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Non-Stationary Spectral Matching

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Abstract

Introduction

Design ground motions are typically prescribed by smooth response spectra. Rather than evaluate a structure for a suite of time histories that are representative of the target spectrum in an ensemble average, engineers often prefer to use a single time history that matches the target design spectrum.

There is an ongoing controvery if a time history with a smooth response spectrum is realistic. After all, no observed time history ever had a smooth response spectrum. However, I do not believe that the fine details of the response spectrum is a good test of "realistic" ground motion. It is possible to generate a time history with a "realistic" response spectrum that is unrealistic in terms of its ground motion. The realistic nature of a time history should be judged in the time domain in terms of the nonstationary character of the acceleration, velocity and displacement. For example, P and S wave arrivals should be present, particulary at the longer periods. Small adjustments can be made to a time history to change its response spectrum from jagged to smooth. If done properly, the resulting time history can be "realistic". In this paper, I present a quantitative definition of realistic timeso we can determine if the spectrum compatible time histories are realistic or not.

Various methods have been developed to modify a reference time history so that its response spectrum is compatible with a specified target spectrum. A review of spectral matching methods is given by Preumont (1984). A commonly used method adjusts the Fourier amplitude spectrum based on the ratio of the target response spectrum to the time history response spectrum while keeping the Fourier phase of the reference time history fixed. While this approach is straight-forward, it has two drawbacks. First, it generally does not have good convergence properties, particularly for multiple damping spectra. Second, it can alter the non-stationary character of the time history if the shape of the Fourier spectrum is changed significantly.

An alternative approach for spectral matching adjusts the time history in the time domain by adding wavelets to the reference time history. A formal optimization procedure for this type of time domain spectral matching was first proposed by Kaul (1978) and was extended to simultaneously match spectra at multiple damping values by Lilhanand and Tseng (1987, 1988). While this procedure is more complicated than the frequency domain approach, it has good convergence properties and in most cases preserves the non-stationary character of the reference time history.

In this paper, I present a modification to the Lilhanand and Tseng algorithm that preserves the non-stationary character of the reference ground motion for a wider range of time histories.

Methodology

Usually, response spectra studies are only concerned with the maximum response of the oscillator; however, for the time domain methods, the time and polarity of the peak response must also be considered. In this paper, I will refer to the *response* to indicate the oscillator time history, not just its maximum value.

Let $a(t)$ be the reference time history and Q_i be the target spectrum for frequency ω_i and damping β_i. Also let R_i be the absolute value of the peak response, P_i be the polarity of the peak response, and t_i be the time of the peak response. The difference between the target spectrum and the computed spectrum is given by

$$\delta R_i = (Q_i - R_i)\, P_i \tag{1}$$

where δR_i includes the polarity of the response.

The basic method is to determine an adjustment time history, $\delta a(t)$, such that the response of $\delta a(t)$ at time t_i is equal to δR_i for all i. Let

$$\delta a(t) = \sum_{j=1}^{N} b_j\, f_j(t) \tag{2}$$

where $f_i(t)$ is a set of adjustment functions, b_i is the set of coefficients to be determined, and N is the number of spectral points (frequency and damping pairs) to match. A restriction on $f_i(t)$ is that $f_i(t) = 0$ for $t<0$. The acceleration response of $\delta a(t)$ at time t_i is given by

$$\delta R_i = \sum_{j=1}^{N} b_j \int_0^{\infty} f_j(\tau)\, h_i(t_i-\tau)\, d\tau \tag{3}$$

where $h_i(t)$ is the acceleration impulse response of the oscillator for frequency ω_i and damping β_i. The acceleration impulse response is given by

$$h_i(t) = \frac{-\omega_i}{\sqrt{1-\beta_i^2}} \exp(-\omega_i\beta_i t) \left[(2\beta_i^2-1) \sin(\omega_i' t) - 2\beta_i\sqrt{1-\beta_i^2} \cos(\omega_i' t) \right] \tag{4}$$

where

$$\omega_i' = \omega_i \sqrt{1-\beta_i^2} \tag{5}$$

and $h_i(t) = 0$ for $t<0$. Next let c_{ij} be the response at time t_i for th ith frequency and damping resulting from the motion $f_j(t)$:

$$c_{ij} = \int_0^{t_i} f_j(\tau)\, h_i(t_i-\tau)\, d\tau \tag{6}$$

(the upper limit of the intergal is t_i because $h_i(t) = 0$ for $t<0$).

Given the spectral misfits, δR_i, and the response coefficients c_{ij}, Eq. 7 can be solved for the b_j. In matrix notation, the solution is simply

$$\mathbf{b} = \mathbf{C}^{-1}\, \delta\mathbf{R} \tag{7}$$

Given, the b_j, the adjustment time history, $\delta a(t)$, can be computed by Eq. 2. The adjusted time history for the first iteration is given by

$$a_1(t) = a_o(t) + \gamma\, \delta a(t) \tag{8}$$

where γ is a relaxation parameter (between 0 and 1) to damp the adjustments. The algorithm is repeated using the adjusted time history until the desired spectral match is achieved.

Selection of the adjustment function

The key to the non-stationarity of the method is the selection of the adjustment function $f_j(t)$. The selection of the form of the adjustment function, $f_j(t)$, is where the seismological considerations enter the problem. What sort of adjustments can be made and yet still yield realistic seismograms? In considering the forms of $f_j(t)$, we need to also consider the numerical stability of the algorithm.

For the method to work efficiently, the timing of $f_j(t)$ should be such that the response of $f_j(t)$ is in phase with the peak response of a(t) (Figure _). For numerical speed, $f_j(t)$ should be chosen so that the elements of **C** given by the integral in Eq. 6 can be analytically. For numerical stability, the off-diagonal terms of **C** should be as small as possible.

As mentioned earlier, in the frequency domain approach, only the Fourier amplitude spectrum is modified. This is equivalent to using

$$f_j(t) = \cos(\omega_j' t + \theta_j) \tag{9}$$

where θ_j is he Fourier phase of the reference time history. The adjustments are computed from the ratio of the computed response to the target response which is equivalent to using $\mathbf{b}=\delta\mathbf{R}$. The poor convergence of this method results from the simplified estimation of **b** which ignores the cross-terms in the **C** matrix (assumes that **C** is the idenity matrix).

In the Lilhanand and Tseng algorithm, $f_j(t)$ is given by

$$f_j(t) = h_j(t_j - t) \tag{10}$$

which is just the oscillator impulse response in reverse time order. Since $h(t)=0$ for $t<0$, this form is one-sided as shown in Figure _.

There are several numerical aspects that make this function attractive. First, it leads to a symmetric C matrix. Second, the abrupt stop insures that the response will peak at time t_j and not resonate to larger values at greater t for all dampings. At high frequency, the f_j has a short duration, but at long periods (T>3sec) f_j has a long duration. If only small adjustments are needed in the low frequencies, then this form works well.

From a seismological point of view, this form of the adjustment function has some undesirable features, particularly at long periods. The adjustment is emergent and stops abruptly. This is contrary to the behavior of strong motion time histories that generally have a sharp initiation and gradual decay for long periods.

As will be shown later, forcing the long period adjustment early in the time history can lead to unrealistic ground motions if a large modification to the response spectral shape is required.

As an alternative, a tapered cosine wave can be used for the adjustment function:

$$f_j(t) = \cos\{\omega_j'(t-t_j+\Delta t_j)\} \exp\{-|t-t_j+\Delta t_j|\, \alpha_j\} \tag{11}$$

where Δt is the time delay between the maximum of $f_j(t)$ and a peak in the response of $f_j(t)$. The α term controls the time duration of $f_j(t)$. The frequency dependence of α can be estimated from the reference time history which helps to preserve the non-stationary character of the reference time history. That is, if the reference time history has a short duration at a perticular frequency, the α is selected such that the adjustment function at that frequnecy will also have a short duration.

Numerical Aspects

Conceptually, the time domain spectral matching method appears straight-forward; however, there are some numerical difficulties. The **C** matrix in Eq. 8 is singular or near-singular for a large number of closely spaced frequencies and multiple dampings. The numerical problem is to find a way to handle this near-singular matrix.

Lilhanand and Tseng subdivided the target spectrum into several smaller subsets that are each about 20 to 30 frequency and damping pairs (Lilhanand, personal communication). Each subset should sample the entire frequency range rather than using low frequency, moderate frequency, high frequency, subgroups. For example, if 4 subsets are used, the first subset contains the 1st, 5th, 9th, ... frequencies. For each subset, the $\mathbf{C}_k$ matrix can be inverted. (stable)

In addition, a singular value decomposition of $\mathbf{C}_k$ is computed. The small eigenvalues are removed (ref). Initially, the smallest eigenvalues are removed until the condition number is less than 10^4.

The method makes narrow band modifications to the time series. Therefore, it is important to use a fine enough frequnecy sampling to ensure that the response spectrum at frequencies not matched will remain smooth. Based on the bandwidth of the oscillator response, about 30 frequencies per decade (equally spaced on the log frequency axis) is sufficient.

Quantification of Realism of Time Histories

Multiple-Damping Spectra

Acknowledgements

Kiat Lilhanand provided explanations of his algorithm. This work was partially supported by Caltrans.

References

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Lilhanand, K. and W. S. Tseng (1987). Generation of synthetic time histories compatible with multiple-damping response spectra, SMiRT-9, Lausanne, K2/10.

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Preumont, A. (1984). The generation of spectrum compatible accelerograms for the design of nuclear power plants, Earth. Engin. Struct. Dyn., **12**, 481-497.

User Guide for Spectral Matching Program RSPMATCH (version 2.2)

Target Spectrum File Format

Line 1: Comment Line (Not used)

Line 2:	nFreq, nDamp		
	nFreq =		Number of frequencies in target.
	nDamp	=	Number of damping values in target.
Line 3:	beta(i),i=1,nDamp		
	beta(i) =		Damping value in fraction of critical (0.05 = 5%)
Line 4 - ...	freq, minTime, Q(i),i=1,nDamp		
	freq	=	frequency (Hz)
	minTime	=	minimum time (seconds) for the adjustment time history. The main use of this parameter is for controlling the timing of the long period waves. If unsure, set this parameter to zero.
	Q(i)	=	target spectral acceleration (in g) for the ith damping value

One line is needed for each frequency.

Reference Accelerogram File Format

Line 1:	Title		
Line 2:	npts, dt		
	npts	=	number of samples
	dt	=	time step (sec)
Line 3 - ...	acc(i), acc(i+1), ...		
	acc	=	acceleration in g. If the accelerogram is not in g, you can use the scaling parameter in the input run parameters.

Input Run Parameters

Line 1: maxIter = maximum number of iterations. Typically set to 20.

Line 2: tol = convergence tolarance for maximum deviation from target (in fraction)
(e.g. tol=0.05 for 5% maximum deviation)

Line 3: gamma = adjustment scale factor.
This sets the fraction of the adjustment that is made at each iteration. Used to help stabilize convergence but slows the convergence.
Recommended default value of 1.0

Line 4: iModel = flag for the model for the functional form of the adjustment time history.
iModel = 1 oscillator impulse response in reverse time order .
iModel = 6 tapered cosine wave
Recommended value: model 6 for initial adjustments, model 1 for final adjustments

Line 5: a1, a2, f1, f2 = parameters for frequency dependence of the taper of the adjustment time history.
The taper is given by function $\alpha(f)$
Model: $\alpha(f) = a1 * f$
for $f < f1$
$\alpha(f) = (a1 + (f\text{-}f1) * (a2\text{-}a1)/(f2\text{-}f1)) * f$
for $f1 \le f \le f2$
$\alpha(f) = a2 * f$
for $f > f2$

Recommended values: 1.25, 0.25, 1.0, 4.0

Line 6: iScale = scaling options
iScale = 0 no scaling
iScale = 1 scale time history to target PGA initially and after each iteration
iScale = 2 scale inital time history to target PGA, but not after each iteration.
Recommended default: iScale=2

Line 7; dtFlag = interpolation factor for reference time history.
Interpolate to 1/dtFlag of the input time step.
The nyquist freq should be about twice the maximum frequency
For example, a maximum freq of 50 Hz, interpolate to 200 samples per sec.

Line 8: evmin = minimum normalized eigenvalue used in singular value decomposition.
This is a control on the convergence. A smaller value gives more rapid, but less stable convergence.
Recommended value: 1.0e-4

Line 9: groupSize = number of spectral values to use in a subgroup.
This is a control on convergence. A smaller value gives more rapid, but less stable convergence.
Recommended value: 25

Line 10: maxFreq = maximum frequency (Hz) for energy content of the accelerogram.
(e.g. frequency at which the spectral acceleration becomes constant.

Line 11: fc1, fc2, nPole Parameters of initial bandpass filter
fc1 = corner frequency of high-pass filter in Hz
fc2 = corner frequency of low-pass filter in Hz
nPole = number of poles for the Butterworth filter (same applied to both high-pass and low-pass)
If fc1=0, then a high-pass filter is not applied
If fc2=0, then a low-pass filter is not applied

Line 12: iModPGA = flag for modifing the peak acceleration
If used, a triangle adjustment function is applied at the time of the PGA. This function can be used to help converge to the PGA. For initial runs, it should not be used.
iModPga=0, No extra modification for the PGA
iModPga=1, Pga modified after each iteration

Line 13: filename of target spectum

Line 14: filename of reference time history.

Line 15: filename of output time history.

Line 16: filename of output spectrum.