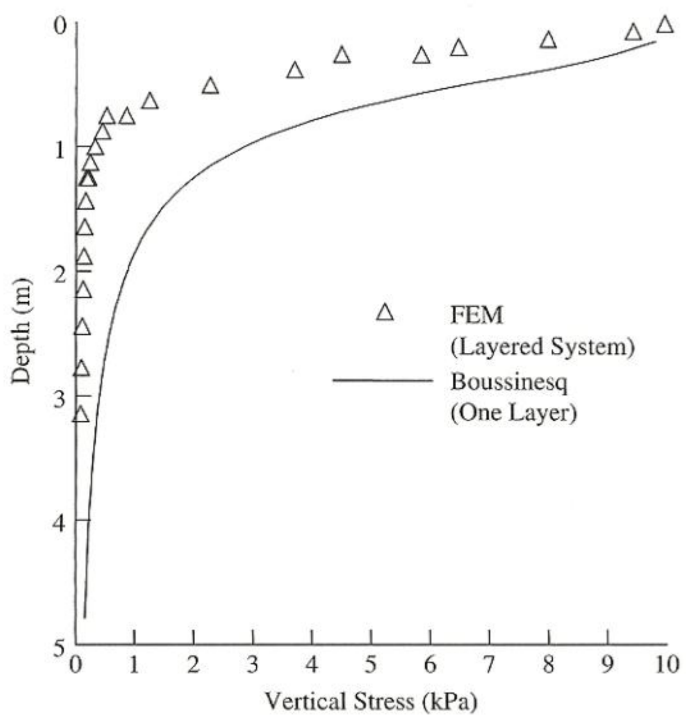


FIGURE 3.24 Comparison between FEM and analytical solution of a layered system with a uniformly loaded circular area.

Calculating Effective Stress in FLAC

Thursday, March 11, 2010
11:43 AM

To calculate the effective vertical stress in FLAC due to changes in groundwater or pore pressure, you can use the adjust total stress feature. This is initiated at the beginning of the FLAC routine by typing the following command:

config ats

However, if the groundwater table is specified at the beginning of the run and is not subsequently changed, then the config ats command is not necessary. It is only required when the user imposes a new watertable or pore pressure condition on the model after the model is initialized.

The adjustment of total stresses for user-specified changes in pore pressure can be made automatic by giving the CONFIG ats command at the beginning of a run. If this is done, then total stresses are adjusted whenever pore pressures are changed with the INITIAL, WATER table or APPLY command, or with the pp(i,j) variable in a user-written FISH function. If CONFIG ats is used, then care should be taken that the initialization of stresses and pore pressures at the beginning of a run is done in the correct order: pore pressure should be set before stresses so that the required values for stresses do not change when a pressure-initialization is made. (FLAC manual).

You must also create a table that specifies the top of the groundwater table. The command below creates table 1 and specifies the coordinates of 0,20 and 20,20 as the ground water surface.

table 1 0,20 20,20; water table

You should specify that the watertable is table 1 as shown below:

water table=1

You should also specify the fluid density (i.e., density of water).

water density=1000.0

These commands must be issued before the solve command.

In addition, remember that the mass density of the soil below the water table should be specified as the saturated mass density.

More Reading

Thursday, March 11, 2010
11:43 AM

- Applied Soil Mechanics Ch. 3
- FLAC v. 5 Manual, Fluid-Mechanical Interaction, Section 1.5.3, Adjust Total Stress

Assignment 4

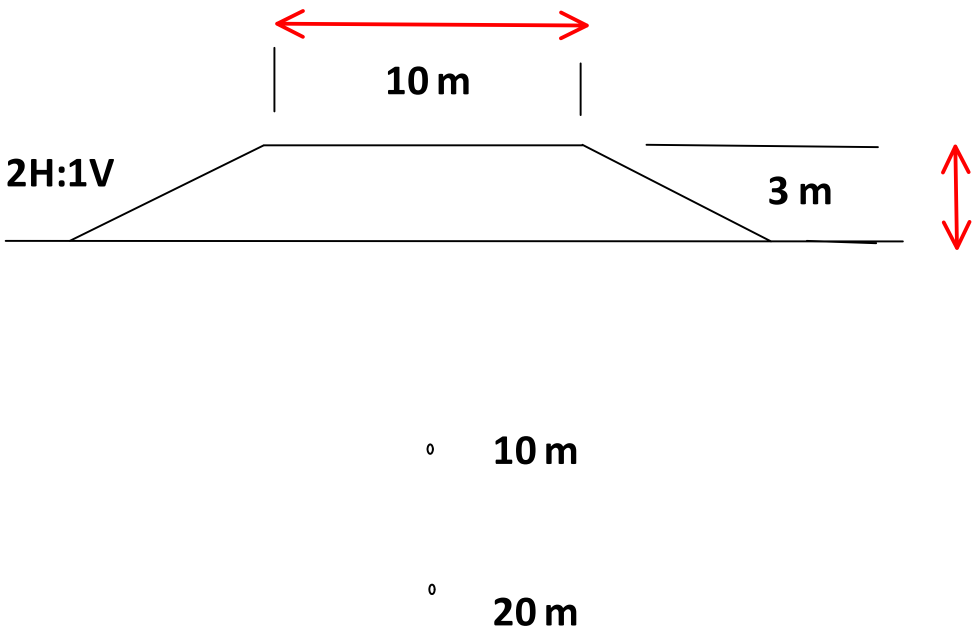
Thursday, March 11, 2010
11:43 AM

1. Use FLAC to determine and contour the total vertical stress for a 20 x 20 m soil column. Assume that total unit weight of the homogenous soil is 2000 kg/m^3. Contour your results and present the plot. Include your FLAC code (10 points).
2. Repeat problem 1, but use FLAC to determine the effective vertical stress assuming that the groundwater is at the ground surface. Contour your results and present the plot. Include your FLAC code (10 points).
3. Solve Example 3.4 (i.e., point load) in the text using the FDM (i.e., FLAC). Graphically compare your FLAC solution at x= 0.1 m with that obtained from Eq. 3.9 for x= 0.1 m. You can create a profile at x=0.1 m by using the profile command in FLAC. Plot the elastic and FDM results from FLAC on the same plot for comparison (20 points).
4. Solve Example 3.5 (i.e., line load) in the text using the FDM (i.e., FLAC). Graphically compare your FLAC solution with Eq. 3.10. To do this, plot the elastic and FDM results from FLAC on the same plot for comparison (20 points).
5. Solve Example 3.7 (i.e., circular load) in the text using the FDM (i.e., FLAC). Graphically compare your solution using Eq. 3.11 and plotting the FLAC results on the same plot (20 points).

Assignment 4 (cont.)

Thursday, March 11, 2010
11:43 AM

6. A highway embankment (shown below) is to be constructed. Calculate the increase in vertical stress due to the placement of the embankment under the centerline of the embankment at depths of 10 and 20 m below the base of embankment. Assume that the average density of the embankment material is 2000 kg per cubic meter (20 points).



7. Solve Example 3.8 for a 4-layered system. Compare this with that obtained in Example 3.7 for a single layered system (20 points).

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Thursday, March 11, 2010
11:43 AM

Consolidation

Tuesday, September 11, 2012
12:43 PM

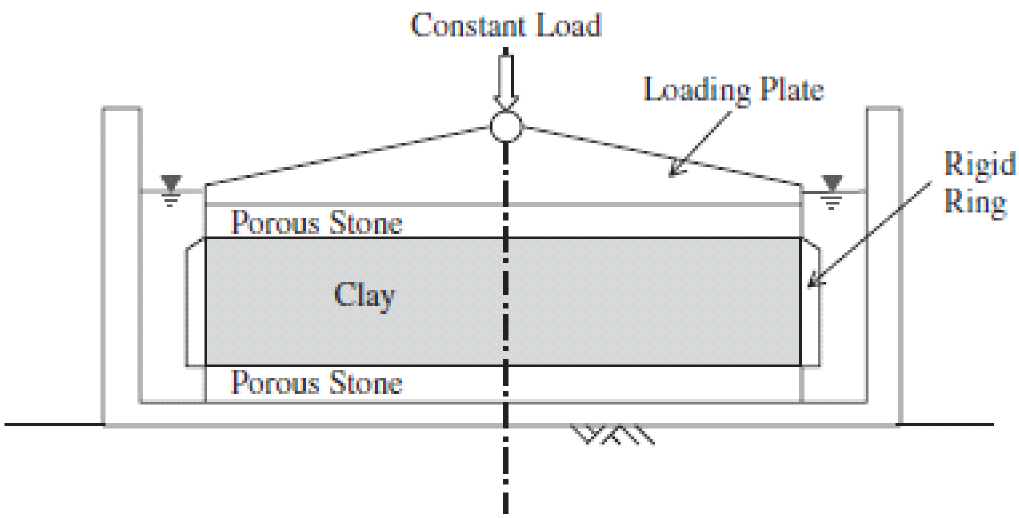


FIGURE 4.3 One-dimensional consolidation test apparatus.

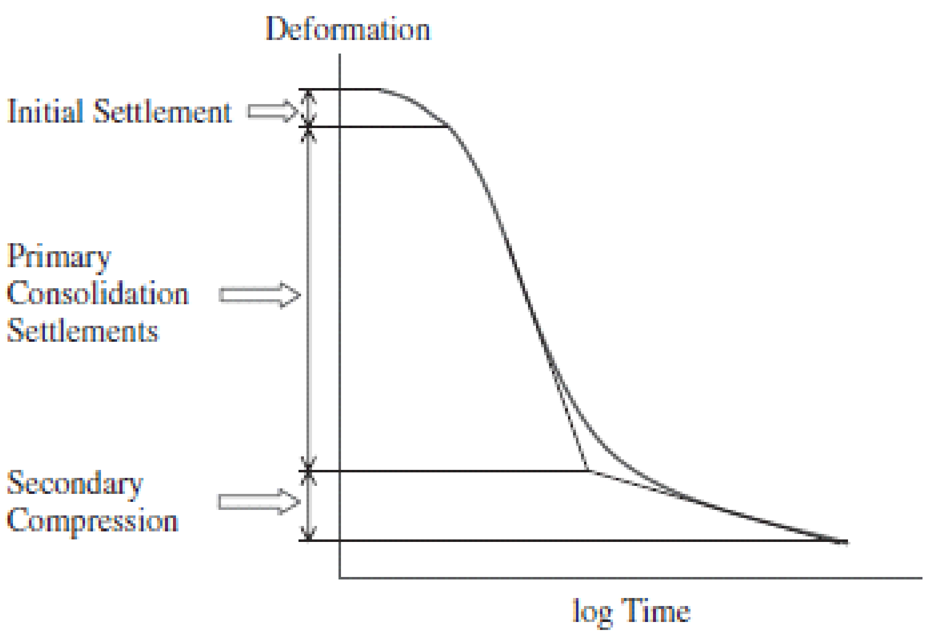


FIGURE 4.4 Deformation versus time curve (semilog).

e vs log σ_v curves

Tuesday, September 11, 2012
12:43 PM

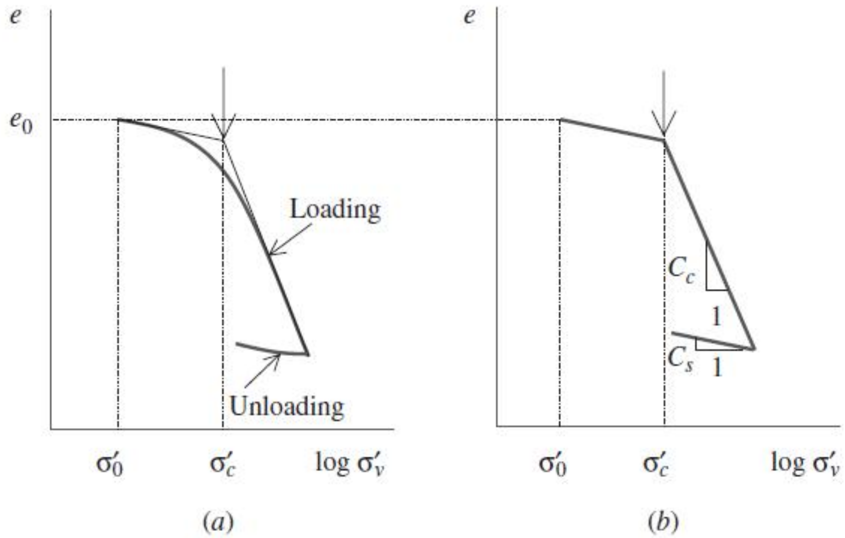


FIGURE 4.5 Void ratio versus vertical effective stress (semilog): (*a*) consolidation test results; (*b*) idealization.

Cc = compression index
Cs = recompression index
σ'_c = preconsolidation stress

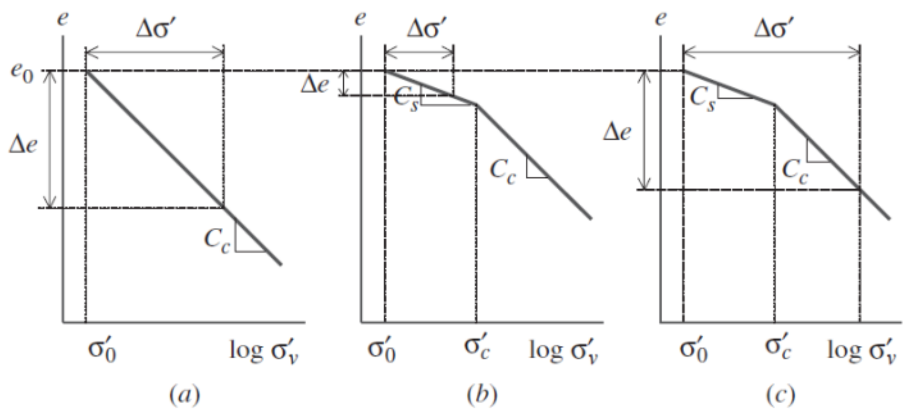


FIGURE 4.7 Calculation of consolidation settlements: (*a*) NC clay; (*b*) OC clay with $\Delta\sigma' + \sigma'_0 < \sigma'_c$; (*c*) OC clay with $\Delta\sigma' + \sigma'_0 > \sigma'_c$.

Settlement Calculations

Tuesday, September 11, 2012
12:43 PM

$$S_c = \frac{\Delta e}{1 + e_0} H$$

Consolidation Settlement

$$\Delta e = C_c \log \frac{\sigma_0' + \Delta\sigma'}{\sigma_0'}$$

Change in void ratio for normally consolidated clay

$$S_c = C_c \frac{H}{1 + e_0} \log \frac{\sigma_0' + \Delta\sigma'}{\sigma_0'}$$

Consolidation settlement for normally consolidated clay

$$\Delta e = C_s \log \frac{\sigma_0' + \Delta\sigma'}{\sigma_0'}$$

Change in void ratio for overconsolidated clay below the preconsolidation stress

$$S_c = C_s \frac{H}{1 + e_0} \log \frac{\sigma_0' + \Delta\sigma'}{\sigma_0'}$$

Consolidation settlement for overconsolidated clay with increase in stress below the preconsolidation stress

$$\Delta\sigma' + \sigma_0' < \sigma_c'$$

$$S_c = \frac{H}{1 + e_0} \left(C_s \log \frac{\sigma_c'}{\sigma_0'} + C_c \log \frac{\sigma_0' + \Delta\sigma'}{\sigma_c'} \right)$$

Consolidation settlement for overconsolidated clay with increase in stress below the preconsolidation stress

$$\Delta\sigma' + \sigma_0' > \sigma_c'$$

Relationships - Elastic and Consolidation

Tuesday, September 11, 2012
12:43 PM

$$M = \frac{(1 - \nu)E}{(1 + \nu)(1 - 2\nu)}$$

Constrained modulus

$$m_v = \frac{1}{M} = \frac{(1 + \nu)(1 - 2\nu)}{(1 - \nu)E}$$

Coefficient of volume compressibility

Also

$$m_v = k/c_v\gamma_w.$$

Coefficient of volume compressibility, where cv is the coefficient of vertical consolidation (defined later).

$$E = \frac{c_v\gamma_w(1 + \nu)(1 - 2\nu)}{k(1 - \nu)}$$

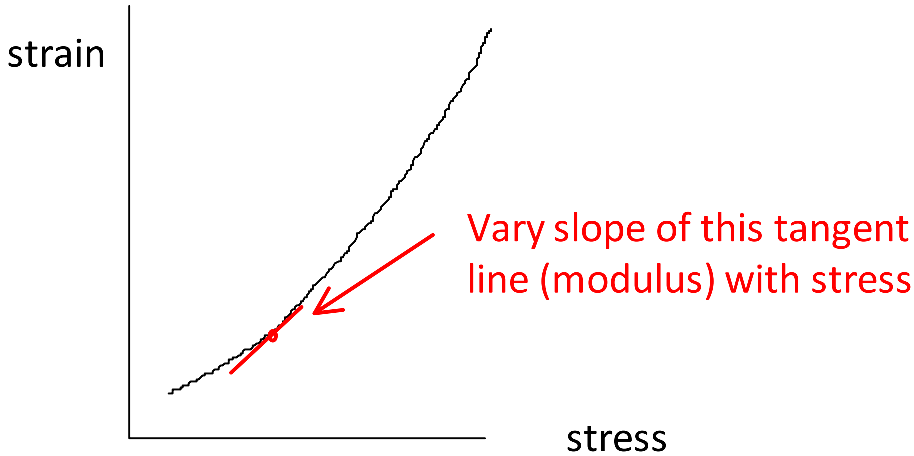
Young's modulus

Note that c_v and k are not constant but vary with void ratio or vertical strain, hence M and E are non linear. c_v is discussed later in this lecture in time rate of consolidation section.

Developing Elastic Model for Consolidation

Tuesday, September 11, 2012
12:43 PM

- Consolidation is a non-linear process that produces a stress-strain relationship where vertical strain is proportional to the log of the change in stress.
- If would be useful to be able to model this process with a simple tangent modulus that varies with the vertical stress.

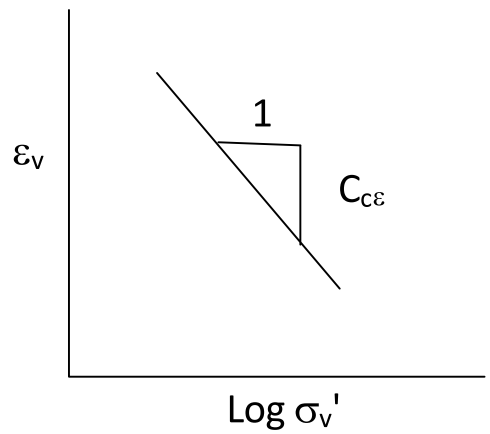


- Because the finite difference techniques uses time steps, it is a simple matter to check the state of stress at any time step and adjust the tangent modulus to follow this non-linear path.
- Thus, we can use an algorithm that allows for the modulus to change as a function of applied stress to mimic this non-linear function if we can determine the relationship that expresses the modulus in terms of the applied stress.

Developing Elastic Model for Consolidation (cont.)

Tuesday, September 11, 2012
12:43 PM

- Step 1 - Define consolidation properties in term of vertical strain, ε_v, instead of void ratio, e.



  - Useful relationships between vertical strain and void ratio
    - $\varepsilon_V = \Delta H/H_o$
    - $\varepsilon_V = e_o - e / (1+e_o)$
    - $C_{c\varepsilon} = C_c / (1+e_o)$

- Step 2 - Find tangent modulus at a point (i.e., derivative)

$y = \log x$
$dy/dx = d \log (x) / dx$
$dy/dx = 1 / (x * \ln(10))$

Let x be σ_v'
Let $dy = C_{c\varepsilon}$

$dy/dx = C_{c\varepsilon} / (\sigma_v' * \ln(10))$

Note that the tangent modulus as defined by dy/dx is also called the constrained modulus, M, for a 1D compression test.

$M = C_{c\varepsilon} / (\sigma_v' * \ln(10))$

Developing Elastic Model for Consolidation (cont.)

Tuesday, September 11, 2012
12:43 PM

- Step 3 - Express Young's modulus E in terms of $C_{c\varepsilon}$ and σ_v'

  $M = (1-\nu)E/[(1+\nu)(1-2\nu)]$

  $\ln(10)\sigma_v'/C_{c\varepsilon} = (1-\nu)E/[(1+\nu)(1-2\nu)]$

  $(1+\nu)(1-2\nu)\sigma_v'\ln(10) = C_{c\varepsilon}(1-\nu)E$

  $E = (1+\nu)(1-2\nu)\sigma_v'\ln(10) / (C_{c\varepsilon}(1-\nu))$

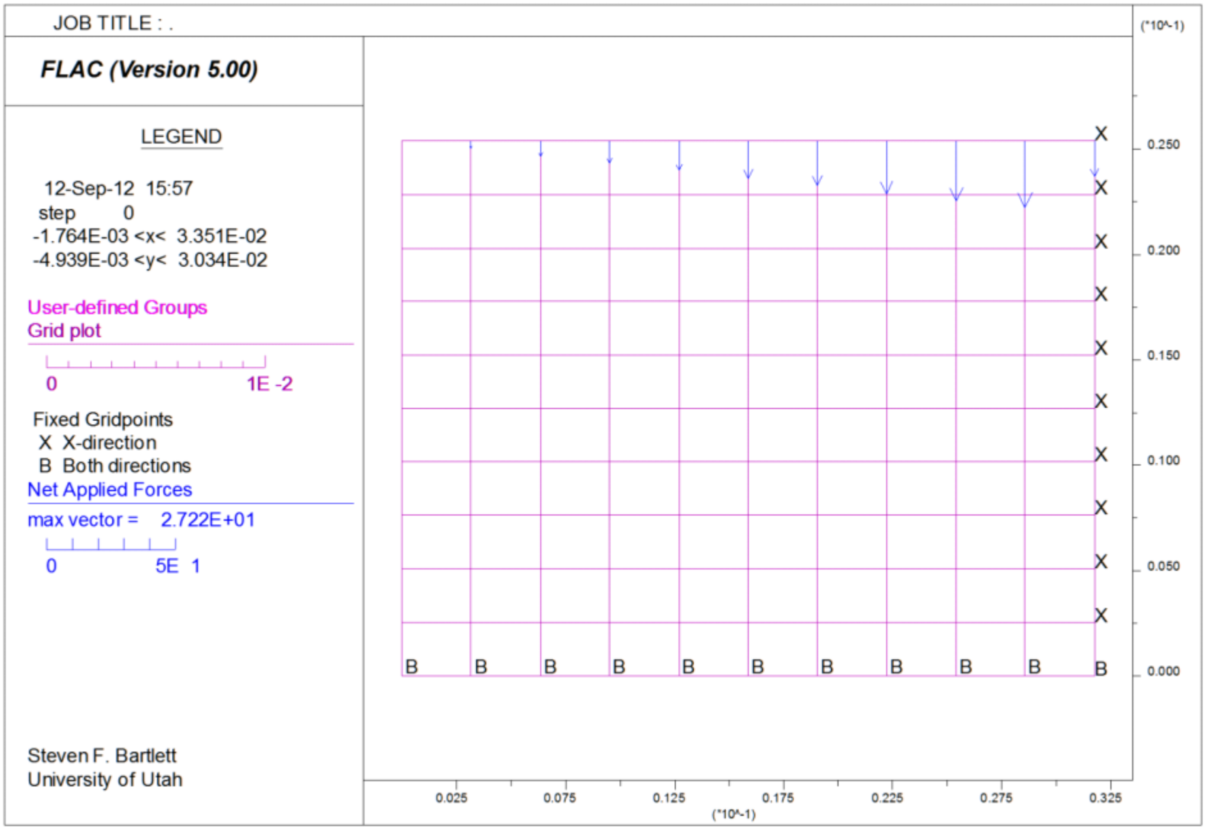
  In the equation above, we have a relationship to define Young's modulus, E, in terms of the vertical stress, $C_{c\varepsilon}$, and ν. The latter two factors are material properties, which can be determined from laboratory tests.

  Thus we have a method to predict how Young's modulus varies non linearly as a function of applied vertical stress for given values of $C_{c\varepsilon}$, and ν.

FLAC Implementation of Elastic Model

Tuesday, September 11, 2012
12:43 PM

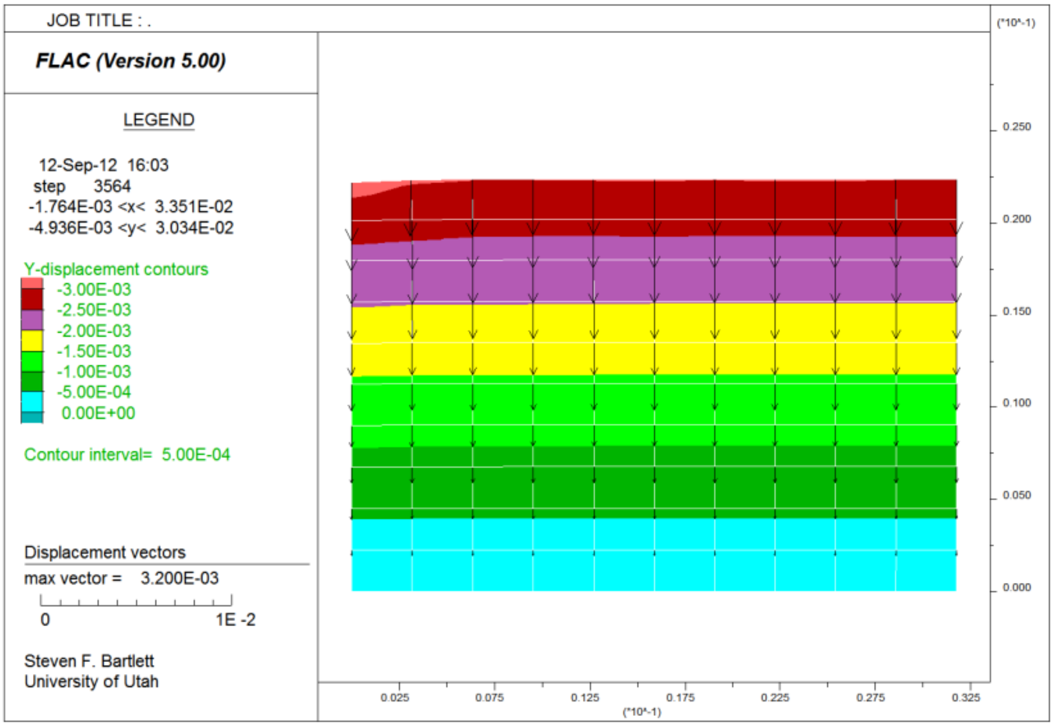
Modeling 1D consolidation test



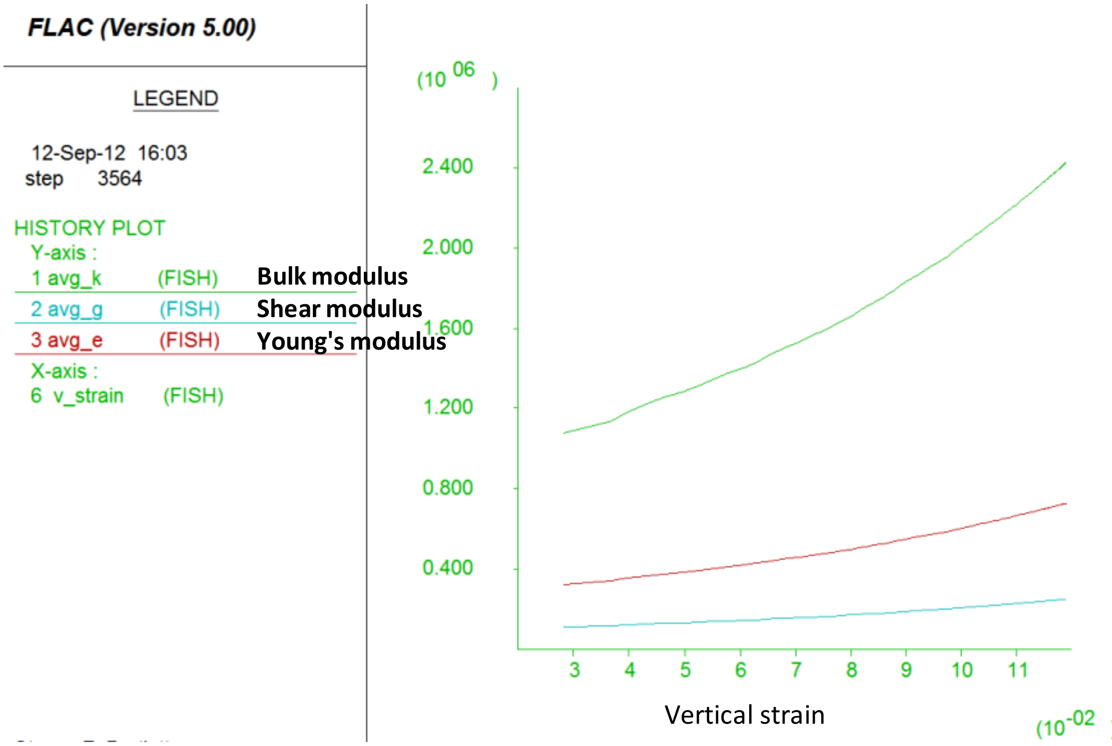
Axisymmetrical model with height = 2.54 cm, radius = 3.175 cm

FLAC Implementation of Elastic Model (cont.)

Tuesday, September 11, 2012
12:43 PM



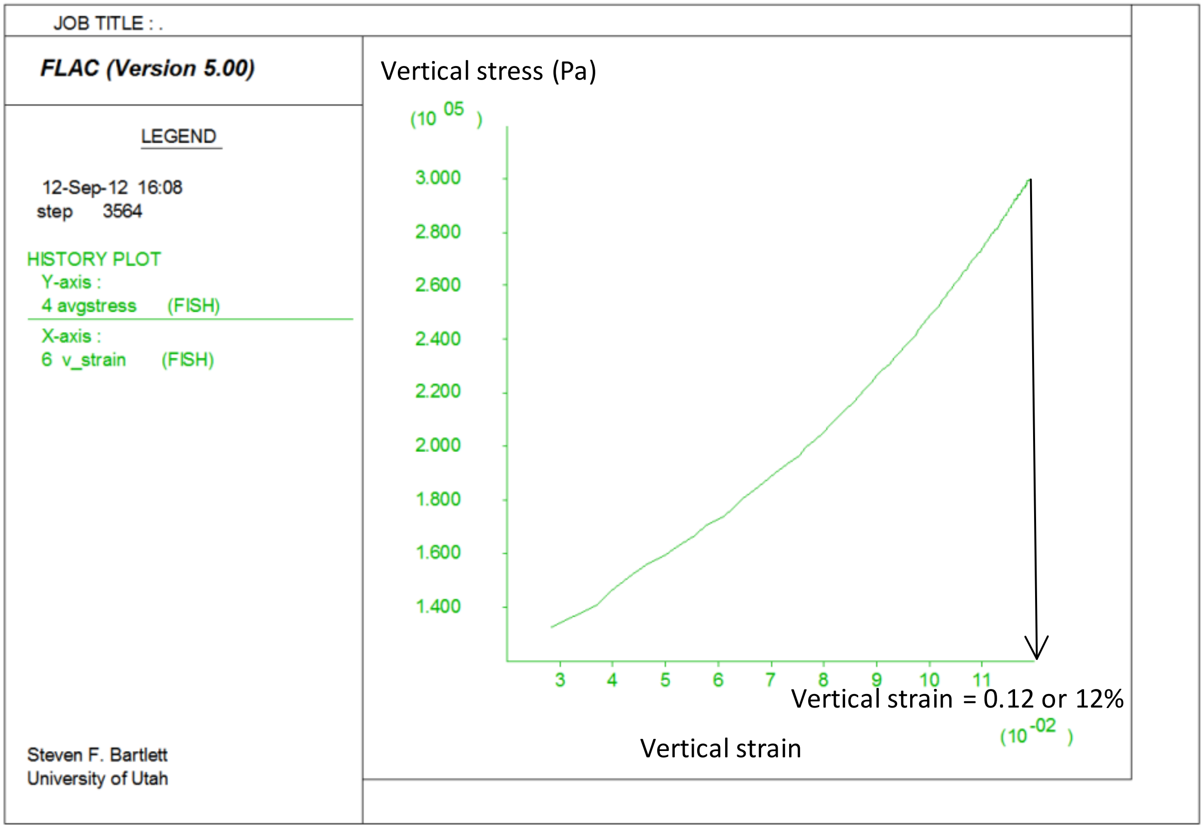
Vertical displacement vectors and displacement after applying 200 kPa, 4 tsf, stress at top of model.



Change in moduli with vertical strain

FLAC Implementation of Elastic Model (cont.)

Tuesday, September 11, 2012
12:43 PM



Check of FLAC results using consolidation theory

$\varepsilon_v = C_{c\varepsilon} \log [(\sigma_o + \Delta\sigma) / \sigma_o]$

$\varepsilon_v = 0.25 \log [(100 + 200)/100]$

$\varepsilon_v = 0.12$ or 12 percent

FLAC Model

Tuesday, September 11, 2012
12:43 PM

```
config axisymmetry ats
set large; let grid deform for high strain problems
grid 10,10
gen 0,0 0,0.0254 0.03175,0.0254, 0.03175,0
define inputs ; subroutine for input values and calcs
   Cce=0.25
   soil_d = 2000 ; soil density
   sigma_c = 100e3*(-1) ; preconsolidation stress
   sigma_f = 300e3*(-1) ; final stress
   P_ratio = 0.45 ; Poisson's ratio
   E_ini = (1+P_ratio)*(1-2*P_ratio)*(-1)*sigma_c*2.3026/(Cce*(1-P_ratio)) ; initial Young's modulus
   K_ini = E_ini/(3*(1-2*P_ratio)) ; initial bulk modulus
   G_ini = E_ini/(2*(1+P_ratio))  ; inital shear modulus
   ;
   new_E = E_ini ; initializes variables used in nonlin
   new_K = K_ini ; initializes variables
   new_G = G_ini ; initializes variables
end
inputs ; runs subroutine
;
model elastic
prop dens = soil_d bu = K_ini sh = G_ini
;
; boundary conditions
fix x y j 1
fix x i 11
;
; initializes preconsolidation stress in model
define preconsol
  loop i (1,izones)
    loop j (1,jzones)
       syy(i,j) = sigma_c
     endloop
 endloop
end
preconsol
```

FLAC Model (cont.)

Tuesday, September 11, 2012
12:43 PM

```
define nonlin ; subroutine to change moduli while stepping
 sumstress = 0
 sum_K = 0
 sum_G = 0
 sum_E = 0
 whilestepping
  loop i (1,izones)
    loop j (1,jzones)
      new_E = (1+P_ratio)*(1-2*P_ratio)*(-1)*syy(i,j)*2.3026/(Cce*(1-P_ratio))
      new_K = new_E/(3*(1-2*P_ratio))
      new_G = new_E/(2*(1+P_ratio))
      bulk_mod(i,j)=new_K
      shear_mod(i,j)=new_G;
      sumstress = sumstress + syy(i,j)
      sum_K = sum_K + new_K
      sum_G = sum_G + new_G
      sum_E = sum_E + new_E
      avgstress = (-1)*sumstress/100 ; average vertical stress in model
      v_strain = ydisp(6,11)*(-1)/0.0254
      avg_K = sum_K/100 ; average bulk modulus in model
      avg_G = sum_G/100 ; average shear modulus in model
      avg_E = sum_E/100
    endloop
  endloop
end
;
; applies new stress at boundary (stress controlled)
set st_damping=local 2.0; required for numerical stability
apply syy sigma_f from 1,11 to 11,11 ; applies vstress at boundary
;
; applies velocity at boundary (strain controlled)
;apply yvelocity -5.0e-6 xvelocity=0 from 1,11 to 11,11 ;applies constant downward velocity to simulate
a strain-controlled test
;
;
; histories
history 1 avg_K ; creates history of bulk modulus
history 2 avg_G ; creates history of shear modulus
history 3 avg_E ; creates history of elastic modulus
history 4 avgstress ; average stess in model
history 5 unbalanced ; creates history of unbalanced forces
history 6 v_strain; vertical strain
;
solve; use this if stress is applied to top boundary (stress controlled)
;cycle 820; use this if velocity is applied to top boundary (strain controlled)
save 1d consolidation.sav 'last project state'
```

Consolidation Settlement Under Strip Footing

Tuesday, September 11, 2012
12:43 PM

Stresses versus depth from elastic theory

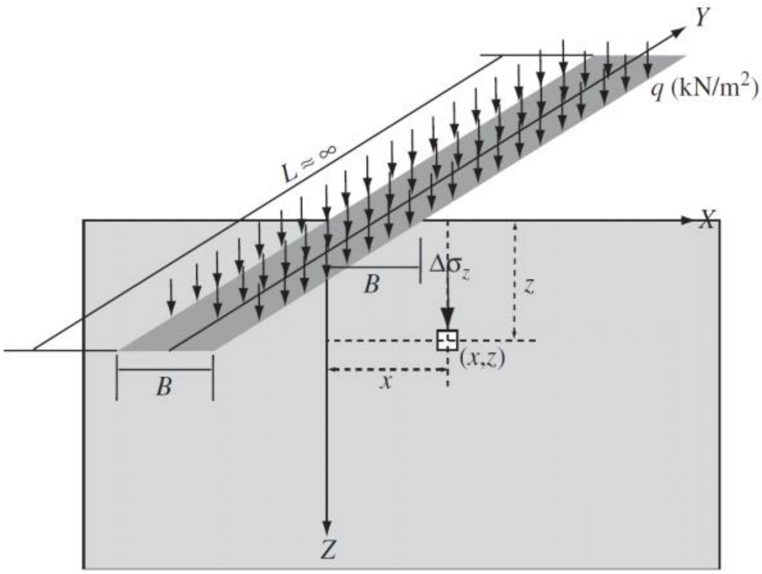


FIGURE 3.25 Stresses caused by a strip load.

Theoretically, a strip foundation is a rectangle of infinite length L and finite width B (i.e., $B/L \approx 0$). But foundations with $L/B > 10$ can be regarded as strip foundations. Examples of strip foundations include foundations for long structures such as retaining walls. A strip load can be thought of as a line load that is applied repeatedly and uniformly along the y-axis covering a width B as illustrated in Figure 3.25. This is a plane strain geometry in which the stresses in the x–z plane are independent of y. The units of a strip load are given as force per unit area, such as kN/m^2.

The vertical stress increase at any point (x,z) is given as:

$$\Delta\sigma_z = \frac{q}{\pi}\left\{\tan^{-1}\left(\frac{x}{z}\right) - \tan^{-1}\left(\frac{x-B}{z}\right) + \sin\left[\tan^{-1}\left(\frac{x}{z}\right) - \tan^{-1}\left(\frac{x-B}{z}\right)\right]\cos\left[\tan^{-1}\left(\frac{x}{z}\right) + \tan^{-1}\left(\frac{x-B}{z}\right)\right]\right\} \quad (3.12)$$

Consolidation Settlement Under Strip Footing (Example 1 - Normally Consolidated Clay)

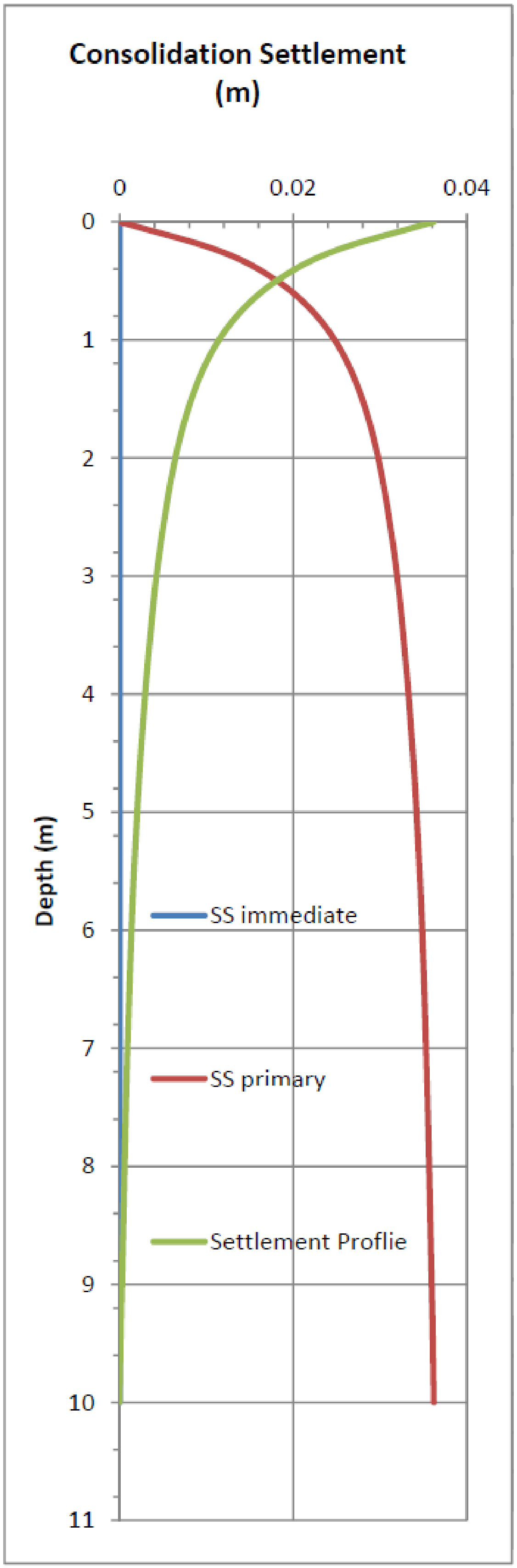
Tuesday, September 11, 2012
12:43 PM

q =	10	stress	(uniform stress)	
x =	0	length	(see Fig. 3.24, x = 0 for ctr. of ftg.)	
B =	0.5	length	(half width of footing)	
Δz =	0.25	length	(depth increment)	
z_w =	2	length	(depth to watertable)	
γ_w =	9.81	force/length^3	(unit weight of water)	(change system of units by changing unit weight of water)

depth	$\Delta\sigma_z$	γ	σ_v'	OCR	σ_p'	C_r	C_c	e_0	$\Sigma S_{immediate}$	$\Sigma S_{primary}$	ΣS_{Total}	Settlement Proflie
0.00001	10.000	20	0	0	0	0.01	0.2	1	0	0	0	3.62E-02
0.25	9.595	20	5.00	1.00	5.00	0.01	0.2	1	0.00E+00	1.16E-02	1.16E-02	2.46E-02
0.5	8.183	20	10.00	1.00	10.00	0.01	0.2	1	0.00E+00	1.81E-02	1.81E-02	1.81E-02
0.75	6.682	20	15.00	1.00	15.00	0.01	0.2	1	0.00E+00	2.21E-02	2.21E-02	1.41E-02
1	5.498	20	20.00	1.00	20.00	0.01	0.2	1	0.00E+00	2.48E-02	2.48E-02	1.14E-02
1.25	4.618	20	25.00	1.00	25.00	0.01	0.2	1	0.00E+00	2.66E-02	2.66E-02	9.61E-03
1.5	3.958	20	30.00	1.00	30.00	0.01	0.2	1	0.00E+00	2.79E-02	2.79E-02	8.26E-03
1.75	3.453	20	35.00	1.00	35.00	0.01	0.2	1	0.00E+00	2.90E-02	2.90E-02	7.24E-03
2	3.058	20	40.00	1.00	40.00	0.01	0.2	1	0.00E+00	2.98E-02	2.98E-02	6.44E-03
2.25	2.740	20	42.55	1.00	42.55	0.01	0.2	1	0.00E+00	3.04E-02	3.04E-02	5.76E-03
2.5	2.481	20	45.09	1.00	45.09	0.01	0.2	1	0.00E+00	3.10E-02	3.10E-02	5.18E-03
2.75	2.265	20	47.64	1.00	47.64	0.01	0.2	1	0.00E+00	3.15E-02	3.15E-02	4.68E-03
3	2.084	20	50.19	1.00	50.19	0.01	0.2	1	0.00E+00	3.20E-02	3.20E-02	4.24E-03
3.25	1.929	20	52.74	1.00	52.74	0.01	0.2	1	0.00E+00	3.24E-02	3.24E-02	3.85E-03
3.5	1.795	20	55.28	1.00	55.28	0.01	0.2	1	0.00E+00	3.27E-02	3.27E-02	3.50E-03
3.75	1.678	20	57.83	1.00	57.83	0.01	0.2	1	0.00E+00	3.30E-02	3.30E-02	3.19E-03
4	1.575	20	60.38	1.00	60.38	0.01	0.2	1	0.00E+00	3.33E-02	3.33E-02	2.91E-03
4.25	1.484	20	62.93	1.00	62.93	0.01	0.2	1	0.00E+00	3.36E-02	3.36E-02	2.66E-03
4.5	1.403	20	65.47	1.00	65.47	0.01	0.2	1	0.00E+00	3.38E-02	3.38E-02	2.43E-03
4.75	1.330	20	68.02	1.00	68.02	0.01	0.2	1	0.00E+00	3.40E-02	3.40E-02	2.22E-03
5	1.265	20	70.57	1.00	70.57	0.01	0.2	1	0.00E+00	3.42E-02	3.42E-02	2.02E-03
5.25	1.205	20	73.12	1.00	73.12	0.01	0.2	1	0.00E+00	3.44E-02	3.44E-02	1.85E-03
5.5	1.151	20	75.66	1.00	75.66	0.01	0.2	1	0.00E+00	3.45E-02	3.45E-02	1.68E-03
5.75	1.102	20	78.21	1.00	78.21	0.01	0.2	1	0.00E+00	3.47E-02	3.47E-02	1.53E-03
6	1.056	20	80.76	1.00	80.76	0.01	0.2	1	0.00E+00	3.48E-02	3.48E-02	1.39E-03
6.25	1.014	20	83.31	1.00	83.31	0.01	0.2	1	0.00E+00	3.50E-02	3.50E-02	1.26E-03
6.5	0.976	20	85.85	1.00	85.85	0.01	0.2	1	0.00E+00	3.51E-02	3.51E-02	1.13E-03
6.75	0.940	20	88.40	1.00	88.40	0.01	0.2	1	0.00E+00	3.52E-02	3.52E-02	1.02E-03
7	0.906	20	90.95	1.00	90.95	0.01	0.2	1	0.00E+00	3.53E-02	3.53E-02	9.12E-04
7.25	0.875	20	93.50	1.00	93.50	0.01	0.2	1	0.00E+00	3.54E-02	3.54E-02	8.11E-04
7.5	0.846	20	96.04	1.00	96.04	0.01	0.2	1	0.00E+00	3.55E-02	3.55E-02	7.15E-04
7.75	0.819	20	98.59	1.00	98.59	0.01	0.2	1	0.00E+00	3.56E-02	3.56E-02	6.26E-04
8	0.794	20	101.14	1.00	101.14	0.01	0.2	1	0.00E+00	3.57E-02	3.57E-02	5.41E-04
8.25	0.770	20	103.69	1.00	103.69	0.01	0.2	1	0.00E+00	3.57E-02	3.57E-02	4.60E-04
8.5	0.747	20	106.23	1.00	106.23	0.01	0.2	1	0.00E+00	3.58E-02	3.58E-02	3.84E-04
8.75	0.726	20	108.78	1.00	108.78	0.01	0.2	1	0.00E+00	3.59E-02	3.59E-02	3.12E-04
9	0.706	20	111.33	1.00	111.33	0.01	0.2	1	0.00E+00	3.60E-02	3.60E-02	2.43E-04
9.25	0.687	20	113.88	1.00	113.88	0.01	0.2	1	0.00E+00	3.60E-02	3.60E-02	1.78E-04
9.5	0.669	20	116.42	1.00	116.42	0.01	0.2	1	0.00E+00	3.61E-02	3.61E-02	1.16E-04
9.75	0.652	20	118.97	1.00	118.97	0.01	0.2	1	0.00E+00	3.62E-02	3.62E-02	5.66E-05
10	0.636	20	121.52	1.00	121.52	-----	-----	-----	0.00E+00	3.62E-02	3.62E-02	0.00E+00

Consolidation Settlement Under Strip Footing (Example 1 cont.)

Tuesday, September 11, 2012
12:43 PM



Consolidation Settlement Under Strip Footing (Example 2 cont.)

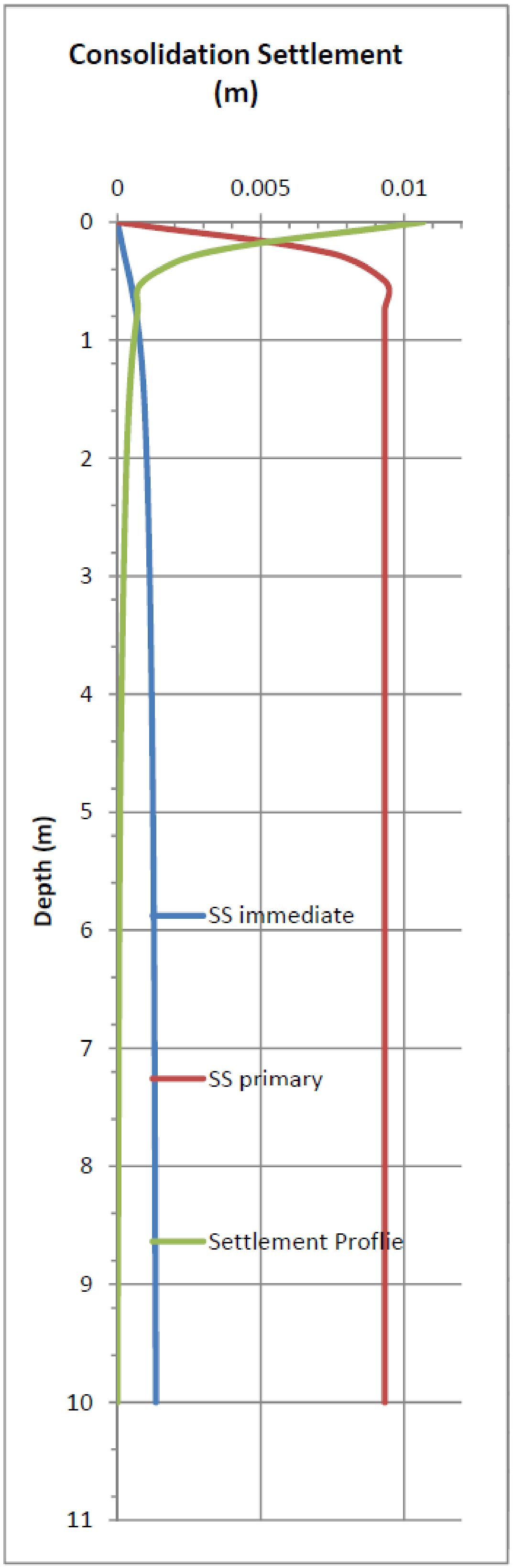
Tuesday, September 11, 2012
12:43 PM

q =	10 stress	(uniform stress)	
x =	0 length	(see Fig. 3.24, x = 0 for ctr. of ftg.)	
B =	0.5 length	(half width of footing)	
Δz =	0.25 length	(depth increment)	
z_w =	2 length	(depth to watertable)	
γ_w =	9.81 force/length^3	(unit weight of water)	(change system of units by changing unit weight of water)

depth	$\Delta\sigma_z$	γ	σ_v'	OCR	σ_p'	C_r	C_c	e_0	$\Sigma S_{immediate}$	$\Sigma S_{primary}$	ΣS_{Total}	Settlement Proflie
0.00001	10.000	20	0	0	0	0.01	0.2	1	0	0	0	1.07E-02
0.25	9.595	20	5.00	1.50	7.50	0.01	0.2	1	2.20E-04	7.23E-03	7.45E-03	3.21E-03
0.5	8.183	20	10.00	1.50	15.00	0.01	0.2	1	4.40E-04	9.32E-03	9.76E-03	9.04E-04
0.75	6.682	20	15.00	1.50	22.50	0.01	0.2	1	6.40E-04	9.32E-03	9.96E-03	7.04E-04
1	5.498	20	20.00	1.50	30.00	0.01	0.2	1	7.72E-04	9.32E-03	1.01E-02	5.72E-04
1.25	4.618	20	25.00	1.50	37.50	0.01	0.2	1	8.64E-04	9.32E-03	1.02E-02	4.80E-04
1.5	3.958	20	30.00	1.50	45.00	0.01	0.2	1	9.31E-04	9.32E-03	1.02E-02	4.13E-04
1.75	3.453	20	35.00	1.50	52.50	0.01	0.2	1	9.82E-04	9.32E-03	1.03E-02	3.62E-04
2	3.058	20	40.00	1.50	60.00	0.01	0.2	1	1.02E-03	9.32E-03	1.03E-02	3.22E-04
2.25	2.740	20	42.55	1.50	63.82	0.01	0.2	1	1.06E-03	9.32E-03	1.04E-02	2.88E-04
2.5	2.481	20	45.09	1.50	67.64	0.01	0.2	1	1.09E-03	9.32E-03	1.04E-02	2.59E-04
2.75	2.265	20	47.64	1.50	71.46	0.01	0.2	1	1.11E-03	9.32E-03	1.04E-02	2.34E-04
3	2.084	20	50.19	1.50	75.28	0.01	0.2	1	1.13E-03	9.32E-03	1.05E-02	2.12E-04
3.25	1.929	20	52.74	1.50	79.11	0.01	0.2	1	1.15E-03	9.32E-03	1.05E-02	1.92E-04
3.5	1.795	20	55.28	1.50	82.93	0.01	0.2	1	1.17E-03	9.32E-03	1.05E-02	1.75E-04
3.75	1.678	20	57.83	1.50	86.75	0.01	0.2	1	1.19E-03	9.32E-03	1.05E-02	1.59E-04
4	1.575	20	60.38	1.50	90.57	0.01	0.2	1	1.20E-03	9.32E-03	1.05E-02	1.45E-04
4.25	1.484	20	62.93	1.50	94.39	0.01	0.2	1	1.21E-03	9.32E-03	1.05E-02	1.33E-04
4.5	1.403	20	65.47	1.50	98.21	0.01	0.2	1	1.22E-03	9.32E-03	1.05E-02	1.21E-04
4.75	1.330	20	68.02	1.50	102.03	0.01	0.2	1	1.23E-03	9.32E-03	1.06E-02	1.11E-04
5	1.265	20	70.57	1.50	105.85	0.01	0.2	1	1.24E-03	9.32E-03	1.06E-02	1.01E-04
5.25	1.205	20	73.12	1.50	109.68	0.01	0.2	1	1.25E-03	9.32E-03	1.06E-02	9.23E-05
5.5	1.151	20	75.66	1.50	113.50	0.01	0.2	1	1.26E-03	9.32E-03	1.06E-02	8.41E-05
5.75	1.102	20	78.21	1.50	117.32	0.01	0.2	1	1.27E-03	9.32E-03	1.06E-02	7.65E-05
6	1.056	20	80.76	1.50	121.14	0.01	0.2	1	1.28E-03	9.32E-03	1.06E-02	6.94E-05
6.25	1.014	20	83.31	1.50	124.96	0.01	0.2	1	1.28E-03	9.32E-03	1.06E-02	6.28E-05
6.5	0.976	20	85.85	1.50	128.78	0.01	0.2	1	1.29E-03	9.32E-03	1.06E-02	5.67E-05
6.75	0.940	20	88.40	1.50	132.60	0.01	0.2	1	1.29E-03	9.32E-03	1.06E-02	5.10E-05
7	0.906	20	90.95	1.50	136.42	0.01	0.2	1	1.30E-03	9.32E-03	1.06E-02	4.56E-05
7.25	0.875	20	93.50	1.50	140.25	0.01	0.2	1	1.30E-03	9.32E-03	1.06E-02	4.05E-05
7.5	0.846	20	96.04	1.50	144.07	0.01	0.2	1	1.31E-03	9.32E-03	1.06E-02	3.58E-05
7.75	0.819	20	98.59	1.50	147.89	0.01	0.2	1	1.31E-03	9.32E-03	1.06E-02	3.13E-05
8	0.794	20	101.14	1.50	151.71	0.01	0.2	1	1.32E-03	9.32E-03	1.06E-02	2.70E-05
8.25	0.770	20	103.69	1.50	155.53	0.01	0.2	1	1.32E-03	9.32E-03	1.06E-02	2.30E-05
8.5	0.747	20	106.23	1.50	159.35	0.01	0.2	1	1.33E-03	9.32E-03	1.06E-02	1.92E-05
8.75	0.726	20	108.78	1.50	163.17	0.01	0.2	1	1.33E-03	9.32E-03	1.06E-02	1.56E-05
9	0.706	20	111.33	1.50	166.99	0.01	0.2	1	1.33E-03	9.32E-03	1.07E-02	1.22E-05
9.25	0.687	20	113.88	1.50	170.82	0.01	0.2	1	1.34E-03	9.32E-03	1.07E-02	8.91E-06
9.5	0.669	20	116.42	1.50	174.64	0.01	0.2	1	1.34E-03	9.32E-03	1.07E-02	5.80E-06
9.75	0.652	20	118.97	1.50	178.46	0.01	0.2	1	1.34E-03	9.32E-03	1.07E-02	2.83E-06
10	0.636	20	121.52	1.50	182.28	-----	-----	-----	1.34E-03	9.32E-03	1.07E-02	0.00E+00

Consolidation Settlement Under Strip Footing (Example 2 - Overconsolidated Clay)

Tuesday, September 11, 2012
12:43 PM



Time Rate of Consolidation

Tuesday, September 11, 2012
12:43 PM

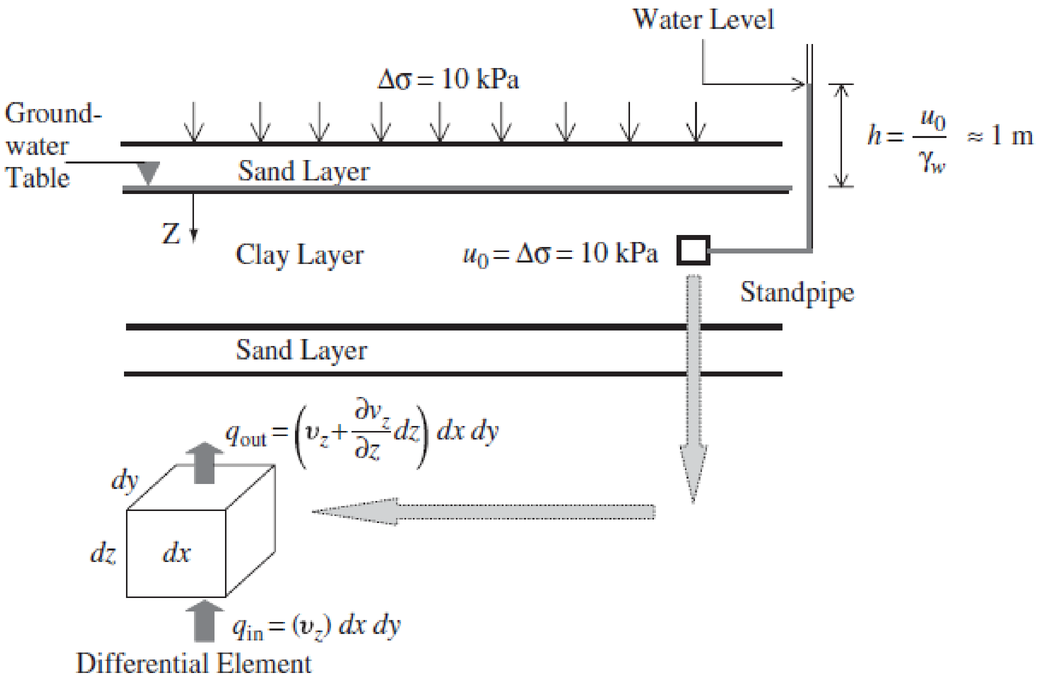


FIGURE 4.1 One-dimensional consolidation.

1. The clay is fully saturated and homogeneous.
2. Water compressibility is negligible.
3. The compressibility of soil grains is also negligible, but soil grains can be rearranged during consolidation.
4. The flow of water obeys *Darcy's law* ($v = ki$), where k is the soil permeability and i is the hydraulic gradient.
5. The total stress ($\Delta\sigma$) applied to the element is assumed to remain constant.
6. The coefficient of volume compressibility, m_v, is assumed to be constant.
7. The coefficient of permeability, k, for vertical flow is assumed to be constant.

$$c_v \frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t}$$

Governing Eq. for 1D Consolidation

We will discuss more about this equation when we cover seepage and groundwater flow

$$c_v = \frac{k}{m_v \gamma_w}$$

Coefficient of Consolidation

Time Rate of Consolidation (cont.)

Tuesday, September 11, 2012
12:43 PM

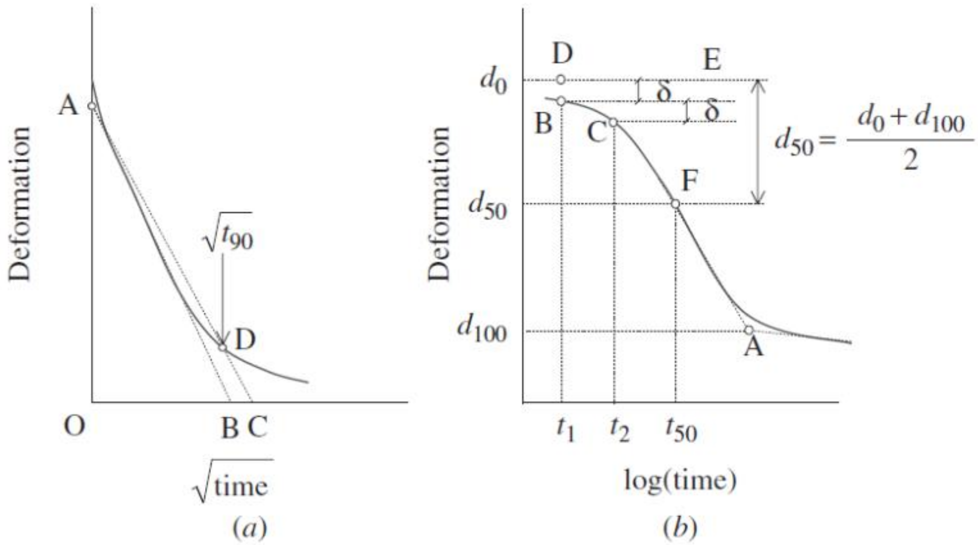


FIGURE 4.6 Graphical procedures for determining the coefficient of consolidation: (*a*) square-root-of-time method; (*b*) log-time method.

Assignment 5

Tuesday, September 11, 2012
12:43 PM

1. Solve Example 3.9 in the book for an infinite strip footing. Solve this using Eq. 3.12 using elastic theory in Excel (10 points) and solve it using FLAC (10 points). Compare the results (5 points).
2. Further develop the Excel spread sheet in problem 1 to perform consolidation settlement calculations for consolidation settlement underneath the strip footing. Verify your spread sheet using the examples found in the lecture notes. (10 points NC case, 10 points OC case)
3. Develop a 2D plane strain FLAC model to perform the settlement calculations for a strip footing placed on a normally consolidated clay. Use the nonlinear elastic method given in these lecture notes to calculate the non-linear elastic modulus as a function of stress level as given the class notes. Verify the 2D plane strain FLAC model results with the settlement calculations for a normally consolidated case from problem 2. For the loading condition, use a 10 kPa stress applied to the footing and a compression ratio, Cc, equal to 0.2. (30 points)

Mohr-Coulomb Model

Tuesday, September 11, 2012
12:43 PM

Mohr-Coulomb

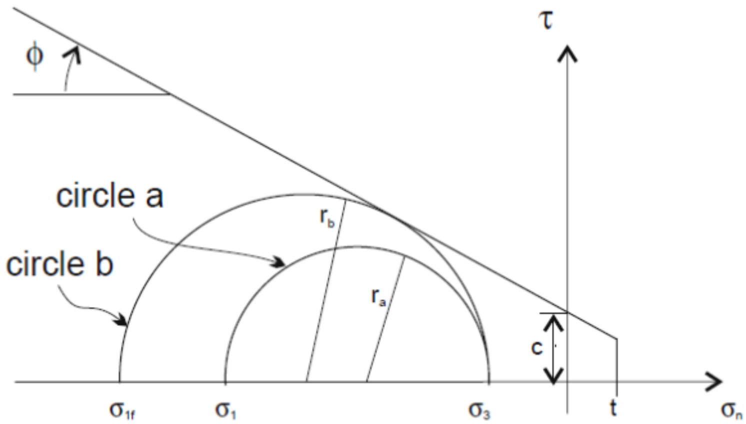


Table 3.7 Selected strength properties (drained, laboratory-scale) for soils [Ortiz et al., 1986]

	Cohesion	Friction Angle	
		Peak	Residual
	(kPa)	(degrees)	(degrees)
gravel	—	34	32
sandy gravel with few fines	—	35	32
sandy gravel with silty or clayey fines	1.0	35	32
mixture of gravel and sand with fines	3.0	28	22
uniform sand — fine	—	32	30
uniform sand — coarse	—	34	30
well-graded sand	—	33	32
low-plasticity silt	2.0	28	25
medium- to high-plasticity silt	3.0	25	22
low-plasticity clay	6.0	24	20
medium-plasticity clay	8.0	20	10
high-plasticity clay	10.0	17	6
organic silt or clay	7.0	20	15

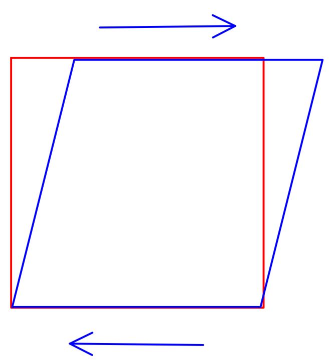
Post-Failure - Dilation Angle

Wednesday, August 17, 2011
12:45 PM

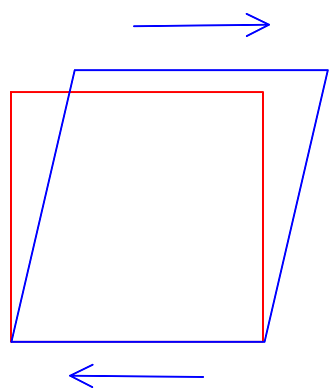
How does dilatancy affect the behavior of soil?

The **angle of dilation** controls an amount of plastic volumetric strain developed during plastic shearing and is assumed constant during plastic yielding. The value of $\psi=0$ corresponds to the volume preserving deformation while in shear. Clays (regardless of overconsolidated layers) are characterized by a very low amount of dilation ($\psi\approx0$). As for sands, the angle of dilation depends on the angle of internal friction. For non-cohesive soils (sand, gravel) with the angle of internal friction $\varphi>30°$ the value of dilation angle can be estimated as $\psi=\varphi-30°$. A negative value of dilation angle is acceptable only for rather loose sands. In most cases, however, the assumption of $\psi = 0$ can be adopted.

Pasted from <http://www.finesoftware.eu/geotechnical-software/help/fem/angle-of-dilation/>



No dilatancy, dilatancy angle = 0. Note that the unit square has undergone distortion solely.

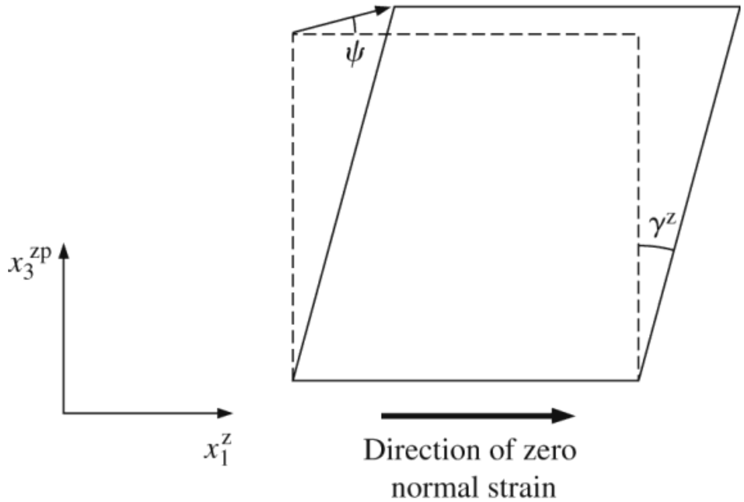


Dilatancy during shear. Note that the unit square has undergone distortion and volumetric strain (change in volume).

Post-Failure - Dilation Angle (cont.)

Wednesday, August 17, 2011
12:45 PM

Soils dilate (expand) or contract upon shearing and the degree of this dilatancy can be explained by the dilatancy angle, ψ.



This element is dilating during shear. **This is plastic behavior**.

(Salgado: The Engineering of Foundations, p. 132)

The dilatancy angle can be calculated from the Mohr's circle of strain, or from the triaxial test, see later. It can also be estimated from the following formulas, if the volumetric and maximum shear strain increments are known.

$$\sin \psi = \frac{OZ}{|OA_1|} = -\frac{\frac{1}{2}(d\varepsilon_1 + d\varepsilon_3)}{\frac{1}{2}(d\varepsilon_1 - d\varepsilon_3)} = -\frac{d\varepsilon_1 + d\varepsilon_3}{d\varepsilon_1 - d\varepsilon_3} = -\frac{d\varepsilon_v}{|d\gamma_{max}|}$$

$$\tan \psi = \frac{OZ}{|ZA_1|} = -\frac{\frac{1}{2}d\varepsilon_v}{|\frac{1}{2}d\gamma^z|} = -\frac{d\varepsilon_v}{|d\gamma^z|}$$

(Salgado: The Engineering of Foundations, p. 132)

Post-Failure Behavior, Dilation Angle from Triaxial Test

Tuesday, September 11, 2012
12:43 PM

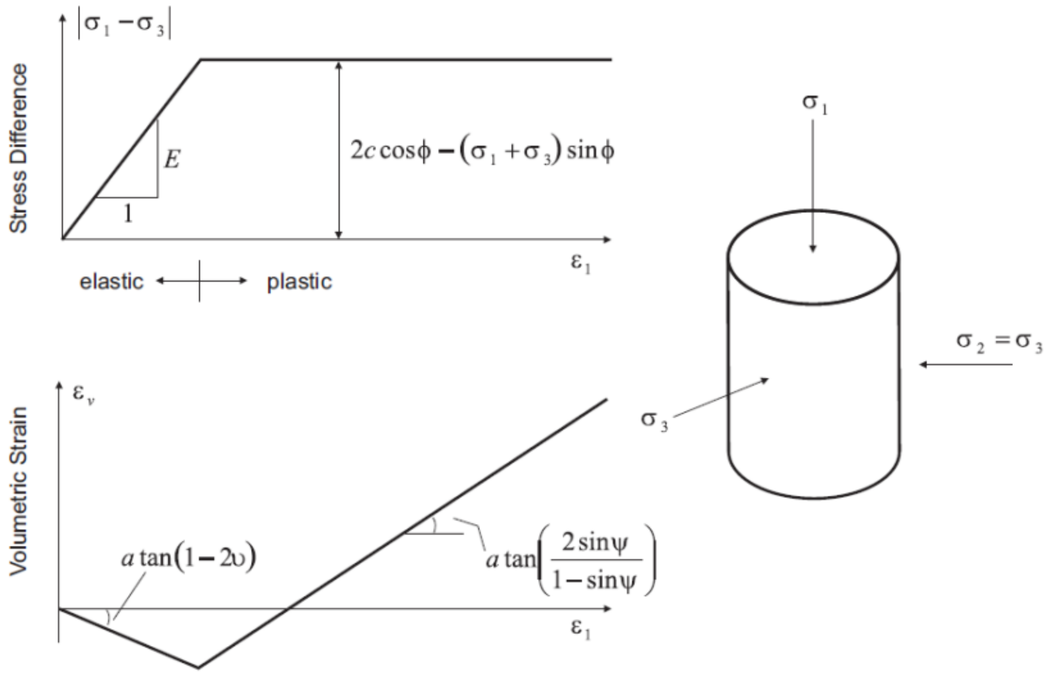


Figure 3.58 Idealized relation for dilation angle, ψ, from triaxial test results [Vermeer and de Borst (1984)]

(Flac v. 5 User Manual)

Shear dilatancy, or dilatancy, is the change in volume that occurs with shear distortion of a material. Dilatancy is characterized by a dilation angle, ψ, which is related to the ratio of plastic volume change to plastic shear strain. This angle can be specified in the Mohr-Coulomb, ubiquitous-joint, strain-hardening/softening, bilinear ubiquitous-joint and double-yield plasticity models. The dilation angle is typically determined from triaxial tests or shear box tests. For example, the idealized relation for dilatancy, based upon the Mohr-Coulomb failure surface, is depicted for a triaxial test in Figure 3.58. The dilation angle is found from the plot of volumetric strain versus axial strain. Note that the initial slope for this plot corresponds to the elastic regime, while the slope used to measure the dilation angle corresponds to the plastic regime.

Table 3.8 Typical values for dilation angle [Vermeer and de Borst (1984)]

dense sand	15°
loose sand	< 10°
normally consolidated clay	0°
granulated and intact marble	12° – 20°
concrete	12°

(Flac v. 5 User Manual)

Post-Failure Behavior, Dilation Angle from Triaxial Test

Tuesday, September 11, 2012
12:43 PM

Bolton (1986) also codified the dependence of the peak friction angle ϕ_p on intrinsic soil variables and soil state variables for both triaxial and plane-strain compression conditions. He found that the following equation quantified the peak friction angle very well for both plane-strain and triaxial compression conditions:

$$\phi_p = \phi_c + A_\psi I_R = \phi_c + [5 - 2(k - 1)]I_R \qquad (5.16)$$

where, as before, $k = 1$ for plane-strain and 2 for triaxial-compression conditions; and so $A_\psi = 3$ for triaxial conditions and $A_\psi = 5$ for plane-strain conditions.[9]

Bolton (1986) also found that, for plane-strain conditions, he could use the following equation to calculate approximate values of ϕ_p:

$$\phi_p = \phi_c + 0.8\psi_p \qquad (5.17)$$

What Eqs. (5.16) and (5.17), considered together, suggest is that, for plane-strain compression, $0.8\psi_p$ is equivalent to $\phi_p - \phi_c = A_\psi I_R$ with $A_\psi = 5$. If we now consider triaxial compression under conditions leading to the same value of I_R, knowing that $A_\psi = 3$, we should have the difference between ϕ_p and ϕ_c be 60% of the value of this difference for plane-strain compression, or $0.6 \times 0.8 = 0.48\psi_p$. So, for triaxial compression, instead of Eq. (5.17), we use the following equation to approximate ϕ_p for a known peak dilatancy angle:

$$\phi_p = \phi_c + 0.48\psi_p \approx \phi_c + 0.5\psi_p \qquad (5.18)$$

(Salgado: The Engineering of Foundations, p. 132)

Plane strain conditions

$\phi_p - \phi_c = 0.8\ \psi_p$

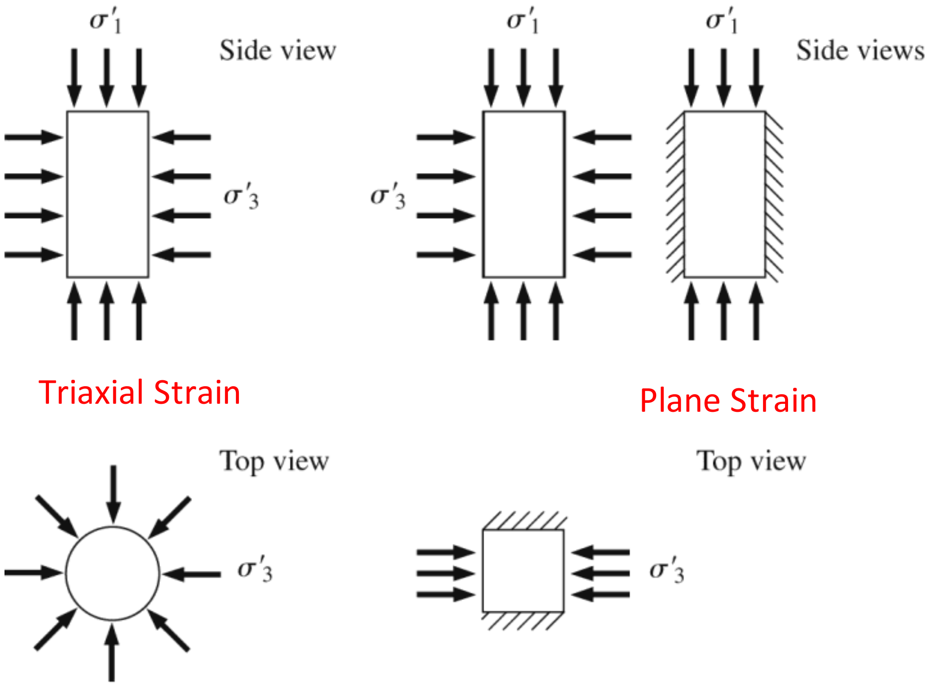
ϕ_P = peak friction angle (used in FLAC as command friction =)
ϕ_C = critical state friction angle (approx. 28 to 36 degrees quartz sand)
ψ_P = peak dilation angle (used in FLAC as dilation =)

Triaxial (i.e., axisymmetrical) conditions

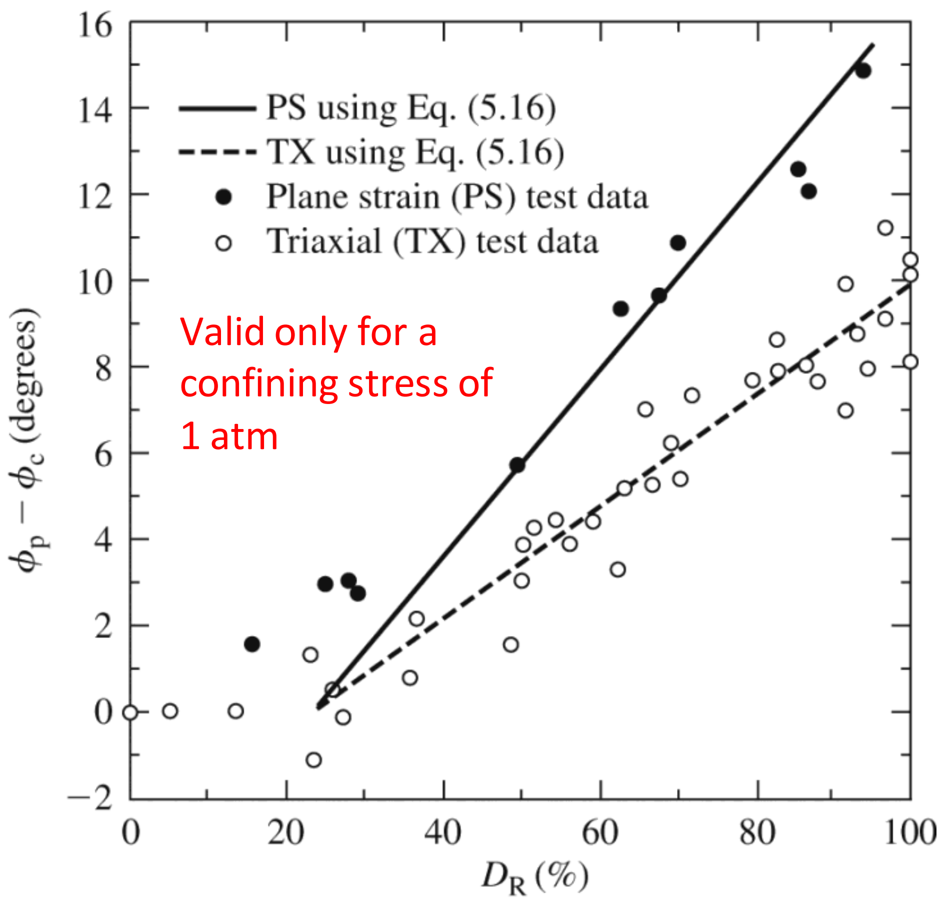
$\phi_p - \phi_c = 0.5\ \psi_p$

Plane Strain vs. Triaxial Strain Conditions

Wednesday, August 17, 2011
12:45 PM



(a) (b)



(See Eq. 5-16 in book to relate ϕ_p and ϕ_c)

ϕ_p = peak friction angle
ϕ_C = critical state friction angle

(Salgado: The Engineering of Foundations)

Wednesday, August 17, 2011
12:45 PM

Iteration to estimate peak friction angle from stress state and void ratio

- Practical application
  - If we know the critical state friction angle of a soil, the horizontal earth pressure coefficient Ko, and the relative density of the deposits, we can estimate the peak friction angle. This is valuable for design because most often, the peak friction angle is used to define the strength of the soil in foundation calculations.

Problem 5-12 A deposit of clean sand has unit weight equal to 22 kN/m^3. The relative density increases approximately linearly from 60% at the surface of the deposit to 75% at a depth of 10 m. K_0 is 0.45 for this deposit. The sand can be assumed to have $\phi_c = 30^\circ$, $Q = 10$ and $R_Q = 1$. Calculate and plot the values of ϕ_p under triaxial and plane-strain compression conditions between 0 and 10 m depth assuming $\sigma'_c = \sigma'_m$. Consider the water table to be very deep (deeper than 10 m).

Let us consider the depth 1m from surface to show the sample calculation.

At 1m vertical stress $\sigma'_v = 1 \times 22 = 22\text{kPa}$

Given K_0 = 0.45 for this deposit. So horizontal stress at 1m

$\sigma'_h = 0.45 \times 22 = 9.9\text{kPa}$

Mean stress $\sigma'_m = \dfrac{\sigma'_v + 2\sigma'_h}{3} = \dfrac{22 + 2(9.9)}{3} = 13.9\text{kPa}$

Now assuming mean stress σ'_m = consolidation stress σ'_c

Let's assume ϕ_p=40°, then:

$$N = \frac{\sigma'_{1p}}{\sigma'_{3p}} = \frac{1+\sin\phi_P}{1-\sin\phi_P} = \frac{1+\sin 40^\circ}{1-\sin 40^\circ} = 4.6 \rightarrow \sigma'_{1p} = 63.9\,\text{kPa}$$

Mean stress at the end of consolidation phase for Ko condition

$$\sigma'_{mp} = \frac{\sigma'_{1p} + 2\sigma'_{3p}}{3} = \frac{63.9 + 2\times 13.9}{3} = 30.6\text{kPa}$$

The mean effective stress (in situ) was used to calculate the average consolidation stress for the sample because the soil has been anisotrophically consolidated in situ. Anisotropic consolidation better represents the actual conditions. Such consolidaton is also called Ko consolidated.

(Salgado: The Engineering of Foundations)

Estimation of the peak friction angle from critical state friction angle

Wednesday, August 17, 2011
12:45 PM

$$D_R\big|_{x=1m} = 60 + \frac{75-60}{10} \times 1 = 61.5\%$$

(Salgado: The Engineering of Foundations)

$$I_D = \frac{D_R}{100} = \frac{61.5}{100} = 0.615$$

$A_\Psi = 3$ (triaxial conditions) , $p_A = 100\text{kPa}$, $Q = 10$ and $R_Q = 1$

Therefore:

$$I_R = I_D\left[Q - \ln\left(\frac{100\sigma'_{mp}}{p_A}\right)\right] - R_Q = 0.615\left[10 - \ln\left(\frac{100 \times 30.6}{100}\right)\right] - 1 = 3.046$$

$$\phi_p = \phi_c + A_\psi I_R = 30° + 3 \times 3.046 = 39.1°$$

Note that in the above example, the peak friction angle calculate from the above equation, is not consistent with the assumed value of 40 degrees. Thus, the mean stress of 30.6 is somewhat inconsistent with the calculated peak friction angle of 39.1 degrees. Hence, another iteration is required. This is done by adjusting the assumed peak friction angle to 39.1 degrees and recalculating the mean stress and resulting friction angle until convergence is reached. In practice, friction angles are usually reported to the nearest whole number, so once the iteration converges to a stable whole number value, then iteration can stop.

OR

We can use the charts on the next page to estimate the difference between the peak and critical state friction angle as a function of effective confining stress.

Estimation of the peak friction angle from critical state friction angle

Wednesday, August 17, 2011
12:45 PM

The calculation shown in Example 5-2 can be done for wide ranges of density and stress so that a chart can be prepared for ϕ_p as a function of D_R and σ'_3. Figure 5-13 shows charts that may be used to estimate ϕ_p given values of D_R, ϕ_c,

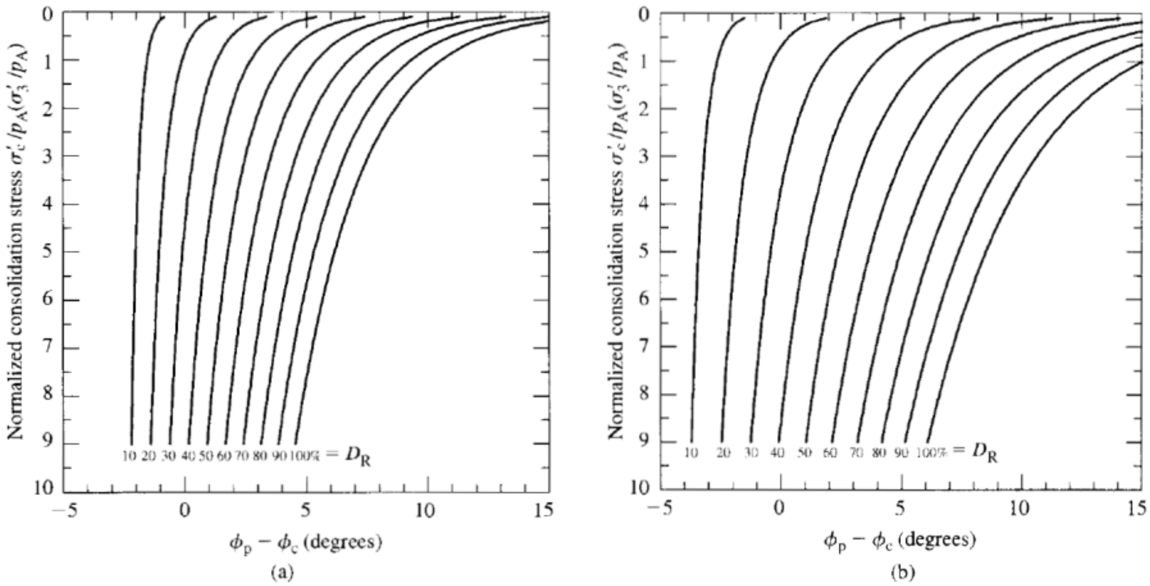


Figure 5-13
Peak friction angle in (a) triaxial compression and (b) plane-strain compression as a function of stress state, relative density, and critical-state friction angle using $Q = 10$ and $R_Q = 1$.

(Salgado: The Engineering of Foundations)

p' vs. q' plots in 2D space

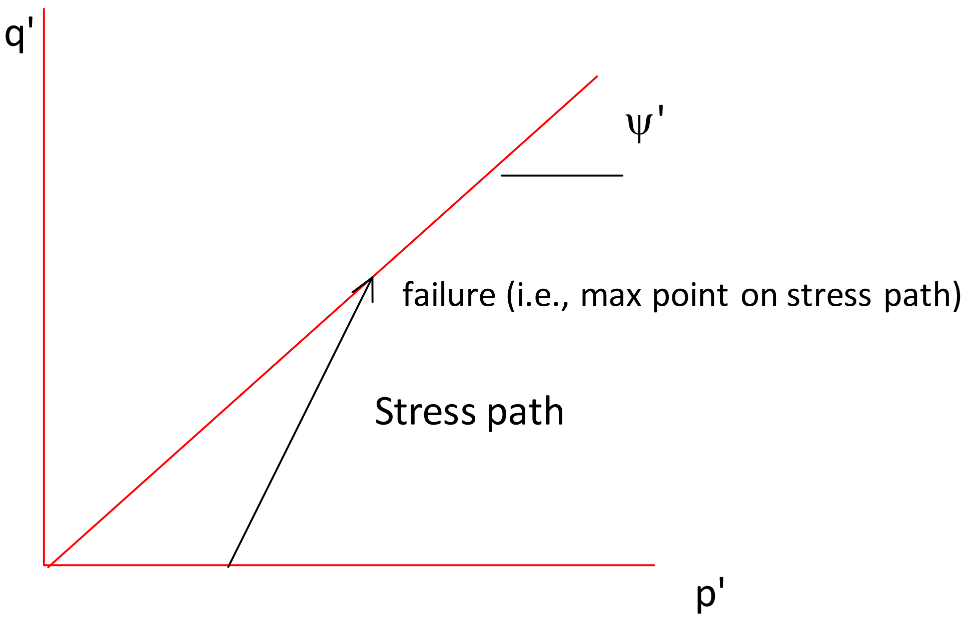
Tuesday, September 11, 2012
12:43 PM

For $\sigma_2 = \sigma_3$ and no pore pressure present in model

then

$p' = (\sigma_1 + \sigma_3) / 2$

$q' = (\sigma_1 - \sigma_3) / 2$



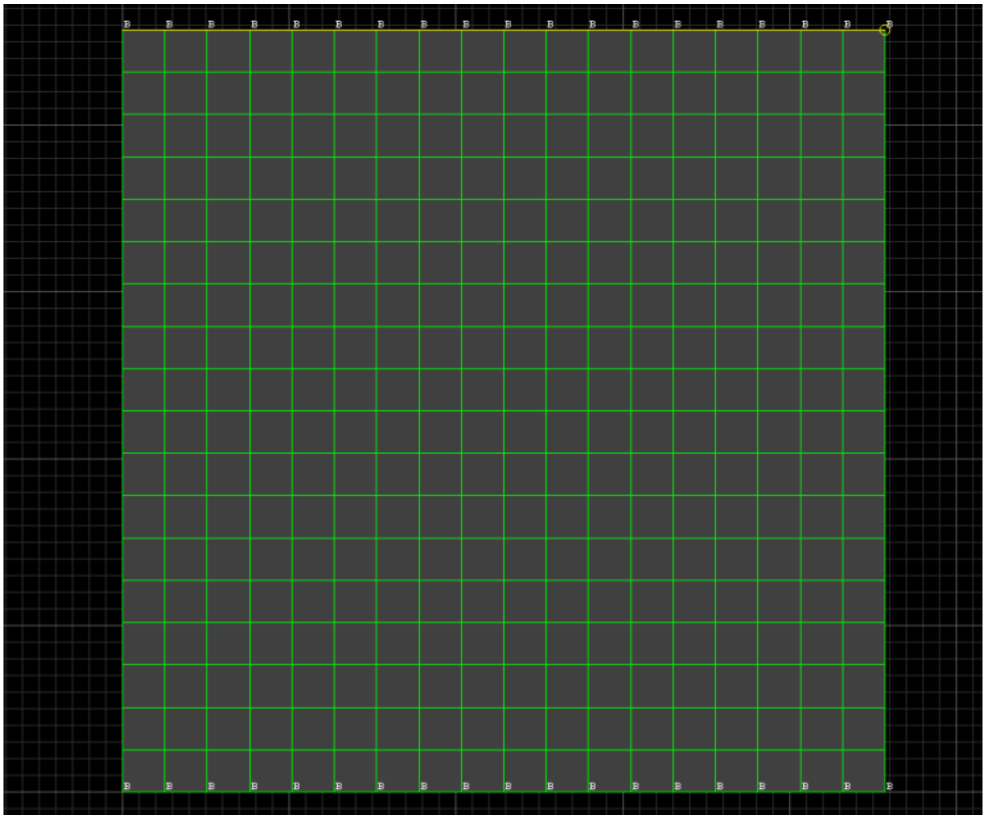
For c = o

$\sin \phi' = \tan \psi'$

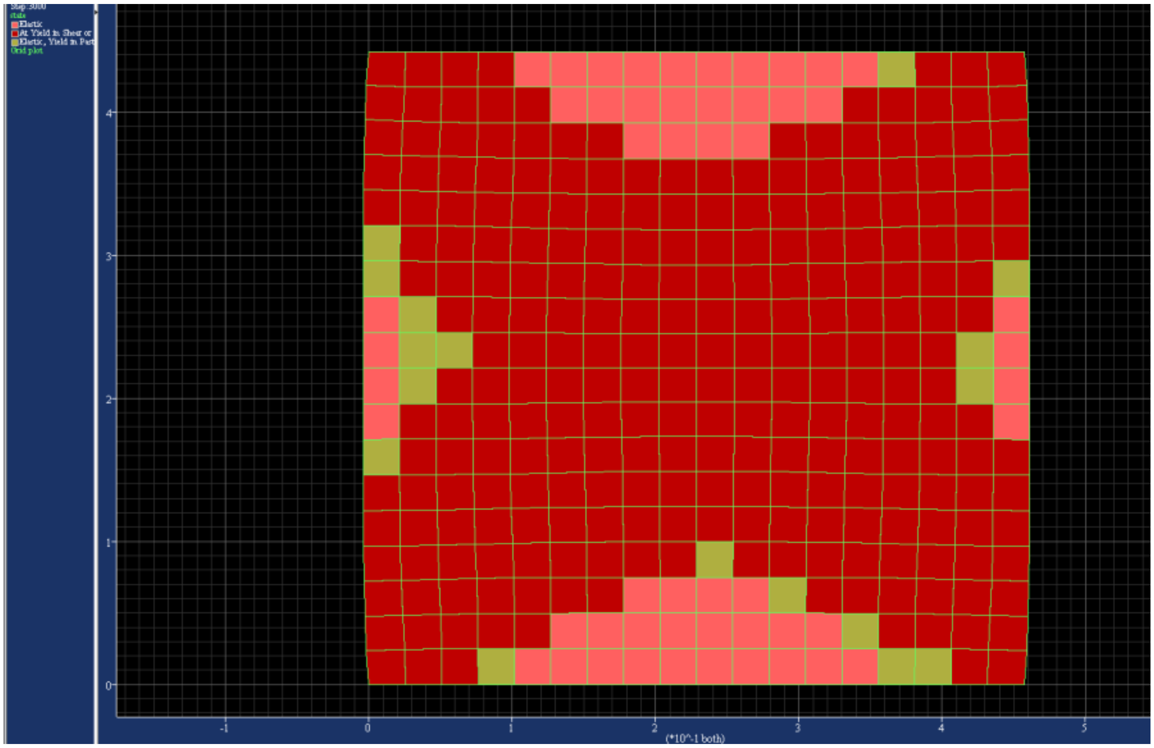
$\sin^{-1} \phi' = q'_f / p'_f$

Mohr - Coulomb Model in FLAC

Tuesday, September 11, 2012
12:43 PM



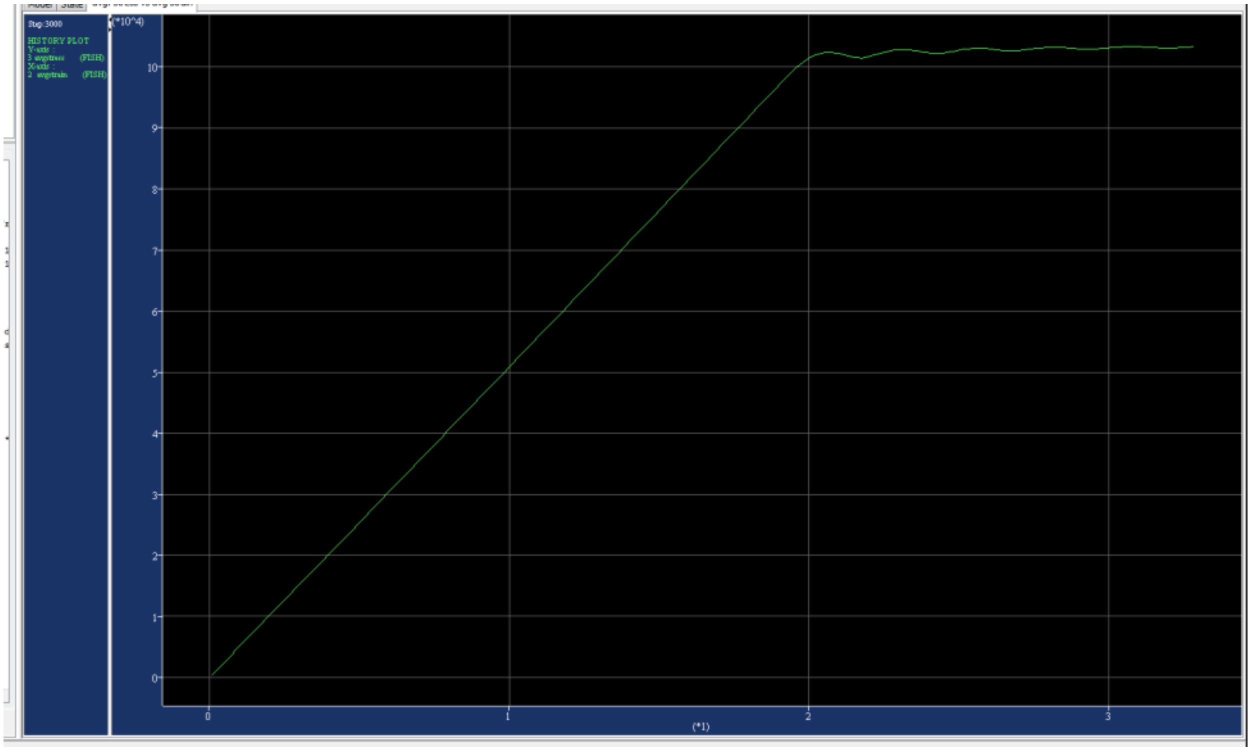
Initial State



Deformed State

Mohr - Coulomb Model in FLAC

Tuesday, September 11, 2012
12:43 PM



Axial Stress versus Axial Strain

Does this relationship show the correct values of:

a. Cohesion at failure?
b. Young's modulus?

Verify these questions by used the above plot to confirm that cohesion and Young's modulus have been appropriately represented.

FLAC Code for Unconfined Compression Test

Tuesday, September 11, 2012
12:43 PM

```
config
set = large; large strain mode
grid 18,18; for 18" x 18" EPS block
model mohr
prop density = 20 bulk = 2.08e6 shear = 2.27e6 cohesion=50e3 friction=0 dilation=0 tension =
100e3; EPS properties
;
ini x mul 0.0254; makes x grid dimension equal to 0.0254 m or 1 inch
ini y mul 0.0254; makes y grid dimension equal to 0.0254 m or 1 inch
;fix y j 1; fixes base in y direction only
fix x y j 1 ;fixes base in x and y direction only
;fix  y i 8 12 j 1 ; fixes only part of base
his unbal 999
;apply yvelocity -5.0e-6 from 1,19 to 19,19 ;applies constant downward velocity to simulate a
strain-controlled test
apply yvelocity -5.0e-6 xvelocity=0 from 1,19 to 19,19 ;applies constant downward velocity to
simulate a strain-controlled test
def verticalstrain; subroutine to calculate vertical strain
 whilestepping
 avgstress = 0
 avgstrain = 0
 loop i (1,izones)
   loop j (1,jzones)
     vstrain = ((0- ydisp(i,j+1) - (0 - ydisp(i,j)))/0.0254)*100 ; percent strain
     vstress = syy(i,j)*(-1)
     avgstrain = avgstrain + vstrain/18/18
     avgstress = avgstress + vstress/18/18
   end_loop
 end_loop
end
his avgstrain 998
his avgstress 997
;step 3000
history 999 unbalanced
cycle 3000
```

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Tuesday, September 11, 2012
12:43 PM

Critical State Soil Mechanics (CSSM)

Thursday, March 11, 2010
11:43 AM

Critical-State Soil Mechanics For Dummies

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2006

- **Critical-state soil mechanics is an effective stress framework describing mechanical soil response.**
- **In its simplest form here, we consider only shear-induced loading.**
- **We tie together two well-known concepts: (1) one-dimensional consolidation behavior, represented by (e-log σ_v') curves; and (2) shear stress-vs. normal stress ($\tau-\sigma_v$') from direct shear box or simple shearing.**
- **Herein, only the bare essence of CSSM concepts are presented, sufficient to describe strength & compressibility response.**

Background

Thursday, March 11, 2010
11:43 AM

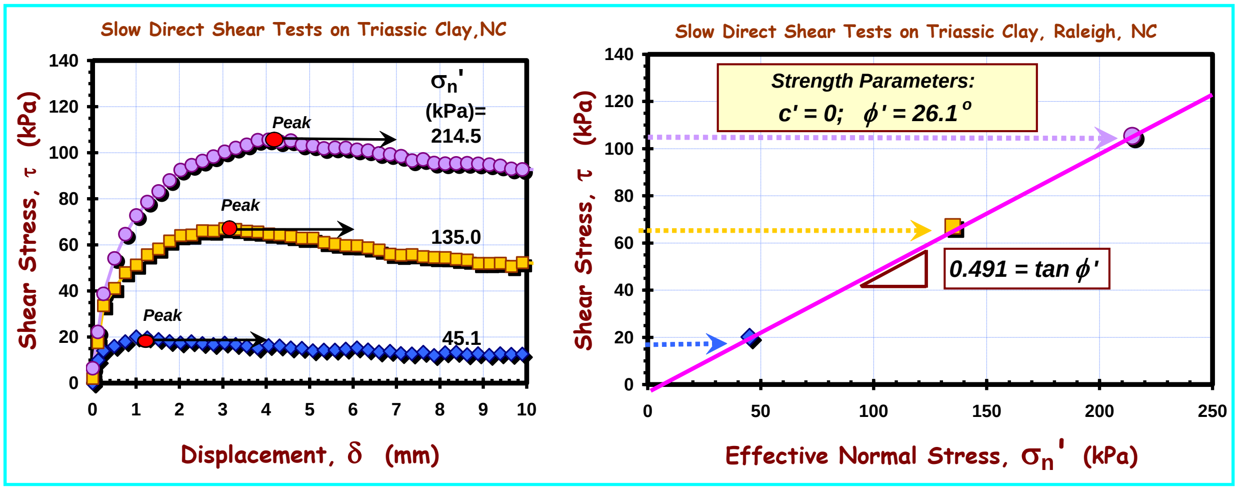
In an attempt to advance soil testing techniques, Kenneth Harry Roscoe of Cambridge University, in the late forties and early fifties, developed a simple shear apparatus in which his successive students attempted to study the changes in conditions in the shear zone both in sand and in clay soils. In 1958 a study of the yielding of soil based on some Cambridge data of the simple shear apparatus tests, and on much more extensive data of triaxial tests at Imperial College, London, led to the publication of the critical state concept (Roscoe, Schofield & Wroth 1958). Subsequent to this 1958 paper, concepts of plasticity were introduced by Schofield and published later in a classic text book (Schofield & Wroth 1968).

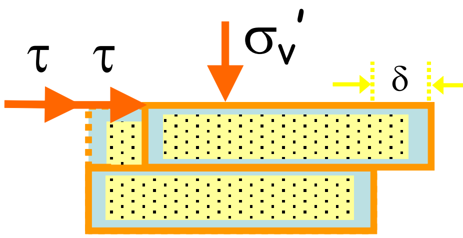
Pasted from <http://en.wikipedia.org/wiki/Critical_state_soil_mechanics>

Shear Strength Theory

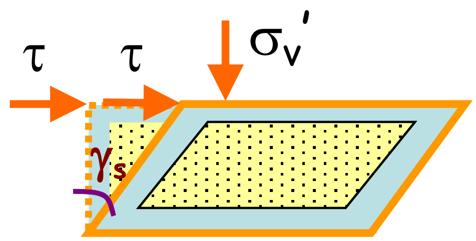
Thursday, March 11, 2010
11:43 AM

Direct Shear Test Results





Direct Shear Box (DSB)



Direct Simple Shear (DSS)

Shear Strength Theory

Thursday, March 11, 2010
11:43 AM

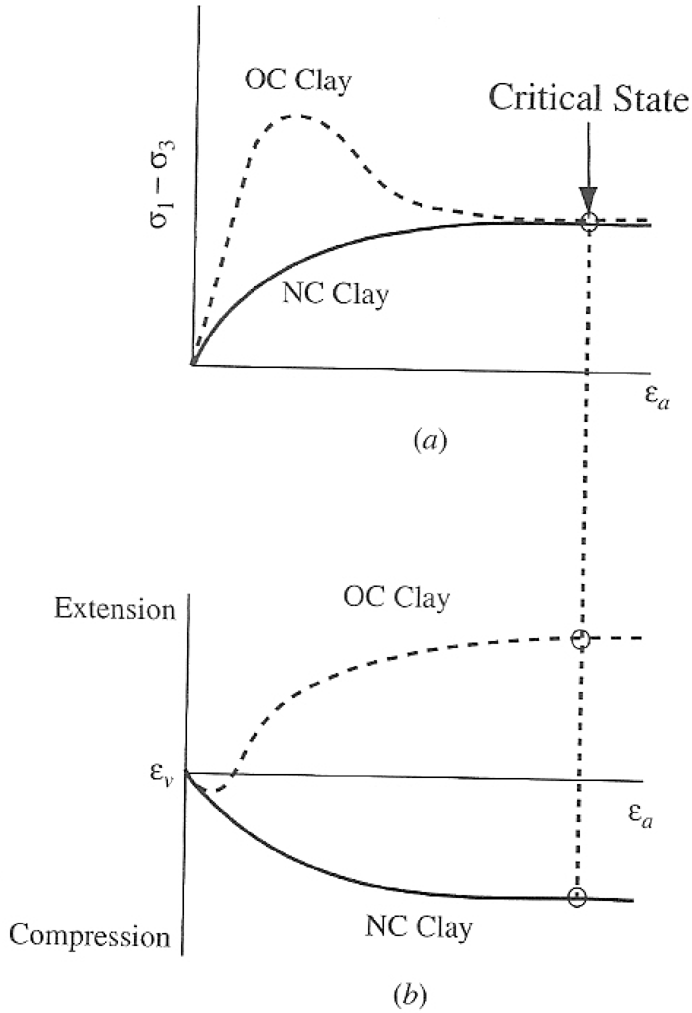


FIGURE 2.8 Critical-state definition.

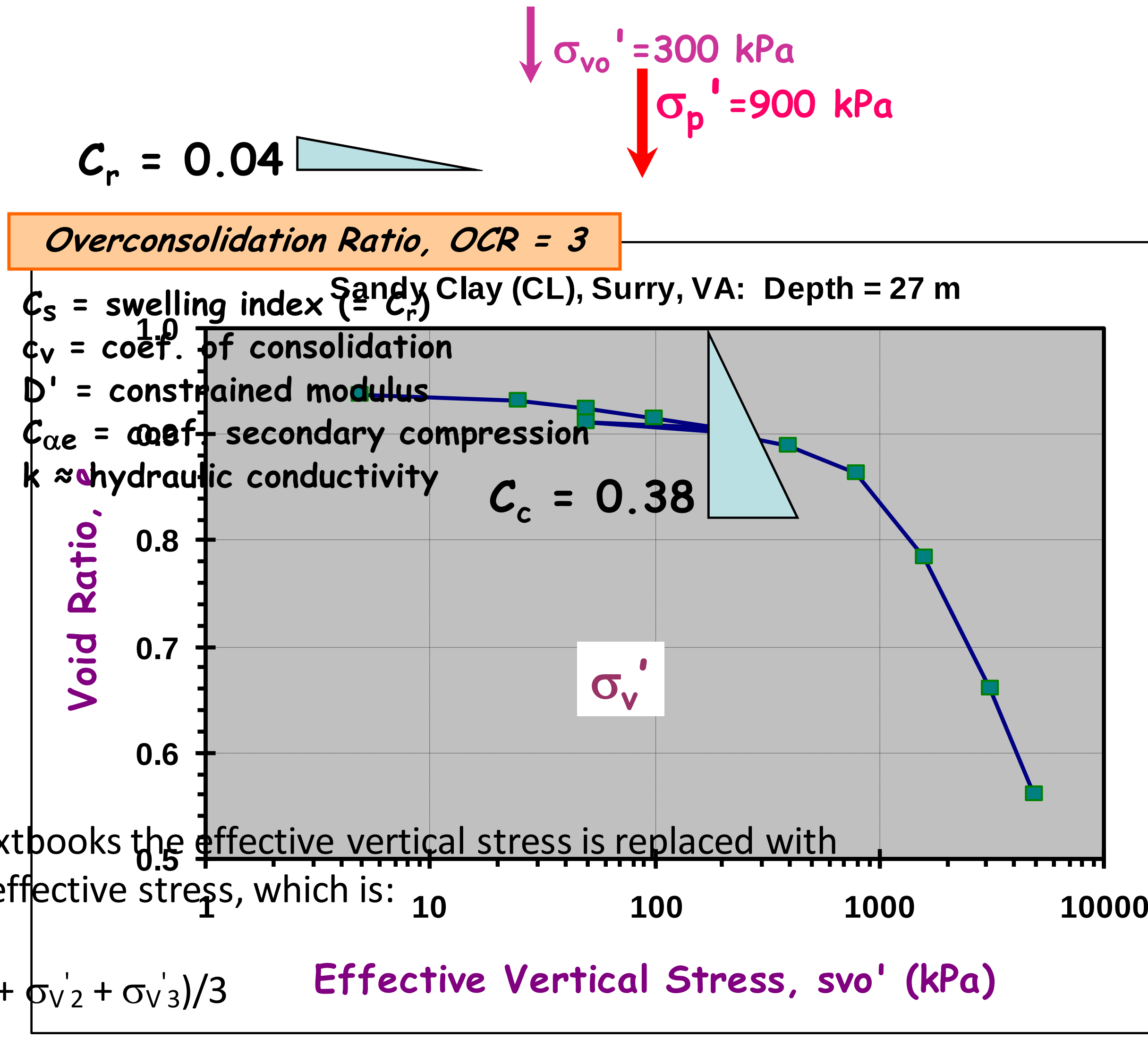
The critical Ssate concept states that soils and other granular materials, if continuously distorted (sheared) until they flow as a frictional fluid, will come into a well-defined critical state. At the onset of the critical state, shear distortions occur without any further changes in mean effective stress or deviatoric stress or void ratio. The void ratio at the critical state is called the critical state void ratio.

Pasted from <http://en.wikipedia.org/wiki/Critical_state_soil_mechanics>

1D Consolidation Theory

Thursday, March 11, 2010
11:43 AM

One-Dimensional Consolidation

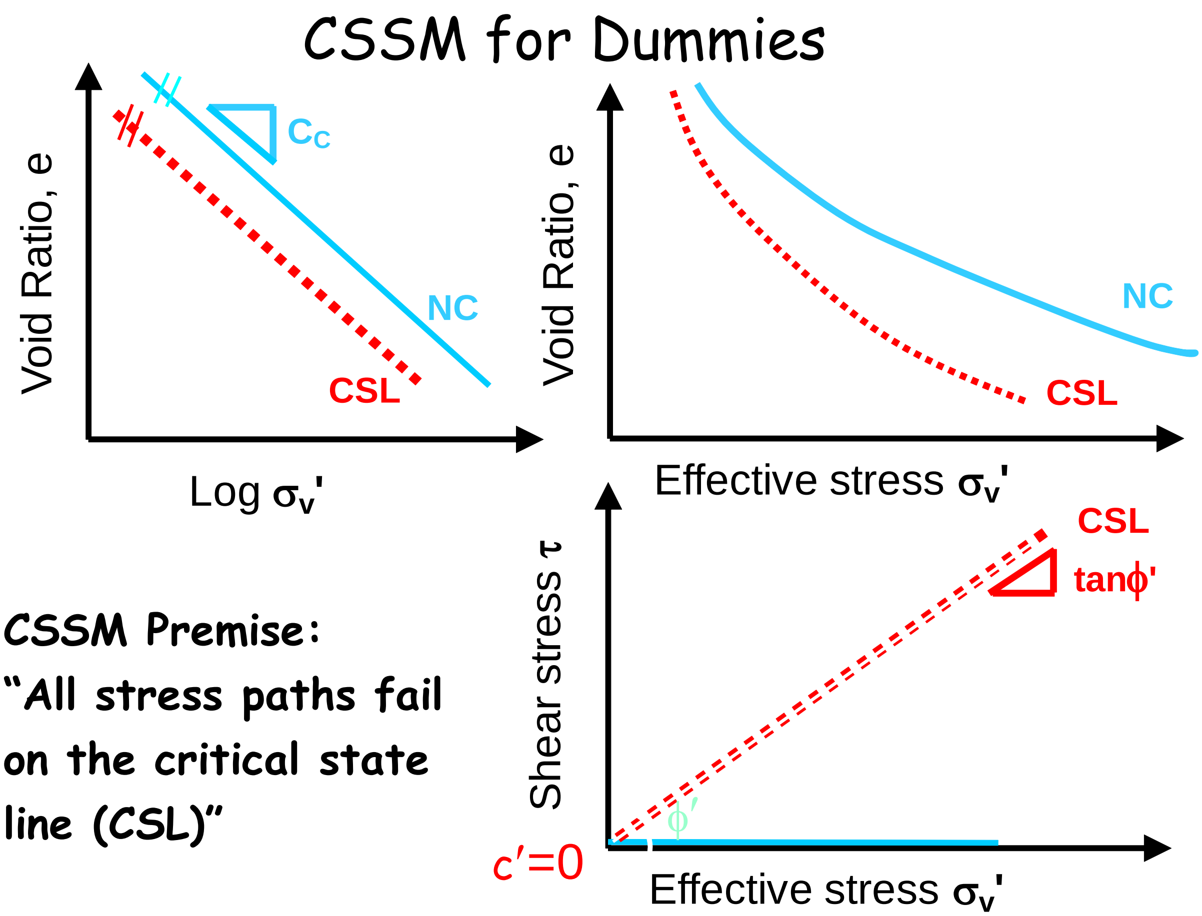


In some textbooks the effective vertical stress is replaced with the mean effective stress, which is:

$\sigma_m' = (\sigma_{V'1} + \sigma_{V'2} + \sigma_{V'3})/3$

Comparison of Critical State Line with Normally Consolidated

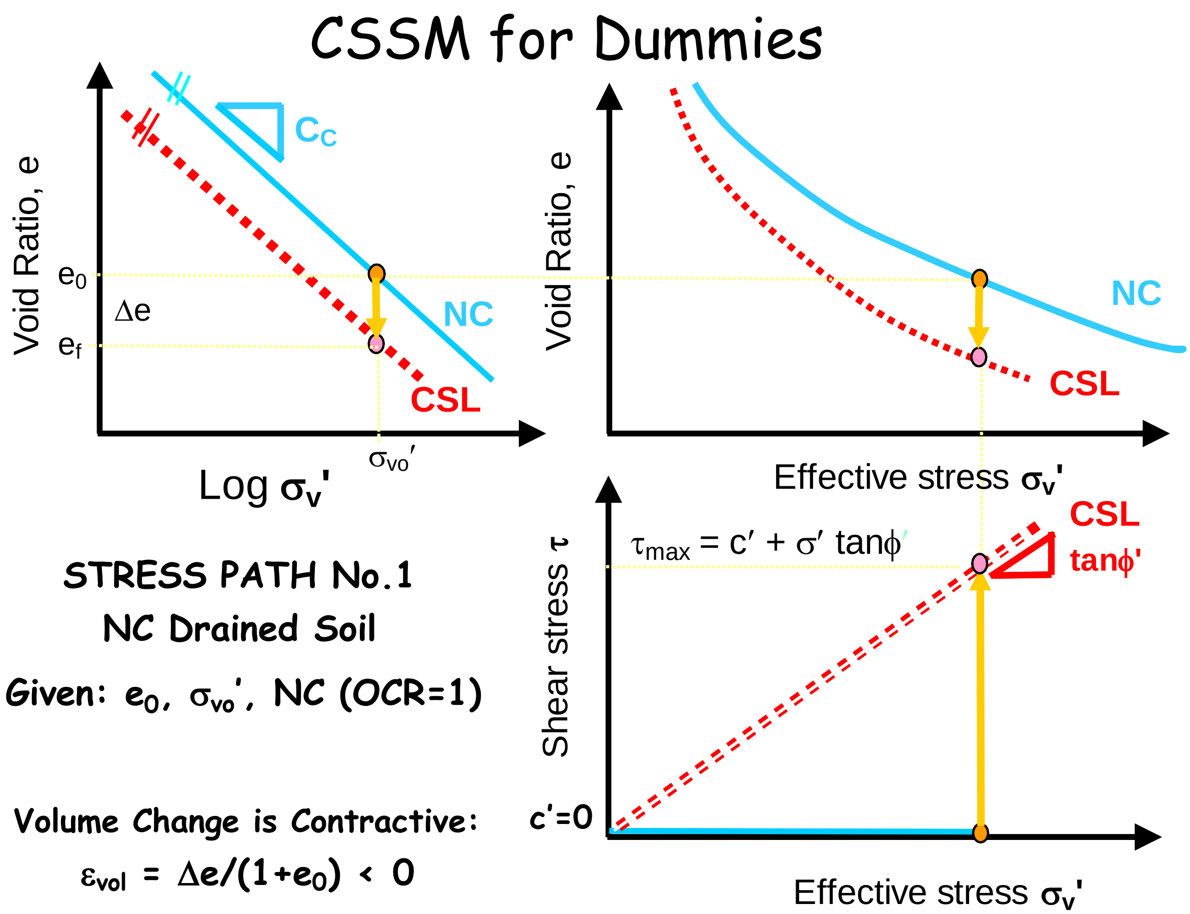
Thursday, March 11, 2010
11:43 AM



The above statement means that the critical state line forms an envelope that defines the failure state of soil. This failure or critical state is a function of the state of stress (vertical or mean effective stress and shear stress) and the void ratio.

Stress Path for Normally Consolidated Drained Soil in DSS

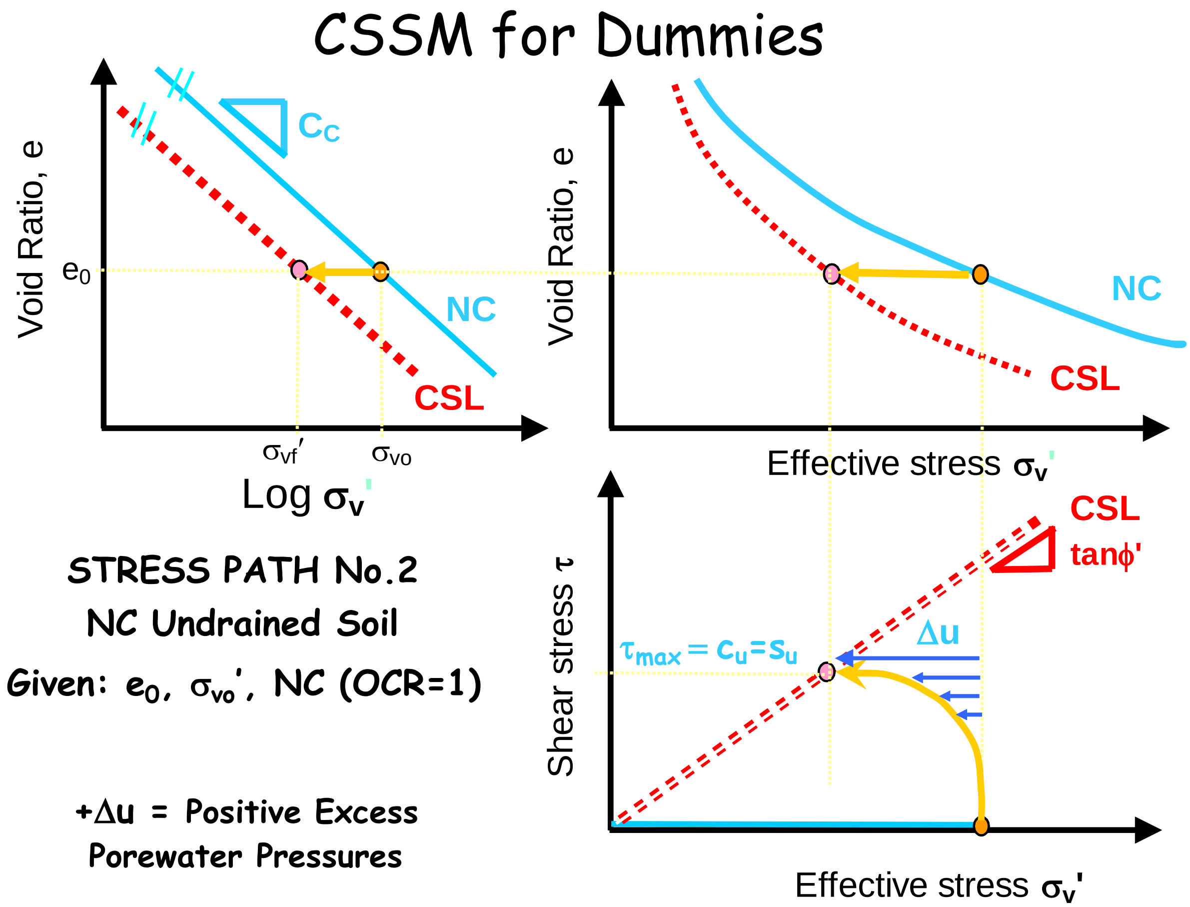
Thursday, March 11, 2010
11:43 AM



The above stress path is straight because there is no excess pore pressure generated during shear because the test is drained and the applied vertical stress is not changing during the direct shear mode of failure.

Stress Path for Normally Consolidated Undrained Soil (DSS)

Thursday, March 11, 2010
11:43 AM

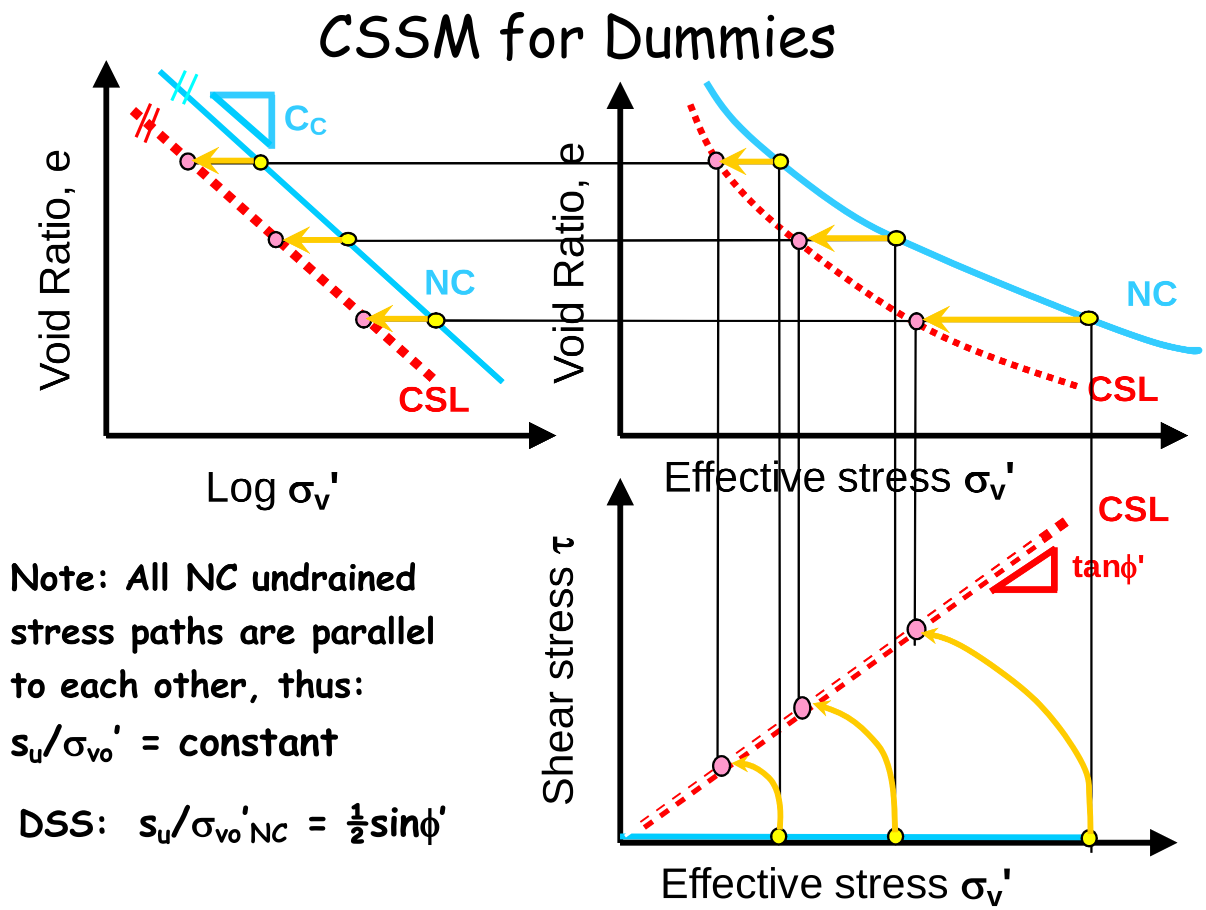


Note that there is no change in void ratio in the above consolidation plots because the test is undrained.

Stress Path for Normally Consolidated Undrained Soil (DSS)

Thursday, March 11, 2010
11:43 AM

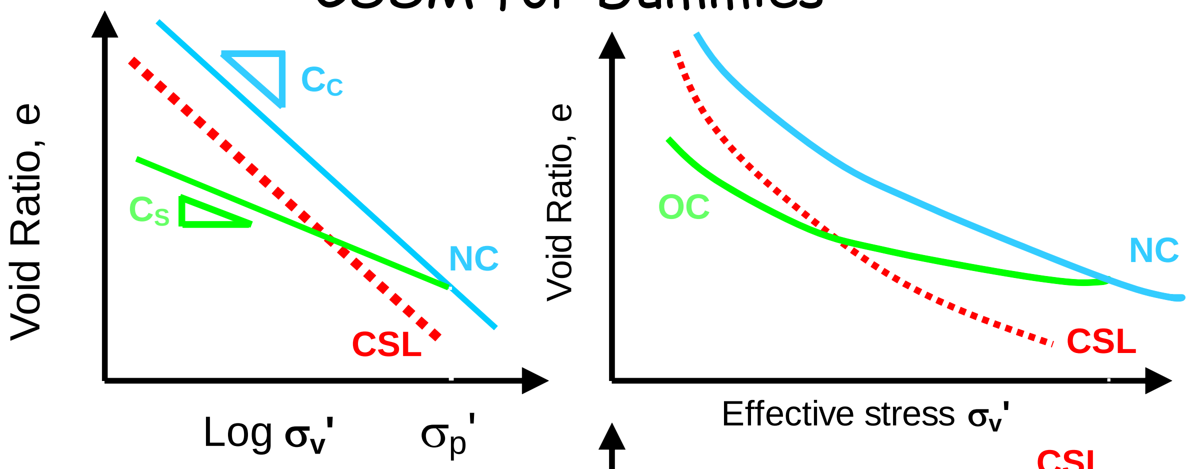
Effect of Increasing effective vertical stress on undrained strength



Critical State Line and Overconsolidated Soils

Thursday, March 11, 2010
11:43 AM

CSSM for Dummies

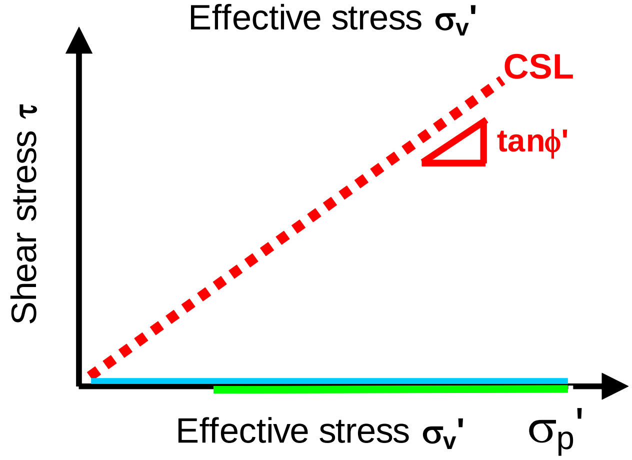


Overconsolidated States:

e_0, σ_{vo}', and OCR = σ_p'/σ_{vo}'

where σ_p' = σ_{vmax}' = P_c' = preconsolidation stress;

OCR = overconsolidation ratio



Stress Path for Undrained Over-consolidated Soil (DSS)

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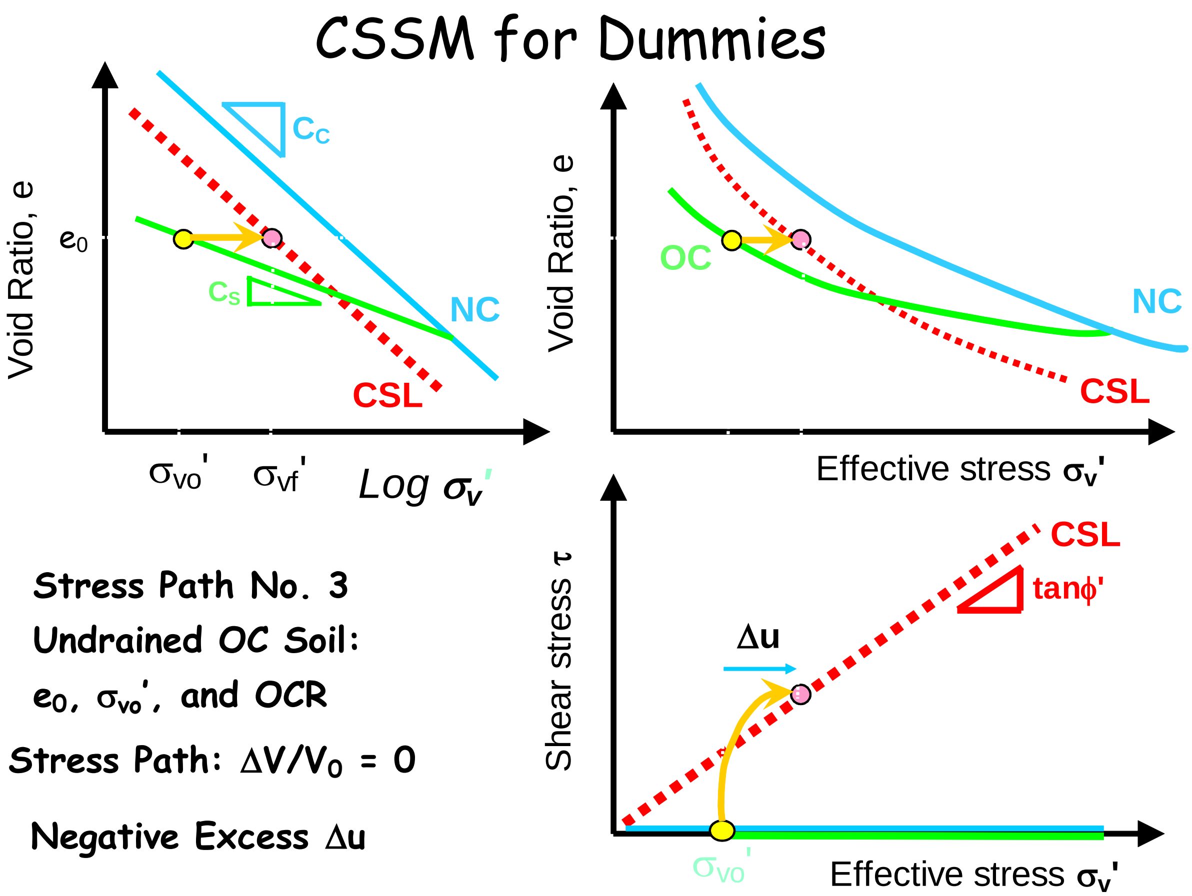
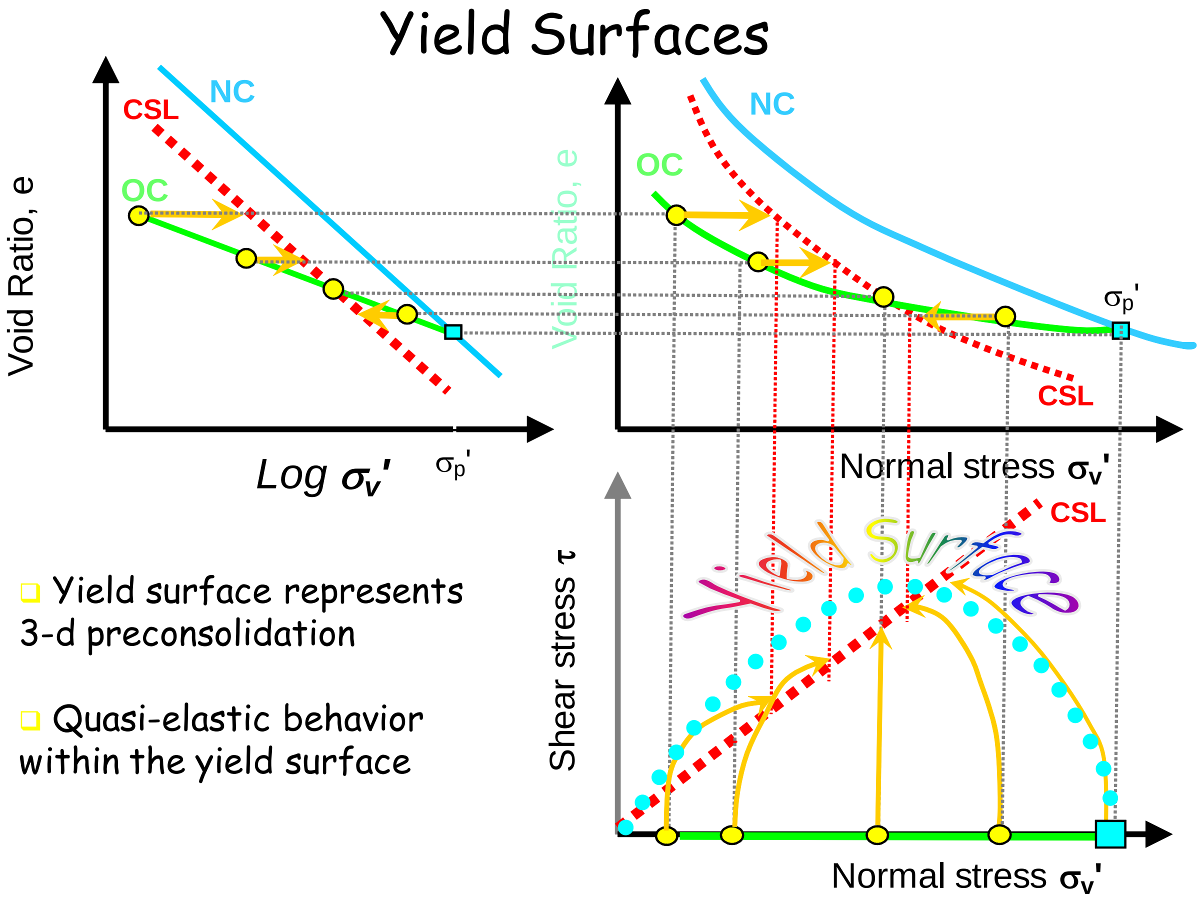
11:43 AM



Yield Surfaces and Stress Paths for Undrained Tests

Thursday, March 11, 2010
11:43 AM



- Yield surface represents 3-d preconsolidation
- Quasi-elastic behavior within the yield surface

The behavior of the soil is elastic until the state of stress on the soil reaches the yield surface. After that, the soil behaves in a plastic manner and undergoes both elastic and plastic strains. The yield surface will expand as the soil dilates or strain hardens: it will contract as the soil contracts or strain softens until the critical state is reached. At this point, this yield surface no longer changes and is known as the yield surface at critical state.

Summary of CSSM

Thursday, March 11, 2010
11:43 AM

- Initial state: e_0, s_{vo}', and OCR = s_p'/s_{vo}'
- Soil constants: f', C_c, and C_s (L = 1-C_s/C_c)
- Using effective stresses, CSSM addresses:
  - NC and OC behavior
  - Undrained vs. Drained (and other paths)
  - Positive vs. negative porewater pressures
  - Volume changes (contractive vs. dilative)
  - $s_u/s_{vo}' = ½ \sin f' \text{ OCR}^L$ where $L = 1-C_s/C_c$
  - Yield surface represents 3-d preconsolidation

Blank

Thursday, March 11, 2010
11:43 AM

Modified Cam Clay (MCC) Model

Thursday, March 11, 2010
11:43 AM

Original Cam-Clay Model

The Original Cam-Clay model is one type of CSSM model and is based on the assumption that the soil is isotropic, elasto-plastic, deforms as a continuum, and it is not affected by creep. The yield surface of the Cam clay model is described by a log arc.

Modified Cam-Clay Model

Professor John Burland was responsible for the modification to the original model, the difference between the Cam Clay and the Modified Cam Clay (MCC) is that the yield surface of the MCC is described by an ellipse and therefore the plastic strain increment vector (which is vertical to the yield surface) for the largest value of the mean effective stress is horizontal, and hence no incremental deviatoric plastic strain takes place for a change in mean effective stress.

Pasted from <http://en.wikipedia.org/wiki/Critical_state_soil_mechanics>

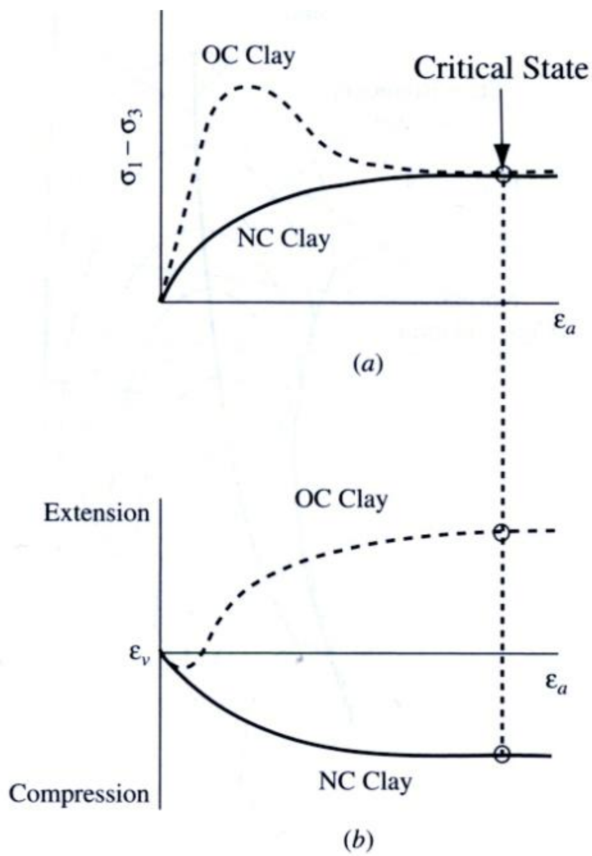
- Explains the pressure-dependent soil strength and the volume change (contraction and dilation) of clayey soils during shear.
- When critical state is reached, then unlimited soil deformations occur without changes in effective stress or volume.
- Formulation of the modified Cam clay model is based on plastic theory which makes it possible to predict volume change due to various types of loading using an associated flow rule

Critical State

Thursday, March 11, 2010
11:43 AM

Critical State and Critical State Line

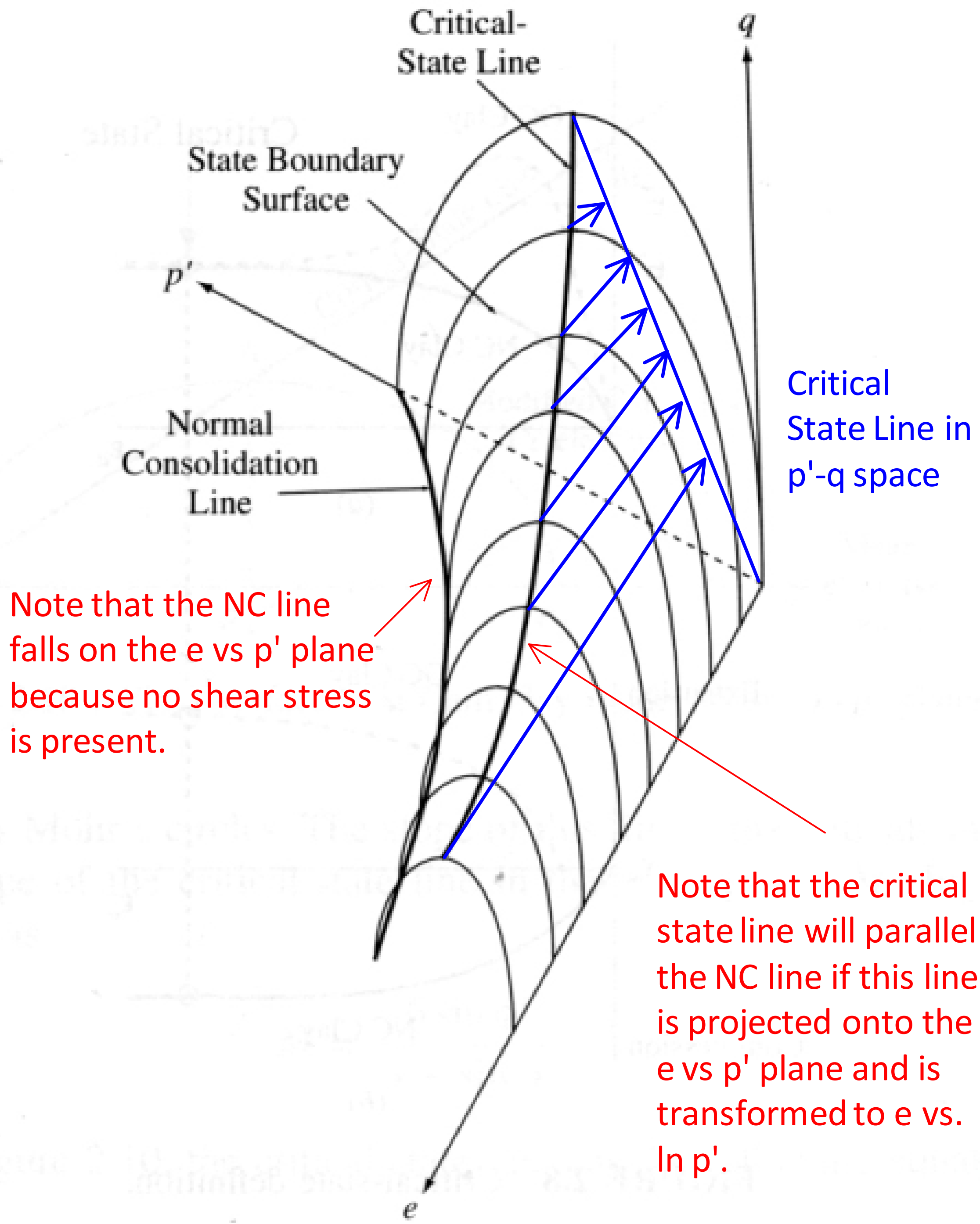
Applying shear stress to a soil will eventually lead to a state where no volume change occurs as the soil is continually sheared. When this condition is reached, it is known as the critical state.



Critical State Boundary Surface

Thursday, March 11, 2010
11:43 AM

Critical state and normally consolidated lines in p'- q - e space.



MCC Model Background

Thursday, March 11, 2010
11:43 AM

State Variables

- Mean effective stress, p'
- Shear stress, q
- Void ratio

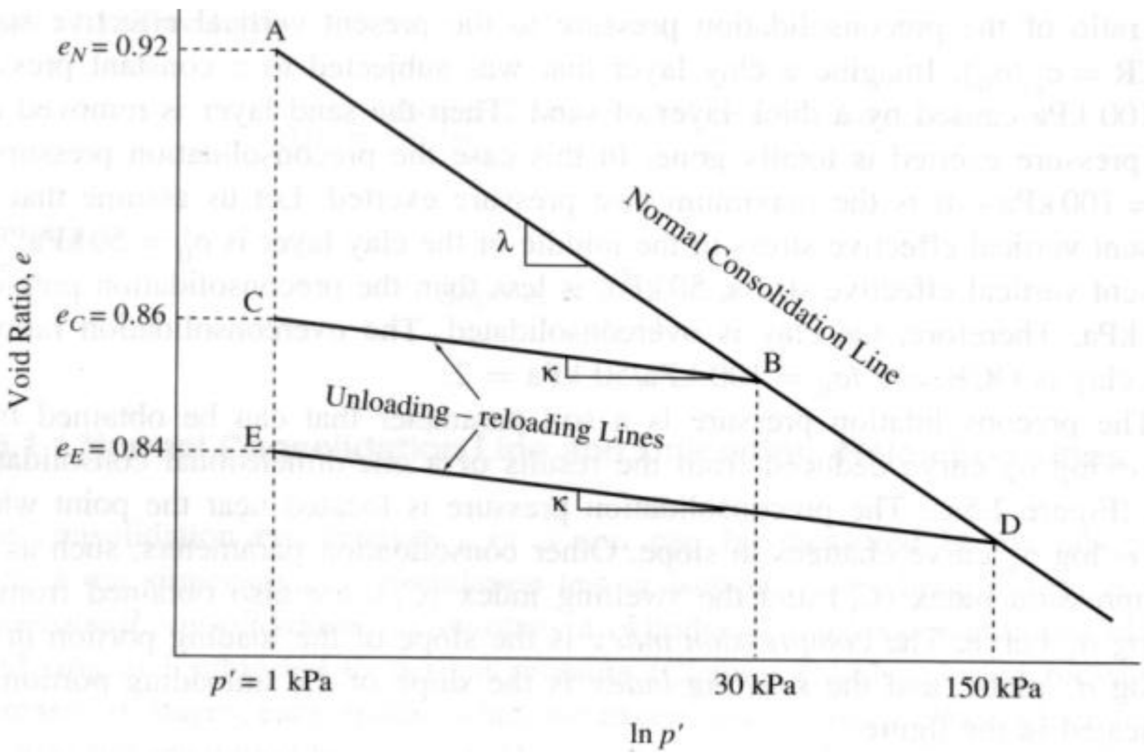
Mean effective stress

$$p' = \frac{\sigma'_1 + \sigma'_2 + \sigma'_3}{3}$$

Shear stress

$$q = \frac{1}{\sqrt{2}}\sqrt{(\sigma'_1 - \sigma'_2)^2 + (\sigma'_2 - \sigma'_3)^2 + (\sigma'_1 - \sigma'_3)^2}$$

Normal Consolidation Line and Unloading and Reloading Curves



MCC Model Background (cont.)

Thursday, March 11, 2010
11:43 AM

Normally consolidated line

$$e = e_N - \lambda \ln p'$$

Unloading - reloading line

$$e = e_C - \kappa \ln p'$$

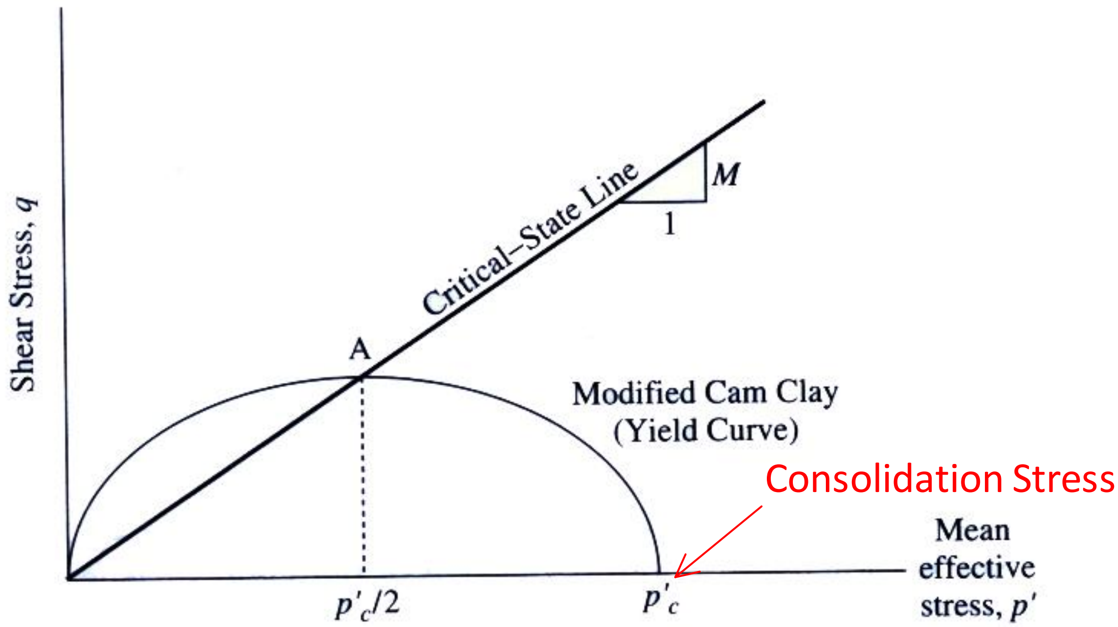
- Any point on the normally consolidated line represents the void ratio and state of stress for a normally consolidated soil.
- Any point on the unloading-reloading line represents and overconsolidated state.
- The material parameters λ κ and e_N are unique for a particular soil.
  - e_N is the void ratio on the normally consolidated line that corresponds to 1 unit stress (i.e., 1 kPa). However, e_N may vary if other stress units are used.
- The slope of the critical state line parallels the normally consolidated line and both have a slope of λ.
- The void ratio of the critical state line at p' = 1 kPa (or other unit pressure) is:

$$e_\Gamma = e_N - (\lambda - \kappa) \ln 2$$

MCC Model Background (cont.)

Thursday, March 11, 2010
11:43 AM

Critical State Line and Yield Curve in p'-q space



- The critical state line is obtained by performing CD triaxial tests
- The slope of the critical state line, M, is related to the critical state friction angle by:

$$M = \frac{6 \sin \phi'}{3 - \sin \phi'}$$

- The shear stress at the critical state can be found from:

$$q_f = M p'_f$$

- The void ratio at failure (i.e., critical state) is found by:

$$e_f = e_\Gamma - \lambda \ln p'$$

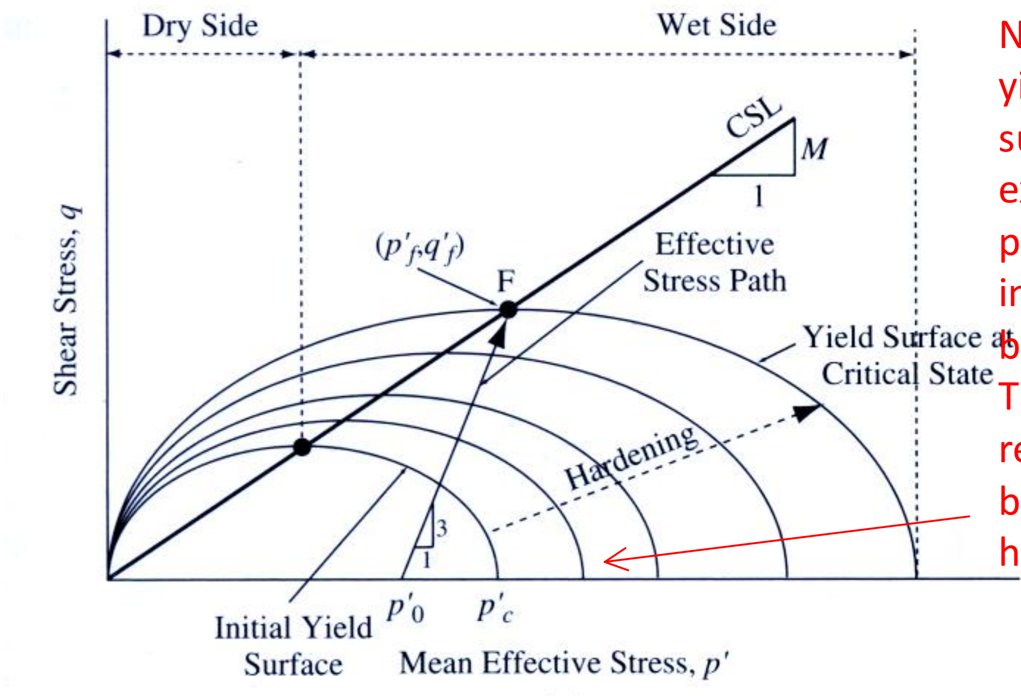
- The yield curve for the MCC model is an ellipse in p'- q space

$$q^2/p'^2 + M(1-p'_c/p') = 0$$

MCC Model Background (cont.)

Thursday, March 11, 2010
11:43 AM

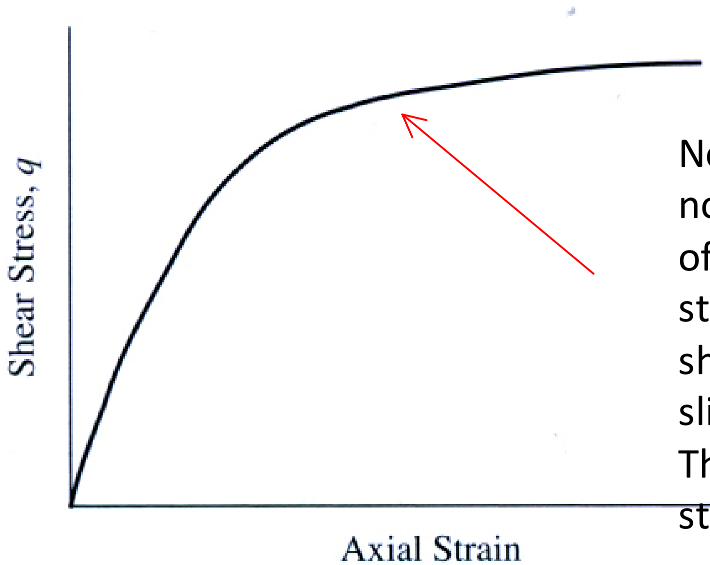
Strain hardening behavior for lightly overconsolidated clay



Note the yield surface expands as p' is increased beyond p'c. This is represented by strain hardening.

The effective stress path for a CD is a 3:1 slope (see text pp. 29-30).

Stress-Strain Curve showing strain hardening

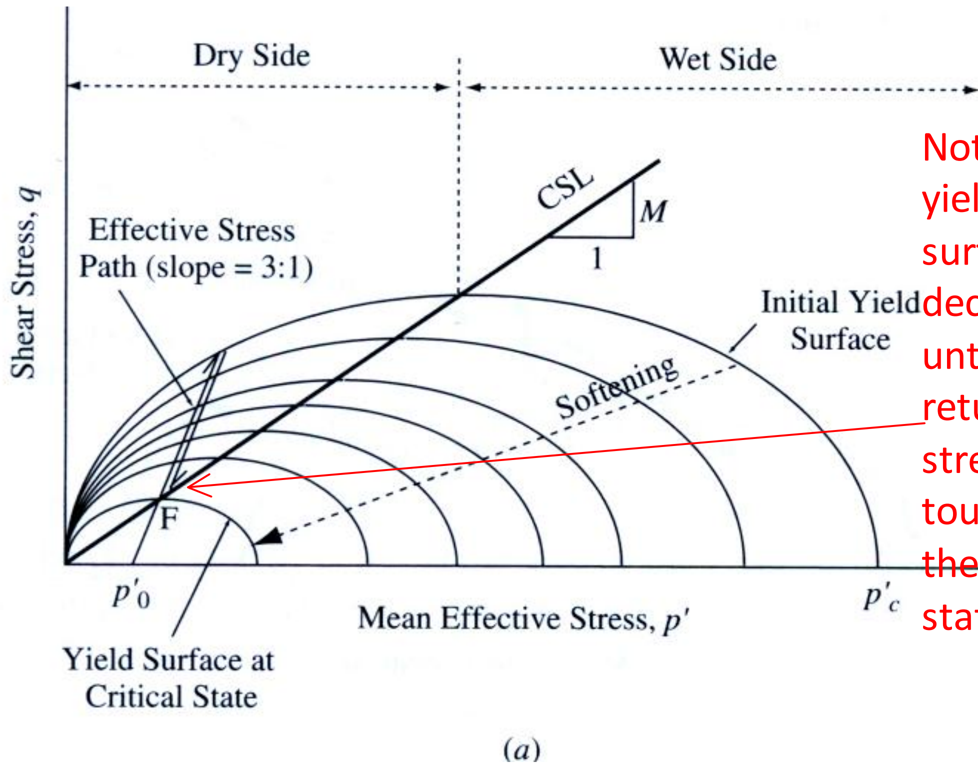


Note that in the non-linear range of this stress-strain curve, the shear resistance is slightly increasing. This represents strain hardening.

MCC Model Background (cont.)

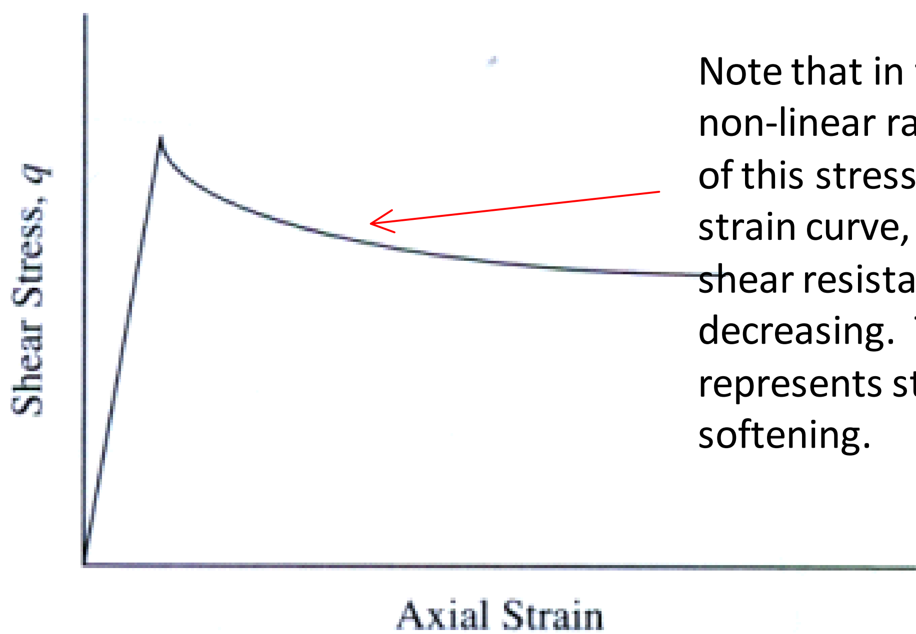
Thursday, March 11, 2010
11:43 AM

Strain softening behavior for heavily overconsolidated clay



Note the yield surface decreases until the returning stress path touches the critical state line.

Stress-Strain Curve showing strain softening



Note that in the non-linear range of this stress-strain curve, the shear resistance is decreasing. This represents strain softening.

MCC Model Background (cont.)

Thursday, March 11, 2010
11:43 AM

Stress dependency of bulk modulus in MCC model

- $K = (1 + e_0)p'/\kappa$

Other elastic parameters expressed in terms of stress dependency

- $E = 3\,(1\text{-}2\nu)(1 + e_0)p'/\kappa$
- $G = 3\,(1\text{-}2\nu)(1 + e_0)p'/\kappa\ /\ (2(1+\nu)\kappa)$

MCC Model Background (cont.)

Thursday, March 11, 2010
11:43 AM

Calculating incremental plastic strains

- Once the yield surface is reached, a part of the strain is plastic (i.e., irrecoverable). The incremental total strain (elastic and plastic parts) can be calculated from:

    - Volumetric strain
      $d\varepsilon_v = d\varepsilon_v^e + d\varepsilon_v^p$

    - Shear strain
      $d\varepsilon_s = d\varepsilon_s^e + d\varepsilon_s^p$

For the triaxial state of stress

- $d\varepsilon_v^p = d\varepsilon_1^p + 2d\varepsilon_3^p$
- $d\varepsilon_s^p = 2/3(d\varepsilon_1^p - d\varepsilon_3^p)$

Roscoe and Burland (1968) derived an associated plastic flow rule which describes the ratio between incremental plastic volumetric strain and incremental plastic shear strain. It is:

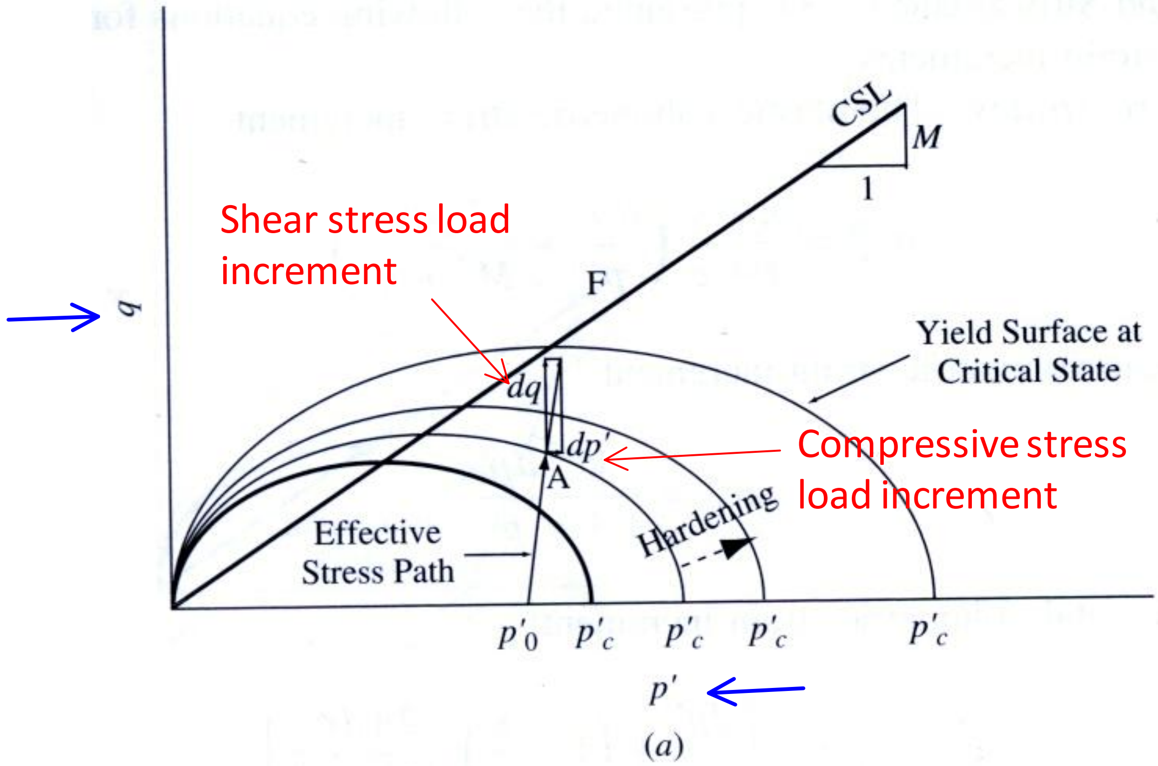
- $d\varepsilon_v^p / d\varepsilon_s^p = (M^2 - \eta^2) / 2\eta$

where $\eta = q/p'$ and at failure $\eta = M$

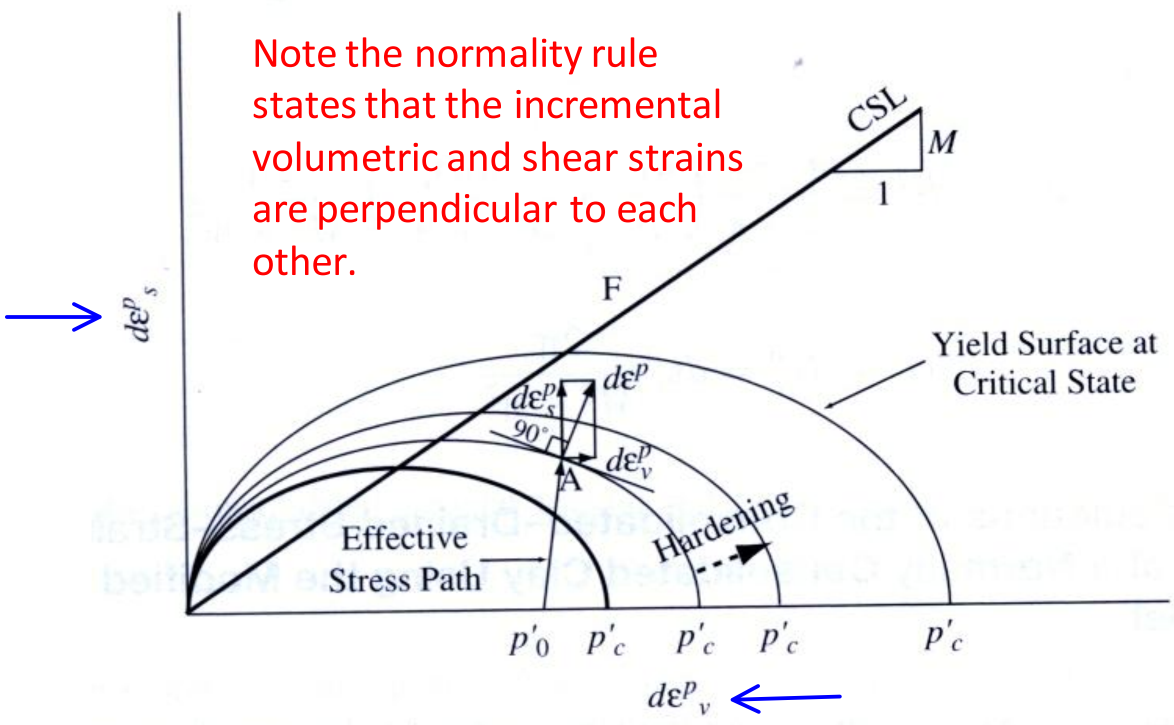
MCC Model Background (cont.)

Thursday, March 11, 2010
11:43 AM

Determination of plastic strain increment



(a)



(b)

MCC Model Background (cont.)

Thursday, March 11, 2010
11:43 AM

Calculation of incremental volumetric and shear strains (plastic and elastic parts = total strain)

Volumetric strains The plastic volumetric strain increment

$$d\varepsilon_v^p = \frac{\lambda - \kappa}{1+e}\left(\frac{dp'}{p'} + \frac{2\eta\, d\eta}{M^2 + \eta^2}\right)$$

The elastic volumetric strain increment

$$d\varepsilon_v^e = \frac{\kappa}{1+e}\frac{dp'}{p'}$$

Thus, the total volumetric strain increment:

$$d\varepsilon_v = \frac{\lambda}{1+e}\left[\frac{dp'}{p'} + \left(1 - \frac{\kappa}{\lambda}\right)\frac{2\eta\, d\eta}{M^2 + \eta^2}\right]$$

Shear strains

$$d\varepsilon_s = d\varepsilon_s^p = \frac{\lambda - \kappa}{1+e}\left(\frac{dp'}{p'} + \frac{2\eta\, d\eta}{M^2 + \eta^2}\right)\frac{2\eta}{M^2 - \eta^2}$$

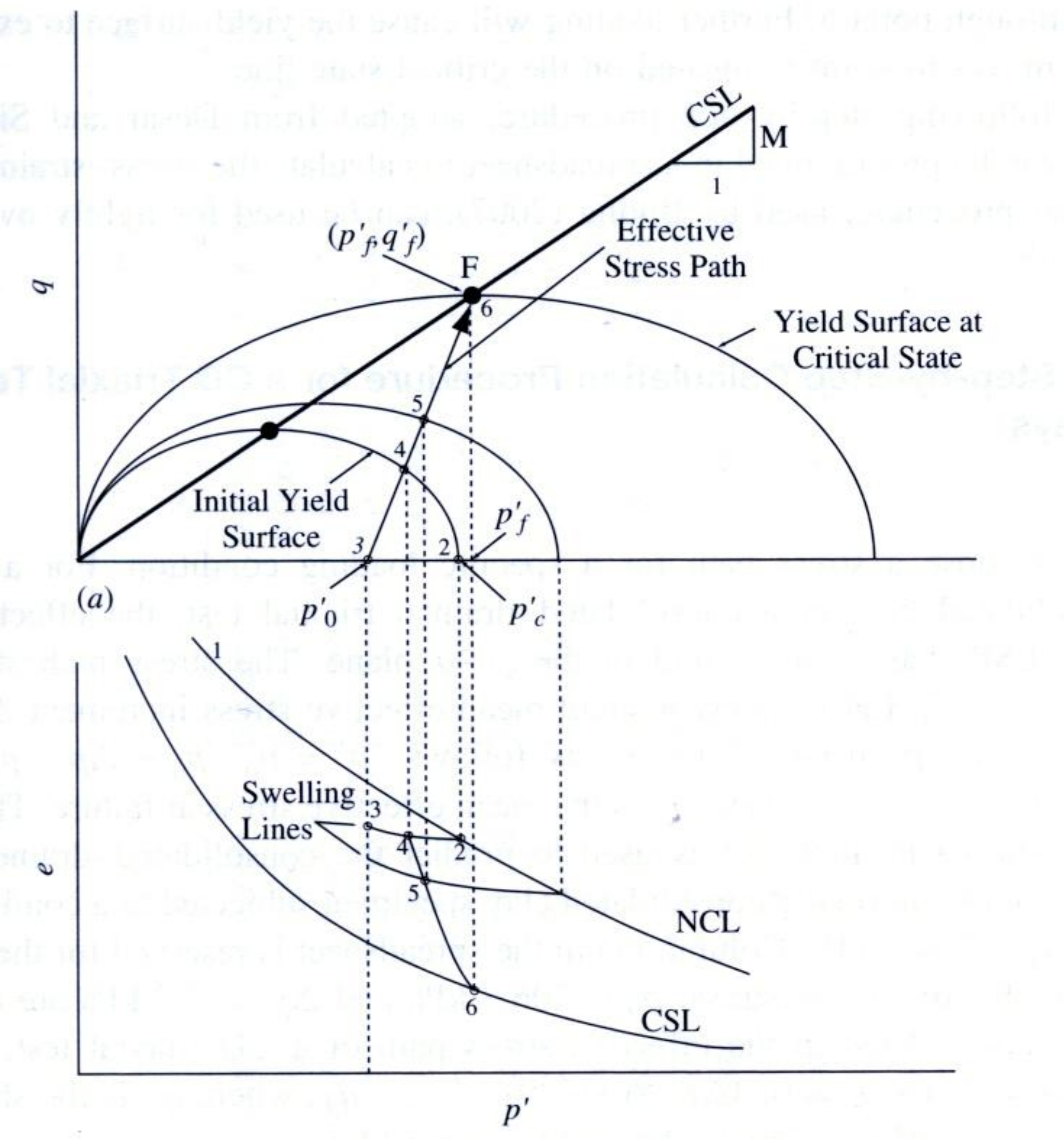
or

$$d\varepsilon_s = d\varepsilon_s^p = d\varepsilon_v^p \frac{2\eta}{M^2 - \eta^2}$$

MCC Model Background (cont.)

Thursday, March 11, 2010
11:43 AM

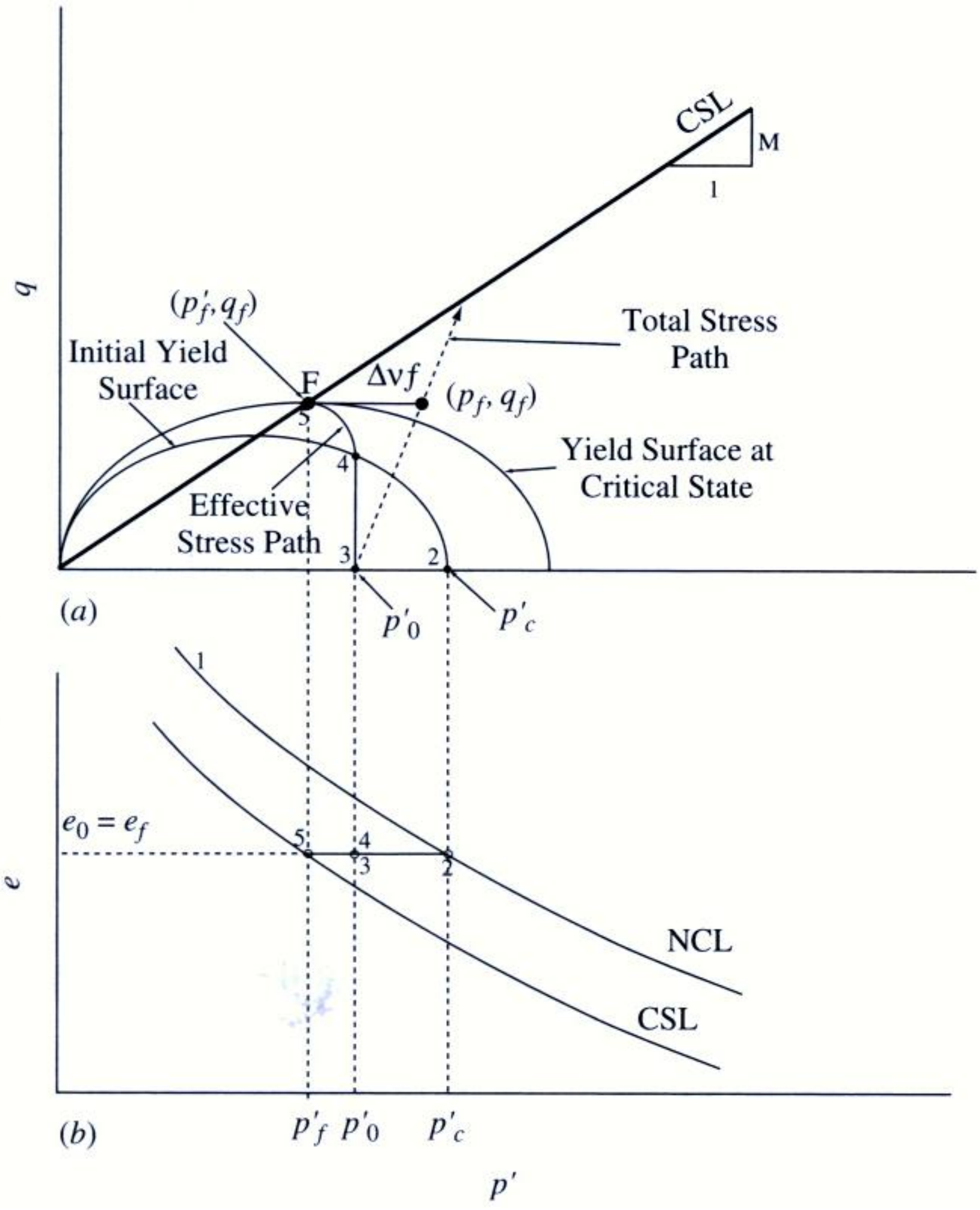
Consolidated Drained Test Behavior of Lightly Overconsolidated Clay



MCC Model Background (cont.)

Thursday, March 11, 2010
11:43 AM

Consolidated Undrained Test Behavior of Lightly Overconsolidated Clay



MCC Model in FLAC

Thursday, March 11, 2010
11:43 AM

FLAC implementation

- incremental hardening/softening elastoplastic model
- nonlinear elasticity and a hardening/softening behavior governed by volumetric plastic strain (“density” driven)
- failure envelopes are similar in shape and correspond to ellipsoids of rotation about the mean stress axis in the principal stress space
- associated shear flow rule
- no resistance to tensile mean stress

MCC Model in FLAC (cont.)

Thursday, March 11, 2010
11:43 AM

Generalized Stress Components in Terms of Principal Stresses

$$p = -\frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$$
$$q = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2} \qquad (2.178)$$

Volumetric and Distortional (i.e., shear) Strain Increments

The incremental strain variables associated with $-p$ and q are the volumetric strain increment Δe and distortional strain increment Δe_q, and we have:

Volumetric strain increment

$$\Delta e = \Delta e_1 + \Delta e_2 + \Delta e_3$$
$$\Delta e_q = \frac{\sqrt{2}}{3}\sqrt{(\Delta e_1 - \Delta e_2)^2 + (\Delta e_2 - \Delta e_3)^2 + (\Delta e_1 - \Delta e_3)^2} \qquad (2.179)$$

Distortional strain increment

where Δe_j, $j = 1, 3$ are principal strain increments. By assumption, the principal strain increments may be decomposed into elastic and plastic parts so that

Principal volumetric strain increments have an elastic and plastic part

$$\Delta e_i = \Delta e_i^e + \Delta e_i^p \qquad i = 1, 3$$

Elastic part　　Plastic part

(Note: e is volumetric strain and not void ratio)

MCC Model in FLAC (cont.)

Thursday, March 11, 2010
11:43 AM

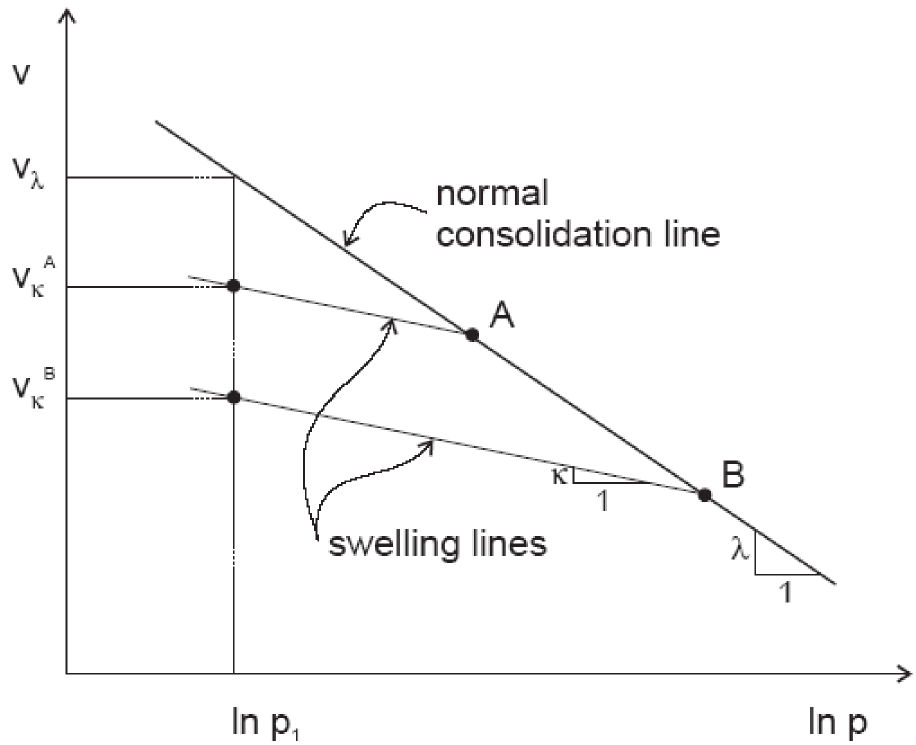


Figure 2.24 Normal consolidation line and unloading-reloading (swelling) line for an isotropic compression test

As the normal consolidation pressure, p, increases, the specific volume, v, of the material decreases. The point representing the state of the material moves along the *normal consolidation line* defined by the equation

$$v = v_\lambda - \lambda \ln \frac{p}{p_1} \tag{2.186}$$

where λ^* and v_λ are two material parameters, and p_1 is a reference pressure. (Note that v_λ is the value of the specific volume at the reference pressure.)

Note that the term "swelling" used above could be replaced with "reloading."

An unloading-reloading excursion, from point A or B on the figure, will move the point along an *elastic swelling line* of slope κ, back to the normal consolidation line where the path will resume. The equation of the swelling lines has the form

$$v = v_\kappa - \kappa \ln \frac{p}{p_1} \tag{2.187}$$

MCC Model in FLAC (cont.)

Thursday, March 11, 2010
11:43 AM

Elastic (i.e., recoverable) change in specific volume

$$\Delta v^e = -\kappa \frac{\Delta p}{p}$$

After dividing by sides by the specific volume produces the relation between elastic changes in specific volume and changes in pressure

$$-\Delta p = \frac{vp}{\kappa} \Delta e^e$$

(The negative sign is needed because increases in pressure cause a decrease in specific volume.)

The tangential bulk modulus can be written as:

$$K = \frac{vp}{\kappa}$$

(Note this is different than an elastic bulk modulus because K is nonlinear function of p (mean effective stress).)

MCC Model in FLAC (cont.)

Thursday, March 11, 2010
11:43 AM

General Loading Conditions with Yielding

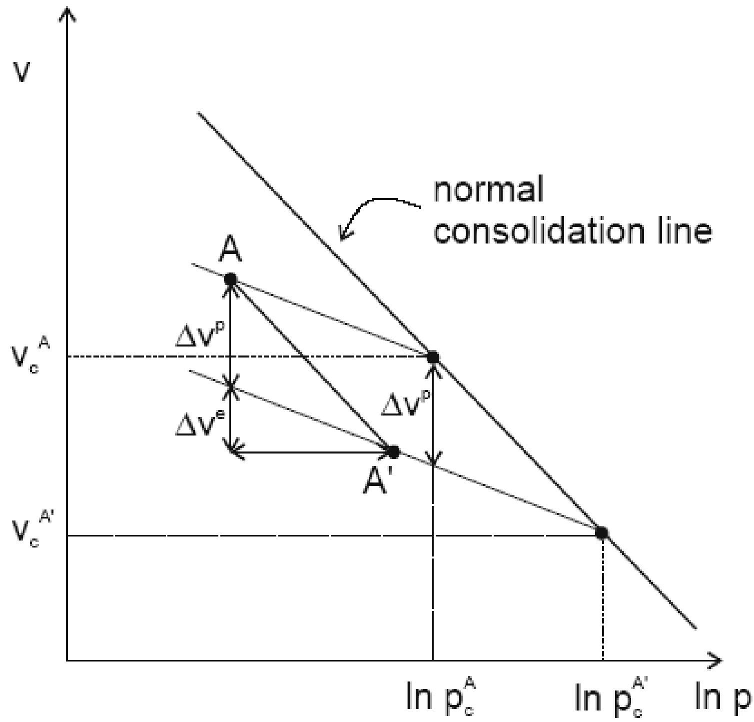


Figure 2.25 Plastic volume change corresponding to an incremental consolidation pressure change

$$\Delta v^p = -(\lambda - \kappa)\frac{\Delta p_c}{p_c}$$

Elastic (recoverable) change in specific volume.

$$\Delta e^p = -\frac{\lambda - \kappa}{v}\frac{\Delta p_c}{p_c}$$

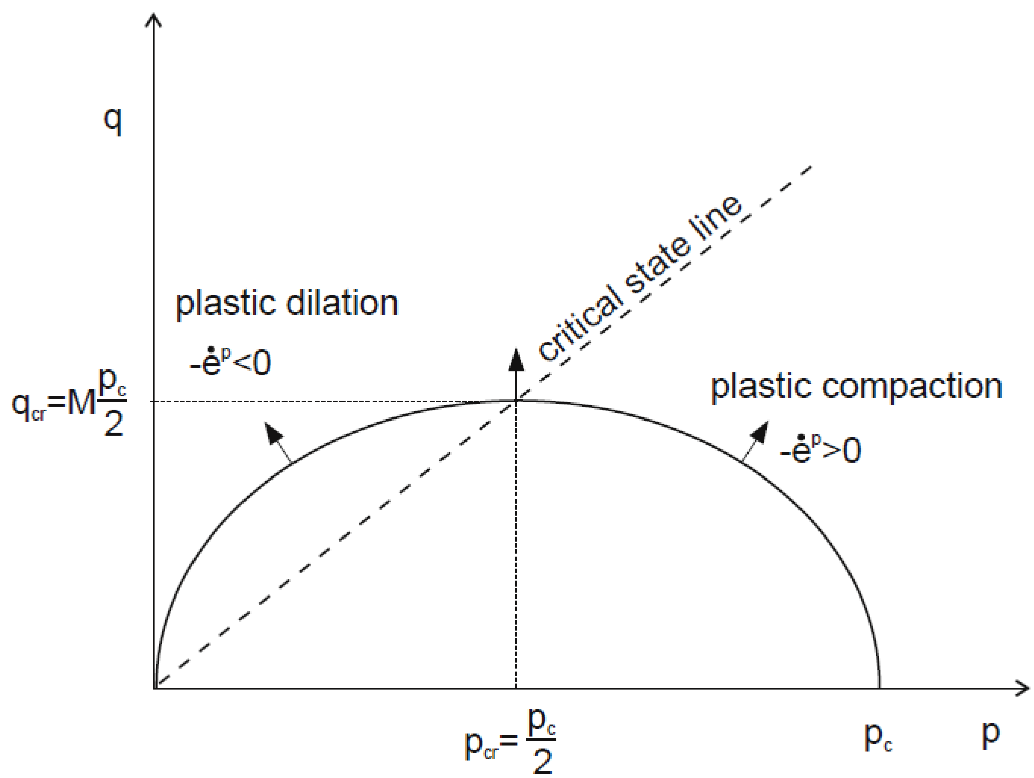
Plastic principal volumetric strain increment

MCC Model in FLAC (cont.)

Thursday, March 11, 2010
11:43 AM

Yield Function

$$f = q^2 + M^2 p(p - p_c)$$



yield condition $f = 0$ is represented by an ellipse

horizontal axis p_c and vertical axis Mp_c in the (q, p) plane

Associated flow rule

$$g = q^2 + M^2 p(p - p_c)$$

MCC Model in FLAC (cont.)

Thursday, March 11, 2010
11:43 AM

(1)	**bulk_mod**	maximum elastic bulk modulus, K_{max}
(2)	**density**	mass density, ρ
(3)	**kappa**	slope of elastic swelling line, κ
(4)	**lambda**	slope of normal consolidation line, λ
(5)	**mm**	frictional constant, M
(6)	**mpc**	preconsolidation pressure, p_c
(7)	**mv0**	initial specific volume, υ_0 (calculated internally, by default)
(8)	**mp1**	reference pressure, p_1
(9)	**mv_l**	specific volume at reference pressure, p_1, on normal consolidation line, υ_λ
(10)	**poiss**	Poisson's ratio, ν
(11)	**shear_mod**	elastic shear modulus, G

Properties required for MCC model in FLAC FLAC names (blue)

bulk modulus(maximum value), K bulk
Poisson's ratio, ν poiss
frictional constant, M mm
slope of normal consolidation line, λ lambda
slope of elastic swelling line, κ kappa
reference pressure, p_1 mp1
specific volume, υ_λ mv_l *Note use mv_l not mv_1*
Preconsolidation stress, mpc

MCC Model in FLAC (cont.)

Thursday, March 11, 2010
11:43 AM

$$M = \frac{6\sin\phi'}{3-\sin\phi'}$$ For triaxial compression

$$M = \frac{6\sin\phi'}{3+\sin\phi'}$$ For triaxial extension

$$\lambda = C_c/\ln(10)$$ From isotropic compression

$$\kappa \approx C_s/\ln(10)$$ Usually 1/5 to 1/3 of λ

$$v_0 = v_\lambda - \lambda\ln\left(\frac{p_{c0}}{p_1}\right) + \kappa\ln\left(\frac{p_{c0}}{p_0}\right)$$

Initial specific volume (Calculated by FLAC) (see below)

$V_\lambda = V/Vs$ — Specific Volume at p_1

V= total volume

Vs = volume of solids

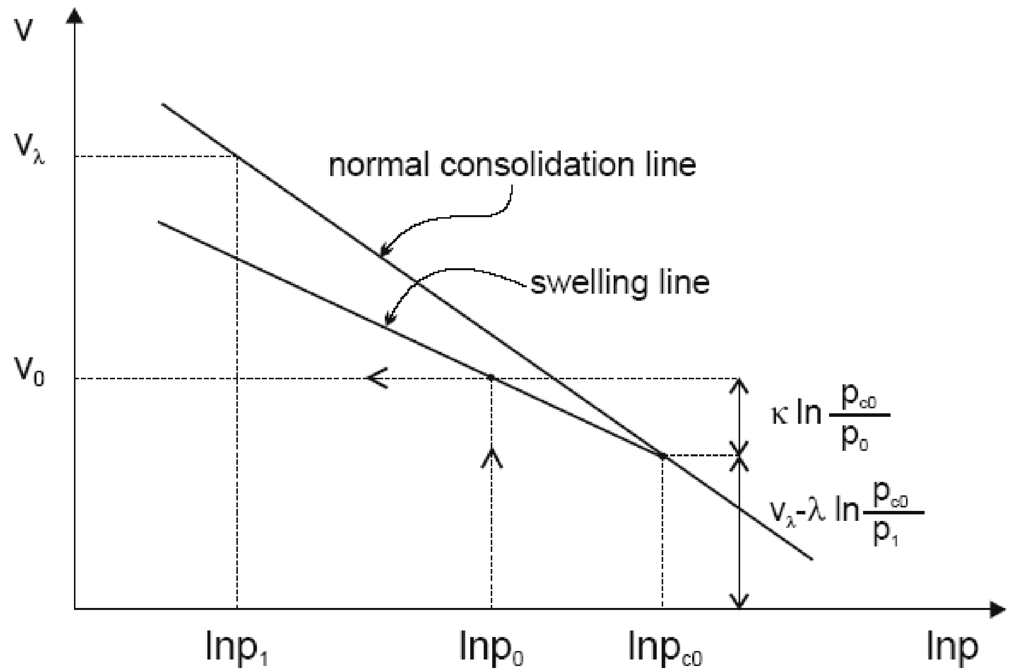


Figure 2.27 Determination of initial specific volume

MCC Model in FLAC (cont.)

Thursday, March 11, 2010
11:43 AM

Notes on Poisson's ratio

If Poisson's ratio, poiss, is not given, and a nonzero shear modulus, shearmod, is specified, then the shear modulus remains constant: Poisson's ratio will change as bulk modulus changes. If a nonzero poiss is given, then the shear modulus will change as the bulk modulus changes: Poisson's ratio remains constant. (The latter case usually applies to most problems.)

Properties for plotting

The following calculated properties can be printed, plotted or accessed via *FISH*.

(12)	**bulk_current**	current elastic bulk_modulus, K
(13)	**cam_p**	effective pressure, p
(14)	**cam_q**	shear stress, q
(15)	**ev_plastic**	accumulated plastic volumetric strain
(16)	**ev_tot**	accumulated total volumetric strain
(17)	**sv**	current specific volume

More Reading

Thursday, March 11, 2010
11:43 AM

- Applied Soil Mechanics with ABAQUS Applications, pp. 28-53
- Applied Soil Mechanics with ABAQUS Applications, Ch. 5
- FLAC User's Manual, Theory and Background, Section 2.4.7

Assignment 6

Thursday, March 11, 2010
11:43 AM

1. Modify the FISH code given below to model an axisymmetrical strain-controlled unconfined compression test on an EPS cylinder with a height of 5 cm and a diameter of 2.5 cm using a 5 x 20 uniform grid. The EPS should be modeled using the M-C using a density of 20 kg/m^3, Young's modulus of 5 MPa, Poisson's ratio of 0.1 and a cohesion of 50 KPa. You should include: a) plot of the undeformed model with boundary conditions, b) plot of the deformed model at approximately 3 percent axial strain, c) plot of axial stress vs axial strain, d) calculation of Young's modulus and unconfined compressive strength from c). (Note that the axial strain should be calculated along the centerline of the specimen.) (20 points).

```
config
set = large; large strain mode
grid 18,18; for 18" x 18" EPS block
model mohr
prop density = 20 bulk = 2.08e6 shear = 2.27e6 cohesion=50e3 friction=0 dilation=0 tension = 100e3;
EP'S
;
ini x mul 0.0254; makes x grid dimension equal to 0.0254 m or 1 inch
ini y mul 0.0254; makes y grid dimension equal to 0.0254 m or 1 inch
fix y j 1; fixes base
;fix y i 8 12 j 1 ; fixes only part of base
his unbal 999
apply yvelocity -5.0e-6 from 1,19 to 19,19 ;applies constant downward velocity to simulate a strain-
controlled test
def verticalstrain; subroutine to calculate vertical strain
 whilestepping
 avgstress = 0
 avgstrain = 0
 loop i (1,izones)
  loop j (1,jzones)
    vstrain = ((0- ydisp(i,j+1) - (0 - ydisp(i,j)))/0.0254)*100 ; percent strain
    vstress = syy(i,j)*(-1)
    avgstrain = avgstrain + vstrain/18/18
    avgstress = avgstress + vstress/18/18
  end_loop
 end_loop
end
his avgstrain 998
his avgstress 997
history 999 unbalanced
cycle 3000
```

Assignment 6 (cont.)

Thursday, March 11, 2010
11:43 AM

2. Modify the FLAC FISH code developed in problem 1 to simulate an axisymmetrical strain-controlled, consolidated drained triaxial compression test on a cylinder of sand that has a height of 5 cm and a diameter of 2.5 cm using a 5 x 20 uniform grid. The sand should be modeled using the M-C using a density of 2000 kg/m^3, Young's modulus of 10 MPa, Poisson's ratio of 0.3 and a drained friction angle of 35 degrees. The sample is first consolidated using a confining stress of 50 kPa and then sheared to failure. Your solution should include: a) plot of the undeformed model with boundary conditions, b) plot of the deformed model at approximately 3 percent axial strain, c) plot of deviatoric stress vs axial strain d) plot of p' vs. q' e) calculation of Young's modulus and drained friction angle from c) and d) (20 points).

3. Change the constitutive relationship in problem 2 to a Modified Cam Clay model where λ is 0.15, κ is 0.03, ν is 0.3 (remains constant) and the initial void ratio of the same is 1.0 at 1 kPa. Model the same test as described in in problem 2 and provide the same required output. In addition, develop a comparative plot of plot of axial stress vs axial strain for the MC and MCC model results (20 points).

Shearing along Interfaces

Thursday, March 11, 2010
11:43 AM

There are several instances in geomechanics in which it is desirable to represent planes on which sliding or separation can occur:

- joint, fault or bedding planes in a geologic medium
- interface between a foundation element and the soil
- contact plane between a bin or chute and the material that it contains
- contact between two colliding objects.

FLAC provides interfaces that are characterized by Coulomb sliding and/or tensile separation. Interfaces have one or more of the following properties:

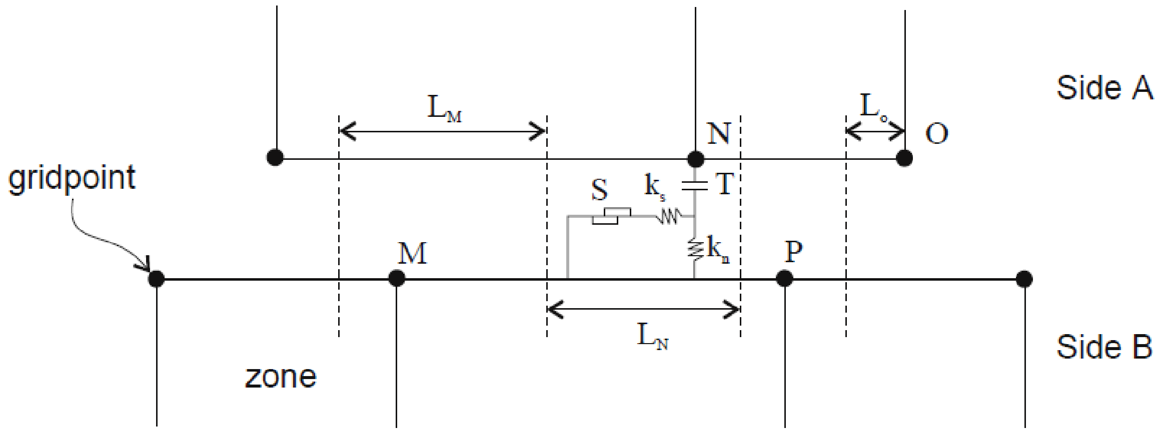
- Friction
- Cohesion
- Dilation
- Normal stiffness
- Shear stiffness
- Tensile Strength

Although there is no restriction on the number of interfaces or the complexity of their intersections, it is generally not reasonable to model more than a few simple interfaces with FLAC because it is awkward to specify complicated interface geometry. The program UDEC (Itasca 2004) is specifically designed to model many interacting bodies; it should be used instead of FLAC for the more complicated interface problems.

An interface can also be specified between structural elements and a grid, or between two structural elements. Interfaces may also be used to join regions that have different zone sizes. In general, the ATTACH
command should be used to join sub-grids together. However, in some circumstances it may be more convenient to use an interface for this purpose. In this case, the interface is prevented from sliding or opening because it does not correspond to any physical entity.

Interfaces (cont.)

Thursday, March 11, 2010
11:43 AM



S = slider
T = tensile strength
k_n = normal stiffness
k_s = shear stiffness
L_n = length associated with gridpoint N
L_m = length associated with gridpoint M
---- denotes limits for joint segments (placed halfway between adjacent gridpoints)

Interface Properties

1. **Glued Interfaces** — If interfaces are declared glued, no slip or opening is allowed, but elastic displacement still occurs, according to the given stiffnesses.

2. **Coulomb Shear-Strength** — The Coulomb shear-strength criterion limits the shear force by the following relation:

$$F_{smax} = cL + \tan\phi \; F_n$$

where c = cohesion (in stress units) along the interface, L = effective contact length (Figure 4.1), and ϕ = friction angle of interface surfaces.

If the criterion is satisfied (i.e., if $|F_s| \geq F_{smax}$), then $F_s = F_{smax}$, with the sign of shear preserved.

In addition, the interface may dilate at the onset of slip (nonelastic sliding). Dilation is governed in the Coulomb model by a specified dilation angle, ψ. Dilation is a function of the direction of shearing. Dilation increases if the shear displacement increment is in the same direction as the total shear displacement, and decreases if the shear increment is in the opposite direction.

Interfaces (cont.)

Thursday, March 11, 2010
11:43 AM

During sliding, shear displacement can cause an increase in the effective normal stress on the interface, according to the relation

$$\sigma_n := \sigma_n + \frac{|F_s|_o - F_{smax}}{L k_s} \tan\psi \, k_n$$

where $|F_s|_o$ is the magnitude of shear force *before* the above correction made.

3. Tension Bond — Two conditions are available for a bonded interface:

a. Bonded interface — If a (positive) tensile bond strength is specified for an interface, each segment of the interface acts as if it is glued (elastic response only), while the magnitude of the tensile normal stress is below the bond strength. If the magnitude of the tensile normal stress of a segment exceeds the bond strength (set with **tbond**), the bond breaks for that segment, and the segment behaves thereafter as unbonded (separation and slip allowed, as described above, in the normal way).

A shear bond strength, as well as the tensile bond strength, can be specified. The bond breaks if the shear stress exceeds the shear bond strength, or the tensile effective normal stress exceeds the normal bond strength. The shear bond strength is set to ***sbr*** times the normal bond strength, using the **sbratio=** ***sbr*** property keyword. The default shear bond strength is 100 times the tensile bond strength.

b. Slip while bonded — There is an optional switch (**bslip=on**) that allows slip to occur for a bonded interface segment, even though separation has not occurred. Shear yield is under the control of the friction and cohesion parameters using the absolute value of the effective normal force. Note that dilation response is suppressed (i.e., $\psi = 0$) when **bslip=on**. By default **bslip=off** if not specified.

Interfaces (cont.)

Thursday, March 11, 2010
11:43 AM

Shear and normal stiffness (cases)

- Interface Used to Join Two Sub-Grids
- Real Interface — Slip and Separation Only
- Real Interface — All Properties Have Physical Significance

Interface Used to Join Two Sub-Grids

Under some circumstances it may be necessary to use an interface to join two sub-grids. This type of interface is declared as **glued** on the **INTERFACE** command, thus preventing any slip or separation; values of friction, cohesion and tensile strength are not needed and are ignored if given. However, shear and normal stiffnesses must be provided. It is tempting (particularly for people familiar with finite element methods) to give a very high value for these stiffnesses to prevent movement on the interface. However, *FLAC* does "mass scaling" (see Section 1.3.5) based on stiffnesses — the response (and solution convergence) will be very slow if very high stiffnesses are specified. It is recommended that the lowest stiffness consistent with small interface deformation be used. A good rule-of-thumb is that k_n and k_s be set to *ten times* the equivalent stiffness of the stiffest neighboring zone. The apparent stiffness (expressed in stress-per-distance units) of a zone in the normal direction is

$$\max\left[\frac{(K + \frac{4}{3}G)}{\Delta z_{\min}}\right] \tag{4.5}$$

where K & G are the bulk and shear moduli, respectively; and

$\Delta z_{\min}$ is the smallest width of an adjoining zone in the normal direction — see Figure 4.7.

The max [ ] notation indicates that the maximum value over all zones adjacent to the interface is to be used (e.g., there may be several materials adjoining the interface).

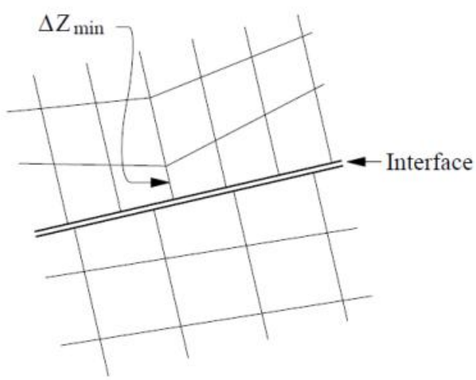


Figure 4.7 Zone dimension used in stiffness calculation

Interfaces (cont.)

Thursday, March 11, 2010
11:43 AM

The prescription given in Eq. (4.5) is reasonable if the material on the two sides of the interface are similar and variations of stiffness occur only in the lateral directions. However, if the material on one side of the interface is much stiffer than that on the other, then Eq. (4.5) should be applied to the *softer* side. In this case, the deformability of the whole system is dominated by the soft side; making the interface stiffness ten times the soft-side stiffness will ensure that the interface has minimal influence on system compliance.

Real Interface — Slip and Separation Only

In this case, we simply need to provide a means for one sub-grid to slide and/or open relative to another sub-grid. The friction (and perhaps cohesion and tensile strength) is important, but the elastic stiffness is not. The approach of Section 4.4.1 is also used here to determine k_n and k_s. However, the other material properties are given real values (see Section 4.4.3 for advice on choice of properties). As an example, we can allow slip in a bin-flow problem (as shown in Figure 4.11), corresponding to the data file in Example 4.8.

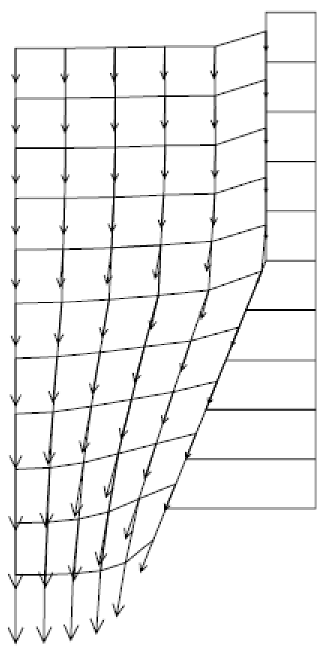


Figure 4.11 Flow of frictional material in a "bin"

Interfaces (cont.)

Thursday, March 11, 2010
11:43 AM

Real Interface — All Properties Have Physical Significance

In this case, properties should be derived from tests on real joints* (suitably scaled to account for size effect), or from published data on materials similar to the material being modeled. However, the comments of Section 4.4.1, with respect to the maximum stiffnesses that are reasonable to use, also apply here. If the physical normal and shear stiffnesses are less than ten times the equivalent stiffnesses of adjacent zones, then there is no problem in using physical values. If the ratio is much more than ten, the solution time will be significantly longer than for the case in which the ratio is limited to ten, without much change in the behavior of the system. Serious consideration should be given to reducing supplied values of normal and shear stiffness to improve solution efficiency. There may also be problems with interpenetration if the normal stiffness, k_n, is very low. A rough estimate should be made of the joint normal displacement that would result from the application of typical stresses in the system ($u = \sigma / k_n$). This displacement should be small compared to a typical zone size. If it is greater than, say, 10% of an adjacent zone size, then there is either an error in one of the numbers or the stiffness should be increased if calculations are to be done in large-strain mode.

Joint properties are conventionally derived from laboratory testing (e.g., triaxial and direct shear tests). These tests can produce physical properties for joint friction angle, cohesion, dilation angle and tensile strength, as well as joint normal and shear stiffnesses. The joint cohesion and friction angle correspond to the parameters in the Coulomb strength criterion* described in Section 4.2.

Values for normal and shear stiffnesses for rock joints typically can range from roughly 10 to 100 MPa/m for joints with soft clay in-filling, to over 100 GPa/m for tight joints in granite and basalt. Published data on stiffness properties for rock joints are limited; summaries of data can be found in Kulhawy (1975), Rosso (1976) and Bandis et al. (1983).

Dilation Angle

Thursday, March 11, 2010
11:43 AM

Shear dilatancy, or dilatancy, is the change in volume that occurs with shear distortion of a material. Dilatancy is characterized by a dilation angle, ψ, which is related to the ratio of plastic volume change to plastic shear strain. This angle can be specified in the Mohr-Coulomb, ubiquitous-joint, strain-hardening/softening, bilinear ubiquitous-joint and double-yield plasticity models. The dilation angle is typically determined from triaxial tests or shear box tests. For example, the idealized relation for dilatancy, based upon the Mohr-Coulomb failure surface, is depicted for a triaxial test in Figure 3.58. The dilation angle is found from the plot of volumetric strain versus axial strain. Note that the initial slope for this plot corresponds to the elastic regime, while the slope used to measure the dilation angle corresponds to the plastic regime.

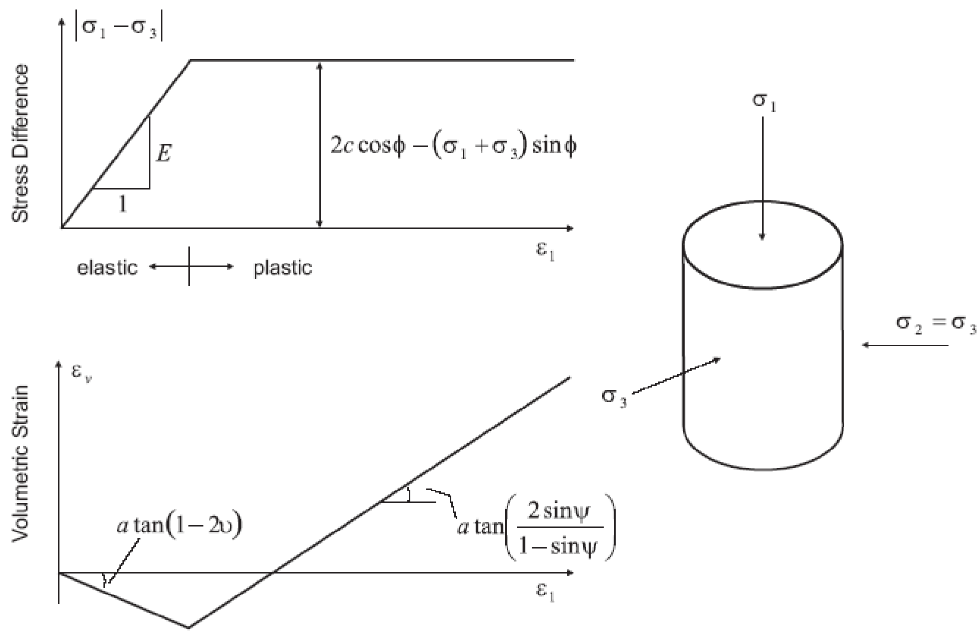


Figure 3.58 Idealized relation for dilation angle, ψ, from triaxial test results [Vermeer and de Borst (1984)]

Dilation Angle (cont.)

Thursday, March 11, 2010
11:43 AM

Solution for dilation angle for Fig. 3.58 in FLAC manual

$$\left(\frac{2\cdot\sin(\text{dilation\_angle})}{1-\sin(\text{dilation\_angle})}\right) = \frac{-(\text{de1}+2\text{de3})}{\text{de1}}$$

Note: A negative sign was added here to be consistent with Salgado Eq. 4-15. Also, the relation between dev and de1 and de3 is from Eq. 4-17 in Salgado

solving for the dilation angle:

$$\begin{bmatrix} -\text{asin}\left[\dfrac{1}{\text{de1}-2\cdot\text{de3}}\cdot(\text{de1}+2\cdot\text{de3})\right] \\ \pi+\text{asin}\left(\dfrac{\text{de1}+2\cdot\text{de3}}{\text{de1}-2\cdot\text{de3}}\right) \end{bmatrix}$$

taking the sin of the dilation angle:

$$\sin\left(\pi+\text{asin}\left(\frac{\text{de1}+2\cdot\text{de3}}{\text{de1}-2\cdot\text{de3}}\right)\right)$$

simplifying:

$$1-\frac{2\cdot\text{de1}}{\text{de1}-2\cdot\text{de3}}$$

from Eq. 4.18 in Salgado

$$\frac{-(\text{de1}+2\text{de3})}{\text{de1}-2\text{de3}}$$

simplifies to:

$$1-\frac{2\cdot\text{de1}}{\text{de1}-2\cdot\text{de3}}$$

the results are the same

Dilation Angle (cont.)

Thursday, March 11, 2010
11:43 AM

For soils, rocks and concrete, the dilation angle is generally significantly smaller than the friction angle of the material. Vermeer and de Borst (1984) report the following typical values for ψ:

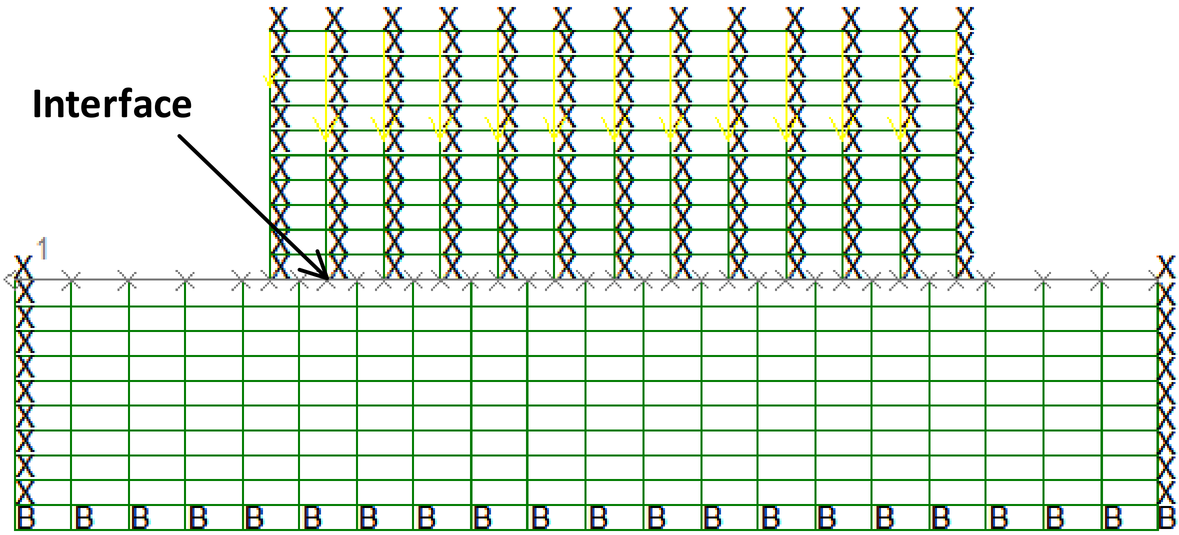
Table 3.8 Typical values for dilation angle [Vermeer and de Borst (1984)]

dense sand	15°
loose sand	$< 10°$
normally consolidated clay	0°
granulated and intact marble	12° − 20°
concrete	12°

Vermeer and de Borst observe that values for the dilation angle are approximately between 0° and 20°, whether the material is soil, rock or concrete. The default value for dilation angle is zero for all the constitutive models in *FLAC*.

Simple Interface Model

Thursday, March 11, 2010
11:43 AM




```
config
set large
g 20 21
model elas
gen 0,0 0,10 21,10 21,0
; scales model to 1 cm
ini x mul 0.01
ini y mul 0.01
; creates horz. gap in grid
model null j 11
; creates gap on both sides of upper part of grid
model null i 1,4 j 12,21
model null i 17,20 j 12,21
; reconnects the grid
ini x add .005 j 12 22
ini y add -.00475 j 12 22
```

Simple Interface Model (cont.)

Thursday, March 11, 2010
11:43 AM

```
; creates interface
int 1 Aside from 1,11 to 21,11 Bside from 5,12 to 17,12
int 1 kn 10e6 ks 10e6 cohesion 0 fric 35 dil 5
; elastic properties for model
prop dens 2000 bulk 8.3e6 shear 3.85e6
; boundary conditions
fix x y j=1
fix x i=1 j 1,11
fix x i=21 j=1,11
; apply pressure at top of model
apply p=50e3 i=5,17 j=22
;
his 999 unb
; consoldates sample under applied pressure
solve
;
; starts shear part of test
ini xvel 5e-7 i= 5,17 j 12,22
fix x i= 5,17 j 12,22
; reinitializes displacements to zero
ini xdis 0.0 ydis 0.0
```

Simple Interface Model (cont.)

Thursday, March 11, 2010
11:43 AM

```
; functions to calculate shear stress and displacements
call int.fin ; this needs to be in default folder
def ini_jdisp
  njdisp0 = 0.0
  sjdisp0 = 0.0
  pnt = int_pnt
  loop while pnt # 0
    pa = imem(pnt+$kicapt)
    loop while pa # 0
      sjdisp0 = sjdisp0 + fmem(pa+$kidasd)
      njdisp0 = njdisp0 + fmem(pa+$kidand)
      pa = imem(pa)
    end_loop
    pa = imem(pnt+$kicbpt)
 loop while pa # 0
   sjdisp0 = sjdisp0 + fmem(pa+$kidasd)
   njdisp0 = njdisp0 + fmem(pa+$kidand)
   pa = imem(pa)
 end_loop
   pnt = imem(pnt)
 end_loop
end
ini_jdisp
;
```

Simple Interface Model (cont.)

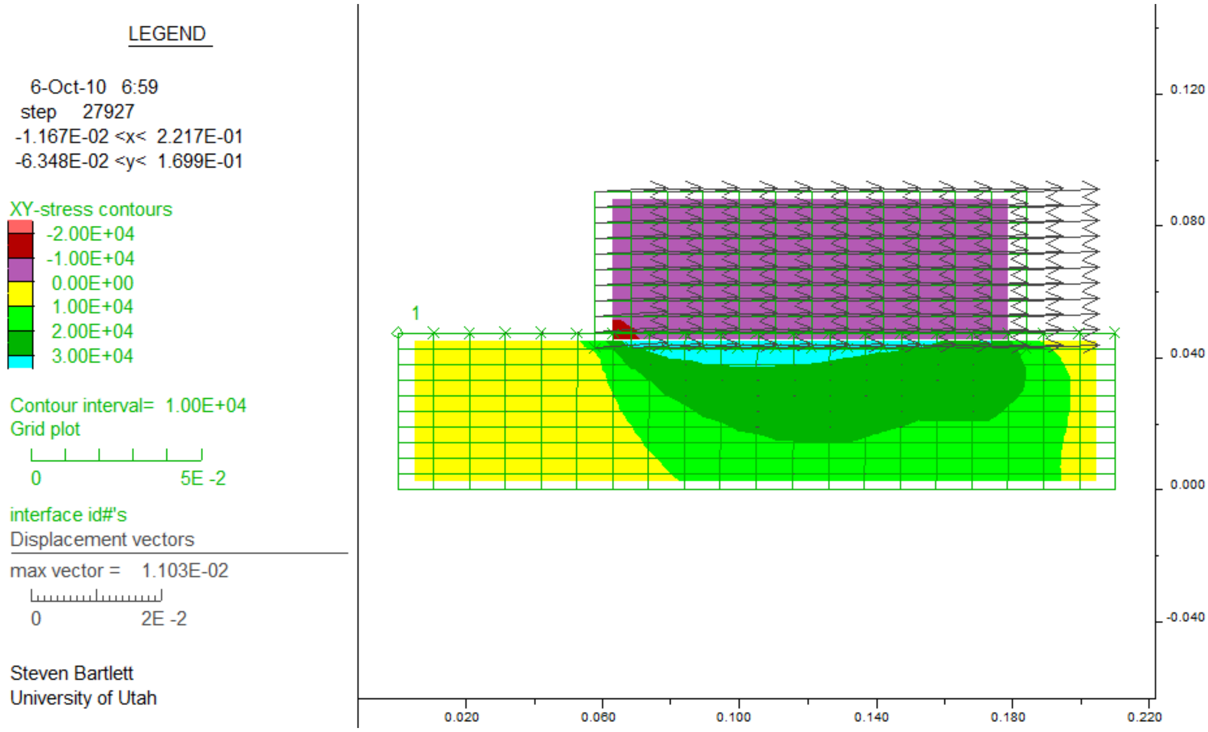
Thursday, March 11, 2010
11:43 AM

```
def av_str
whilestepping
  sstav = 0.0
  nstav = 0.0
  njdisp = 0.0
  sjdisp = 0.0
  ncon = 0
  jlen = 0.0
  pnt = int_pnt
  loop while pnt # 0
   pa = imem(pnt+$kicapt)
    loop while pa # 0
      sstav = sstav + fmem(pa+$kidfs)
      nstav = nstav + fmem(pa+$kidfn)
      jlen = jlen + fmem(pa+$kidlen)
      sjdisp = sjdisp + fmem(pa+$kidasd)
      njdisp = njdisp + fmem(pa+$kidand)
      pa = imem(pa)
    end_loop
    pa = imem(pnt+$kicbpt)
  loop while pa # 0
     ncon = ncon + 1
     sstav = sstav + fmem(pa+$kidfs)
     nstav = nstav + fmem(pa+$kidfn)
    jlen = jlen + fmem(pa+$kidlen)
    sjdisp = sjdisp + fmem(pa+$kidasd)
    njdisp = njdisp + fmem(pa+$kidand)
    pa = imem(pa)
  end_loop
  pnt = imem(pnt)
 end_loop
 if ncon # 0
   sstav = sstav / jlen
   nstav = nstav / jlen
   sjdisp = (sjdisp-sjdisp0) / (2.0 * ncon)
   njdisp = (njdisp-njdisp0) / (2.0 * ncon)
endif
end
```

Simple Interface Model (cont.)

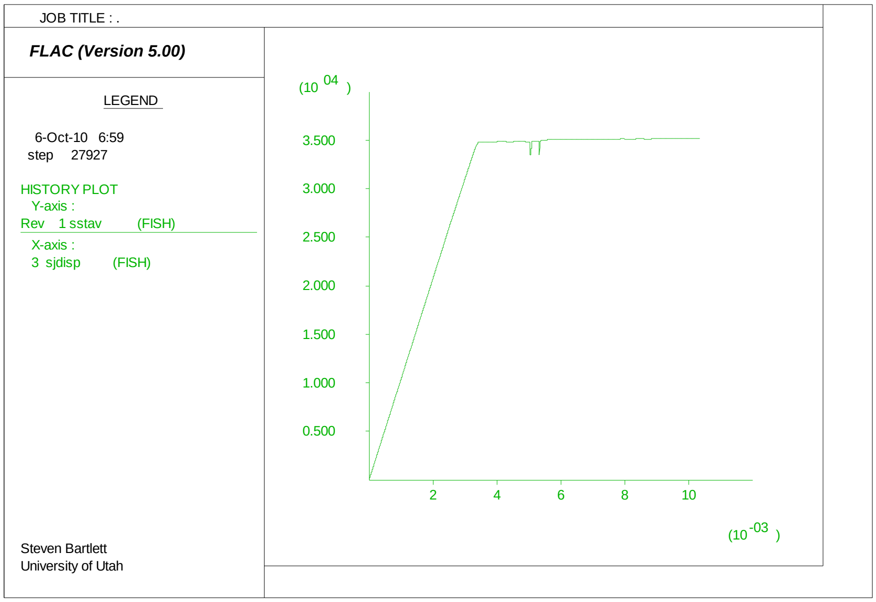
Thursday, March 11, 2010
11:43 AM

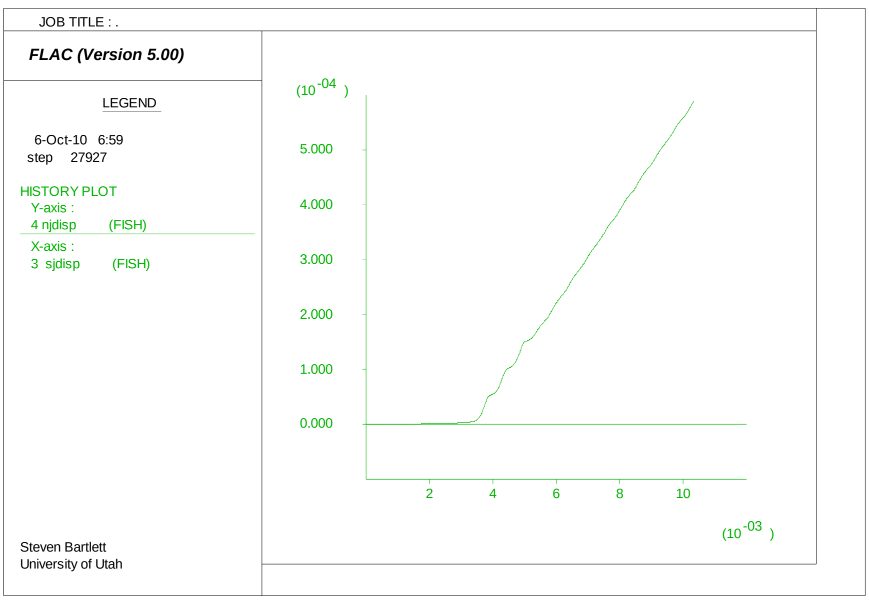
hist sstav nstav sjdisp njdisp
step 22000
save directshear.sav 'last project state'



Simple Interface Model (cont.)

Thursday, March 11, 2010
11:43 AM





Interface Example

Thursday, March 11, 2010
11:43 AM

Typical freestanding geofoam embankment at bridge approach. Note that continuous horizontal planes are created by the block placement pattern. Question: Could sliding occur along these interface planes during a major earthquake?

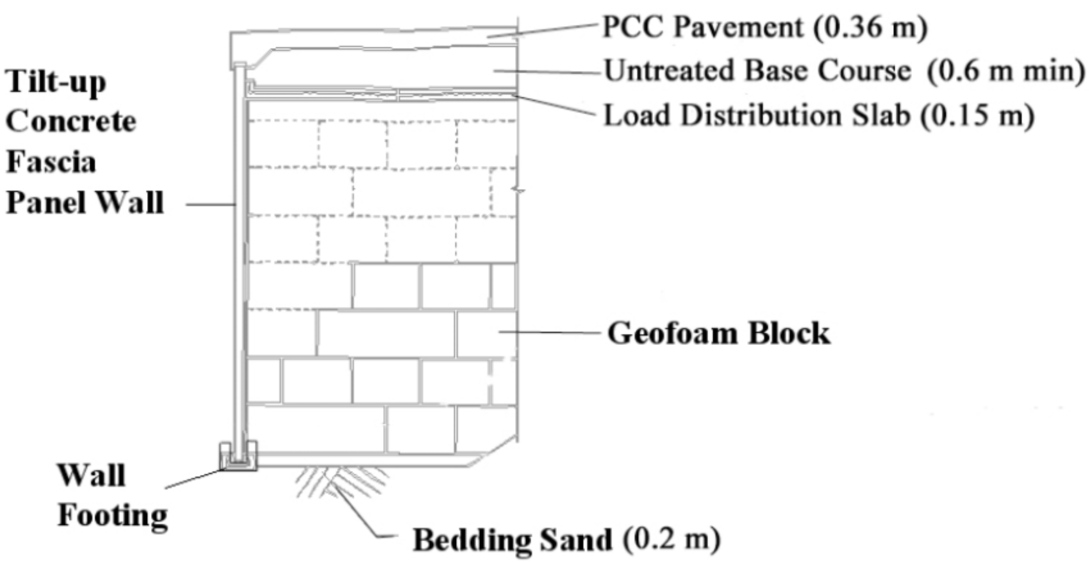
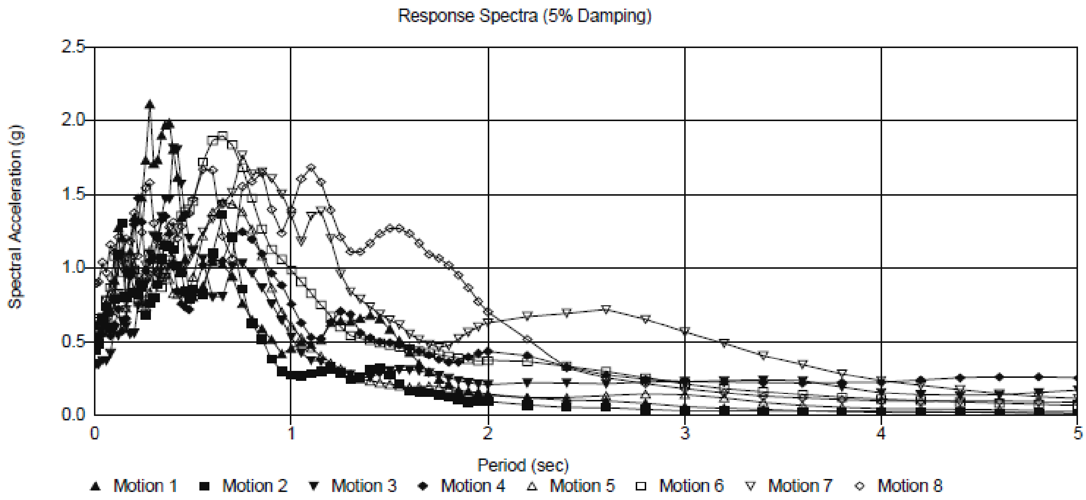


Figure 4. Typical geofoam cross-section used for the I-15 Reconstruction Project.

Interface Example (cont.)

Thursday, March 11, 2010
11:43 AM



e 1. Five percent damped horizontal acceleration response spectra for the evaluation time histories.

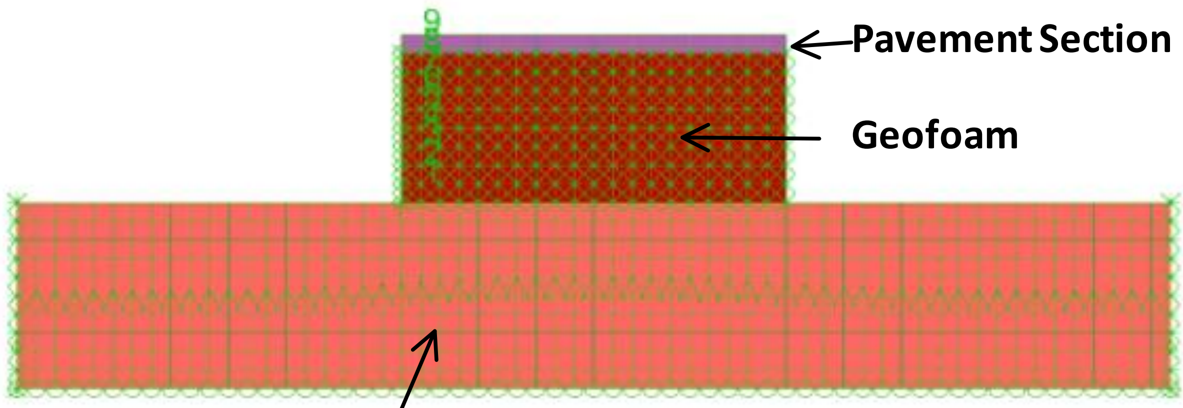
Horizontal acceleration response spectra for earthquakes used in sliding evaluation.

TABLE I. HORIZONTAL STRONG MOTION RECORDS SELECTED FOR EVALUATIONS.

Motion	Earthquake	M	R (km)	Component	PGA (g)
1	1989 Loma Prieta, CA	6.9	8.6	Capitola 000	0.52
2	1989 Loma Prieta, CA	6.9	8.6	Capitola 090	0.44
3	1999 Duzce, Turkey	7.1	8.2	Duzce 180	0.35
4	1999 Duzce, Turkey	7.1	8.2	Duzce 270	0.54
5	1992 Cape Mendocino, CA	7.1	9.5	Petrolia 000	0.59
6	1992 Cape Mendocino, CA	7.1	9.5	Petrolia 090	0.66
7	1994 Northridge, CA	6.7	6.2	Sylmar 052	0.61
8	1994 Northridge, CA	6.7	6.2	Sylmar 142	0.90

Interface Example (cont.)

Thursday, March 11, 2010
11:43 AM



FLAC X-sectional model (2D plane strain)

TABLE III. INITIAL ELASTIC MODULI AND PROPERTIES FOR THE FLAC MODEL

Material Type	Layer No.	ρ (kg/m³)[4]	E (MPa)[5]	ν[6]	K (MPa)[7]	G (MPa)[8]
Foundation Soil	1-10	1840	174	0.4	290.0	62.1
Geofoam	11-18	18	10	0.103	4.2	4.5
UTBC[1]	19	2241	570	0.35	633.3	211.1
LDS[2] & PCCP[3]	19	2401	30000	0.18	15625.0	12711.9

[1] Untreated base course, [2] Load distribution slab, [3] Portland concrete cement pavement, [4] Mass density, [5] Initial Young's modulus, [6] Poisson's ratio, [7] Bulk modulus, [8] Shear modulus

Interface Example (cont.)

Thursday, March 11, 2010
11:43 AM

More on interface properties

Normal and shear stiffness at the interfaces are also required by *FLAC.* These are spring constants that represent the respective stiffness between two planes that are in contact with each other. Interfacial stiffness is often used in *FLAC* to represent the behavior of rock joints where some elastic deformation in the joint is allowed before slippage occurs. However for geofoam block placed in layers, such elastic behavior before slippage occurs is probably small. Thus, for the case where only slippage and separation are considered at the interface (i.e., one geofoam subgrid is allowed to slide and/or open relative to another subgrid), the normal and shear stiffnesses used in the *FLAC* model are not important (Itasca. 2005). For this case, the *FLAC* user's manual recommends that the normal and shear interface stiffness (kn and ks, respectively) be set to ten times the stiffness of the neighboring zone.

$$k_n = k_s = 10\ [(K + 4/3G)/\Delta z_{min}]$$

TABLE IV. INTERFACIAL PROPERTIES USED FOR SLIDING EVALUATION IN THE *FLAC* MODEL.

Contact Surface	Interface number (bottom to top)	Normal and Shear Stiffness ($k_n = k_s$) (MPa)	Friction angle (degrees)
Geofoam-soil	1	102	31[1]
Geofoam-Geofoam	2-8	102	38
Geofoam-Lump Mass	9	102	38[2]

[1] A glued interface was used for interface 1 in FLAC because the geofoam is abutted against the panel wall footing and cannot slide. [2] Neglects any tensile or shear bonding that may develop between the top of geofoam and base of the load distribution slab.

Interface Example (cont.)

Thursday, March 11, 2010
11:43 AM

More on interface properties

Normal and shear stiffness at the interfaces are also required by *FLAC.* These are spring constants that represent the respective stiffness between two planes that are in contact with each other. Interfacial stiffness is often used in *FLAC* to represent the behavior of rock joints where some elastic deformation in the joint is allowed before slippage occurs. However for geofoam block placed in layers, such elastic behavior before slippage occurs is probably small. Thus, for the case where only slippage and separation are considered at the interface (i.e., one geofoam subgrid is allowed to slide and/or open relative to another subgrid), the normal and shear stiffnesses used in the *FLAC* model are not important (Itasca. 2005). For this case, the *FLAC* user's manual recommends that the normal and shear interface stiffness (kn and ks, respectively) be set to ten times the stiffness of the neighboring zone.

$$k_n = k_s = 10\,[(K + 4/3G)/\Delta z_{min}]$$

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Interface Example (cont.)

Thursday, March 11, 2010
11:43 AM

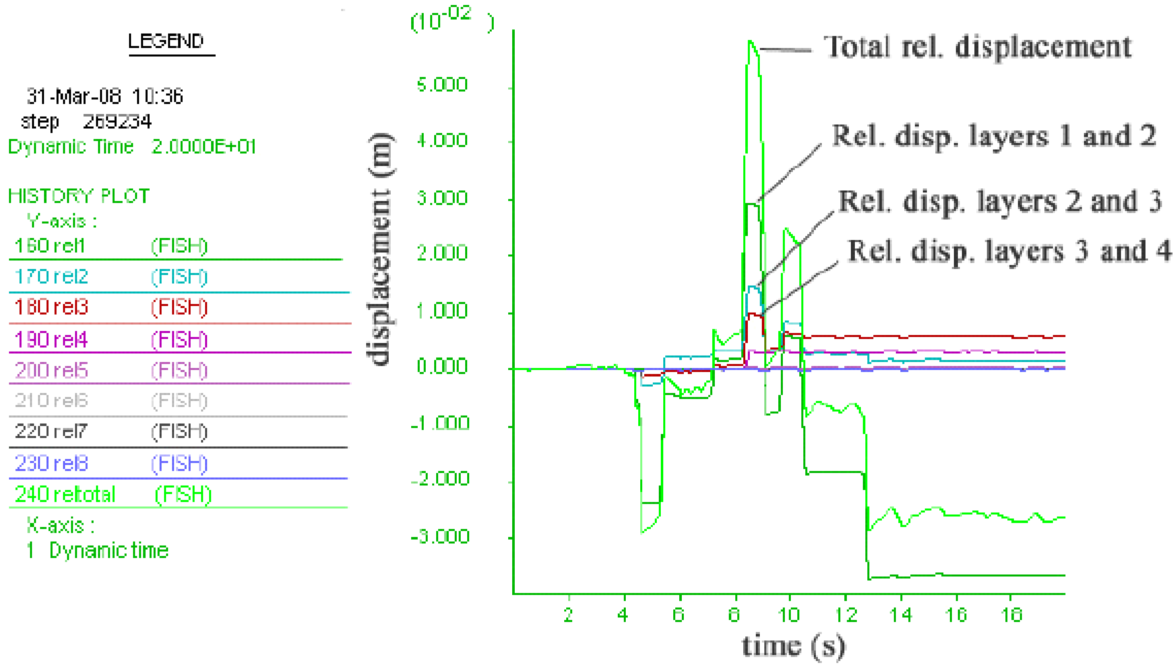
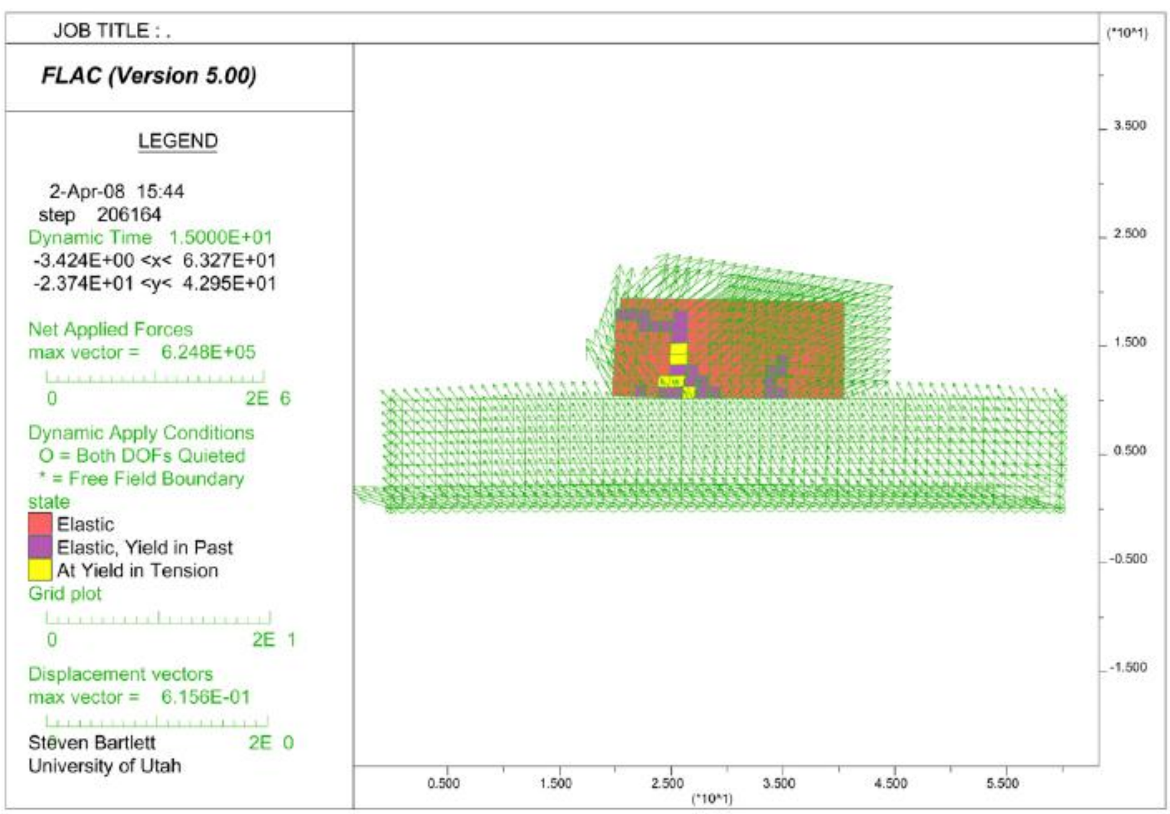


Figure 7. Relative sliding displacement plot for various geofoam layers for case 1a.



Interface Example (cont.)

Thursday, March 11, 2010
11:43 AM

CONCLUSIONS

In general. the majority of the evaluated cases suggest that interlayer sliding is within tolerable limits (0.01 to 0.1 m) however, two input time histories produced interlayer sliding that was greater than *0.5* in.. which is considered unacceptable from a performance standpoint Because the model predicted a wide range of interlayer sliding displacement for the cases analyzed, this suggests that sliding is a highly nonlinear process and is strongly governed by the frequency content and long period displacement pulses present in the input time histories.

The model also suggests that interlayer sliding displacement can, in some cases1 increase when the vertical component of strong motion is included in the analysis For cases where interlayer sliding is just initiating, the sliding displacement increases by a factor of 2 to *5* times when the vertical component of strong motion is added to the analyses However, when the interlayer sliding displacements are larger. the presence of the vertical component in the model is less important and the displacements remain the same or only slightly increase. Thus, we conclude that it is generally unconservative to ignore the vertical component of strong motion when estimating sliding displacement, but *its* inclusion is less important when the interlayer sliding displacement is well developed. All models showed that the interlayer sliding is generally concentrated in the basal layers and diminishes greatly in the higher layers. The potential for interlayer sliding displacement in geofoam embankments can be resolved by constructing shear keys within the geofoam mass to disrupt continuous horizontal layers that are being created by current construction practices

The numerical model also suggests that internal deformation caused by rocking and sway can cause local tensile yielding of some blocks within the embankment, usually near the base.. In some cases, this yielding can propagate upward and cause the embankment to begin to decouple dynamically. Consideration should be given to using blocks with higher strengths than Type VIII geofoam in the basal zones of geofoam embankments undergoing high levels of strong motion.

More Reading

Thursday, March 11, 2010
11:43 AM

- FLAC User's Manual, Theory and Background, Section 4 Interfaces
- FLAC User's Manual, Section 3.4.7.1, Shear Dilatancy
- Bartlett, S. F. and Lawton E. C., 2008, “Evaluating the Seismic Stability and Performance of Freestanding Geofoam Embankment,” 6^{th} National Seismic Conference on Bridges and Highways, Charleston, S.C., July 27^{th} –30^{th} 2008, 17 p.

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Thursday, March 11, 2010
11:43 AM

Earth Pressure Theory

Thursday, March 11, 2010
11:43 AM

Examples of Retaining Walls

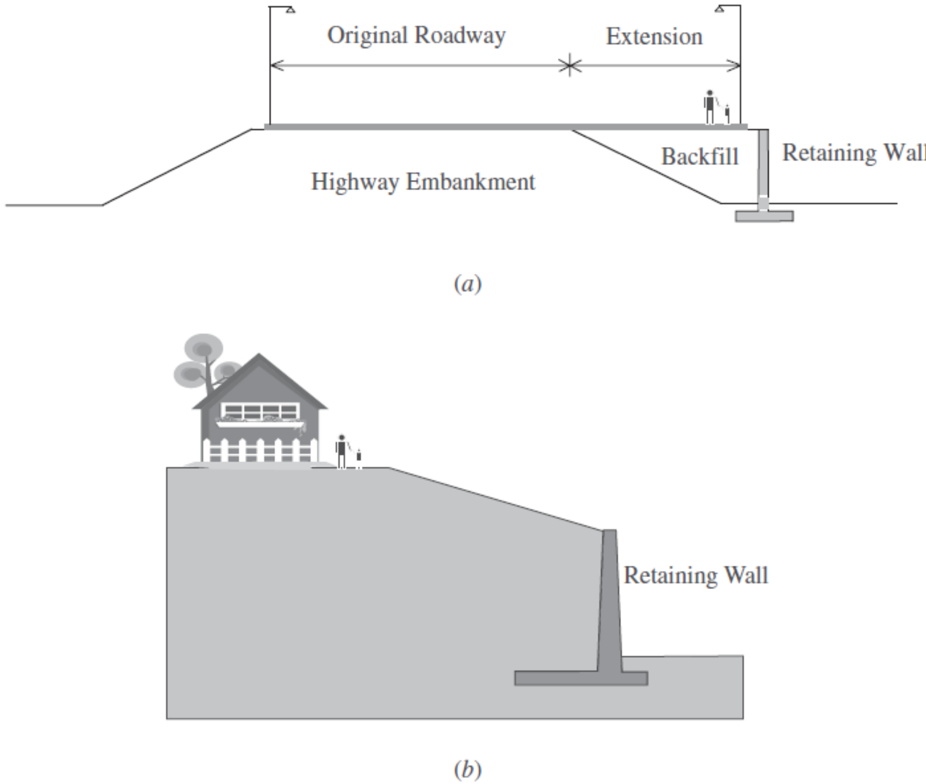
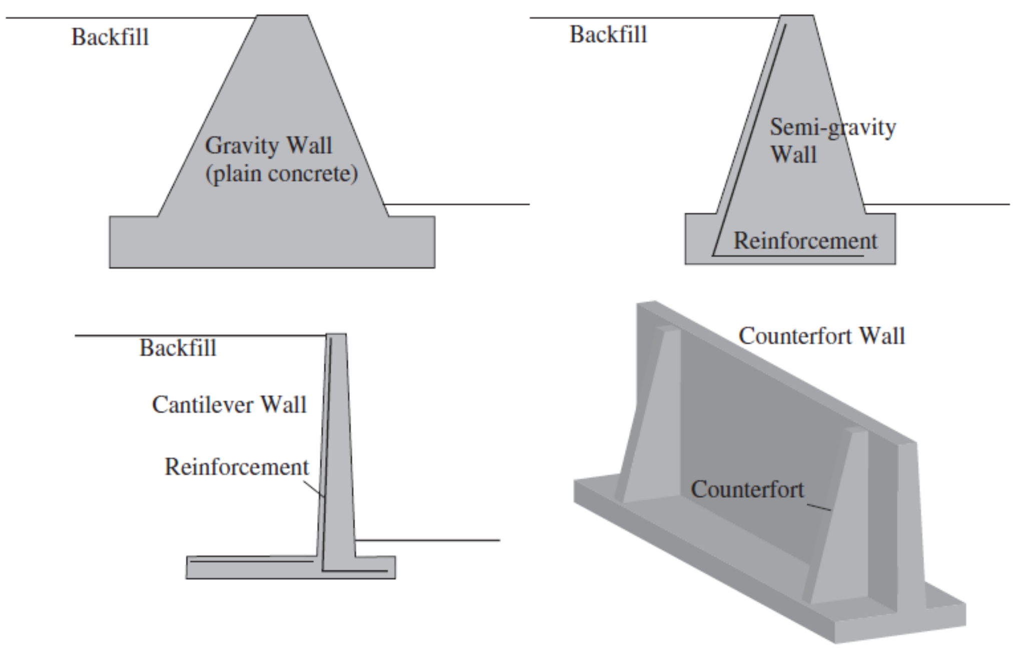
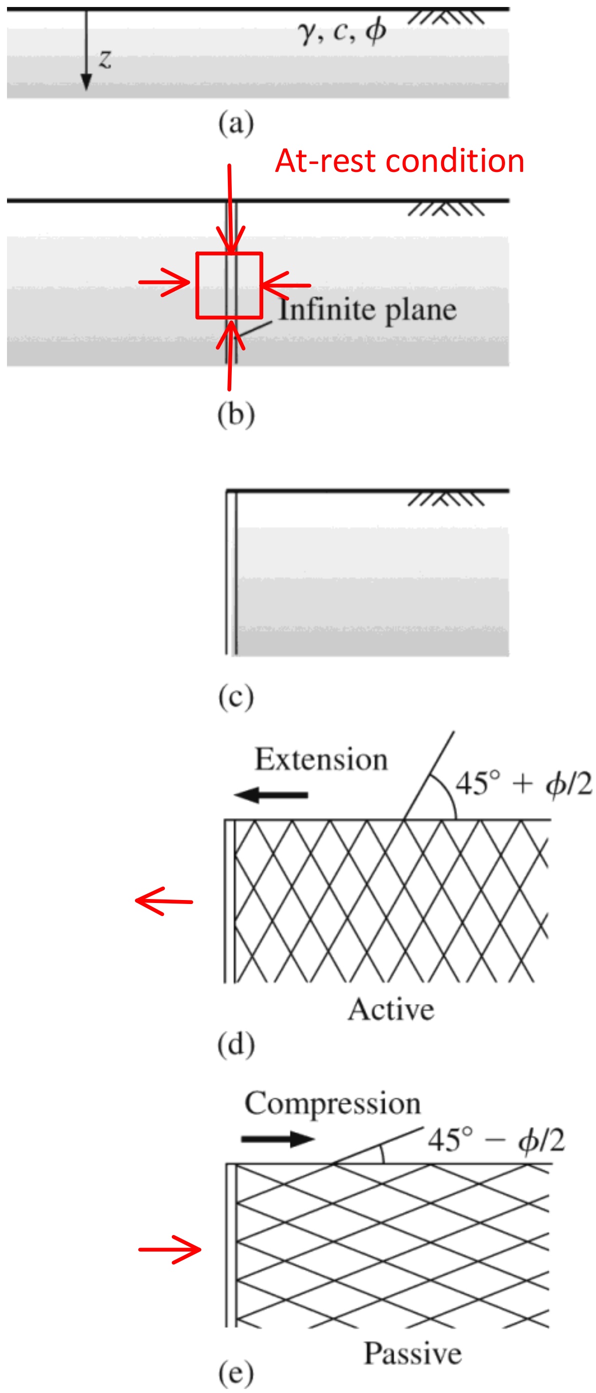


FIGURE 7.1 Retaining walls.



At-Rest, Active and Passive Earth Pressure

Wednesday, August 17, 2011
12:45 PM



At-rest earth pressure:

a. Shear stress are zero.
b. $\sigma_V = \sigma_1$
c. $\sigma_H = \sigma_3$
d. $\sigma_H = K_o\sigma_1$
e. $K_0 = 1 - \sin\phi$ (Normally consolidated
f. $K_0 = (1 - \sin\phi)\ OCR^{-1/2}$
g. $OCR = \sigma'_{vp}/\sigma'_v$
h. $K_0 = \nu / (1-\nu)$

Let us assume that:

a) wall is perfectly smooth (no shear stress develop on the interface between wall and the retained soil)
b) no sloping backfill
c) back of the wall is vertical
d) retained soil is a purely frictional material (c=0)

At-Rest, Active and Passive Earth Pressure (cont.)

Thursday, March 11, 2010
11:43 AM

Earth pressure is the lateral pressure exerted by the soil on a shoring system. It is dependent on the soil structure and the interaction or movement with the retaining system. Due to many variables, shoring problems can be highly indeterminate. Therefore, it is essential that good engineering judgment be used.

At-Rest Earth Pressure

At rest lateral earth pressure, represented as K_0, is the *in situ* horizontal pressure. It can be measured directly by a dilatometer test (DMT) or a borehole pressure meter test (PMT). As these are rather expensive tests, empirical relations have been created in order to predict at rest pressure with less involved soil testing, and relate to the angle of shearing resistance. Two of the more commonly used are presented below.

Jaky (1948) for normally consolidated soils:

$$K_{0(NC)} = 1 - \sin \phi'$$

Mayne & Kulhawy (1982) for overconsolidated soils:

$$K_{0(OC)} = K_{0(NC)} * OCR^{(\sin \phi')}$$

The latter requires the OCR profile with depth to be determined

Pasted from <http://en.wikipedia.org/wiki/Lateral_earth_pressure>

Earth Pressure Theory (cont)

Thursday, March 11, 2010
11:43 AM

The at-rest earth pressure coefficient (Ko) is applicable for determining the in situ state of stress for undisturbed deposits and for estimating the active pressure in clays for systems with struts or shoring. Initially, because of the cohesive property of clay there will be no lateral pressure exerted in the at-rest condition up to some height at the time the excavation is made. However, with time, creep and swelling of the clay will occur and a lateral pressure will develop. This coefficient takes the characteristics of clay into account and will always give a positive lateral pressure. This method is called the Neutral Earth Pressure Method and is covered in the text by Gregory Tschebotarioff. This method can be used in FLAC to establish the at-rest condition in the numerical model.

$$K_o = \frac{v}{1 - v}$$

v = The Poisson's Ratio. It is determined by a Laboratory test (Maximum value = 0.5)

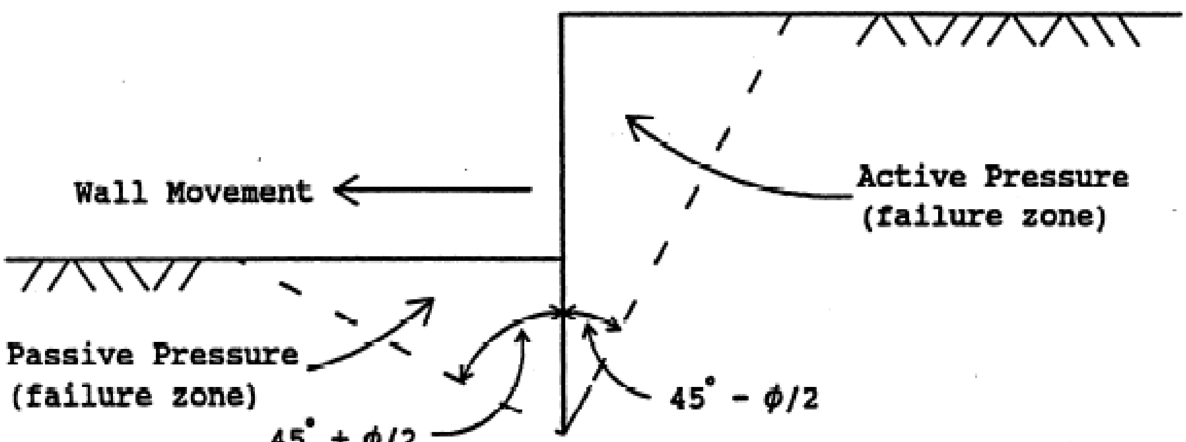
A Poisson's ratio of 0.5 means that there is no volumetric change during shear (i.e., completely undrained behavior).

Soil Type	Typical Value for Poisson's Ratio *	K_o
Clay, saturated	0.40 - 0.50	0.67 - 1.00
Clay, unsaturated	0.10 - 0.30	0.11 - 0.42
Sandy Clay	0.20 - 0.30	0.25 - 0.42
Silt	0.30 - 0.35	0.42 - 0.54
Sand		
Dense	0.20 - 0.40	0.25 - 0.67
Coarse		
(void ratio 0.4 - 0.7)	0.15	0.18
Fine-grained		
(void ratio 0.4 - 0.7)	0.25	0.33
Rock	0.10 - 0.40	0.11 - 0.67

Active and Passive Cases

Thursday, March 11, 2010
11:43 AM

Active and passive earth pressures are the two stages of stress in soils which are of particular interest in the design or analysis of shoring systems. Active pressure is the condition in which the earth exerts a force on a retaining system and the members tend to move toward the excavation. Passive pressure is a condition in which the retaining system exerts a force on the soil. Since soils have a greater passive resistance, the earth pressures are not the same for active and passive conditions. When a state of oil failure has been reached, active and passive failure zones, approximated by straight planes, will develop as shown in the following figure (level surfaces depicted).



Rankine Theory - Active and Passive Cases

Thursday, March 11, 2010
11:43 AM

The Rankine theory assumes that there is no wall friction and the ground and failure surfaces are straight planes, and that the resultant force acts parallel to the backfill slope (i.e., no friction acting between the soil and the backfill). The coefficients according to Rankine's theory are given by the following expressions:

$$K_a = \cos\beta \left[\frac{\cos\beta - [\cos^2\beta - \cos^2\phi]^{1/2}}{\cos\beta + [\cos^2\beta - \cos^2\phi]^{1/2}} \right]$$

$$K_p = \cos\beta \left[\frac{\cos\beta + [\cos^2\beta - \cos^2\phi]^{1/2}}{\cos\beta - [\cos^2\beta - \cos^2\phi]^{1/2}} \right]$$

If the backslope of the embankment behind the wall is level (i.e., $\beta = 0$) the equations are simplified as follows:

$$K_a = \frac{1 - \sin\phi}{1 + \sin\phi} = \tan^2(45^\circ - \phi/2)$$

$$K_p = \frac{1 + \sin\phi}{1 - \sin\phi} = \tan^2(45^\circ + \phi/2)$$

The Rankine formula for passive pressure can only be used correctly when the embankment slope angle equals zero or is negative. If a large wall friction value can develop, the Rankine Theory is not correct and will give less conservative results. Rankine's theory is not intended to be used for determining earth pressures directly against a wall (friction angled does not appear in equations above).
The theory is intended to be used for determining earth pressures on a vertical plane within a mass of soil.

Rankine Theory - Active Case and Displacements

Thursday, March 11, 2010
11:43 AM

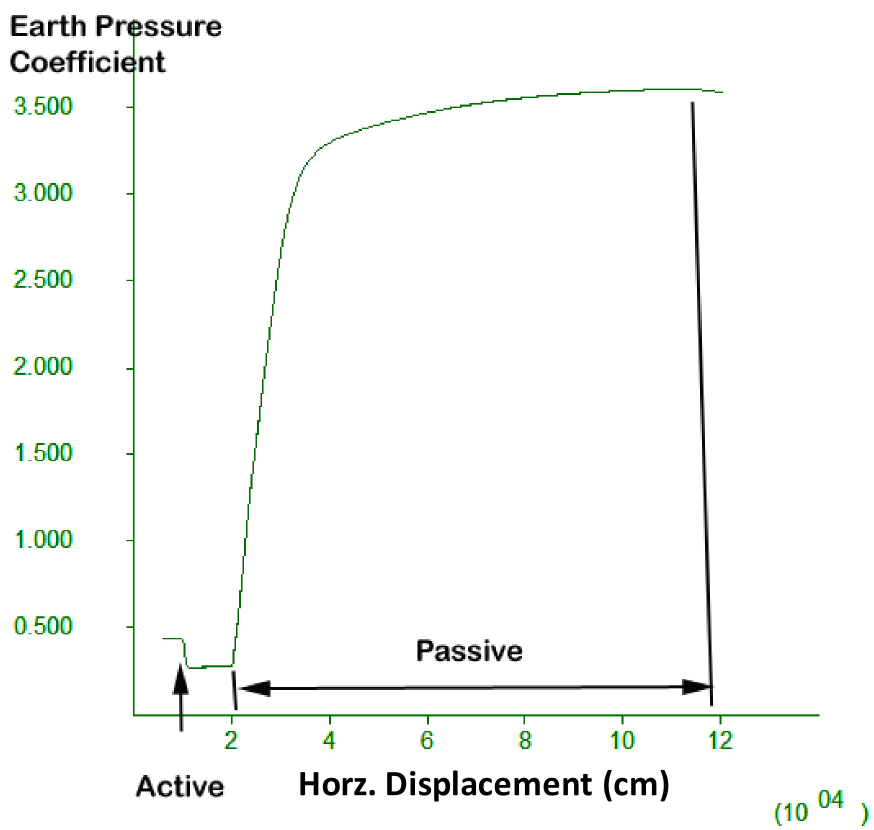
MOVEMENT OF WALL NECESSARY TO PRODUCE ACTIVE PRESSURES

Soil Type	Wall Yield
Cohesionless, dense	0.001 H
Cohesionless, loose	0.001 - 0.002 H
Clay, firm	0.010 - 0.020 H
Clay, soft	0.020 - 0.050 H

* New Zealand Department of Public Works Retaining Wall Manual

H = height of wall

The amount of displacement to mobilize full passive resistance is about 10 times larger than active (see below).



Coulomb Theory

Thursday, March 11, 2010
11:43 AM

Coulomb theory provides a method of analysis that gives the resultant horizontal force on a retaining system for any slope of wall, wall friction, and slope of backfill provided This theory is based on the assumption that soil shear resistance develops along the wall and failure plane. The following coefficient is for a resultant pressure acting at angle δ.

$$K_a = \frac{\cos^2(\phi - \omega)}{\{\cos^2\omega\}\{\cos(\delta + \omega)\}\left[1 + \sqrt{\frac{\{\sin(\phi + \delta)\}\{\sin(\phi - \beta)\}}{\{\cos(\delta + \omega)\}\{\cos(\beta - \omega)\}}}\right]^2}$$

The passive K_p value for sloping embankment is not listed since this value can be drastically overestimated.

The following coefficients are for a horizontal resultant pressure and a vertical wall:

$$K_a = \frac{\cos^2\phi}{\cos\delta\left[1 + \sqrt{\frac{\{\sin(\phi + \delta)\}\{\sin(\phi - \beta)\}}{(\cos\delta)(\cos\beta)}}\right]^2}$$

$$K_p = \frac{\cos^2\phi}{\cos\delta\left[1 - \sqrt{\frac{\{\sin(\phi + \delta)\}\{\sin(\phi + \beta)\}}{(\cos\delta)(\cos\beta)}}\right]^2}$$

δ is the interface friction angle between the soil and the backwall.
β is the angle of the backslope

Interface Friction Angles and Adhesion

Thursday, March 11, 2010
11:43 AM

Wall friction angle (δ) varies from 0° to 22°, but is always less than the internal angle of friction of the soil (ϕ). It is accepted practice to assume a value of $\delta = 1/3(\phi)$ to $2/3(\phi)$. For systems subject to dynamic loading (adjacent railroads, pile driving operations, etc.) use $\delta = 0$. It is important to note that as wall friction increases, lateral pressures decrease, but the vertical load on the shoring system increase. Vertical load components must be considered in shoring design. **TABLE 14** lists friction of select soil types acting against various structural materials.

Interface Friction Angles and Adhesion

Thursday, March 11, 2010
11:43 AM

ULTIMATE FRICTION FACTORS AND ADHESION FOR DISSIMILAR MATERIALS

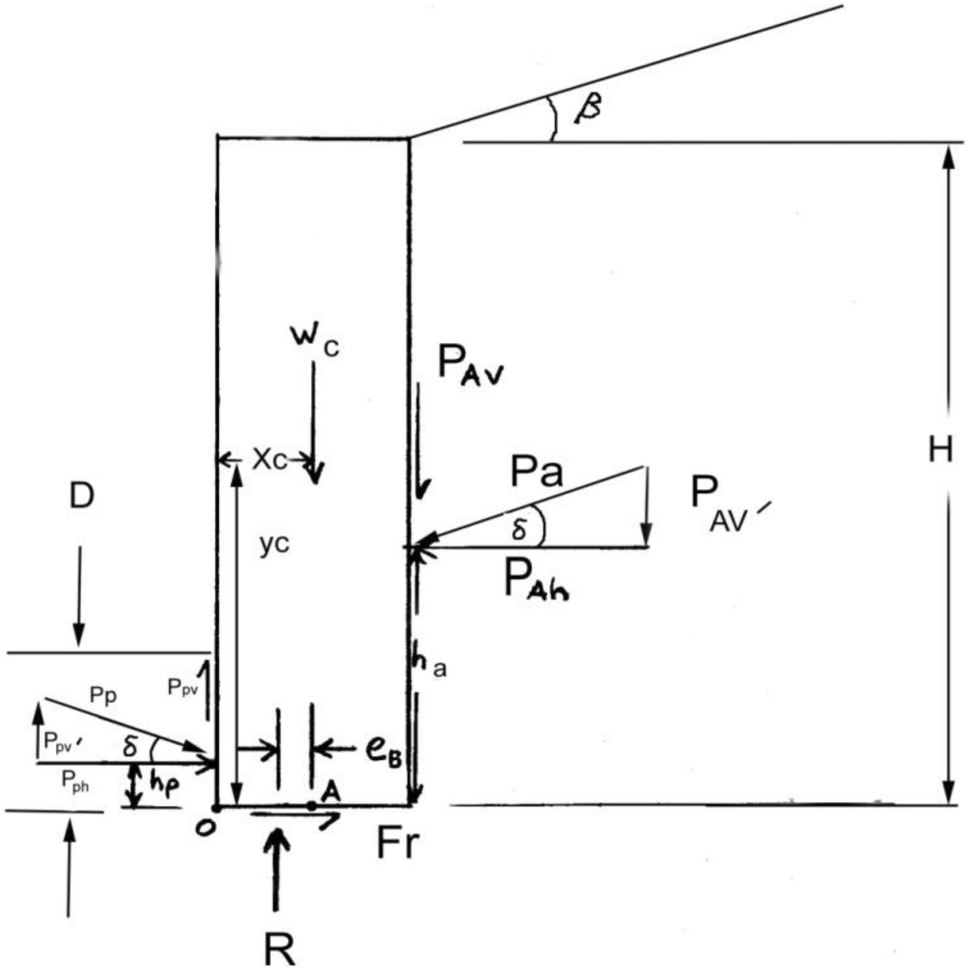
INTERFACE MATERIALS	FRICTION ANGLE, δ DEGREES
Steel sheet piles against the following soils:	
Clean gravel, gravel-sand mixtures, well-graded rock fill with spalls................................	22
Clean sand, silty sand-gravel mixture, single size hard rock fill......................................	17
Silty sand, gravel or sand mixed with silt or clay....	14
Fine sandy silt, nonplastic silt......................	11
Formed concrete or concrete sheet piling aginst the following soils:	
Clean gravel, gravel-sand mixture, well-graded rock fill with spalls................................	22 to 26
Clean sand, silty sand-gravel mixture, single size hard rock fill......................................	17 to 22
Silty sand, gravel or sand mixed with silt or clay....	17
Fine sandy silt, nonplastic silt......................	14
Mass concrete on the following materials:	
Clean sound rock......................................	35
Clean gravel, gravel-sand mixtures, coarse sand.......	29 to 31
Clean fine to medium sand, silty medium to coarse sand, silty or clayey gravel.......................	24 to 29
Clean fine sand, silty or clayey fine to medium sand..	19 to 24
Fine sandy silt, nonplastic silt......................	17 to 19
Very stiff and hard residual or preconsolidated clay..	22 to 26
Medium stiff and stiff clay and silty clay............	17 to 19
(Masonry on foundation materials has same friction factors.)	
Various structural materials:	
Masonry on masonry, ingneous and metamorphic rocks:	
Dressed soft rock on dressed soft rock..............	35
Dressed hard rock on dressed soft rock..............	33
Dressed hard rock on dressed hard rock..............	29
Masonry on wood (cross grain).........................	26
Steel on steel at sheet pile interlocks...............	17

INTERFACE MATERIALS (COHESION)	ADHESION C_a (PSF)
Very soft cohesive soil (0 - 250 psf)	0 - 250
Soft cohesive soil (250 - 500 psf)	250 - 500
Medium stiff cohesive soil (500 - 1000 psf)	500 - 750
Stiff cohesive soil (1000 - 2000 psf)	750 - 950
Very stiff cohesive soil (2000 - 4000 psf)	950 - 1,300

TABLE 14 - WALL FRICTION AND ADHESION

Gravity Wall Design

Thursday, March 11, 2010
11:43 AM



Wall Dimensions				**Fill Properties**			
Top	3	ft		$\beta_{backfill}$ deg	20	0.349	*radians*
Bottom	3	ft		β_{toe} deg	0	0.000	*radians*
$\gamma_{concrete}$	150	pcf		φ deg	40	0.698	*radians*
H	10	ft		δ deg	20	0.349	*radians*
D	2	ft		$Q_{backwall}$ deg	0	0.000	*radians*
				$Q_{frontwall}$ deg	0	0.000	*radians*
x_c	1.500	ft		$\gamma_{backfill}$	100		*pcf*
y_c	5.000	ft					

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Gravity Wall Design (cont.)

Thursday, March 11, 2010
11:43 AM

Earth Pressures	Coulomb Theory			
K_A	0.2504			
K_P	11.7715			
Forces				
Pa	1252.1	lb/ft		
P_{ah}	1176.6	lb/ft		
$P_{av'}$	428.2	lb/ft		
P_{av}	428.2	lb/ft		
W_c	4500	lb/ft		
R	4928.2	lb/ft	$W_c + P_{av'}$	
F_r	4135.3	lb/ft	R tan (d or f)	
0.5Pp	1177.1	lb/ft	(half of P_p)	
P_{ph}	1106.16	lb/ft		
$P_{pv'}$	402.6	lb/ft		
P_{pv}	402.6	lb/ft		

Resisting Moments on Wall		
P_{av} * B	1284.7	
P_{ph} * D/3	737.4	
W_c * x_c	6750	
SM_r	**8772.2**	
Overturning Moments on Wall		
P_{ah} * h_a	**3921.9**	
SM_o	**3921.9**	
Factors of Safety		
$FS_{sliding}$	**4.455**	
FS_{oturn}	**2.237**	

Building Systems Incrementally

Thursday, March 11, 2010
11:43 AM

For multilayer systems or systems constructed in lifts or layers, it is sometimes preferable to place each layer and allow FLAC to come to equilibrium under the self weight of the layer before the next layer is placed.

This incremental placement approach is particularly useful when trying to determine the initial state of stress in multilayered systems with marked differences in stiffness (e.g., pavements).

It can also be used to replicate the construction process or to determine how the factor of safety may vary versus fill height when analyzing embankments or retaining wall.

This approach is shown in the following pavement system example

Note this approach is not required for homogenous media.

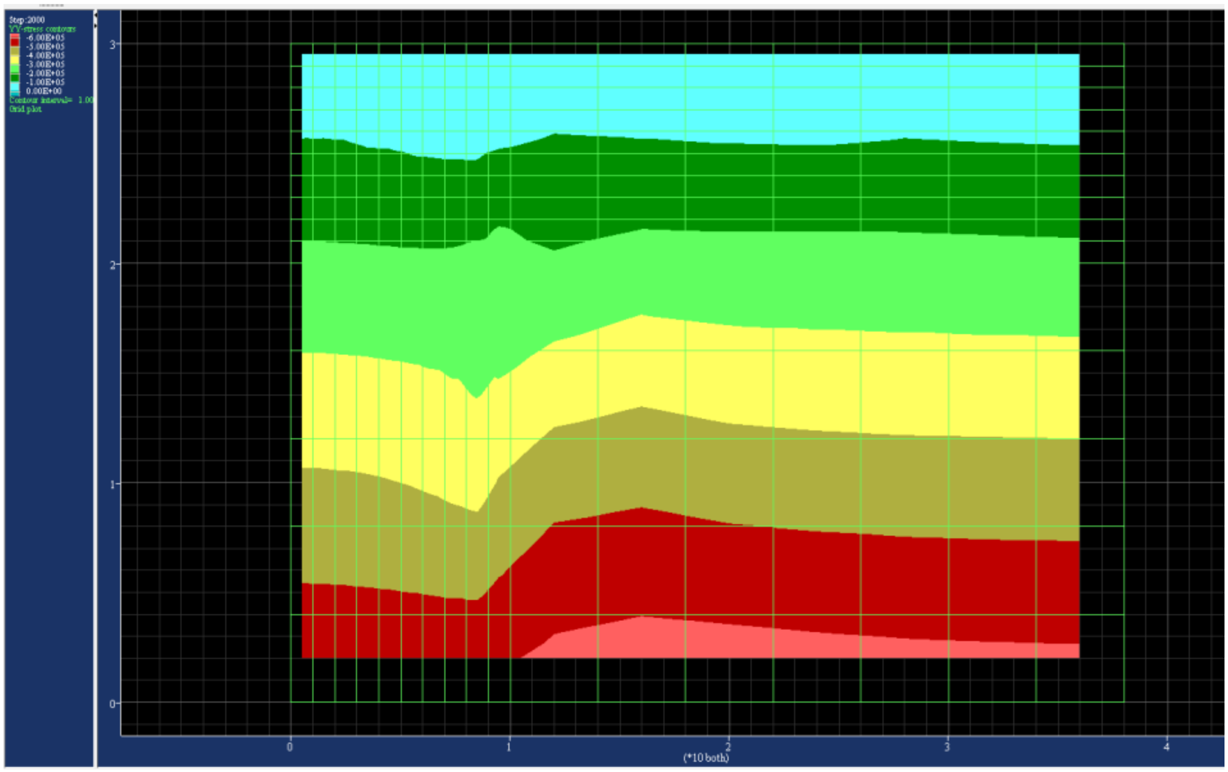
Building Systems Incrementally (cont.)

Thursday, March 11, 2010
11:43 AM

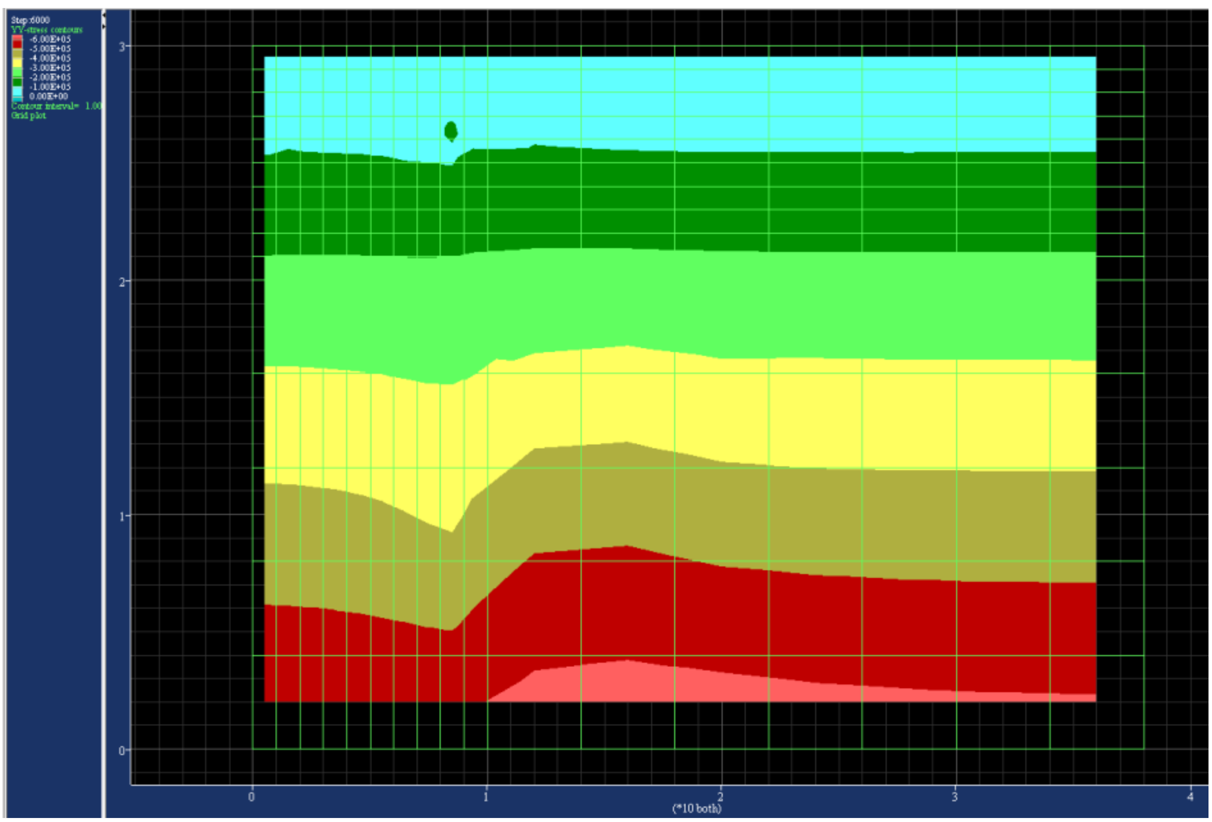
```
;flac 1 - incremental loading
config
grid 17,15
model mohr
gen same 0 20 10 20 same i 1 11 j 1 6
gen same 0 25 10 25 same i 1 11 j 6 11
gen same 0 30 10 30 same i 1 11 j 11 16
gen same same 38 20 38 0 i 11 18 j 1 6
gen same same 38 25 same i 11 18 j 6 11
gen same same 38 30 same i 11 18 j 11 16
mark j 6 ; marked to determine regions
mark j 11 ;marked to determine regions
prop density=2160.5 bulk=133.33E6 shear=44.4444E6 cohesion=0 friction=35.0 reg i 2 j 2 ; region
command
prop density=2400.5 bulk=41.67E6 shear=19.23E6 cohesion=25e3 friction=25.0 reg i 2 j 8
prop density=2240.5 bulk=833.33E6 shear=384.6E6 cohesion=0 friction=30.0 reg i 2 j 12
set gravity=9.81
fix x i=1
fix x i=18
fix y j=1
his unbal
; nulls out top two layers
model null reg i 2 j 8 ; second layer
model null reg i 2 j 12 ; third layer
step 2000 ; solves for stresses due to first layer
model mohr reg i 2 j 8; assign properties to 2nd layer
prop density=2400.5 bulk=41.67E6 shear=19.23E6 cohesion=25e3 friction=25.0 reg i 2 j 8
step 2000
model mohr reg i 2 j 12; assign properties to 3rd layer
prop density=2240.5 bulk=833.33E6 shear=384.6E6 cohesion=0 friction=30.0 reg i 2 j 12
step 2000
save incremental load.sav 'last project state'
```

Building Systems Incrementally

Thursday, March 11, 2010
11:43 AM



Vertical stress for 3 layers placed all at one time



Vertical stress for 3 layers placed incrementally

More Reading

Thursday, March 11, 2010
11:43 AM

- Applied Soil Mechanics with ABAQUS Applications, Ch. 7

Assignment 7

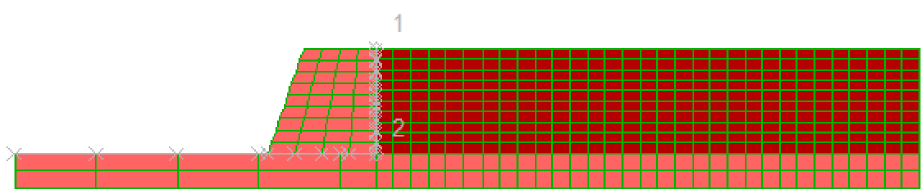
Thursday, March 11, 2010
11:43 AM

1. Develop a FLAC model of a concrete gravity wall (3-m high, 2-m wide (top) 3-m wide (base)) resting on a concrete foundation. Use the model to calculate the earth pressures for the cases shown below using the given soil properties. To do this, show a plot of the average earth pressure coefficient that develops against the backwall versus dytime. Report your modeling answers to 3 significant figures (30 points). Compare the modeling results with those obtained from Rankine theory.

   a. At-rest
   b. Active
   c. Passive

   Backfill (Mohr-Coulomb)
   Density = 2000 kg/m^3
   Bulk modulus = 25 Mpa
   Friction angle = 35 degrees
   Dilation angle = 5 degrees
   Cohesion = 0

   Concrete (Elastic)
   prop density=2400.0 bulk=1.5625E10 shear=1.27119E10



2. Repeat problem 1a, b and c but assume that the friction acting against the back wall of the retaining wall is phi (backfill) divided by 2. (10 points). Compare your results with Coulomb theory.

Assignment 7

Thursday, March 11, 2010
11:43 AM

3. Using the results of problem 1 from FLAC, calculate the factor of safety against sliding and overturning assuming that there is no friction acting between the backfill and the back wall and that the interface friction at the base of the wall is 35 degrees.

   To calculate the factors of safety, you must use the horizontal stresses (converted to forces) that act on the back wall of the gravity wall from the FLAC results. This can be obtained by using histories commands and converted to forces by multiplying by the contributing area. You can also calculate the basal stresses along the bottom of the wall in a similar manner (10 points).

4. Repeat problem 3, but use limit equilibrium methods to calculate the appropriate forces from Rankine theory (10 points).

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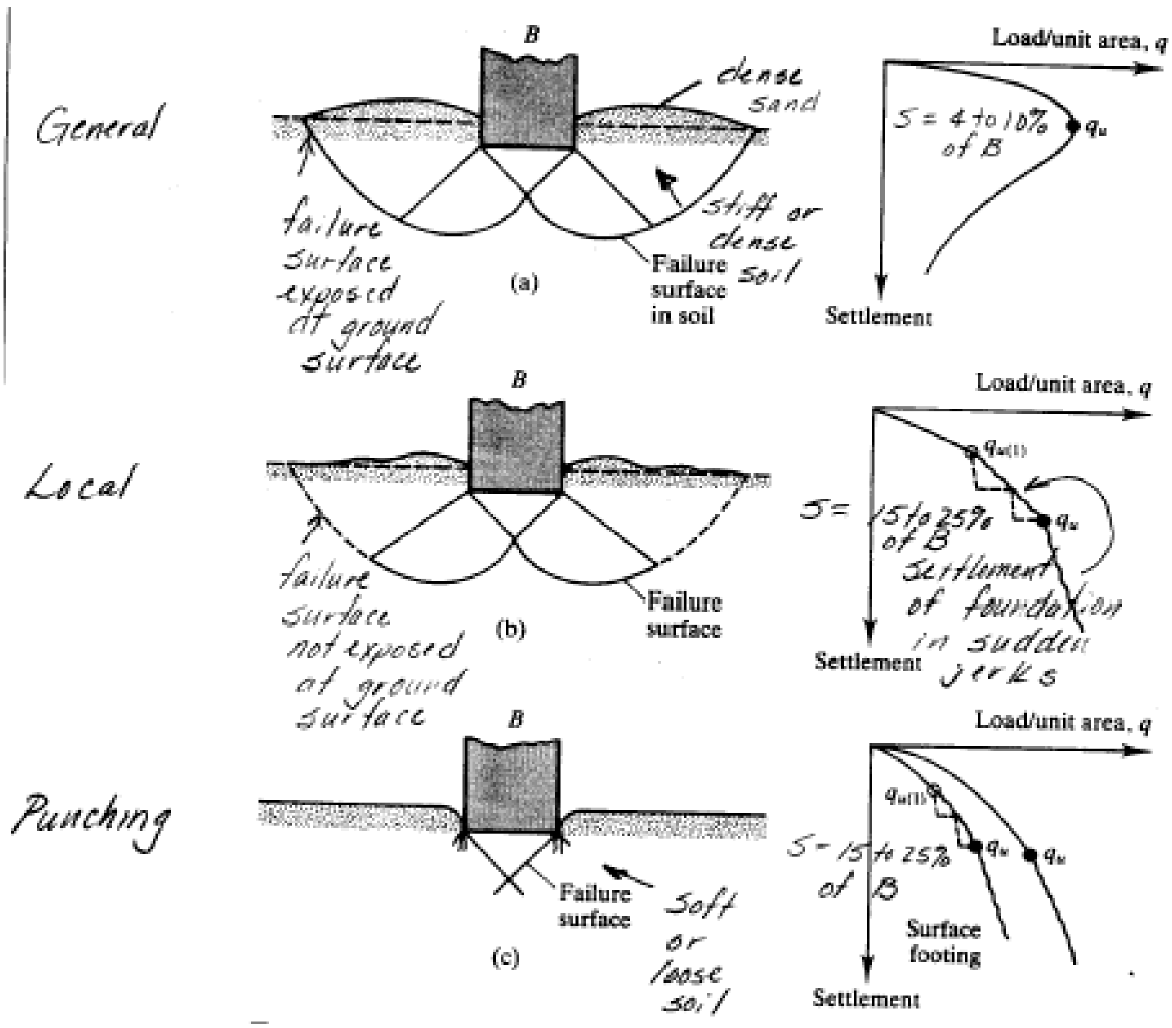
Thursday, March 11, 2010
11:43 AM

Shallow Foundations

Thursday, March 11, 2010
11:43 AM

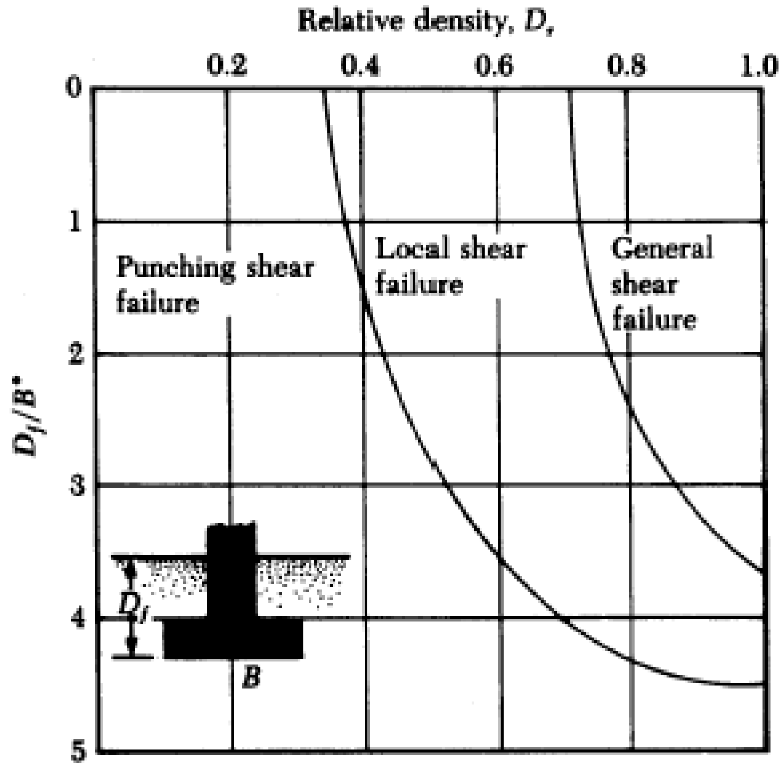
Modes of Failure

- **General Shear Failure**
- **Local Shear Failure**
- **Punching**



Modes of Failure

Thursday, March 11, 2010
11:43 AM



▼ FIGURE 3.3 Modes of foundation failure in sand (after Vesic, 1973)

D_r = relative density

D_f = depth of foundation (ground surface to bottom of footing)

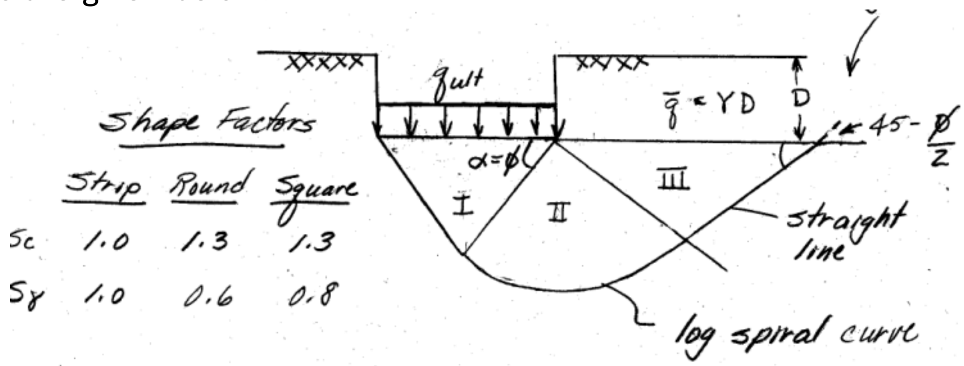
$$B^\circ = \frac{2BL}{B+L}$$

where B = width of footing
L = length of footing

Terzaghi's Bearing Capacity Theory

Thursday, March 11, 2010
11:43 AM

Karl von Terzaghi was the first to present a comprehensive theory for the evaluation of the ultimate bearing capacity of rough shallow foundations. This theory states that a foundation is shallow if its depth is less than or equal to its width.[3] Later investigations, however, have suggested that foundations with a depth, measured from the ground surface, equal to 3 to 4 times their width may be defined as shallow foundations(Das, 2007). Terzaghi developed a method for determining bearing capacity for the general shear failure case in 1943. The equations are given below.



For square foundations:

$$q_{ult} = 1.3c'N_c + \sigma'_{zD}N_q + 0.4\gamma'BN_\gamma$$

For continuous foundations:

$$q_{ult} = c'N_c + \sigma'_{zD}N_q + 0.5\gamma'BN_\gamma$$

For circular foundations:

$$q_{ult} = 1.3c'N_c + \sigma'_{zD}N_q + 0.3\gamma'BN_\gamma$$

where

$$N_q = \frac{e^{2\pi(0.75-\phi'/360)\tan\phi'}}{2\cos^2(45+\phi'/2)}$$

$$N_c = 5.7$$

for ϕ' = 0

$$N_c = \frac{N_q - 1}{\tan\phi'}$$

for ϕ' > 0

$$N_\gamma = \frac{\tan\phi'}{2}\left(\frac{K_{p\gamma}}{\cos^2\phi'} - 1\right)$$

Terzaghi's Bearing Capacity Theory (cont.)

Thursday, March 11, 2010
11:43 AM

c' is the effective cohesion.
σ_{zD}' is the vertical effective stress at the base of the foundation
γ ′is the effective unit weight when saturated or the total unit weight when not fully saturated.
B is the width or the diameter of the foundation.
φ′ is the effective internal angle of friction.

$K_{p\gamma}$ is obtained graphically. Simplifications have been made to eliminate the need for $K_{p\gamma}$. One such was done by Coduto, given below, and it is accurate to within 10%.[2]

$$N_\gamma = \frac{2\left(N_q + 1\right)\tan\phi'}{1 + 0.4\sin 4\phi'}$$

For foundations that exhibit the local shear failure mode in soils, Terzaghi suggested the following modifications to the previous equations. The equations are given below.

For square foundations:
$$q_{ult} = 0.867c'N_c' + \sigma_{zD}'N_q' + 0.4\gamma' B N_\gamma'$$
For continuous foundations:
$$q_{ult} = \frac{2}{3}c'N_c' + \sigma_{zD}'N_q' + 0.5\gamma' B N_\gamma'$$
For circular foundations:
$$q_{ult} = 0.867c'N_c' + \sigma_{zD}'N_q' + 0.3\gamma' B N_\gamma'$$

N'_c, N'_q *and* N'_γ, the modified bearing capacity factors, can be calculated by using the bearing capacity factors equations(for N_c, N_q, *and* N_γ, respectively) by replacing the effective internal angle of friction(ϕ') by a value equal to
: $tan^{-1}\left(\frac{2}{3}tan\phi'\right)$

Pasted from <http://en.wikipedia.org/wiki/Bearing_capacity>

FLAC Model of Shallow Footing on Cohesive Soil

Thursday, March 11, 2010
11:43 AM

The prediction of collapse loads under steady plastic flow conditions is one that can be difficult for a numerical model to simulate accurately (Sloan and Randolph 1982). A simple example of a problem involving steady-flow is the determination of the bearing capacity of a footing on an elastic-plastic soil. The bearing capacity is dependent on the steady plastic flow beneath the footing, thereby providing a measure of the ability of FLAC to model this condition.

Strip-Footing Problem — The bearing capacity for a strip footing is from the solution to "Prandtl's wedge" as given by Terzaghi and Peck (1967):

$$q = (2 + \pi)c$$

or

$$q = 5.14c \qquad (6.1)$$

where c is the cohesion of the material, and q is the bearing capacity stress at failure. The solution is based on the mode of failure, as shown in Figure 6.1.

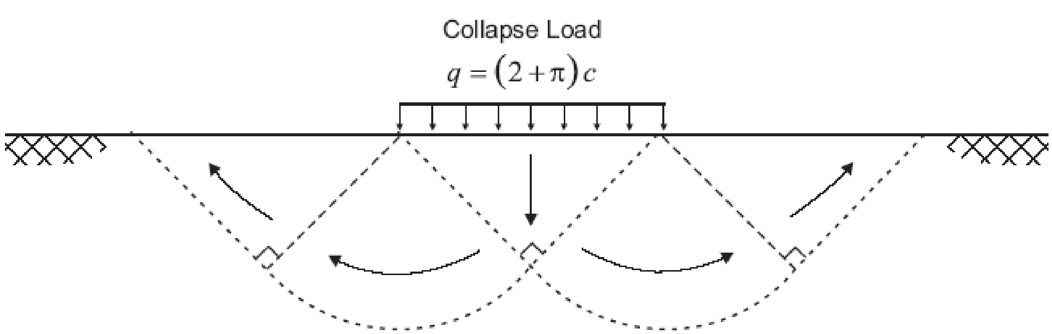


Figure 6.1 Prandtl's wedge problem of a strip footing on a frictionless soil

FLAC Model of Shallow Footing on Cohesive Soil (cont.)

Thursday, March 11, 2010
11:43 AM

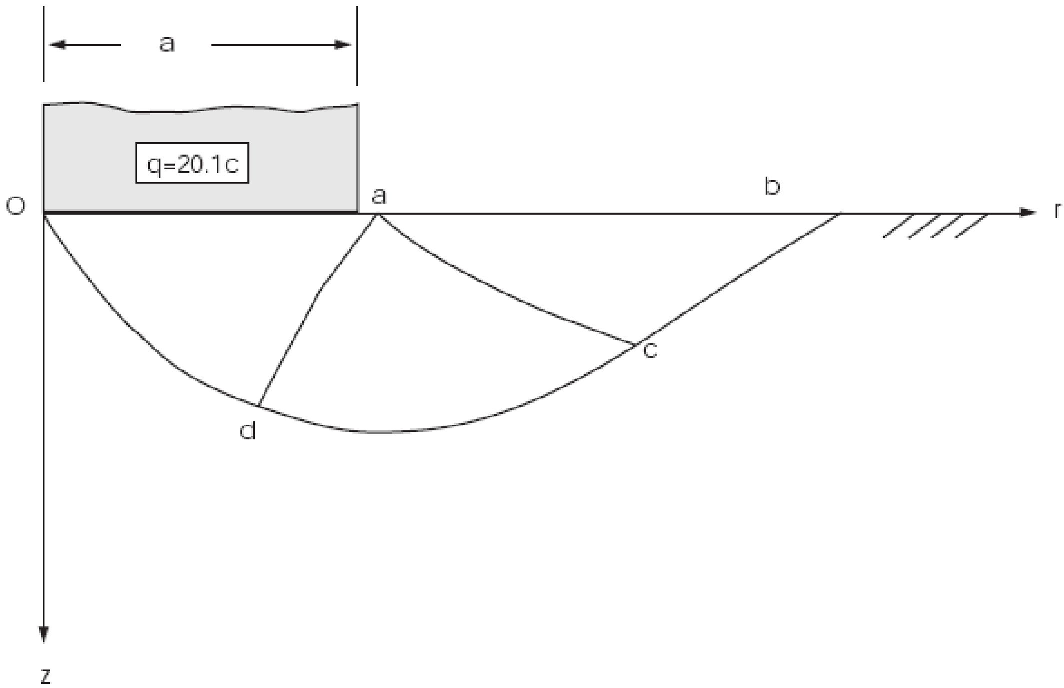


Figure 6.2 Cox slip-line net for a smooth circular footing ϕ = 20°

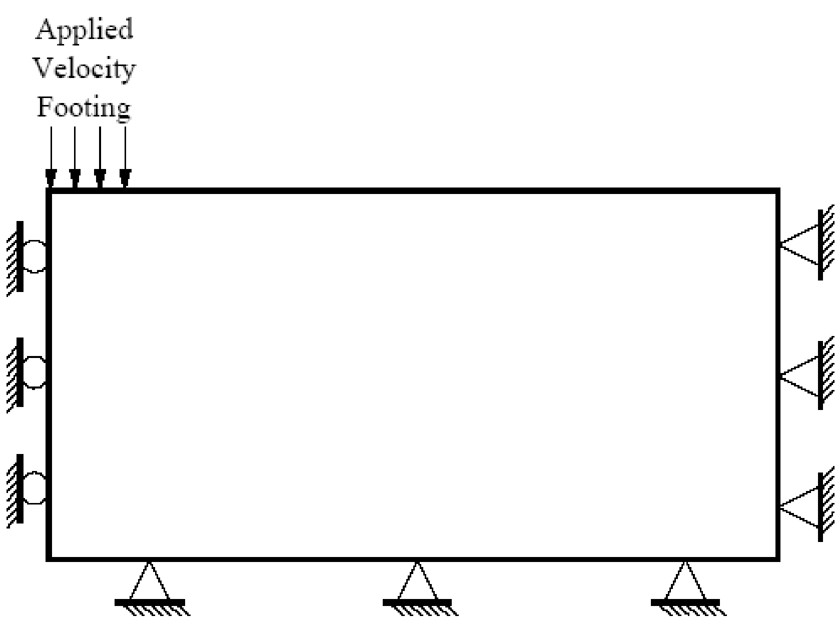


Figure 6.3 FLAC model boundary conditions

FLAC Model of Shallow Footing on Cohesive Soil (cont.)

Thursday, March 11, 2010
11:43 AM

```
config extra 8
; --- geometry ---
g 20 10
; --- constitutive model ---
model mohr
pro s=1e8 bul=2e8 d 1000 coh 1e5 fric 0.0 ten 1e10
; --- boundary conditions ---
fix x i=1
fix x y i=21
fix x y j=1
fix x y i=1,4 j=11
ini yv -2.5e-5 i=1,4 j=11
; --- comparison to analytical solution ---
def load
 sum =0.0
 loop i (1,4)
 sum =sum + yforce(i,11)
end_loop
load = sum/(0.5*(x(4,11)+x(5,11))); v stress see note below
disp = -ydisp(1,11)
end
def err
sol=(2+pi)*1e5; or 5.14*c from Terzaghi Theory
err=(load-sol)/sol*100; percent error
end
; -----------
; Histories
; -----------
hist unbal
hist load
hist err
hist sol
hist disp
step 5000
save terzaghi_strip.sav 'last project state'
```

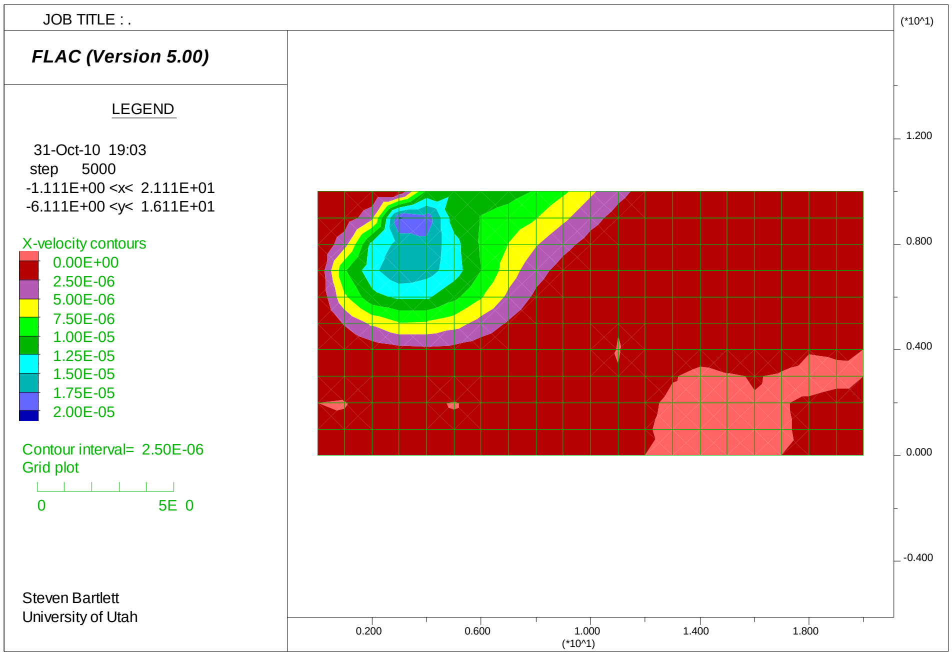
* When a velocity is applied to gridpoints to simulate a footing load, the bearing area is found by assuming that the velocity varies linearly from the value at the last applied gridpoint, to zero at the next gridpoint. The half-width area is then

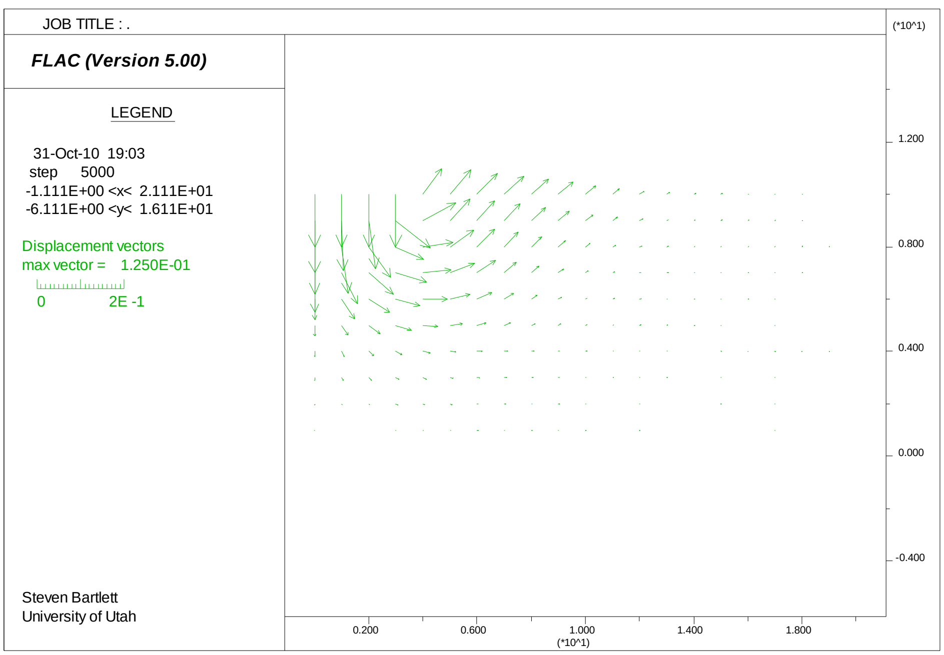
$$A = 0.5(x_l + x_{l+1}) \tag{6.3}$$

where x_l is the x-location of the last applied gridpoint velocity, and x_{l+1} is the x-location of the gridpoint adjacent to x_l.

FLAC Model of Shallow Footing on Cohesive Soil (cont.)

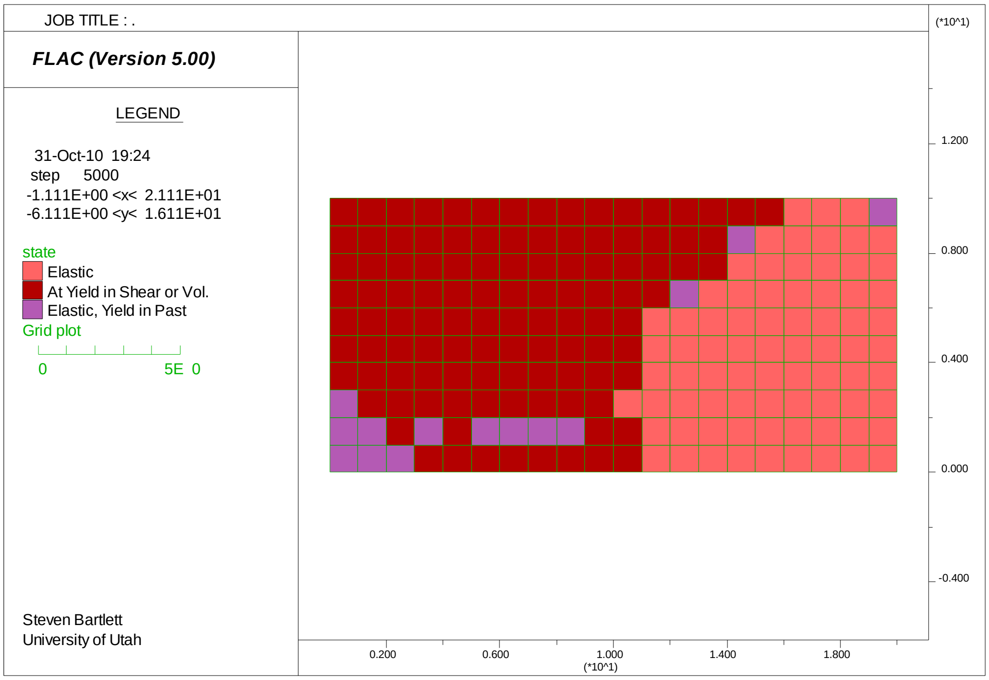
Thursday, March 11, 2010
11:43 AM

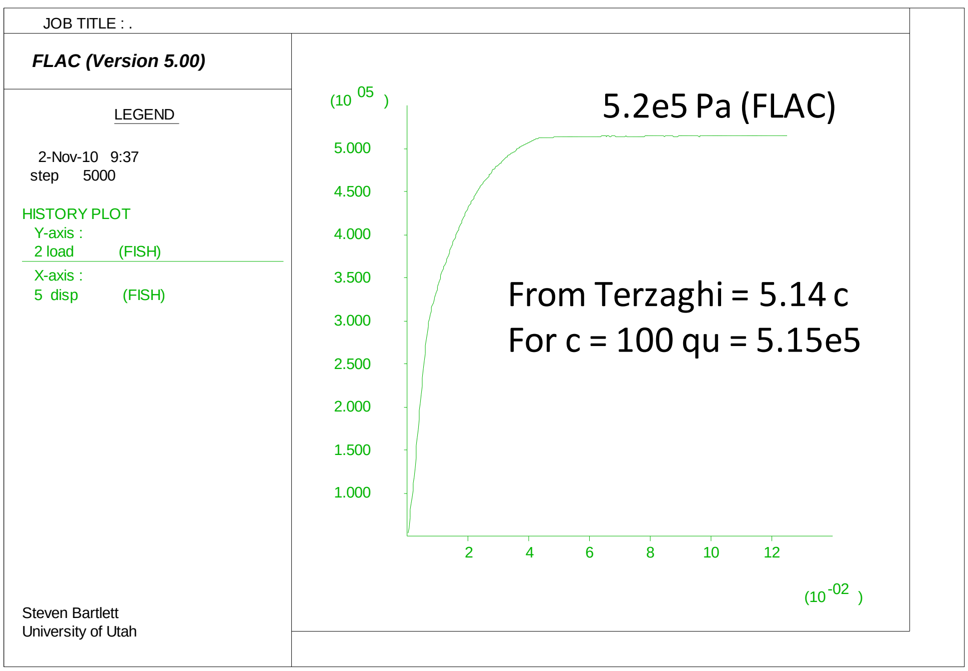




FLAC Model of Shallow Footing on Cohesive Soil (cont.)

Thursday, March 11, 2010
11:43 AM





Cohesionless Soil and Modification of Bearing Capacity Equation

Thursday, March 11, 2010
11:43 AM

Modification of Terzaghi's Bearing Capacity Equation

- Effects of embedment
- Different shapes (shape factors)
- Inclined loads (inclination factors)

Hansen Equation - Cohesionless Soil

Thursday, March 11, 2010
11:43 AM

Bearing Capacity, Hansen Equation (*Valid only for ϕ>0)
Reference: American Society of Civil Engineers, US Army Corps of Engineers Technical Engineering and Design Guide No. 7, Bearing Capacity of Soils, 1993, pp. 26-32.
Friction Angle estimate: Peck, Hansen, and Thornburn, Foundation Engineering, 1974, p. 310.

$pcf := 1 \cdot \frac{lb}{ft^3}$ $\quad psf := 1 \cdot \frac{lb}{ft^2}$ $\quad ksf := 1000 \cdot psf$

Effective Friction Angle	$\phi = 35.0\,deg$	Footing Width	$B_1 := 16.4 \cdot ft$
Cohesion	$C := 0 \cdot psf$	Footing Depth	$D_1 := 0 \cdot ft$
Effective unit weight of soil above footing	$\gamma_H := 124.8 \cdot pcf$	Footing Length	$L_1 := 1640 \cdot ft$
		Factor of Safety	$FS := 1$
Effective unit weight of soil beneath footings	$\gamma_D := 124.8 \cdot pcf$	N-value	$N = 26.205$

Bearing Capacity Factors

$N_q := e^{\pi \cdot \tan(\phi)} \cdot \left(\frac{1+\sin(\phi)}{1-\sin(\phi)}\right)$ $\qquad N_q = 33.3$

$N_\gamma := 1.5 \cdot (N_q - 1) \cdot \tan(\phi)$ $\qquad N_\gamma = 33.92$

$N_c := (N_q - 1) \cdot \cot(\phi)$ $\qquad N_c = 46.12$

Shape Factors

$s_c := 1 + \frac{N_q}{N_c} \cdot \frac{B_1}{L_1}$ $\qquad s_q := 1 + \frac{B_1}{L_1} \cdot \tan(\phi)$ $\qquad s_\gamma := 1 - 0.4 \cdot \frac{B_1}{L_1}$

$s_c = 1.01$ $\qquad s_q = 1.01$ $\qquad s_\gamma = 1.00$

Depth Factors $\qquad k := if\left(\frac{D_1}{B_1} \le 1, \frac{D_1}{B_1}, atan\left(\frac{D_1}{B_1}\right)\right)$ $\quad k = 0$

$d_c := 1 + 0.4 \cdot k$ $\qquad d_q := 1 + 2 \cdot \tan(\phi) \cdot (1 - \sin(\phi))^2 \cdot k$ $\qquad d_\gamma := 1.0$

$d_c = 1.00$ $\qquad d_q = 1.00$ $\qquad d_\gamma = 1.00$

Allowable Bearing Capacity, q_a

$q_{u2} := C \cdot N_c \cdot s_c \cdot d_c + D_1 \cdot \gamma_D \cdot N_q \cdot s_q \cdot d_q + 0.5 \cdot \gamma_H \cdot B_1 \cdot N_\gamma \cdot s_\gamma \cdot d_\gamma$

$q_{a1} := \frac{q_{u2}}{FS}$ $\qquad q_{a1} = 34.57\,ksf$

Vesic Equation - Cohesionless Soil

Thursday, March 11, 2010
11:43 AM

Bearing Capacity, Vesic Equation (*Valid only for ϕ>0)
Reference: American Society of Civil Engineers, US Army Corps of Engineers Technical Engineering and Design Guide No. 7, Bearing Capacity of Soils, 1993, pp. 26-32.
Friction Angle estimate: Peck, Hansen, and Thornburn, Foundation Engineering, 1974, p. 310.

Effective Friction Angle	$\phi = 35\,deg$	Footing Width	$B_1 = 16.4\,ft$
Cohesion	$C = 0\,psf$	Footing Depth	$D_1 = 0\,ft$
Effective unit weight of soil above footing	$\gamma_H = 124.8\,pcf$	Footing Length	$L_1 = 1.64 \times 10^3\,ft$
Effective unit weight of soil beneath footings	$\gamma_D = 124.8\,pcf$	Factor of Safety	$FS = 1$
		N-value	$N = 26.205$

Bearing Capacity Factors

$$N_q := e^{\pi \cdot \tan(\phi)} \cdot \left(\frac{1+\sin(\phi)}{1-\sin(\phi)}\right) \qquad N_q = 33.3$$

$$N_\gamma := 2 \cdot (N_q + 1) \cdot \tan(\phi) \qquad N_\gamma = 48.03$$

$$N_c := (N_q - 1) \cdot \cot(\phi) \qquad N_c = 46.12$$

Shape Factors

$$s_c := 1 + \frac{N_q}{N_c} \cdot \frac{B_1}{L_1} \qquad s_q := 1 + \frac{B_1}{L_1} \cdot \tan(\phi) \qquad s_\gamma := 1 - 0.4 \cdot \frac{B_1}{L_1}$$

$$s_c = 1.01 \qquad s_q = 1.01 \qquad s_\gamma = 1.00$$

Depth Factors

$$k := \text{if}\left(\frac{D_1}{B_1} \le 1, \frac{D_1}{B_1}, \text{atan}\left(\frac{D_1}{B_1}\right)\right) \qquad k = 0$$

$$d_c := 1 + 0.4 \cdot k \qquad d_q := 1 + 2 \cdot \tan(\phi) \cdot (1 - \sin(\phi))^2 \cdot k \qquad d_\gamma := 1.0$$

$$d_c = 1.00 \qquad d_q = 1.00 \qquad d_\gamma = 1.00$$

Allowable Bearing Capacity, q_a

$$q_{u2} := C \cdot N_c \cdot s_c \cdot d_c + D_1 \cdot \gamma_D \cdot N_q \cdot s_q \cdot d_q + 0.5 \cdot \gamma_H \cdot B_1 \cdot N_\gamma \cdot s_\gamma \cdot d_\gamma$$

$$q_{a2} := \frac{q_{u2}}{FS} \qquad q_{a2} = 48.95\,ksf$$

Meyerhof Equation - Cohesionless Soil

Thursday, March 11, 2010
11:43 AM

Bearing Capacity, Meyerhof Equation (*Valid only for ϕ>0)
Reference: American Society of Civil Engineers, US Army Corps of Engineers Technical Engineering and Design Guide No. 7, Bearing Capacity of Soils, 1993, pp. 26-32.
Friction Angle estimate: Peck, Hansen, and Thornburn, Foundation Engineering, 1974, p. 310.

Effective Friction Angle	$\phi = 35\,\text{deg}$	Footing Width	$B_1 = 16.4\,\text{ft}$
Cohesion	$C = 0\,\text{psf}$	Footing Depth	$D_1 = 0\,\text{ft}$
Effective unit weight of soil above footing	$\gamma_H = 124.8\,\text{pcf}$	Footing Length	$L_1 = 1.64 \times 10^3\,\text{ft}$
Effective unit weight of soil beneath footings	$\gamma_D = 124.8\,\text{pcf}$	Factor of Safety	$FS = 1$
		N-value	$N = 26.205$

Bearing Capacity Factors

$N_\phi := \left(\frac{1+\sin(\phi)}{1-\sin(\phi)}\right)$ $N_\phi = 3.69$

$N_q := e^{\pi \cdot \tan(\phi)} \cdot N_\phi$ $N_q = 33.3$

$N_\gamma := (N_q - 1) \cdot \tan(1.4 \cdot \phi)$ $N_\gamma = 37.15$

$N_c := (N_q - 1) \cdot \cot(\phi)$ $N_c = 46.12$

Shape Factors

$s_c := 1 + 0.2 \cdot N_\phi \cdot \frac{B_1}{L_1}$ $s_q := 1 + 0.1 \cdot N_\phi \cdot \frac{B_1}{L_1}$ $s_\gamma := 1 + 0.1 \cdot N_\phi \cdot \frac{B_1}{L_1}$

$s_c = 1.01$ $s_q = 1.00$ $s_\gamma = 1.00$

Depth Factors

$d_c := 1 + 0.2 \cdot N_\phi^{0.5} \cdot \frac{D_1}{B_1}$ $d_q := 1 + 0.1 \cdot N_\phi^{0.5} \cdot \frac{D_1}{B_1}$ $d_\gamma := 1 + 0.1 \cdot N_\phi^{0.5} \cdot \frac{D_1}{B_1}$

$d_c = 1.00$ $d_q = 1.00$ $d_\gamma = 1.00$

Allowable Bearing Capacity, q_a

$q_{u3} := C \cdot N_c \cdot s_c \cdot d_c + D_1 \cdot \gamma_D \cdot N_q \cdot s_q \cdot d_q + 0.5 \cdot \gamma_H \cdot B_1 \cdot N_\gamma \cdot s_\gamma \cdot d_\gamma$

$q_{a3} := \frac{q_{u3}}{FS}$ $q_{a3} = 38.16\,\text{ksf}$

Comparison of Methods

Thursday, March 11, 2010
11:43 AM

<u>Method</u>	<u>Allowable Bearing Capacity</u>
1. Hansen	$q_{a1} = 34.6\,\text{ksf}$
2. Vesic	$q_{a2} = 49.0\,\text{ksf}$
3. Meyerhof	$q_{a3} = 38.2\,\text{ksf}$

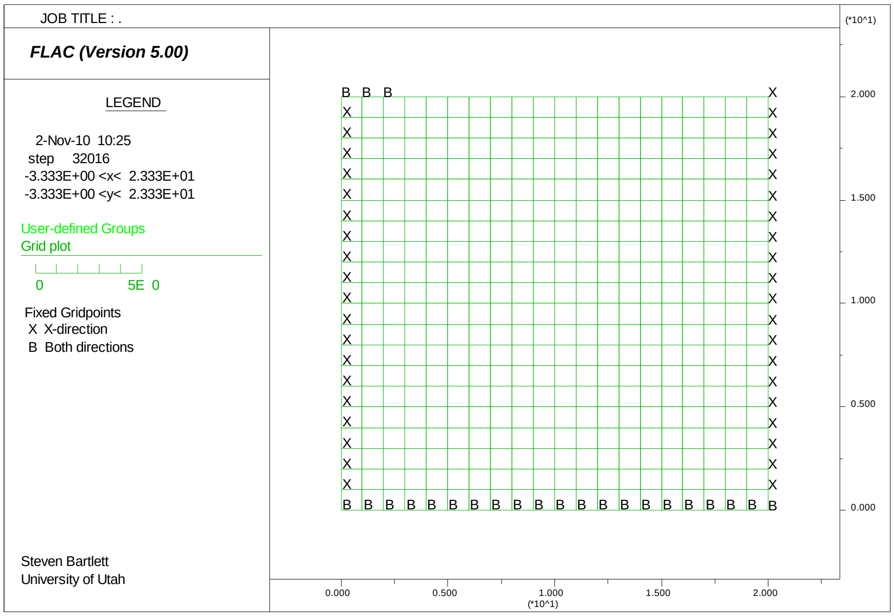
$$\text{Average} := g \cdot \frac{q_{a1} + q_{a2} + q_{a3}}{3}$$

$$\text{Average} = 1.942 \times 10^{6}\,\text{m}^{-1} \cdot \text{kg} \cdot \text{s}^{-2} \quad \text{or Pa}$$

1942 kPa

FLAC Model of Shallow Footing on Cohesionless Soil

Thursday, March 11, 2010
11:43 AM



FLAC Model of Shallow Footing on Cohesionless Soil (cont.)

Thursday, March 11, 2010
11:43 AM

```
config extra 8
; --- geometry ---
g 20 20
; --- constitutive model ---
model mohr
pro s=40e6 bul=80e6 d 2000 coh 0 fric 35.0 dilation 0 ten 0
; --- boundary conditions ---
fix x i=1
fix x i=21
fix x y j=1
;
set gravity 9.81
solve
;
; start pushing footing downward
fix x y i=1,3 j=21
ini yv -2.5e-5 i=1,3 j=21
;
def load
 sum =0.0
 loop i (1,3)
 sum =sum + yforce(i,21)
end_loop
load = sum/(0.5*(x(3,21)+x(4,21))); v stress see note below
disp = -ydisp(1,21)
end
; -----------
; Histories
; -----------
hist unbal
hist load
hist disp
step 40000
save bearing_capacity_strip.sav 'last project state'
```

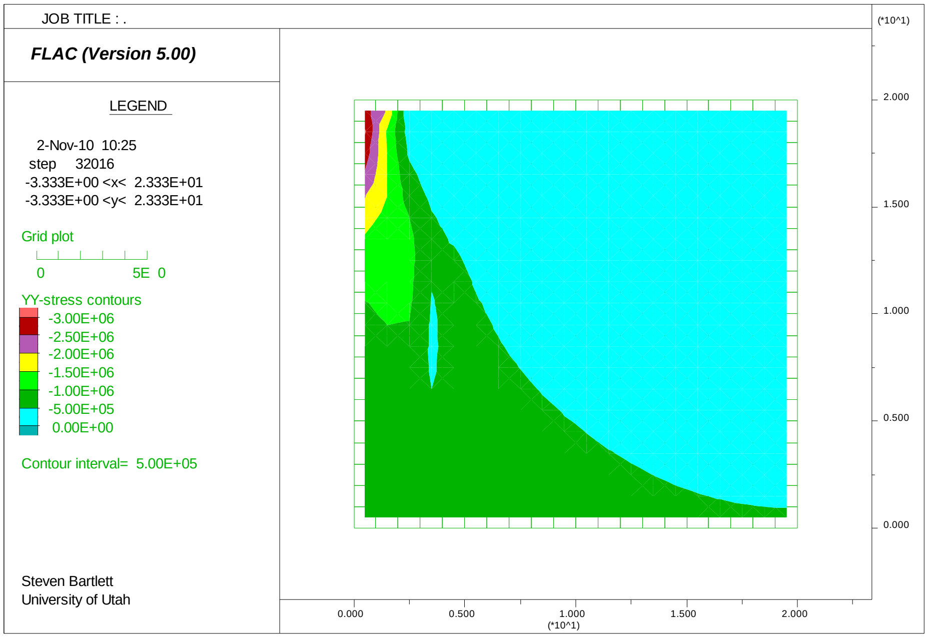
* When a velocity is applied to gridpoints to simulate a footing load, the bearing area is found by assuming that the velocity varies linearly from the value at the last applied gridpoint, to zero at the next gridpoint. The half-width area is then

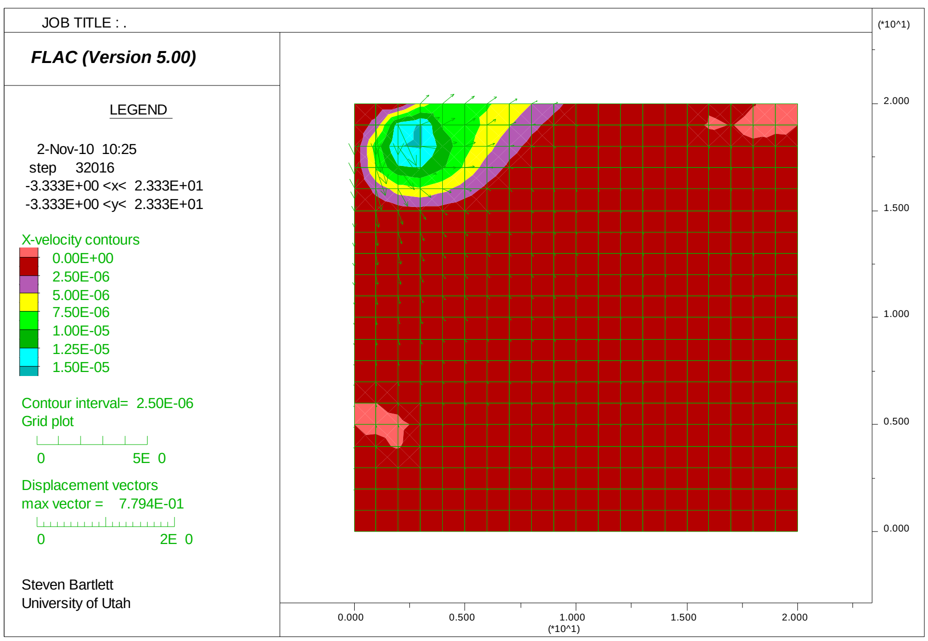
$$A = 0.5(x_l + x_{l+1}) \qquad (6.3)$$

where x_l is the x-location of the last applied gridpoint velocity, and x_{l+1} is the x-location of the gridpoint adjacent to x_l.

FLAC Model of Shallow Footing on Cohesionless Soil (cont.)

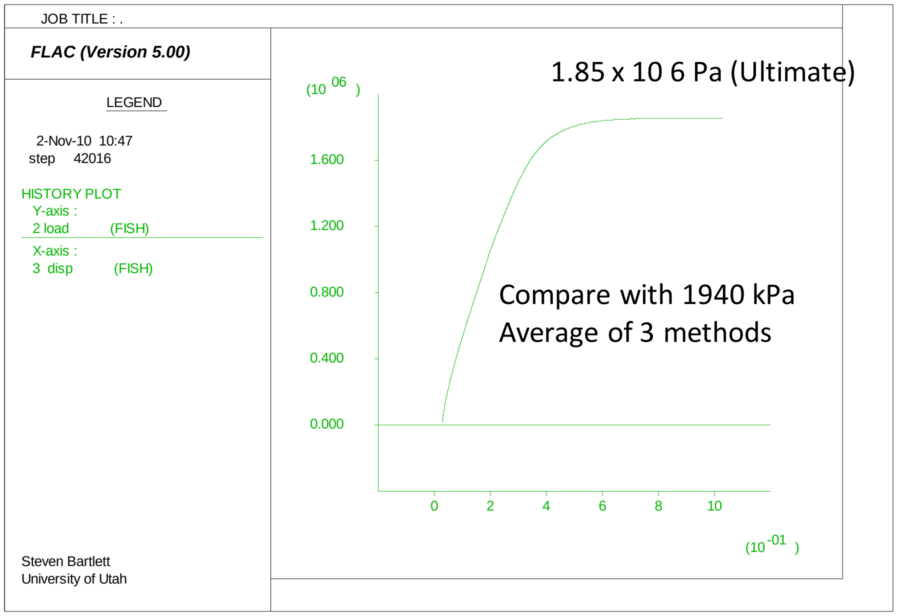
Thursday, March 11, 2010
11:43 AM





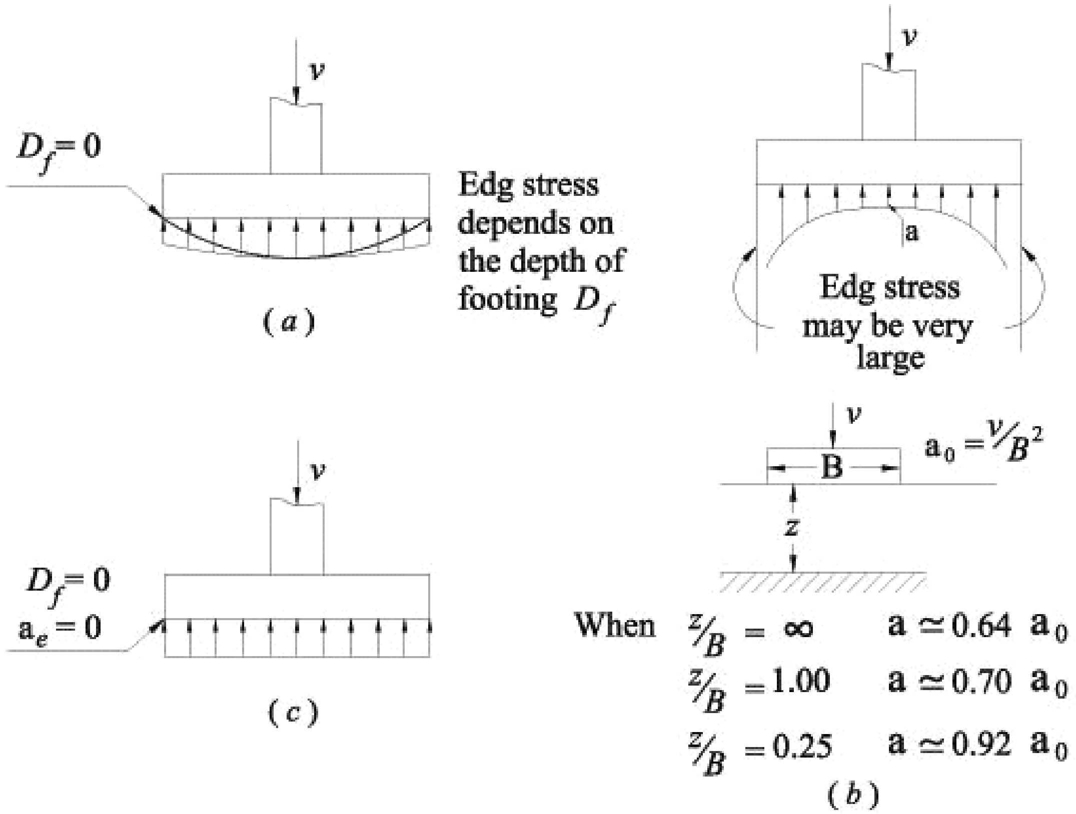
FLAC Model of Shallow Footing on Cohesionless Soil (cont.)

Thursday, March 11, 2010
11:43 AM



Stress Distribution for Footings

Thursday, March 11, 2010
11:43 AM



- Probable pressure distribution beneath a rigid footing :
  - (a) On a loose cohesionless soil
  - (b) Generally for cohesive soils
  - (c) Usual linear pressure distribution

Fig. (6)

The theory of elasticity analysis indicates that the stress distribution beneath footings, symmetrically loaded, is not uniform. The actual stress distribution depends on the type of material beneath the footing and the rigidity of the footing. For footings on loose cohesion-less material, the soil grains tend to displace laterally at the edges from under the load, whereas in the center the soil is relatively confined. This results in a pressure diagram somewhat as indicated in Fig.6. For the general case of rigid footings on cohesive and cohesion-less materials, Fig.6 indicates the probable theoretical pressure distribution. The high edge pressure may be explained by considering that edge shear must take place before settlement can take place.

Pasted from <http://osp.mans.edu.eg/sfoundation/foundtion.htm>

Beam Elements for Modeling Structural Frames

Thursday, March 11, 2010
11:43 AM

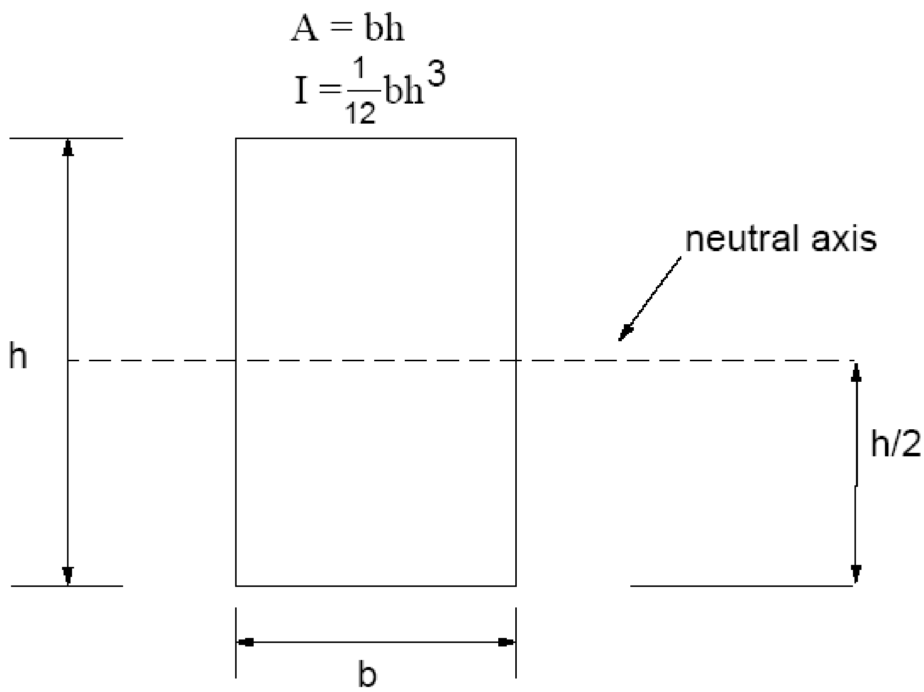


Figure 1.3 ***Rectangular beam cross-section with second moment of area, I, and cross-sectional area, A***

Beam Elements—Beam elements are two-dimensional elements with three degrees of freedom (x-translation, y-translation and rotation) at each end node. Beam elements can be joined together with one another and/or the grid. Beam elements are used to represent a structural member, including effects of bending resistance and limited bending moments. Tensile and compressive yield strength limits can also be specified. Beams may be used to model a wide variety of supports, such as support struts in an open-cut excavation and yielding arches in a tunnel. Interface elements can be attached on both sides of beam elements in order to simulate the frictional interaction of a foundation wall with a soil or rock. Beam elements attached to sub-grids via interface elements can also simulate the effect of geotextiles. They can also be used to model a footing on a soil foundation.

Beam Elements (cont.)

Thursday, March 11, 2010
11:43 AM

<u>Beam-Element Properties</u>

The beam elements used in FLAC require the following input parameters:
(1) elastic modulus [stress];
(2) cross-sectional area [length^2];
(3) second moment of area [length^4] (commonly referred to as the moment of inertia= 1/12*b*h^3);
(4) spacing [length] (optional—if not specified, *beams are considered to be continuous* in the out-of-plane direction);
(5) plastic moment [force-length] (optional—if not specified, the moment capacity is <u>*assumed to be infinite*</u>);
(6) axial peak tensile yield strength [stress] (optional—if not specified, the tensile yield strength is <u>*assumed to be infinite*</u>);
(7) axial residual tensile yield strength [stress] (optional—if not specified, the r<u>*esidual tensile yield strength is zero*</u>);
(8) axial compressive yield strength [stress] (optional—if not specified, the compressive yield strength is <u>*assumed to be infinite*</u>);
(9) density [mass/volume] (<u>*optional*</u> — used for dynamic analysis and gravity loading); and
(10) thermal expansion coefficient (<u>*optional*</u> — used for thermal analysis).

Beam-element properties are easily calculated or obtained from handbooks. For example, typical values for structural steel are 200 GPa for Young's modulus, and 0.3 for Poisson's ratio. For concrete, typical values are 25 to 35 GPa for Young's modulus, 0.15 to 0.2 for Poisson's ratio, and 2100 to 2400 kg/m3 for mass density.

Beam Elements and Bearing Capacity (FLAC Example)

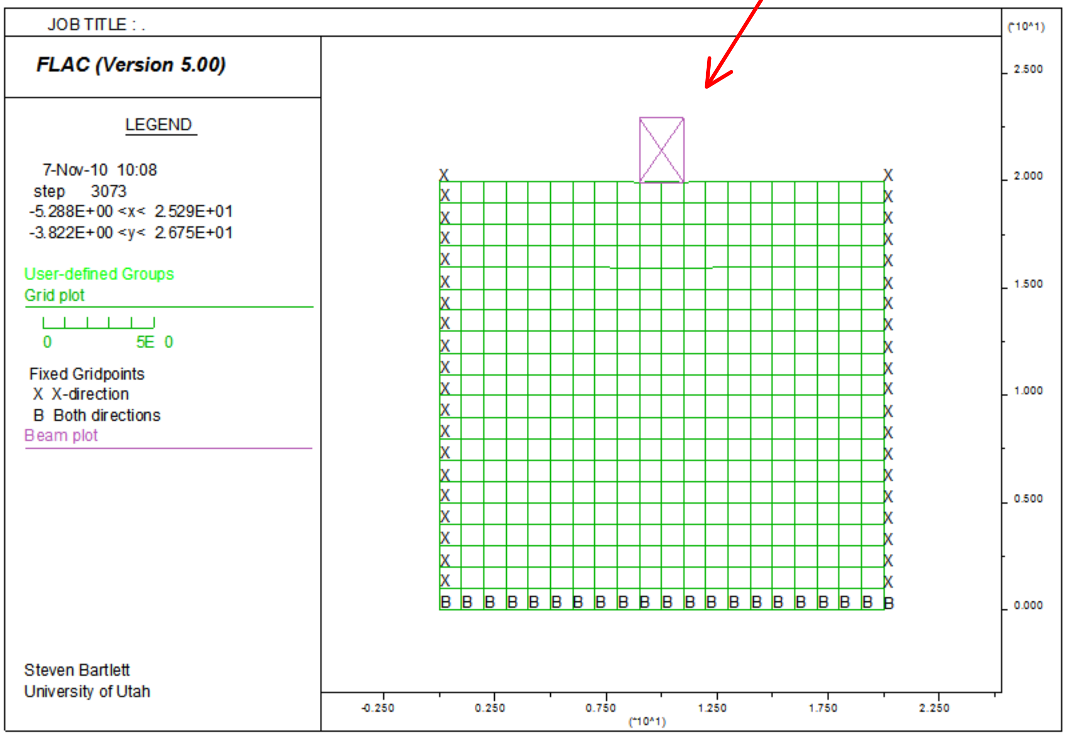
Thursday, March 11, 2010
11:43 AM

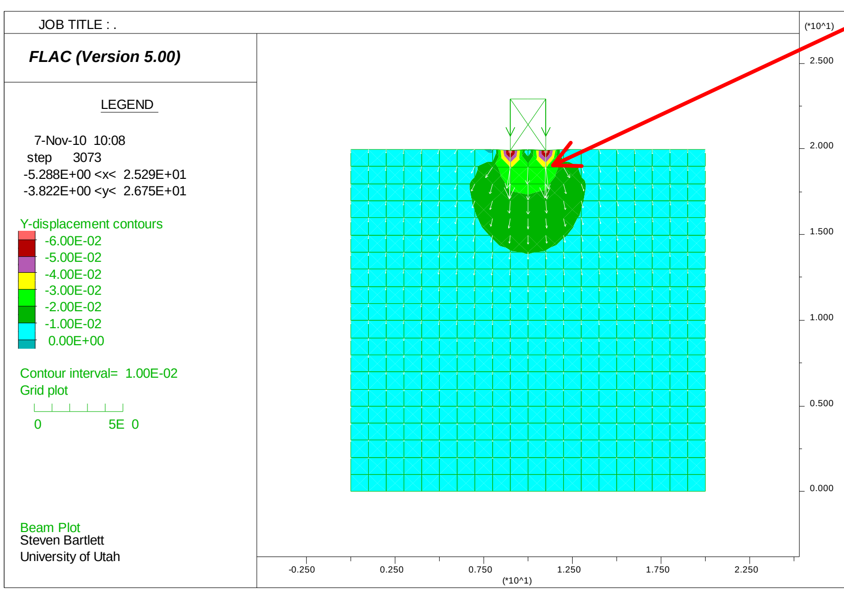
```
config
set large
; --- geometry ---
g 20 20
model mohr
pro s=40e6 bul=80e6 d 2000 coh 25e3 fric 35.0 dilation 0 ten 10e3 j 1 20
; --- boundary conditions ---
fix x i=1
fix x i=21
fix x y j=1
;
set gravity 9.81
step 2000
;
ini xdisp 0
ini ydisp 0
;
; concrete slab
struc prop=1001 E=17.58e9 I=0.0104 a=.5
struc prop=1002 E=200e9 I=2.3e-5 a=4.8e-3
struc beam beg gr 10,21 end gr 12,21 seg=1 pr=1001; slab
struc beam beg node 1 end 9,23 seg=2 pr=1002; left wall
struc beam beg 9,23 end 11,23 seg=2 pr=1002; roof
struc beam beg 11,23 end 11,20 seg=2 pr=1002; right wall
struc node=9 10.0,21.5; middle of frame
struc beam beg node=9 end node=1 seg=1 pr=1002; forms frame
struc beam beg node=9 end node=4 seg=1 pr=1002
struc beam beg node=9 end node=6 seg=1 pr=1002
struc beam beg node=9 end node=2 seg=1 pr=1002
struc node=1 fix r
struc node=2 fix r
struc node=4 Load 0 -7.0e5 0; use -2e7 and m e to show flexing of structure
struc node=6 Load 0 -7.0e5 0
;
; -----------
; Histories
; -----------
solve
save bearing_capacity_w_box.sav 'last project state'
```

Beam Elements and Bearing Capacity (FLAC Example)

Thursday, March 11, 2010
11:43 AM

Frame established using beam elements



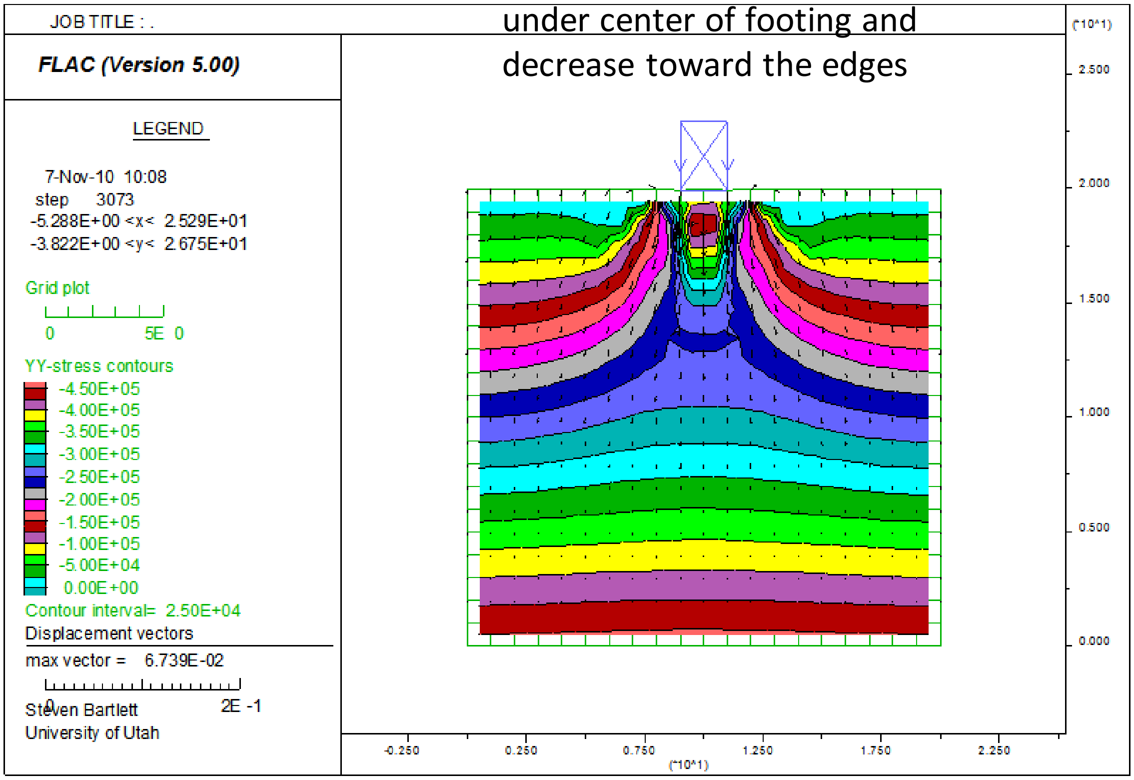


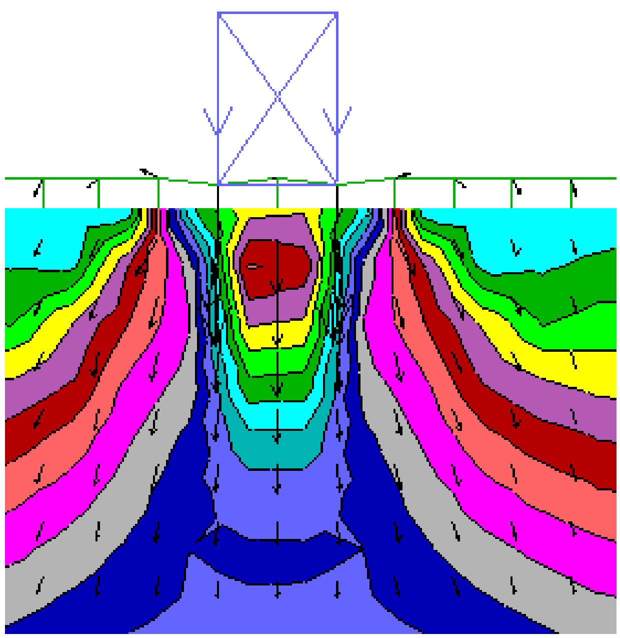
Note: High displacement occurs under walls of frame because of flexible floor. This is shows soil-structure interaction in that the flexibility of the structure changes the stress distribution in the soil.

Beam Elements and Bearing Capacity (FLAC Example)

Thursday, March 11, 2010
11:43 AM

Note: Stresses are highest under center of footing and decrease toward the edges



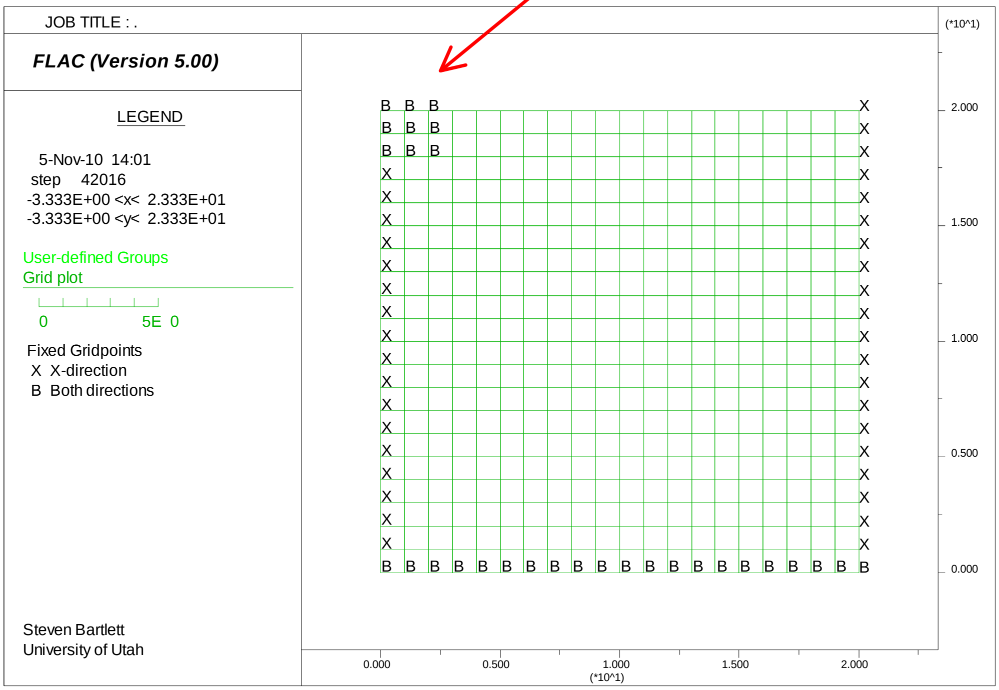


Vertical stress profile underneath structural frame with -7e5 N load on each wall.

Embedment of Footing

Thursday, March 11, 2010
11:43 AM

Ftg. Embedded 2 m
(No Interfaces)



properties s=40e6 bul=80e6 d 2000 coh 0 fric 35.0 dilation 0 ten 0

Embedment of Footing

Thursday, March 11, 2010
11:43 AM

Bearing Capacity, Hansen Equation (*Valid only for ϕ>0)
Reference: American Society of Civil Engineers, US Army Corps of Engineers Technical Engineering and Design Guide No. 7, Bearing Capacity of Soils, 1993, pp. 26-32.
Friction Angle estimate: Peck, Hansen, and Thornburn, Foundation Engineering, 1974, p. 310.

$pcf := 1 \cdot \frac{lb}{ft^3}$ $psf := 1 \cdot \frac{lb}{ft^2}$ $ksf := 1000 \cdot psf$

Effective Friction Angle	$\phi := 35deg$	Footing Width	$B_1 := 16.4 \cdot ft$
Cohesion	$C := 0 \cdot psf$	Footing Depth	$D_1 := 6.56 \cdot ft$
Effective unit weight of soil above footing	$\gamma_H := 124.8 \cdot pcf$	Footing Length	$L_1 := 1640 \cdot ft$
		Factor of Safety	$FS := 1$
Effective unit weight of soil beneath footings	$\gamma_D := 124.8 \cdot pcf$	N-value	$N = 26.205$

Bearing Capacity Factors

$N_q := e^{\pi \cdot \tan(\phi)} \cdot \left(\frac{1 + \sin(\phi)}{1 - \sin(\phi)} \right)$ $N_q = 33.3$

$N_\gamma := 1.5 \cdot (N_q - 1) \cdot \tan(\phi)$ $N_\gamma = 33.92$

$N_c := (N_q - 1) \cdot \cot(\phi)$ $N_c = 46.12$

Shape Factors

$s_c := 1 + \frac{N_q}{N_c} \cdot \frac{B_1}{L_1}$ $s_q := 1 + \frac{B_1}{L_1} \cdot \tan(\phi)$ $s_\gamma := 1 - 0.4 \cdot \frac{B_1}{L_1}$

$s_c = 1.01$ $s_q = 1.01$ $s_\gamma = 1.00$

Depth Factors $k := if\left(\frac{D_1}{B_1} \le 1, \frac{D_1}{B_1}, atan\left(\frac{D_1}{B_1} \right) \right)$ $k = 0.4$

$d_c := 1 + 0.4 \cdot k$ $d_q := 1 + 2 \cdot \tan(\phi) \cdot (1 - \sin(\phi))^2 \cdot k$ $d_\gamma := 1.0$

$d_c = 1.16$ $d_q = 1.10$ $d_\gamma = 1.00$

Allowable Bearing Capacity, q_a

$q_{u2} := C \cdot N_c \cdot s_c \cdot d_c + D_1 \cdot \gamma_D \cdot N_q \cdot s_q \cdot d_q + 0.5 \cdot \gamma_H \cdot B_1 \cdot N_\gamma \cdot s_\gamma \cdot d_\gamma$

$q_{a1} := \frac{q_{u2}}{FS}$ $q_{a1} = 64.82 \cdot ksf$

Embedment of Footing

Thursday, March 11, 2010
11:43 AM

Bearing Capacity, Vesic Equation (*Valid only for ϕ>0)
Reference: American Society of Civil Engineers, US Army Corps of Engineers Technical Engineering and Design Guide No. 7, Bearing Capacity of Soils, 1993, pp. 26-32.
Friction Angle estimate: Peck, Hansen, and Thornburn, Foundation Engineering, 1974, p. 310.

Effective Friction Angle	$\phi = 35 \cdot deg$	Footing Width	$B_1 = 16.4 \cdot ft$
Cohesion	$C = 0 \cdot psf$	Footing Depth	$D_1 = 6.56 \cdot ft$
Effective unit weight of soil above footing	$\gamma_H = 124.8 \cdot pcf$	Footing Length	$L_1 = 1.64 \times 10^3 \cdot ft$
		Factor of Safety	$FS = 1$
Effective unit weight of soil beneath footings	$\gamma_D = 124.8 \cdot pcf$	N-value	$N = 26.205$

Bearing Capacity Factors

$N_q := e^{\pi \cdot \tan(\phi)} \cdot \left(\frac{1 + \sin(\phi)}{1 - \sin(\phi)}\right)$ $\quad N_q = 33.3$

$N_\gamma := 2 \cdot (N_q + 1) \cdot \tan(\phi)$ $\quad N_\gamma = 48.03$

$N_c := (N_q - 1) \cdot \cot(\phi)$ $\quad N_c = 46.12$

Shape Factors

$s_c := 1 + \frac{N_q}{N_c} \cdot \frac{B_1}{L_1}$ $\quad s_q := 1 + \frac{B_1}{L_1} \cdot \tan(\phi)$ $\quad s_\gamma := 1 - 0.4 \cdot \frac{B_1}{L_1}$

$s_c = 1.01$ $\quad s_q = 1.01$ $\quad s_\gamma = 1.00$

Depth Factors $\quad k := \text{if}\left(\frac{D_1}{B_1} \le 1, \frac{D_1}{B_1}, \text{atan}\left(\frac{D_1}{B_1}\right)\right)$ $\quad k = 0.4$

$d_c := 1 + 0.4 \cdot k$ $\quad d_q := 1 + 2 \cdot \tan(\phi) \cdot (1 - \sin(\phi))^2 \cdot k$ $\quad d_\gamma := 1.0$

$d_c = 1.16$ $\quad d_q = 1.10$ $\quad d_\gamma = 1.00$

Allowable Bearing Capacity, q_a

$q_{u2} := C \cdot N_c \cdot s_c \cdot d_c + D_1 \cdot \gamma_D \cdot N_q \cdot s_q \cdot d_q + 0.5 \cdot \gamma_H \cdot B_1 \cdot N_\gamma \cdot s_\gamma \cdot d_\gamma$

$q_{a2} := \frac{q_{u2}}{FS}$ $\quad q_{a2} = 79.20 \cdot ksf$

Embedment of Footing

Thursday, March 11, 2010
11:43 AM

Bearing Capacity, Meyerhof Equation (*Valid only for ϕ>0)
Reference: American Society of Civil Engineers, US Army Corps of Engineers Technical Engineering and Design Guide No. 7, Bearing Capacity of Soils, 1993, pp. 26-32.
Friction Angle estimate: Peck, Hansen, and Thornburn, Foundation Engineering, 1974, p. 310.

Effective Friction Angle	$\phi = 35 \cdot deg$	Footing Width	$B_1 = 16.4 \cdot ft$
Cohesion	$C = 0 \cdot psf$	Footing Depth	$D_1 = 6.56 \cdot ft$
Effective unit weight of soil above footing	$\gamma_H = 124.8 \cdot pcf$	Footing Length	$L_1 = 1.64 \times 10^3 \cdot ft$
Effective unit weight of soil beneath footings	$\gamma_D = 124.8 \cdot pcf$	Factor of Safety	$FS = 1$
		N-value	$N = 26.205$

Bearing Capacity Factors

$N_\phi := \left(\frac{1+\sin(\phi)}{1-\sin(\phi)}\right)$ $\quad N_\phi = 3.69$

$N_q := e^{\pi \cdot \tan(\phi)} \cdot N_\phi$ $\quad N_q = 33.3$

$N_\gamma := (N_q - 1) \cdot \tan(1.4 \cdot \phi)$ $\quad N_\gamma = 37.15$

$N_c := (N_q - 1) \cdot \cot(\phi)$ $\quad N_c = 46.12$

Shape Factors

$s_c := 1 + 0.2 \cdot N_\phi \cdot \frac{B_1}{L_1}$ $\quad s_q := 1 + 0.1 \cdot N_\phi \cdot \frac{B_1}{L_1}$ $\quad s_\gamma := 1 + 0.1 \cdot N_\phi \cdot \frac{B_1}{L_1}$

$s_c = 1.01$ $\quad s_q = 1.00$ $\quad s_\gamma = 1.00$

Depth Factors

$d_c := 1 + 0.2 \cdot N_\phi^{0.5} \cdot \frac{D_1}{B_1}$ $\quad d_q := 1 + 0.1 \cdot N_\phi^{0.5} \cdot \frac{D_1}{B_1}$ $\quad d_\gamma := 1 + 0.1 \cdot N_\phi^{0.5} \cdot \frac{D_1}{B_1}$

$d_c = 1.15$ $\quad d_q = 1.08$ $\quad d_\gamma = 1.08$

Allowable Bearing Capacity, q_a

$$q_{u3} := C \cdot N_c \cdot s_c \cdot d_c + D_1 \cdot \gamma_D \cdot N_q \cdot s_q \cdot d_q + 0.5 \cdot \gamma_H \cdot B_1 \cdot N_\gamma \cdot s_\gamma \cdot d_\gamma$$

$q_{a3} := \frac{q_{u3}}{FS}$ $\quad q_{a3} = 70.55 \cdot ksf$

Embedment of Footing

Thursday, March 11, 2010
11:43 AM

Summary

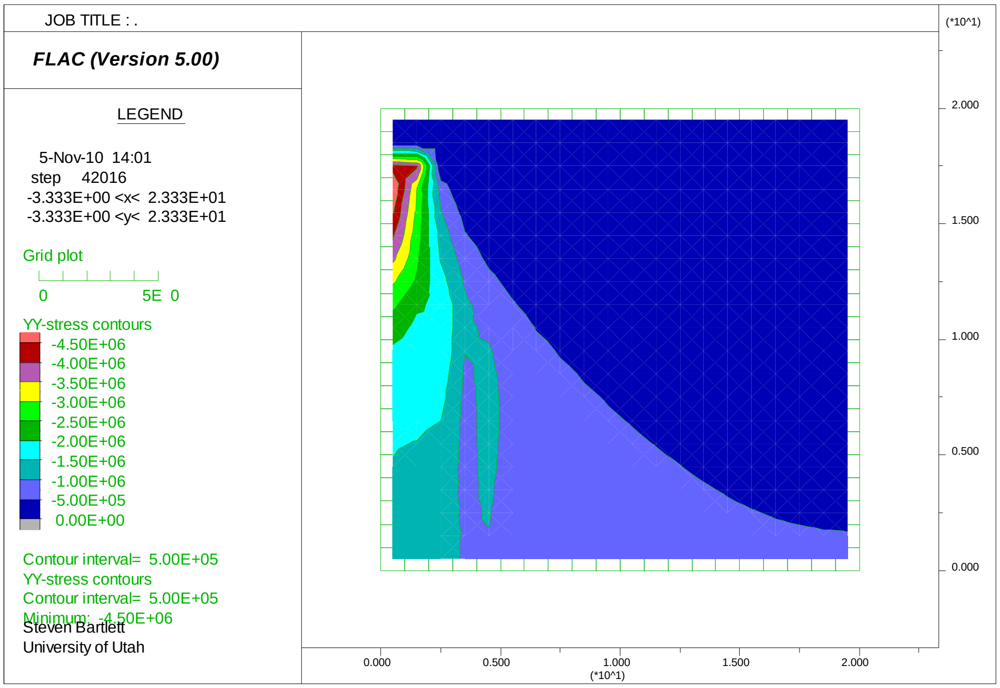
Method	Allowable Bearing Capacity
1. Hansen	$q_{a1} = 64.8 \cdot ksf$
2. Vesic	$q_{a2} = 79.2 \cdot ksf$
3. Meyerhof	$q_{a3} = 70.6 \cdot ksf$
4. Meyerhof (settlement = 1 in.)	$q_{a4} = 8.3 \cdot ksf$

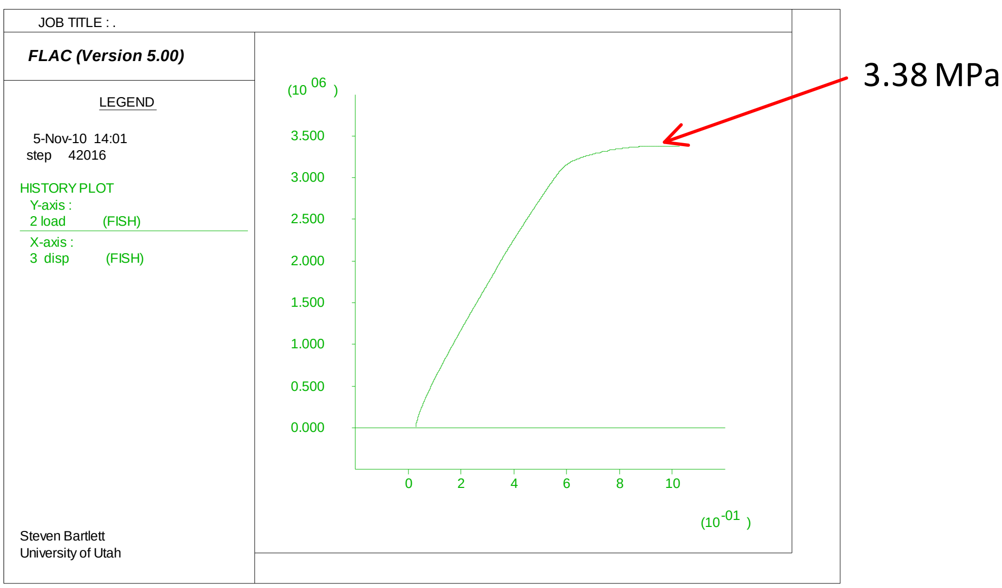
$$Average := g \cdot \frac{q_{a1} + q_{a2} + q_{a3}}{3}$$

$$Average = 3.425 \times 10^{6} m^{-1} \cdot kg \cdot s^{-2}$$ or Pa

Embedment of Footing

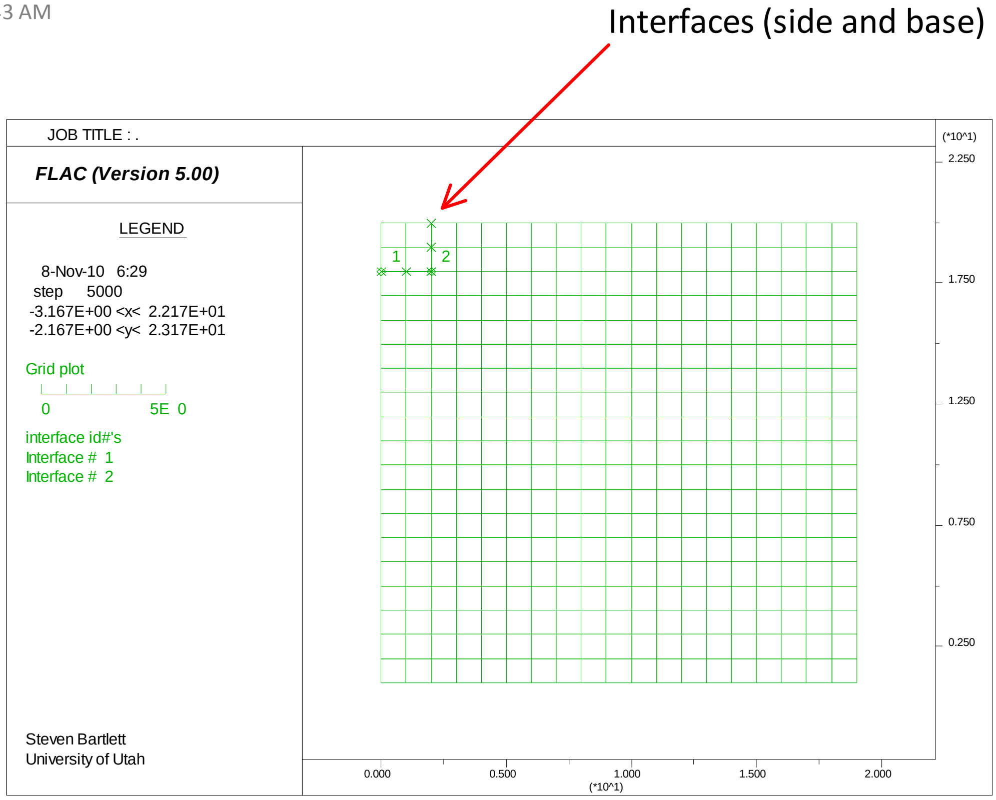
Thursday, March 11, 2010
11:43 AM





Embedment of Footing with Interfaces

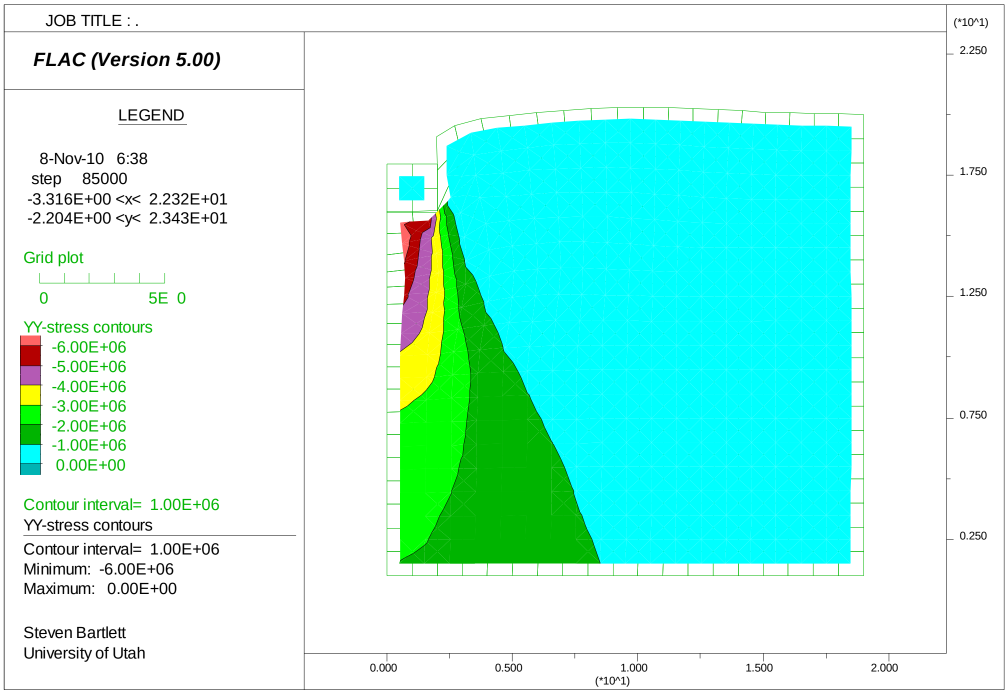
Thursday, March 11, 2010
11:43 AM

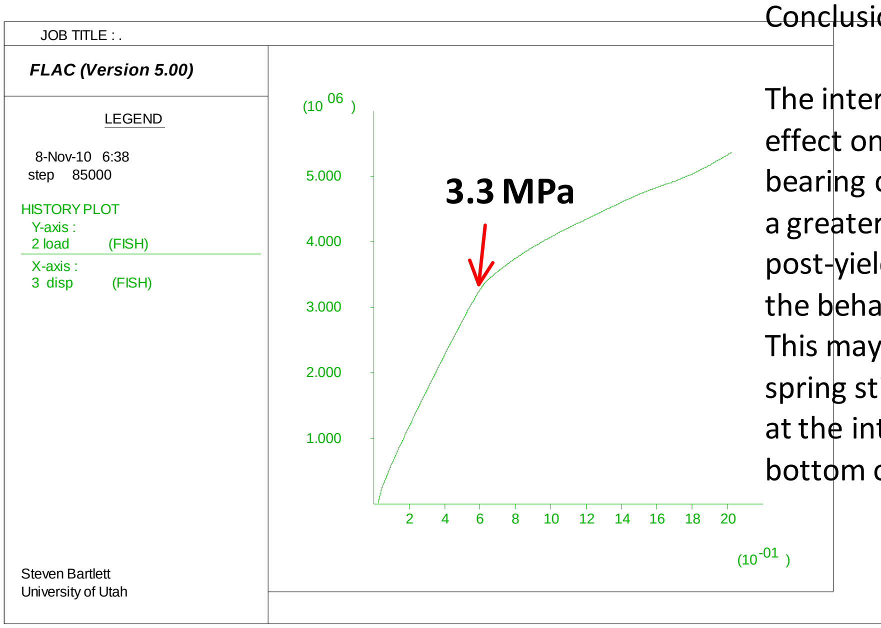



```
pro s=40e6 bul=80e6 d 2000 coh 0 fric 35.0 dilation 0 ten 0
interface 1 aside from 2,18 to 4,18 bside from 1,19 to 3,19
interface 1 unglued kn=133e6 ks=133e6 friction = 35
interface 2 aside from 4,18 to 4,20 bside from 3,19 to 3,21
interface 2 unglued kn=133e6 ks=133e6 friction = 0
```

Embedment of Footing

Thursday, March 11, 2010
11:43 AM





Conclusion:

The interface has a small effect on the ultimate bearing capacity and has a greater effect on the post-yield behavior (i.e., the behavior is stiffer). This may be due to the spring stiffness assigned at the interface at the bottom of the footing.

More Reading

Thursday, March 11, 2010
11:43 AM

- **Applied Soil Mechanics with ABAQUS Applications, Ch. 6**

Assignment 8

Thursday, March 11, 2010
11:43 AM

1. Problem 6.1 (Text) (10 points).
2. Problem 6.2. (Text) Use FDM instead of FEM. Use Mohr-Coulomb Model for the elastoplastic behavior instead of the Cap Model. (20 points).
3. Problem 6.3 (Text). Assume the footing is a strip footing that is 2 m wide. (10 points).
4. Problem 6.4 (Text). Use FDM instead of FEM. Assume the footing is a strip footing that is 2 m wide. (20 points).

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Thursday, March 11, 2010
11:43 AM

Reinforced Walls and Slopes

Thursday, March 11, 2010
11:43 AM

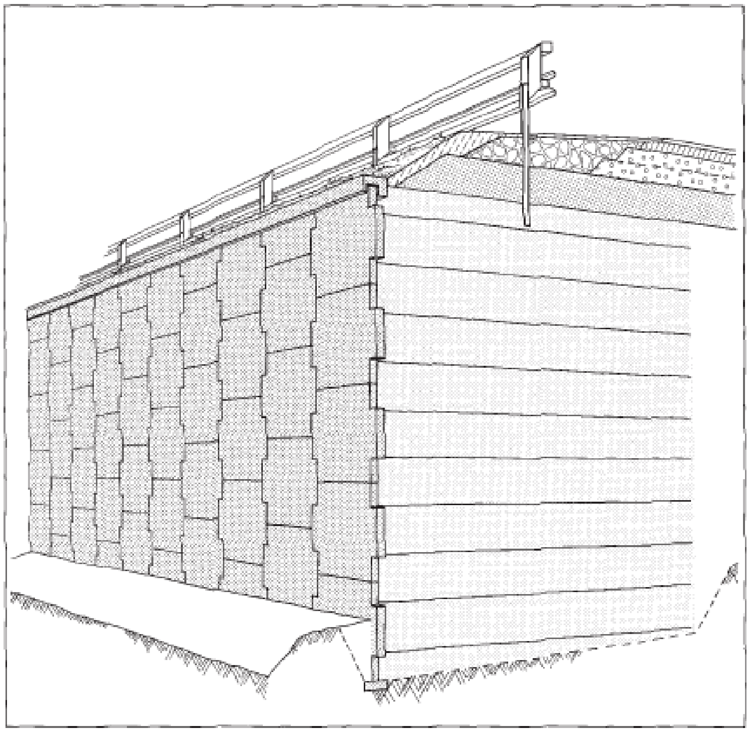


Figure 1.75 Cut-away view of a typical reinforced earth retaining wall showing strip reinforcement

Mechanically stabilized earth or MSE is soil constructed with artificial reinforcing. It can be used for retaining walls, bridge abutments, dams, seawalls, and dikes.[1][2] Although the basic principles of MSE has been used throughout history, MSE was developed in its current form in the 1960s. The reinforcing elements used can vary but include steel and geosynthetics. The reinforcement materials of MSE can vary. Originally, long steel strips 50 to 120 mm (2 to 5 in) wide were used as reinforcement. These strips are sometimes ribbed, although not always, to provide added friction. Sometimes steel grids or meshes are also used as reinforcement. Several types of geosynthetics can be used including geogrids and geotextiles. The reinforcing geosynthetics can be made of high density polyethylene, polyester, and polypropylene. These materials may be ribbed and are available in various size.

Pasted from <http://en.wikipedia.org/wiki/Mechanically_stabilized_earth>

General Design Considerations from FHWA NHI

1. Establish the geometric, loading, and performance requirements for design.
2. Determine the engineering properties of the in-situ soils.
3. Determine the properties of reinforced fill and, if different, the retained fill.
4. Evaluate design parameters for the reinforcement
5. Design reinforcement to provide a stable slope (internal stability).
   a. Reinforcement length (horz.)
   b. Reinforcement spacing (vert.)
   c. Reinforcement tensile strength
6. Check internal stability
   a. Pullout failure
   b. Rupture failure
   c. Connections
7. Check external stability
   a. Failure behind and underneath the wall (slope stability)
   b. Compound failure (behind and through reinforced zone)
   c. Toppling
   d. Bearing Capacity Failure
   e. Excessive Settlement
8. Check seismic stability.
9. Evaluate requirements for subsurface and surface water runoff control.

Failure Modes

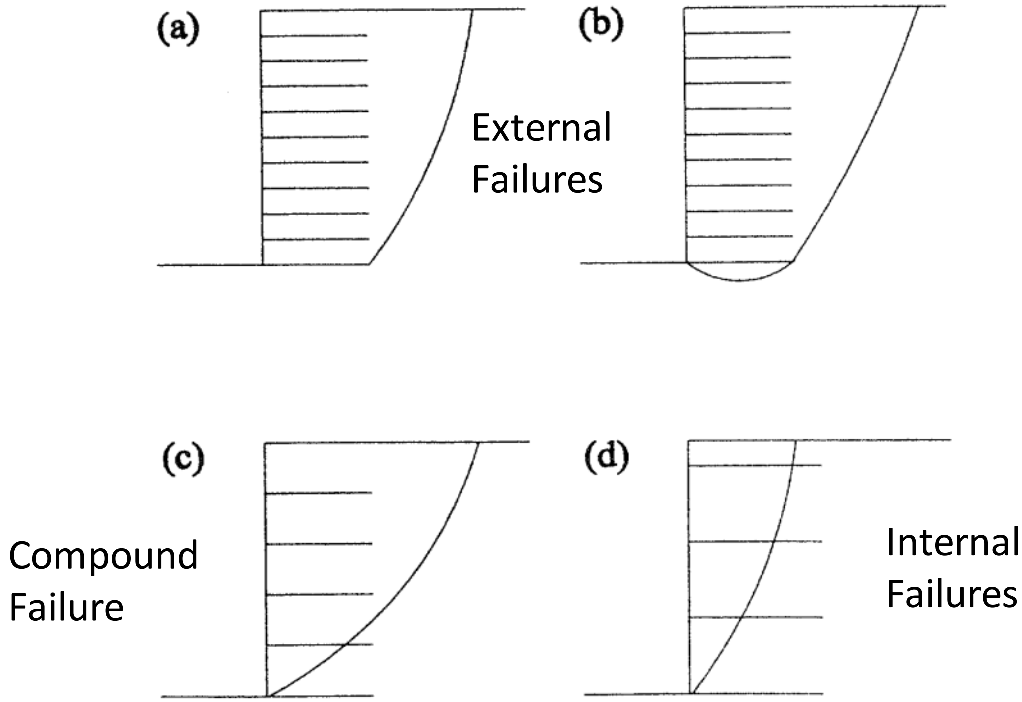


Figure 4.3 Slip Surface Types: (a) External Slip Surface; (b) Deep-Seated Slip Surface; (c) Compound Slip Surface; (d) Internal Slip Surface.

Failure Example

MSE wall failure - Philippines

MSE wall failure - Philippines

Failure Example (cont.)

MSE wall failure - Philippines

MSE wall failure - Philippines

Failure Example (cont.)

MSE wall reconstruction - Philippines

Comparison of LE and FLAC Methods for Internal Stability Evaluations of MSE Walls

Thursday, March 11, 2010
11:43 AM

LIMIT EQUILIBRIUM AND CONTINUUM MECHANICS-BASED NUMERICAL METHODS FOR ANALYZING STABILITY OF MSE WALLS

Jie Han1 (Member, ASCE) and Dov Leshchinsky2 (Member, ASCE)

17th ASCE Engineering Mechanics Conference
June 13–16, 2004, University of Delaware, Newark, DE

EM2004

ABSTRACT

Limit equilibrium (LE) methods have been commonly used to analyze stability of geosynthetic-reinforced slopes. LE methods assume a potential slip surface, the soils along this slip surface providing shear resistance, and geosynthetic reinforcement providing tensile forces and resisting moments. **Continuum mechanics-based numerical method has become increasingly used in recent years for slope stability analysis**.

Continuum mechanics-based numerical method assumes a reduction of soil strength by a factor to reach a critical state prior to failure. Both methods yield factors of safety of the system. This paper presents a study to investigate the stability of MSE walls (vertical or 20 degree batter) using LE and numerical methods. The comparisons of the critical slip surface and the factor of safety are made when the predicted factor of safety using the LE approach is equal to 1.0. It is concluded that properly adopted LE approach can be used to analyze the stability of MSE walls.

Limit Equilibrium Methods

Thursday, March 11, 2010
11:43 AM

Limit equilibrium methods

- Used for decades to safely design major geotechnical structures
- Bishop's simplified method, utilizing a circular arc slip surface, is probably the most popular limit equilibrium method. (Bishop's method is not rigorous in a sense that it does not satisfy horizontal force limit equilibrium, it is simple to apply and, in many practical problems, and it yields results close to rigorous limit equilibrium methods.)
- Bishop's simplified method was modified to include reinforcement as a horizontal force intersecting the slip circle. This approach considers the reinforcement producing a tensile force to generate a resisting moment as well as affect the normal force on the slip surface thus affecting shear resistance. This modified formulation is consistent with the original formulation by Bishop (1955).
- The mobilized reinforcement strength at its intersection with the slip circle depends on its long-term strength, its rear-end pullout capacity (or connection strength), and Bishop's factor of safety.
- The analysis assumes that when the soil and reinforcement strengths are reduced by the factor of safety, a limit equilibrium state is achieved (i.e., the system is at the verge of failure), meaning that under this state, the soil and reinforcement mobilize their respective strengths simultaneously.

<u>Limit Equilibrium Methods</u> (cont.)

Thursday, March 11, 2010
11:43 AM

Bishop's Simplified method

- The Modified (or Simplified) Bishop's Method is a method for calculating the stability of slopes. It is an extension of the Method of Slices. By making some simplifying assumptions, the problem becomes statically determinate and suitable for hand calculations:
- Forces on the sides of each slice are horizontal

The method has been shown to produce factor of safety values within a few percent of the "correct" values from more rigorous methods.

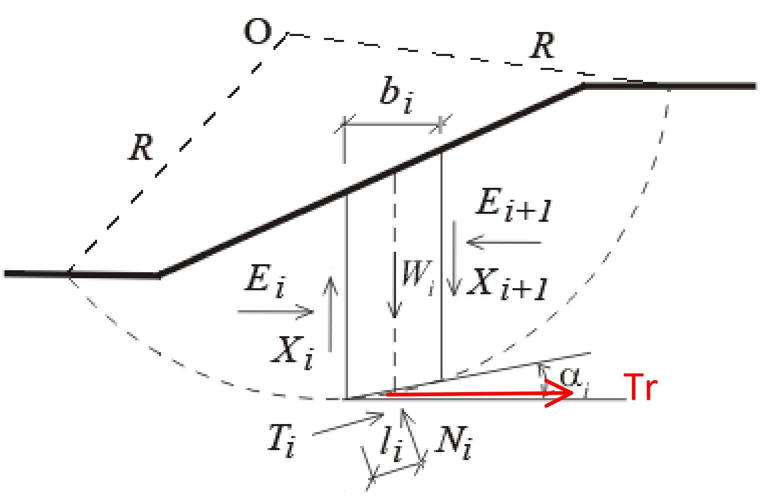
$$F = \frac{\sum\left[\frac{c' + ((W/b) - u)\tan\phi'}{\psi}\right]}{\sum[(W/b)\sin\alpha]}$$

where

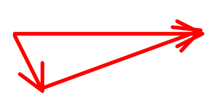
$$\psi = \cos\alpha + \frac{\sin\alpha\tan\phi}{F}$$

c' is the effective cohesion
ϕ' is the effective internal angle of internal friction
b is the width of each slice, assuming that all slices have the same width
W is the weight of each slice
u is the water pressure at the base of each slice

Pasted from <http://en.wikipedia.org/wiki/Slope_stability>



The component of Tr that is parallel to Ni increases the normal force on the base of the slice and hence increases the frictional resistance.



Pasted from <http://en.wikipedia.org/wiki/Slope_stability>

Continuum Mechanics Methods (e.g., FLAC)

Thursday, March 11, 2010
11:43 AM

Cundall (2002) compared the characteristics of numerical solutions and limit equilibrium methods in solving the factor of safety of slopes and concluded that continuum mechanics-based numerical methods have the following advantages:

1. No pre-defined slip surface is needed
2. The slip surface can be of any shape
3. Multiple failure surfaces are possible
4. No statical assumptions are needed
5. Structures (such as footings, tunnels, etc.) and/or structural elements (such as beams, cables, etc.) and interfaces can be included without concern about compatibility
6. Kinematics is satisfied (i.e., dynamic problems can be modeled)

Wall Properties Analyzed by Han and Leshchinsky (2004)

Thursday, March 11, 2010
11:43 AM

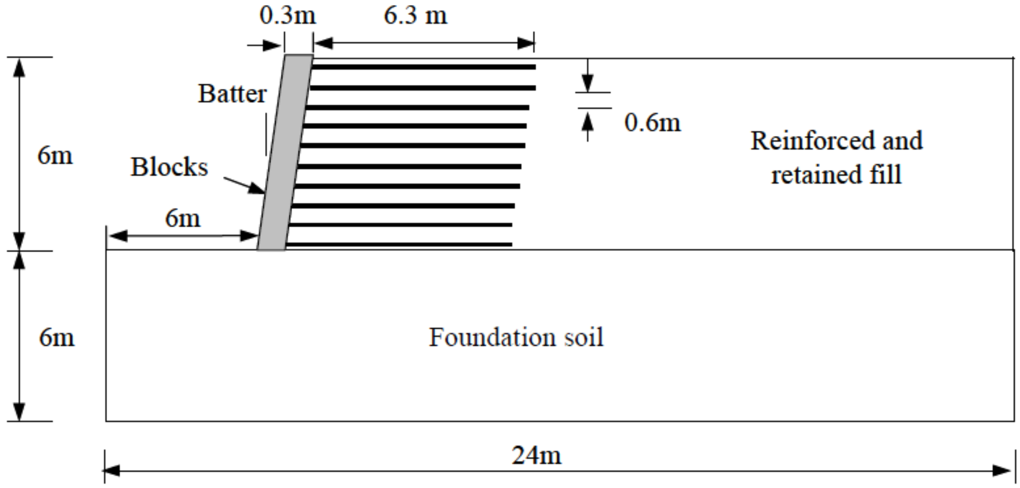


Fig. 1. Model for limit equilibrium and numerical analyses

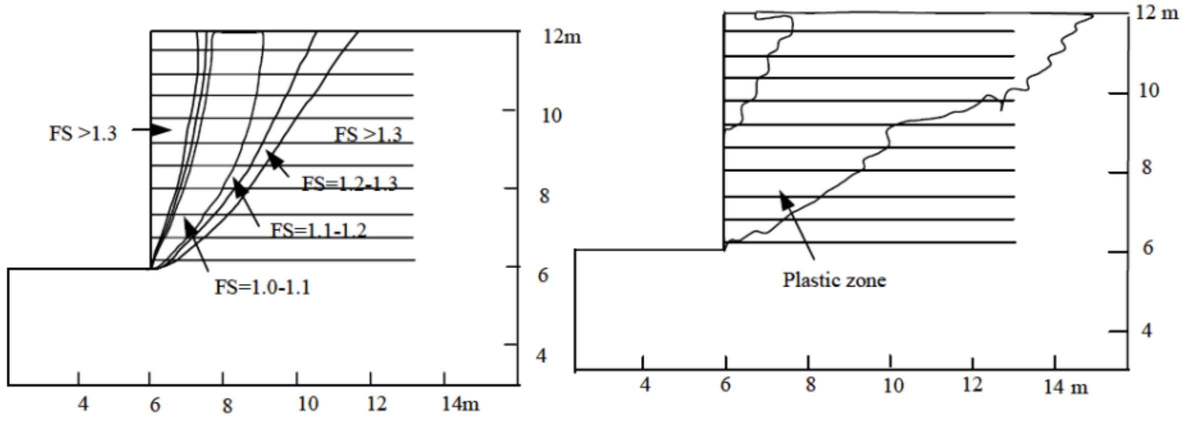
TABLE 1. Material Properties Used in the Analyses

Materials	Blocks	Reinforced and retaining fill	Foundation soil	Reinforcement
Properties	γ =18kN/m3, c=2.5kPa, ϕ=34^0	γ=18kN/m3, c=0kPa, ϕ=34^0	γ=18kN/m3, c=10 kPa, ϕ=34^0	T_a = 11.1kN/m (vertical wall) or T_a = 6.2kN/m (20° batter), C_i = 0.8

γ = unit weight, c = cohesion, ϕ = friction angle, T_a = design tensile strength of reinforcement, and C_i = interaction coefficient of reinforcement and soil.

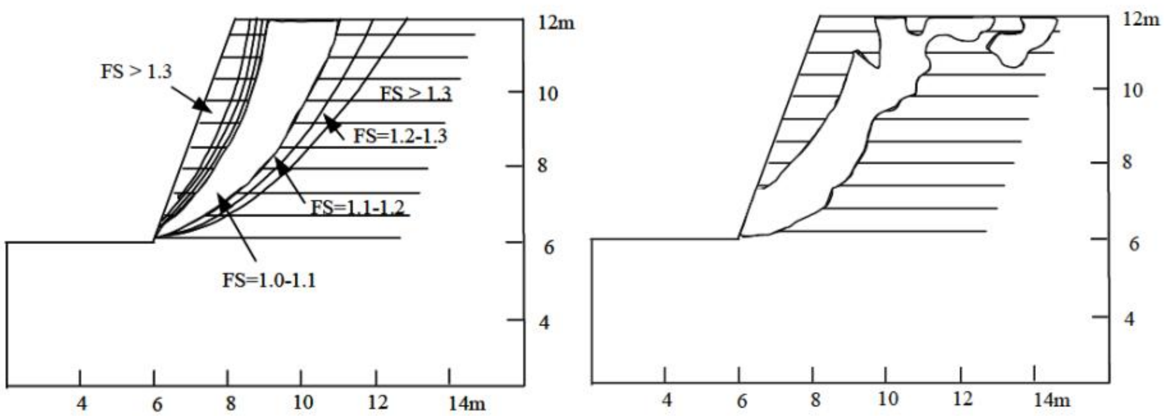
Comparison of Factor of Safety (LE) and Plasticity Results (FLAC)

Thursday, March 11, 2010
11:43 AM



(a) Factor of safety map by ReSSA (b) Plasticity by FLAC

Fig. 2. Spatial distribution of factors of safety and plasticity (vertical wall)

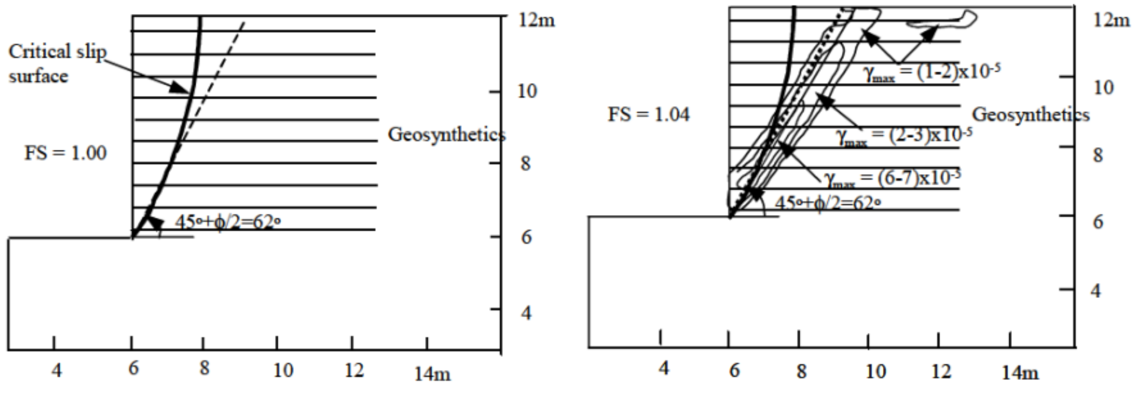


(a) Factor of safety map by ReSSA (b) Plasticity by FLAC

Fig.3. Spatial distribution of factors of safety and plasticity (20° batter MSE wall)

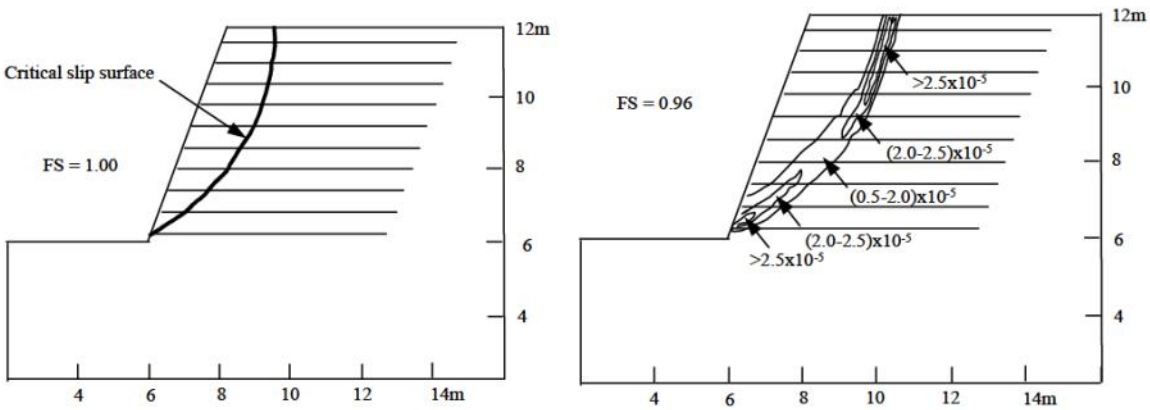
Comparison of Factor of Safety (LE) and Plasticity Results (FLAC)

Thursday, March 11, 2010
11:43 AM



(a) Critical slip surface and FS by ReSSA (b) Maximum shear strain rate and FS by FLAC

Fig. 4. Critical slip surfaces and minimal factors of safety (vertical wall)



(a) Critical slip surface and FS by ReSSA (b) Maximum shear strain rate and FS by FLAC

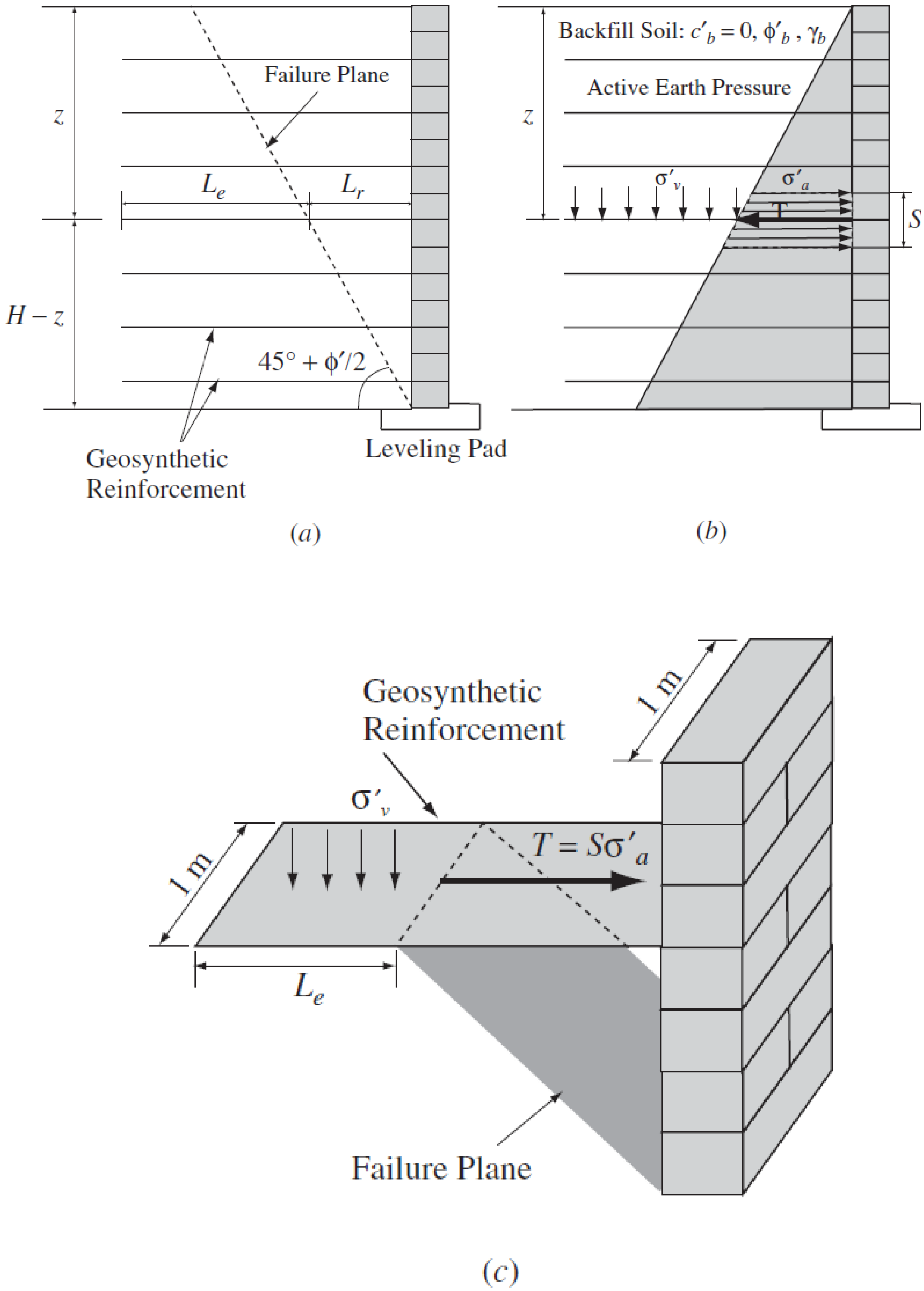
Fig. 5. Critical slip surfaces and minimal factors of safety (20° batter MSE wall)

CONCLUSIONS

In this study, limit equilibrium (LE) and continuum mechanics-based numerical methods were used to investigate the stability of MSE walls. This investigation indicates that there is a difference in the location of the critical slip surface predicted by the LE method and the numerical method. The difference becomes less when the batter of the MSE wall increases. In spite of the difference in the critical slip surface, the factors of safety computed by the LE method and the numerical method are very close. Since the factor of safety is the key to designing MSE walls in terms of stability, properly adopted LE approach can be used to fulfill this purpose.

Internal Stability Calculations - Helwany

Thursday, March 11, 2010
11:43 AM



Internal Stability Calculations - Helwany (cont.)

Thursday, March 11, 2010
11:43 AM

- Calculation of the active earth pressure versus depth

$$\sigma'_a = K_a \sigma'_v = K_a \gamma_b z$$

$$K_a = \tan^2\left(45^\circ - \frac{\phi'_b}{2}\right)$$

- Obtain rupture strength of geosynthetic, T_R (kN/m) from vendor literature

- Calculate the vertical spacing S between two geosynthetic layers from the following equation by setting FS_R equal to 1.5.

$$\mathrm{FS}_R = \frac{T_R}{\sigma'_a S} = \frac{T_R}{K_a \gamma_b z S}$$

rearranged

$$S = \frac{T_R}{K_a \gamma_b z \cdot \mathrm{FS}_R}$$

- Determine the required length of reinforcement, L, located at depth z.

$$L = L_r + L_e$$

L_r is the length within the Rankine's failure wedge

L_e is the extended length beyond Rankine's failure wedge

Internal Stability Calculations - Helwany (cont.)

Thursday, March 11, 2010
11:43 AM

$$\tan\left(45^\circ + \frac{\phi'_b}{2}\right) = \frac{H - z}{L_r}$$

$$L_r = \frac{H - z}{\tan(45^\circ + \phi'_b/2)}$$

- Factor of safety against pullout can be calculated from

$$\text{FS}_P = \frac{2L_e\gamma_b z \tan\phi'_{int}}{K_a\gamma_b z S} = \frac{2L_e \tan\phi'_{\text{int}}}{K_a S}$$

$$\phi'_{\text{int}} = \tfrac{2}{3}\phi'_b$$

$$L_e = \frac{SK_a \cdot \text{FS}_P}{2\tan\phi'_{\text{int}}}$$

$$L = \frac{H - z}{\tan(45^\circ + \phi'_b/2)} + \frac{SK_a \cdot FS_P}{2\tan\phi'_{\text{int}}}$$

L should not be less than 0.5*H

Factors of safety used in the calculations vary by according to oversight agency. Make sure that you check the requirements of these agencies.

Internal Stability Calculations - Helwany (cont)

Thursday, March 11, 2010
11:43 AM

Obtain dimensions of reinforced zone from internal stability

Note: To find the required vertical spacing, z was set to H (i.e., base of reinforcement).

Internal Stability of Geosynthetic MSE Wall
(after Helwany, 2007 , p. 272 and 275)

Inputs

$\phi := 20$ drained friction angle of backfill

$\phi_1 := \frac{2}{3} \cdot \phi$ interface friction for reinforcement and backfill interface (2/3 phi w/o data)

$Tr := 1.25 klf$ reinforcement rupture strength

$Tr = 1.824 \times 10^4 \frac{kg}{s^2} \left(\frac{N}{m}\right)$

$FSr := 1.5$ factor of safety against rupture

$FSp := 4$ factor of safety against pullout

$z := 5 \cdot m$ depth **(Check req. spacing at bottom z = H) (Check L for z = Sactual)**

$He := 5 \cdot m$ height of reinforced zone

$\gamma := 20 \frac{kN}{m^3}$ unit weight of backfill

Calculations

$$Ka := \left(\tan\left(45deg - \frac{\phi \cdot deg}{2}\right)\right)^2$$

$Ka = 0.49$

$$S := \frac{Tr}{Ka \cdot \gamma \cdot z \cdot FSr}$$

$S = 0.248 m$ required spacing

$Sactual := 0.25 \cdot m$ **(must be greater than S for z = H)**

$$Lr := \frac{He - z}{\tan\left(45deg + \frac{\phi \cdot deg}{2}\right)}$$

$Lr = 0 m$ Length of reinforcement in the active failure zone

$$Le := \frac{Sactual \cdot Ka \cdot FSp}{2 \cdot \tan(\phi_1 \cdot deg)}$$

$Le = 1.034 m$ Length of reinforcement behind the active failure zone

$L := Lr + Le$

$L = 1.034 m$

Internal Stability Calculations - Helwany (cont.)

Thursday, March 11, 2010
11:43 AM

Note: To find the required length of reinforcement, z was set to Sactual (i.e., depth corresponding to top layer of reinforcement).

Internal Stability of Geosynthetic MSE Wall
(after Helwany, 2007 , p. 272 and 275)

Inputs

$\phi := 20$ drained friction angle of backfill

$\phi\iota := \frac{2}{3}\cdot\phi$ interface friction for reinforcement and backfill interface (2/3 phi w/o data)

$Tr := 1.25klf$ reinforcement rupture strength

$Tr = 1.824\times10^{4}\frac{kg}{s^{2}}\ \left(\frac{N}{m}\right)$

$FSr := 1.5$ factor of safety against rupture

$FSp := 4$ factor of safety against pullout

$z := 0.25\cdot m$ depth **(Check req. spacing at bottom z = H) (Check L for z = Sactual)**

$He := 5\cdot m$ height of reinforced zone

$\gamma := 20\frac{kN}{m^{3}}$ unit weight of backfill

Calculations

$$Ka := \left(\tan\left(45deg - \frac{\phi\cdot deg}{2}\right)\right)^{2}$$

$Ka = 0.49$

$$S := \frac{Tr}{Ka\cdot\gamma\cdot z\cdot FSr}$$

$S = 4.961\,m$ required spacing

$Sactual := 0.25\cdot m$ **(must be greater than S for z = H)**

$$Lr := \frac{He - z}{\tan\left(45deg + \frac{\phi\cdot deg}{2}\right)}$$

$Lr = 3.326\,m$ Length of reinforcement in the active failure zone

$$Le := \frac{Sactual\cdot Ka\cdot FSp}{2\cdot\tan(\phi\iota\cdot deg)}$$

$Le = 1.034\,m$ Length of reinforcement behind the active failure zone

$L := Lr + Le$

$L = 4.36\,m$

Internal Stability Evaluations Using Strip Elements in FLAC

Thursday, March 11, 2010
11:43 AM

The strip element is a type of structural element specifically designed to simulate the behavior of thin, flat reinforcing strips placed in layers within a soil embankment to provide support. Figure 1.75 (see first page) shows a typical reinforced earth retaining wall containing layers of strip reinforcement.

The strip can yield in compression and tension, and a rupture limit can be defined, similar to the rock bolt behavior. Strips provide shear resistance but cannot sustain bending moments, similar to cables. In addition, the shear behavior at the strip/soil interface is defined by a nonlinear shear failure envelope that varies as a function of a user-defined transition confining pressure.

Characteristics of Strip Elements

Thursday, March 11, 2010
11:43 AM

Strip elements represent the behavior of thin reinforcing strips placed in layers within a soil embankment to provide structural support. The strip element is similar to the rockbolt element in that strips can yield in tension or compression, and a tensile failure strain limit can be defined. Strips cannot sustain a bending moment. The shear behavior at the strip/interface is defined by a nonlinear shear failure envelope that varies as a function of a user-defined transition confining pressure. Strip elements are designed to be used in the simulation of reinforced earth retaining walls.

The strip model was developed in collaboration with Terre Armée/Reinforced Earth Company, Soiltech R & D Division, Nozay, France. The model was developed to represent the behavior of the Terre Armée reinforcing strips.

Characteristics of Strip Elements (cont.)

Thursday, March 11, 2010
11:43 AM

1. The reinforcing strips are prescribed by the number of strips (nstrips) per calculation width (calwidth), measured out-of-plane. The individual strip thickness (strthickness) and strip width (strwidth) are also input.
2. The elastic stiffness of the strip is defined by the cross-sectional area of the strip per calculation width (out-of-plane) and the Young's modulus (E) of the strip material.
3. The strip may yield in tension (defined by the strip tensile yield-force limit, stryield) and in compression (defined by the strip compressive yield-force limit, strcomp).
4. Strip breakage is simulated with a user-specified tensile failure strain limit (tfstrain). The strain measure is based on the accumulated plastic strain calculated at each strip segment along the length of the strip. The strip breakage formulation is similar to that used for rock bolts (see Eq. (1.41)), except that bending strain is not included in the strip breakage calculation. If the plastic strain at a segment exceeds the tensile failure strain limit, the strip segment is assumed to have failed, the forces in the strip segment are set to zero, and the segment is separated into two segments.
5. The shear behavior of the strip/soil interface is defined by a nonlinear shear failure envelope that varies as a function of confining pressure. The maximum shear force F_s^{max} is determined from the equations presented later.
6. Softening of the strip/interface strength as a function of shear displacement for the interface cohesion and apparent friction can be prescribed via user-defined tables, strsctable (for cohesion) and strsftable (for apparent friction).

Note that forces calculated for strip elements are "scaled" forces (i.e., they are forces per unit model thickness out-of-plane). Actual forces in a strip can be derived from the scaled forces, the calculation width, calwidth, and the number of strips per width, nstrips.

Characteristics of Strip Elements (cont.)

Thursday, March 11, 2010
11:43 AM

(1) calculation width (**calwidth**) [length];

(2) density of the strip (**density**) [mass/volume] (optional – used for dynamic analysis and gravity loading);

(3) elastic modulus (**e**) [stress] of the strip;

(4) initial apparent friction coefficient at the strip/interface (f_0^*) (**fstar0**);

(5) minimum apparent friction coefficient at the strip/interface (f_1^*) (**fstar1**);

(6) number of strips per calculation width (**nstrips**);

(7) transition confining pressure (σ'_{c0}) (**sigc0**) [stress];

(8) strip/interface shear stiffness (**strkbond**) [force/strip length/displ.];

(9) strip/interface cohesion (**strsbond**) [force/strip length];

(10) number of table relating strip/interface cohesion to plastic relative shear displacement (**strsctable**);

(11) number of table relating strip/interface apparent friction angle to plastic relative shear displacement (**strsftable**);

(12) strip thickness (**strthickness**) [length];

(13) strip width (**strwidth**) [length];

(14) strip compressive yield-force limit (**strycomp**) [force];

(15) strip tensile yield-force limit (**stryield**) [force]; and

(16) tensile failure strain limit of strip (**tfstrain**).

Characteristics of Strip Elements (cont.)

Thursday, March 11, 2010
11:43 AM

maximum shear force F_s^{max}

$$\frac{F_s^{\max}}{L} = S_{\text{bond}} \qquad if \;\; \sigma'_c < 0 \qquad (1.42)$$

$$\frac{F_s^{\max}}{L} = S_{\text{bond}} + \sigma'_c \times f^* \times \text{perimeter} \qquad if \;\; \sigma'_c \geq 0 \qquad (1.43)$$

where:

$$f^* = f_0^* - (f_0^* - f_1^*) \times \frac{\sigma'_c}{\sigma'_{c0}} \qquad if \;\; 0 \leq \sigma'_c < \sigma'_{c0} \qquad (1.44)$$

$$f^* = f_1^* \qquad if \;\; \sigma'_c \geq \sigma'_{c0} \qquad (1.45)$$

and:

L	=	strip element length;
S_{bond}	=	strip/interface cohesion;
σ'_c	=	effective confining stress normal to the strip;
perimeter	=	perimeter of strip;
f_0^*	=	initial apparent friction coefficient;
f_1^*	=	minimum apparent friction coefficient; and
σ'_{c0}	=	transition confining pressure.

The effective confining pressure acting normal to the flat strip is

$$\sigma'_c = -\sigma_{nn} - p \qquad (1.46)$$

where:

p	=	pore pressure;
σ_{nn}	=	$\sigma_{xx}\, n_1^2 + \sigma_{yy}\, n_2^2 + 2\,\sigma_{xy}\, n_1\, n_2$; and
n_i	=	unit vector normal to the strip.

Characteristics of Strip Elements (cont.)

Thursday, March 11, 2010
11:43 AM

The perimeter of a strip element is calculated from the strip width (**strwidth**), the number of strips (**nstrips**), and the calculation width (**calwidth**):

$$\text{perimeter} = \frac{2 \times \textbf{strwidth} \times \textbf{nstrips}}{\textbf{calwidth}} \tag{1.47}$$

The cohesion, S_{bond}, at the strip/interface is calculated from the cohesion of the individual strip (**strbond**), the number of strips (**nstrips**), and the calculation width (**calwidth**):

$$S_{\text{bond}} = \frac{\textbf{strbond} \times \textbf{nstrips}}{\textbf{calwidth}} \tag{1.48}$$

Single Strip with Confinement

Thursday, March 11, 2010
11:43 AM

The effect of confining stress is evaluated in this test. A single horizontal strip is placed within a grid, as shown in Figure 1.81. The strip/interface has an initial apparent friction coefficient of 1.5, and a minimum apparent friction coefficient of 0.727. The grid is fixed in the x- and y-directions at the base, and in the x-direction along the sides. A uniform, vertical confining pressure of 80 kPa is applied to the top of the model. After the model is brought to equilibrium for the specified confining stress, the strip is pulled in the negative x-direction by applying a small constant velocity to the left-end node of the strip. The axial force in the left-end segment of the strip is monitored and plotted versus the relative x-displacement of the left-end node. Figure 1.82 shows the results. Note that, for this case, the transition confining pressure (sigc0) is 120 kPa.

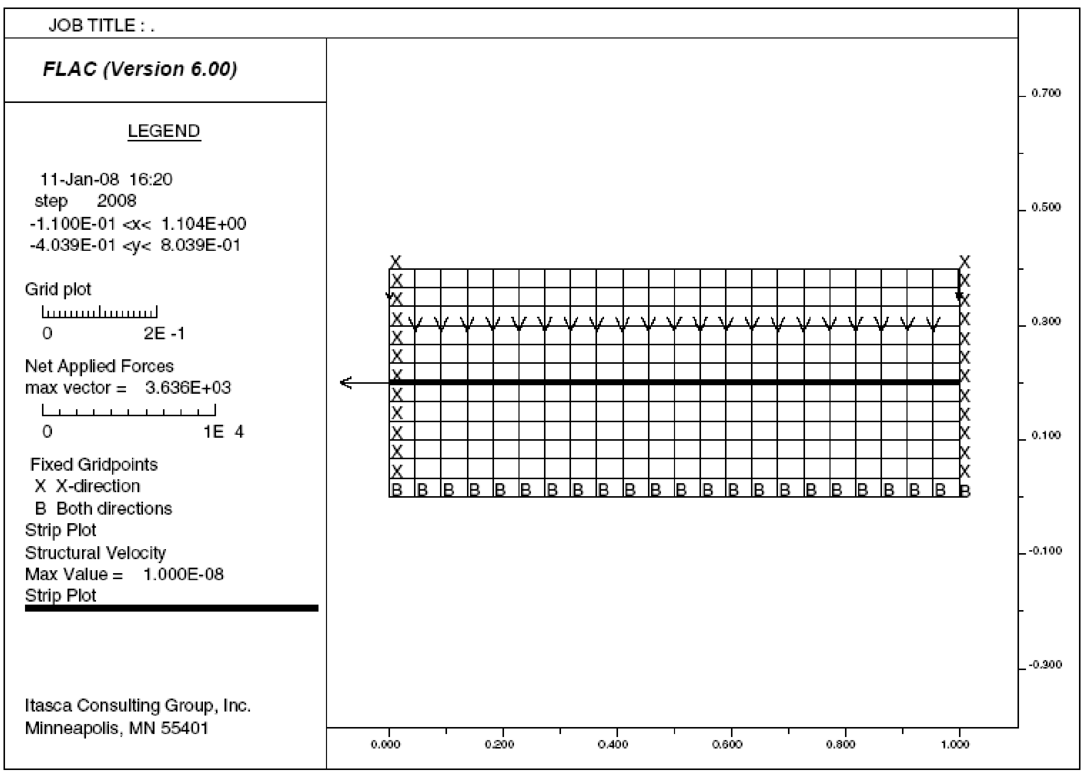


Figure 1.81 Strip element in grid: vertical confining pressure and x-velocity applied at end node

Single Strip with Confinement (cont.)

Thursday, March 11, 2010
11:43 AM

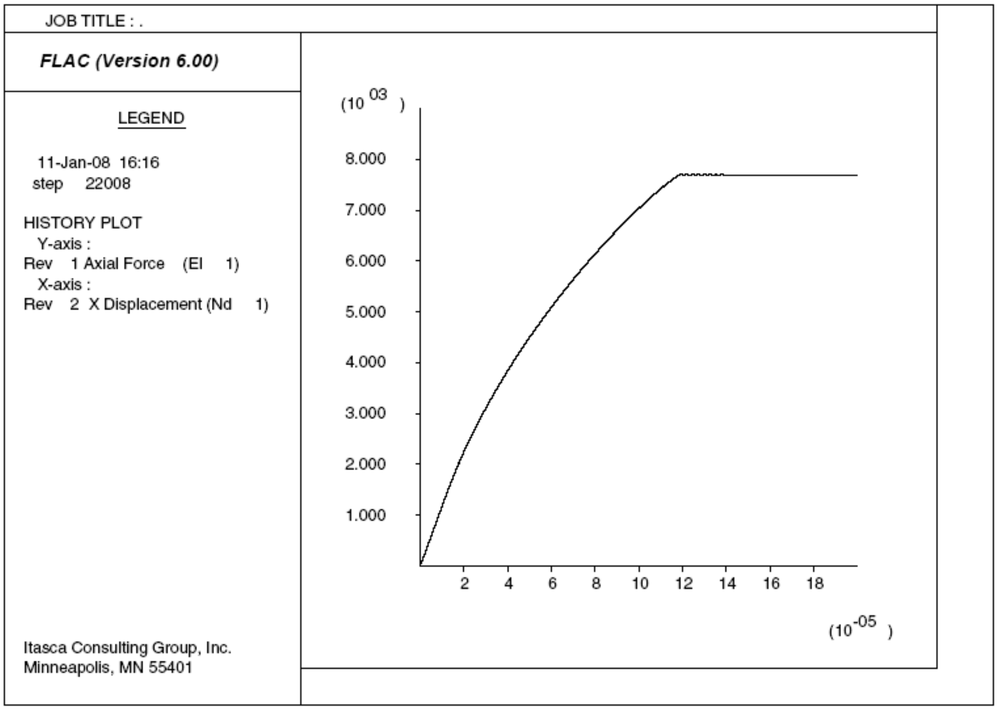


Figure 1.82 Strip axial force versus axial displacement – sigc0 = *120 kPa*

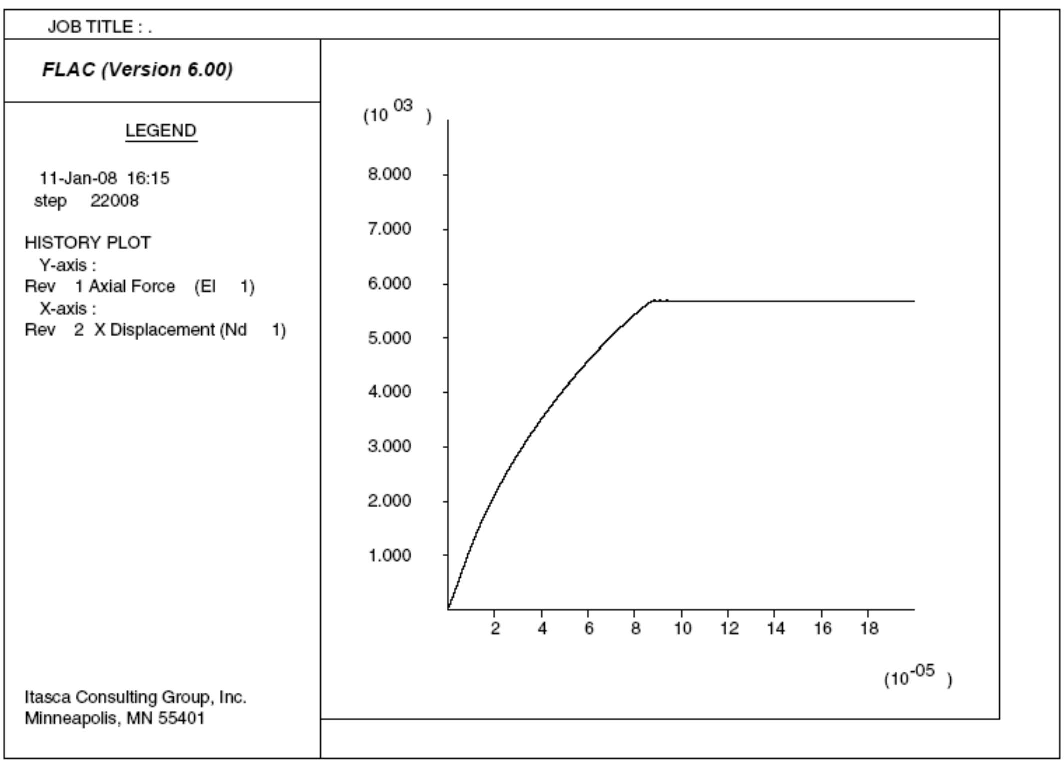


Figure 1.83 Strip axial force versus axial displacement – sigc0 = *70 kPa*

Single Strip with Confinement (cont.)

Thursday, March 11, 2010
11:43 AM

```
config
grid 22,12
gen (0.0,0.0) (0.0,0.4) (1.0,0.4) (1.0,0.0) i=1,23 j=1,13
model elastic
prop density=2000.0 bulk=1.0E8 shear=3.0E7
fix x y j 1
fix x i 23
fix x i 1
apply pressure 80000.0 from 1,13 to 23,13
struct node 1 0.0,0.2
struct node 2 1.0,0.2
struct strip begin node 1 end node 2 seg 22 prop 7001
struct prop 7001
struct prop 7001 e 2.1E11 calwidth 1.0 nstrips 1.0 strwidth 0.05 strthickness 0.0040 stryield 52000.0
& strycomp 52000.0 strkbond 1.0E9 strsbond 0e3 fstar0 1.5 fstar1 0.727 sigc0 120e3
history 999 unbalanced
solve
struct node 1 fix x initial xvel=1.0E-8 yvel=0.0
history 1 element 1 axial
history 2 node 1 xdisplace
set st_damp struc=combined 0.8
cycle 20000
save MSE strip elements.sav 'last project state'
```

External Stability Using FLAC

Thursday, March 11, 2010
11:43 AM

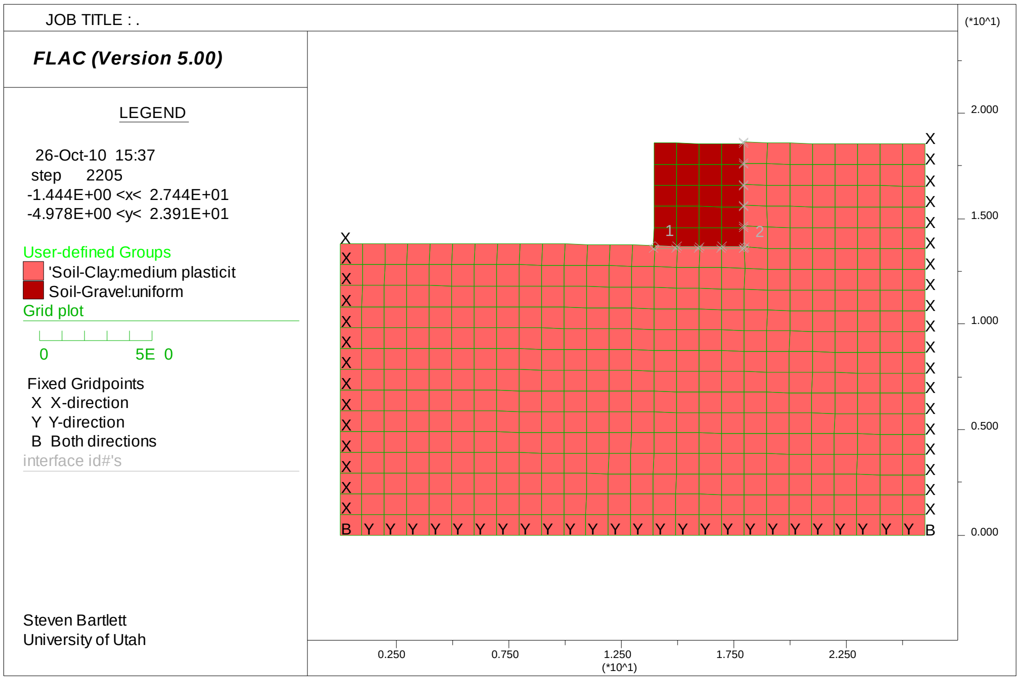
<u>Simplified Method</u> of Assessing External Stability (for cases where internal stability of the MSE zone has already been determined by other calculations using limit equilibrium methods).

1. Use wall geometry determined from internal stability calculations
   a. Input wall height into numerical model
   b. Input wall width (i.e., reinforcement length) into numerical model.
   c. Fill in the remaining part of the model geometry with the foundation soil and back slope soil according to evaluation case.
2. Assign an MC model to the reinforced zone (i.e., backfill) with the appropriate density, bulk modulus and shear modulus; <u>assign a very high cohesion</u> to the reinforced zone to prevent failure in this zone.
3. Assign MC model and appropriate soil properties to the foundation and back slope material.
4. Create interfaces along the base and back of the reinforced zone.
5. Assign appropriate interface properties
6. Execute model using solve
7. Execute model using solve fos
8. Verify that external stability is achieved and the fos is accceptable.

FLAC model for Assessing External Stability (cont.)

Thursday, March 11, 2010
11:43 AM

Based on internal stability considerations, the reinforced zone is 5 m high and 4 m wide. This geometry is input into FLAC and the reinforced zone is given sufficient cohesion to prevent failure through this zone. Subsequently, the FLAC model is used to evaluate global stability.



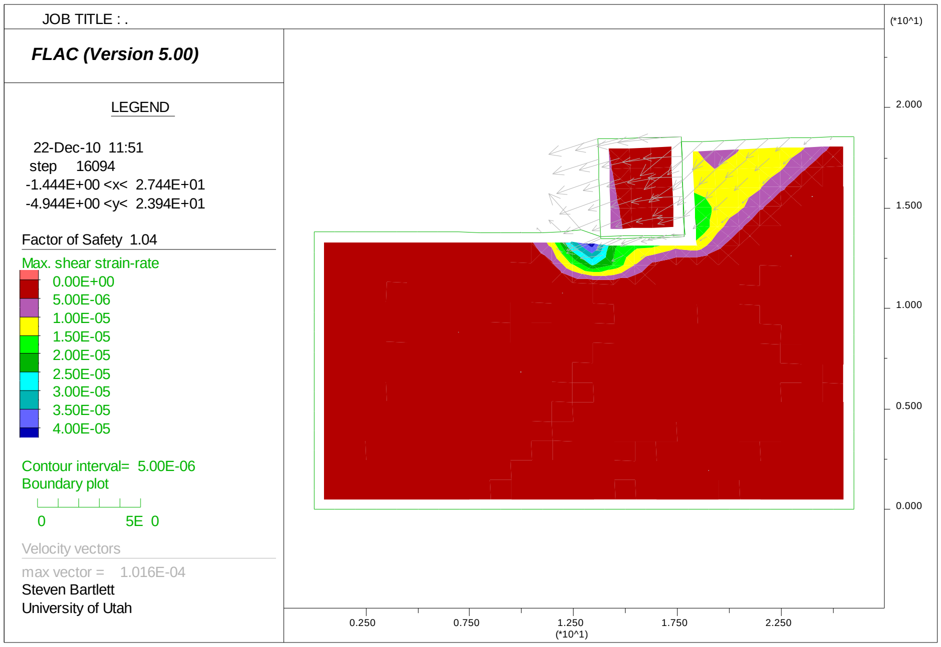
FLAC model for Assessing External Stability (cont.)

Thursday, March 11, 2010
11:43 AM

```
config
set large
grid 26,20
model elastic
model null i 1 13 j 15 20
group 'null' i 1 13 j 15 20
group delete 'null'
model null i 14 17 j 15
group 'null' i 14 17 j 15
group delete 'null'
model null i 18 j 15 20
group 'null' i 18 j 15 20
group delete 'null'
model null i 19 30 j 20
group 'null' i 19 30 j 20
group delete 'null'
;
;
ini x add 1 i 14 18 j 16 21
ini y add -1 i 14 18 j 16 21
;
group 'Soil-Clay:medium plasticity' j 1 14 ; base
model mohr group 'Soil-Clay:medium plasticity'
prop density=1800.0 bulk=5.56E5 shear=1.85E5 cohesion=0 friction=20 dilation=0.0 tension=0.0 group
'Soil-Clay:medium plasticity'
group 'Soil-Clay:medium plasticity' i 19 30 j 15 19 ; backwall
model mohr group 'Soil-Clay:medium plasticity'
prop density=1800.0 bulk=5.56E6 shear=1.85E6 cohesion=0  friction=20 dilation=0.0 tension=0.0 group
'Soil-Clay:medium plasticity'
group 'Soil-Gravel:uniform' i 14 17 j 16 20 ; reinforced zone
model mohr group 'Soil-Gravel:uniform'
prop density=1600.0 bulk=2.67E7 shear=1.6E7 cohesion=1e6 group 'Soil-Gravel:uniform'
;
interface 1 aside from 15,15 to 19,15 bside from 14,16 to 18,16 ; base
interface 1 unglued kn=8e5 ks=8e5 cohesion=0 dilation=0 friction=20 tbond=0.0 bslip=Off
interface 2 aside from 19,15 to 19,20 bside from 18,16 to 18,21 ; backwall
interface 2 unglued kn=8e6 ks=8e6 cohesion=0 dilation=0 friction=20 tbond=0.0 bslip=Off
set gravity=9.81
history 999 unbalanced
fix x y j 1
fix x i 1 j 1 15
fix x i 27 j 1 21
solve elastic; used to initialize stress in slope
solve fos
```

FLAC model for Assessing External Stability (cont.)

Thursday, March 11, 2010
11:43 AM



Factor of Safety for global stability = 1.04, friction angle = 20 deg., no cohesion is present in model.

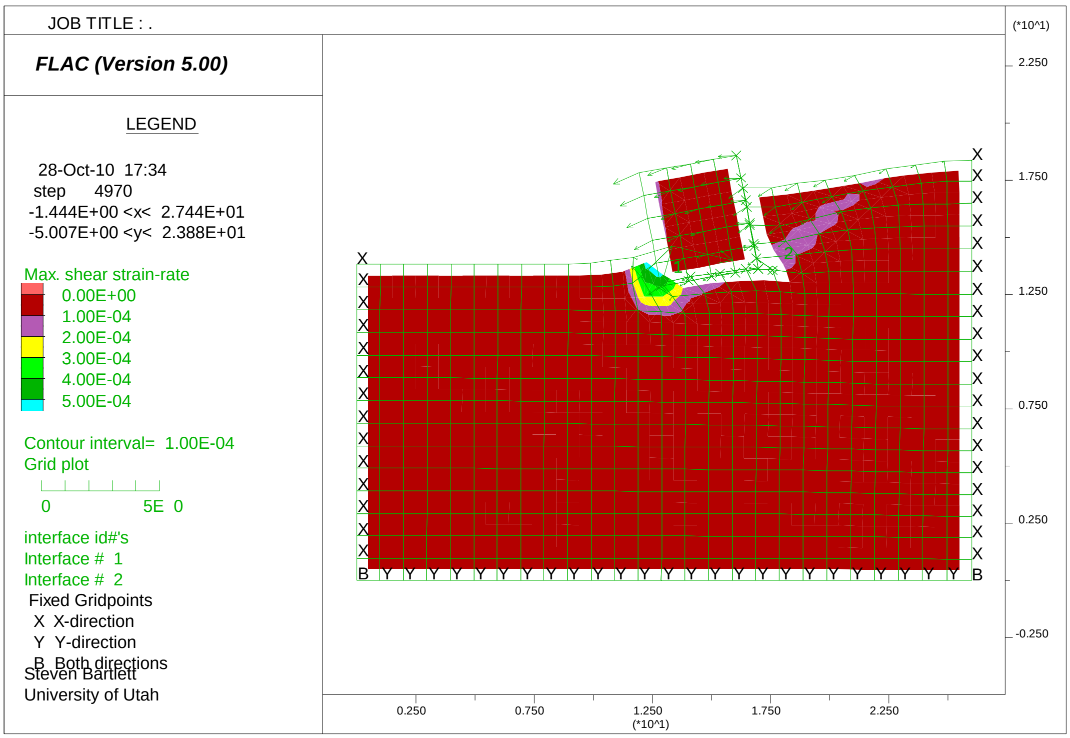
This plot can be generated using Plot FOS command that is in the run menu. If you cannot find this tab, make sure that you have included factor of safety calculations in the File/Model Options menu.

FLAC model for Assessing External Stability (cont.)

Thursday, March 11, 2010
11:43 AM

Finding the critical failure zone

- Lower the strength (i.e., friction angle) until a FS = 1.00 is obtained using trial and error.
- Use the Plot FOS command to show the maximum shear strain rate.
- Inspect the plot for the zone of high shear strain rate. This best defines the failure plane



FOS FLAC Methodology for calculating factor or safety

Thursday, March 11, 2010
11:43 AM

A shear strength reduction technique was adopted in FLAC to solve for a factor of safety of slope stability. Dawson et al. (1999) exhibited the use of the shear strength reduction technique in this finite difference program and verified numerical results with limit equilibrium results for simple slopes. In this technique, a series of trial factors of safety are used to adjust the cohesion, c and the friction angle, ϕ, of soil as follows:

$$c_{trial} = \frac{1}{FS_{trial}} c \qquad (1)$$

$$\phi_{trial} = \arctan\left(\frac{1}{FS_{trial}} \tan \phi\right) \qquad (2)$$

Adjusted cohesion and friction angle of soil layers are re-inputted in the model for equilibrium analysis. The factor of safety is sought when the specific adjusted cohesion and friction angle make the slope become instability from a verge stable condition or verge stable from an unstable condition.

More Reading

Thursday, March 11, 2010
11:43 AM

- Applied Soil Mechanics with ABAQUS Applications, Ch. 7.6
- FLAC v. 5 Manual, Fluid-Mechanical Interaction, Structural Elements, Section 1.7
- LIMIT EQUILIBRIUM AND CONTINUUM MECHANICS-BASED NUMERICAL METHODS FOR ANALYZING STABILITY OF MSE WALLS by Jie Han1 (Member, ASCE) and Dov Leshchinsky2 (Member, ASCE)

Additional Reference (not required reading)

- EFFECTS OF GEOYSNTHETIC REINFORCEMENT SPACING ON THE PERFORMANCE OF MECHANICALLY STABILIZED EARTH WALLS (FHWA).

Assignment 9

Thursday, March 11, 2010
11:43 AM

1. Use the methodology described in Helwany (2007) to calculate the internal stability of a MSE wall.
   a. Calculate the acceptable tensile reinforcement vertical spacing for a reinforcement that has an ultimate tensile strength of 3 kips/ft (10 points)
   b. Calculate the required length of the reinforcement, L, based on the tensile strength calculated in 1a and the required vertical spacing. (10 points)

   Horizontal backfill
   Phi (backfill) = 20 degrees
   Phi (interface) = 2/3 phi backfill
   FS rupture = 1.5
   FS pulllout = 4
   Height of reinforced zone = 8 m
   Unit weight of backfill = 20 kN/m^3

2. Develop a FLAC model to calculate the external stability of the wall given in Problem 1. Assume that the native soil behind wall and in the foundation is homogeneous and has the following properties. Give the factor of safety associated with these properties. (20 points)

   model mohr group 'Soil-Clay:medium plasticity'
   prop density=1800.0 bulk=5.56E5 shear=1.85E5 cohesion=5e3 friction=20 dilation=0.0 tension=0.0 group 'Soil-Clay:medium plasticity'

3. Use the same FLAC model develop in 2 and reduce the factor of safety to 1.00. Show the critical shear zone that develops behind and underneath the MSE zone. (10 points)

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Thursday, March 11, 2010
11:43 AM

Seepage

Thursday, March 11, 2010
11:43 AM

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Excel based finite difference modeling of ground water flow

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Mathematical Background

- Darcy's Law

$$q_x = -K\frac{\partial h}{\partial x}\ ;\ q_y = -K\frac{\partial h}{\partial y}\quad ;\ q_z = -K\frac{\partial h}{\partial z} \qquad \ldots\ldots\ldots\ldots(1)$$

q_x, q_y, q_z are the specific discharge in the x, y, z

- Continuity Equation

$$\frac{\partial q_x}{\partial x}+\frac{\partial q_y}{\partial y}+\frac{\partial q_z}{\partial z}=0 \qquad \ldots\ldots\ldots(2)$$

(flow into and out of a unit cube must be zero if no source or sinks are present)

Combining Eq (1) and (2) and assuming the K is independent of x,y and z (i.e., K is homogenous and isotropic then:

$$\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}+\frac{\partial^2 h}{\partial z^2}=0 \qquad \ldots\ldots\ldots(3)$$

Laplace's Equation 3D

$$\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}=0 \qquad \ldots\ldots\ldots\ldots\ldots(4)$$

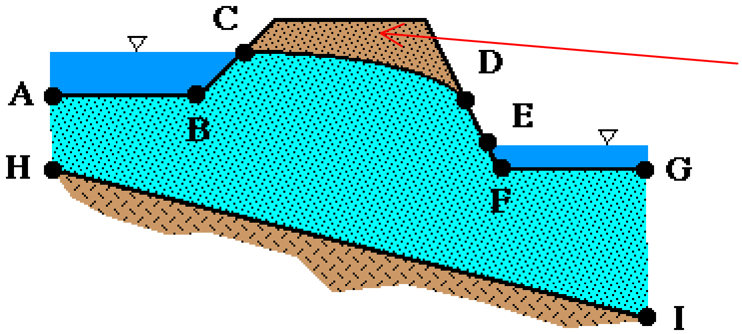
Laplace's Equation 2D

Types of Boundary Conditions

Thursday, March 11, 2010
11:43 AM

Boundary Types

Specified Head: a special case of constant head (ABC, EFG)
Constant Head: could replace (ABC, EFG)
Specified Flux: could be recharge across (CD) (Infiltration)
No Flow (Streamline): a special case of specified flux (HI)
Head Dependent Flux: could replace (ABC, EFG)
Free Surface: water-table, phreatic surface (CD)
Seepage Face: h = z; pressure = atmospheric at the ground surface (DE)



Note: Infiltration may occur between C and D.

DIRICHLET
Constant Head & Specified Head Boundaries

- Specified Head: Head (H) is defined as a function of time and space.
- Constant Head: Head (H) is constant at a given location.

Implications: *Supply Inexhaustible, or Drainage Unfillable*

NEUMANN
No Flow and Specified Flux Boundaries

- Specified Flux: Discharge (Q) varies with space and time.
- No Flow: Discharge (Q) equals 0.0 across boundary.

Implications: *H will be calculated as the value required to produce a gradient to yield that flux, given a specified hydraulic conductivity (K). The resulting head may be above the ground surface in an unconfined aquifer, or below the base of the aquifer where there is a pumping well; neither of these cases are desirable.*

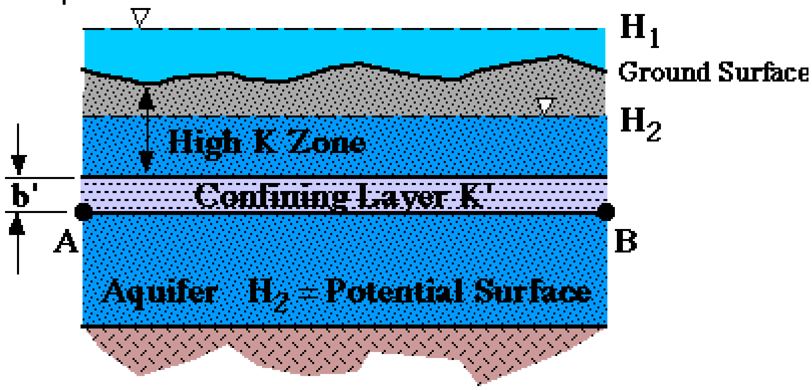
pasted from <http://igwmc.mines.edu/thought/boundary/?CMSPAGE=igwmc/thought/boundary/ >

Types of Boundary Conditions (cont.)

Thursday, March 11, 2010
11:43 AM

CAUCHY
Head Dependent Flux

- Head Dependent Flux:



H_1 = Specified head in reservoir
H_2 = Head calculated in model

$$\text{Flux into aquifer} = q = \frac{H_1 - H_2}{b'} K'A$$

Implications:

- *If H_2 is below AB, q is a constant and AB is the seepage face, but model may continue to calculate increased flow.*
- *If H_2 rises, H_1 doesn't change in the model, but it may in the field.*
- *If H_2 is less than H_1, and H_1 rises in the physical setting, then inflow is underestimated.*
- *If H_2 is greater than H_1, and H_1 rises in the physical setting, then inflow is overestimated.*

Types of Boundary Conditions (cont.)

Thursday, March 11, 2010
11:43 AM

Free Surface

- Free Surface: h = Z, or H = f(Z)
  e.g. the water table h = z
  or a salt water interface
  Note, the position of the boundary is not fixed!

  Implications: *Flow field geometry varies so transmissivity will vary with head (i.e., this is a nonlinear condition). If the water table is at the ground surface or higher, water should flow out of the model, as a spring or river, but the model design may not allow that to occur.*

Seepage Surface

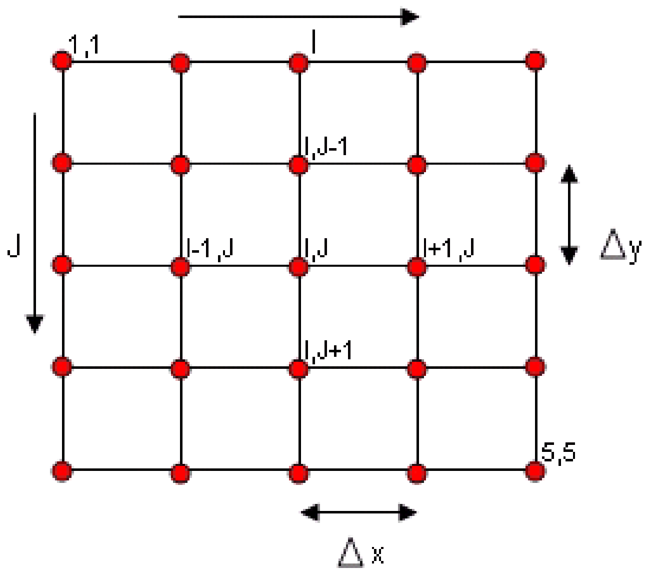
- Seepage Surface: The saturated zone intersects the ground surface at atmospheric pressure and water discharges as evaporation or as a downhill film of flow.
  The location of the surface is fixed, but its length varies (unknown a priori).

  Implications: *A seepage surface is not a head or flowline, and often can be neglected in large scale models.*

pasted from <http://igwmc.mines.edu/thought/boundary/?CMSPAGE=igwmc/thought/boundary/>

FDM 2-D

Thursday, March 11, 2010
11:43 AM



Central difference formula

The head at node i,j is $h_{i,j}$.

$$\frac{\partial^2 h}{\partial x^2} \approx \frac{h_{i-1,j} - 2h_{i,j} + h_{i+1,j}}{(\Delta x)^2}$$

$$\frac{\partial^2 h}{\partial y^2} \approx \frac{h_{i,j-1} - 2h_{i,j} + h_{i,j+1}}{(\Delta y)^2}$$

(For derivation, see FDM_Seepage.pdf in reading assignment)

For a square grid

$$h_{i-1,j} + h_{i+1,j} + h_{i,j-1} + h_{i,j+1} - 4h_{i,j} = 0$$

Solve for $h_{i,j}$

$$h_{i,j} = \frac{h_{i-1,j} + h_{i+1,j} + h_{i,j-1} + h_{i,j+1}}{4}$$

general recursive equation

(For revising general recursive equation for boundary conditions and for anisotropy, download FDM_Seepage.pdf from the course website)

Steady State Flow Example

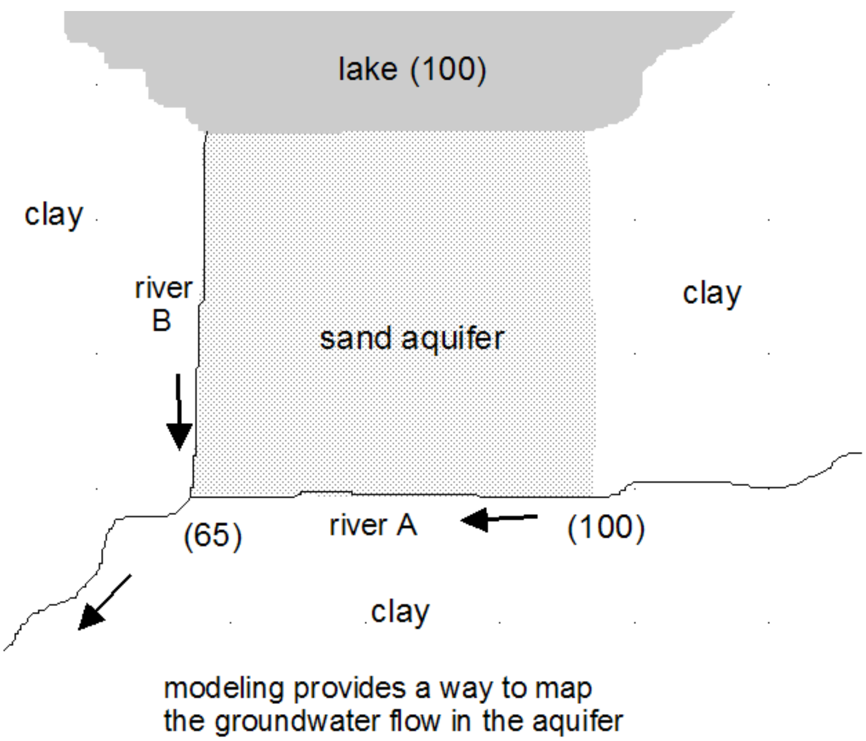
Thursday, March 11, 2010
11:43 AM

Introduction: Groundwater will flow from areas of high to low water level. A basic understanding of differential calculus is necessary to derive the important equations and formulas that map the flow, but we will use an approximate algebraic approach to model the flow.

To model the system, it needs to be divided into small grid cells, which is easy in EXCEL. To analyze groundwater flow, the nature of boundary cells around the system must be *specified*. Boundary cells can either be "no-flow," across which groundwater is *not allowed to flow*, or a "constant head," where the water level is *always fixed* and groundwater can freely flow either in or out.

Example Analysis: In this example analysis, we modify a model written by Dr. Rex Hodges of Clemson University. Figure 1 shows the layout of the aquifer. To the north lies a lake at 100. On the west side a river drains the lake and flows south and joins a river from the east at an elevation of 65. The south river has an elevation of 100 at the southeast corner of the aquifer. We want to use Excel to find the distribution of water levels in the aquifer.

Pasted from
<http://www.geology.und.nodak.edu/gerla/gge220/finite_difference.htm>



Steady State Flow Example (cont.)

Thursday, March 11, 2010
11:43 AM

Set Up the Iteration: Under Formulas and Calculation Options, turn off Automatic and make it Manual. Click the Office button in the upper left corner, select Excel Options, Formulas. Enable iterative calculation and change the maximum iterations to “1”. In cell A1 write "TWO-DIMENSIONAL STEADY-STATE MODEL", and in cell A2 enter your name and the date. We will construct an 8x8 cell model.

Creating the Boundary Cells: The top, bottom, and left side will be constant head boundaries, and the right edge will be a no-flow boundary as shown in Fig. 1. First, enter the values of the boundaries in the cells as shown below:

Pasted from <http://www.geology.und.nodak.edu/gerla/gge220/finite_difference.htm>

West Side		North Side		South Side	
Cell	Value	Cell	Value	Cell	Value
B11	100	C10	100	C19	65
B12	95	D10	100	D19	70
B13	90	E10	100	E19	75
B14	85	F10	100	F19	80
B15	80	G10	100	G19	85
B16	75	H10	100	H19	90
B17	70	I10	100	I19	95
B18	65	J10	100	J19	100

For the no-flow boundary on the right side of the model, the head at the boundary is set equal to the head in the adjacent cell of the model. This forces no slope on the water level and, hence, no flow can occur across the boundary. Enter the following formulas into the cells on the boundary. The "K" cells do not really exist in the physical model, but are there to stop the flow at the east boundary of the model, which lies on the east side of the J cells.

Steady State Flow Example (cont.)

Thursday, March 11, 2010
11:43 AM

in Cell	use Formula
K11	=J11
K12	=J12
K13	=J13
K14	=J14
K15	=J15
K16	=J16
K17	=J17
K18	=J18

Filling in the Model's Interior Cells: Next, enter a finite-difference relationship for each of the model's interior cells. This situation is steady-state flow with no recharge, which can be done by solving the Laplace equation using finite-differences. Thus, for each cell, sum the head in the four adjacent cells, and then divide by 4. For example, the finite-difference equation for cell F13 would be =(F12+E13+F14 +G13)/4; just the average of its nearest neighbors.

But first we need to insert an initial value in each of the interior cells as a starting point for the iteration. This can be done with a shortcut in the cell's formula. We will put an initial value in a cell outside the model grid, and then with Excel's logical operator "IF" enter this value into each interior cell. The complete formula for cell F13 becomes =IF(A3= "ic",A4,(F12 +E13 +F14+G13)/4), meaning that if "ic" is entered in cell A3, then whatever value is found in cell A4 is placed in cell F13. If "ic" is not found in cell A3, then the finite-difference equation is used to calculate a value for cell F13.

Steady State Flow Example

Thursday, March 11, 2010
11:43 AM

First, format the interior cells and the no-flow boundary. Move the cell pointer to cell K18, press the left mouse button, and hold it down. While holding down the left mouse button, move the cell pointer to cell C11 and release the left mouse button. The interior cells of the model and the no-flow boundary should be shaded. Now right click, select Format Cells, Number, and change the number of decimal digits to two. Then click OK. Next type "ic" into cell A3 and a "4" into cell A4. The following equations must be entered into the interior cells of the model, but read on before you enter them!

C11	=IF(A3="ic",A4,(C10 + B1l +C12+ D11)/4)
C12	=IF(A3="ic",A4,(C11 +B12+C13 +D12)/4)
C13	=IF(A3="ic",A4,(C12+B13+C14+D13)/4)
C14	=IF(A3="ic",A4,(C13 + B14+C15+D14)/4)
C15	=IF(A3="ic",A4,(C14+ B15+C16+D15) /4)
C16	=IF(A3="ic",A4,(C15+B16+C17+D16)/4)
C17	=IF(A3="ic",A4,(Cl6+B17+C18+D17)/4)
C18	=IF(A3="ic",A4,(C17+B18 +C19 +D18)/4)
D11	=IF(A3="ic",A4,(D10+C1l +D12 +E11)/4)
D12	=IF(A3="ic",A4,(D11 +C12+ D13 + E12) /4)
D13	=IF(A3="ic",A4,(D12+C13+ D14+E13)/4)
D14	=IF(A3="ic",A4,(D13 +C14 +Dl5 +E14)/4)
D15	=IF(A3="ic",A4,(D14+C15 +D16+E15)/4)
D16	=IF(A3="ic",A4,(D15+C16+D17+E16)/4)
D17	=IF(A3="ic",A4,(D16+C17+D18+E17)/4)
D18	=IF(A3="ic",A4,(D17+C18+D19+E18)/4)
E11	=IF(A3="ic",A4,(E10+ D11+E12+F11) /4)
E12	=IF(A3= "ic",A4,(E11 +D12+E13+F12)/4)

Etc.

Steady State Flow Example (cont.)

Thursday, March 11, 2010
11:43 AM

Continue in the above pattern until all interior cells are filled in, which includes column J11 through J18. You can GREATLY simplify this process by using the copy and paste. Type the correct formula for cell C11 and press Enter. Then move the cell pointer to cell C11, right click, and "Copy". Next, move the cell pointer back to cell C12, press and hold the left mouse button and drag the cell pointer to cell J18. Release the left mouse button and you will have highlighted all the interior cells. Now right click, "Paste", and the correct equations should be entered throughout. Once you have copied from C11 to the remaining cells, check a few of the cells to see if they are correct. Excel should change the formula in each cell to reflect its relative position.

Calculating the Water Levels: Press the F9 key and the “4” should appear in each of the interior cells and the no-flow right-side boundary. At this point, save your spreadsheet. Now go to cell A3 delete the “ic”. The conditional IF statement now activates the finite-difference equation. Do one iteration by hitting the F9 key. Notice that a new value for the hydraulic head in each interior cell and the right-side no-flow boundary has been calculated. If you press the F9 key again, Excel will perform a second iteration, and the interior cell values will again change. Now keep pressing the F9 key until there are no further changes in the interior cells. You are manually going though the iteration. (Think about what a royal pain this would be if you had to do it with a calculator!) Notice that the interior cells of the model now reflect the general shape set by the boundary conditions --- groundwater flows from the lake on the north toward the southwest corner.

Automating the Iterations: Click on the “Office” button in the upper left and then Excel Options, Calculation Options, and change the maximum number of iterations to 100. Click OK on the bottom of the menu. Reset the initial conditions by putting “ic” back into cell A3. The interior nodes should reset to 4 after you hit the F9 key. Now remove “ic” from A3 and press F9. The model should iterate.

Steady State Flow Example (cont.)

Thursday, March 11, 2010
11:43 AM

Sink of Source of Water: The model can be used to see how an well or a recharge basin affects the water levels. For example, move the cell pointer to interior cell Gl5. Type in "30" and press Enter. You have now replaced the finite-difference formula with a constant head of 30, which represents the water drawdown level in a pumping well at that location. Reset the initial conditions in the model. Notice that cell G15 is still set at 30, since it is now a constant head cell. Run the model with maximum iterations of 100. The shape of the water surface in the interior now reflects the drawdown around the well. You can determine the effect of a recharge basin on the water level surface by putting a constant head of 92 in cell G15 and rerunning the model.
Plotting the Results: Us the following steps to create a *three-dimensional surface chart* of the water level surface head:

1. Select the block of cells you want to map (for this part of the groundwater flow assignment it would be C11 : J18)
2. Click on INSERT, Other Charts, Surface (use the 3rd selection with the colored interval). Note that (1) the plot slopes NW, not southwest like it should, and (2) the contour interval is too large (20 units) and should start at, say, 65 (not zero).
3. Change the vertical axis by going to Chart Tools, Layout, Axes, Primary Vertical Axis, More Options, Axes Options. Set the Minimum = fixed and 65, Maximum = fixed and 100, Major Unit = fixed and 5, Minor Unit = fixed and 1. Apply the changes.
4. Finally, change the depth axis by reversing the order. Choose Depth Axis, More Options, Axis Options. Put a check in the Series in Reverse Order box.
5. You can change the color, lines, shading, line patterns, etc. under the Chart Tools Format Selection menu item.

FLAC Modeling of Fluid Flow

Thursday, March 11, 2010
11:43 AM

Introduction

FLAC models the flow of fluid (e.g., groundwater) through a permeable solid, such as soil.

The flow modeling may be done by itself (non-coupled, flow only), independent of the usual mechanical calculation of FLAC, or it may be done in parallel with the mechanical modeling (coupled), so as to capture the effects of fluid/solid interaction (e.g., consolidation).

The basic flow scheme handles both fully saturated flow (confined) and flow in which a phreatic surface (i.e., water-table) develops (unconfined). In this case, pore pressures are zero above the phreatic surface, and the air phase is considered to be passive. This logic is applicable to coarse materials when capillary effects can be neglected.

In order to represent the evolution of an internal transition between saturated and unsaturated zones, the flow in the unsaturated region must be modeled so that fluid may migrate from one region to the other. A simple law that relates the apparent permeability to the saturation is used. The transient behavior in the unsaturated region is only approximate (due to the simple law used), but the steady-state phreatic surface should be accurate.

FLAC Model Characteristics for Fluid Flow

Thursday, March 11, 2010
11:43 AM

1. The fluid transport law corresponds to both isotropic and anisotropic permeability.
2. Different zones may have different fluid-flow properties.
3. Fluid pressure, flux and impermeable boundary conditions may be prescribed.
4. Fluid sources (wells) may be inserted into the material as either point sources (INTERIOR discharge) or volume sources (INTERIOR well). These sources correspond to either a prescribed inflow or outflow of fluid and vary with time.
5. Both explicit and implicit fluid-flow solution algorithms are available.
6. Any of the mechanical models may be used with the fluid-flow models. In coupled problems, the compressibility of the saturated material is allowed.

FLAC Model Characteristics for Fluid Flow

Thursday, March 11, 2010
11:43 AM

Table 1.3 Recommended procedure to select a modeling approach for fluid-mechanical analysis

Time Scale	Imposed Process Perturbation	Fluid vs Solid Stiffness	Modeling Approach & Main Calculation Commands	Adjusted Fluid Bulk Modulus (M^a or K_w^a)	Examples (6)
$t_s >>> t_c$ (steady-state analysis)	mechanical or pore pressure	any R_k	*Effective Stress (1)* with no fluid flow or	no fluid	E.A. 1 (SLOPE.DAT) E.A. 7 (WATER.DAT) E.A. 10 (ROCKSL.DAT)
t_s as the required time scale of the analysis. t_c as the characteristic time of the coupled diffusion process			*Effective Stress (2)* **CONFIG gw** **SET flow off** **SET mech on**	$M^a = 0.0$ or $K_w^a = 0.0$	E.A. 1 (SLOPEGW.DAT) E.A. 11 (DIAP.DAT) E.A. 14 (EXC.DAT) E.A. 17 (LINER.DAT) E.A. 18 (EDAM.DAT)
$t_s <<< t_c$ (undrained analysis)	mechanical or pore pressure	any R_k	*Pore Pressure Generation (3)* **CONFIG gw** **SET flow off** **SET mech on**	realistic value for M^a or K_w^a	E.A.4 (CAV.DAT) E.A.7 (WATER_GW.DAT) E.A.15 (WHARF.DAT) E.A.16 (PEMBANK.DAT) V.P.15 (CAM2.DAT) V.P.21 (EBANK.DAT)
t_s in the range of t_c	pore pressure	any R_k	*Uncoupled Flow-Mechanical (4)* **CONFIG gw** Step 1. **SET flow on** **SET mech off** Step 2. **SET flow off** **SET mech on**	$M^a = \frac{1}{\frac{1}{M} + \frac{\alpha^2}{K+4G/3}}$ or $K_w^a = \frac{n}{\frac{n}{K_w} + \frac{1}{K+4G/3}}$ $M^a = 0.0$ or $K_w^a = 0.0$	V.P. 14 (BH.DAT)
t_s in the range of t_c	mechanical	any R_k	*Coupled Flow-Mechanical (5)* **CONFIG gw** **SET flow on** **SET mech on**	adjust M^a (or K_w^a) so that $R_k \leq 20$	V.P.9 (H1.DAT) V.P.18 (MANDEL.DAT) E.A.13 (EMC.DAT)

Effective Stress for Pre-defined Water Table

Thursday, March 11, 2010
11:43 AM

In some calculations, the pore pressure distribution is important only because it is used in the computation of effective stress at all points in the system. For example, in modeling slope stability, we may be given a pre-defined fixed water table (i.e., steady-state). To represent this system with FLAC, it is sufficient to specify a pore pressure distribution that is unaffected by any mechanical deformations that may subsequently occur. Because no change in pore pressure is involved, *we do not need to configure the grid for groundwater flow (do not need to use CONFIG gw command)*. In this approach, the strength of the material will be controlled by effective stress parameters.

To use this approach, we use the WATER table command to specify the fixed phreatic surface (denoted by a table of (x,y) values), which generates a hydrostatic pore pressure distribution for all zones beneath the given surface. Alternatively, the INITIAL command or a FISH function may be used to generate the required static pore pressure distribution. Either way, we must supply the saturated density below the water table and the moist or dry density above the water table).

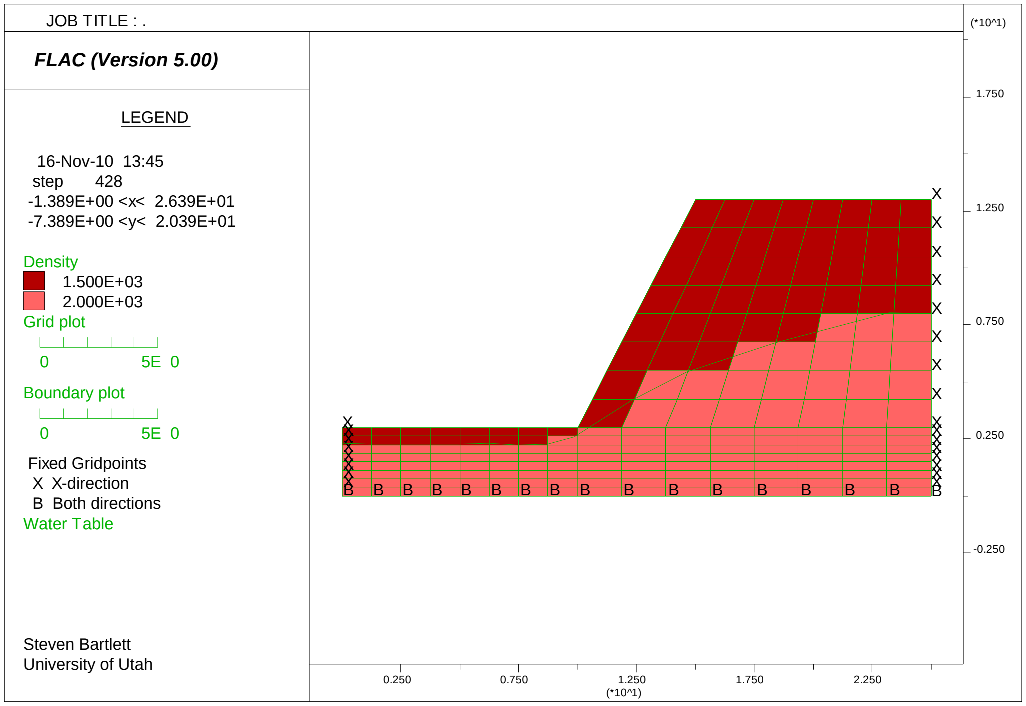
Effective Stress for Pre-defined Water Table

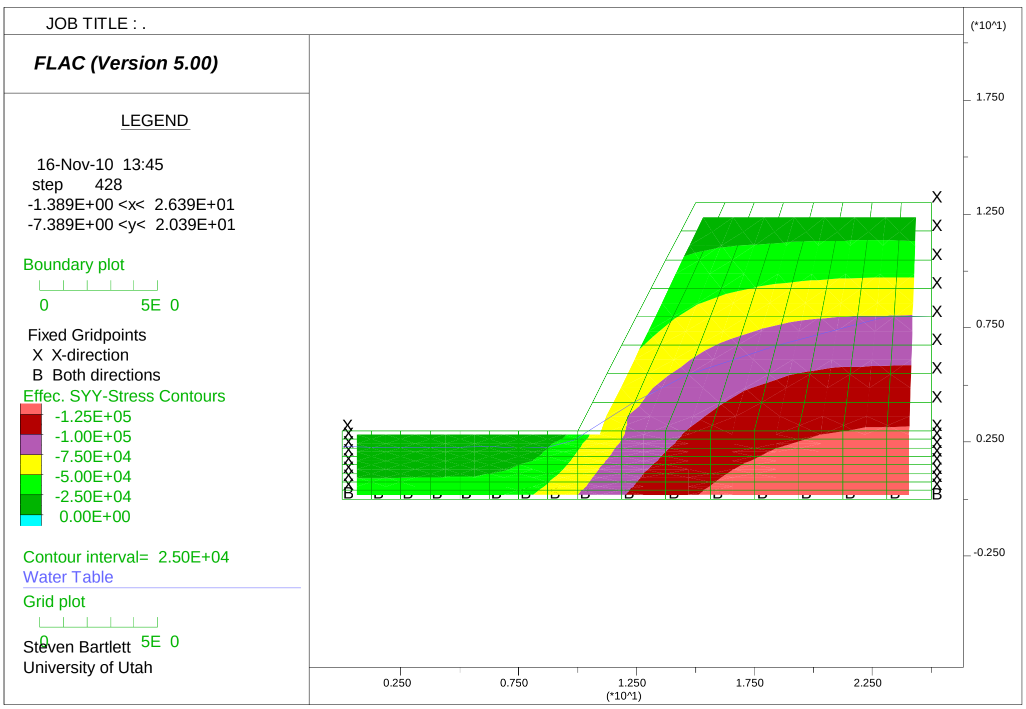
Thursday, March 11, 2010
11:43 AM

```
config
grid 16,16
gen (0.0,0.0) (0.0,3.0) (10.0,3.0) (10.0,0.0) i 1 9 j 1 9
gen (10.0,0.0) (10.0,3.0) (25.0,3.0) (25.0,0.0) i 9 17 j 1 9
gen (10.0,3.0) (15.0,13.0) (25.0,13.0) (25.0,3.0) i 9 17 j 9 17
model mohr i=1,8 j=1,8
model mohr i=9,16 j=1,8
model mohr i=9,16 j=9,16
;
; SOIL PROPERTIES
;
prop density = 1500 bulk = 20e7 shear = 10e7 friction = 30 cohesion = 100e3 i 1 16 j 1 16
;
; BOUNDARY CONDITIONS
fix x y j 1
fix x i 17 j 2 17
fix x i 1 j 2 9
;
; DEFINE WATERTABLE
table 1 0.03395 2.298 1.194 2.275 2.517 2.275 3.725 2.251 4.932 2.275 6.232 2.298 7.462 2.228 8.716
2.251 9.853 2.600 12.29 4.178 14.63 5.455 18.39 6.708 23.22 8.055 25.01 8.008
water table = 1
water density=1000.0
;
; ASSIGN SATURATED SOIL UNIT WEIGHTS
def wet den
loop i (1,izones)
 loop j (1,jzones)
  if model(i,j)>1 then
     xa=(x(i,j)+x(i+1,j)+x(i+1,j+1)+x(i,j+1))
     xc=0.25*xa
     ya=(y(i,j)+y(i+1,j)+y(i+1,j+1)+y(i,j+1))
     yc=0.25*ya
     if yc < table(1,xc) then
       density(i,j) = 2000; saturated unit weight
    endif
   endif
 endloop
endloop
end
wet den
;
SET GRAVITY = 9.81
solve
save slope_w_gw_no_flow.sav 'last project state'
```

Effective Stress for Pre-defined Water Table

Thursday, March 11, 2010
11:43 AM

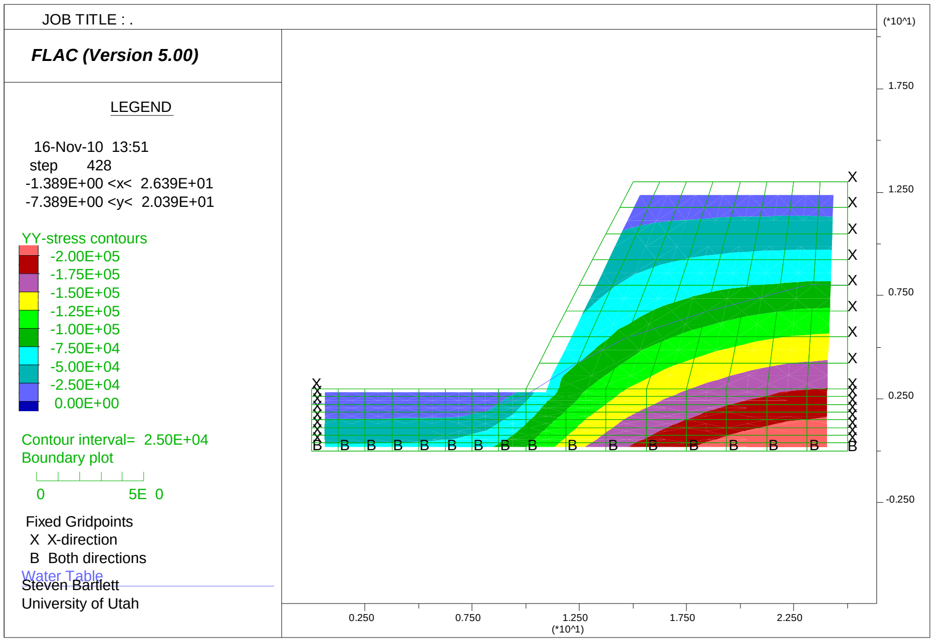




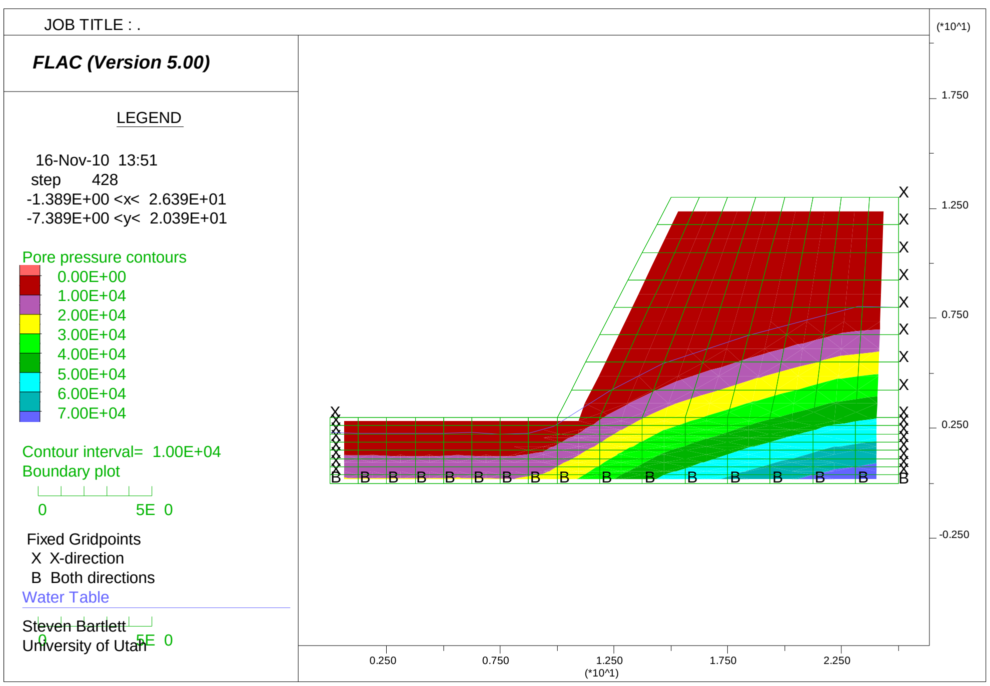
Effective Stress (esyy)

Effective Stress for Pre-defined Water Table

Thursday, March 11, 2010
11:43 AM



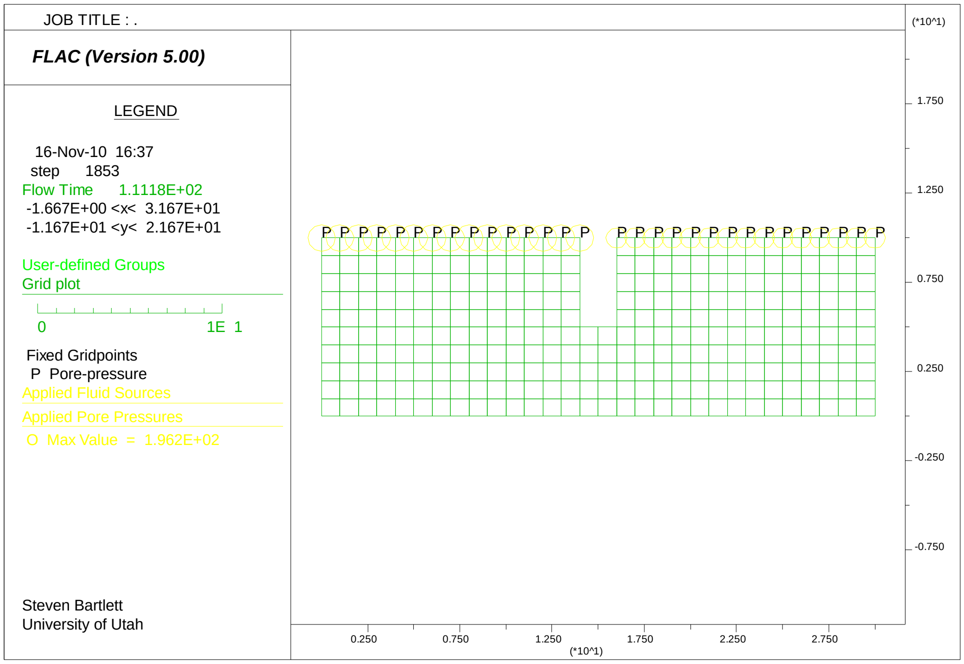
Total Stress (syy)

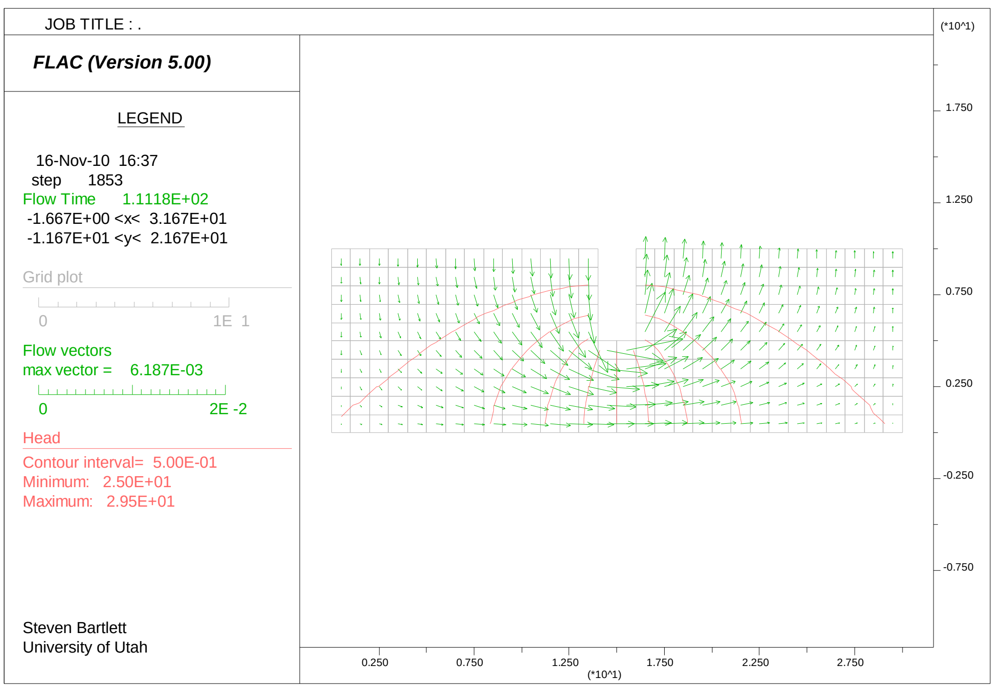


Pore Pressure

Confined Groundwater Flow Around an Embedded Structure

Thursday, March 11, 2010
11:43 AM





Confined Groundwater Flow Around an Embedded Structure

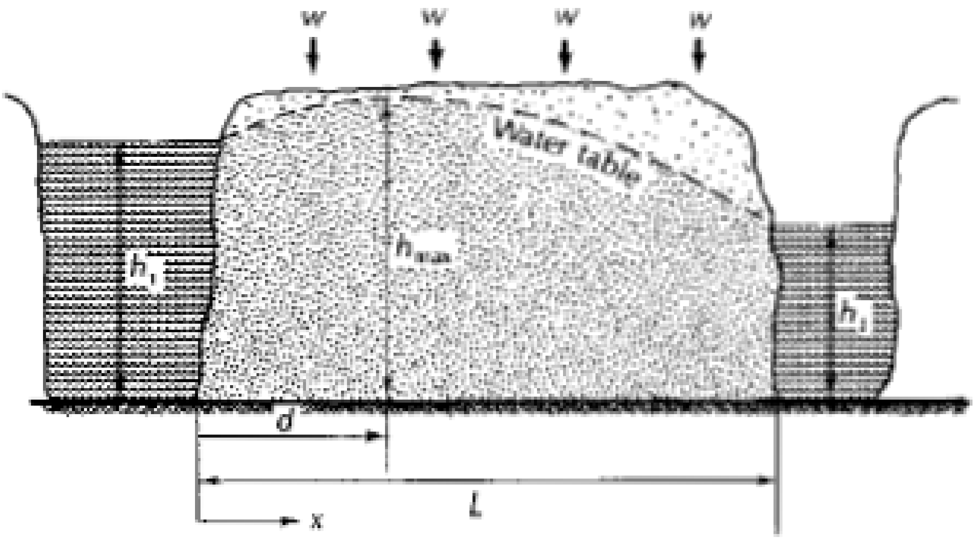
Thursday, March 11, 2010
11:43 AM

```
config gwflow
g 30 10
;
; INITIALIZES INPUT
def ini_modelsize
 h1 = 10  ; height of model
 b1 = 30  ; base of model
 ck = 1e-3 ; hyd. conductivity
 rw = 1000 ; water density
 gr = 9.81 ; gravity
end
ini_modelsize
;
; GENERATES MODEL GEOMETRY
;
gen 0 0 0 h1 b1 h1 b1 0
model elastic
;
;  CREATES CUTOFF WALL
;
model null i 15 16 j 6 11
group 'null' i 15 16 j 6 11
group delete 'null'
;
;   PROPERTIES
;
prop por .3 perm=ck den 2000
water den=rw bulk 1e3
;
;  FLOW BOUNDARY CONDITIONS
;
apply discharge 0.0 from 1,1 to 1,11 ; left side
apply discharge 0.0 from 31,1 to 31,11 ; right side
apply discharge 0.0 from 1,1 to 31,1 ; base
apply discharge 0.0 from 15,11 to 17,11; cutoff wall
apply pp 196.20e3 from 1,11 to 15,11; 20 m pore pressure head
apply pp 147.15e3 from 17,11 to 31,11; 15 m pore pressure head
;
;   SETTINGS
;
set mech off
set grav=gr
step 50
;
; SOLVE FOR STEADY STATE
solve
save GW_flow_cutoff_small.sav 'last project state'
```

Steady State Flow - Unconfined Aquifer - Dupuit's Equation

Thursday, March 11, 2010
11:43 AM

- Dupuit's Equation can be used to solve for steady-state groundwater flow between two constant head boundaries in an unconfined aquifer.



$$h^2 = h_1^2 - \frac{(h_1^2 - h_2^2)x}{L}$$

h^2 at any point

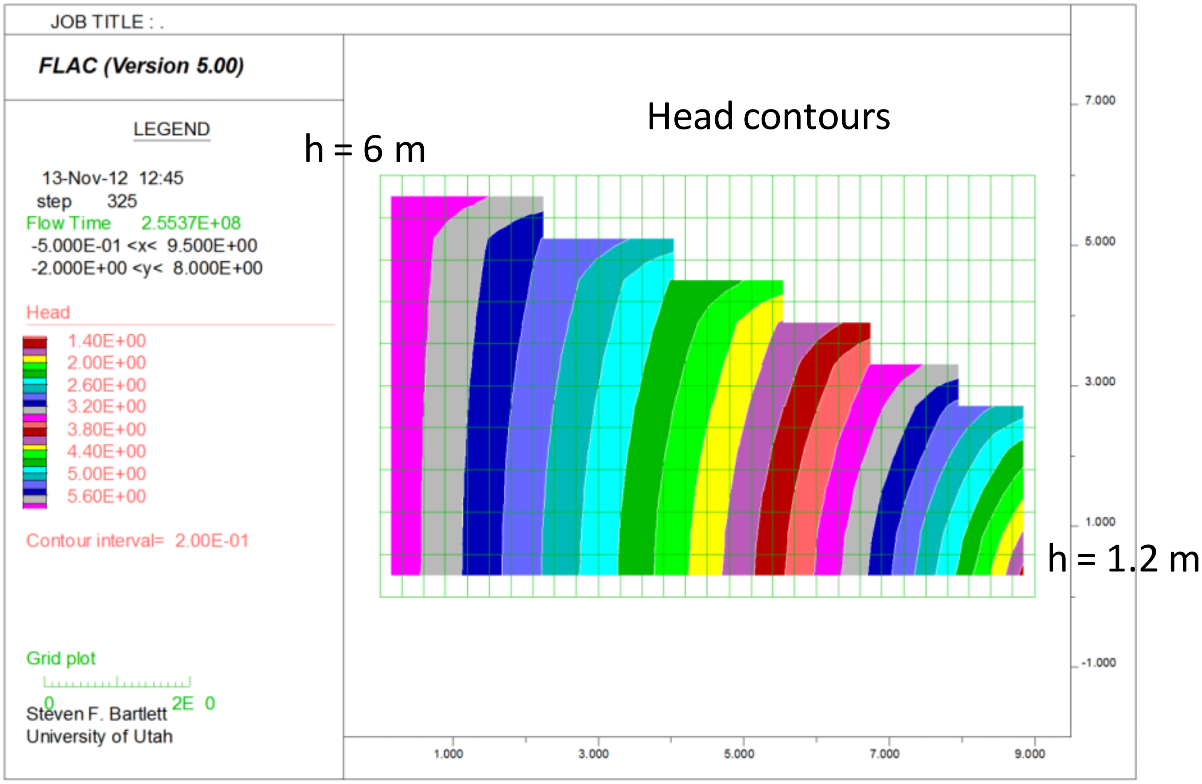
$$d = \frac{L}{2} - \frac{K}{w}\frac{(h_1^2 - h_2^2)}{2L}$$

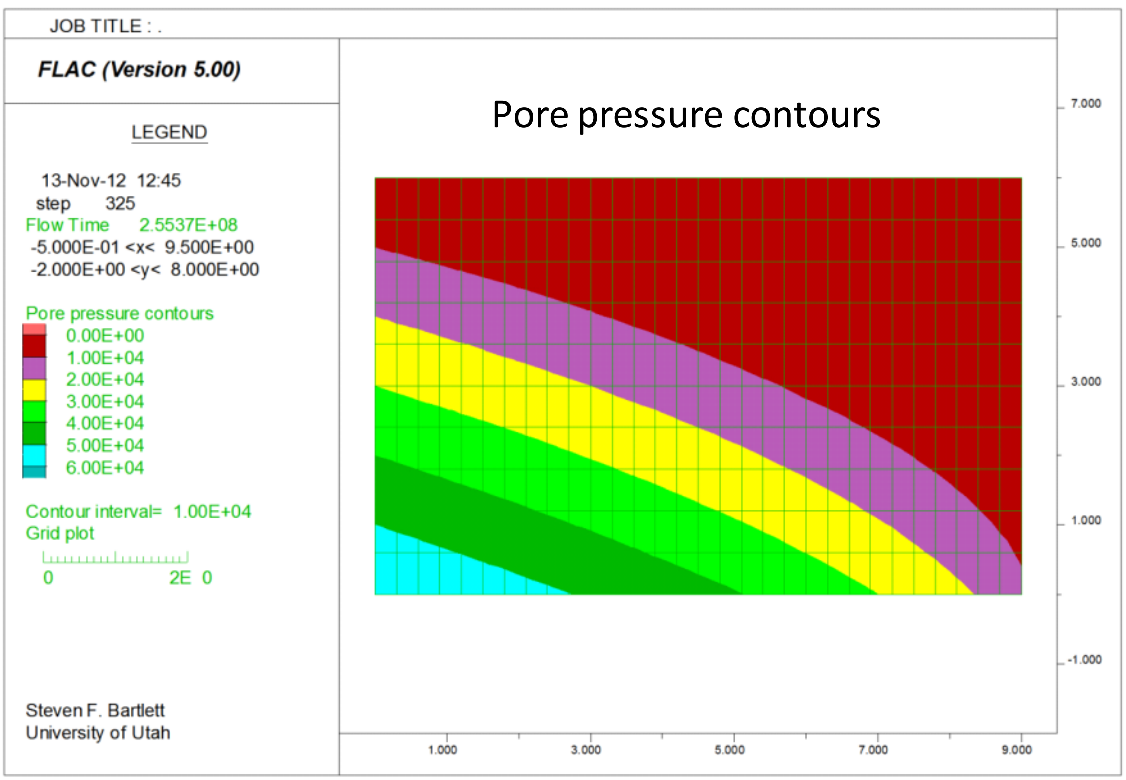
Distance to groundwater divide (see above)

(For more information, see more reading)

Steady State Flow - Unconfined Aquifer - Dupuit's Equation (cont)

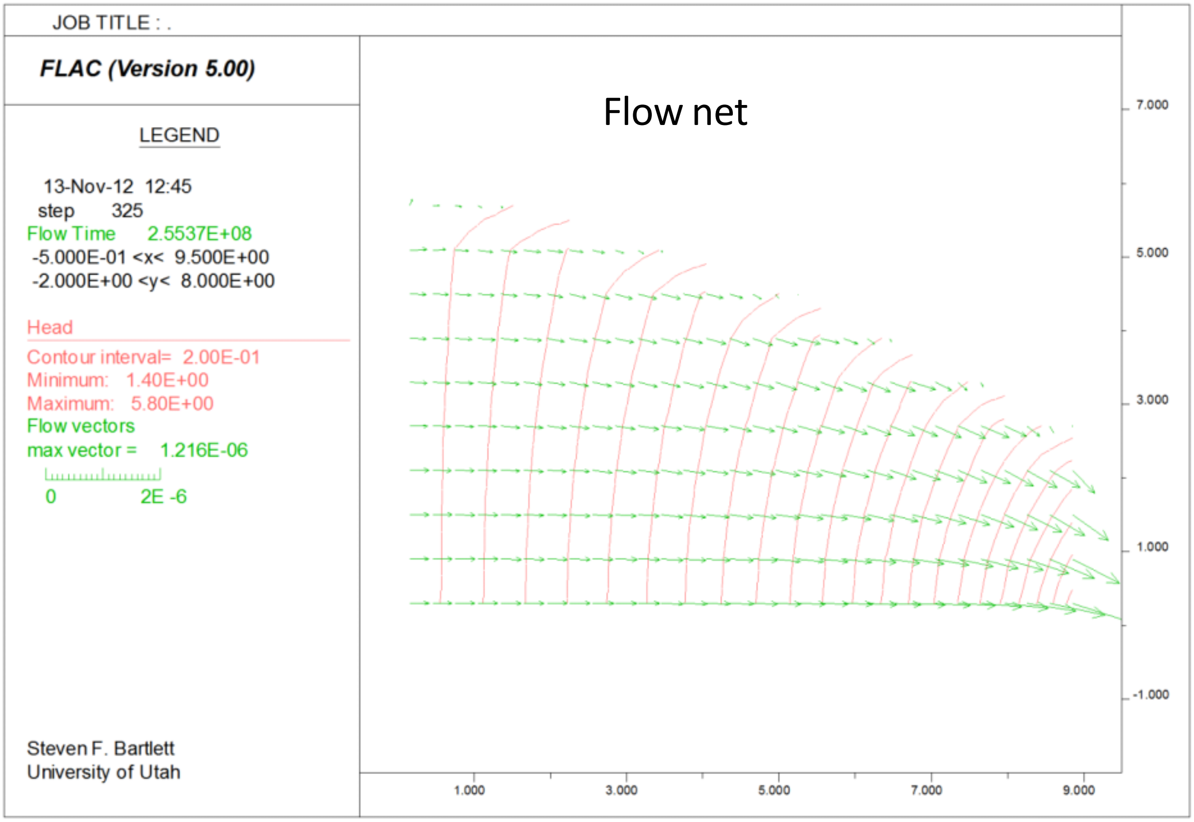
Thursday, March 11, 2010
11:43 AM

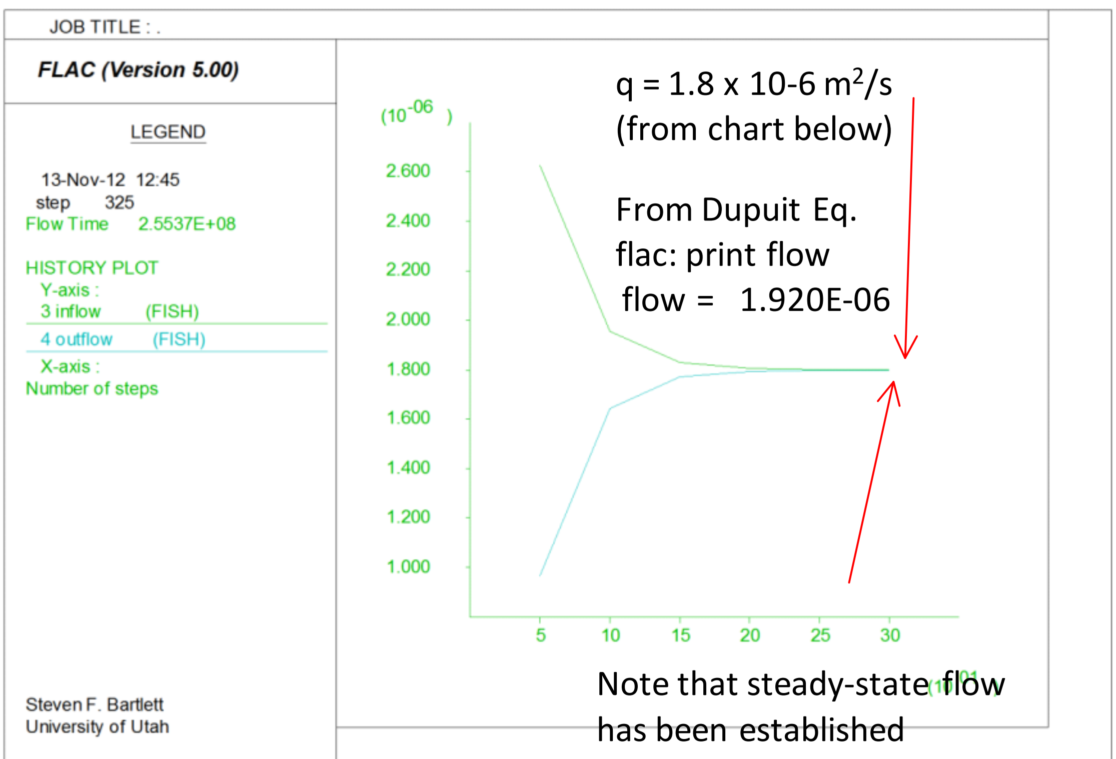




Steady State Flow - Unconfined Aquifer - Dupuit's Equation (cont)

Thursday, March 11, 2010
11:43 AM





Note that steady-state flow has been established because inflow is equal to outflow and is not changing with time

Steady State Flow - Unconfined Aquifer - Dupuit's Equation (cont)

Thursday, March 11, 2010
11:43 AM

```
config gw
g 30 10
;
def ini_h2
h1 = 6.; head on left side of box
h2 = 1.2; head on right side of box
bl = 9.; length of base
ck = 1e-10; k or permeability
rw = 1e3;mass density of water
gr = 10.; gravity
qt = ck*rw*gr*(h1*h1 - h2*h2)/(2.0*bl); flow from Dupuit Eq.
end
ini_h2
;
gen 0 0 0 h1 bl h1 bl 0 ; note scaling to a predefine variable
model elastic
;
; --- Properties ---
prop por .3 perm=ck den 2000
water den=rw bulk 1e3
;
; --- Initial conditions ---
ini sat 0
; --- Boundary conditions ---
ini pp 6e4 var 0 -6e4 i 1; pore pressure left side
ini pp 1.2e4 var 0 -1.2e4 i 31 j 1 5; pore pressure right side
fix pp i 1; fix the above p. pressure on left side
fix pp i 31; fix the above p. pressure on right side
ini sat 1 i 1; saturate right side
ini sat 1 i 31 j 1 5; saturate side
; --- Settings ---
set mech off
set grav=gr
set funsat on
```

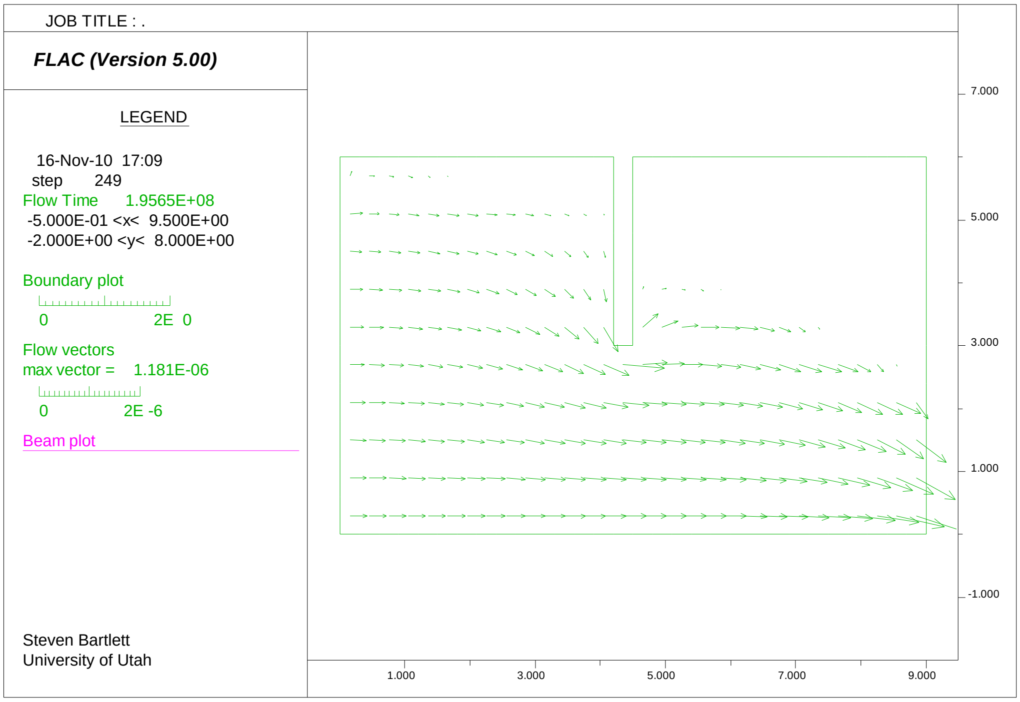
Steady State Flow - Unconfined Aquifer - Dupuit's Equation (cont)

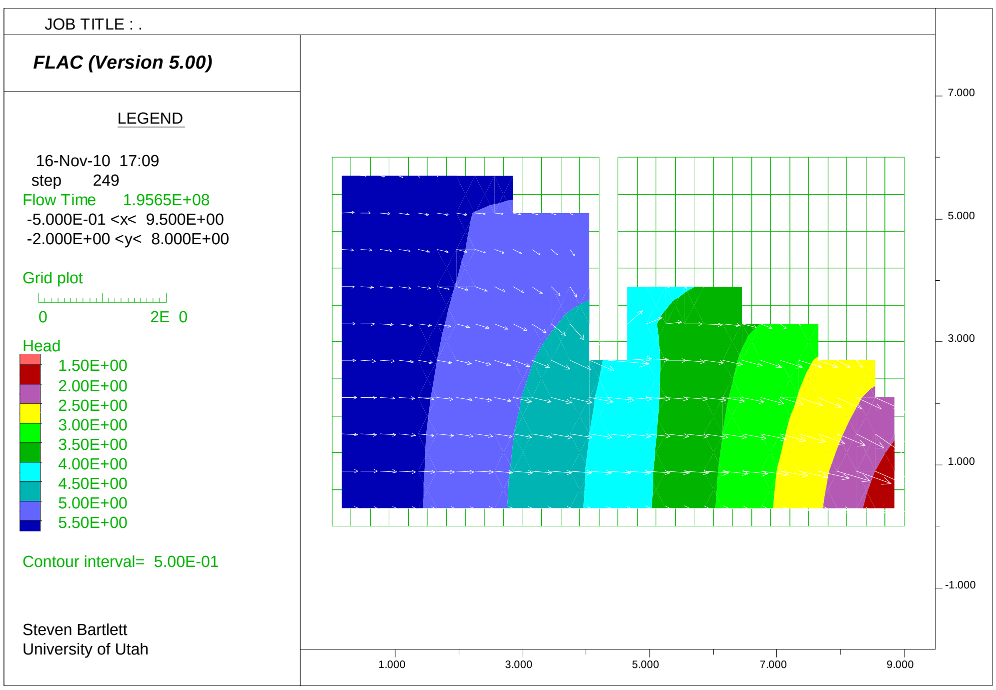
Thursday, March 11, 2010
11:43 AM

```
; --- Fish functions ---
def flow; flow calculations
  inflow=0.0
  outflow=0.0
 loop j (1,jgp)
 inflow=inflow+gflow(1,j)
 outflow=outflow-gflow(31,j)
end_loop
flow=qt
end
; --- Histories ---
hist nstep 50
hist pp i 15 j 1
hist flow
hist inflow
hist outflow
; --- Step ---
step 50
; --- Step to steady-state ---
solve
```

Steady State Flow in Unconfined Aquifer with Cutoff Wall

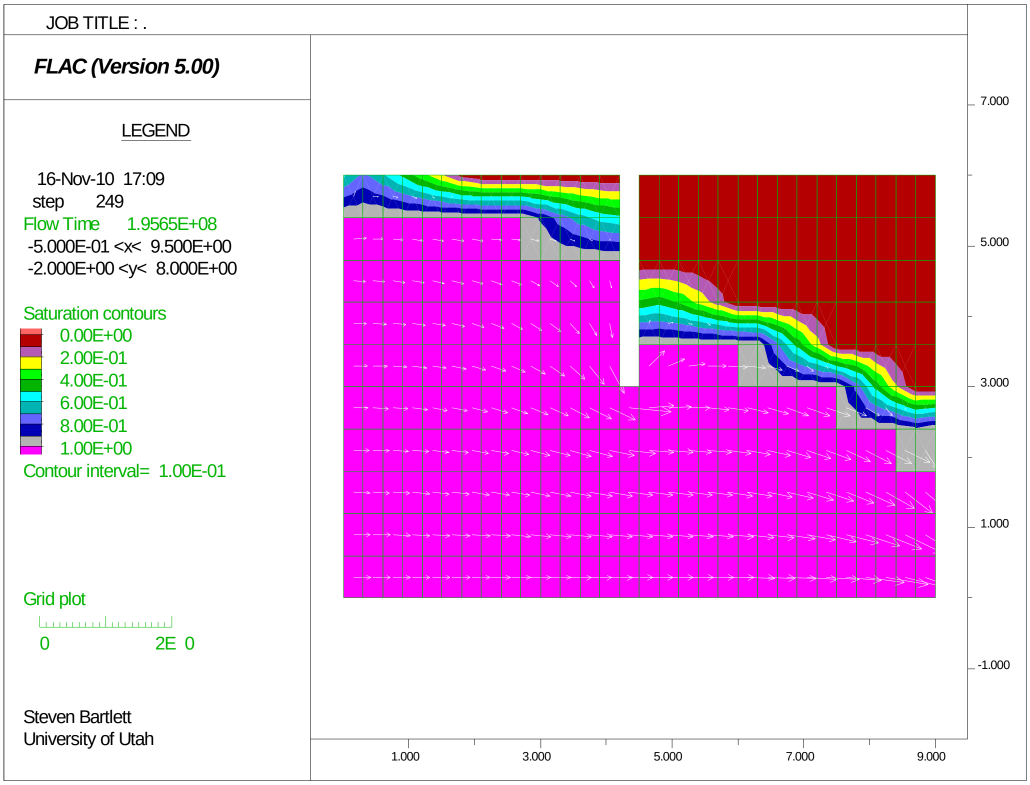
Thursday, March 11, 2010
11:43 AM





Steady State Flow in Unconfined Aquifer with Cutoff Wall

Thursday, March 11, 2010
11:43 AM



Steady State Flow in Unconfined Aquifer with Cutoff Wall

Thursday, March 11, 2010
11:43 AM

```
config gw
g 30 10
def ini_h2
h1 = 6.; head on left side of box
h2 = 1.2; head on right side of box
bl = 9.; length of base
ck = 1e-10; k or permeability
rw = 1e3;mass density of water
gr = 10.; gravity
qt = ck*rw*gr*(h1*h1 - h2*h2)/(2.0*bl); flow from Dupuit Eq.
end
ini_h2
gen 0 0 0 h1 bl h1 bl 0
mo el
m null i=15 j 6 11
; --- Properties ---
prop por .3 perm=ck den 2000
water den=rw bulk 1e3
; --- Initial conditions ---
ini sat 0
; --- Boundary conditions ---
apply discharge 0.0 from 15,11 to 16,11; no flow in cutoff
ini pp 6e4 var 0 -6e4 i 1; pore pressure left side
ini pp 1.2e4 var 0 -1.2e4 i 31 j 1 5; pore pressure right side
fix pp i 1; fix the above p. pressure on left side
fix pp i 31; fix the above p. presure on right side
ini sat 1 i 1; saturate right side
ini sat 1 i 31 j 1 5; saturate side
; --- Settings ---
set mech off
set grav=gr
set funsat on
; --- Fish functions ---
def flow; flow calculations
 inflow=0.0
 outflow=0.0
 loop j (1,jgp)
 inflow=inflow+gflow(1,j)
 outflow=outflow-gflow(31,j)
end_loop
flow=qt
end
```

Steady State Flow in Unconfined Aquifer with Cutoff Wall

Thursday, March 11, 2010
11:43 AM

```
; --- Histories ---
hist nstep 50
hist pp i 15 j 1
hist flow
hist inflow
hist outflow
; --- Step ---
step 50
save ff1_16a.sav
; --- Step to steady-state ---
solve
save GW_flow_cutoff_unconfined.sav 'last project state'
```

Undrained Footing Load

Thursday, March 11, 2010
11:43 AM

The "instantaneous" pore pressures produced by a footing load can be computed where flow is prevented but mechanical response is allowed. If the command SET flow off is given and the fluid bulk modulus is given a realistic value (comparable with the mechanical moduli., drained bulk modulus), then pore pressures will be generated as a result of mechanical deformations.

If the fluid bulk modulus is much greater than the solid bulk modulus (i.e., drained bulk modulus), convergence will be slow for the reasons stated in Section 1.8.1 of the FLAC manual. The data file given on the next page illustrates pore pressure build-up produced by a footing load on an elastic/plastic material contained in a box. The left boundary of the box is a line of symmetry. By default, the porosity is 0.5; however, permeability is not needed, since flow is not calculated. Note that the pore pressures are fixed at zero at grid points along the top of the grid. This is done because at the next stage of this model a coupled, drained analysis will be performed (see Section 1.8.6) in which drainage will be allowed at the ground surface. The zero pore pressure condition is set now to provide the compatible pore pressure distribution for the second stage. The saturation is also fixed at the top of the model to prevent desaturation from occurring during the drainage stage.

Undrained Footing Load

Thursday, March 11, 2010
11:43 AM

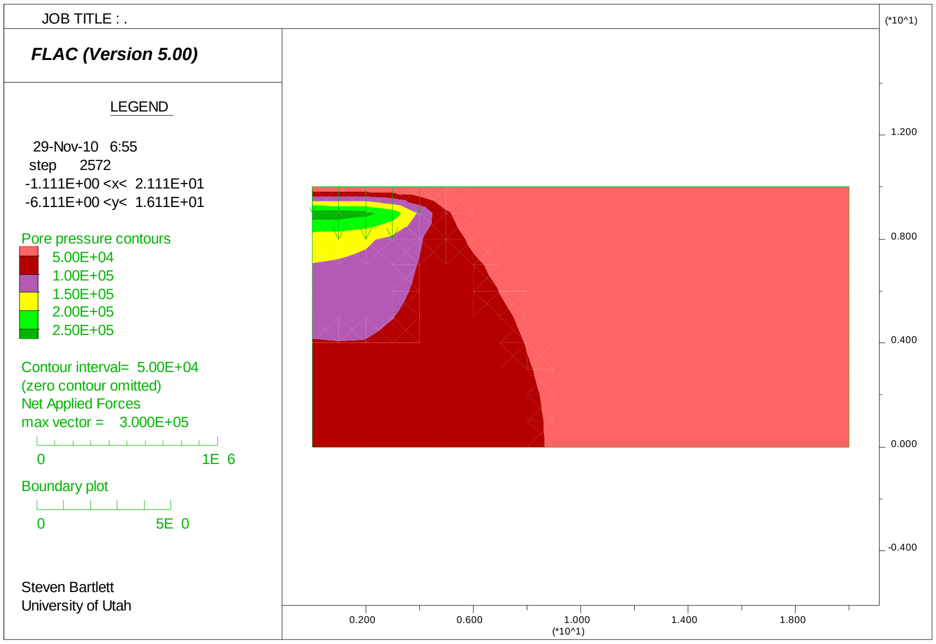
```
config gwflow
grid 20,10
model elastic
group ’soil’ notnull
model mohr notnull group ’soil’
prop density=2000.0 bulk=5E8 shear=3E8 cohesion=100000.0 friction=25.0 dilation=0.0 tension=
1e10 notnull group ’soil’
fix x i 1
fix x i 21
fix y j 1
def ramp
 ramp = min(1.0,float(step)/200.0) ; ramp function from 0 to 1 in 200 steps
end
apply nstress -300000.0 hist ramp from 1,11 to 5,11 ; applies normal stress using ramp function
history 1 pp i=2, j=9
; set fastflow on
set flow=off
water bulk=2.0E9
initial pp 0.0 j 11
fix pp j 11
history 999 unbalanced
history 998 ramp
solve elastic
```

Note:

nstress v
stress component v, applied in the normal direction to the model boundary (compressive stresses are negative)

Undrained Footing Load

Thursday, March 11, 2010
11:43 AM



As a large amount of plastic flow occurs during loading, the normal stress is applied gradually, by using the FISH function ramp to supply a linearly varying multiplier to the APPLY command. The above figure 1 shows pore pressure contours and vectors representing the applied forces. It is important to realize that the plastic flow will occur in reality over a very short period of time (on the order of seconds); the word "flow" here is misleading since, compared to groundwater flow, it occurs instantaneously. Hence, the undrained analysis (with SET flow=off) is realistic.

Note that the pore pressures generated by mechanical loading may be somewhat inaccurate at locations where the grid is distorted. The effect is evident at the inner and outer boundaries of an axisymmetric grid: these gridpoints show deviations from the mean pore pressure generation. As the grid is refined, these anomalies become less important.

Transient Flow - Introduction

Thursday, March 11, 2010
11:43 AM

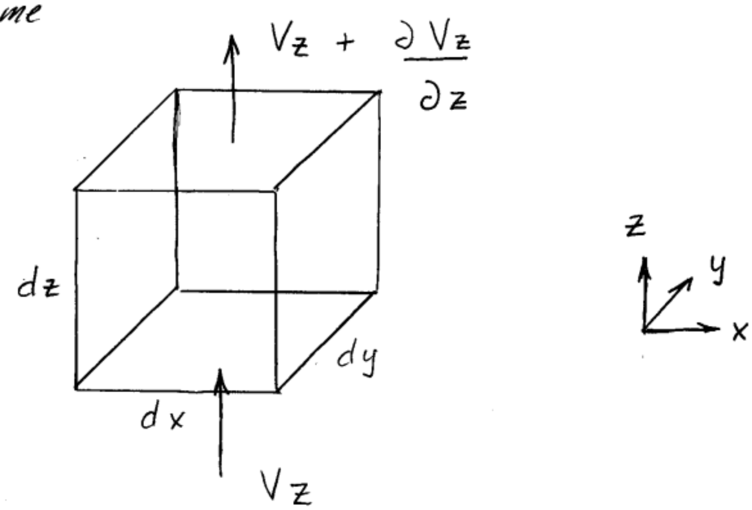
The presence of a freely moving fluid in a porous rock modifies its mechanical response. Two mechanisms play a key role in this interaction between the interstitial fluid and the porous rock: (i) an increase of pore pressure induces a dilation of the rock, and (ii) compression of the rock causes a rise of pore pressure, if the fluid is prevented from escaping the pore network. These coupled mechanisms bestow an apparent time-dependent character to the mechanical properties of the rock. Indeed, if excess pore pressure induced by compression of the rock is allowed to dissipate through diffusive fluid mass transport, further deformation of the rock progressively takes place. It also appears that the rock is more compliant under drained conditions (when excess pore pressure is completely dissipated) than undrained ones (when the fluid cannot escape the porous rock) Interest in the role of these coupled diffusion-deformation mechanisms was initially motivated by the problem of "consolidation"–the progressive settlement of a soil under surface surcharge. However, the role of pore fluid has since been explored in scores of geomechanical processes: subsidence due to fluid withdrawal, tensile failure induced by pressurization of a borehole, propagation of shear and tensile fractures in fluid-infiltrated rock with application to earthquake mechanics, in situ stress determination, sea bottom instability under water wave loading, and hydraulic fracturing, to cite a few.

The earliest theory to account for the influence of pore fluid on the quasi-static deformation of soils was developed in 1923 by Terzaghi who proposed a model of one-dimensional consolidation. This theory was generalized to three-dimensions by Rendulic in 1936. However, it is Biot who in 1935 and 1941 first developed a linear theory of poroelasticity that is consistent with the two basic mechanisms outlined above. Essentially the same theory has been reformulated several times by Biot himself, by Verruijt in a specialized version for soil mechanics, and also by Rice and Cleary who linked the poroelastic parameters to concepts that are well understood in rock and soil mechanics. In particular, the presentation of Rice and Cleary emphasizes the two limiting behaviors, drained and undrained, of a fluid-filled porous material; this formulation considerably simplifies the interpretation of asymptotic poroelastic phenomena. Alternative theories have also been developed using the formalism of mixtures theory, but in practice they do not offer any advantage over the Biot theory (from Fundamentals of Poroelasticity by Emmanuel Detournay and Alexander H.-D. Cheng).

Transient Flow and Consolidation

Thursday, March 11, 2010
11:43 AM

- Assumptions (cont.)
  - flow is 1-D; usually considered vertical
  - D'Arcy's law is valid
- Derivation
  - consider flow in and out of unit volume



V_z = velocity of fluid flow in z direction into unit volume

$V_z + \frac{\partial V_z}{\partial z}$ = velocity of fluid flow in z direction out of unit volume

- equate rate of flow in and out to rate of volume change of cube (conservation of mass for incompressible fluid)

rate flow out − rate flow in = rate vol. change

(1) $$\left(V_z + \frac{\partial V_z}{\partial z} dz\right) dx\, dy - V_z\, dx\, dy = \frac{\partial S}{\partial t}$$

where $\frac{\partial S}{\partial t}$ = change of volume or storage with time

Transient Flow and Consolidation

Thursday, March 11, 2010
11:43 AM

- Derivation (cont.)
  - Previous equation simplifies to:

(2) $$\frac{\partial v_z}{\partial z} \, dz \, dx \, dy = \frac{\partial S}{\partial t}$$

- This essentially says change volume or storage with time in the soil medium will produce a respective change in the velocity of flow in the medium.
- Changes in flow velocity versus time means changes in the hydraulic gradient with time. This condition is called transient flow. It is also called non-steady state flow.

- Write previous equation in terms of D'Arcy's law.

V_z = velocity z direction = $K_z \, i_z$

K_z = hydraulic conductivity, z direction

i_z = hydraulic gradient, z direction

$$V_z = K_z \, i_z$$

also

$$i_z = \frac{\partial h_z}{\partial z}$$

where $\frac{\partial h_z}{\partial z}$ is the change in head in the z direction

also $h_z = u \, \gamma_w$

Transient Flow and Consolidation

Thursday, March 11, 2010
11:43 AM

- Derivation (cont.)

  u = pore pressure

  γ_w = unit weight of water

thus

(3) $$\frac{k_z}{\gamma_w} \frac{\partial}{\partial z}\left(\frac{\partial u}{\partial z}\right) dz\, dx\, dy = \frac{\partial S}{\partial t}$$

- Write ∂S in terms of strain

  $\partial S = \partial \epsilon_z V_0$

  where: ϵ_z = strain z direction, 1-D compres.

  V_0 = initial volume = $dx \cdot dy \cdot dz$

(4) $$\partial S = \partial \epsilon_z \, dx \cdot dy \cdot dz$$

- Substitute eq. (4) into eq. (3)

$$\frac{k_z}{\gamma_w} \frac{\partial^2 u}{\partial z^2} dz\, dx\, dy = \frac{\partial \epsilon_z}{\partial t} dx\, dy\, dz$$

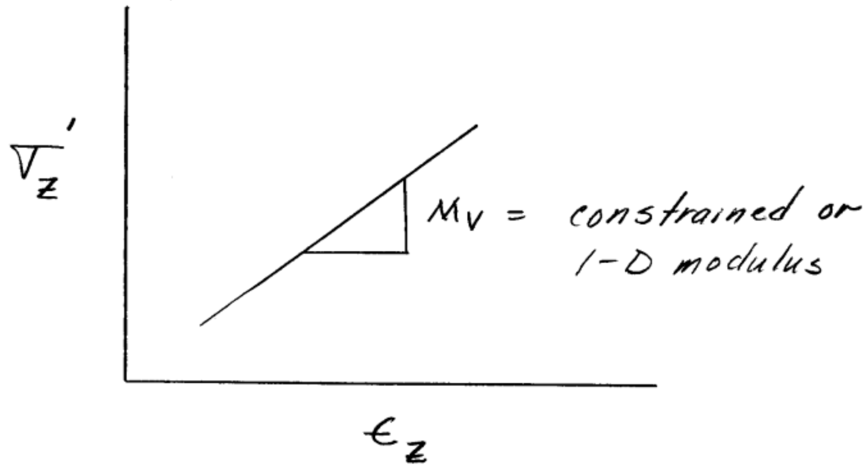
(5) $$\frac{k_z}{\gamma_w} \frac{\partial^2 u}{\partial z^2} = \frac{\partial \epsilon_z}{\partial t}$$

- During consolidation, strain ϵ_z is caused by increase in vertical effective stress as pore water pressure decreases during drainage

Transient Flow and Consolidation

Thursday, March 11, 2010
11:43 AM

- We can use the modulus, M, to relate how changes in effective stress cause strain.



(6) $$M_v = \frac{\Delta \sigma_z'}{\Delta \epsilon_z}$$

- Note that the assumption that stress and strain are related by a constant (linear) modulus is not strictly true of soils. It is an approximation. Real soils are nonlinear.

- Because changes in effective vertical stress are due to dissipation, we can write

(7) $$\Delta \sigma_v' = \Delta u$$

where Δu = change in pore water pressure.

- Substituting eq. (7) into eq. (6) produces

(8) $$M_v = \frac{\Delta u}{\Delta \epsilon_z} = \frac{\partial u}{\partial z}$$

Transient Flow and Consolidation

Thursday, March 11, 2010
11:43 AM

- Solving eq. (8) in terms of $\Delta \epsilon_z$

(9) $$\partial \epsilon_z = \frac{\partial u}{M_v}$$

- Substituting eq. (9) into eq. (5)

$$\frac{K_z}{\gamma_w} \frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t} \cdot \frac{1}{M_v}$$

- Rearrange

$$\frac{K_z M_v}{\gamma_w} \frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t}$$

- Let $\frac{K_z M_v}{\gamma_w} = C_v$ (C_v = coeff. of consolidation)

- $$\boxed{\frac{\partial u}{\partial t} = C_v \frac{\partial^2 u}{\partial z^2}}$$ ← Terzaghi's 1-D Consolidation equation

Diffusion and Storage Coefficient as Used in FLAC

Thursday, March 11, 2010
11:43 AM

1. *fluid storage:*

S = storage coefficient
M = Biot's coefficient
K_w = Bulk modulus of water

$$S = \frac{1}{M}$$

$$= \frac{n}{K_w} \quad (if\ \alpha = 1)$$

fluid compressibility only

S is the increase of the amount of fluid (per unit volume of soil/rock as a result of a unit increase of pore pressure, under constant volumetric strain.

2. *phreatic storage:*

Unconfined aquifers are sometimes also called *water table* or *phreatic* aquifers, because their upper boundary is the water table or phreatic surface. Pasted from <http://en.wikipedia.org/wiki/Aquifer

Fluid storage   Dewatering

$$S = \frac{1}{M} + \frac{n}{\rho_w g L_p}$$

$$= \frac{n}{K_w} + \frac{n}{\rho_w g L_p} \quad (if\ \alpha = 1)$$

fluid compressibility and water stored in voids in soil fabric (unconfined)

$\alpha = \Delta V_f/\Delta V$, if $\alpha = 1$ then porous medium is incompressible

3. *elastic storage:*

Confined aquifers have very low storage coefficients (much less than 0.01, and as little as 10^{-5}), which means that the aquifer is storing water using the mechanisms of aquifer matrix expansion and the compressibility of water, which typically are both quite small quantities. **Pasted from <http://en.wikipedia.org/wiki/Aquifer>**

Fluid storage   Elastic storage

$$S = \frac{1}{M} + \frac{\alpha^2}{K + 4/3G}$$

$$= \frac{n}{K_w} + \frac{1}{K + 4/3G} \quad (if\ \alpha = 1)$$

fluid compressibility and water stored from elastic changes in soil fabric (confined)

where n is porosity, K_w is fluid bulk modulus, M is the Biot modulus, K is drained bulk modulus, G is shear modulus, α is the Biot coefficient, ρ_w is fluid density, g is gravity and L_p is characteristic storage length (i.e., the average height of the medium available for fluid storage).

Diffusion and Storage Coefficient as Used in FLAC Related to Cv

Thursday, March 11, 2010
11:43 AM

Storage coefficient and coefficient of 1D vertical consolidation

Flow Equation for Transient Flow Confined Aquifer

$$\frac{k'}{\mu}\frac{\partial^2 P_p}{\partial z^2} = \left[\frac{1}{M} + \frac{\alpha^2}{(\lambda + 2G)}\right]\frac{\partial P_p}{\partial t} = S\frac{\partial P_p}{\partial t}.$$

$$\frac{k'}{\mu}\frac{\partial^2 P_p}{\partial z^2} = S\frac{\partial P_p}{\partial t}.$$

k' = permeability (m^2)

μ = fluid viscosity ~ (0.001 Pa * s) at 20 deg C.

K_v = hydraulic conductivity in vertical direction

$K_v = \gamma k'/\mu$

Compare $\frac{\partial u}{\partial t} = C_v \frac{\partial^2 u}{\partial z^2}$ with $\frac{k'}{\mu}\frac{\partial^2 P_p}{\partial z^2} = S\frac{\partial P_p}{\partial t}.$

thus,

$C_v = k'/(\mu * S)$

C_v = coefficient of consolidation in the vertical direction

$C_v = K_v * \mu/\gamma/(\mu * S)$

$C_v = K_v/(\gamma * S)$

where S = storage coefficient (see next page).

Diffusion and Storage Coefficient as Used in FLAC

Thursday, March 11, 2010
11:43 AM

If one considers a general ground water system that is free to deform in all directions, there is clearly no justification to interpret the storage coefficient solely in terms of vertical strain. In such systems, the storage should be interpreted as related to the volumetric strain and not the vertical strain.

This means that S values estimated from C_v values obtained from 1D consolidation tests should not be used strictly for isotropic, 3D changes in stress.

For isotropic changes in stress in an elastic medium

$$S = \frac{1}{M} + \frac{\alpha^2}{K + 4/3G}$$

(M does not equal M_v; in this case M is Biot's modulus and is used for isotropic changes in stress.)

The *Biot coefficient*, α, is defined as the ratio of the fluid volume gained (or lost) in a material element to the volume change of that element when the pore pressure is changed. It can be determined in the same drained test as that used to determine the drained bulk modulus, K, of the material. Its range of variation is between $\frac{3n}{2+n}$ and 1, where n is the porosity. In the particular case of an incompressible solid constituent, $\alpha = 1$. This value is the default value adopted by *FLAC*.

$\alpha = \Delta V_f/\Delta V$

where:

ΔV_f = loss or gain in the fluid volume

ΔV = total change in the soil volume when subjected to a change in pressure

Diffusion and Storage Coefficient as Used in FLAC

Thursday, March 11, 2010
11:43 AM

For saturated flow-only calculations, S is fluid storage and c is the fluid diffusivity:

No mechanical calculations used.

$$c = kM$$

$$= k\frac{K_w}{n} \quad (if\ \alpha = 1)$$

Note that c (fluid diffusivity) and k (mobility coefficient) are related to each other by the bulk modulus of water and porosity.

The "permeability" used in *FLAC* is the *mobility coefficient*: the coefficient of the pore pressure term in Darcy's law. It is defined as the ratio of intrinsic permeability to fluid dynamic viscosity. See Section 1.7.1 for the relation of *FLAC*'s permeability to other definitions of permeability.

$$k = \frac{k_H}{g\rho_w}$$

hydraulic conductivity k_H (e.g., in m/sec)
g is the gravitational acceleration
ρ_w is the fluid mass density

"intrinsic permeability," κ, (e.g., in m^2) is related to k and k_H as follows:

$$\kappa = \frac{\mu k_H}{g\rho_w} = \mu k$$

For coupled, saturated, deformation-analysis, c (generalized coefficient of consolidation) is calculated as:

$$c = \frac{k}{\frac{1}{M} + \frac{\alpha^2}{K+4G/3}}$$

$$= \frac{k}{\frac{n}{K_w} + \frac{1}{K+4G/3}} \quad (if\ \alpha = 1)$$

Diffusion and Storage Coefficient as Used in FLAC

Thursday, March 11, 2010
11:43 AM

Other relations for Biot coefficient:

$$\alpha = 1 - \frac{K}{K_s}$$

K is the drained bulk modulus of the porous medium and Ks is the bulk modulus of the solid component of the porous medium (no voids).

Relations for Biot Modulus, M:

$$M = \frac{K_w}{n + (\alpha - n)(1 - \alpha)\frac{K_w}{K}}$$

$$M = \frac{K_u - K}{\alpha^2}$$

K_u is the undrained bulk modulus

$$K_u = K + \alpha^2 M$$
$$= K + \frac{K_w}{n} \quad (if\ \alpha = 1)$$

Introduction to Coupled Analysis - Characteristic Time

Thursday, March 11, 2010
11:43 AM

It is often useful when planning a simulation involving fluid flow or coupled flow calculations with FLAC to estimate the time scales associated with the different processes involved. Knowledge of the problem time scales and diffusivity help in the assessment of maximum grid extent, minimum zone size, time step magnitude and general feasibility. Also, if the time scales of the different processes are very different, it may be possible to analyze the problem using a simplified (uncoupled) approach. Time scales may be appreciated using the definitions of characteristic time given below. These definitions, derived from dimensional analyses, are based on the expression of analytical continuous source solutions. They can be used to derive approximate time scales for FLAC analysis.

Characteristic time of the diffusion process:

$$t_c^f = \frac{L_c^2}{c} \tag{1.55}$$

where L_c is the characteristic length (i.e., the average length of the flow path through the medium) and c is the diffusivity, defined as mobility coefficient k divided by storativity S:

$$c = \frac{k}{S} \tag{1.56}$$

Characteristic time of the mechanical process

$$t_c^m = \sqrt{\frac{\rho}{K_u + 4/3G}}\, L_c$$

where K_u is undrained bulk modulus, G is shear modulus, ρ is mass density, and L_c is characteristic length (i.e., the average dimension of the medium).

Introduction to Coupled Analysis - Undrained mechanical process

Thursday, March 11, 2010
11:43 AM

By default, FLAC will do a coupled flow and mechanical calculation if the grid is configured for flow, and if the fluid bulk modulus and permeability are set to realistic values. The relative time scales associated with consolidation and mechanical loading should be appreciated. Mechanical effects occur almost instantaneously: on the order of seconds or fractions of seconds. However, fluid flow is a long-term process: the dissipation associated with consolidation takes place over hours, days or weeks.

Relative time scales may be estimated by considering the ratio of characteristic times for the coupled and undrained processes. The characteristic time associated with the undrained mechanical process is found by:

$$\frac{t_c^f}{t_c^m} = \sqrt{\frac{M + K + 4/3G}{\rho}}\ \frac{L_c}{k}\left(\frac{1}{M} + \frac{1}{K + 4/3G}\right)$$

Lc is characteristic length (i.e., the average dimension of the medium).

where $M = K_w/n$. In most cases, M is approximately 10^{10} Pa, but the mobility coefficient, k, may differ by several orders of magnitude; typical values are:

10^{-19} m^2/Pa-sec for granite;

10^{-17} m^2/Pa-sec for limestone;

10^{-15} m^2/Pa-sec for sandstone;

10^{-13} m^2/Pa-sec for clay; and

10^{-7} m^2/Pa-sec for sand.

Values of mobility coefficient (similar to hydraulic conductivity)

(In practice, mechanical effects can then be assumed to occur instantaneously when compared to diffusion effects; this is also the approach adopted in the basic flow scheme in FLAC(see Section 1.3), where no time is associated with any of the mechanical sub-steps taken in association with fluid-flow steps in order to satisfy quasi-static equilibrium. The use of the dynamic option in FLAC may be considered to study the fluid-mechanical interaction in materials such as sand, where mechanical and fluid time scales are comparable.)

Buoyancy and Seepage Force Calculations (General Approaches)

Thursday, March 11, 2010
11:43 AM

There are 2 general approaches to modeling buoyancy, heave and seepage forces:

1. Uncoupled analysis
2. Coupled analysis

For the uncoupled analysis, FLAC offers two alternative methods for performing this analysis:

1. Config ats with a water table or with the ini command to initialize pore water pressure.
2. Config gw and use the ini command to initialize pore water pressure.

We will consider the options for the uncoupled analyses first, because of their simplicity.

Buoyancy and Seepage Force Calculations (Uncoupled)(Config ats)

Thursday, March 11, 2010
11:43 AM

The CONFIG ats configuration offers a convenient way to model the effect, on heave or settlement, of a soil layer resulting from raising or lowering of a water table. For such problems, it is computationally advantageous to account directly for the stress changes associated with a change of pore pressures imposed on the model by an INITIAL pp or WATER table command. This can be done without having to conduct a fluid flow simulation.

In this approach, we specify a hydrostatic pore pressure distribution corresponding to the new water level by either using the WATER table command or the INITIAL pp command, and we specify a wet bulk density for the soil beneath the new water level. Finally, we cycle the model to static equilibrium.

By using the CONFIG ats command, the effect of the pore pressure change on soil deformation is automatically taken into account. In this configuration, any pore pressure increments or changes taking place in the model as a result of issuing the INITIAL pp command, for example, will generate stress changes and deformations, as appropriate.

For the example, we consider a layer of soil of large lateral extent, and thickness H = 10 meters, resting on a rigid base. The layer is elastic, the drained bulk modulus K is 100 MPa, and the shear modulus G is 30 MPa. The bulk density of the dry soil, ρ, is 1800 kg/m3, and the density of water, ρw, is 1000 kg/m3. The porosity, n, is uniform; the value is 0.5. Also, gravity is set to 10 m/sec2. Initially, the water table is at the bottom of the layer, and the dry layer is in equilibrium under gravity. We evaluate the heave of the layer when the water level is raised to the soil surface. However, instead of conducting a coupled fluid-mechanical simulation, we use CONFIG ats.

Buoyancy and Seepage Force Calculations (Uncoupled)(Config ats)

Thursday, March 11, 2010
11:43 AM

After defining the input parameters via a subroutine called setup and generating the grid, we set the initial saturation to zero in the model and establish force equilibrium in the dry layer by specifying the horizontal and vertical stress for the unsaturated soil column. The vertical and horizontal stresses are initialized using the INITIAL sxx, INITIAL syy and INITIAL szz commands. The values horizontal stress are initialized by using a Ko value of 0.5714 (calculated from (K − 2G/3)/(K +4G/3)).

ini syy -1.8e5 var 0 1.8e5

This command initiates the vertical stress as compression (negative sign). The var command varies the stress starting at the base with -1.8e5 - 0 and at the top with -1.8e5 - 1.8e5 or zero.

The internal variable sratio is used in conjunction with the SOLVE command to detect the steady state in flow-only calculations. For example, the run will be terminated when the value of sratio falls below 0.01 (i.e., when the balance of flows is less than 0.001%) if the following command is issued: solve sratio 0.00001

The water table can be raised two ways, either using the initialize pp command or the water table command.

```
; --- raise water
level ---
water density w_d
; (we can do it this
way ...)
ini pp 1e5 var 0 -1e5
; (or this way ...)
;water table 11
;table 11 (-1,_H) (3,_H)
; --- use wet density
below water table ---
prop dens m_rho
```

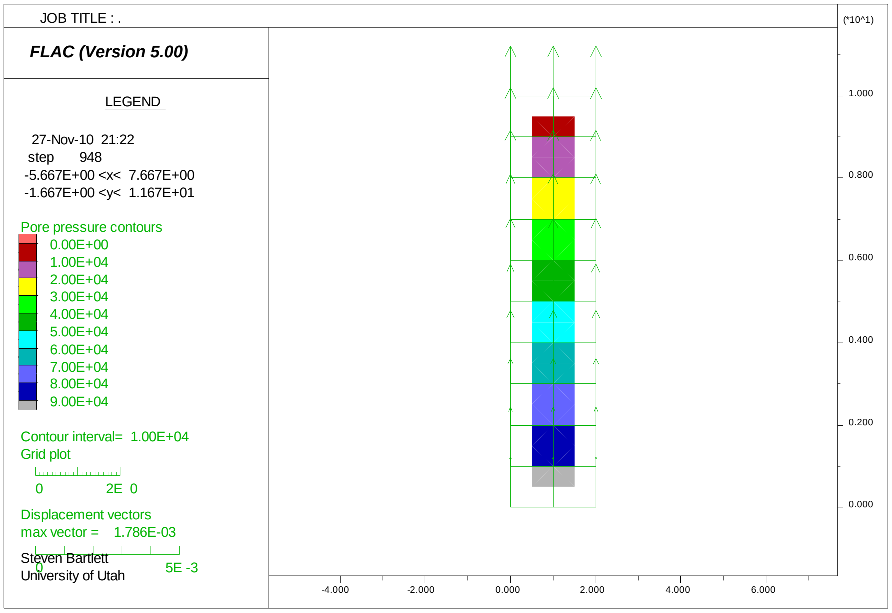
Buoyancy and Seepage Force Calculations (Uncoupled)(Config ats)

Thursday, March 11, 2010
11:43 AM

```
config ats
def setup
 m_bu  = 1e8   ; drained bulk modulus
 m_sh  = 0.3e8  ; shear modulus
 m_d   = 1800.  ; material dry mass density
 m_n   = 0.5   ; porosity
 w_d   = 1000.  ; water mass density
 _grav = 10.   ; gravity
 _H    = 10.   ; height of column
; --- derived quantities ---
 m_rho = m_d+m_n*w_d   ; material bulk wet density
end
setup
g 2 10
gen 0 0 0 10 2 10 2 0
m e
prop bu m_bu sh m_sh
; --- column is dry ---
prop density m_d
; --- boundary conditions ---
fix y j=1
fix x i=1
fix x i=3
; --- gravity ---
set grav=_grav
; --- histories ---
his ydisp i=1 j=5
his ydisp i=1 j=11
; --- initial equilibrium ---
ini syy -1.8e5 var 0 1.8e5; vertical stress
ini sxx -1.029e5 var 0 1.029e5; horizontal stress for Ko
ini szz -1.029e5 var 0 1.029e5
;
solve sratio 1e-5
; --- raise water level to surface---
water density w_d
; (we can do it this way ...)
ini pp 1e5 var 0 -1e5; given by user
; (or this way ...)
;water table 11
;table 11 (-1,_H) (3,_H)
; --- use wet density below water table ---
prop dens m_rho
; --- static equilibrium ---
solve sratio 1e-5
save buoyancy1.sav 'last project state'
```

Buoyancy and Seepage Force Calculations (Uncoupled)(Config ats)

Thursday, March 11, 2010
11:43 AM



Displacement at top of model is $1.78e^{-3}$ m.

Verification

$$u_y = \frac{(1-n)\rho_w g}{(K+4G/3)}\left[H - \frac{y}{2}\right]y$$

(((1-0.5)*1000*10)/(1e8+4*0.3e8*1/3))*(10-10/2)*10=0.0018

(This solution for u_y (vertical displacement at the top of the model) will be derived in the coupled analysis section. For now, we will use this equation without its derivation.

Buoyancy and Seepage Force Calculations (Uncoupled)(Config GW)

Thursday, March 11, 2010
11:43 AM

If we use the CONFIG gw command, we must still specify a hydrostatic pore pressure distribution corresponding to the new water level using the INITIAL pp command. (Note that the WATER table command cannot be applied in CONFIG gw mode.) The saturation is initialized to 1 below the water level. However, we do not update the soil density to account for the presence of water beneath the new water level. (The adjustment is automatically accounted for by FLAC when in CONFIG gw mode.) Finally, we SET flow off and cycle the model to static equilibrium.

```
config gw ats
def setup
  m_bu  = 1e8    ; drained bulk modulus
  m_sh  = 0.3e8  ; shear modulus
  m_d   = 1800.  ; material dry mass density
  m_n   = 0.5    ; porosity
  w_d   = 1000.  ; water mass density
  _grav = 10.    ; gravity
  _H    = 10.    ; height of column
; --- derived quantities ---
  m_rho = m_d+m_n*w_d   ; material bulk wet density
end
setup
g 2 10
gen 0 0 0 10 2 10 2 0
m e
prop bu m_bu sh m_sh
; --- column is dry ---
; (must initialize sat at 0)
ini sat 0
prop density m_d
; --- boundary conditions ---
fix y j=1
fix x i=1
fix x i=3
; --- gravity ---
set grav=_grav
; --- histories ---
his ydisp i=1 j=5
his ydisp i=1 j=11
```

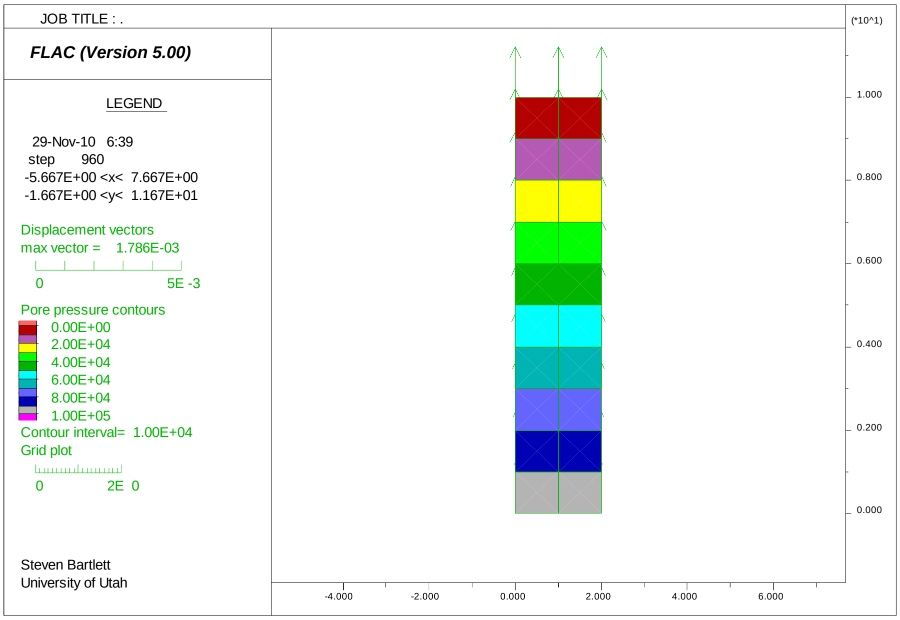
Buoyancy and Seepage Force Calculations (Uncoupled)(Config GW)

Thursday, March 11, 2010
11:43 AM

```
; --- initial equilibrium ---
ini syy -1.8e5 var 0 1.8e5
ini sxx -1.029e5 var 0 1.029e5
ini szz -1.029e5 var 0 1.029e5
;
set flow off mech on
water bulk 0
solve sratio 1e-5
save buoyancy2.sav 'last project state'
; --- raise water level ---
; (initialize sat at 1 below the water level)
ini sat 1
water density w_d
; (cannot use water table command in config gw)
; (initialize pp instead)
ini pp 1e5 var 0 -1e5
; --- no need to specify wet density below water table ---
;prop dens m_rho
; --- static equilibrium ---
set flow off mech on
water bulk 0
solve sratio 1e-5
```

Buoyancy and Seepage Force Calculations (Uncoupled)(Config GW)

Thursday, March 11, 2010
11:43 AM



Verification

Displacement at top of model is 1.78e^{-3} m. (same answer as previously obtained.)

Buoyancy and Seepage Forces (coupled analysis)

Thursday, March 11, 2010
11:43 AM

<u>In FLAC, stress equilibrium expressed in terms of total stress</u>

$$\frac{\partial \sigma_{ij}}{\partial x_j} + \rho_s g_i = 0 \qquad (1.73)$$

ρ_s is the undrained density, and g_i is gravitational vector.

$$\rho_s = \rho_d + ns\rho_w \qquad (1.74)$$

drained density, ρ_d,

n is porosity, and s is saturation

fluid density, ρ_w,

$$\sigma_{ij} = \sigma'_{ij} - p\delta_{ij} \qquad (1.75)$$

(definition of effective stress, note that total stress has been primed instead of effective stress)

Substitution of Eqs. (1.74) and (1.75) in Eq. (1.73) gives,

$$\frac{\partial \sigma'_{ij}}{\partial x_j} + \rho_d g_i - (1-n)\frac{\partial p}{\partial x_i} - n\gamma_w \frac{\partial \phi}{\partial x_i} = 0 \qquad (1.76)$$

Force from change in total stress with respect to x

Gravitational force from material solid weight

Force from change in pore water pressure (from buoyancy)

Force from change in head or position (from seepage force)

fluid unit weight, γ_w $\qquad \gamma_w = \rho_w g$

piezometric head, ϕ

Buoyancy and Seepage Forces (coupled analysis)

Thursday, March 11, 2010
11:43 AM

A simple coupled model is given in the following pages to illustrate the contribution of these individual terms in the context of FLAC methodology. For this example, we consider a layer of soil of large lateral extent and thickness, H = 10 m, resting on a rigid base. The layer is elastic, the drained bulk modulus, K, is 100 MPa, and the shear modulus, G, is 30 MPa. The density of the dry soil, ρd , is 500 kg/m3. The porosity, n, is uniform with a value of 0.5. The mobility coefficient, k, is 10−8 m2/(Pa-sec). The fluid bulk modulus, Kw, is 2 GPa and gravity is set to 10 m/sec2.

Initially, the water table is at the bottom of the layer, and the layer is in equilibrium under gravity. In this example, we study the heave of the layer when the water level is raised, and also the heave and settlement under a vertical head gradient.

This example is run using the groundwater configuration (CONFIG gw). The coupled groundwater mechanical calculations are performed using the basic fluid-flow scheme

The one dimensional incremental stress-strain relation for this problem condition is:

$$\Delta\sigma_{yy} + \alpha\Delta p = (K + 4G/3)\Delta\epsilon_y \qquad (1.77)$$

Change in v. stress — **Change in pressure** **=** **Modulus * change in vertical strain**

α is the Biot coefficient

K is the drained bulk modulus

G is the shear modulus.

ϵ_y is the vertical strain.

Buoyancy and Seepage Forces (coupled analysis)

Thursday, March 11, 2010
11:43 AM

```
config gw ats
def setup
 m_bu  = 1e8   ; drained bulk modulus
 m_sh  = 0.3e8 ; shear modulus
 m_d   = 500.0 ; material dry mass density
 m_n   = 0.5   ; porosity
 w_d   = 1000. ; water mass density
 _grav = 10.   ; gravity
 _H    = 10.   ; height of column
end
setup
grid 2 10
gen 0 0 0 10 2 10 2 0
m e
prop bu m_bu sh m_sh
prop density m_d
;
; SOLID WEIGHT
; --- (column is dry) ---
ini sat 0
; --- boundary conditions ---
fix y j=1
fix x i=1
fix x i=3
; --- gravity ---
set grav=_grav
; --- histories ---
his gwtime
his ydisp i=1 j=5
his ydisp i=1 j=11
; --- initial equilibrium ---
set sratio 1e-5
set flow off mech on
solve
;
; BOUYANCY 1
ini xdis 0 ydis 0
; --- add water ---
ini sat 1
fix sat j 11
water den w_d bulk 2e8
prop poro=m_n perm 1e-8
; --- boundary conditions ---
fix pp j=11
; --- static equilibrium ---
set flow on mech on
; --- we can run this simulation coupled, using ---
solve auto on age 5e2
```

Buoyancy and Seepage Forces (coupled analysis)

Thursday, March 11, 2010
11:43 AM

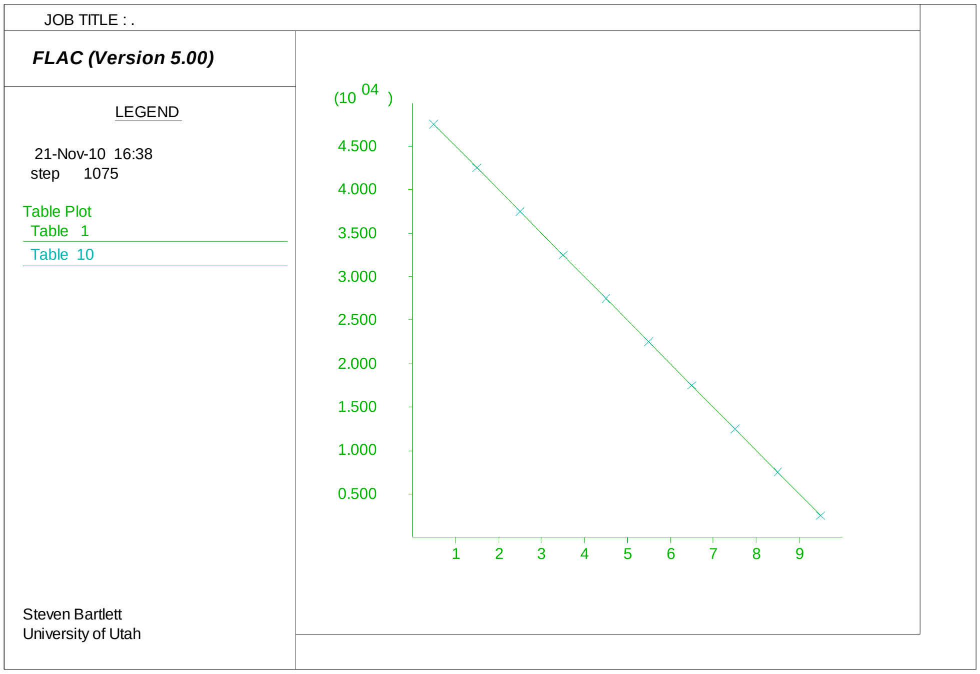
```
; BOUYANCY 2
ini xdis 0 ydis 0
; --- fluid boundary conditions ---
apply pp 2e5 j=11
; --- apply pressure of water ---
apply pressure 2e5 j=11
; --- static equilibrium ---
solve auto on age 1e3
;
;SEEPAGE FORCE 1
ini xdis 0 ydis 0
; --- flush fluid up ---
apply pp 5e5 j=1
; --- static equilibrium ---
solve auto on age 3e3
;SEEPAGE FORCE 2
ini xdis 0 ydis 0
; --- flush fluid up ---
apply pp 1e5 j=1
; --- static equilibrium ---
solve auto on age 6e3
;
save buoyancy.sav 'last project state'
```

Buoyancy and Seepage Forces (coupled analysis)

Thursday, March 11, 2010
11:43 AM

Solid Weight — We first consider equilibrium of the dry layer. The dry density of the material is assigned, and the saturation is initialized to zero (the default value for saturation is 1 in CONFIG gw mode). The value of fluid bulk modulus is zero for this stage, the flow calculation is turned off, and the mechanical calculation is on. The model is cycled to equilibrium. By integration of Eq. (1.73) applied to the dry medium, we obtain:

$$\sigma_{yy}^{(1)} = -\rho_d g(H - y)$$



Vertical stress (Pa) versus elevation (m) for dry layer

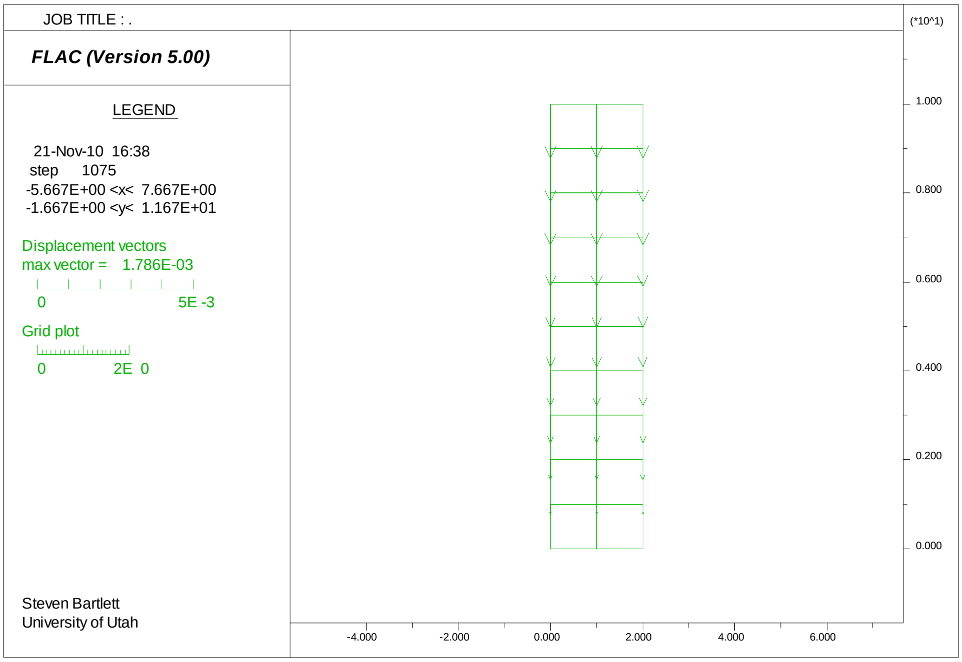
Verification

500 g * gravity * depth

For full depth, then 500 kg * 10 m/s^2 * 10-0 m = 5e4 N

Buoyancy and Seepage Forces (coupled analysis)

Thursday, March 11, 2010
11:43 AM



The vertical displacement at the model surface is found from the equation

$$u_y = -\frac{\rho_d g H^2}{2(K + 4G/3)}$$

Verification

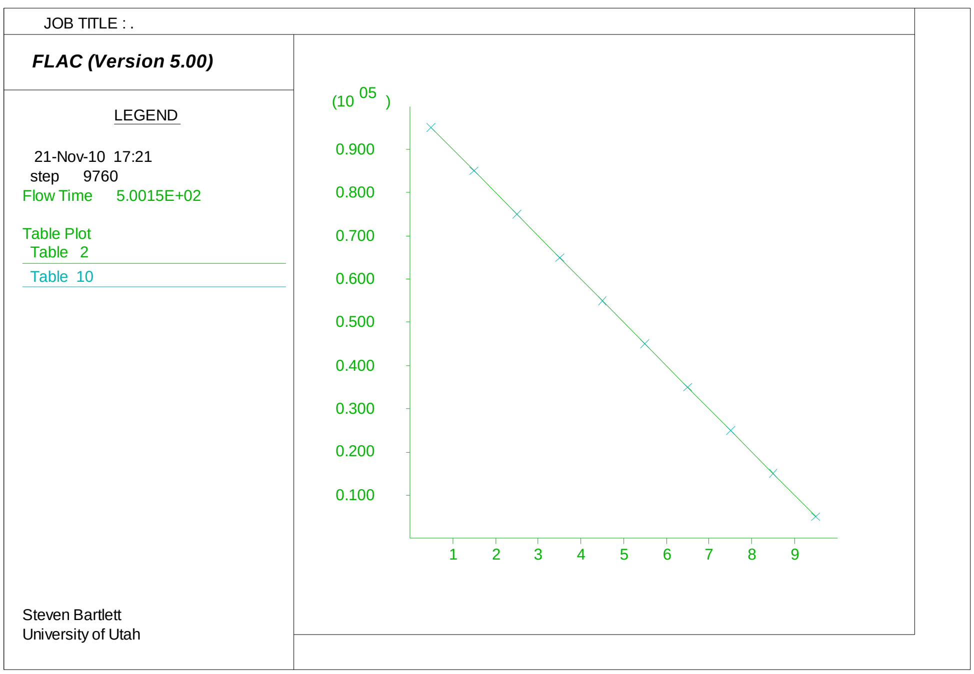
u_y = - (500*10*10^2)/(2*(1e^8+4*0.3e^8/3))

u_y = 1.7957e-3 m (compares with plot above)

Buoyancy and Seepage Forces (coupled analysis)

Thursday, March 11, 2010
11:43 AM

Buoyancy 1—We continue this example by raising the water table to the top of the model. We reset the displacements to zero, and assign the fluid properties. The pore pressure is fixed at zero at the top of the model, and the saturation is initialized to 1 throughout the grid and fixed at the top. The saturation is fixed at 1 at the top to ensure that all zones will stay fully saturated during the fluid-flow calculations. (Note that a fluid-flow calculation to steady state is faster if the state starts from an initial saturation 1 instead of a zero saturation.) Fluid-flow and mechanical modes are both on for this calculation stage, and a coupled calculation is performed to reach steady state. The saturated density is used for this calculation, as determined from Eq. (1.74). By integration of Eq. (1.73) for the saturated medium, we obtain:



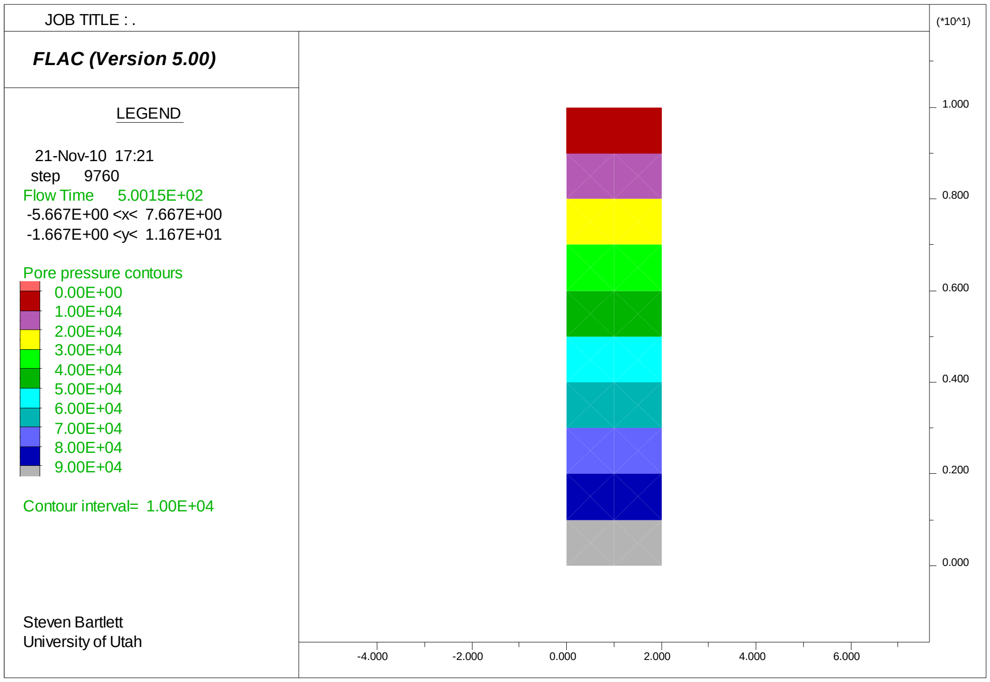
Verification

$$\sigma_{yy}^{(2)} = -\rho_s g(H - y)$$

500*10*(10 - 0) = 50,000 (see above)

Buoyancy and Seepage Forces (coupled analysis)

Thursday, March 11, 2010
11:43 AM



Pore pressure for saturated layer (Pa)

Vertical displacement (upward due to heave)

$$(\sigma_{yy}^{(2)} - \sigma_{yy}^{(1)}) + (p^{(2)} - p^{(1)}) = (K + 4G/3)\frac{\partial u}{\partial y}$$ (see Eq. 1.77)

$$-\Big[(\rho_s - \rho_w) - \rho_d\Big]g(H - y) = (K + 4G/3)\frac{\partial u}{\partial y}$$

$(\rho_s - \rho_w)$ is the *buoyant density*

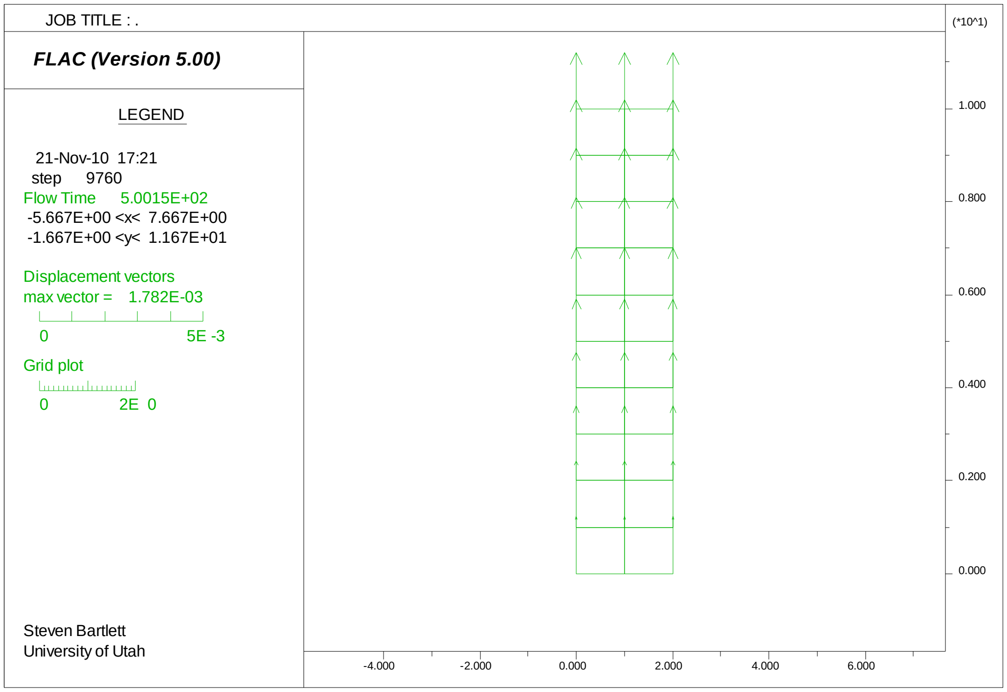
$$(1 - n)\rho_w g(H - y) = (K + 4G/3)\frac{\partial u}{\partial y}$$

after integration

$$u_y = \frac{(1 - n)\rho_w g}{(K + 4G/3)}\Big[H - \frac{y}{2}\Big]y$$

Buoyancy and Seepage Forces (coupled analysis)

Thursday, March 11, 2010
11:43 AM



Verification

$$u_y = \frac{(1-n)\rho_w g}{(K+4G/3)}\left[H - \frac{y}{2}\right]y$$

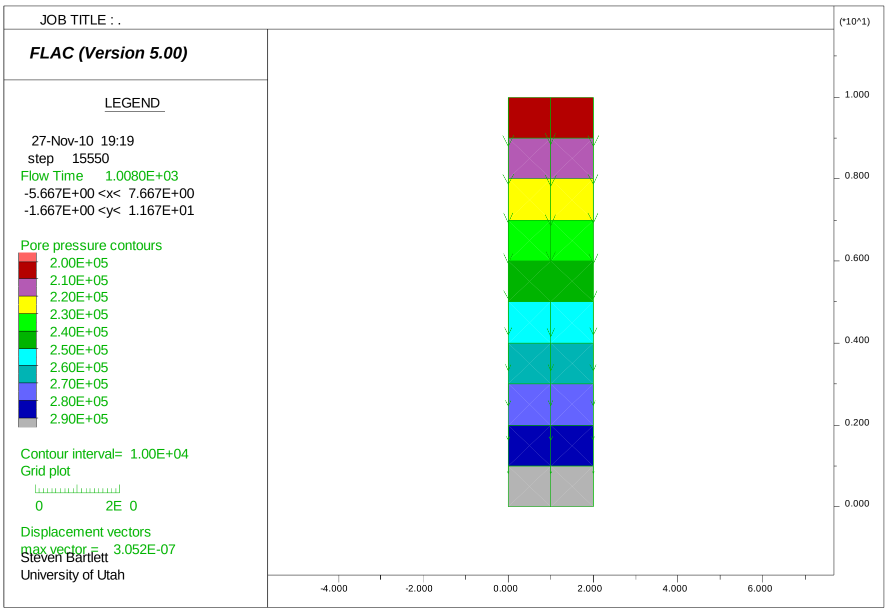
(((1-0.5)*1000*10)/(1e8+4*0.3e8*1/3))*(10-10/2)*10=0.0018

(see above)

Buoyancy and Seepage Forces (coupled analysis)

Thursday, March 11, 2010
11:43 AM

Buoyancy 2 - Additional Rise in Water Table—We continue from this stage and model the effect of an additional rise in the water level on the layer. This time the water table is raised to 20 m above the top of the model. The corresponding hydrostatic pressure is p = ρwgh where h is 20 m, and p = 0.2 MPa. We reset displacements to zero and apply a pressure of 0.2 MPa at the top of the model. A fluid pore pressure is applied (with APPLY pp), as is a mechanical pressure (with APPLY pressure), along the top boundary. We now perform the coupled calculation again for an additional 500 seconds of fluid-flow time. *No further movement of the model is calculated. This is because the absolute increase in σyy is balanced by the increase in pore pressure, and the Biot coefficient is set to 1.*



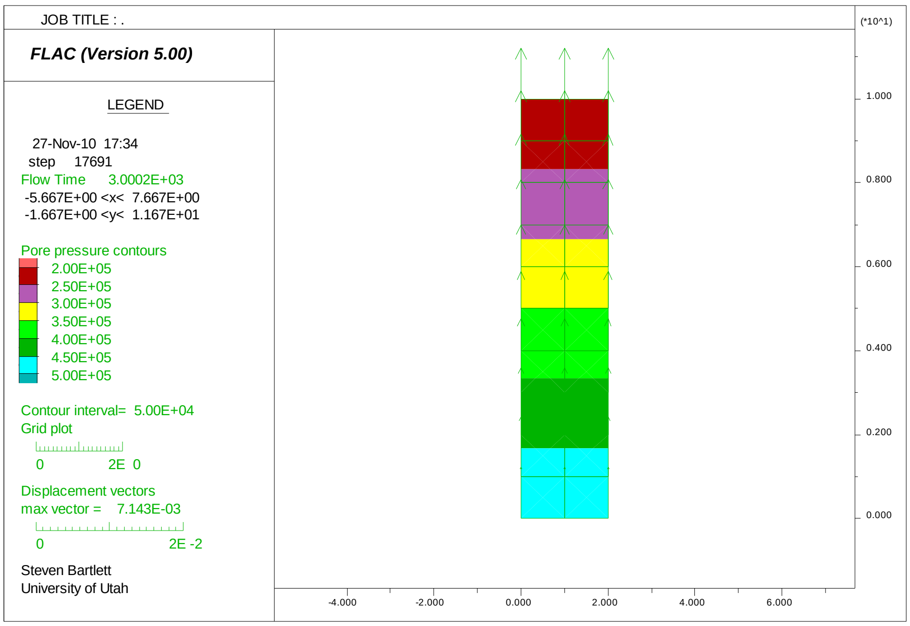
Verification

The maximum displacement for this case is very small 3e^{-7} m

Buoyancy and Seepage Forces (coupled analysis)

Thursday, March 11, 2010
11:43 AM

Seepage Force 1 (Upwards Flow) — We now study the scenario in which the base of the layer is in contact with a high-permeability over-pressured aquifer. The pressure in the aquifer is 0.5 MPa. We continue from the previous stage, reset displacements to zero, and apply a pore pressure of 0.5 MPa at the base (APPLY pp). The coupled mechanical-flow calculation is performed until steady state is reached.



Verification

$$u = \frac{p^{(4)} - p^{(3)}}{(K + 4G/3)} y \left[1 - \frac{y}{2H}\right] \qquad p^{(3)} = p_b^{(3)}\left[1 - \frac{y}{H}\right] + p_t \frac{y}{H}$$

For $y = H$, we obtain

$$u = \frac{p^{(4)} - p^{(3)}}{(K + 4G/3)} \frac{H}{2}$$

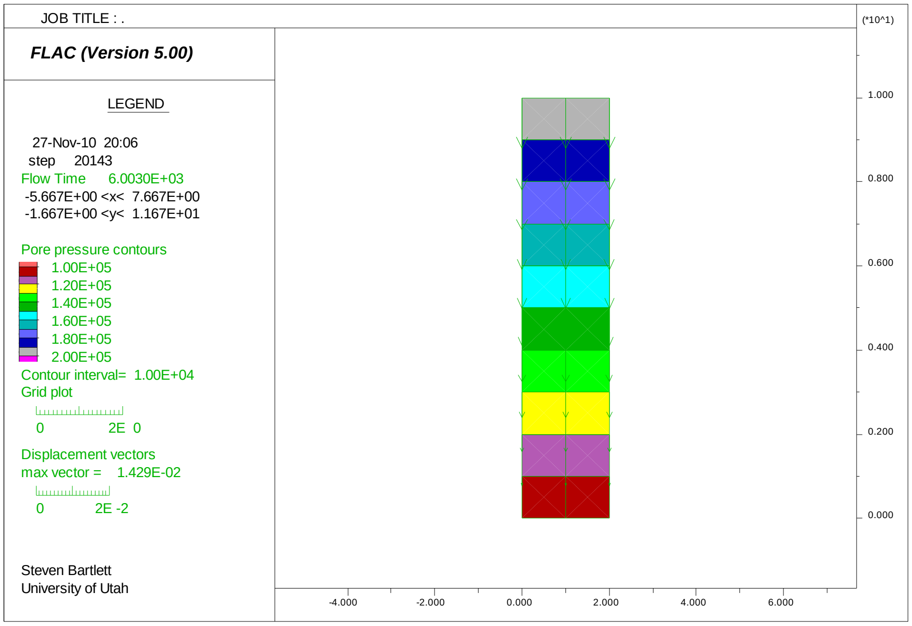
$p^{(3)}$ = $pb^{(3)}$ = $3e^5$ for H = 0 from previous example
$p^{(4)}$ = $pb^{(4)}$ = $5e^5$ for H = 0 from current example

$((5e^5-3e^5)/(1e^8+4*0.3e^8/3))*(10/2)=0.0071$ (upward)

Buoyancy and Seepage Forces (coupled analysis)

Thursday, March 11, 2010
11:43 AM

Seepage Force 2 (Downwards Flow)—The seepage force case is repeated for the scenario in which the base of the layer is in contact with a high-permeability under-pressured aquifer. This time a pressure value of p(5) = 0.1MPa is specified at the base. The displacements are reset and the coupled calculation is made. The layer settles in this case.



Verification

$$u = \frac{p^{(4)} - p^{(3)}}{(K + 4G/3)} \frac{H}{2}$$

$p^{(3)} = pb^{(3)} = 5e^5$ for H = 0 from previous example
$p^{(4)} = pb^{(4)} = 1e^5$ for H = 0 from current example

$((1e^5-5e^5)/(1e^8+4*0.3e^8/3))*(10/2)=-0.0143$ (downward)

More Reading

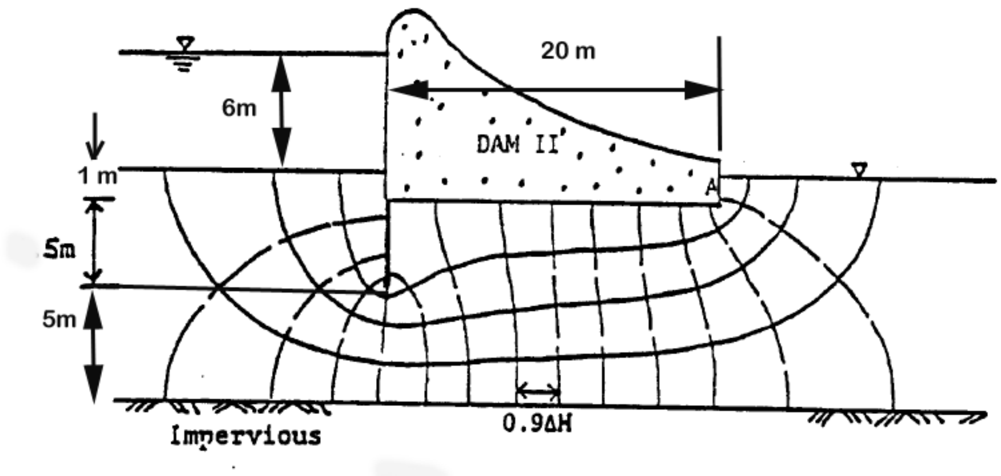
Thursday, March 11, 2010
11:43 AM

- FDM_Seepage.pdf
- FLAC Manual FLUID-MECHANICAL INTERACTION — SINGLE FLUID PHASE
  - 1.1 Introduction
  - 1.5 Calculation Modes and Commands for Fluid-Flow Analysis
  - 1.8.3 Fixed Pore Pressure (Used in Effective Stress Calculation)
  - 1.8.4 Flow Calculation to Establish a Pore Pressure Distribution
  - 1.8.5 No Flow — Mechanical Generation of Pore Pressure (e.g., pore pressures from loading a footing)
  - 1.8.6 Coupled Flow and Mechanical Calculations
  - 1.8.7 Uncoupled Approach for Coupled Analysis
  - 1.9.6 Fluid Barrier Provided by a Structure
- Steady State Flow in an Unconfined Aquifer (Fetter, p. 132 - 139).
- Applied Soil Mechanics with ABAQUS Applications, Ch. 9

Assignment 10

Thursday, March 11, 2010
11:43 AM

1. Use an FDM excel spreadsheet to develop a flow net similar to that shown below.



2. Use FLAC to develop a numerical model for a flow net similar to that shown above.

Blank

Thursday, March 11, 2010
11:43 AM

Slope Stability

Thursday, March 11, 2010
11:43 AM

Slope and excavation stability analyses are used in a wide variety of geotechnical engineering problems, including, but not limited to, the following:

• Determination of stable cut and fill slopes

• Assessment of overall stability of retaining walls, including global and compound stability (includes permanent systems and temporary shoring systems)

• Assessment of overall stability of shallow and deep foundations for structures located on slopes or over potentially unstable soils, including the determination of lateral forces applied to foundations and walls due to potentially unstable slopes

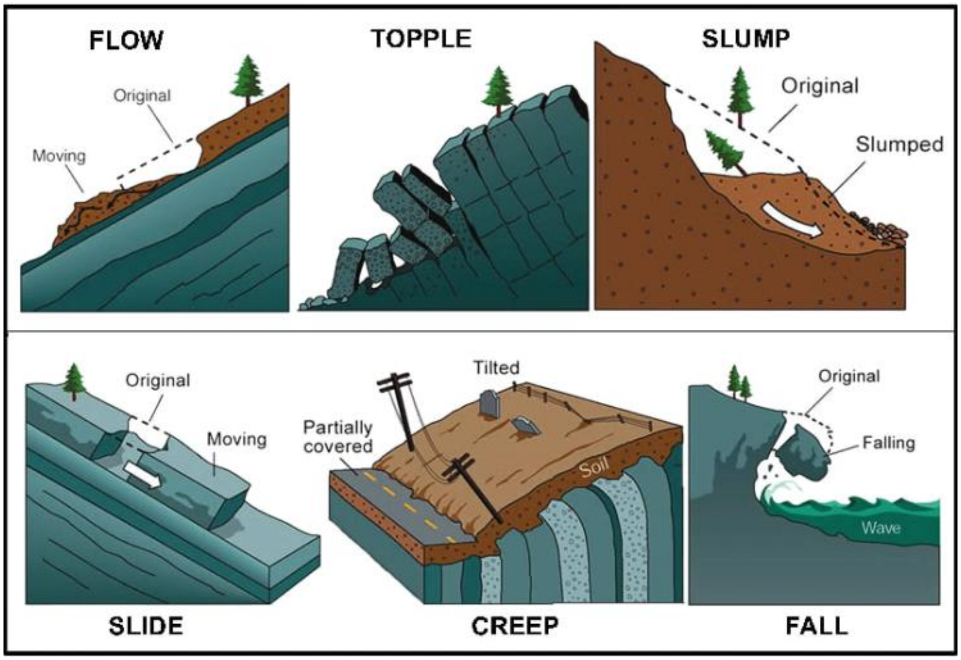
• Stability assessment of landslides (mechanisms of failure, and determination of design properties through back-analysis), and design of mitigation techniques to improve stability

• Evaluation of instability due to liquefaction

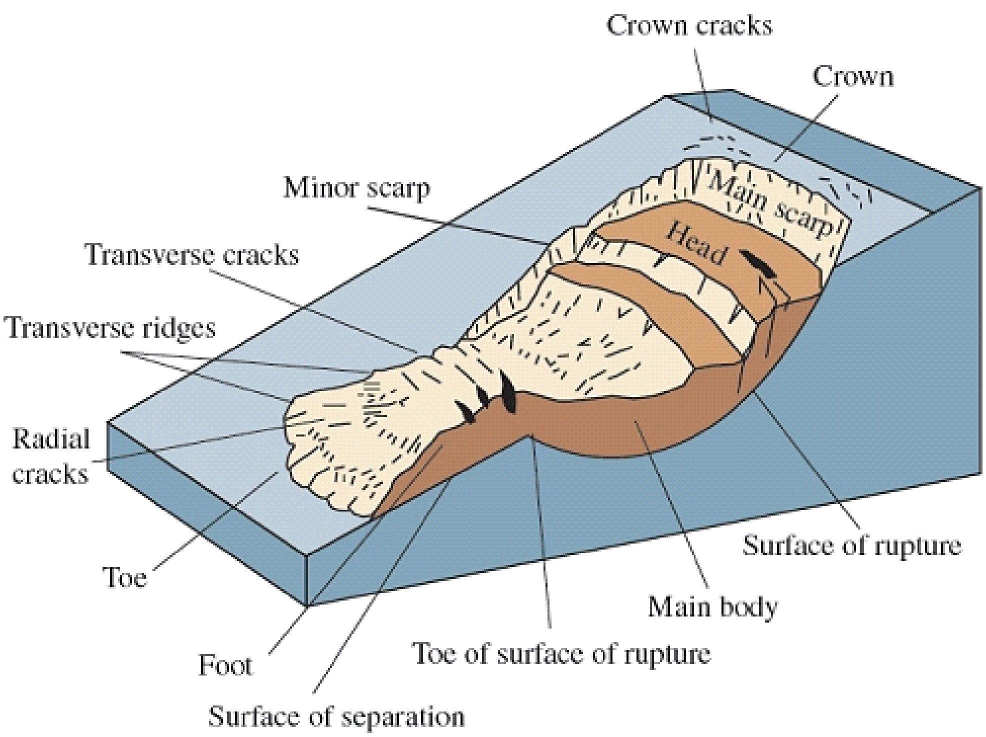
(From WASDOT manual of instruction)

Types of Mass Movement (i.e., Landsliding)

Thursday, March 11, 2010
11:43 AM



General Types of Mass Movement



Morphology of a Typical Soil Slump

Required Soil Parameters

Thursday, March 11, 2010
11:43 AM

Whether long-term or short-term stability is in view, and which will control the stability of the slope, will affect the selection of soil and rock shear strength parameters used as input in the analysis.

For short-term stability analysis, undrained shear strength parameters should be obtained. Short-term conditions apply for rapid loadings and for cases where construction is completed rapidly (e.g. rapid raise of embankments, cutting of slopes, etc.)

For long-term stability analysis, drained shear strength parameters should be obtained. Long-term conditions imply that the pore pressure due to the loading have dissipated and the equilibrium pore pressures have been reached.

For assessing the stability of landslides, residual shear strength parameters will be needed, since the soil has in such has typically deformed enough to reach a residual value. This implies that the slope or soil has previously failed along a failure plane and the there is potential for reactivation of the failure along this plane.

For highly overconsolidated clays, such as the Seattle clays (e.g., Lawton Formation), if the slope is relatively free to deform after the cut is made or is otherwise unloaded, residual shear strength parameters should be obtained and used for the stability analysis.

Factors of Safety for Slopes and Embankments

Thursday, March 11, 2010
11:43 AM

Factors of safety for slopes other than the slopes of dams should be selected consistent with the uncertainty involved in the parameters such as shear strength and pore water pressures that affect the calculated value of factor of safety and the consequences of failure. When the uncertainty and the consequences of failure are both small, it is acceptable to use small factors of safety, on the order of 1.3 or even smaller in some circumstances.

When the uncertainties or the consequences of failure increase, larger factors of safety are necessary. Large uncertainties coupled with large consequences of failure represent an unacceptable condition, no matter what the calculated value of the factor of safety.

The values of factor of safety listed in Table 3-1 provide guidance but are not prescribed for slopes other than the slopes of new embankment dams. Typical minimum acceptable values of factor of safety are about 1.3 for end of construction and multistage loading, 1.5 for normal long-term loading conditions, and 1.1 to 1.3 for rapid drawdown in cases where rapid drawdown represents an infrequent loading condition. In cases where rapid drawdown represents a frequent loading condition, as in pumped storage projects, the factor of safety should be higher. (from US Army Corp EM 1110-2-1902)

Reliability analysis techniques can be used to provide additional insight into appropriate factors of safety and the necessity for remediation.
(from US Army Corp EM 1110-2-1902)

Table 3-1
Minimum Required Factors of Safety: New Earth and Rock-Fill Dams

Analysis Condition[1]	Required Minimum Factor of Safety	Slope
End-of-Construction (including staged construction)[2]	1.3	Upstream and Downstream
Long-term (Steady seepage, maximum storage pool, spillway crest or top of gates)	1.5	Downstream
Maximum surcharge pool[3]	1.4	Downstream
Rapid drawdown	1.1-1.3[4,5]	Upstream

e. Loads on slopes. Loads imposed on slopes, such as those resulting from structures, vehicles, stored materials, etc. should be accounted for in stability analyses.

Note that for long-term stability of natural or cut slopes, a factor of safety of 1.5 is usually selected for cases where failure of the slope could affect safety or property.

Analysis Methods - Limit Equilibrium

Thursday, March 11, 2010
11:43 AM

Limit Equilibrium

- Most common LE method is the method of slices
- Methods/Researchers
  - Ordinary Method of Slices
  - Modified or Simplified Bishop
  - Taylor
  - Spencer
  - Spencer-Wright
  - Janbu
  - Fellenius (Swedish)
  - Morgenstern
  - Morgenstern-Price
  - US Army Corp of Engineers
  - Bell
  - Sharma
  - General Limit Equilibrium Methods (GLE)

Table C-7
Comparison of Features of Limit Equilibrium Methods

Feature	Ordinary Method of Slices	Simplified Bishop	Spencer	Modified Swedish	Wedge	Infinite Slope
Accuracy		X	X			X
Plane slip surfaces parallel to slope face						X
Circular slip surfaces	X	X	X	X		
Wedge failure mechanism			X	X	X	
Non-circular slip surfaces – any shape			X	X		
Suitable for hand calculations	X	X		X	X	X

Table C-2
Limitations of Limit-Equilibrium Methods

1. The factor of safety is assumed to be constant along the potential slip surface.
2. Load-deformation (stress-strain) characteristics are not explicitly accounted for.
3. The initial stress distribution within the slope is not explicitly accounted for.
4. Unreasonably large and or negative normal forces may be calculated along the base of slices under certain conditions (SectionC-l0.b and C-10.c).
5. Iterative, trial and error, solutions may not converge in certain cases (Section C-10d).

LE Analysis Methods - References

Thursday, March 11, 2010
11:43 AM

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Bishop, A. W., and Morgenstern, N. 1960. "Stability Coefficients for Earth Slopes," *Geotechnique*, Vol 10, No. 4, pp 129-150.

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Simplified Bishops Method

Thursday, March 11, 2010
11:43 AM

Simplified Bishop's Method (from US Army Corp EM 1110-2-1902)

a. Assumptions. The Simplified Bishop Method was developed by Bishop (1955). This procedure is based on the assumption that the interslice forces are horizontal, as shown in Figure C-11. A circular slip surface is also assumed in the Simplified Bishop Method. Forces are summed in the vertical direction. The resulting equilibrium equation is combined with the Mohr-Coulomb equation and the definition of the factor of safety to determine the forces on the base of the slice. Finally, moments are summed about the center of the circular slip surface to obtain the following expression for the factor of safety:

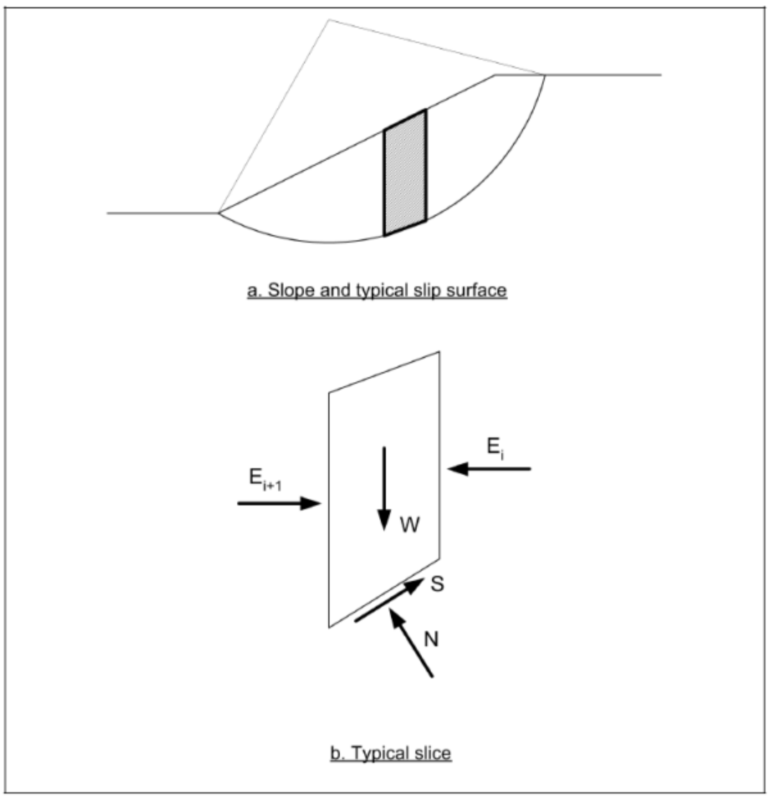


Figure C-11. Typical slice and forces for Simplified Bishop Method

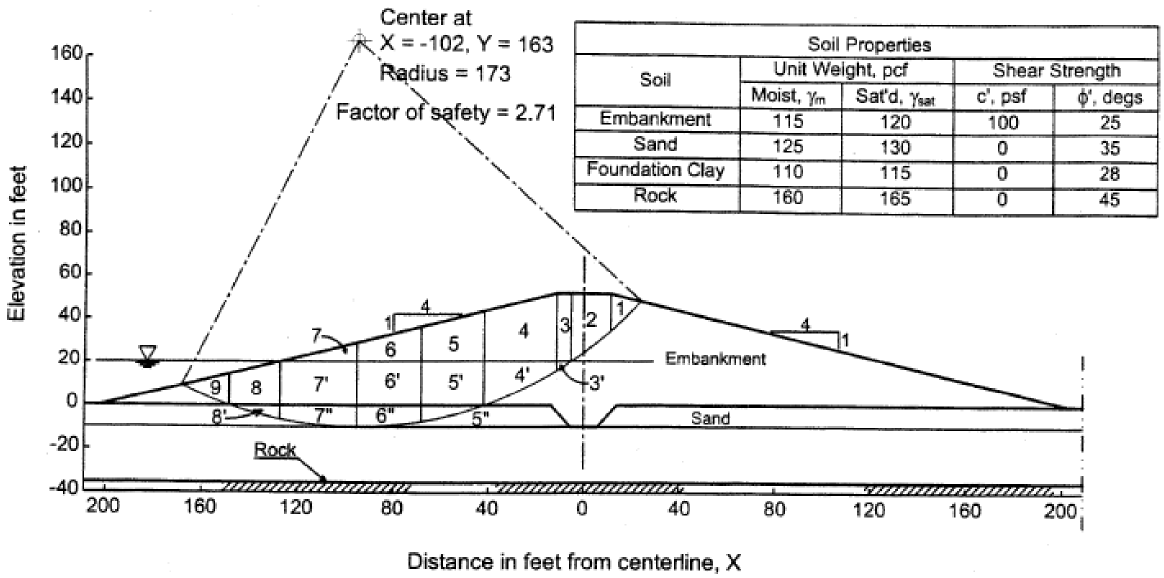
Table C-4
Unknowns and Equations for the Simplified Bishop Method

Unknowns	Number of Unknowns for n Slices
Factor of safety (F)	1
Normal forces on bottom of slices (N)	n
TOTAL NUMBER OF UNKNOWNS	n + 1
Equations	**Number of Equations for n Slices**
Equilibrium of forces in the vertical direction, $\Sigma F_y = 0$	n
Equilibrium of moments of the entire soil mass	1
TOTAL NUMBER OF EQUILIBRIUM EQUATIONS	n + 1

Simplified Bishops Method Example

Thursday, March 11, 2010
11:43 AM

(from US Army Corp EM 1110-2-1902)



Soil Properties				
Soil	Unit Weight, pcf		Shear Strength	
	Moist, γ_m	Sat'd, γ_{sat}	c', psf	ϕ', degs
Embankment	115	120	100	25
Sand	125	130	0	35
Foundation Clay	110	115	0	28
Rock	160	165	0	45

Slice Number	Horizontal Width (b), ft	Average Slice Height, ft	Slice Area, sq. ft	Total Unit Weight (γ), pcf	Partial Weights, kips	Total Weight (W), kips	Base Inclination (α), deg	W sin(α)	Ht. of Surface Water, ft	Avg. Surface Press., ksf	Base Inclination (α), deg	Surface Load (P), kips/ft	Horiz Moment Arm (d_h), ft	Vert Moment Arm (d_v), ft	Moment (MP), kip-ft	Piezometric Height, ft	Pore Water Pressure (u), ksf	Cohesion (c), ksf	Friction Angle (φ'), deg	c'b + (W + Pcosβ - ub)tanφ'	m_α (F=2.71)	[c'b+(W + Pcos β - ub)tan φ')] m_α
1	11	7	83	115	10	10	45	7	-	-	-	-	-	-	-	0	0.00	0.1	25	6	0.83	7
2	20	22	436	115	50	50	38	31	-	-	-	-	-	-	-	0	0.00	0.1	25	25	0.89	28
3 3'	10	30 3	311 34	115 120	36 4	40	32	21	-	-	-	-	-	-	-	3	0.20	0.1	25	19	0.94	20
4 4'	29	26 13	766 385	115 120	88 46	134	25	57	-	-	-	-	-	-	-	13	0.83	0.1	25	54	0.98	55
5 5' 5''	29	19 20 4	554 579 109	115 120 130	64 70 14	147	15	37	-	-	-	-	-	-	-	24	1.48	0	35	73	1.03	71
6 6' 6''	29	12 20 9	344 580 254	115 120 130	40 70 33	142	5	12	-	-	-	-	-	-	-	29	1.80	0	35	63	1.02	62
7 7' 7''	33	4 20 8	136 660 278	115 120 130	16 79 36	131	-6	-13	-	-	-	-	-	-	-	28	1.77	0	35	51	0.97	52
8 8'	25	17 3	421 85	120 130	51 11	62	-15	-16	3	0.19	14	5	45	146	399	23	1.46	0	35	21	0.90	23
9	20	7	139	120	17	17	-23	-7	9	0.55	14	11	68	152	1176	16	0.98	0.1	25	6	0.85	7
								129							1574							326

$$F = \frac{326}{129 - \frac{1574}{173}} = 2.71$$

Simplified Bishops Method - Misc.

Thursday, March 11, 2010
11:43 AM

Limitations. Horizontal equilibrium of forces is not satisfied by the Simplified Bishop Method. Because horizontal force equilibrium is not completely satisfied, the suitability of the Simplified Bishop Method for pseudo-static earthquake analyses where an additional horizontal force is applied is questionable. The method is also restricted to analyses with circular shear surfaces.

Recommendation for use. It has been shown by a number of investigators that the factors of safety calculated by the Simplified Bishop Method compare well with factors of safety calculated using more rigorous equilibrium methods, usually within 5 percent. Furthermore, the procedure is relatively simple compared to more rigorous solutions, computer solutions execute rapidly, and hand calculations are not very time-consuming. The method is widely used throughout the world, and thus, a strong record of experience with the method exists. The Simplified Bishop Method is an acceptable method of calculating factors of safety for circular slip surfaces. It is recommended that, where major structures are designed using the Simplified Bishop Method, the final design should be checked using Spencer's Method.

Verification procedures. When the Simplified Bishop Method is used for computer calculations, results can be verified by hand calculations using a calculator or a spreadsheet program, or using slope stability charts. An approximate check of calculations can also be performed using the Ordinary Method of Slices, although the OMS will usually give a lower value for the factor of safety, especially if ϕ is greater than zero and pore pressures are high.

Numerical Methods

Thursday, March 11, 2010
11:43 AM

51st Rankine Lecture
Geotechnical Stability Analysis
Professor Scott W Sloan
University of Newcastle,NSW, Australia

ABSTRACT

Historically, geotechnical stability analysis has been performed by a variety of approximate methods that are based on the notion of limit equilibrium. Although they appeal to engineering intuition, these techniques have a number of major disadvantages, not the least of which is the need to presuppose an appropriate failure mechanism in advance. This feature can lead to inaccurate predictions of the true failure load, especially for cases involving layered materials, complex loading, or three-dimensional deformation.

This lecture will describe recent advances in stability analysis which avoid these shortcomings. Attention will be focused on new methods which combine the limit theorems of classical plasticity with finite elements to give rigorous upper and lower bounds on the failure load. These methods, known as finite element limit analysis, do not require assumptions to be made about the mode of failure, and use only simple strength parameters that are familiar to geotechnical engineers. The bounding properties of the solutions are invaluable in practice, and enable accurate solutions to be obtained through the use of an exact error estimate and automatic adaptive meshing procedures. The methods are extremely general and can deal with layered soil profiles, anisotropic strength characteristics, fissured soils, discontinuities, complicated boundary conditions, and complex loading in both two and three dimensions. Following a brief outline of the new techniques, stability solutions for a number of practical problems will be given including foundations, anchors, slopes, excavations, and tunnels.

Numerical Methods

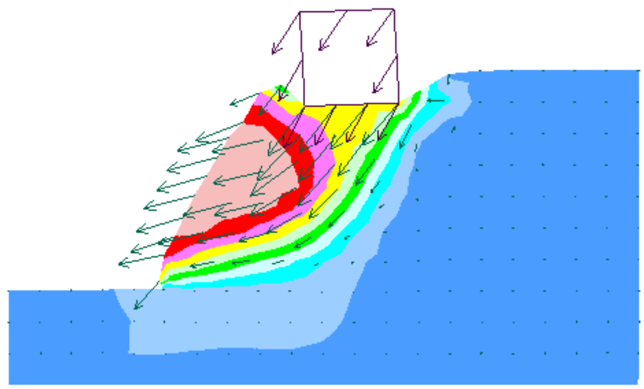
Thursday, March 11, 2010
11:43 AM

Numerical Modeling (FDM and FEM)

Numerical model such as FLAC offers these advantages over Limit Equilibrium methods:

- Any failure mode develops naturally; there is no need to specify a range of trial surfaces in advance.
- No artificial parameters (e.g., functions for inter-slice angles) need to be given as input.
- Multiple failure surfaces (or complex internal yielding) evolve naturally, if the conditions give rise to them.
- Structural interaction (e.g., rock bolt, soil nail or geogrid) is modeled realistically as fully coupled deforming elements, not simply as equivalent forces.
- Solution consists of mechanisms that are feasible kinematically.

Pasted from <http://www.itascacg.com/flacslope/overview.html>



There are a number of methods that could have been employed to determine the factor of safety using FLAC. The FLAC shear strength reduction (SSR) method of computing a factor of safety performs a series of computations to bracket the range of possible factors of safety. During SSR, the program lowers the strength (angle) of the soil and computes the maximum unbalanced force to determine if the slope is moving. If the force unbalance exceeds a certain value, the strength is increased and the original stresses returned to the initial value and the deformation analyses recomputed. This process continues until the force unbalance is representative of the initial movement of the slope and the angle for this condition is compared to the angle available for the soil to compute the factor of safety.

FLAC modeling - Total Stress vs. Effective Stress Analysis

Thursday, March 11, 2010
11:43 AM

Short-term behavior

If t_s is very short compared to the characteristic time, t_c, of the coupled diffusion process, the influence of fluid flow on the simulation results will probably be negligible, and an undrained simulation can be performed with *FLAC* (**CONFIG gw**, **SET flow off**). No real time will be involved in the numerical simulation (i.e., $t_s <<< t_c$), but the pore pressure will change due to volumetric straining if the fluid bulk modulus is given a realistic value. The footing load simulation in Example 1.4 is an example of this approach. Alternatively, a "dry" simulation may be conducted using the undrained bulk modulus for the material (see Section 1.9.4.2).

Long-term behavior

If $t_s >>> t_c$ and drained behavior prevails at $t = t_s$, then the pore pressure field can be uncoupled from the mechanical field. The steady-state pore pressure field can be determined using a flow-only simulation (**SET flow on**, **SET mech off**) (the diffusivity will not be representative), and the mechanical field can be determined next by cycling the model to equilibrium in mechanical mode with $M = 0$, or $K_w = 0$, (**SET mech on**, **SET flow off**). (Strictly speaking, this engineering approach is only valid for an elastic material because a plastic material is path-dependent.) This approach is used in Example 1.1.

Short-term analysis (Immediate or sudden changes in load)

- Effective stress analysis (drained parameters) (if pore pressure due to loading can be estimated, howeever often difficult to do this)
- Total stress analysis (undrained parameters) (if pore pressure are not estimated and not present in the model)

Long-term analysis (pore pressure from change in loading have dissipated)

- Effective stress analysis

In FLAC, the yield criterion for problems involving plasticity is expressed in terms of effective stresses. The strength parameters used for input in a fully coupled mechanical-fluid flow problem are drained properties. Also, whenever CONFIG gw is selected: a) the drained bulk modulus of the material should be used if the fluid bulk modulus is specified; and b) the dry mass density of the material should be specified when the fluid density is given. The apparent volumetric and strength properties of the medium will then evolve with time, because they depend on the pore pressure generated during loading and dissipated during drainage. The dependence of apparent properties on the rate of application of load and drainage is automatically reflected in a coupled calculation, even when constant input properties are specified.

FLAC modeling - Effective Stress Analysis

Thursday, March 11, 2010
11:43 AM

Initializing Stress and Pore Pressures for Horizontally Layered Systems

Initializing Equilibrium Stress Distribution in Groundwater Problems for Media with Voids

The *FISH* function "INIV.FIS" initializes stresses and pore pressures as a function of depth, taking into account the presence of voids in the model. It is assumed that line **j = jgp** is the free surface and that stresses depend on the vertical distance below this. Pore pressures are set to vary linearly from a free ground surface; a given free-water surface is not recognized. The grid must be configured for **gw**.

One input parameter must be set:

k0 ratio of effective horizontal stress to effective vertical stress

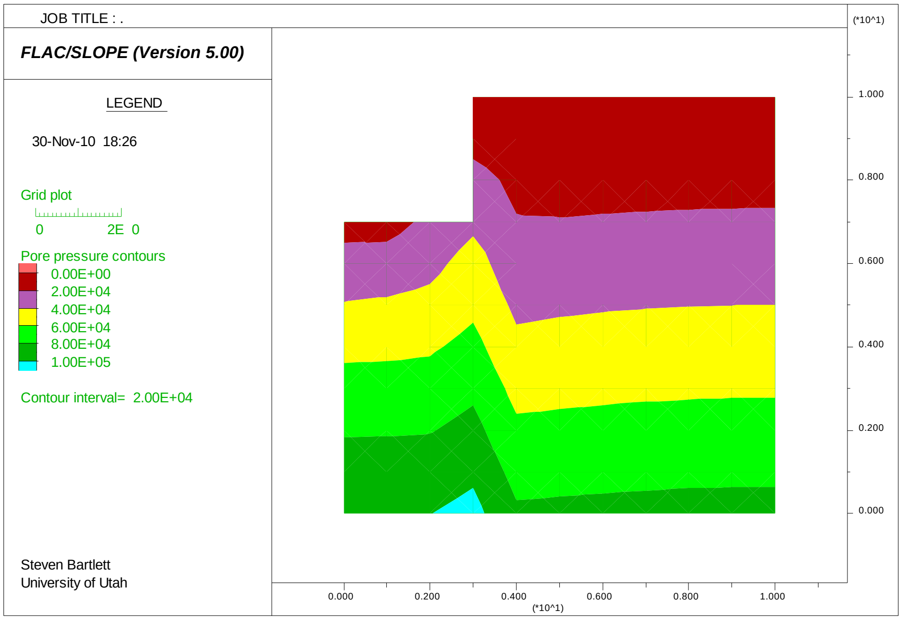
The following example illustrates the initialization of stresses in a model with a surface excavation. Note that there still is an unbalanced force as a result of the excavation; however, the stress state is close to equilibrium.

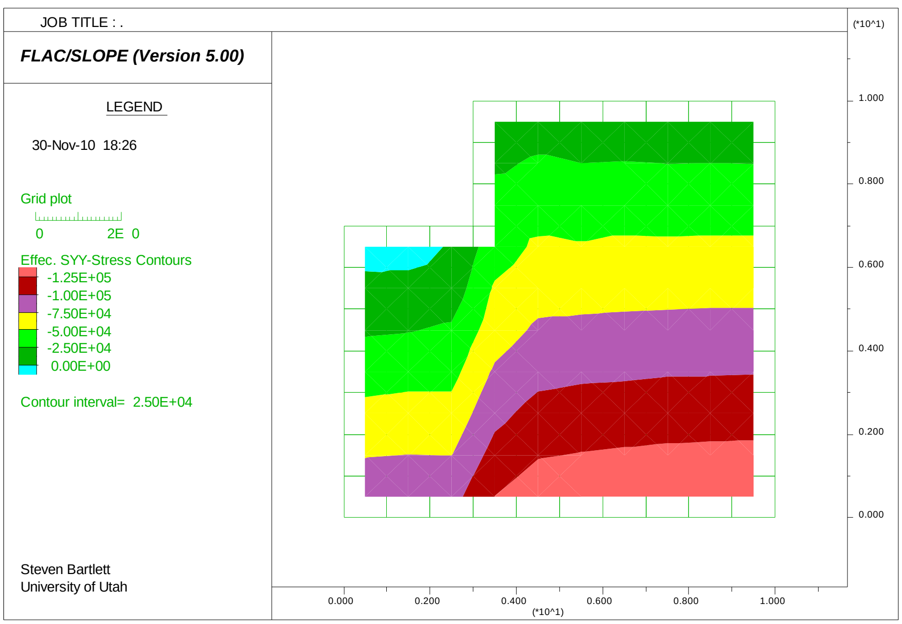
```
config gw ex=4
g 10 10
mo e
pro bulk 3e8 she 1e8 den 2000 por .4
pro den 2300 por .3 j 3 5
pro den 2500 por .2 j 1 2
pro perm 1e-9
mo null i=1,3 j=8,10
water bulk 2e9 den 1000
set g=9.8
call iniv.fis; this file must be present in project file folder
set k0=0.7
i_stress
fix x i 1
fix x i 11
fix y j 1
hist unbal
set flow off
step 1
save iniv.sav
solve
ret
FLAC
save initiate.sav 'last project state'
```

FLAC modeling - Effective Stress Analysis

Thursday, March 11, 2010
11:43 AM

Initializing Stress and Pore Pressures for Horizontally Layered Systems





FLAC modeling - Interface Considerations

Thursday, March 11, 2010
11:43 AM

If a model containing interfaces is configured for groundwater flow, effective stresses (for the purposes of slip conditions) will be initialized along the interfaces (i.e., the presence of pore pressures will be accounted for within the interface stresses when stresses are initialized in the grid). To correctly account for pore pressures, CONFIG gw must be specified. For example, the WATER table command (in non-CONFIG gw mode) will not include pore pressures along the interface, because pore pressures are not defined at gridpoints for interpolation to interface nodes for this mode. Note that flow takes place, without resistance, from one surface to the other surface of an interface, if they are in contact. Flow along an interface (e.g., fracture flow) is not computed, and the mechanical effect of changing fluid pressure in an interface is not modeled. If the interface pore pressure is greater than the total stress acting across the interface (i.e., if the effective stress tends to be tensile), then the effective stress is set to zero for the purpose of calculating slip conditions.

FLAC Modeling - Total Stress Analysis

Thursday, March 11, 2010
11:43 AM

For clayey material, the time required for dissipation of excess pore pressures developed by application of the load may be so long that undrained conditions may exist not only during, but for a long time after, loading. In this time scale, the influence of fluid flow on the system response may be neglected; if the fluid is stiff compared to the clay material (Kw >>> K + (4/3)G, where K and G are drained moduli), the generation of pore pressures under volumetric strain may strongly influence the soil behavior. In this situation, an undrained analysis can be applied. If the primary emphasis is on the determination of failure, and assuming a Mohr-Coulomb material with no dilation, two modeling approaches may be adopted in FLAC:

1. WET SIMULATION - The groundwater configuration (CONFIG gw) is adopted with a no-flow condition. Dry density, drained bulk and shear elastic moduli, and drained cohesion and friction angle are used in the input. In this approach effective stress strength properties are used because pore pressures are initialized in the model and the increase in pore pressure for the applied load is calculated by the model. Because pore pressures are present, then effective stress are appropriate and calculated for the undrained loading.)

2. DRY SIMULATION - The slope or foundation soil may be analyzed without taking the fluid explicitly into account. For this approach, total unit weight and undrained strength properties should be used throughout the model. For this simulation, the fluid is not explicitly taken into consideration, but its effect on the stresses is accounted for by assigning the medium an undrained bulk modulus. The groundwater configuration is not selected in this simulation, and a wet density ρ_u must be assigned to the saturated medium. In the following example, we make use of the material undrained shear strength; it is applicable if the following conditions hold:

1) plane-strain condition;
2) undrained condition;
3) undrained Poisson's ratio ν_u is equal to 0.5; and
4) Skempton's pore pressure coefficient B is equal to one.

FLAC Modeling - Dry Simulation

Thursday, March 11, 2010
11:43 AM

So that a dry simulation will yield comparable results to a wet simulation, the undrained cohesion must be calculated so that it is comparable to the drained friction angle, drained cohesion at the appropriate stress level (i.e., initial mean effective stress.) Because mean effective stress varies with depth, this means that the undrained cohesion must also vary with depth. This describes how this is done in FLAC and the limitations of a dry simulation.

The emphasis of the simulation is on failure detection. As mentioned before, the undrained shear strength of a material is a function of the mean effective stress $\sigma'_m = (\sigma'_1 + \sigma'_3)/2$ at failure, where σ'_1 and σ'_3 are minor and major principal stresses. For a plane-strain undrained problem, it can be shown that σ'_m remains constant and equal to its initial value $\sigma^{I'}_m$ up to (but maybe not after) incipient failure, provided $\nu_u = 0.5$ and $B = 1$ (see below). In this case, the undrained shear strength remains constant and, using the Mohr-Coulomb criterion and geometric considerations, its expression can be shown to be

$$C_u = -\sigma^{I'}_m \sin(\phi) + C \cos(\phi) \tag{1.135}$$

where ϕ is the friction angle and C is the cohesion. In total stress space, the material behavior will be seen as frictionless and cohesive. In the dry simulation, the material is assigned a zero friction and a cohesion value evaluated from the initial conditions using Eq. (1.135). The model is cycled to equilibrium after gradual application of the embankment load. Contours of vertical displacements, vertical displacement histories at four monitoring points, and plastic state at the end of the numerical simulation are presented in Figures 1.41 to 1.43. They can be compared to the results obtained previously.

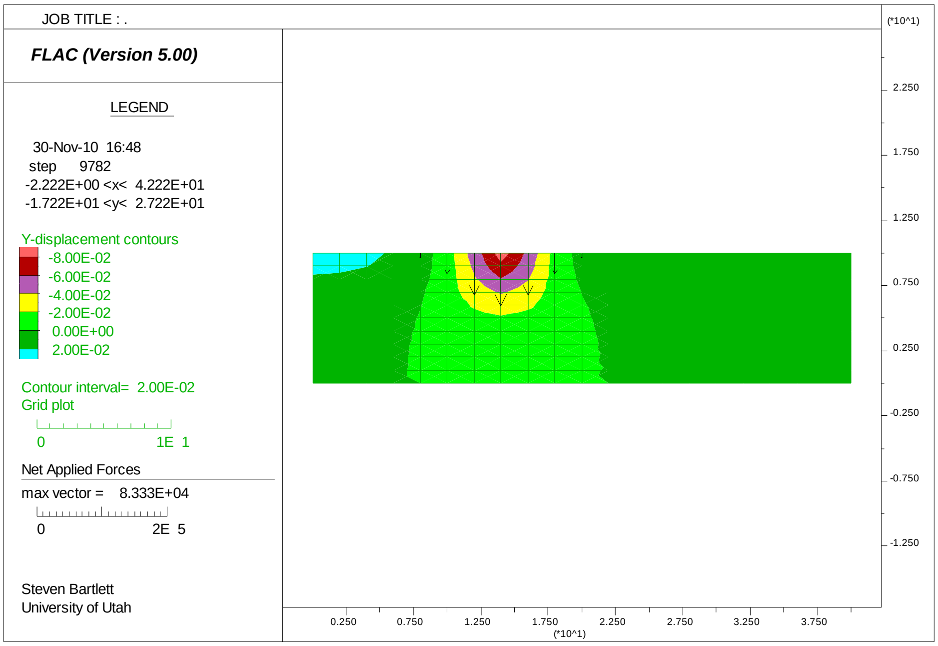
Undrained Analysis - Wet Simulation

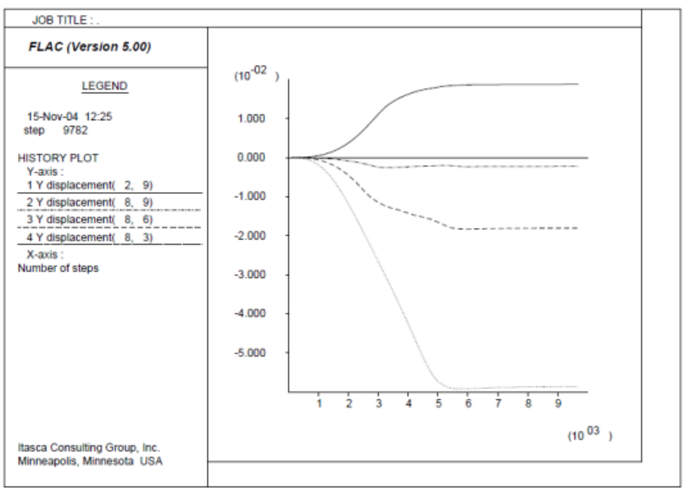
Thursday, March 11, 2010
11:43 AM

```
; WET SIMULATION ****
config gw ex 5
grid 20 10
model mohr
def prop_val
w_bu = 2e9 ; water bulk modulus
d_po = 0.5 ; porosity
d_bu = 2e6 ; drained bulk modulus
d_sh = 1e6 ; shear modulus
d_de = 1500 ; dry density
w_de = 1000 ; water density
b_mo = w_bu / d_po ; Biot modulus, M
d_fr = 25.0 ; friction
d_co = 5e3 ; cohesion
end
prop_val
;
ini x mul 2
prop dens=d_de sh=d_sh bu=d_bu; drained properties
prop poros=d_po fric=d_fr coh=d_co tens 1e20
water dens=w_de bulk=w_bu tens=1e30
set grav=10
; --- boundary conditions ---
fix x i=1
fix x i=21
fix x y j=1
; --- initial conditions ---
ini syy -2e5 var 0 2e5
ini sxx -1.5e5 var 0 1.5e5
ini szz -1.5e5 var 0 1.5e5
ini pp 1e5 var 0 -1e5
set flow=off
;
; --- surcharge from embankment ---
def ramp
  ramp = min(1.0,float(step)/4000.0)
end
apply syy=0 var -5e4 0 his ramp i=5,8 j=11
apply syy=-5e4 var 5e4 0 his ramp i=8,11 j=11
;
; --- histories ---
his nstep 100
his ydisp i=2 j=9
his ydisp i=8 j=9
his ydisp i=8 j=6
his ydisp i=8 j=3
; --- run ---
solve
save wet.sav
```

Undrained Analysis - Wet Simulation - Displacements

Thursday, March 11, 2010
11:43 AM





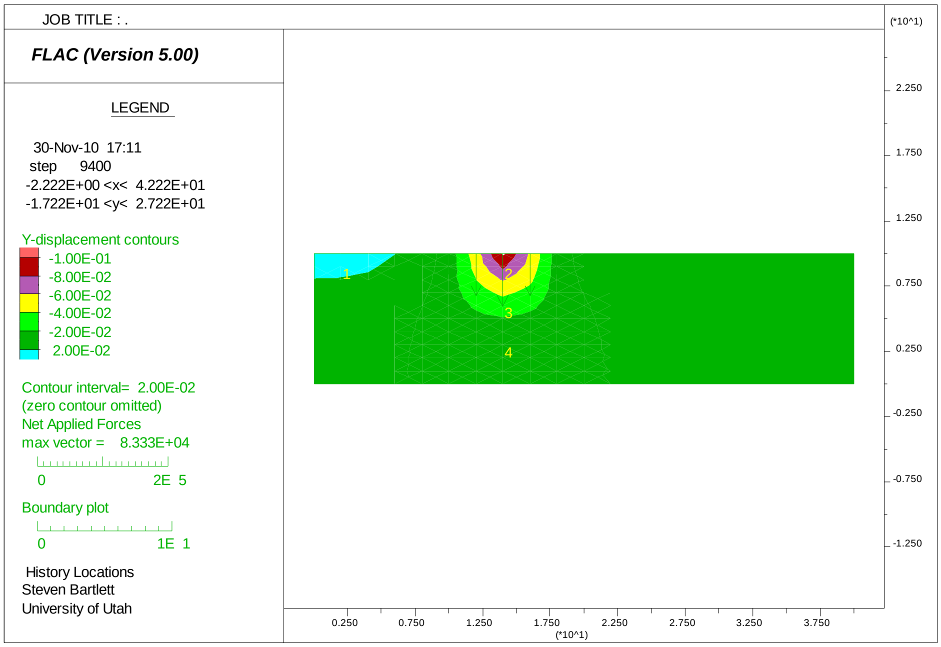
Undrained Analysis - Dry Simulation

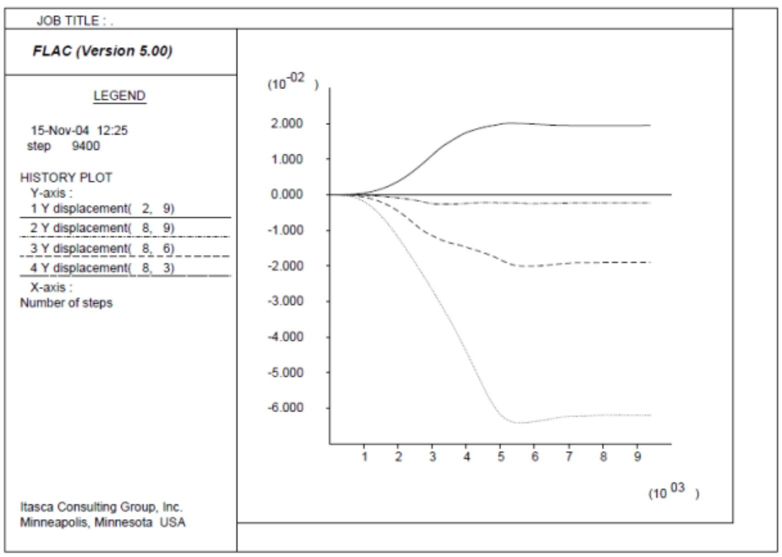
Thursday, March 11, 2010
11:43 AM

```
; DRY SIMULATION ****
; this simulation uses undrained parameters and
;undrained shear strength and undrained bulk
;modulus
config ex 5
grid 20 10
model mohr
def prop_val
  w_bu = 2e9 ; water bulk modulus
  d_po = 0.5 ; porosity
  d_bu = 2e6 ; drained bulk modulus
  d_sh = 1e6 ; shear modulus
  d_de = 1500 ; dry density
  w_de = 1000 ; water density
  b_mo = w_bu / d_po ; Biot modulus, M
  u_bu = d_bu + b_mo ; undrained bulk modulus
  u_de = d_de + d_po * w_de ; wet density
  d_fr = 25.0 ; friction
  d_co = 5e3 ; cohesion
  skempton = b_mo / u_bu ; Skempton coefficient
  nu_u = (3.*u_bu-2.*d_sh)/(6.*u_bu+2.*d_sh)
  ; undrained poisson's ratio
end
prop_val
ini x mul 2
; --- assign wet density
;  and undrained bulk modulus ---
prop dens=u_de sh=d_sh bu=u_bu
; --- first assign 'dry' friction and cohesion
prop fric=d_fr coh=d_co tens 1e20
; --- setting ---
set grav=10
; --- boundary conditions ---
fix x i=1
fix x i=21
fix x y j=1
; --- initial conditions ---
ini ex_1 1e5 var 0 -1e5 ; <--- pore pressure
; pore pressure initialized and not calculated
ini syy -2e5 var 0 2e5
ini sxx -1.5e5 var 0 1.5e5
ini szz -1.5e5 var 0 1.5e5
; --- assign undrained c and no friction ---
; (only for plane strain, Skempton=1,
; undrained Poisson's ratio = 0.5)
def ini_u_co
 loop ii (1,izones)
  loop jj (1,jzones)
  if model(ii,jj) = 3 then
   c_fr = friction(ii,jj)*degrad
   ; mean effective pressure in plane
   c_p = ex_1(ii,jj)+ex_1(ii+1,jj)+ex_1(ii,jj+1)
   c_p = (c_p + ex_1(ii+1,jj+1))*0.25
   emp = -(sxx(ii,jj)+syy(ii,jj))*0.5  - c_p
   u_co = emp * sin(c_fr) + cohesion(ii,jj) * cos(c_fr)
   if u_co < 0.0 then
    iii=out(' warning: invalid undrained cohesion')
    u_co = 0.0
  end_if
  command
  prop coh=u_co fric=0.0 tension=1e20 i=ii,ii j=jj,jj
  end_command
  end_if
 end_loop
 end_loop
end
ini_u_co
; --- surcharge from embankment ---
def ramp
  ramp = min(1.0,float(step)/4000.0)
end
apply syy=0 var -5e4 0 his ramp i=5,8 j=11
apply syy=-5e4 var 5e4 0 his ramp i=8,11 j=11
his nstep 100
his ydisp i=2 j=9
his ydisp i=8 j=9
his ydisp i=8 j=6
his ydisp i=8 j=3
solve
save dry.sav 'last project state'
```

Undrained Analysis - Dry Simulation - Displacements

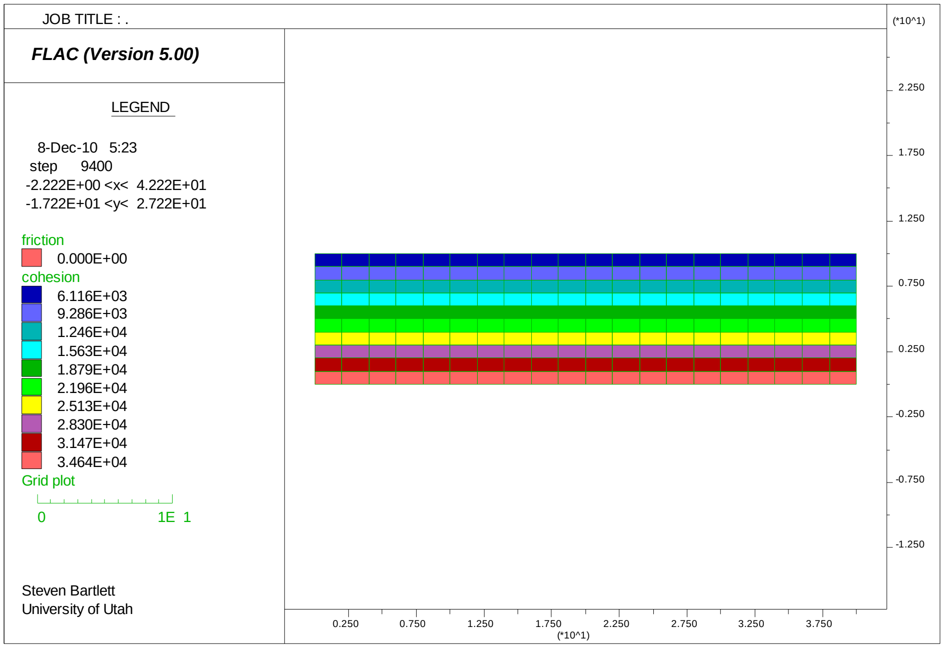
Thursday, March 11, 2010
11:43 AM





Undrained Analysis - Dry Simulation - Displacements

Thursday, March 11, 2010
11:43 AM



Note how the undrained cohesion varies with depth to account for changes in the mean effective stress.

$$C_u = -\sigma_m^{I\,\prime} \sin(\phi) + C \cos(\phi)$$

initial value $\sigma_m^{I\,\prime}$

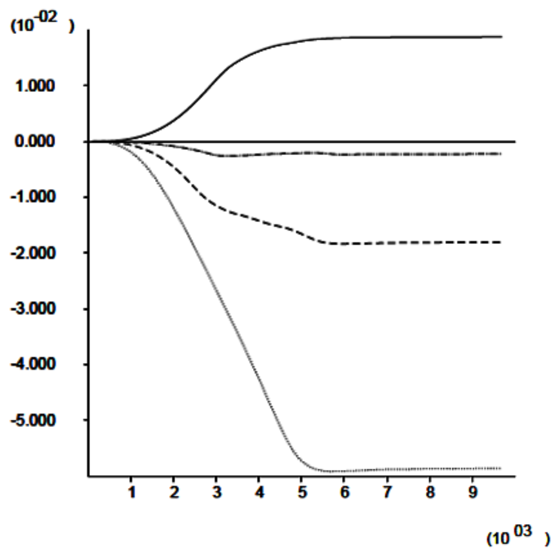
$$\sigma_m' = (\sigma_1' + \sigma_3')/2$$

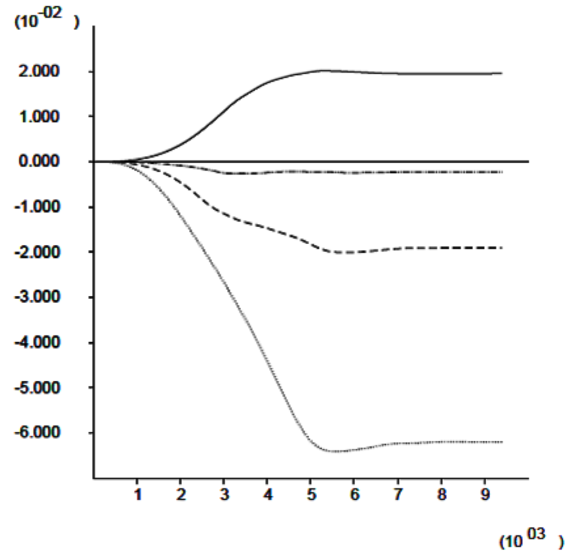
ϕ is the friction angle and C is the cohesion. (drained)

In total stress space, the material behavior will be seen as frictionless and cohesive. In the dry simulation, the material is assigned a zero friction and a cohesion value calculated from the initial conditions using the above relation.

Comparison of Displacements (Dry vs. Wet Simulation)

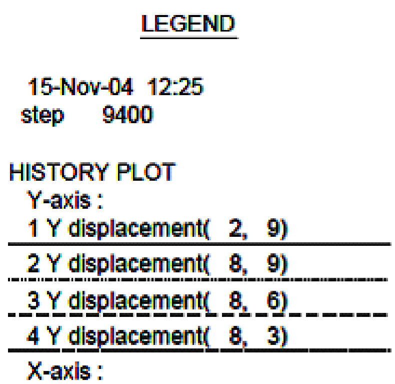
Thursday, March 11, 2010
11:43 AM





Wet Simulation

Dry Simulation



Slope Stability Example - No Groundwater

Thursday, March 11, 2010
11:43 AM

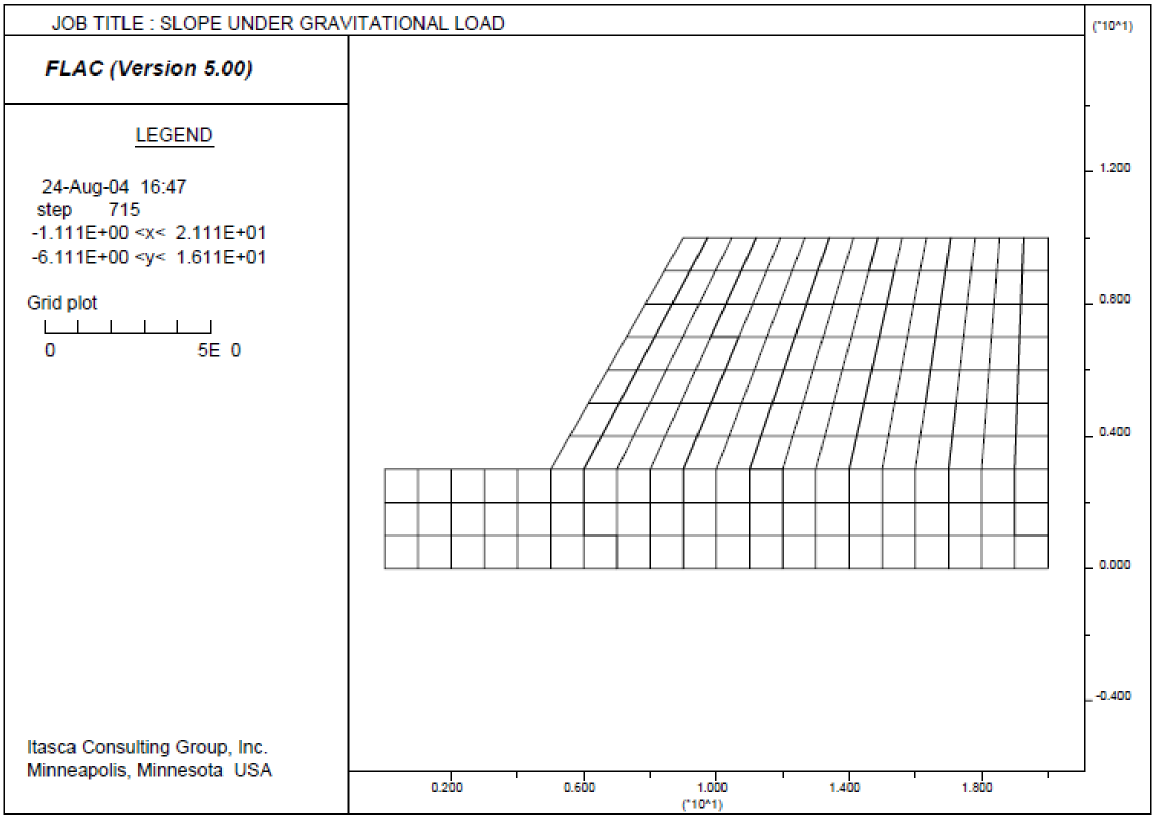


Figure 1.1 Grid plot of initial slope

A Mohr-Coulomb constitutive model is assigned to all zones (assumed because no range is given) with the following properties:

density	1500 kg/m^3
shear modulus	0.3×10^8 Pa
bulk modulus	10^8 Pa
friction angle	20°
cohesion	10^{10} Pa
tensile strength	10^{10} Pa

Note that a high cohesion and tensile strength are assigned to prevent slope failure during the initialization of gravitational stresses in the model (see below).

Slope Stability - No Groundwater (cont.)

Thursday, March 11, 2010
11:43 AM

Generating the slope

The first **GENERATE** command defines the base of the slope, and the second **GENERATE** command creates the slope. Note that the zones are aligned with the angle of the slope so that the zones along the slope face are all quadrilateral-shaped. This is recommended because all zones are then composed of two overlaid sets of triangular elements. These zones are well-suited for plasticity analysis (see Section 1.3.3.2 in **Theory and Background**). It is also possible to create a slope using the **GENERATE line** command. However, with this command, single triangular zones will be created along the slope face; these zones are not as accurate for plasticity analysis.

The area directly to the left of the slope face is excavated by declaring the appropriate zones as null. This is done by creating a "region" (i.e., the grid is divided into two regions separated by a boundary) that is defined by "marking" selected gridpoints as boundaries between regions. The following commands mark the boundary of the excavated region and then null the zones within that region:

```
mark i = 1,6   j = 4
mark i = 6     j = 4,11
model null region 1,10
```

The marked boundaries can be verified by issuing the **PRINT mark** command. The **MODEL null** command will delete zones in the region containing zone (1,10). Figure 1.1 shows the resulting *FLAC* grid.

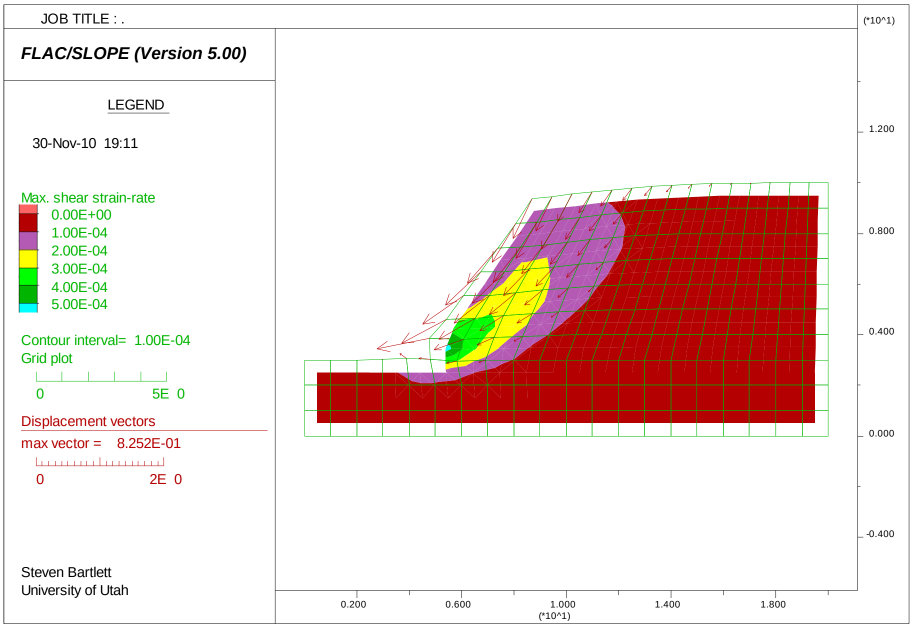
Slope Stability - No Groundwater (cont.)

Thursday, March 11, 2010
11:43 AM

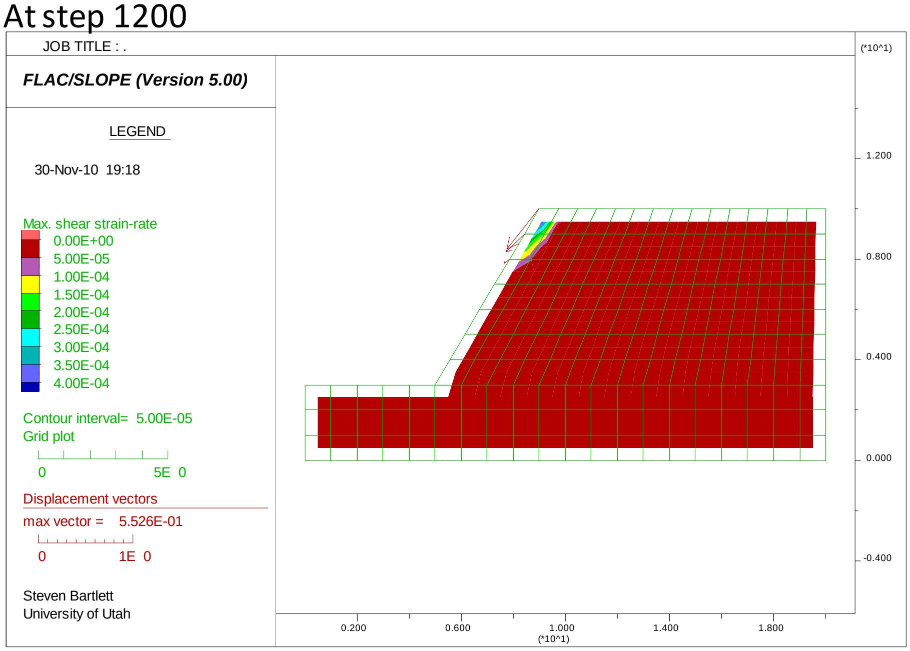
```
config ats
grid 20,10
;Mohr-Coulomb model
m m
; soil properties --- note large cohesion to force initial elastic
; behavior for determining initial stress state. This will prevent
; slope failure when initializing the gravity stresses
prop s=.3e8 b=1e8 d=1500 fri=20 coh=1e10 ten=1e10
; warp grid to form a slope :
gen 0,0 0,3 20,3 20,0 j 1,4
gen same 9,10 20,10 same i 6 21 j 4 11
mark i=1,6 j=4
mark i=6 j=4,11
model null region 1,10
; displacement boundary conditions
fix x i=1
fix x i=21
fix x y j=1
; apply gravity
set grav=9.81
; displacement history of slope
his ydis i=10 j=10
; solve for initial gravity stresses
solve
;
; reset displacement components to zero
ini xdis=0 ydis=0
; set cohesion to 0
; this is done to explore the failure mechanism in the cohesionless slope
prop coh=0
; use large strain logic
set large
step 1200; comment this line out to calculate factor of safety of undeformed slope
solve fos
save dry_slope.sav 'last project state'
```

Slope Stability - No Groundwater (cont.)

Thursday, March 11, 2010
11:43 AM



At step 1200

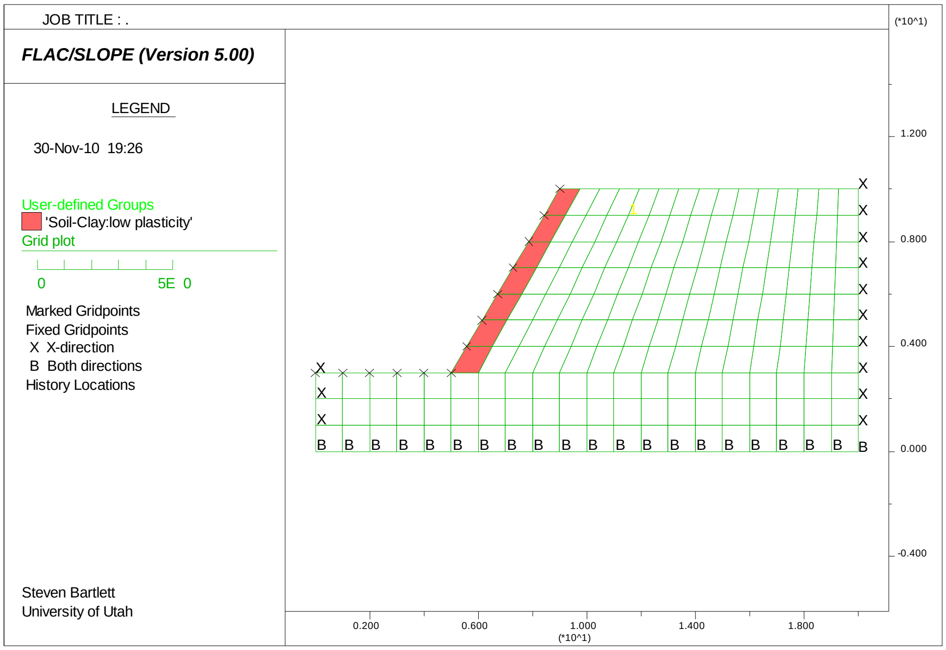


Factor of safety = 0.27 (However, this is surficial slip is not of particular interest. This slip surface will be eliminated, see next page.)

Slope Stability - No Groundwater (cont.)

Thursday, March 11, 2010
11:43 AM

Note that the surficial failure at the top of the slope can be prevented by slightly increasing the cohesive strength of the soil at the slope face. This often done to explore deeper failure surfaces in the soil mass.

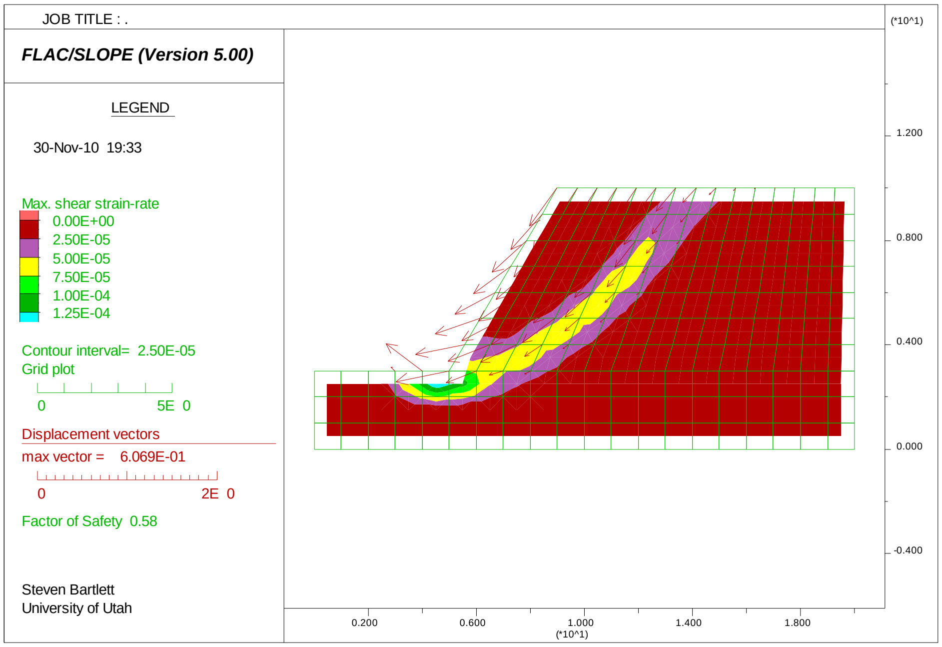


The last part of the FLAC code has been modified to look like this:

```
; set cohesion to 0
prop coh=0
group 'Soil-Clay:low plasticity' i 6 j 4 10
model mohr group 'Soil-Clay:low plasticity'
prop density=1900.0  bulk=1.33E6  shear=8E5  cohesion=100e3  friction=30.0  dilation=0.0  tension=0.0
group 'Soil-Clay:low plasticity'
; use large strain logic
set large
;step 1200
solve fos
```

Slope Stability - No Groundwater (cont.)

Thursday, March 11, 2010
11:43 AM



Factor of safety = 0.58

(This is the true factor of safety of the slope for a rotation, slump failure.)

Slope Stability - Groundwater - Cohesionless Soil

Thursday, March 11, 2010
11:43 AM

The groundwater flow option in FLAC can be used to find the phreatic surface and establish the pore pressure distribution before the mechanical response is investigated. The model is run in groundwater flow mode by using the CONFIG gw command.

We turn off the mechanical calculation (SET mech off) in order to establish the initial pore pressure distribution. We apply pore pressure boundary conditions to raise the water level to 5 m at the left boundary, and 9 m at the right. The slope is initially dry (INI sat 0). We also set the bulk modulus of the water to a low value (1.0×10^4) because our objective is to reach the steady-flow state as quickly as possible. The groundwater time scale is wrong in this case, but we are not interested in the transient time response. The steady-flow state is determined by using the SOLVE ratio command. When the groundwater flow ratio falls below the set value of 0.01, steady-state flow is achieved.

Mechanical equilibrium is then established including the pore pressure by turning flow off and mechanical on. These commands turn off the flow calculation, turn on the mechanical calculation, apply the weight of the water to the slope surface, and set the bulk modulus of the water to zero. This last command also prevents pore pressures from generating as a result of mechanical deformation.

Slope Stability - Groundwater - Cohesionless Soil

Thursday, March 11, 2010
11:43 AM

Groundwater added to the model

```
config gw ats ex 1
grid 20,10
;Mohr-Coulomb model
m m
; soil properties --- note large cohesion to force initial elastic
; behavior for determining initial stress state. This will prevent
; slope failure when initializing the gravity stresses
prop s=.3e8 b=1e8 d=1500 fri=20 coh=1e10 ten=1e10
; warp grid to form a slope :
gen 0,0 0,3 20,3 20,0 j 1,4
gen same 9,10 20,10 same i 6 21 j 4 11
mark i=1,6 j=4
mark i=6 j=4,11
model null region 1,10
prop perm 1e-10 por .3
water den 1000 bulk 1e4
; note bulk modulus of water assigned low value to speed up flow calc.
; displacement boundary conditions
fix x i=1
fix x i=21
fix x y j=1
; pore pressure boundary conditions
apply pp 9e4 var 0 -9e4 i 21 j 1 10
apply pp 5e4 var 0 -3e4 i 1 j 1 4
ini pp 2e4 var 0 -2e4 mark i 1 6 j 4 6
fix pp mark
; apply gravity
set grav=9.81
;call qratio.fis
;hist gwtime
;hist qratio
;hist inflow
;hist outflow
; these lines have been commented out because do not want to inspect flow calculations
set mech off flow on
solve
;
set flow off mech on
app press 2e4 var 0 -2e4 from 1 4 to 6 6
water bulk 0.0
; displacement history of slope
hist reset
his ydis i=10 j=10
solve
```

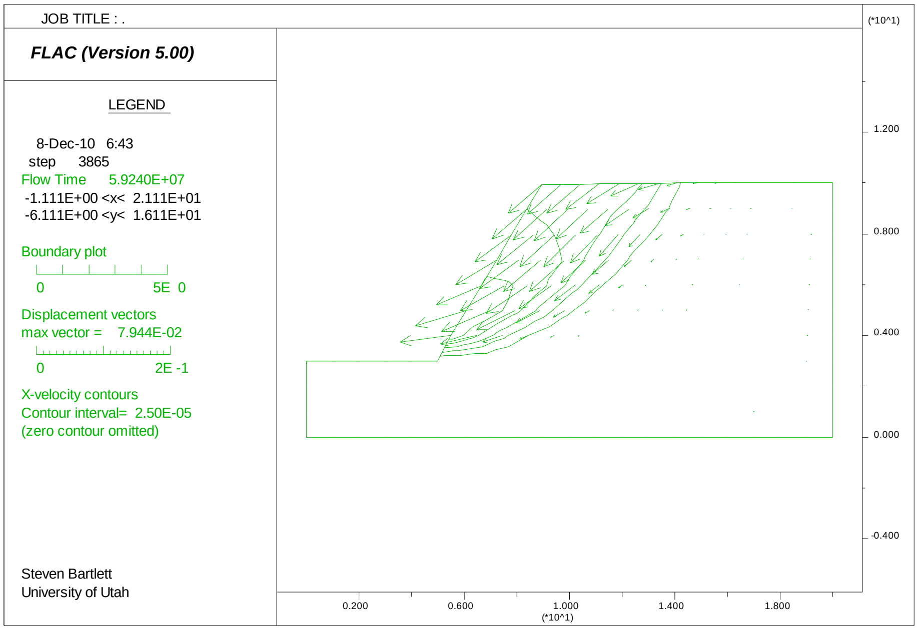
Slope Stability - Groundwater - Cohesionless Soil

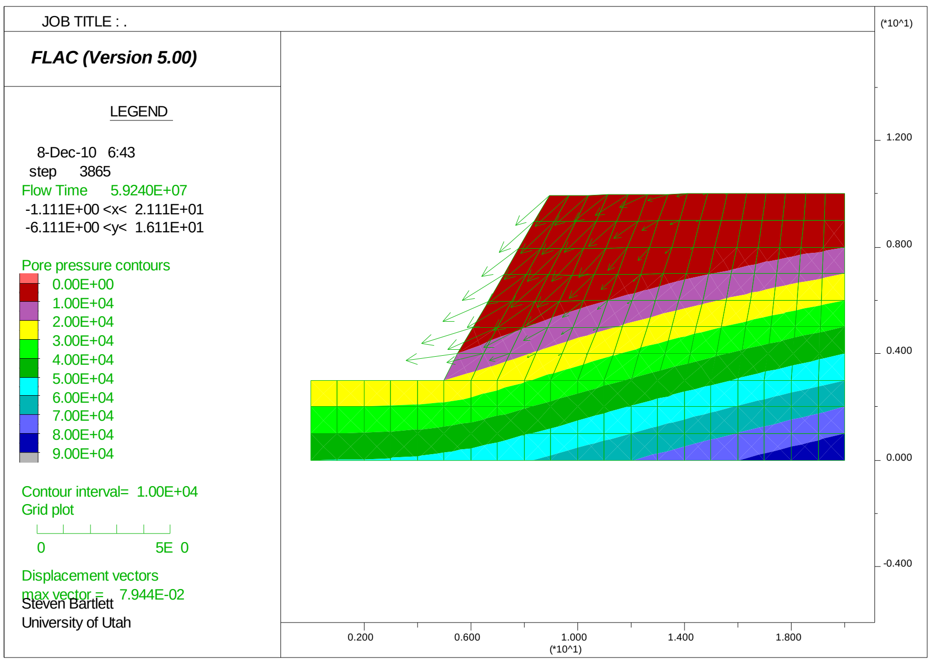
Thursday, March 11, 2010
11:43 AM

```
ini xdis 0.0 ydis 0.0
prop coh 1e4 ten 0.0
set large
step 1000
sclin 1 19 0 19 10
Save wet_slope.sav
;*** plot commands ****
;plot name: grid
plot hold grid
;plot name: Displacement vectors
plot hold bound displacement xvel zero
;plot name: Water Table
plot hold density fill inv grid water apply lmagenta
;plot name: Pore pressure distribution
plot hold bound velocity pp
;plot name: Steady-state flow
plot hold bound flow saturation alias 'phreatic surface' min 0.0 max 0.5 &
int 0.5 lmagenta
```

Slope Stability - Groundwater - Cohesionless Soil

Thursday, March 11, 2010
11:43 AM





Displacement vectors and X-velocity contours

Pore pressure distribution and displacement vectors

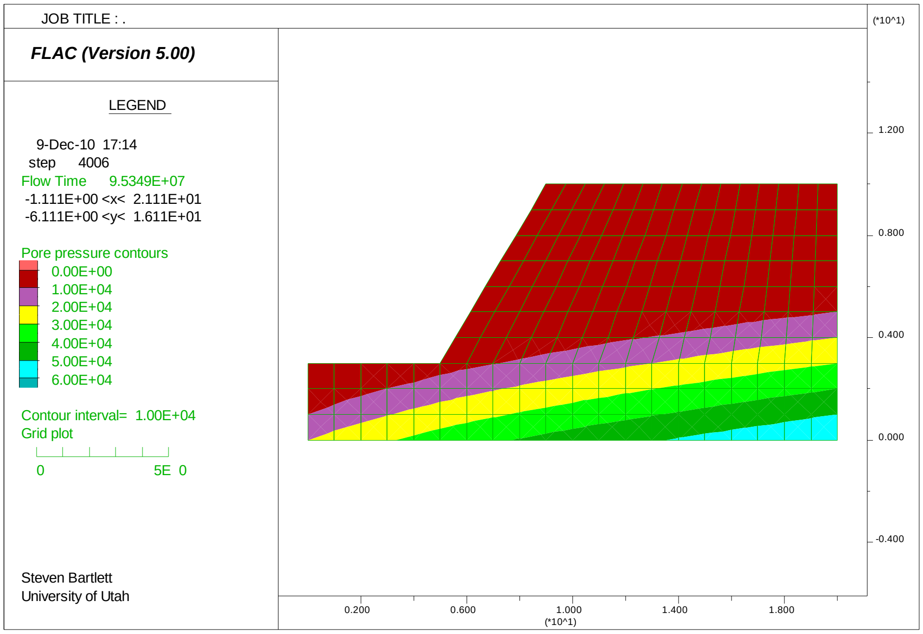
Slope Stability - Groundwater - Cohesionless Soil - FS Calc

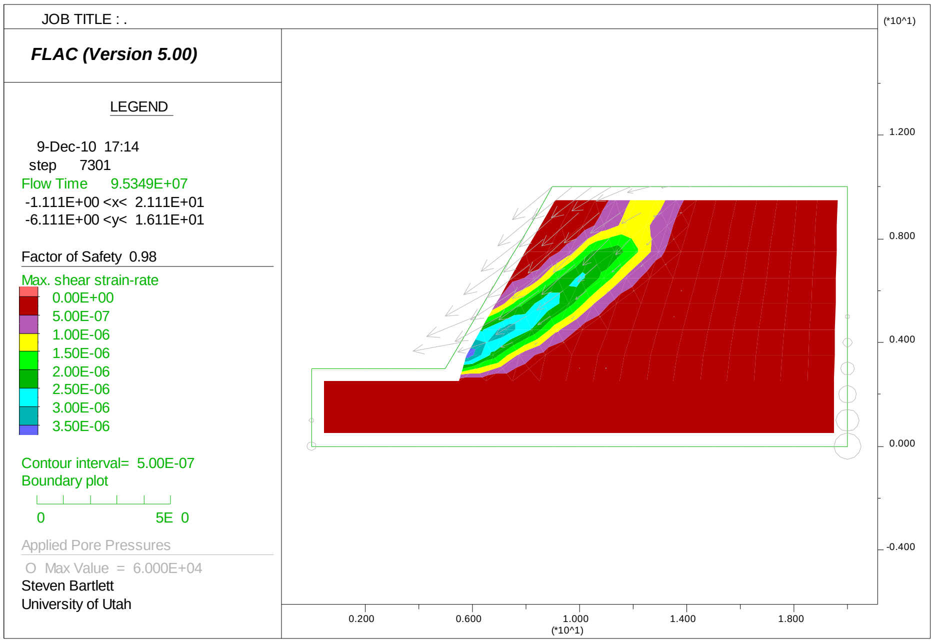
Thursday, March 11, 2010
11:43 AM

```
config gw ats ex 1
grid 20,10
;Mohr-Coulomb model
m m
; soil properties
prop s=.3e8 b=1e8 d=1500 fri=20 coh=10e10 ten=10e10
; warp grid to form a slope :
gen 0,0 0,3 20,3 20,0 j 1,4
gen same 9,10 20,10 same i 6 21 j 4 11
mark i=1,6 j=4
mark i=6 j=4,11
model null region 1,10
prop perm 1e-10 por .3
water den 1000 bulk 1e4
; note bulk modulus of water assigned low value to speed up flow calc.
; displacement boundary conditions
fix x i=1
fix x i=21
fix x y j=1
; pore pressure boundary conditions
apply pp 6e4 var 0 -6e4 i 21 j 1 7
apply pp 2e4 var 0 -2e4 i 1 j 1 3
; apply gravity
set grav=9.81
set mech off flow on
solve
;
set flow off mech on
water bulk 0.0
; initializes stress in slope with high cohesion
solve
;
; reduces cohesion for factor of safety calculations
prop coh = 10e3 ten = 0
; resets displacements in slope
ini xdisp=0
ini ydisp =0
; displacement history of slope
his ydis i=10 j=10
solve fos
save wet_slope_w_fos.sav 'last project state'
```

Slope Stability - Groundwater - Cohesionless Soil - FS Calc

Thursday, March 11, 2010
11:43 AM





Assignment 11

Thursday, March 11, 2010
11:43 AM

1. A roadway widening project is planned atop an existing natural slope as shown in Figure 1. The proposed roadway X-section is also shown, which consists of a 6-m high vertical embankment at the edge of the roadway. The vertical elevation of the widened roadway must match the elevation of the top of the slope. In addition, the construction limits for the project are also shown in Figure 1. Most importantly, construction on the left side of the X-section cannot extend beyond the shown boundary because right-of-way has not been obtained at the base of the slope. (You include the weight of the 1-m pavement section in your stability calculations which has an average density of 2300 kg/m^3.)

   Geofoam is being considered for this project to widen the roadway. You are to examine the design cases given in Table 1 and report the factor of safety for each design case with the steady-state watertable present as defined by the boundary conditions. For the final case (case 4) you are to find a geofoam/slope configuration that meets the design criteria with a factor of safety of 1.2.

   The hydraulic and fluid properties for the soil/groundwater are given in Table 3. (You may use these hydraulic properties for all zone in the model regardless of the material type.)

   For the earthquake analysis, the design basis earthquake will produce 0.33 g horizontal acceleration to the potential slide mass. You must account for this in your analyses. In addition, assume that any dynamically-induced pore pressure will be negligible. Thus, the only pore pressure present will be from the steady-state ground water conditions.

Assignment 11 (cont.)

Thursday, March 11, 2010
11:43 AM

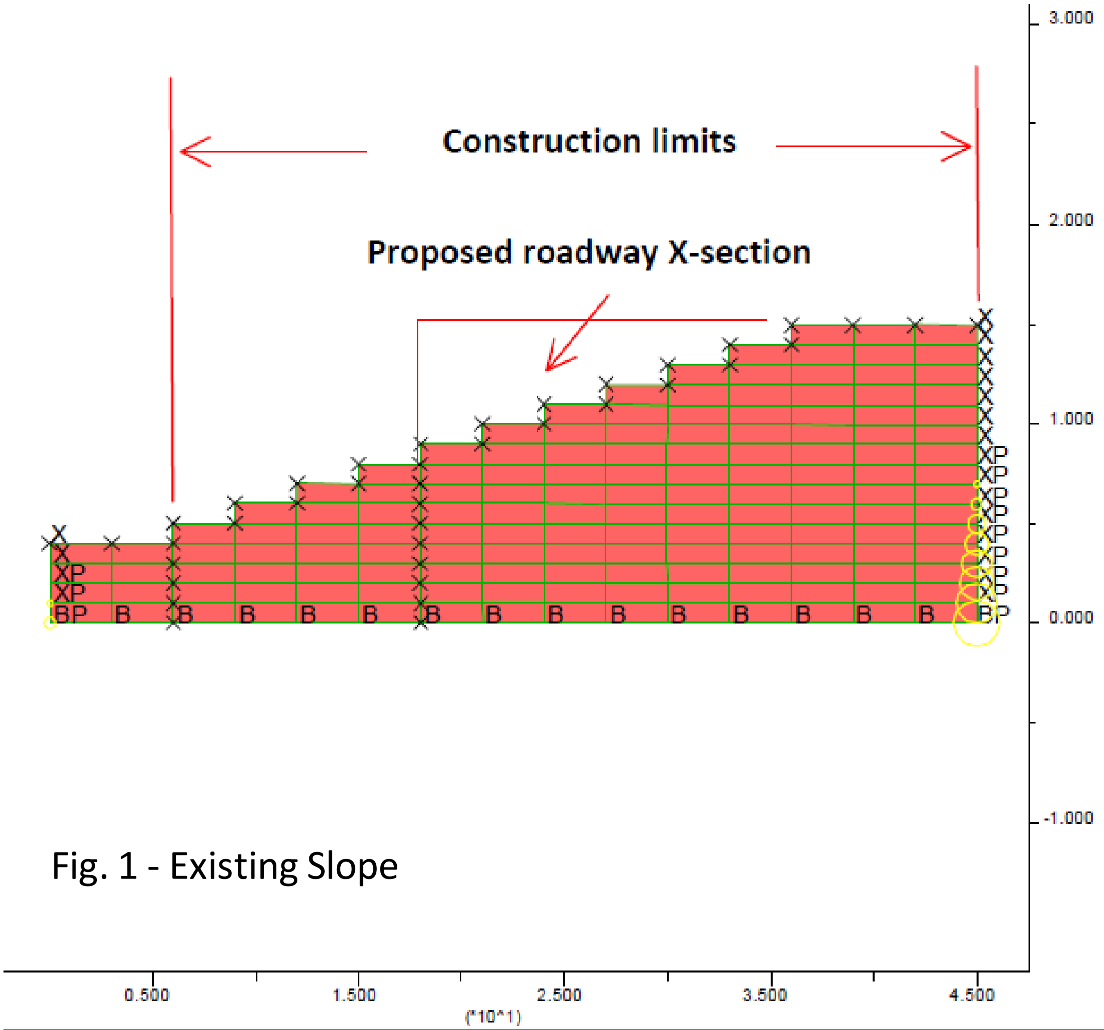


Fig. 1 - Existing Slope

Soil Type	FLAC Properties
Native soil (effective stress)	prop density=1600.0 bulk=1.67E7 shear=1E7 cohesion= 10e3 friction=20.0 dilation=0.0 tension=0.
Native soil (total stress)	prop density=1600 bulk=1.67E7 shear=1E7 cohesion= 10e3 (unsaturated) friction=0 cohesion=0.3 x sigma v' tension=0.0; note use density = 2000 below water table
EPS (geofoam)	prop density=20.0 bulk=2.08333E6 shear=2.27273E6 cohesion=50e3 friction=0.0 dilation=0.0 tension=50e3 group 'User:EPS20'

Hydraulic Properties (for all zones in the model)
prop perm 1e-7 por 0.3 j 1 15 water den 1000 bulk 1e4; lowered to allow faster solutioin

Assignment 11 (cont.)

Thursday, March 11, 2010
11:43 AM

Case	Description	Ground Water Conditions
1	Long-term stability with initial conditions of existing unmodified slope with existing low groundwater conditions. (10 points)	2 m left side, 8 m right side
2	Long-term stability with high groundwater of existing conditions in unmodified, existing slope. (10 points)	2 m left side, 12 m right side
3	Short-term stability with initial conditions of existing unmodified slope with existing low groundwater conditions. (10 points)	2 m left side, 8 m right side
4	Long-term stability of a 6-m high vertical EPS embankment with high groundwater conditions in modified slope. (10 points)	2 m left side, 12 m right side
5	Effective stress analysis of a 6-m high vertical EPS embankment with 0.33 g horizontal ground acceleration earthquake. Assume negligible dynamic pore pressure generation in slope. (10 points)	2 m left side, 8 m right side

Calculation/Plot requirements for Problem 1

(<u>You should provide each of these items for each case)</u>

- Plot of model grid with bulk moduli for all zones
- Plot of model grid with pore pressures calculated by FLAC for steady state conditions
- Plot of factor of safety including maximum shear strain rate and velocity vectors using the plot FOS menu
- Hard copy of the FLAC code
- Txt file that includes the FLAC code for each case

Assignment 11 (cont.)

Thursday, March 11, 2010
11:43 AM

FLAC code for generating mesh for slope and marking construction limits

```
grid 15 15
set large
model elastic
ini x mult 3
model null i 1 2 j 5 15
group 'null' i 1 2 j 5 15
group delete 'null'
model null i 3 j 6 15
group 'null' i 3 j 6 15
group delete 'null'
model null i 4 j 7 15
group 'null' i 4 j 7 15
group delete 'null'
model null i 5 j 8 15
group 'null' i 5 j 8 15
group delete 'null'
model null i 6 j 9 15
group 'null' i 6 j 9 15
group delete 'null'
model null i 7 j 10 15
group 'null' i 7 j 10 15
group delete 'null'
model null i 8 j 11 15
group 'null' i 8 j 11 15
group delete 'null'
model null i 9 j 12 15
group 'null' i 9 j 12 15
group delete 'null'
model null i 10 j 13 15
group 'null' i 10 j 13 15
group delete 'null'
model null i 11 j 14 15
group 'null' i 11 j 14 15
group delete 'null'
model null i 12 j 15
group 'null' i 12 j 15
group delete 'null'
;
;
```

```
mark i 3 j 1 15
mark i 7 j 1 15
mark i 3 4 j 6
mark i 4 j 6 7
mark i 4 5 j 7
mark i 5 j 7 8
mark i 5 6 j 8
mark i 6 j 8 9
mark i 6 7 j 9
mark i 7 j 9 10
mark i 7 8 j 10
mark i 8 j 10 11
mark i 8 9 j 11
mark i 9 j 11 12
mark i 9 10 j 12
mark i 10 j 12 13
mark i 10 11 j 13
mark i 11 j 13 14
mark i 11 12 j 14
mark i 12 j 14 15
mark i 12 13 j 15
mark i 13 j 15 16
mark i 13 16 j 16
mark i 1 3 j 5
```

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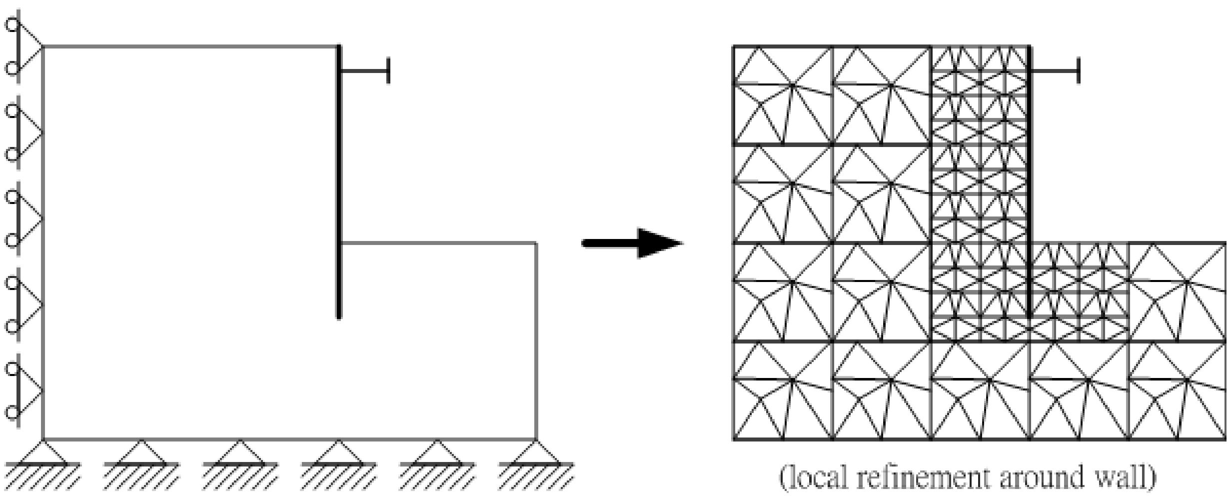
Thursday, March 11, 2010
11:43 AM

2D Finite Element Modeling - Steps

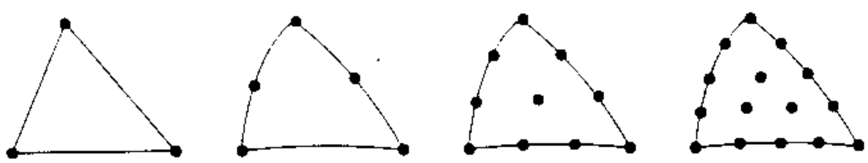
Thursday, March 11, 2010
11:43 AM

1. Discretize the Continuum
2. Select Interpolation Functions
3. Find the Element Properties
4. Assemble the Element Properties to Obtain the System Equations
5. Impose the Boundary Conditions
6. Solve the System of Equations

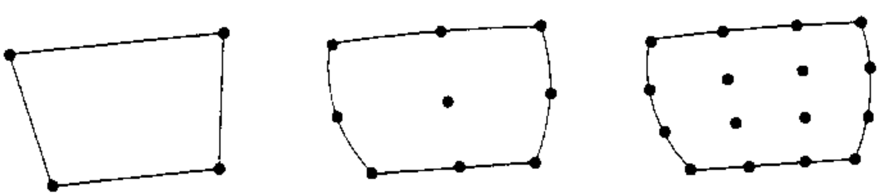
1. Discretize the Continumm



(local refinement around wall)



(a) Triangular elements



(b) Lagrange elements

2D Finite Element Modeling - Steps (continued)

Thursday, March 11, 2010
11:43 AM

2. Select Interpolation Functions

- **To interpolate is to devise a continuous function that satisfies prescribed conditions at a finite number of points.**
- **The points are the nodes of the elements & the prescribed conditions are the nodal values of the field variable.**
- **Polynomials are the usual choice for FEM**
  - **Easy to integrate and differentiate**
  - **Order of polynomial depends on:**
    - **Number of nodes for the element**
    - **Nature and number of unknowns**
    - **Continuity requirements imposed at nodes**

The polynomial function $\phi(x)$ is used to interpolate a field variable based on its values at n-points

$$\phi(x) = \sum_{i=0}^{n} a_i x^i \qquad \text{or} \qquad \phi = \lfloor \mathbf{X} \rfloor \{\mathbf{a}\}^T$$

$$\lfloor \mathbf{X} \rfloor = \lfloor 1 \quad x \quad x^2 \quad \ldots \quad x^n \rfloor \quad \text{and} \quad \{\mathbf{a}\} = \lfloor a_0 \quad a_1 \quad a_2 \quad \ldots \quad a_n \rfloor$$

The number of terms in the polynomial is chosen to match the number of given quantities at the nodes.

With one quantity per node, we calculate a_i's using the n-equations resulting from the expressions for ϕ_i at each of the n-known points

$$\phi(x_j) = \sum_{i=0}^{n} a_i x_j^i \qquad \{\phi_e\} = [\mathbf{A}]\{\mathbf{a}\} \qquad \{\mathbf{a}\} = [\mathbf{A}]^{-1}\{\phi_e\}$$

Traditional interpolation takes the following steps

1. Choose a interpolation function
2. Evaluate interpolation function at known points
3. Solve equations to determine unknown constants

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Thursday, March 11, 2010
11:43 AM

Final Exam

Thursday, March 11, 2010
11:43 AM

Problem 2. For final configuration (case 7), <u>calculate the required ultimate tensile strength for the MSE reinforcement in N/m</u> (i.e., Newtons per 1 m width of reinforcement). The reinforcement will be placed at a vertical spacing of 0.5 m (maximum) and will have a of 5.9 m (minimum). Use the following assumptions for your internal stability calculations. (10 points)

Phi (soil) = 20 deg.
Mass density (soil) = 1600 kg/m3
Phi (interface) = 2/3 phi of MSE reinforced zone
FS rupture = 1.5
FS pulllout = 4

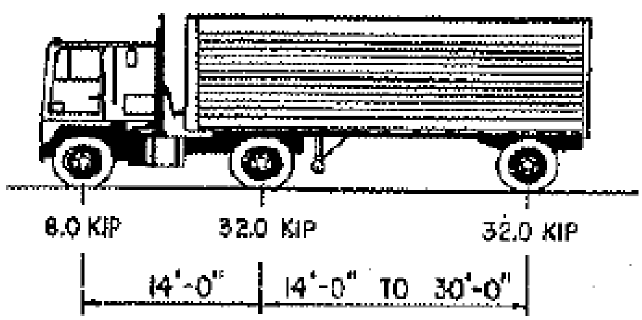
Final Exam

Thursday, March 11, 2010
11:43 AM

Problem 3. **The rear axial tire loading produced by a rear 32-kip single axial haul truck (i.e., 16-kips per each set of rear dual tires) is given below. The tire pressure for the haul truck is 100 psi. This vehicle will be trafficking atop a 3-m thick geofoam embankment that is covered by a 1.33-m thick layer of untreated base course (UTBC) (i.e., road base). The design properties for the geofoam and UTBC are given in Table 1. (Use a fixed base condition underneath the geofoam at the base of the FLAC model.)**

Use FLAC modeling results and the principal of superposition to determine the average vertical stress induced in the geofoam from the 32-kip rear axial loading at a point underneath the left set of rear dual tires in the top of the geofoam.

In calculating the average vertical stress at this point, your solution must include the increase in vertical stress from the axial loading from both sets of duals and the weight of the overlying UTBC. All of these loading sources must be present in your final estimate. Report them as follows: A) increase in vertical stress in the geofoam from the left set of dual tires, B) increase in stress in the geofoam from the adjacent set of dual tires located 6 ft (1.8 m) to the right, C) weight of the UTBC, D) total stress from the 32-kip axial loading and UTBC combined (20 points).



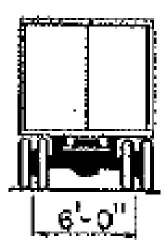


Figure 3.6.1.2.2-1 Characteristics of the Design Truck.

Final Exam

Thursday, March 11, 2010
11:43 AM

Material Properties

	ϱ (kg/m^3)	γ (lb/ft^3)	E (MPa)	v	K (MPa)	G (MPa)	Thickness (m)
Goafoam	24	1.50	7.5	0.1	3.1	3.4	**3**
UTBC/Soil	2241	140.00	104	0.35	116	39	**1.33**

Table 1. Layer properties of pavement section.

For the case of a single axle with dual tires, the contact area can be estimated by converting the set of duals into a singular circular area by assuming that the circle has an area equal to the contact area of the duals, as indicated by Equation 34. The radius of contact is given by Equation 35. Equation 34 yields a conservative value, i.e., smaller area, for the contact area because the area between the duals is not included.

$$A_{CD} = \frac{Q_D}{q} \qquad (34)$$

$$r = \left(\frac{A_{CD}}{\pi}\right)^{\frac{1}{2}} \qquad (35)$$

Where

A_{CD} = contact area of dual tires,
Q_D = live load on dual tires, and
q = contact pressure on each tire = tire pressure.

Calculation of contact area for a set of dual tires (from AASHTO).